# MATHEMATICS FOR STUDENTS OF CASUALTY ACTUARIAL SCIENCE*

BY

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The Educational Committee has requested me to write a paper on the mathematics required of students of our Society. Similar papers have been written dealing with Law, Statistics, and Economics.

It seems logical to devote our attention at first to some of the reasons why the mathematics required in the examinations is included. It perhaps is inappropriate here to go into detail as to why mathematics is taught in schools and colleges as a matter of general education and not merely to students who wish to make practical use of them. College courses in Principles of Education, Psychological Basis of Education, and other similar courses are partly devoted to such analyses. Suffice it to suggest that the concepts of mathematics are very different from those in other subjects. Of course logic must be used in every subject but it is generally used so unobtrusively that its presence is not noted. Logic is the essence of mathematics. It is possible that logic should be studied as a separate subject more than it is, but my experience indicates that such a requirement for actuarial students, for instance, would be more difficult than the mathematics.

Another concept of mathematics is that of precision. In most branches of knowledge there is so much truth in different viewpoints that only general conclusions can be drawn, and even these are largely a matter of judgment, different authorities sometimes reaching different conclusions or at least placing different emphasis on the relative importance of conclusions. Of course the same principles of thought apply in mathematics, but here the assumptions are made explicit, so that when they are once understood the conclusions must be reached by every one capable of following the reasoning. Perhaps my thought will be

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clearer if we attempt to imagine a student asked to discuss the influence of insurance on the country. Almost any high school pupil could give such a discussion on the basis of almost no knowledge of the subject, and the surprising feature would be that many of them that gave very poor discussions would consider that they had given very good answers, and would even consider themselves unjustly treated if they were graded on their answers from the viewpoint of what a really good answer should be. In other words, students in general in most subjects have little conception of the difference between a thoroughly satisfactory answer and a halfway answer, but in mathematics there is no middle ground. Even the poorest student, with the possible exception of some of subnormal intelligence, knows whether a proof in mathematics is correct or not, of course within the limits of his knowledge. The precise reasoning must be followed. The student realizes the futility of bluffing. Appreciation of these concepts is fundamental to good education.

Without going further into the reasons why mathematics is almost universally approved by educators and included in their curricula, we may note that elementary algebra and plane geometry are required in practically all high schools in the country, except possibly where a school is teaching a trade largely with the idea of holding pupils in school who would not otherwise go to high school. Algebra is even introduced in the grammar school or in the early years of the junior high school in many of the progressive cities. These subjects are also almost universally required in college entrance examinations. Solid geometry, trigonometry, and advanced algebra are required at entrance of the best engineering colleges. Is it surprising, therefore, that this Society, one of whose aims is to uphold actuarial standards for casualty insurance practice, requires about as much mathematics for the completion of its course as engineering colleges require for the entrance to theirs, when we remember the fundamental mathematics back of some of the casualty actuarial work to which reference will be made later?

From an entirely different viewpoint the student who wishes to develop into an actuary might consider the training that the men now prominent as actuaries have had. I am not in a position to estimate the part that has been taken by different actuaries in the actual calculation of premium rates and other work of casu-

alty bureaus, nor to judge their relative abilities in their company positions, but an analysis of the education had by writers of the papers that have been presented to the *Proceedings* is illuminating. A large share of these papers have been written by Fellows who are also members of the Actuarial Society of America, the examination requirements for admission to which have thus far included considerably more mathematics than the Casualty Society has required of its students. Most of the other papers have been written by college graduates. In general the papers have been written by men who have either received special training or have given themselves enough special training so that they are recognized authorities in their lines.

I fully recognize the success of executives who have had no opportunities for special training, except what they could dig out themselves. In the last analysis success in any line is due principally to perseverance, hard work, and personality as well as to intelligence. But the question for the young man is not whether the executive officers of companies are mathematicians or statisticians, but whether or not they might be better officers if they had had greater opportunities to master the insurance sciences. To be more specific the young man's concern should not, from the present standpoint, be whether he has the qualities that will later make him a success in any event, but whether or not acquiring the mastery of mathematical and statistical tools will increase his usefulness and the consequent probabilities of his greater success. I will not discuss whether or not complete mastery of a person's work, including the fundamental principles, may not improve his personality by instilling confidence and self-respect, and also whether or not the increased respect of his co-workers may also have a favorable reflex action on his personality.

