Introduction to Exposure Rating

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Section 1: Introduction

- Exposure rating is the process of using industry-based loss curves, as applied to a cedent's exposures written, to determine the portion of total losses that corresponds to the layer(s) being priced.

- The portion of the losses that are applicable to the layer is multiplied by the anticipated loss to premium ratio, and the result is a loss cost as a percentage of cedent premium.

Section 1: Introduction

- This is done separately for all classes of business, and the techniques vary slightly by line (property, casualty, and workers comp) because of:
    - The nature of the lines
    - How the rating bureaus deliver industry aggregate data on losses by layer
    - How Policy limits (if any) are tabulated

Section 1: Introduction

- Yet the mathematical theory underlying the different calculations is consistent across lines. The industry based curves that are most consistent with the risks the cedent writes are used to allocate losses to a layer.

Section 1: Introduction

- Exposure Rating:
    - Estimates a loss cost based on the premiums, risks, and limits expected to be exposed to a treaty during a prospective treaty period
    - Reflects the current risk profile, which may differ from the profiles in past years

Section 1: Introduction

- Exposure Rating:
    - Can be done most of the time, and usually is
    - Analogous to a primary manual rate, before adjustments for cedent experience
    - Is credibility weighted with the experience rating to estimate expected losses to a treaty.

Section 1: Introduction

- Exposure Rating, and when we do it:
  - The Reinsurance Pricing Paradox
  - New Book of Business with inadequate history
  - Experience Rating has less than full credibility
  - Changes in the business during the history period makes experience rating less relevant

Section 1: Introduction

- Exposure Rating, and when we do it:
    - Difficult to get accurate understanding of the rate changes during the history period
    - Excess Loss Development is unstable

Section 1: Introduction

- We generally denote excess layers as Limit xs Attachment Point.

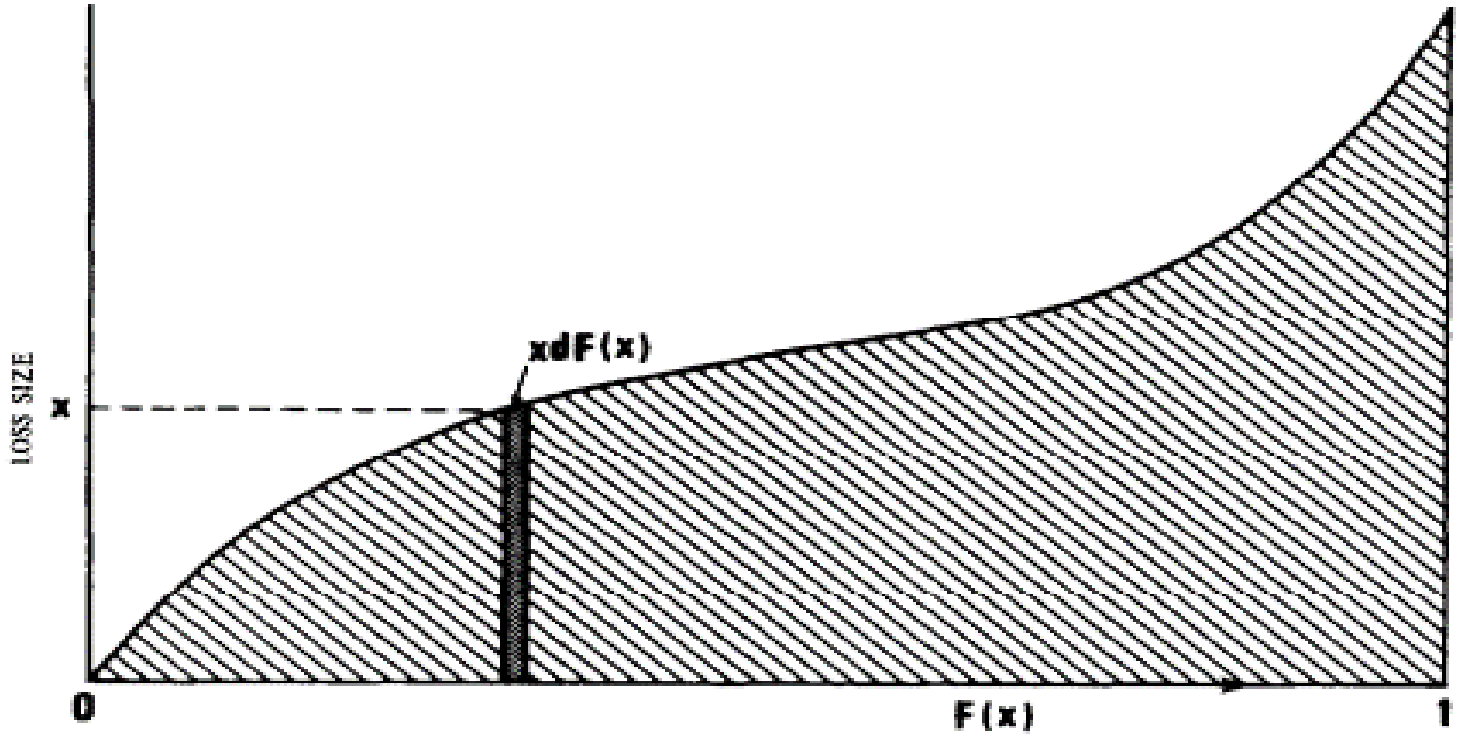
- Know that 200,000 xs 300,000 represents losses greater than 300,000 but capped at 500,000 from the ground.

