Section 6: Discussions of Issues

In this section, some of the important issues associated with our analysis are discussed. These issues generally deal with specific questions or decisions which were faced in performing the analysis and developing the model.

Equilibrium vs. Arbitrage Free Models

One of the primary processes in a financial scenario generator is a term structure model. A tremendous variety of models is available for both practitioners and researchers. (For a discussion of many of the available models see Yan (2001)). No single term structure model has yet proven itself worthy for all possible applications (see the discussion in Chapman and Pearson (2001)). In virtually all cases, the user of a term structure model has a tradeoff to consider: complexity of the model vs. accuracy. This tradeoff depends on the specific application of the term structure model.

There are two important issues to consider when choosing among term structure models. The first consideration is related to the theoretical background of the model. Specifically, there are equilibrium models and arbitrage-free models. Equilibrium models typically begin with an assumption for short-term interest rates, which are usually derived from more general assumptions about the state variables that describe the overall economy. Using the assumed process for short-term rates, one can determine the yield on longer-term bonds by looking at the expected path of interest rates until the bond's maturity. One of the primary advantages of equilibrium models is that the prices of many popular securities have closed-form analytic solutions. Another advantage is that equilibrium models are fairly easy to use. On the negative side, equilibrium term structure models generate yield curves which are inconsistent with current market prices. While the parameters of these models may be selected carefully, there is no guarantee that the resulting term structure will generate observed market prices.

Contrary to equilibrium models, arbitrage-free term structure models project future interest rate paths that emanate from the existing yield curve. For applications using arbitrage-free term structure models, resulting prices will be based on the concept of arbitrage. Unfortunately, arbitrage-free term structure models are frequently more difficult to use than their equilibrium counterparts.

Outside of the previously mentioned items, there are other benefits and costs associated with equilibrium and arbitrage-free term structure models.

- Pricing accuracy
    - Arbitrage-free models are more useful for pricing derivatives. Since derivatives are priced against the underlying assets, a model that explicitly captures the market prices of those underlying assets is superior to models that more or less ignore market values. Jegadeesh (1998) looks at the pricing of interest rate caps and determines that arbitrage-free models price interest rate caps more accurately than equilibrium models. Hull (2000) and Tuckman (1994) also comment that given that arbitrage-free

term structure models are founded upon the absence of arbitrage, using these models to price derivative contracts is more plausible then equilibrium approaches.
    - Unfortunately, comments revolving around the pricing accuracy of arbitrage-free term structure models are based on a short pricing horizon. There have been no formal long-term tests of accuracy.
- Internal consistency
    - Exploding models – Arbitrage-free models can "explode" over long periods of time. With many arbitrage-free models, the forward rate plays a central role in the expected path of interest rates. Forward rates are related to the slope of the yield curve. Depending on the existing slope of the yield curve, forward rates may exhibit strange behavior impacting projections of interest rate paths in arbitrage-free term structure models. For steeply sloped yield curves, the forward rate may become very large. For inverted yield curves, the forward rate may even become negative. Especially for long-term projections, simulation paths may become extreme.
    - Arbitrage-free models also suffer from inconsistency across time. (see Wilmott (1998) and Tuckman (1994)). The underlying assumption of many arbitrage-free term structure models is that the risk-free rate is closely related to the forward rate curve. If the model were correct, the forward rates would be the perfect predictors of future spot rates. At time 0, the known term structure implies future spot rates and volatilities throughout the projection period; all of the mean reversion levels and volatilities in the future are known on the projection date, without any regard to any risk premium that may be contained in these values. Equilibrium models provide more consistent statements about interest rates over time.
- Data issues
    - Isolating the term structure – Determining the "true" term structure for input into an arbitrage-free model is difficult. One usually considers risk-free securities such as U.S. Treasuries. There are several difficulties in looking at U.S. Treasury data. First, market data gathered from strip data is noisy; term structures that are created from this data are not smooth. An alternative source for long-term interest rate data is to look at long-term U.S. Treasury coupon bonds. Even when there was a regular history of issuing 30-year bonds, liquidity affected long-term rates. When on-the-run Treasuries were issued, the securities typically had higher liquidity and therefore higher price, forcing down long-term yields. The result is a forward rate curve that initially increased until liquidity issues dominated. The end of the forward rate curve dipped, leading to a strange forward rate curve. Aside from liquidity concerns, the future of 30-year bonds is uncertain, given the federal government's termination of 30-year bond issues.
    - Noisy data and interpolation - When there is sparse data available (which is typically the case for long-term Treasuries), there are fewer points to interpolate the term structure. This makes arbitrage-free models very sensitive to the market data or any inefficiencies in market quotes that is due to noisy data.
    - Market price of risk – While these issues affect the input for arbitrage-free models, equilibrium models require some measure of the market price of risk. This information can be harder to obtain than spot rates.

