A Global Framework for Insurer Solvency Assessment

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for Insurer Solvency Assessment

A Report by the Insurer Solvency Assessment
Working Party
of the
International Actuarial Association

Table of Contents

Preface

Acting in support of the International Association of Insurance Supervisors (IAIS) the Insurance Regulation Committee of the International Actuarial Association (IAA) formed the Insurer Solvency Assessment Working Party (WP) in early 2002 to prepare a report on insurer solvency assessment. This Report represents the culmination of that mandate and is meant to assist in the development of a global framework for insurer solvency assessment and the determination of insurer capital requirements. The IAA considers this Report to represent useful educational material. The Report is not intended to express a unique or absolute point of view with regard to the issues which surround the topic of insurer solvency assessment. The materials contained in the Report will need to be enhanced over time in light of new developments.

In the course of its mandate, the WP made several presentations on the work of the WP before a variety of insurance supervisory and professional actuarial meetings. The WP met with the IAIS Technical Sub-Committee on Solvency and Other Actuarial Issues, the insurance internal market directorate of the European Commission, the Conference of European Insurance Supervisors, as well as numerous professional actuarial associations. Feedback from these presentations has been both positive and constructive.

The WP wishes to extend its thanks to all those individuals and organizations who have provided commentary on this report. Of particular note are the contributions of the Casualty Actuarial Society and the Society of Actuaries who have provided assistance with the editing of this report.

The WP members also extend their sincere gratitude to those who have contributed to this report with their wisdom, insight and practical examples. In particular, we would like to recognize the work of Peter-Paul Hoogbruin, Christoph Hummel, John Manistre, Greg Martin, Ulrich Mueller, Martin Paino, Les Rehbeli, Shawn Stackhouse, Erik von Schilling and Brent Walker for their contributions to this report. The Chair expresses special thanks to Julie Silva for her special talents in assembling and formatting this extensive report and set of appendices. Finally, the WP members appreciate the support of their employers and actuarial associations throughout this project. The Working Party looks forward to wider discussion of the issues discussed in this report.

IAA Insurer Solvency Assessment Working Party

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1. Introduction

1.1   One of the current initiatives of the International Association of Insurance Supervisors (IAIS) is to develop a global framework for insurer capital requirements. Acting in support of the IAIS, the International Actuarial Association (IAA) has formed an Insurer Solvency Assessment Working Party to prepare a paper on the structure for a risk-based solvency assessment system for insurance. The terms of reference of the Working Party (WP) are as follows:

- The WP should describe the principles and methods involved in quantifying the total funds needed to provide a chosen level of confidence to policyholders and shareholders that the insurer's policyholder obligations will be met.

- The paper should be specific and practical enough that its recommended principles and methods could be used as a foundation for a global risk-based solvency capital system for consideration by the IAIS.

- The paper should, starting from a coherent risk framework, identify risk measures that can be explicitly or implicitly used to measure the exposure to loss from risk and also any risk dependencies. The paper should also identify measures that are not effective in this regard.

- In balancing its focus between practical versus sophisticated methodologies, the working party will place greater weight on those methodologies with the greatest likelihood of practical implementation. However, since simple methodologies that can be applied to many insurers in a territory or across territories may prove insufficiently reliable or capital efficient, the WP should consider whether risk models developed internally by insurers can provide a useful and reliable approach.

1.2   The focus of prudential regulation and supervision of financial institutions is usually defined as the protection of the rights of policyholders and depositors. Since this includes oversight of the continuing ability of insurance companies to meet their contractual and other financial obligations to their policyholders, the supervisor has a strong interest in the continuing solvency of both insurers and reinsurers under its jurisdiction. The application of this report is intended for both direct writing insurers as well as reinsurers. Throughout this report, "insurer" will be used to refer to both direct writing insurance companies as well as to reinsurers.

1.3   This report deals with methods the supervisor might use to assess the current financial position as well as to understand the possible future financial positions of insurers. Its primary focus is on capital requirements and practices that strengthen the ability of a company to successfully manage its risk in a way to lessen its need for capital.