Section 1: Lee Diagrams

- The following graphs are from the Yoong-Sin Lee paper, *"The Mathematics of Excess of Loss Coverages and Retrospective Rating - A Graphical Approach"*

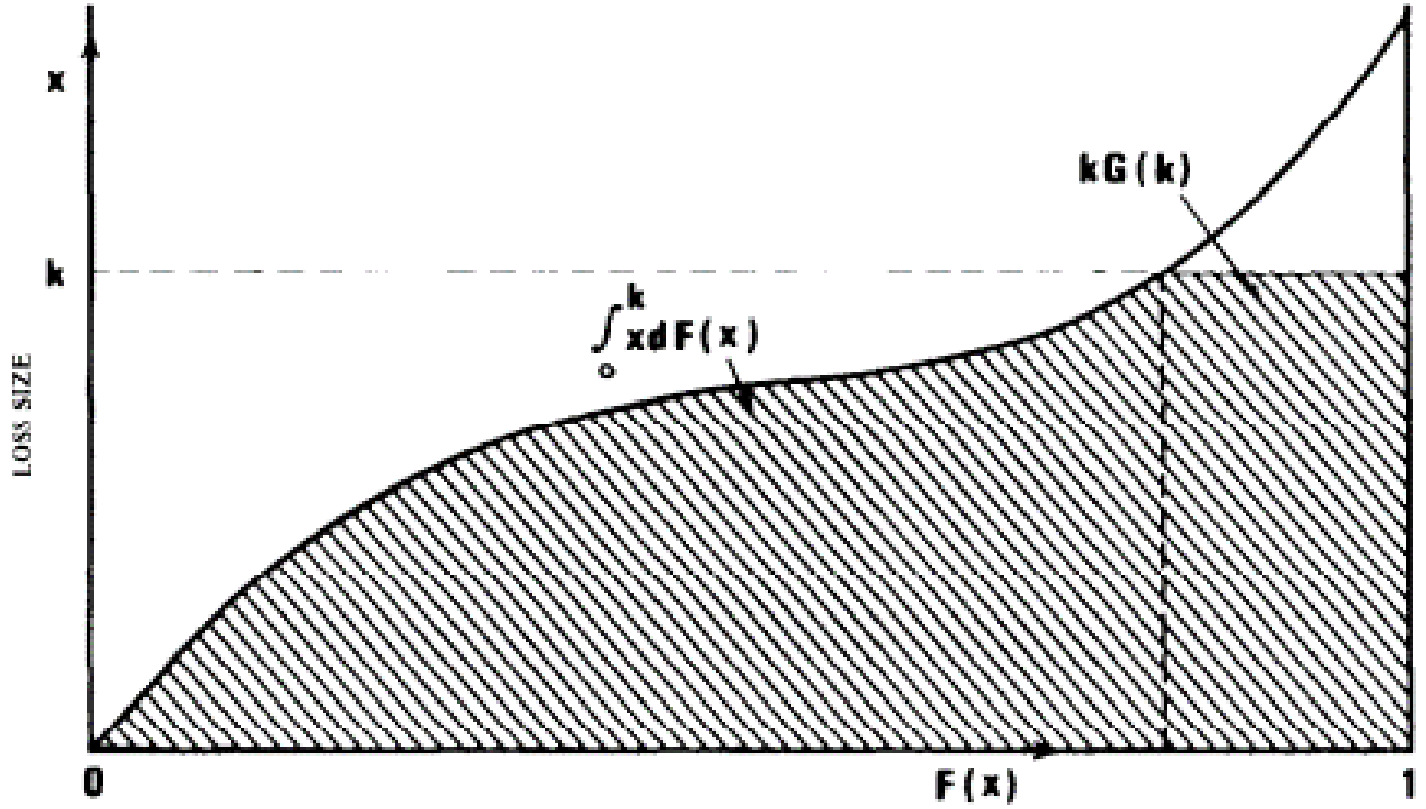
Section 1: Lee Diagrams

FIGURE 2
CDF CURVE AND EXPECTATION



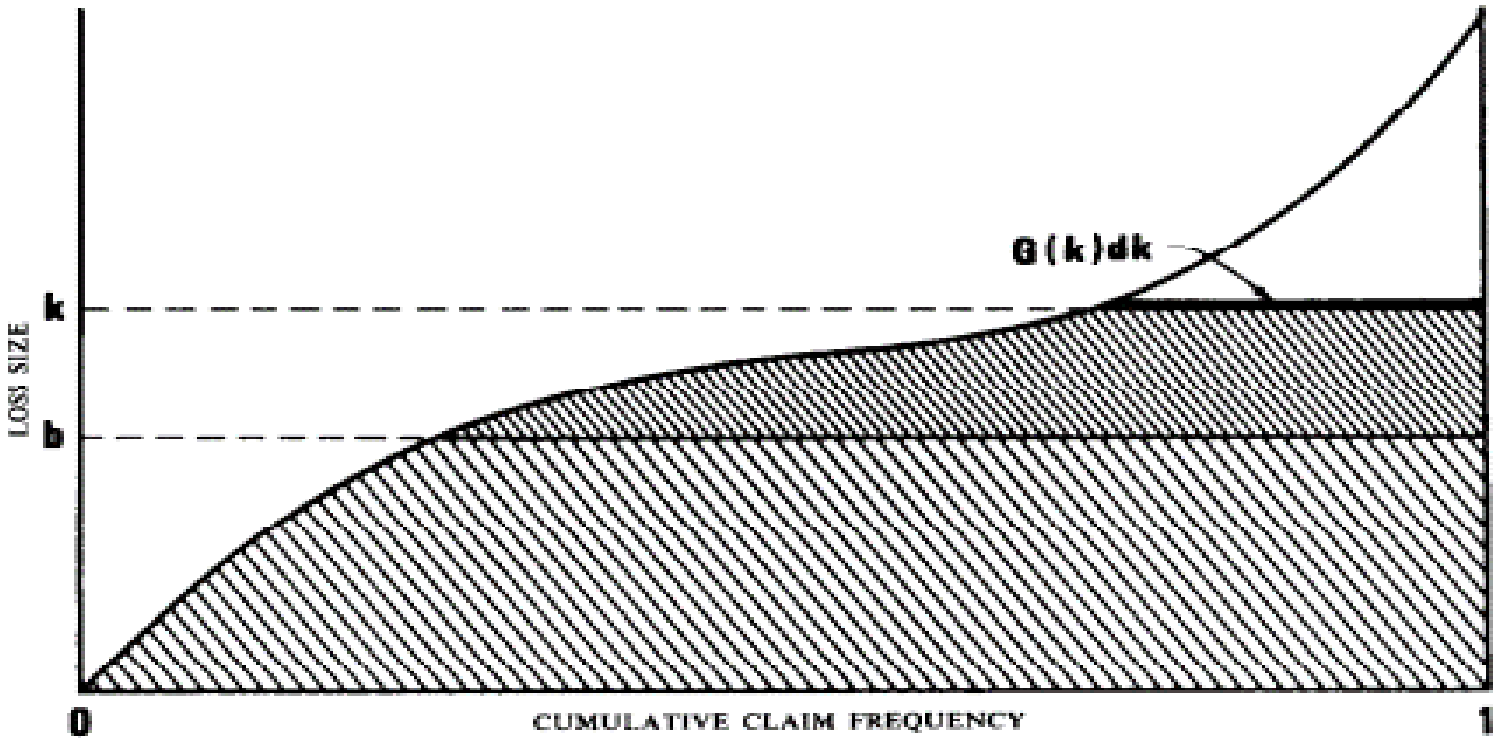
Section 1: Lee Diagrams

FIGURE 5
LOSSES WITH INDEMNITY LIMITED TO k



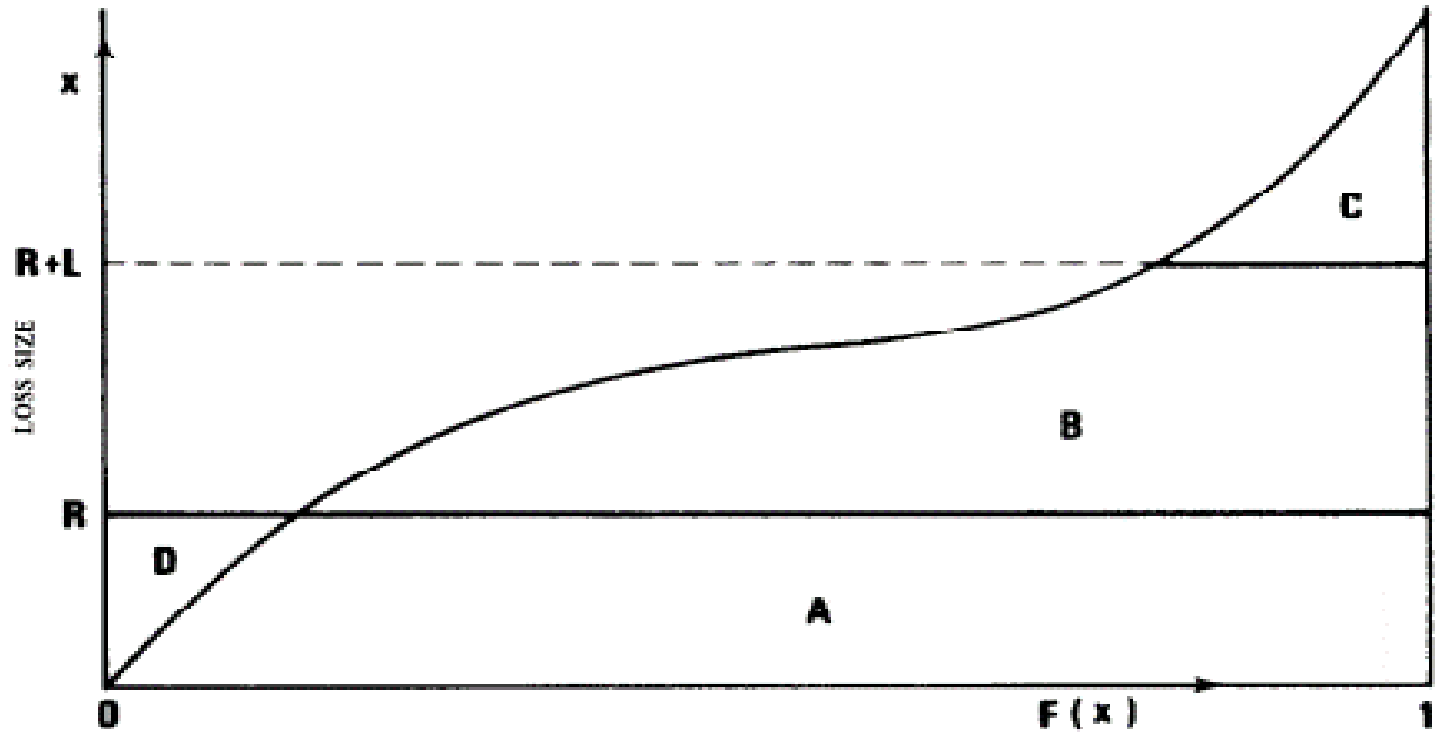
Section 1: Lee Diagrams

FIGURE 6
INCREASED LIMITS FACTOR



Section 1: Lee Diagrams

FIGURE 9
EXCESS OF LOSS COVERAGE



Section 2: Property

The industry size of loss distributions are given in a table of exposure factors.

The exposure factor “represents the amount of loss capped at a given percent of insured value relative to the total value of loss”