Further, I am convinced that while broad actuarial concepts sufficed for the earliest casualty work, that day is past, and that adequate solutions of problems will increasingly demand greater actuarial technique and insight. In general business a grammar school education was sufficient for men now in their prime. Now a high school education is almost necessary in some lines, and many with the best promise of future success have a college education. This situation is particularly accentuated in casualty actuarial science which has developed so recently. Statisticians and actuaries originally had almost no strictly pertinent data to

which they could apply mathematical principles if they had had mastery of them. At present, I believe, the data are extensive enough so that the full value is not generally derived from it because of lack of understanding of the possibilities of analysis. This situation can be but accentuated in the future.

From the Society's standpoint one of the main objects of its existence is to raise the standards of present day casualty work. This process to be effective means requiring more training of the individual prospective members and to some extent the selection of those showing the best promise of success. Whether or not an examination in mathematics is an entirely satisfactory method of selection, it is about the only method available that can be applied impartially to young men to test this phase of their development. Of course the selection could be on the basis of obtaining executive positions in individual companies or bureaus. This method is applied to some extent by our Society, but it cannot be an effective method of giving the young men a chance. Individual recommendation by members would not be feasible. The mathematical examinations required seem to be a reasonable minimum test of mental ability. The other examinations are minimum tests of practical knowledge of the casualty business and of ability to use that knowledge. Of course great knowledge, or even ability in mathematics, is not a true measure of the possibility of success in the profession. The latter only comes from experience and development of judgment. The examinations are merely a means of selecting those who show most promise of development. From the Society's viewpoint, and at least in some cases from companies' viewpoints, the examinations indicate those who may be worthy of further chances for development. From the employee's viewpoint they may be considered to be indicative of the individual's willingness and ability to better prepare himself for his work and therefore the passing of them an evidence of his having fitted himself for advancement.

Probably the reason for the mathematics in the examination that most interests the student is how it can be practically applied. In discussing the individual subjects I will attempt to illustrate some of their interrelations and uses. My object in this discussion is twofold: first, to show what I believe it is advisable to study for the examinations and, second, indicate what might be studied to more thoroughly master the subjects.

One of the most important branches is the ELEMENTS OF THE THEORY OF LIFE CONTINGENCIES. Of course a comprehensive and thoroughly efficient knowledge of this subject as required by a life actuary is not necessary, but a considerable knowledge of life and morbidity contingencies, etc., is desirable, among other things for pension work and valuation of disability and death claim benefits in workmen's compensation and personal accident and sickness insurance. No text book has been written from the standpoint of the casualty student, so that the only books recommended are very elementary. Though they include some material not really necessary, I believe that they do not give an adequate preparation in certain particulars. More of the theory of life annuities and joint lives should be included and considerable additional work on multiple decrement tables, such as total permanent disability tables and remarriage tables should be included. A real understanding of proper calculation of premiums and reserves for this necessitates considerably more mathematical training than is required for our examinations. A person just able to pass perfectly all the mathematics in the examinations will need to dig much deeper into these very subjects before he can attain competency in premium or reserve calculations where multiple or new contingencies are involved, or in the most pressing problem of the interpretation of past and present data as guides to underwriting for most probable future success. The examinations are intended to cover the subject of non-cancellable accident and health insurance. I doubt whether a man with a mastery of only the required mathematics will have a sufficient fundamental understanding of the calculus and of analytical reasoning to fully appreciate Mr. Hezlett's paper on "Premiums and Reserves for Temporary and Total Disability Benefits Incorporated in Life Contracts" in Volume XXIV of the *Transactions* of the Actuarial Society. Of course this paper is written from the standpoint of life insurance but the same principles are applicable to non-cancellable accident and health insurance, and I believe that the companies should study this problem from every conceivable standpoint.

It will thus be seen that the Syllabus requirements of *Mathematics of Life Insurance* by L. W. Dowling, *Life Assurance Primer* by Henry Moir, and *Mathematical Theory of Life Insurance* by C. H. Forsyth are minimum. Most of the books men-

tioned later under Interest and Annuities include at least an introductory chapter on this subject. Older books replaced by the modern ones just mentioned are *Actuarial Science* by Ninian Glen and *Notes on Life Insurance* by Gustavus W. Smith.