1.4   Working within the terms of reference, this report is organized as follows:

- Section 3 – "Capital Requirements" reviews the purpose of capital and important principles for the determination of appropriate levels of risk; describes defensive tactics for solvency protection and their role in the design of a capital requirement

- Section 4 – "Framework for Solvency Assessment" provides an introduction to the WP's suggested approach towards insurer capital requirements

- Section 5 – "Insurer Risks" describes the key insurer risks and the key considerations in measuring them

- Section 6 – "Standardized Approaches" describes the considerations involved in the design of standardized approaches to solvency assessment

1

- Section 7 – "Advanced Approaches" describes the considerations involved in the design of advanced or company-specific approaches to solvency assessment

- Section 8 – "Reinsurance" outlines the unique considerations involved with reinsurer solvency assessment

- Section 9 – "Total Company Requirement" describes the additional considerations involved in developing a combined approach to solvency assessment for an entire company or group of companies

- Appendix A – "Life Insurance Case Study" provides a life insurance numerical example of the most important elements of this report

- Appendix B – "Non-Life (P&C) Insurance Case Study" provides a non-life insurance numerical example of the most important elements of this report

- Appendix C – "Health Insurance Case Study" provides a health insurance numerical example of the most important elements of this report

- Appendix D – "Market Risk" provides an in-depth discussion of this risk as it affects insurers

- Appendix E – "Credit Risk" provides an in-depth discussion of this risk as it affects insurers

- Appendix F – "Lessons from Insurer Failures" provides insights from sample insurer failures

- Appendix G – "Introduction to Insurance Risk" provides a layman's introduction to the risks faced by insurers

- Appendix H – "Analytic Methods" provide proven mathematical methods for estimating loss distributions

- Appendix I – "Copulas" describes the key features of these mathematical techniques for approximating risk dependencies

2. Executive Summary

2.1   This paper has been prepared for the International Association of Insurance Supervisors (IAIS) to explore the elements needed for an international capital standard for insurers and to provide a "best practices" approach available to all supervisors. It deals with methods the supervisor might use to assess the current financial position as well as to understand the range of possible future financial positions of insurers. Its primary focus is on capital requirements for insurers.

2.2   To assist in the development of a global framework for insurer solvency assessment and the determination of insurer capital requirements, the WP proposes a number of guiding principles to be used in their design. In summary, these principles focus on:

- A "three-pillar" approach to supervision (see Section 4.1)
- Principles versus rules-based approach (see Section 4.2)
- Total balance sheet approach (see Section 3.1.7 and 4.3)
- Degree of protection (see Section 3.1.5 and 4.4)
- Appropriate time horizon (see Section 3.1.6 and 4.5)
- Types of risks to be included (see Section 5.1.2, 5.1.3 and 5.2)
- Appropriate risk measures (see Section 4.5 and 5.3)
- Risk dependencies (see Section 6.2.1 and 9.3)
- Risk management (see Section 3.2.2)
- Standardized approaches (see Section 4.6 and 6)
- Advanced or company-specific models (see Section 4.6 and 7)
- Market efficient capital requirements (see Section 3.1.1)

"Three Pillar" Approach

2.3   The WP believes that a multi-pillar supervisory regime is essential for the successful implementation of the global framework proposed in this report. The conclusions of this report are consistent with the "three pillar" approach to the regulation of financial service entities that is reflected in the Basel Accord for the regulation of banks internationally.

2.4   The approach envisaged would have three pillars consisting of:

> **Pillar I:** Minimum financial requirements
> **Pillar II:** Supervisory review process
> **Pillar III:** Measures to foster market discipline.

The definition of these pillars needs to reflect the specific features of insurance.

2.5   Pillar I (minimum financial requirements) involves the maintenance of a) appropriate technical provisions (policy liabilities), b) appropriate assets supporting those obligations and c) a minimum amount of capital (developed from a set of available and required capital elements) for each insurer. Of primary interest to the WP in this report are the capital requirements. To the greatest extent possible given the sophistication of the approach chosen and the insurer's ability to model them, it is the WP's view that these calculations must reflect a comprehensive view of the insurer's own risks.

2.6   Pillar II (supervisory review process) is needed, in addition to the first pillar, since not all types of risk can be adequately assessed through solely quantitative measures. Even for those risks that can be assessed quantitatively, their determination for solvency purposes will require independent review by the supervisor or by a designated qualified party. This is especially true for those determined using internal models. The second pillar is intended to ensure not only that insurers have adequate capital to support all the risks in their business but also to encourage insurers to develop and use better risk management techniques reflective of the insurer's risk profile and in monitoring and managing these risks. Such review will enable supervisory intervention if an insurer's capital does not sufficiently buffer the risks.

2.7   Pillar III serves to strengthen market discipline by introducing disclosure requirements. It is expected that through these requirements, industry "best practices" will be fostered.

2.8   The actuarial profession can assist supervisors within the second pillar by providing independent peer review of the determination of policy liabilities, risk management, capital requirements, current financial position, future financial condition etc., where these entail the use of substantial judgement or discretion. Assistance can also be provided within the third pillar in the design of appropriate disclosure practices to serve the public interest.

2.9   The WP believes that while customization of the individual pillars is needed as they are applied to insurers, the use of a "three-pillar" approach similar to that used by the banks makes sense and is extremely useful given,

- the common features shared by the two financial sectors
- that many insurance supervisors are part of integrated financial supervisory agencies, and are well acquainted with the Basel Accord.