Section 2: Property

- The portion of the expected loss on the risk which falls into the treaty layer is given by:

- EF ((Retention + Limit) / Insured Value) – EF(Retention / Insured Value)

Section 2: Property

Exposure Factor Table

Percent of Insured Value	Exposure Factor
0%	0%
10%	37%
20%	49%
30%	57%
40%	64%
50%	70%
60%	76%
70%	81%
80%	85%
90%	89%
100%	93%
110%	97%
120%	100%

What percent of loss dollars are expected for losses on a $200,000 building if the losses are capped at 60K?

60/200 = 30%

EF(at 30%) = 57%

So 57% of loss dollars are for all losses less than or capped at 60k.

Section 2: Property

Exposure Factor Table

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What percent of loss dollars are expected in excess of 250k on a building worth 500k?

Section 2: Property

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70%	81%
80%	85%
90%	89%
100%	93%
110%	97%
120%	100%

What percent of loss dollars are expected in excess of 250k on a building worth 500k?

250/500 = 50%

EF (at 50%) = 70%

100%-70% = 30% of loss dollars are in excess of 250k.

Section 2: Property

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120%	100%

What percent of loss dollars are expected in the excess layer 700 xs 300 on a building worth 1M?

Section 2: Property

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70%	81%
80%	85%
90%	89%
100%	93%
110%	97%
120%	100%

What percent of loss dollars are expected in the excess layer 700 xs 300 on a building worth 1M?

