COURSE 1 – REVISED SAMPLE EXAM

A table of values for the normal distribution will be provided with the Course 1 Exam.

Revised August 1999

Problem # 1

A marketing survey indicates that 60% of the population owns an automobile, 30% owns a house, and 20% owns both an automobile and a house.

Calculate the probability that a person chosen at random owns an automobile or a house, but not both.

A. 0.4

B. 0.5

C. 0.6

D. 0.7

E. 0.9

Problem # 2

In a country with a large population, the number of people, N, who are HIV positive at time t is modeled by

$$N = 1000 \ln(t+2), \quad t \geq 0.$$

Using this model, determine the number of people who are HIV positive at the time when that number is changing most rapidly.

A. 0

B. 250

C. 500

D. 693

E. 1000

Problem # 3

Ten percent of a company=s life insurance policyholders are smokers. The rest are nonsmokers.

For each nonsmoker, the probability of dying during the year is 0.01. For each smoker, the probability of dying during the year is 0.05.

Given that a policyholder has died, what is the probability that the policyholder was a smoker?

A. 0.05

B. 0.20

C. 0.36

D. 0.56

E. 0.90

Problem # 4

Let X and Y be random losses with joint density function

$$f(x, y) = e^{-(x+y)} \text{ for } x > 0 \text{ and } y > 0.$$

An insurance policy is written to reimburse $X+Y$.

Calculate the probability that the reimbursement is less than 1.

A. e^{-2}

B. e^{-1}

C. $1 - e^{-1}$

D. $1 - 2e^{-1}$

E. $1 - 2e^{-2}$

Problem # 5

The rate of change of the population of a town in Pennsylvania at any time t is proportional to the population at time t. Four years ago, the population was 25,000. Now, the population is 36,000.

Calculate what the population will be six years from now.

A. 43,200

B. 52,500

C. 62,208

D. 77,760

E. 89,580

Problem # 6

Calculate $\int_0^\infty \int_0^x (1+x^2+y^2)^{-2}\, dy\, dx$.

A. 0

B. $\dfrac{\pi}{16}$

C. $\dfrac{\pi}{8}$

D. $\dfrac{\pi}{4}$

E. π

Problem # 7

As part of the underwriting process for insurance, each prospective policyholder is tested for high blood pressure.

Let X represent the number of tests completed when the first person with high blood pressure is found. The expected value of X is 12.5.

Calculate the probability that the sixth person tested is the first one with high blood pressure.

A. 0.000

B. 0.053

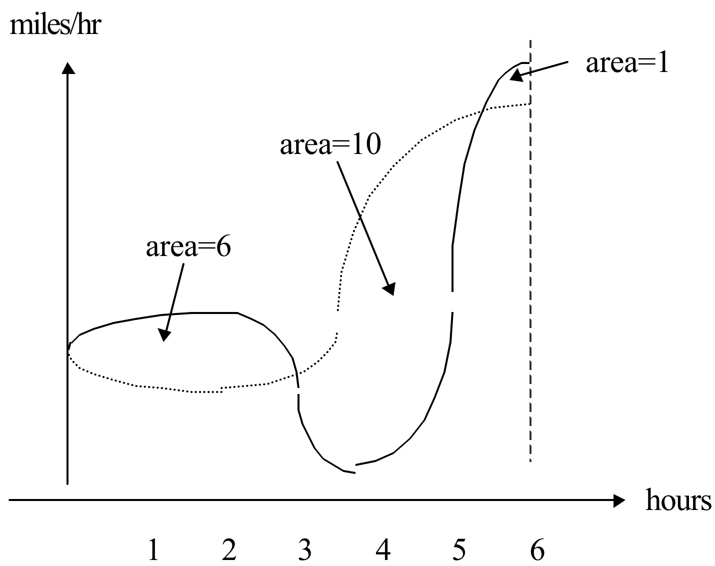
C. 0.080

D. 0.316

E. 0.394

Problem # 8

At time $t = 0$, car A is five miles ahead of car B on a stretch of road. Both cars are traveling in the same direction. In the graph below, the velocity of A is represented by the solid curve and the velocity of B is represented by the dotted curve.



Determine the time(s), t, on the time interval (0, 6], at which car A is exactly five miles ahead of car B.

A. at $t = 2$.

B. at $t = 3$.

C. at some t, $3 < t < 5$, which cannot be determined precisely from the information given.

D. at $t = 3$ and at $t = 5$.

E. Car A is never exactly five miles ahead of Car B on (0,6].

Problem # 9

The distribution of loss due to fire damage to a warehouse is:

Amount of Loss	Probability
0	0.900
500	0.060
1,000	0.030
10,000	0.008
50,000	0.001
100,000	0.001

Given that a loss is greater than zero, calculate the expected amount of the loss.

A. 290

B. 322

C. 1,704

D. 2,900

E. 32,222

Problem # 10

An insurance policy covers the two employees of ABC Company. The policy will reimburse ABC for no more than one loss per employee in a year. It reimburses the full amount of the loss up to an annual company-wide maximum of 8000.

The probability of an employee incurring a loss in a year is 40%. The probability that an employee incurs a loss is independent of the other employee's losses.

The amount of each loss is uniformly distributed on [1000, 5000].

Given that one of the employees has incurred a loss in excess of 2000, determine the probability that losses will exceed reimbursements.

A. $\frac{1}{20}$

B. $\frac{1}{15}$

C. $\frac{1}{10}$

D. $\frac{1}{8}$

E. $\frac{1}{6}$

Problem # 11

The risk manager at an amusement park has determined that the expected cost of accidents is a function of the number of tickets sold. The cost, C, is represented by the function

$$C(x) = x^3 - 6x^2 + 15x, \text{ where } x \text{ is the number of tickets sold (in thousands).}$$

The park self-insures this cost by including a charge of 0.01 in the price of every ticket to cover the cost of accidents.

Calculate the number of tickets sold (in thousands) that provides the greatest margin in the insurance charges collected over the expected cost of accidents.

A. 0.47

B. 0.53

C. 2.00

D. 3.47

E. 3.53

Problem # 12

An investor invests 100. The value, I, of the investment at the end of one year is given by the equation

$$I = 100\left(1+\frac{c}{n}\right)^n$$

where c is the nominal rate of interest and n is the number of interest compounding periods in one year.

Determine I if there are an infinite number of compounding periods in one year.

A. 100

B. $100ec$

C. $100e^c$

D. $100e^{\frac{1}{c}}$

E. ∞

Problem # 13

Let X and Y be discrete random variables with joint probability function

$$p(x,y)=\begin{cases}\dfrac{2x+y}{12} & \text{for } (x,y)=(0,1),\ (0,2),\ (1,2),\ (1,3)\\ 0, & \text{otherwise.}\end{cases}$$

Determine the marginal probability function of X.

