The Actuarial Committee of the Mutual Insurance Rating Bureau has recommended to the Automobile Rating Committee that the experience utilized in determining Mutual Bureau private passenger automobile liability rates be based on the combined experience of all companies reporting to the Mutual Bureau and National Bureau on both the family automobile policy form and the package automobile policy form for bodily injury and property damage liability coverages. The medical payment component of the two policy forms will be separately determined. It is quite possible that the suggestion made by Mr. Lange for the use of ratio estimation will be helpful. It will require further study and tests, but certainly the experience developed under the package policy forms should no longer be ignored in the making of package policy rates. This situation also exists in rating the special multi-peril policy forms. There is some hope that with the development of the Commercial Risk Statistical Plan by the National Insurance Actuarial and Statistical Association data will be available to test Mr. Lange's suggestions in the rating of commercial package policies.

DISCUSSION BY DALE NELSON

This paper is another in a series of studies on the application of contemporary mathematical developments to the problems of the actuarial sciences both in terms of providing the theoretical justification for, and introducing new techniques into, actuarial practices. Specifically, this paper is concerned with the application of two techniques of sampling theory—stratification and ratio estimation—to (package) ratemaking. My remarks will be confined to a critique of the statistical theory involved, and I will leave the practical aspects of the implied ratemaking process for others to discuss. It might be observed in passing, though, that Mr. Lange has presented some persuasive arguments in favor of sampling theory in package ratemaking: the ability to incorporate more accurate trend, credibility, and loss development factors as well as to analyze the design of package policies, among others.

The two basic ideas discussed in this paper are in fact, if not in name, well-known to all of us. For example, ratio estimation is used, among other places, in the derivation of loss development factors. Similarly, the classification of risks by territory and class grouping is nothing other than stratification. However, it should be pointed out that this form of stratification has a different purpose from that in statistical sampling. In ratemaking (the non-packaged variety), we are directly interested in the characteristics (e.g. pure premiums) for the various strata and only mildly interested—

if at all—in the aggregate characteristics. This is contrary to the situation in sampling design where the strata are set up primarily to yield a more efficient estimate of the aggregate characteristic. Thus, it would be somewhat misleading to justify this form of actuarial stratification on the grounds that it minimizes the variance of the aggregate estimates, since that part of the statistical theory never comes into play.

Now in package ratemaking, we are interested in the most efficient estimate of the aggregate characteristics; and at first glance it appears that the decomposition of the aggregate experience by coverage or by layer provides the desired stratification for minimizing the sampling variance. However, contrary to Mr. Lange's contention, this decomposition does not necessarily constitute a stratification in the technical sense. The latter term, by definition, is reserved for the decomposition of a population into ,mutually exclusive subpopulations. For example, if we were picking a sample from the population of claims, a breakdown by coverage would be a stratification; on the other hand, the population of policies cannot be uniquely classified by coverage or by kind of loss. Consequently, the decomposition by coverage of the experience compiled by policy—while meaningful—does not satisfy the requirements of statistical stratification.

Well then, what is the justification for making this kind of decomposition; it certainly seems to be a reasonable thing to do. It turns out that the theoretical advantage is tied directly to the other technique discussed by Mr. Lange. Basically, it stems from the fact that this kind of decomposition provides the means for obtaining the maximum efficiency from ratio estimation. In a recent paper in the *Journal of the American Statistical Association*,[1] it was shown that:

> "The precision of ratio estimates is substantially improved if the correlated variables are decomposed into the sum of several components which are pairwise more highly correlated than the original variables."

Thus, it makes sense to split up the package experience by coverage since the latter is presumably more highly correlated to the corresponding non-packaged experience than is the combined experience for the two forms. However, it seems advisable that these presumptions be tested, since the claimed advantages to ratio estimation are dependent on their validity. Unfortunately, I suspect that this would be a rather difficult task to perform.

Finally, I would like to stress what is perhaps the most important

[1] Robson, D. S. and Vithayasai, C., "Unbiased Componentwise Ratio Estimation," *JASA* Vol. 56, P. 350.

point made by Mr. Lange: that "insurance statistics may be viewed as samples of what might have occurred." As a corollary to this observation, the whole of sampling theory is then available in the actuary's tool kit. How much use can or will be made of this fact is only touched on lightly in the paper under review; it is hoped that future papers, by Mr. Lange and others, will delve into these applications in more detail.

DISCUSSION BY CHARLES F. COOK

It is rare for a paper presenting essentially new material to have such broad scope as this one. The author begins with a lucid discussion of his statistical philosophy of ratemaking and insurance. Next, he presents two technical sections: a discussion of the value of stratification in sampling design, and an introduction to ratio estimation, with special consideration of its power for inference and the control of its bias. Then he applies both techniques to package policy ratemaking in general and to an example, the Special Automobile Package Policy. Leaving the personal lines, he reviews the Bailey, Hobbs, Hunt, and Salzmann paper, "Commercial Package Policies — Rating and Statistics,"[1] mediates the indivisible premium-component rating debate, and shows that the key to the whole problem is a good multiple-line statistical plan.

His presentation of stratification and ratio estimation is clear and accurate; the applications and example are well presented and reasonable. Unfortunately, in the common statistical sense of the terms, the ratio of package to non-package pure premiums is not a ratio estimate, and the subdivision of experience by coverage and layer of loss is not stratification. With this exception (to be developed later) the author is very convincing. Not only am I convinced by what he says, but by several things he does not say. He flirts with some interesting ideas and potential applications of his sampling tools, raising questions in the reader's mind without having space to develop them. I hope he will not mind if I add a bit here.

SAMPLING THEORY AND RATEMAKING

"Those who refuse to go beyond the facts
rarely get as far as facts." — Thomas Huxley

This section presents a good case for treating the entire population of losses as a sample of a greater population of potential losses. It should be

[1] Bailey, R., Hobbs, E., Hunt, F., and Salzmann, R., "Commercial Package Policies — Rating and Statistics," *PCAS*, L, p. 87.