A more thorough knowledge of the subject would involve reading principally the Institute of Actuaries Text Book on *Life Contingencies*, by E. F. Spurgeon. Chapter XXI dealing with multiple decrement tables would be specially valuable. This book replaces the *Text Book II* by George King which was the standard for about thirty-five years. *Actuarial Theory* by Robertson and Ross supplements this, *Practical Lessons in Actuarial Science* by Miles M. Dawson and *Graduated Exercises and Examples* by Thomas G. Ackland and George F. Hardy are older books of intermediate grade. More searching books are *Mortality Laws and Statistics* by Robert Henderson, *Sources and Characteristics of the Principal Mortality Tables*, *Construction of Mortality Tables from the Records of Insured Lives*, and *Total and Permanent Disability Benefits in Relation to Life Insurance*, the latter three being among the actuarial studies published by the Actuarial Society of America. The casualty actuary uses mortality tables comparatively little, except when combined with remarriage rates or occasionally with disability rates, so that parts of these books may well be passed over for such future reference as he may require. If he wishes to be thoroughly familiar with the sources and characteristics of present mortality tables and to be in a position to employ the principles in tabulations of his own, more extensive and intensive study of these books would be very valuable indeed. *Construction of Mortality and Sickness Tables* by W. P. Elderton and Richard C. Fippard is the most elementary book dealing particularly with its subject. Numerous papers appear in the *Journal of the Institute of Actuaries*, The *Transactions of the Faculty of Actuaries*, the *Record of the American Institute of Actuaries*, and the *Transactions of the Actuarial Society of America*. The papers on disability benefits in the latter are particularly valuable.

The whole subject of life contingencies, as indeed the whole subject of insurance, is dependent upon the THEORY OF PROBABILITY. In the language of college catalogs it is a "prerequisite" of life contingencies. The references given in the *Recommendations* are all very elementary, and deal entirely with *a posteriori*

probabilities. The chapter in Hall & Knight's *Higher Algebra* requires most thought and previous knowledge. The other references are included largely because it is felt that solution of many simple problems is almost as valuable, especially in such a subject as this, as working a few difficult problems. Not only the original algebraic treatment should be considered but the relationship of probabilities to statistics, and especially to the normal probability curve and its characteristics should be included. Here again chapters recommended, even including that in Hall & Knight, give only an elementary concept of the subject. Books such as *Choice and Chance*, by William A. Whitworth, based on the algebraic treatment, *Mathematical Theory of Probabilities* by Arne Fisher, introducing the student to *a priori* probabilities, and to the modern Scandinavian methods of approach, and *A Treatise on Probability* by John M. Keynes, devoted largely to the logical bases, may be consulted to obtain a more thorough knowledge. The most readable of the more comprehensive books is *An Introduction to Mathematical Probability* by Julian L. Coolidge. Little effort has as yet been made in texts to apply the theory of probabilities to casualty lines of insurance, but Professor Whitney indicated some of the possibilities in his papers on "Theory of Schedule Rating, Particularly with Reference to Fire Insurance" and "An Inquiry into the Nature of the Fundamental Principles of the Contract of Indemnity" in the *Transactions*. Albert H. Mowbray in early papers in the *Proceedings*: "How Extensive a Pay Roll Exposure is Necessary to Give a Dependable Pure Premium"—"A New Criterion of Adequacy of Exposure"—and "Notes on Poisson's Exponential and Charlier's Curves" contributes partial solutions of the most perplexing problem of insurance of any line: how extensive an exposure is necessary in order that premium rates calculated upon it may be found proper. It is my belief that extension of these methods or further applications of probabilities and mathematical theory of statistics will be very fruitful in the future.

The DIFFERENTIAL AND INTEGRAL CALCULUS in the Examinations include only the elementary applications of differentiation and integration, with some knowledge of maxima and minima. For several years the questions have been set so that not only differentiation and integration of trigonometric functions

has not been required, but not even has the use of trigonometry in differentiation and integration of algebraic functions been involved. In other words, only a fractional part of an ordinary college course is presupposed. The student may well use the text with which he is already familiar. The principal application of calculus is in Makeham's law of mortality in life contingencies which is the fundamental principle made use of in most tables of joint life annuities. The integral calculus is also extensively used in obtaining expressions for complicated benefits in life contingencies. Fundamental formulas expressing the value of life annuities payable momently are first derived in the form of integrals and then approximations obtained for the actual evaluation. Of course the integral calculus is also involved in studying the normal probability curve. It might be mentioned that most French and German texts on actuarial science are developed largely from the view point of the calculus.