1000/1000 = 100%

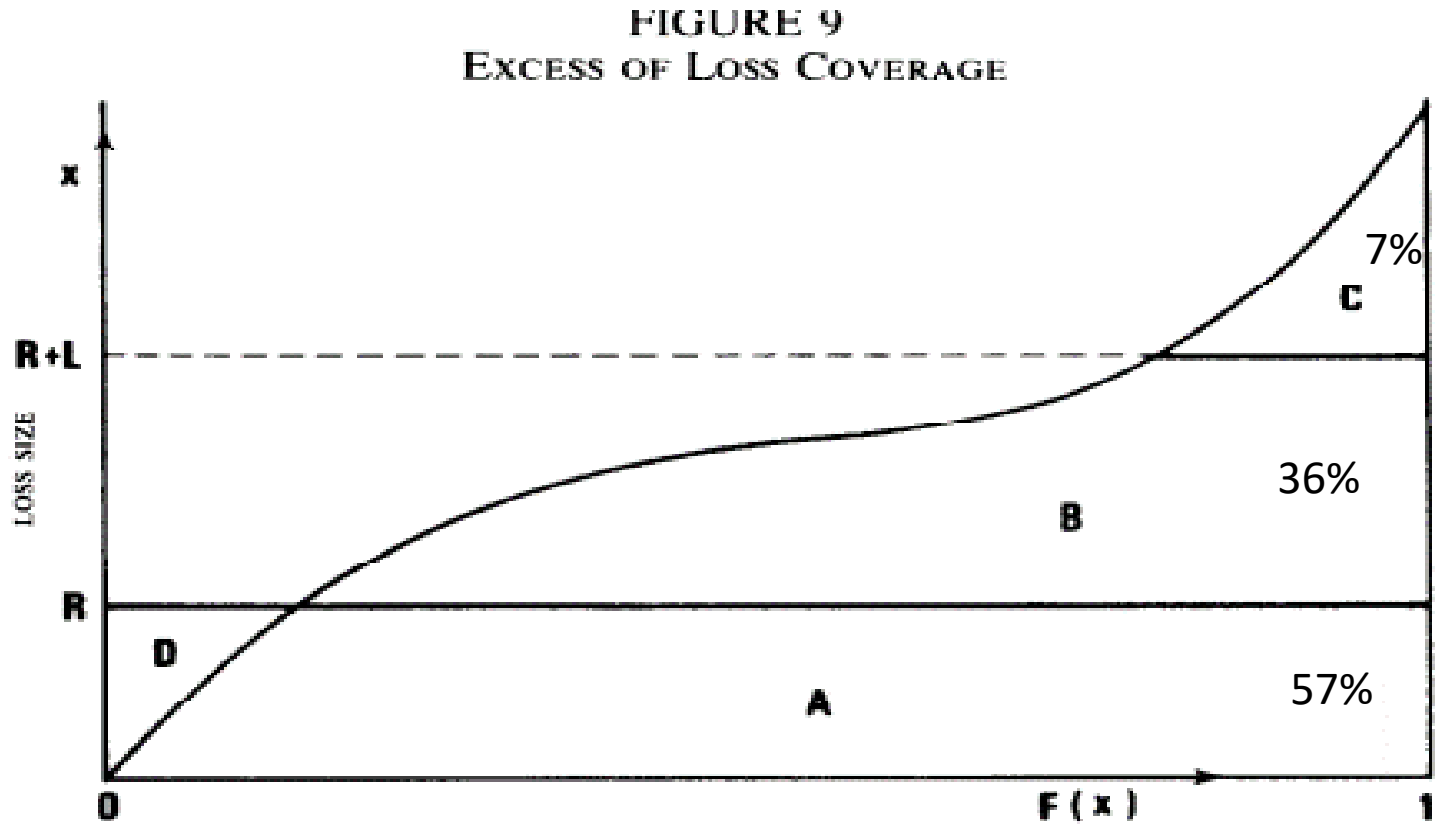
EF (at 100%) = 93%

300/1000 = 30%

EF (at 30%) = 57%

93% - 57% = 36% of loss dollars

Section 1: Lee Diagrams



Section 2: Property

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90%	89%
100%	93%
110%	97%
120%	100%

What is the 600k xs 400k layer price given the following data:

Expected Loss Ratio = 60%

Range of Insured Values (in 1000's)		Mid Pt	Premium
100 to	300	200	2,800
300 to	700	500	1,500
700 to	1300	1000	500
1300 to	2700	2000	200
			5,000

Section 2: Property

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120%	100%

What is the 600k xs 400k layer price
given the following data:
Expected Loss Ratio = 60%

Range of Insured Values (in 1000's)		Mid Pt	Premium	Ret + Limit as % IV	Retention as % IV
100 to	300	200	2,800	500%	200%
300 to	700	500	1,500	200%	80%
700 to	1300	1000	500	100%	40%
1300 to	2700	2000	200	50%	20%
			5,000		