A. $$p(x)=\begin{cases}\dfrac{1}{6} & \text{for } x=0\\ \dfrac{5}{6} & \text{for } x=1\\ 0, & \text{otherwise.}\end{cases}$$

B. $$p(x)=\begin{cases}\dfrac{1}{4} & \text{for } x=0\\ \dfrac{3}{4} & \text{for } x=1\\ 0, & \text{otherwise.}\end{cases}$$

C. $$p(x)=\begin{cases}\dfrac{1}{3} & \text{for } x=0\\ \dfrac{2}{3} & \text{for } x=1\\ 0, & \text{otherwise.}\end{cases}$$

D. $$p(x)=\begin{cases}\dfrac{2}{9} & \text{for } x=1\\ \dfrac{3}{9} & \text{for } x=2\\ \dfrac{4}{9} & \text{for } x=3\\ 0, & \text{otherwise.}\end{cases}$$

E. $$p(x)=\begin{cases}\dfrac{y}{12} & \text{for } x=0\\ \dfrac{2+y}{12} & \text{for } x=1\\ 0, & \text{otherwise.}\end{cases}$$

Problem # 14

Workplace accidents are categorized into three groups: minor, moderate and severe. The probability that a given accident is minor is 0.5, that it is moderate is 0.4, and that it is severe is 0.1.

Two accidents occur independently in one month.

Calculate the probability that neither accident is severe and at most one is moderate.

A. 0.25

B. 0.40

C. 0.45

D. 0.56

E. 0.65

Problem # 15

An insurance company issues insurance contracts to two classes of independent lives, as shown below.

Class	Probability of Death	Benefit Amount	Number in Class
A	0.01	200	500
B	0.05	100	300

The company wants to collect an amount, in total, equal to the 95th percentile of the distribution of total claims.

The company will collect an amount from each life insured that is proportional to that life's expected claim. That is, the amount for life j with expected claim $E[X_j]$ would be $kE[X_j]$.

Calculate k.

A. 1.30

B. 1.32

C. 1.34

D. 1.36

E. 1.38

Problem # 16

Micro Insurance Company issued insurance policies to 32 independent risks. For each policy, the probability of a claim is 1/6. The benefit amount given that there is a claim has probability density function

$$f(y) = \begin{cases} 2(1-y), & 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of total benefits paid.

A. $\frac{16}{9}$

B. $\frac{8}{3}$

C. $\frac{32}{9}$

D. $\frac{16}{3}$

E. $\frac{32}{3}$

Problem # 17

An actuary is reviewing a study she performed on the size of claims made ten years ago under homeowners insurance policies.

In her study, she concluded that the size of claims followed an exponential distribution and that the probability that a claim would be less than \$1,000 was 0.250.

The actuary feels that the conclusions she reached in her study are still valid today with one exception: every claim made today would be twice the size of a similar claim made ten years ago as a result of inflation.

Calculate the probability that the size of a claim made today is less than \$1,000.

A. 0.063

B. 0.125

C. 0.134

D. 0.163

E. 0.250

Problem # 18

A large company has determined that the function

$$f(x) = 3x^2,\ 0 \le x \le 1,$$

serves as a payroll density function. That is, the distribution of payroll, $F(x)$, which is the proportion of total payroll earned by the lowest paid fraction x of employees, $0 \le x \le 1$, relates to $f(x)$ in the same way that probability distributions and probability densities relate.

Gini's index, G, defined as

$$G = 2\int_0^1 \left| x - F(x) \right| dx$$

is a measure of how evenly payroll is distributed among all employees.

Calculate G for this large company.

A. 0.2

B. 0.4

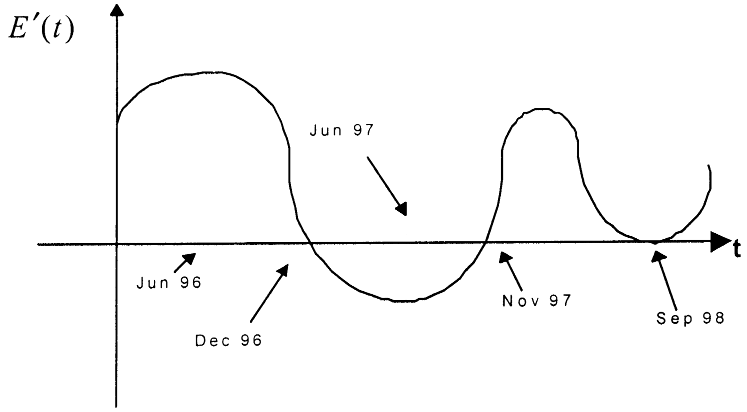
C. 0.5

D. 0.8

E. 1.0

Problem # 19

According to classical economic theory, the business cycle peaks when employment reaches a relative maximum. Employment can be approximated as a function of time, t, by a differentiable function $E(t)$. The graph of $E'(t)$ is pictured below.



Which of the following points represents a peak in the business cycle?

A. Jun 96

B. Dec 96

C. Jun 97

D. Nov 97

E. Sep 98

Problem # 20

Let $f(x) = \int_{2}^{3x} \sin^2 t \, dt$.

Calculate $f''(x)$.

A. $\sin^2(3x)$

B. $3\sin^2(3x)$

C. $2\sin(3x)\cos(3x)$

D. $6\sin(3x)\cos(3x)$

E. $18\sin(3x)\cos(3x)$

Problem # 21

An economist defines an "index of economic health," D, as follows

$$D = 100E^2(1 - I)$$

where: E is the portion of the working-age population that is employed and
I is the rate of inflation (both expressed in decimal form).

On June 30, 1996, employment is at 0.95 and is increasing at a rate of 0.02 per year and the rate of inflation is at 0.06 and is increasing at a rate of 0.03 per year.

Calculate the rate of change of D on June 30, 1996.

A. -0.9503 per year

B. -0.9500 per year

C. 0.0000 per year

D. 0.8645 per year

E. 1.7860 per year

Problem # 22

A dental insurance policy covers three procedures: orthodontics, fillings and extractions. During the life of the policy, the probability that the policyholder needs:

- orthodontic work is 1/2
- orthodontic work or a filling is 2/3
- orthodontic work or an extraction is 3/4
- a filling and an extraction is 1/8

The need for orthodontic work is independent of the need for a filling and is independent of the need for an extraction.

Calculate the probability that the policyholder will need a filling or an extraction during the life of the policy.

A. 7/24

B. 3/8

C. 2/3

D. 17/24

E. 5/6

Problem # 23

The value, v, of an appliance is based on the number of years since purchase, t, as follows:

$$v(t) = e^{(7-0.2t)}$$

If the appliance fails within seven years of purchase, a warranty pays the owner the value of the appliance. After seven years, the warranty pays nothing.

The time until failure of the appliance failing has an exponential distribution with mean 10.

Calculate the expected payment from the warranty

A. 98.70

B. 109.66

C. 270.43

D. 320.78

E. 352.16

Problem # 24

An automobile insurance company divides its policyholders into two groups: good drivers and bad drivers. For the good drivers, the amount of an average claim is 1400, with a variance of 40,000. For the bad drivers, the amount of an average claim is 2000, with a variance of 250,000. Sixty percent of the policyholders are classified as good drivers.

Calculate the variance of the amount of a claim for a policyholder.

A. 124,000

B. 145,000

C. 166,000

D. 210,400

E. 235,000

Problem # 25

Let S be the region in the first quadrant of the xy-plane bounded by $y = \sqrt{x},\ x - y = 2$ and the x-axis.

Calculate $\iint\limits_{S} y\, dA$.

A. 13/12

B. 4/3

C. 9/4

D. 8/3

E. 10/3

Problem # 26

Let X be a random variable with moment generating function

$$M(t)=\left(\frac{2+e^t}{3}\right)^9, \quad -\infty < t < +\infty$$

Calculate the variance of X.