Adequacy of a Two-Factor Interest Rate / Inflation Model

The number of factors to use in modeling interest rates is a decision which frequently elicits passionate debate. Here, it is important to keep in mind the purpose toward which this research is working: to produce reasonable distributions of future variable values. Our work is NOT intended for security-trading purposes. This is a hugely important context to keep in mind – it has implications for the type of interest rate model used, the number of parameters employed, etc. Furthermore, there is often a misunderstanding as to the types and movements of yield curves that are available from two-factor (and with respect to some issues, even one-factor) models. For example, humped curves are indeed possible. (A good paper for considering the types of yield curve movements that predominate historically is an article by Litterman and Schenkman.) We believe that the two-factor model we have employed is a reasonable selection in view of both historical and parsimony considerations.

Building a Model which Uses @Risk Software

@Risk, the stochastic simulation software which we use (a product of Palisades Corp.), provides a variety of useful output statistics relative to simulated interest rate scenarios (as well as many other useful metrics). However, during our project, concern was expressed about our use of this software, as not all potential users might have access to it. While we appreciate this concern, we believe that leveraging off an existing and widely available simulation spreadsheet add-on package is the most effective approach. The programming required to do the same things in Excel alone, with respect to both modeling capabilities and output metrics, would be considerable. Furthermore, we sincerely believe that any organization truly interested in generating economic and financial scenarios, presumably to enhance strategic planning and operational decision-making, should commit the resources to purchasing and understanding such a package.

Nevertheless, a potential alternative is also provided. We have created a database which contains scenario values, predicated upon default model parameters, for a variety of economic variables for use by others. In other words, for example, for a particular set of input assumptions, we have created a spreadsheet containing hundreds of scenario simulations, or iterations, for each economic series. (See Appendix C: *Simulated Financial Scenario Data*.)

Model Parameters

All of the default parameters in the model are identified, and changeable by the user. This includes the ability to input certain specific economic / financial scenarios. Thus, for example, the model has the ability to accept, as inputs, specifically identified nominal interest rate scenarios (for example, as specified in NY Regulation 126).

Degree of "Fit" of Model Relative to Historical Data

This is certainly a large and challenging area, relating to all aspects of modeling and the underlying data – e.g., the possibility that there might be measurement errors in some data series, the underlying reasons for the possible presence of "mean reversion," etc. In general, we believe our theoretical framework provides a parsimonious approach to closed-form solutions of particular variables of interest. Having said that, investigation into historical correlations among variables, and the resulting potential impact upon model parameter selection, should continue as new series data emerges over time.

Regime-Shifting Values for Equity Returns

Examination of historical equity data (for the S&P 500) shows that a normal distribution does not adequately reflect the outliers in historical S&P 500 equity returns. A better fit has been found to involve a regime-switching model (incorporated in our Financial Scenario Model spreadsheet) in which one regime has a relatively low standard deviation and the other a relatively high standard deviation. The key point is that no one knows which regime they are in at any given time, and they cannot elect to invest only when the higher expected values are occurring.

Stochastic Simulation versus User-Specified Scenarios

In general, the financial scenario model provides for stochastic simulation of future economic variables, based upon user-specified parameters for the various economic processes. However, there are instances – particularly relating to regulatory tests and to sensitivity testing – where it is desirable to allow the user to input specific scenarios for the future values of certain processes. In particular, it was decided to provide a scenario specification option for three economic variables in the model: nominal interest rates, inflation, and equity returns. For example, with respect to nominal interest rates, each of the "New York 7" regulatory interest rate tests are pre-programmed into the model and may be selected by the user; the user may also specify a scenario of her/his own creation for any of the three economic processes. More specific information regarding each of these scenario specification options is provided in *Appendix A – User's Guide*.

Exclusion of Negative Values

Much discussion occurred regarding the need for, and the most appropriate approach to, limiting the future simulation of certain variables to non-negative values. Most of the concern centered around nominal and real interest rates, but inflation was also discussed within this context. The issue was further complicated by the fact that nominal interest rates are modeled as a function of *both* real interest rates and inflation. Ultimately, it was decided to provide the user with two options:

- *Placing lower bounds on the levels of inflation and real interest rates*. The model then simulates these processes "normally" (i.e., as if there were no lower bound), but selects the maximum of the lower bound and the normal simulated value as the final simulated value of the process.
- *Eliminating the potential for negative nominal interest rates.* In this case, the model uses the standard inflation simulation, but effectively places a lower bound on the real interest rates such that the resulting nominal interest rate is non-negative.