Section 2: Property

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100%	93%
110%	97%
120%	100%

EF(500%)	100% -	EF(200%)	100% =	0%
EF(200%)	100% -	EF(80%)	85% =	15%
EF(100%)	93% -	EF(40%)	64% =	27%
EF(50%)	70% -	EF(20%)	49% =	21%

0%	*	2800	* 60%	= 0
15%	*	1500	* 60%	= 135
27%	*	500	* 60%	= 81
21%	*	200	* 60%	= 25
				241 k

Section 2: Property

Exposure Factor Table

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110%	97%
120%	100%

241 / 5000 = 4.82%

Section 3 Casualty

Percent of Losses Exposed

= (E[x; min(PL,AP + Lim)] –E[x;min(PL,AP)]) / E[x; PL]

- Where :
- PL = primary policy limit
- AP = Attachment point of the treaty
- Lim = Limit of the treaty

- This seems more complicated than it really is. In practice, this is pretty straightforward. The percent of losses exposed is directed by the overlap of the treaty layer and the primary policy limit as a portion of the primary policy expected losses.

Section 3: Casualty

Philo Insurance Company writes a $10M commercial auto book at a loss ratio of 65% all at $1M policy limits. What's the loss cost for 250k xs 250k? For a 750k xs 250k layer?

ILF Table Policy Limit	Reinsurer's ILF Table	Limited Avg Severity
100,000	1.000	6,255
250,000	1.200	7,506
500,000	1.325	8,288
750,000	1.400	8,757
1,000,000	1.450	9,070

Section 3: Casualty

$$\frac{\text{ILF @500k - ILF @250}}{\text{ILF @1M}}$$

$$\frac{1.325 - 1.200}{1.450} = 8.62\%$$ percent of losses exposed to layer

ILF Table Policy Limit	Reinsurer's ILF Table	Limited Avg Severity
100,000	1.000	6,255
250,000	1.200	7,506
500,000	1.325	8,288
750,000	1.400	8,757
1,000,000	1.450	9,070

Section 3: Casualty

8.62%	*	10M	*	65%	
=		560,300	or	5.60%	
				of Subj Prem	

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250,000	1.200	7,506
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Section 3: Casualty

Solution: for 750 xs 250

$$\frac{\text{LAS @1M - LAS @250}}{\text{LAS @1M}}$$

$$\frac{9070 - 7506}{9070} = 17.24\%$$

Percent of Losses Exposed

ILF Table Policy Limit	Reinsurer's ILF Table	Limited Avg Severity
100,000	1.000	6,255
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500,000	1.325	8,288
750,000	1.400	8,757
1,000,000	1.450	9,070

Section 3: Casualty

17.24%	*	10M	*	65%
=		1,120,600	or	11.21% of Subj Prem

ILF Table Policy Limit	Reinsurer's ILF Table	Limited Avg Severity
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250,000	1.200	7,506
500,000	1.325	8,288
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Section 3: Casualty

Philo Insurance Company has a GL book of $10M at 60% ELR with a distribution of policy limits at 250k, 500k, and 1M.
What is the loss cost for a for a 750k xs 250k layer?

ILF Table Policy Limit	Reinsurer's ILF Table	Limited Avg Severity
100,000	1.000	11,730
250,000	1.300	15,249
500,000	1.550	18,182
750,000	1.750	20,528
1,000,000	1.900	22,287

Limit Profile	Policy Counts	Policy Count %	Premium	Premium %
250,000	1250	12.5%	935,252	9.4%
500,000	2500	25.0%	2,230,216	22.3%
1,000,000	6250	62.5%	6,834,533	68.3%
total	10000	100.0%	10,000,000	100.0%

Section 3: Casualty

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total	10000	100.0%	10,000,000	100.0%