A. 2

B. 3

C. 8

D. 9

E. 11

Problem # 27

The annual number of claims filed under a block of disability income insurance policies has been constant over a ten-year period, but the number of claims outstanding does exhibit seasonal fluctuations. The number of outstanding claims peaks around the first of the year, declines through the first two quarters of the year, reaches its lowest level around July 1, then climbs again to regain its peak level on January 1.

Which of the following functions best represents the number of outstanding claims, as a function of time, t, where t is measured in months and $t = 0$ on January 1, 1987?

A. $k\cos\left(\frac{\pi t}{12}\right)$ where k is a constant greater than zero

B. $k\cos\left(\frac{\pi t}{6}\right)$ where k is a constant greater than zero

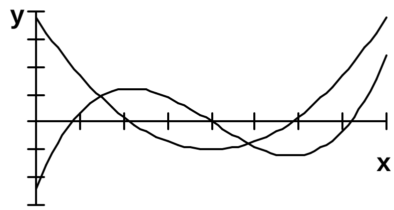
C. $k\cos\left(\frac{\pi t}{6}\right)+c$ where c and k are constants greater than zero

D. $k\cos\left(\frac{\pi t}{6}\right)+ct$ where c and k are constants greater than zero

E. $k\cos\left(\frac{\pi t}{12}\right)+ct$ where c and k are constants greater than zero

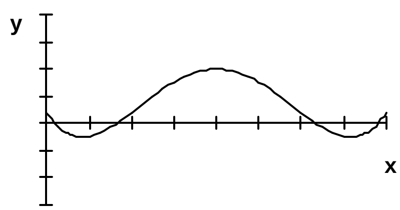
Problem # 28

The graphs of the first and second derivatives of a function are shown below, but are not identified from one another.

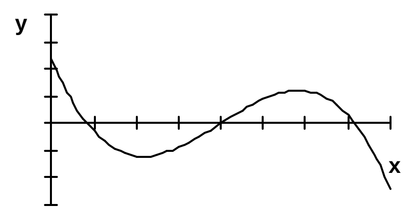


Which of the following could represent a graph of the function?

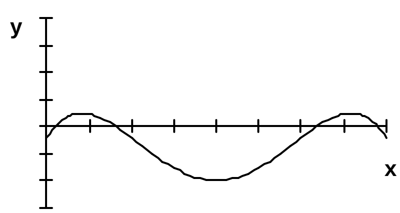
A.



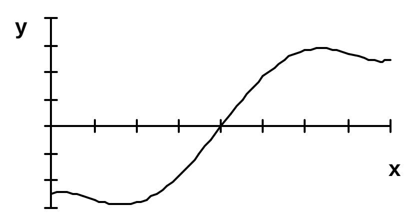
D.



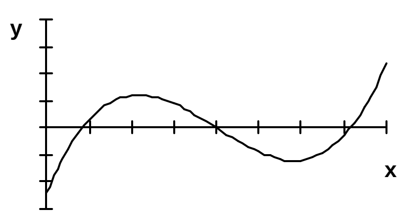
B.



E.



C.



Problem # 29

An insurance company designates 10% of its customers as high risk and 90% as low risk.

The number of claims made by a customer in a calendar year is Poisson distributed with mean q and is independent of the number of claims made by a customer in the previous calendar year.

For high risk customers $q = 0.6$, while for low risk customers $q = 0.1$.

Calculate the expected number of claims made in calendar year 1998 by a customer who made one claim in calendar year 1997.

A. 0.15

B. 0.18

C. 0.24

D. 0.30

E. 0.40

Problem # 30

Calculate $\lim_{x \to 0^+} \left(e^x + 3x\right)^{\frac{1}{x}}$.

A. 1

B. 3

C. 4

D. e^3

E. e^4

Problem # 31

Let X and Y be random losses with joint density function

$$f(x,y) = 2x \text{ for } 0 < x < 1 \text{ and } 0 < y < 1.$$

An insurance policy is written to cover the loss $X+Y$. The policy has a deductible of 1.

Calculate the expected payment under the policy.

A. 1/4

B. 1/3

C. 1/2

D. 7/12

E. 5/6

Problem # 32

Curve C_1 is represented parametrically by $x = t + 1,\ y = 2t^2$. Curve C_2 is represented parametrically by $x = 2t + 1,\ y = t^2 + 7$.

Determine all the points at which the curves intersect.

A. (3,8) only

B. (1,0) only

C. (3,8) and (-1,8) only

D. (1,0) and (1,7) only

E. The curves do not intersect anywhere.

Problem # 33

The number of clients a stockbroker has at the end of the year is equal to the number of new clients she is able to attract during the year plus the number of last year's clients she is able to retain.

Because working with existing clients takes away from the time she can devote to attracting new ones, the stockbroker acquires fewer new clients when she has many existing clients.

The number of clients the stockbroker has at the end of year n is modeled by

$$C_n = \frac{2}{3} C_{n-1} + \frac{9000}{C_{n-1}^2} .$$

The stockbroker has five clients when she starts her business.

Determine the number of clients she will have in the long run.

A. 3

B. 5

C. 10

D. 30

E. 363

Problem # 34

Under a group insurance policy, an insurer agrees to pay 100% of the medical bills incurred during the year by employees of a small company, up to a maximum total of one million dollars. The total amount of bills incurred, X, has probability density function

$$f(x) = \begin{cases} \dfrac{x(4-x)}{9} & \text{for } 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

where x is measured in millions.

Calculate the total amount, in millions of dollars, the insurer would expect to pay under this policy.

A. 0.120

B. 0.301

C. 0.935

D. 2.338

E. 3.495

Problem # 35

Suppose the remaining lifetimes of a husband and wife are independent and uniformly distributed on the interval [0,40]. An insurance company offers two products to married couples:

One which pays when the husband dies; and
One which pays when both the husband and wife have died.

Calculate the covariance of the two payment times.

A. 0.0

B. 44.4

C. 66.7

D. 200.0

E. 466.7

Problem # 36

An index of consumer confidence fluctuates between -1 and 1. Over a two-year period, beginning at time $t = 0$, the level of this index, c, is closely approximated by

$$c(t) = \frac{t\cos(t^2)}{2}, \text{ where } t \text{ is measured in years.}$$

Calculate the average value of the index over the two-year period.

A. $-\frac{1}{4}\sin(4)$

B. 0

C. $\frac{1}{8}\sin(4)$

D. $\frac{1}{4}\sin(4)$

E. $\frac{1}{2}\sin(4)$

Problem # 37

An insurance contract reimburses a family's automobile accident losses up to a maximum of two accidents per year.

The joint probability distribution for the number of accidents of a three person family (X,Y,Z) is

$$p(x.\, y, z) = k(x + 2y + z)$$

where

$x = 0,1$

$y = 0,1,2$

$z = 0,1,2$ and

x, y and z are the number of accidents incurred by X, Y and Z, respectively.

Determine the expected number of unreimbursed accident losses given that X is not involved in any accidents.

A. 5/21

B. 1/3

C. 5/9

D. 46/63

E. 7/9

Problem # 38

An investor bought one share of a stock. The stock paid annual dividends. The first dividend paid one dollar. Each subsequent dividend was five percent less than the previous one.

After receiving 40 dividend payments, the investor sold the stock.

Calculate the total amount of dividends the investor received.