The CALCULUS OF FINITE DIFFERENCES parallels the differential and integral calculus. It embraces two practical processes; interpolation and summation, of which the former is most useful, as it can be used to obtain more accurately than by any other method intermediate values of functions such as the reserves at individual ages in Mr. Cammack's table of "Reserves for Noncancelable Accident and Health Insurance," *Proceedings*, Volume VII, page 300. It can frequently be employed to save considerable work in calculation: as for instance, in obtaining the values of complex annuities, it may be accurate enough and avoid considerable work with complicated formulas to obtain the values for quinquennial or decennial ages and interpolate intermediate values. The principles of interpolation are fundamental in the more complicated processes of graduation, by which mortality, disability, and remarriage rates, or any important numerical experience data may be smoothed out to eliminate the effects of accidental fluctuations due to inadequacy of exposure and thus disclose a more meaningful empirical or even physical law of progression, so that the data will serve more conveniently and more dependably in the calculation of premiums, reserves and other values. Reading of *Lectures on the Theory of Construction of Tables of Mortality* by G. F. Hardy—*Frequency Curves and Correlation* by Elderton—*Graduation of Mortality and Other Tables* by Robert Henderson—*Interpolation*

by J. F. Steffensen about to be published, and numerous papers in the *Journal of the Institute of Actuaries*, the *Transactions of the Actuarial Society of America*, and the *Record of the American Institute of Actuaries* presupposes more mathematics than these examinations now require.

Again the casualty student is unfortunate in that no book or article on finite differences has been written from his viewpoint. *A Treatise on the Calculus of Finite Differences* by George Boole is the classic and has just been reprinted, but is more suitable for advanced study. *Elements of Finite Differences* by J. Burn and E. H. Brown, and the chapters in the second edition of *Text Book II* of the Institute of Actuaries are perhaps the best to study, but these are out of print and may not be available to everybody. Up to the present the more complicated sections, particularly of the latter, have not been required in the actual examinations. The chapters in the *Introduction to the Mathematical Analysis of Statistics* by C. H. Forsyth are probably inadequate, so that *Calculus and Probability for Actuarial Students* by Alfred Henry must almost necessarily be resorted to by many students, though the examinations so far have not presupposed mastery of everything included therein. The best advice to the ordinary student, to enable him to pass the examinations, appears to be to study carefully the type of questions previously asked and regulate his study of Henry accordingly, trying to include somewhat more of the easier operations than has been included in the past, because of the possibility of the standard being raised slightly.

The possibilities for the casualty executive's use of DESCRIPTIVE AND ANALYTICAL STATISTICS have been well suggested by Edwin W. Kopf in "Statistics in the Service of Insurance Administration". *Proceedings*, Volume XI, Page 102*. It perhaps will be sufficient to note here that while considerable value may be derived from intelligent use of statistics without conscious application of mathematical principles, dependable utilization of statistical measures commonly involves considerable mathematics entirely

*Since the above was written several interesting contributions to the subject of "Requirements for Statisticians and Their Training" have appeared in the December 1926 number of the *Journal of the American Statistical Association*. The one by H. L. Rietz and A. R. Crathorne on "Mathematical Background for the Study of Statistics" is most pertinent to this paper.

within the range of that complex known as the calculus of observations, and yet in many cases beyond the present requirements of our examinations. *Principles and Methods of Statistics* by R. E. Chaddock, *Economic Statistics* by W. L. Crum and A. C. Patton, *Introduction to the Theory of Statistics* by G. Udny Yule are included in the *Recommendations*. Among other elementary books on statistics that may be consulted to clear up doubtful points and throw additional light on other points are: *A First Course in Statistics* by D. C. Jones: *Elements of Statistics* by Arthur L. Bowley. More comprehensive books include *Statistical Method* by Truman L. Kelley, *Handbook of Mathematical Statistics* by H. L. Rietz, Editor in Chief; *The Combination of Observations* by David Brunt, and *The Calculus of Observations* by Whittaker and Robinson. *Introduction to Mathematical Statistics* by Carl West, and *Introduction to Mathematical Statistics* by James W. Glover and Harry C. Carver contribute little additional material. It may be anticipated that *Mathematical Statistics*, by H. L. Rietz, to be published soon, will be very helpful.