Solution:			Percent of Losses Exposed
for 250k policies:	$\frac{0}{1.3}$	=	0%
for 500k policies:	$\frac{1.55 - 1.3}{1.550}$	=	16.13%
for 1M policies:	$\frac{1.90 - 1.3}{1.900}$	=	31.58%

Section 3: Casualty

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total	10000	100.0%	10,000,000	100.0%

So Percent of losses exposed:

0.00%	of	9.4%
16.13%	of	22.3%
31.58%	of	68.3%
		25.18% of total loss dollars

25.18% * 10,000,000 * 60% = 1,510,804

or 15.1% of subject premium

Section 4: WC

Given this table of Excess Loss Factors for the state of East Kentuckiana, calculate the expected loss cost (in percentage)
for a 1.2M xs 800k Layer of Workers Comp business, when the expected loss ratio is 62.5%.

LIMIT	Percent of Losses XS
0	100.00%
200,000	32.10%
400,000	18.80%
600,000	12.40%
800,000	8.80%
1,000,000	6.60%
1,200,000	5.20%
1,400,000	4.30%
1,600,000	3.60%
1,800,000	3.10%
2,000,000	2.70%
999,999,999	0.00%

Section 4: WC

Exposure Factor = ELF (@ap) - ELF (@ap + limit)
attachment point = 800k
attachment point + limit = 2000k

ELF (800k)	8.80%	
ELF (2000k)	2.70%	
difference of	6.10%	is portion of total losses in layer
6.1% * .625 =	3.81%	loss cost as % of cedent premium

LIMIT	Percent of Losses XS
0	100.00%
200,000	32.10%
400,000	18.80%
600,000	12.40%
800,000	8.80%
1,000,000	6.60%
1,200,000	5.20%
1,400,000	4.30%
1,600,000	3.60%
1,800,000	3.10%
2,000,000	2.70%
999,999,999	0.00%

Section 5: The $1 trick

given:

1) The frequency of losses in excess of a given attachment point is fixed no matter what the limit of the excess layer is.
2) We can exposure rate any limit given a decent interpolation function on our loss curves

Let's refer back to the GL example:
What happens if we exposure rate $1.00 in excess of 500k and we get an indication of 0.0000299%?

Section 5: The $1 trick

10,000,000 * .000000299 = 2.99

That means that we need $2.99 cents to pay for the expected value of losses in excess of 500,000 and limited to $1.

What else has a limit of 1 for a layer?

The expected frequency.

Section 5: The $1 trick

So we can use that information to determine estimates for the frequency and severity of losses to the layer.
1,510,804 in expected losses to the 750 xs 250 layer
We expect 2.99 claims in excess of $250,000
1510804 / 2.99 = 505,286 is the expected severity of losses to the layer for this example.

This can be helpful in fitting loss distrubtions to the losses to the layer for the evaluation of various pricing features such as Reinstatements, Annual Aggregate Deductibles, Profit Commissions, etc.

Section 6: Concluding Thoughts and Review

- You should know that:
  - When to do Exposure Rating: Almost Always if you have the data; It's a reference rate
  - For a given Limit, the price should decrease as the attachment point rises
  - For a given attachment point, the frequency is the same no matter the limit
  - Industry Exposure tables are proportionate to loss propensity

Section 6: Concluding Thoughts and Review

- You should know that:
    - Loss ratio estimate should be a robust estimate of losses on the premium for the exposure period
    - Expected Loss Dollars = Premium * RobustLossRatio * Percent of Losses Exposed to Layer
    - Exposure Profiles by Counts instead of Premium will understate excess exposure

Section 6: Concluding Thoughts and Review

- You should know that:
    - Your exposure indication is only as solid as the data that you have at your disposal
    - You may want to investigate manners to weight out the exposure and experience Rating. Generally speaking, the higher the volume of experience loss dollars expected, the less weight to be applied to the Exposure rating. Remember the Reinsurance Pricing Paradox