A. $ 8.03

B. $17.43

C. $20.00

D. $32.10

E. $38.00

Problem # 39

The loss amount, X, for a medical insurance policy has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{9}\left(2x^2 - \frac{x^3}{3}\right), & 0 \le x \le 3 \\ 1, & x > 3. \end{cases}$$

Calculate the mode of the distribution.

A. 2/3

B. 1

C. 3/2

D. 2

E. 3

Problem # 40

A small commuter plane has 30 seats. The probability that any particular passenger will not show up for a flight is 0.10, independent of other passengers. The airline sells 32 tickets for the flight.

Calculate the probability that more passengers show up for the flight than there are seats available.

A. 0.0042

B. 0.0343

C. 0.0382

D. 0.1221

E. 0.1564

Problem	Key
1	B
2	D
3	C
4	D
5	C
6	C
7	B
8	C
9	D
10	B
11	E
12	C
13	B
14	E
15	E
16	A
17	C
18	C
19	B
20	E
21	D
22	D
23	D
24	D
25	D
26	A
27	C
28	A
29	C
30	E
31	A
32	C
33	D
34	C
35	C
36	C
37	E
38	B
39	D
40	E

Problem # 12

An investor invests 100. The value, I, of the investment at the end of one year is given by the equation

$$I = 100\left(1 + \frac{c}{n}\right)^n$$

where c is the nominal rate of interest and n is the number of interest compounding periods in one year.

Determine I if there are an infinite number of compounding periods in one year.

A. 100

B. $100ec$

C. $100e^c$

D. $100e^{\frac{1}{c}}$

E. ∞

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An investor invests 100. The value, I, of the investment at the end of one year is given by the equation

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Determine I if there are an infinite number of compounding periods in one year.

A. 100

B. $100ec$

C. $100e^{c}$

D. $100e^{\frac{1}{c}}$

E. ∞

COURSE 1 SOLUTIONS

Solution #1: B

Let A be the event that a person owns an automobile. Let H be the event that a person owns a house.

We want $\mathrm{P}(A \cup H) - \mathrm{P}(A \cap H)$.

$$\mathrm{P}(A \cup H) = \mathrm{P}(A) + \mathrm{P}(H) - \mathrm{P}(A \cap H)$$
$$\mathrm{P}(A \cup H) = 0.6 + 0.3 - 0.2$$
$$\mathrm{P}(A \cup H) = 0.7$$
$$\mathrm{P}(A \cup H) - \mathrm{P}(A \cap H) = 0.7 - 0.2 = 0.5$$

Solution #2: D

$\frac{dN}{dt} = \text{change in number of people who are HIV positive.}$

We want to maximize $\frac{dN}{dt}$.

$$\frac{dN}{dt} = 1000 \times \frac{1}{t+2} = \frac{1000}{t+2}.$$

$\frac{1000}{t+2}$ is strictly decreasing for $t \geq 0$.

Therefore, N is maximized when $t = 0$.

So, $N = \ln(t+2) = \frac{1000}{2} = 1000 \times 6.93 = 693.$

Solution #3: C

$$P(S|D) = \frac{P(S \cap D)}{P(D)}$$

$$P(S \cap D) = (0.1)(0.05) = 0.005$$

$$P(\overline{S} \cap D) = (0.9)(0.01) = 0.009$$

$$P(D) = P(S \cap D) + P(\overline{S} \cap D) = 0.014$$

$$P(S|D) = \frac{0.005}{0.014} = 0.36$$

Solution #4: D

$$\int_0^1 \int_0^{1-x} e^{-(x+y)} dy\, dx = 1 - 2e^{-1}$$

Solution #5: C

$$\frac{dP}{dt} = kP(t)$$

Separation of variables $= \frac{dP}{P} = kdt \Rightarrow \ln P = kt + c$

We are given: $P(0) = 25{,}000$
$P(4) = 36{,}000$

$$\ln(25{,}000) = k \cdot 0 + c$$

$$c = \ln(25{,}000) = 10.12663$$

$$\ln(36{,}000) = k \cdot 4 + \ln(25{,}000)$$

$$k = \frac{\ln(36{,}000) - \ln(25{,}000)}{4} = 0.091161$$

$$\ln[P(10)] = (0.091161)(10) + 10.12663$$

$$\ln P = 11.03824$$

$$P = e^{11}11.03824 = 62{,}208$$

Solution #6: C

$r^2 - x^2 + y^2$
$y = r\sin q$
$x = r\cos q$

Change to polar coordinates $\int_0^{p/4}\int_0^{\infty}\left(1+r^2\right)^{-2} r\,dr\,dq = \frac{p}{8}$

Solution #7: B

$\mathrm{E}(X) = 12.5$

P (person has high blood pressure) $= \frac{1}{\mathrm{E}(X)} = \frac{1}{12.5} = 0.08$

P (sixth person has high blood pressure)

= P (first five <u>don't</u> have high blood pressure) P (sixth has high blood pressure)
$= (1-0.08)^5(0.08)$
$= 0.053$

Solution #8: C

A starts out 5 miles ahead of B. The distances traveled are given by the areas under the respective graph. Therefore, the distance between A and B increases by the area between the solid graph and the dotted graph, when the solid graph is above the dotted and decreases by the area when the solid graph is below the dotted. Therefore, at t = 3, the cars are 11 miles apart. At t = 5, the cars are 1 mile apart. So the cars must be 5 miles apart at some time between t=3 and t = 5.

Solution #9: D

$$\frac{(500)(0.06)+(1000)(0.03)+(10{,}000)(0.008)+(50{,}000)(0.001)+(100{,}000)(0.001)}{(0.06+0.03+0.008+0.001+0.001)}$$
$$= \frac{30+30+80+50+100}{0.1}$$
$$= 2900$$

Solution #10: B

Let C_j = the claim for employee j, for $j = 1,2$

$\mathrm{P}(C_j > 0) = 0.40$

$$\mathrm{P}(C_j > x \mid C_j > 0) = \int_x^{5000} \frac{1}{4000}\,dx = 5000 - \frac{x}{4000},\ 1000 \le x \le 5000$$

Note: if $C_j \le 3000$, then total losses cannot exceed 8000.

$$\mathrm{P}(C_1 + C_2 > 8000 \mid C_1 > 2000) = 0.40 \int_{3000}^{5000} \frac{1}{3000} \int_{8000-y}^{5000} \frac{1}{4000}\,dx\,dy = 0.40 \int_{3000}^{5000} \frac{y-3000}{12 \cdot 10^6}\,dy$$

$$= 0.40\left[\frac{y^2}{24 \cdot 10^6} - \frac{3000y}{12 \cdot 10^6}\right]_{3000}^{5000} = 0.40\left[\frac{25-9}{24} - \frac{15-9}{12}\right] = \frac{1}{15}$$

Solution #11: E

Let $M(x)$ = the margin as a function of the number of tickets sold.