The subject of COMPOUND INTEREST AND ANNUITIES CERTAIN is similar to Descriptive and Analytical Statistics, and Elements of Accounting in the fact that it is advantageous in many lines of business, and therefore probably will be useful even to students who fail to make good in actuarial work and turn to other lines of work. It is also similar to the Statistics in that there were almost no satisfactory books covering the ground ten or at least fifteen years ago, while now there are a large number of elementary books, and courses are now given along this line in many colleges. The actuary must not only be able to compute the value of bonds or the rate of yield of bonds by the mere use of bond tables, but he must understand the principles involved enough to perform similar calculations on bonds with peculiar terms or where for other reasons the bond tables are not sufficient. In some companies he may find occasion to draw up amortization schedules of bonds. He must be able to understand the computation of rates of interest earned on investments more fundamentally than he could without such a course. He must realize the significance of the interest factor if premiums are paid more often than annually or if discount is allowed for in a single premium to cover several years' insurance. Of course this subject is also fundamental to any problem involving life contingencies

and any problem involving computation of reserve to cover benefits continuing a long time.

This subject is one that many students have to learn themselves, and here again it seems best to read several of the easy texts mentioned and work a great number of easy problems. *Mathematical Theory of Investment* by E. B. Skinner, *Mathematics of Finance* by Rietz, Crathorne and Rietz, *Mathematics of Investment* by W. L. Hart and *Interest and Bond Values* by M. A. Mackensie are recommended. Other similar texts are: *Mathematical Principles of Finance*, by Frederick C. Kent, *Mathematics of Finance* by L. L. Smail, and the first part of *Actuarial Theory* by Robertson and Ross. *The Mathematics of Finance*, by H. W. Kuhn and C. C. Morris has just been advertised as relating the whole subject to a small number of fundamental formulas and their simple transformations. The standard work is the Institute of Actuaries' Text Book—Volume I, but this is written from the English viewpoint and also from the standpoint of logic without much consideration to developing the student's concepts gradually. It is a book greatly to be admired by a person already familiar with the subject. The earlier English work on "The Theory of Finance" by George King is much more readable and still valuable where available. Books not especially suitable are *Finance and Life Insurance* by William A. Dudley, *Mathematics of Accounting and Finance* by Seymour Walton and H. A. Finney, *Mathematical Theory of Finance* by T. M. Putnam, and *Mathematics for the Accountant* by Eugene R. Vinal. Most of the elementary books on actuarial science include introductions to interest and annuities.

The ADVANCED ALGEBRA needed for the examinations corresponds in general to an intermediate high school course, but also includes selected chapters from high school advanced algebra. Special emphasis is placed on geometrical progressions, because these are needed to understand the interest and annuities. The subject of logarithms is very important, as they are necessary in some problems in interest and annuities, and statistics; are needed in the calculus and are useful elsewhere. Convergency and divergency of series is included to assure some conception of the limitations under which algebraic processes are valid. Summation of series is included to form an introduction to the calculus of finite differences and to suggest some of the other methods

of summation not treated therein. Treatment of the binomial theorem is fundamental to all subsequent mathematics. In particular, consideration of the coefficients immensely simplifies various problems in probabilities. Permutations and combinations are one of the most important features of the algebra, because they constitute the foundation on which probabilities is largely built. For this subject the student can use any high school or college algebra with which he is already familiar, but the inclusion of Hall & Knight's *Higher Algebra* in the *Recommendations* indicates that a more thorough treatment of the special topics recommended is implied than would be obtained from most American texts.