2.10   Some reasons for the differences in approach to be used for insurance would include 1) the nature of insurance risks and the techniques to assess them in Pillar I, 2) the need for multi-period review under Pillar II and 3) the definition of relevant information for purposes of disclosure in Pillar III.

Principles Versus Rules-Based Approach

2.11   Solvency assessment should be based on sound principles. Implementation of solvency assessment will require rules developed from these principles. However, the WP considers that the rules used should include provisions to allow their adaptation to current or unforeseen circumstances with the prior agreement of the relevant supervisor.

Total Balance Sheet Approach

2.12   The application of a common set of capital requirements will likely produce different views of insurer strength for each accounting system used because of the different ways accounting systems can define liability and asset values. In the view of the WP, these definitions may create a hidden surplus or deficit that must be appropriately recognized for the purpose of solvency assessment.

2.13   The WP believes that a proper assessment of an insurer's true financial strength for solvency purposes requires appraisal of its total balance sheet on an integrated basis under a system that depends upon realistic values, consistent treatment of both assets and liabilities and does not generate a hidden surplus or deficit.

Degree of Protection

2.14   It is impossible for capital requirements, by themselves, to totally prevent failures. The establishment of extremely conservative capital requirements, well beyond economic capital levels, would have the impact of discouraging the deployment of insurer capital in the jurisdiction.

2.15   In forming its recommendation for an appropriate degree of protection for insurer solvency assessment purposes, the WP considered the role of rating agencies in assessing insurers and the substantial volume of credit rating and default data available from these agencies. The WP also noted the relation between the degree of protection and the time horizon considered. In addition, the specific manner of applying the capital requirement risk measure may also affect the degree of protection chosen. The WP's recommendation for degree of protection is therefore linked with its recommendation for an appropriate time horizon for solvency assessment as shown in the following paragraphs.

Appropriate Time Horizon

2.16   A reasonable period for the solvency assessment time horizon, for purposes of determining an insurer's current financial position (Pillar I capital requirements), is about one year. This assessment time horizon should not be confused with the need to consider, in such an assessment, the full term of all of the assets and obligations of the insurer.

2.17   The amount of required capital must be sufficient with a high level of confidence, such as 99%, to meet all obligations for the time horizon as well as the present value at the end of the time horizon of the remaining future obligations (e.g., best estimate value with a moderate level of confidence such as 75%).

2.18   Due to the long term and complex nature of some insurer risks, the insurer should consider valuing its risks for their lifetime using a series of consecutive one year tests with a very high level of confidence (say 99%) and reflecting management and policyholder behaviour (but no new business). Alternatively, this test can be conducted with a single equivalent, but lower (say 90% or 95%), level of confidence for the entire assessment time horizon. This lower level of confidence over a longer time horizon is consistent with the application of a series of consecutive higher level one-year measures.

Types of Risk Included

2.19   In principle, the WP recommends that all significant types of risk should be considered (implicitly or explicitly) in solvency assessment. However, there may be valid reasons why certain risks do not lend themselves to quantification and can only be supervised under Pillar II. The WP believes that the types of insurer risk to be addressed within a Pillar I set of capital requirements are underwriting, credit, market and operational risks.

Appropriate Fisk Measures

2.20   A risk measure is a numeric indicator that can be used to determine the solvency capital requirement for an insurance company. The most appropriate risk measures for solvency assessment will exhibit a variety of desirable properties (e.g., consistency). Of course, it is difficult for one risk measure to adequately convey all the information needed for a particular risk. One risk measure that exhibits several desirable properties for various (but not all) risks is Tail Value at Risk (also called TVaR, TailVar, Conditional Tail Expectation, CTE or even Policyholders' Expected Shortfall). In many situations, this risk measure is better suited to

insurance than Value at Risk (VaR), a risk measure commonly used in banking, since it is common in insurance for their risk event distributions to be skewed.

Risk Dependencies

2.21   The solvency assessment method should recognize the impact of risk dependencies, concentration and diversification. This has implications for the desirable properties of the appropriate risk measure.

2.22   Risk dependencies within an insurer can have a very significant impact on the overall net effect of its risks (compared to the gross effect without taking account of their dependencies). Even the most basic fixed-ratio method should implicitly allow for risk dependencies. Currently, required capital formulas in Japan and the U.S. incorporate some recognition of dependencies, concentration and diversification. However, in many countries, diversification between different risk types is not recognized in the formulas for required capital.

2.23   The concept of describing dependencies between risks, and particularly by using a technique based on copulas, is discussed in this report and its Appendices.