$$\begin{aligned} M(x) &= 1000 \cdot x \cdot (0.01) - C(x) \\ &= 10x - \left(x^3 - 6x^2 + 15x\right) \\ &= -x^3 + 6x^2 - 5x \end{aligned}$$

$$0 = M'(x) = -3x^2 + 12x - 5$$
$$0 = 3x^2 - 12x + 5$$

$$\begin{aligned} x = \frac{12 \pm \sqrt{144-60}}{6} &= \frac{6 \pm \sqrt{21}}{3} \\ &= 2 \pm \frac{\sqrt{21}}{3} \\ &= 3.53 \text{ or } 0.47 \end{aligned}$$

x	$M(x)$
0.47	-1.13
3.53	13.13

Solution #12: C

$$I = \lim_{n\to\infty} 100\left(1+\frac{c}{n}\right)^n$$

$$= 100\lim_{n\to\infty}\left(1+\frac{c}{n}\right)^n$$

$$= 100\cdot\lim_{n\to\infty}\left[\left(1+\frac{c}{n}\right)^{\frac{n}{c}}\right]^c$$

$$= 100\cdot e^c$$

Solution #13: B

$$\mathrm{P}(X=0) = \mathrm{P}(0,1)+\mathrm{P}(0,2) = \frac{1}{12}+\frac{2}{12} = \frac{1}{4}$$

$$\mathrm{P}(X=1) = \mathrm{P}(1,2)+\mathrm{P}(1,3) = \frac{2+2}{12}+\frac{2+3}{12} = \frac{3}{4}$$

Solution #14: E

Type	Probability
Minor	0.5
Moderate	0.4
Severe	0.1

Probability that both are minor = $(0.5)(0.5) = 0.25$

Probability that 1 is minor and 1 is moderate = $2\cdot(0.5)(0.4) = 0.40$

0.25 + 0.40 = 0.65

Solution #15: E

$$X_j \text{ claim distribution } \begin{cases} 0 & 780/800 \\ 100 & 15/800 \\ 200 & 5/800 \end{cases}$$

$$m = \mathrm{E}(X_j) = 100 \cdot \frac{3}{160} + 200 \cdot \frac{1}{160} = \frac{500}{160} = \frac{50}{16} = \frac{25}{8} = 3.125$$

$$\mathrm{E}(X_j^2) = 100^2 \cdot \frac{3}{160} + 200^2 \cdot \frac{1}{160} = 437.5$$

$$\mathrm{Var}(X_j) = \mathrm{E}(X_j^2) - \mathrm{E}(X_1)^2 = 427.734375$$

$$s = \sqrt{\mathrm{Var}(X_j)} = 20.6817$$

Let $n = 800$

$$T = \sum_{j=1}^{n} X_j \qquad \mathrm{E}(T) = 800 \cdot \mathrm{E}(X_j) = 2500$$

Find t so that:

$$0.95 = \mathrm{P}(T < t) = \mathrm{P}\left(\frac{T - nm}{\sqrt{n}\, s} < \frac{t - nm}{\sqrt{n}\, s} \right)$$

$$\approx \mathrm{P}\left[N(0,1) < \frac{t - nm}{\sqrt{n}\, s} \right] \approx \mathrm{P}\left[N(0,1) < 1.645 \right]$$

$$1.645 = \frac{t - nm}{\sqrt{n}\, s}$$

$$t = (1.645)\sqrt{n}\, s + nm = 3462.27$$
$$k \cdot \mathrm{E}(T) = t$$
$$k = \frac{t}{\mathrm{E}(T)} = \frac{3462.27}{2500} = 1.3849$$

Solution #16: A

For each policy k:

B_k is the random variable representing the benefit amount, given that there is a claim.
I_k is the claim indicator random variable.
$C_k = I_k \cdot B_k$ is the claim random variable.
$T = \sum_{k=1}^{32} C_k$ is the total claims.

$$\mathrm{E}(I_k) = \frac{1}{6}$$

$$\mathrm{E}(B_k) = \int_0^1 y \cdot 2 \cdot (1-y)dy = \frac{1}{3}$$

$$\mathrm{E}(T) = \sum_{k=1}^{32} \mathrm{E}(C_k) = 32 \cdot \frac{1}{18} = \frac{16}{9}$$

Solution #17: C

Exponential Distribution:

Density $f(x) = \frac{1}{l} e^{-x/l}$ for $0 \le x < \infty$.

Distribution $F(x) = \int_0^x f(t)dt = 1 - e^{-x/l}$.

Today $0.25 = F(2000) = 1 - e^{-x/l}$

$$l = \frac{-2000}{\ln(0.75)}$$

Problem:

Find $F(1000) = 1 - e^{-x/l}$

$$\begin{aligned} &= 1 - e^{\frac{\ln(0.75)}{2}} \\ &= 1 - \sqrt{3/4} \\ &= 1 - \frac{\sqrt{3}}{2} \\ &= 1 - \frac{1.732}{2} \\ &= 0.134 \end{aligned}$$

Solution #18: C

$$\begin{aligned} F(x) = \mathrm{P}(X \le x) &= \int_0^x f(t)dt \\ &= \int_0^x 3t^2 dt \\ &= t^3\Big|_0^x \\ &= x^3 \end{aligned}$$

$$\begin{aligned} G &= 2\int_0^1 \left|x - x^3\right| dx \\ &= 2\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Bigg|_0^1 \\ &= 2\left(\frac{1}{2} - \frac{1}{4}\right) \\ &= \frac{1}{2} \end{aligned}$$

Solution #19: B

Find points where slope is zero, changing from positive to negative. Dec. 96 is the only one.

Solution #20: E

Set $g(u) = \int_2^u \sin^2 t \; dt$

$g'(u) = \sin^2(u)$ Fundamental theorem of calculus.

$$\begin{aligned} f(x) &= g(3x) \\ f'(x) &= g'(3x) \cdot 3 \\ &= 3 \cdot g'(3x) \\ &= 3 \cdot \sin^2(3x) \\ f''(x) &= 18\sin(3x)\cos(3x) \end{aligned}$$

Solution #21: D

$$D(t) = 100\big[E(t)\big]^2\big[1 - I(t)\big]$$

$$\frac{dD}{dt} = 100\left[2E\,\frac{dE}{dt}(1 - I) + E^2\left(-\frac{dI}{dt}\right)\right]$$

If $t = t_0$ on June 30, 1996, we are given:

$E(t_0) = 0.95$ $\qquad$ $\dfrac{dE(t_0)}{dt} = 0.02$

$I(t_0) = 0.06$ $\qquad$ $\dfrac{dI(t_0)}{dt} = 0.03$

$$\frac{dD(t_0)}{dt} = 100\big[2(0.95)(0.02)(1 - 0.06) + (0.95)^2(-0.03)\big] = 0.8645$$

Solution #22: D

Let: $\mathrm{P}(O) =$ probability of orthodontic work
$\mathrm{P}(F) =$ probability of a filling
$\mathrm{P}(E) =$ probability of an extraction

We are given: $\mathrm{P}(O) = 1/2$
$\mathrm{P}(O \cup F) = 2/3$
$\mathrm{P}(O \cup E) = 3/4$
$\mathrm{P}(F \cap E) = 1/8$

Since O and F are independent, $\mathrm{P}(O \cap F) = \mathrm{P}(O) \times \mathrm{P}(F)$
and since O and E are independent, $\mathrm{P}(O \cap E) = \mathrm{P}(O) \times \mathrm{P}(E)$.