To summarize, I might suggest that the mathematics needed for the Society's examinations cover approximately as much ground as a good high school course. This statement needs some explanation, because a high school graduate although he has had all the mathematics there, could not pass the examinations without considerable extra studying, and also because calculus and finite differences are not as a rule taught in high schools. As a matter of fact most of the matter required under calculus is the mechanical operations of differentiation and integration of algebraic and exponential functions. Little of the fundamental theory is required and none of the applications found in ordinary text books except the maxima and minima. Furthermore I understand that corresponding high school courses in Europe frequently introduce such concepts in mathematics corresponding to our high school grade. There is also a tendency in this country to introduce courses giving an elementary conception of such branches of mathematics in one course instead of devoting all of an ordinary student's time to a more comprehensive study of one or two subjects. Finite differences is also not given in high school, but if only the subject matter required were presented in one article I doubt whether it would occupy more than twenty-five pages. Again these extra subjects may offset the plane geometry, solid geometry, trigonometry, and a large share of the advanced algebra taught in high school because most of the time in the usual such course in the latter, after a review of previous algebra studied, is devoted to the theory of equations. This is eliminated entirely from our *Recommendations*.

I wish I could include helpful suggestions on how actually to

study mathematics, but I am afraid that learning how to study is an accomplishment that must be attained by the individual. One of the principal points is learning to recognize the most important features, and devote correspondingly more effort to these. As far as the examinations of the Society are concerned, I have already tried to indicate which subjects and sections of subjects are so elementary that everything included is important, and which texts are of the more advanced nature so that selection should be exercised. The principal criterion here, from the standpoint of the immediate present, is which subjects appear to be most closely connected with the examination questions previously asked.

Of course merely reading the text, even understandingly, in mathematics has comparatively little value. As soon as the student has read it sufficiently to understand it he should put aside the text and try to reproduce the reasoning, preferably not in the exact words of the text but so as to bring out clearly the significance of each step. Furthermore only a partial understanding of the subject is obtained unless many if not all of the problems have been worked. Of course it is not necessary to work every problem where the subject is being reviewed and the student really has a good knowledge of it, but practically every problem should be worked on, the subject matter that is new or not thoroughly understood.

It may assist the student to secure a better perspective to read a small amount of material ahead without thoroughly understanding every detail, but only a limited amount of the subject matter should be covered in this way before time is taken to thoroughly understand what has been read. Whether or not the subconscious mind works on problems that have seriously engaged the attention of the conscious mind, it does seem to me that it is worth while to review a subject several times. Matter studied one night assumes additional meaning the second night. The subject reviewed a few weeks or months after the first study attains a much richer meaning. It therefore seems especially advisable to commence preparation for the examinations early enough so that perhaps a month will be left for merely a review.

The object of this paper is partly to help students realize the reasons why elementary mathematics is required of candidates seeking admission to the Casualty Actuarial Society and to sug-

gest the possibilities in their study for the examinations, and partly to indicate how they may conduct their own post-graduate studying. It should be emphasized that the essential point in the examination, as well as later, is what knowledge and power of reasoning a student *has* and not what he may have *had* sometime in the past that counts. In general the applicants for admission to the Society that have the best attributes of success are those who tackle the subjects recommended with the idea of mastering the sections required instead of only brushing up merely enough to pass the examinations. A man that masters whatever he undertakes is developing himself so as to be entrusted with more difficult problems in his business. A man who does merely enough work to get by stands much less chance of success; in fact, I believe that the examinations should be considered as one of the man's principal opportunities of proving how much brain power and ability to apply it he has acquired. Most college graduates need only to review some of the subjects and find it easy to study the new subjects required. A high school graduate will probably have to work considerably harder to master the subjects, but the examinations furnish the opportunity of testing his ability and showing that he is able to overcome the handicap with which he starts. Our recommendations are designed to guide him to an equivalent of the parts of a college course most vital to this profession.

Of course the examinations are not entirely an end in themselves. Some men will pass who will not be able to develop the judgment and breadth of view necessary for real success. My conception of the examinations is that they are a means of selecting groups of promising young men who have given evidence of more than usual ability and which groups will make eventually a large proportion, though not all, of future actuaries. They may be likened to the qualifying rounds of a golf tournament. Among the qualifiers may be a few who play above their standard and will not be reckoned eventually among the successful. A few others that fail to qualify may do so in the succeeding year. How great the eventual success of those passing will be, depends mostly on how well they apply their ability to study all problems they meet in their business and all allied problems. The examinations are merely one stage in their development intended to indicate fitness for further study. Part of the emphasis of this

paper is upon the fact that the matter required of candidates is only the elements in each subject. Future development depends upon acquiring a further breadth of view and ability to look beyond the immediate present, which with actual experience should develop the judgment that is the ultimate goal of study and one of the attributes of success.