We are asked to find $\mathrm{P}(F \cup E) = \mathrm{P}(F) + \mathrm{P}(E) - \mathrm{P}(F \cap E)$.

$$\tfrac{3}{4} = \mathrm{P}(O \cup E) = \mathrm{P}(O) + \mathrm{P}(E) - \mathrm{P}(O \cap E) = \tfrac{1}{2} + \mathrm{P}(E) - \tfrac{1}{2}\mathrm{P}(E)$$

So $\tfrac{1}{2}\mathrm{P}(E) = \tfrac{1}{4}$, $\mathrm{P}(E) = \tfrac{1}{2}$.

$$\tfrac{2}{3} = \mathrm{P}(O \cup F) = \mathrm{P}(O) + \mathrm{P}(F) - \mathrm{P}(O \cap F) = \tfrac{1}{2} + \mathrm{P}(F) - \tfrac{1}{2}\mathrm{P}(F)$$

So $\tfrac{1}{2}\mathrm{P}(F) = \tfrac{1}{6}$, $\mathrm{P}(F) = \tfrac{1}{3}$.

Then $\mathrm{P}(F \cap E) = \mathrm{P}(F) + \mathrm{P}(E) - \mathrm{P}(F \cap E) = \tfrac{1}{3} + \tfrac{1}{2} - \tfrac{1}{8} = \tfrac{17}{24}$

Solution #23: D

If $f(t)$ is the time to failure, we are given $f(t) = \frac{1}{m} e^{-\frac{t}{m}}$ where $m = 10$.

The expected payment

$$
\begin{aligned}
&= \int_0^7 v(t) f(t) dt \\
&= \int_0^7 e^{7-0.2t} \frac{1}{10} e^{-\frac{t}{10}} dt \\
&= \frac{1}{10} \int_0^7 e^{7-0.2t} e^{-0.1t} dt \\
&= \frac{1}{10} \int_0^7 e^{7-0.3t} dt \\
&= \frac{1}{10} \left(\frac{-1}{0.3} \right) e^{7-0.3t} \Big]_0^7 \\
&= \frac{-1}{3} \left(e^{7-2.1} - e^7 \right) \\
&= \frac{-1}{3} (134.29 - 1096.63) \\
&= 320.78
\end{aligned}
$$

Solution #24: D

Let: X = amount of a claim.
Y = 0 or 1 according to whether the policyholder is a good or bad driver.

Apply the relationship $\mathrm{Var}(X) = \mathrm{E}\left[\mathrm{Var}(X|Y)\right] + \mathrm{Var}\left[\mathrm{E}(X|Y)\right]$.

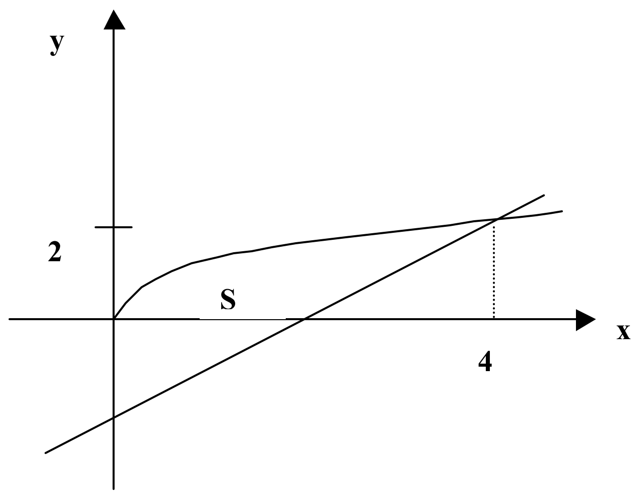
We are given:

$$\mathrm{P}(Y=0)=0.6 \qquad \mathrm{E}(X|Y=0)=1400 \qquad \mathrm{Var}(X|Y=0)=40{,}000$$
$$\mathrm{P}(Y=1)=0.4 \qquad \mathrm{E}(X|Y=1)=2000 \qquad \mathrm{Var}(X|Y=1)=250{,}000$$

Then:

$$\mathrm{E}\left[\mathrm{Var}(X|Y)\right] = 0.6(40{,}000) + 0.4(250{,}000) = 124{,}000$$
$$\mathrm{E}\left[\mathrm{E}(X|Y)\right] = 0.6(1400) + 0.4(2000) = 1640$$
$$\mathrm{Var}\left[\mathrm{E}(X|Y)\right] = 0.6(1400-1640)^2 + 0.4(2000-1640)^2 = 86{,}400$$
$$\mathrm{Var}(X) = 124{,}000 + 86{,}400 = 210{,}400$$

Solution #25: D



From $y = \sqrt{x}$

$y^2 = x$

Substitute $x - y = 2$

$$y^2 - y - 2 = 0$$
$$(y-2)(y+1) = 0$$
$$y = 2$$

$$\iint_S y\,dA = \int_0^2 \int_{y^2}^{y+2} y\,dx\,dy$$

$$= \int_0^2 yx\Big]_{x=y^2}^{x=y+2} dy$$

$$= \int_0^2 y\left(y+2-y^2\right)dy$$

$$= \int_0^2 y^2 + 2y - y^3 dy$$

$$= \left(\frac{1}{3}y^3 + y^2 - \frac{1}{4}y^4\right)\Bigg]_{y=0}^{y=2}$$

$$= \left(\frac{8}{3} + 4 - 4\right) - (0)$$

$$= \frac{8}{3}$$

Solution #26: A

$$M(t) = 1 + \mathrm{E}\big[X\big]t + \mathrm{E}\big[X^2\big]\frac{t^2}{2} + \mathrm{E}\big[X^3\big]\frac{t^3}{3!} + \dots$$

$$\mathrm{E}\big[X\big] = \left.\frac{dM(t)}{dt}\right]_{t=0}$$

$$\frac{dM}{dt} = \frac{d}{dt}\left(\frac{2+e^t}{3}\right)^9 = 9\left(\frac{2+e^t}{3}\right)\frac{1}{3}e^t$$

$$\left.\frac{dM}{dt}\right]_{t=0} = 9\left(\frac{2+1}{3}\right)^8 \frac{1}{3}\cdot 1 = 3$$

$$\mathrm{E}\big[X^2\big] = \left.\frac{d^2M(t)}{dt^2}\right]_{t=0}$$

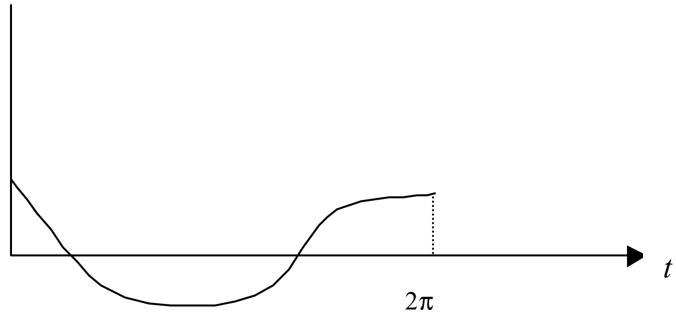
$$\frac{d^2M(t)}{dt^2} = \frac{d}{dt}3e^t\left(\frac{2+e^t}{3}\right)^8 = 3e^t\left(\frac{2+e^t}{3}\right)^8 + 3\cdot 8e^t\left(\frac{2+e^t}{3}\right)\frac{1}{3}e^t$$

$$\left.\frac{d^2M(t)}{dt^2}\right]_{t=0} = 3 - 1\cdot\left(\frac{2+1}{3}\right)^8 + 3\cdot 8\cdot 1\left(\frac{2+1}{e}\right)^7 \frac{1}{3}\cdot 1 = 3 + 8 = 11$$

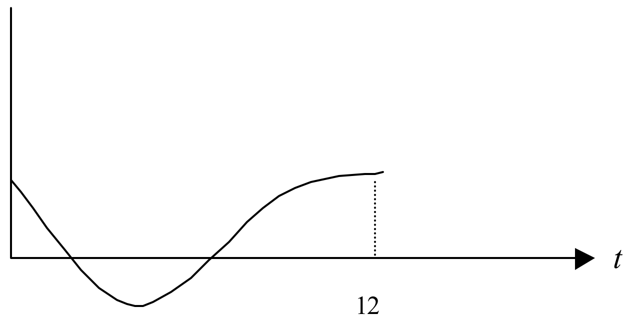
$$\mathrm{Var}\, X = \mathrm{E}\left(X^2\right) - \big[\mathrm{E}(X)\big]^2 = 11 - 3^2 = 2$$

Solution #27: C

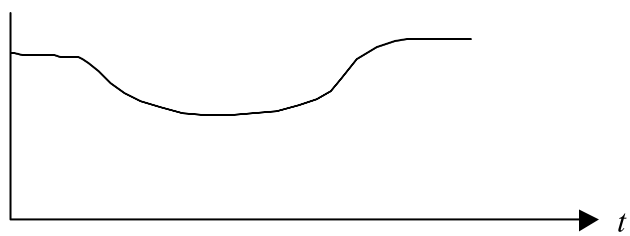
Graph = $\cos t$



$k\cos\frac{pt}{6}$



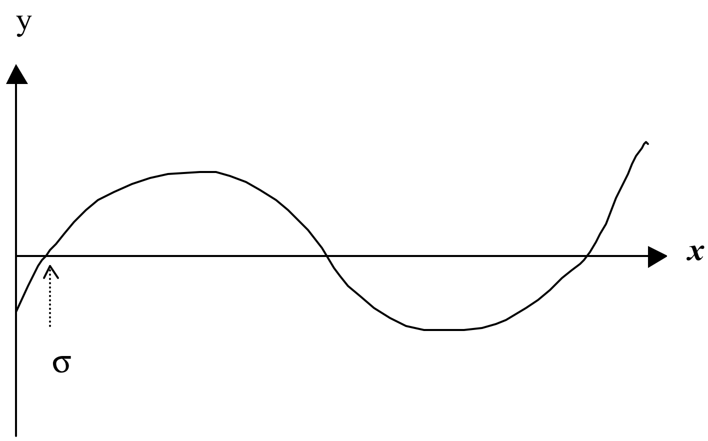
$k\cos\frac{p\,t}{6}+c$



Solution #28: A

Let $f(x)$ be the function
$g(x) = f'(x)$
$h(x) = g'(x) = f''(x).$

Then in the given graph, looking for example, at $x = 2$, the function with $y = 0$ must be the derivative of the function with a relative maximum. Therefore, the graph of $g(x)$ is



Let a be the value on the x axis so that $g(a) = 0,\ 0 < a < 1.$.

Then:

$g(x) < 0$ for $0 < x < a$
$g(x) > 0$ for $a < x < 4$

So:

$f(x)$ is decreasing for $0 = x < a$.
$f(x)$ is increasing for $a < x < 4$.

Only alternative "A" satisfies these conditions.

Solution #29: C

Let: S = the sample space of customers.
H = the subset of S consisting of high-risk customers.
L = the subset of S consisting of low-risk customers.
N_y = the number of claims in year y for customer C.

Then the problem is to find $\mathrm{E}[N_{98}]$.

$$\mathrm{E}[N_{98}] = \mathrm{P}(c \in H | N_{97} = 1)(0.6) + \mathrm{P}(c \in L | N_{97} = 1)(0.1)$$

We have $\mathrm{P}(N_{97} = k) = \dfrac{q^k e^{-q}}{k!}$ where k = 0, 1, 2, 3 … and q is the mean of N.

$$\mathrm{P}(N_{97} = 1 | c \in H) = \frac{0.6e^{-0.6}}{1} = 0.3293$$
$$\mathrm{P}(N_{97} = 1 | c \in L) = \frac{0.1e^{-0.1}}{1} = 0.0905$$

H and L partition S, applying Bayes Theorem

$$\mathrm{P}(c \in H | N_{97} = 1) = \frac{(0.1)(0.3293)}{(0.1)(0.3293) + 0.9(0.0905)} = 0.2879$$

$$\mathrm{P}(c \in L | N_{97} = 1) = 1 - \mathrm{P}(c \in H | N_{97} = 1) = 0.7121$$

And:

$$\mathrm{E}[N_{98}] = (0.2879)(0.6) + (0.7121)(0.1) = 0.2440$$

Solution #30: E

Consider $\ln\left(e^x+3x\right)^{\frac{1}{x}}=\frac{1}{x}\ln\left(e^x+3x\right)$

as $x\to 0^+\ \ln\left(e^x+3x\right)\to 0$

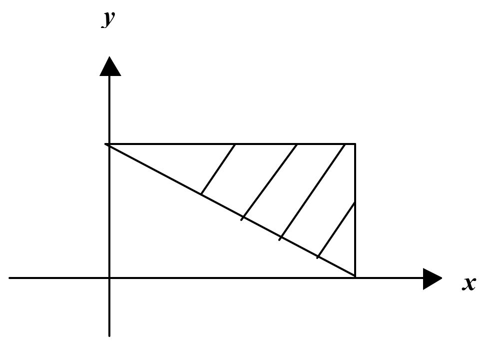
$$so\ \lim\frac{\ln\left(e^x+3x\right)}{x}=\lim_{x\to 0^+}\frac{\frac{d}{dx}\ln\left(e^x+3x\right)}{\frac{d}{dx}x}$$

$$=\lim_{x\to 0^+}\frac{1}{e^x+3x}\left(e^x+3\right)$$

$$=\frac{1+3}{1+0}=4$$

Therefore: $\lim_{x\to 0^+}\left(e^x+3x\right)^{\frac{1}{x}}=\lim_{x\to 0^+}e^{\ln\left(e^x+3x\right)^{\frac{1}{x}}}=e^4$

Solution #31: A



Expected payment is:

$$\int_0^1 \int_{1-y}^1 (2x)(x+y-1)\,dx\,dy$$

$$= \int_0^1 \int_{1-y}^1 2x^2 + 2xy - 2x \;\, dx\;dy$$

$$= \left[\frac{2x^3}{3} + x^2y - x^2\right]_{1-y}^{1} = \tfrac{2}{3} + y - 1 - \tfrac{2}{3}(1-y)^3 - (1-y)^2 y + (1-y)^2$$

$$= \int_0^1 \left(-\tfrac{1}{3} + y + \tfrac{2}{3}y^3 - 2y^2 + 2y - \tfrac{2}{3} - y^3 + 2y^2 - y + 1 - 2y + y^2\right)dy$$

$$= \int_0^1 \frac{-y^3}{3} + y^2 dy = \left[\frac{-y^4}{12} + \frac{y^3}{3}\right]_0^1 = \tfrac{1}{3} - \tfrac{1}{12} = \tfrac{3}{12} = \tfrac{1}{4}$$

Solution #32: C

C_1: $x = t+1, \quad y = 2t^2$, thus $t = x-1 + t > \pm\sqrt{y/2}$

$\therefore x-1 = \pm\sqrt{y/2}, \quad x^2 - 2x + 1 = \frac{1}{2}y$ or

$y = 2x^2 - 4x + 2$

C_2: $x = 2t+1, \quad y = t^2 + 7 \therefore t = \dfrac{x-1}{2} + t = \pm\sqrt{y-7}$

$\therefore \dfrac{x-1}{2} = \pm\sqrt{4-7}$, or $\dfrac{x^2-2x+1}{4} = y-7$

$y = x^2 - \dfrac{2x}{4} + 1 + 7, \quad y = \dfrac{x^2}{4} - \dfrac{x}{2} + \dfrac{29}{4}$

$y = \frac{1}{4}\left(x^2 - 2x + 29\right)$

$\therefore 2x^2 - 4x + 2 = \frac{1}{4}\left(x^2 - 2x + 29\right)$,

$8x^2 - 16x + 8 = x^2 - 2x + 29$

$7x^2 - 14x - 21 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$ or $x = 3, \ x = -1$

$x = 3, \ y = 8 \qquad\qquad x = -1, \ y - 8$

Solution #33: D

Long run number of clients stabilizes at $C = \lim\limits_{n\to\infty} C_n$.

$$C = \tfrac{2}{3}C + \frac{9000}{C^2}$$

$$\tfrac{1}{3}C = \frac{9000}{C_{n-1}^2}$$

$$C^3 = 27{,}000$$

$$C = \sqrt[3]{27{,}000} = 30$$

Solution #34: C

$$\mathrm{E}(X) = \int_0^1 \frac{x^2(4-x)}{9}dx + 1\int_1^3 \frac{x(4-x)}{9}dx$$

$$= \int_0^1 \frac{4x^2 - x^3}{9}dx + \int_1^3 \frac{4x - x^2}{9}dx$$

$$= \left[\frac{4x^3}{(3)(9)} - \frac{x^4}{(9)(4)}\right]_0^1 + \left[\frac{4x^2}{18} - \frac{x^3}{27}\right]_1^3$$

$$= \frac{4}{27} - \frac{1}{36} + \frac{36}{18} - 1 - \frac{4}{18} + \frac{1}{27}$$

$$= \frac{5}{27} + \frac{32}{18} - 1 - \frac{1}{36}$$

$$= 0.185 + 1.778 - 1 - 0.028$$

$$= 0.935$$

Solution #35: C

Let H = time to death of the husband

W = time to death of the wife

J = time to payment after both husband and wife have died = max(H,W)

$$f_H(h) = f_W(w) = \frac{1}{40}$$

$$\mathrm{E}[H] = \mathrm{E}[W] = 20$$

$$\mathrm{P}(\max(H,W) < j) = \int_0^j \int_0^j \left(\frac{1}{40}\right)\left(\frac{1}{40}\right) dh\,dw = \frac{j^2}{1600}$$

$$f_J(j) = \frac{j}{800}$$

$$\mathrm{E}[J] = \int_0^{40} \frac{j^2}{800} dj = \frac{40^3}{2400} = \frac{80}{3}$$

$$f_{J,H}(j,h) = \begin{cases} 0 & j < h \\ f_W(j) f_H(h) = \dfrac{1}{40^2} & j > h \\ F_W(h) f_H(h) = \dfrac{h}{40^2} & j = h \end{cases}$$

$$\mathrm{E}[J,H] = \int_0^{40}\int_h^{40} \frac{jh}{40^2} dj\,dh + \int_0^{40} h^2 \cdot \frac{h}{40^2} dh = \int_0^{40} \frac{40^2 h^2 - h^3}{2 \cdot 40^2} + \frac{h^3}{40^2} dh =$$

$$\frac{40^4}{4 \cdot 40^2} + \frac{40^4}{8 \cdot 40^2} = 600$$

$$\mathrm{cov}[J,H] = \mathrm{E}[J,H] - \mathrm{E}[J]\mathrm{E}[H] = 600 - \frac{1600}{3} = 66\frac{2}{3}$$

Solution #36: C

$$\text{Average value} = \frac{\text{value of integral}}{\text{width of interval}}$$

$$= \frac{\int_0^2 \frac{t\cos\left(t^2\right)}{2}dt}{2} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\frac{\int_0^2 (2)t\ \cos\left(t^2\right)}{2}$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\frac{\left[\left(\sin\left(t^2\right)\right)\right]_0^2}{2}$$

$$= \frac{\sin 4}{8}$$

Solution #37: E

Distribution is:

P (0,0,0) = 0	P (1,0,0) = k
P (0,0,1) = k	P (1,0,1) = 2k
P (0,0,2) = 2k	P (1,0,2) = 3k
P (0,1,0) = 2k	P (1,1,0) = 3k
P (0,1,1) = 3k	P (1,1,1) = 4k
P (0,1,2) = 4k	P (1,1,2) = 5k
P (0,2,0) = 4k	P (1,2,0) = 5k
P (0,2,1) = 5k	P (1,2,1) = 6k
P (0,2,2) = 6k	P (1,2,2) = 7k

K = 1/63

P (0,0/X = 0)	P (2,0/X = 0) = 4/27
P (0,1/X =0) = 1/27	P (2,1/X = 0) = 5/27
P (0,2/X = 0) = 2/27	P (2,2/X =0) = 6/27
P (1,0/X = 0) = 2/27	
P (1,1/X = 0) = 3/27	
P (1,2/x = 0) = 4/27	

Expected number of unreimbursed equals
(1 accident) (9/27) + (2 accidents) (6/27) = 9/27 + 12/27 = 21/27 = 7/9.

Solution #38: B

First dividend paid = \$1.00
Second dividend paid = (0.95) (1.00)
Third dividend paid = $(0.95)^2$ (1.00)
.
.
.
40^{th} dividend paid = $(0.95)^{39}$ (1.00)

Total dividends = 1.00 + 0.95 + $(0.95)^2$ + … + $(0.95)^{39}$

Geometric series;

$$S_{40} = \frac{1-(0.95)^{40}}{1-0.95} = \frac{1-0.1285}{1-0.95} = \frac{0.8715}{0.05} = \$17.43$$

Solution #39: D

Mode is max of probable distribution $f(x) = \frac{4x}{9} - \frac{x^2}{9}$.

Maximize $f(x)$, so take derivative:

$$f(x) = \frac{4}{9} - \frac{2x}{9} = 0$$

$$2x = 4$$
$$x = 2$$

Solution #40: E

Let X = number of passengers that show for a flight.

You want to know $P(X = 31)$ and $P(X = 32)$.
All passengers are independent.

Binomial distribution:

$$\mathrm{P}(X = x) = \binom{n}{x}\mathrm{P}^x(1-\mathrm{P})^{n-x}$$

$\mathrm{P} = 90$

$1 - \mathrm{P} = 0.10$

$n = 32$

$$\mathrm{P}(X = 31) = \binom{32}{31}(0.90)^{31}(0.10)^1 = (32)(0.90)^{31}0.10 = 0.1221$$

$$\mathrm{P}(X = 32) = \binom{32}{32}(0.90)^{32}(0.10)^2 = 0.0343$$

P (more show up than seats) $= 0.1221 + 0.0343 = 0.1564$