2.24   For purposes of solvency, it is imperative to find methods or models to describe dependencies both in the absence of reliable or scarce data as well the "increasing" dependency in extreme events (i.e., in the tails of the probability distributions which describe the risks). The latter is very important to solvency assessment as the events in the tail of the distribution are those which can jeopardize the financial position of an enterprise most.

Risk Management

2.25   The solvency assessment method should recognize appropriately the impact of various risk transfer or risk sharing mechanisms used by the insurer.

2.26   The actuarial control cycle referred to in this report is a continuous review process that is fundamental to any soundly based enterprise risk monitoring process. The control cycle provides information to improve the company's ability to manage its risks and make more effective business decisions. Some of the ways in which an insurer can manage its risks, beyond the fundamentals of prudent claim management, include

- risk reduction
- risk integration
- risk diversification
- risk hedging
- risk transfer
- risk disclosure

2.27   While many of these types of risk management serve to reduce the risk in question, it is important to note that some of them create additional risk related to the technique itself. For example, both hedging and reinsurance create counterparty risk, a form of credit risk.

2.28   Regardless of the risk management process used by the insurer for its risks, including full retention of its risks, effective management of these risks is encouraged by appropriate disclosure of the extent of the risks and their management by the company. Appropriate audiences for such disclosure include the stakeholders of the insurer including the supervisors.

Standardized Approaches

2.29   Many of the discussions comparing different solvency assessment methods (e.g., fixed-ratio versus risk-based capital - RBC - versus scenario-based, etc.) do not adequately explain the optimum conditions that must be present for each method to be reliable. Supervisors considering new methods should be alerted to the conditions needed for the new methods to be a success. The WP believes these concepts are worthy of note and appropriate inclusion in our report.

2.30   Simple risk measures are appropriate when it is recognized that the risk in question is important from a solvency perspective but there does not currently exist a generally accepted view of how the risk should be assessed. They are also appropriate if the risk is of minor importance.

2.31   Sophisticated risk measures are appropriate for material risks where one or more of the following conditions exist:

- The risk in question is very important from a solvency perspective and cannot be adequately assessed through the use of simple risk measures,
- There is sound technical theory for the risk to be assessed and the risk measure to be used,
- Sufficient technical skills and professionalism are present among the staff,
- Relevant and sufficient data is present or the knowledge about the risks is otherwise reliable,
- The risk is actually managed in accordance with the risk measure used,
- Risk management practices are evident to a high degree.

Advanced (Company-Specific) Approaches

2.32   For stronger, more technically able companies with effective risk management programs, it may be appropriate to introduce advanced (or company-specific) models that can incorporate all types of quantifiable risks. An internal model can also incorporate all types of interactions among risks if those interactions are understood and quantifiable. However, in practice, many aspects of risk are not well understood, particularly in the case of extreme events for which little history exists (and which are most important for solvency assessment). Hence, internal models provide a model of risks faced by an insurer that can, at best, be described as representing reality in an approximate way. In building an internal model, care must be given to capture the most important risk variables.

2.33   Required capital can be thought of as a second line of defence protecting an insurance company's solvency and its policyholders. The first line of defence is solid risk management. If trouble develops that cannot be prevented through management of a risk, then capital should be available to cover the financial losses that emerge. It follows that in order for a supervisor to be content with a lower amount of required capital under a company-specific approach, there must be some assurance that the particular source of risk is under control, its effects are well mitigated and there is a reduced need for the required capital. Therefore, in approving a company's use of an advanced or company-specific approach, the supervisor should confirm that the company has in place appropriate risk management processes together with a satisfactory reporting structure.

2.34   A particular strength of internal models is their ability to capture the impact of combinations of risks beyond a simple aggregation of individual risk factors that cannot accurately assess risk interaction effects.

Market Efficient Capital Requirements

2.35   Excessive minimum capital requirements, while affording additional solvency protection, will also serve to impede capital investment in insurers because of the perceived additional cost of capital required in the business, beyond that required by economic levels of capital, that may not be recoverable in product pricing.

3. Capital Requirements

3.1 The Purpose of Capital

3.1 In this report, the WP sets out a consistent framework for capital requirements and risk oversight for insurance companies that could be applied in almost all jurisdictions world-wide to suit the circumstances of each jurisdiction. Under this framework, the capital requirements and risk oversight process in two jurisdictions with similar business, legal, economic and demographic environments and supervisory philosophy and controls should be comparable. The resulting capital requirements may differ materially between jurisdictions that have significantly different environments for their insurance markets and companies. Nonetheless, because these requirements are based on a consistent set of principles, the differences between them should be explainable as a function of the different environments.

3.2 To set a target or requirement for the amount of capital and surplus that should be held by an insurance company requires a clear vision of the purposes for which capital is held. This then clarifies how the requirement should be determined. This section is devoted to reviewing the purpose of capital and the important principles for determining appropriate levels of capital.

3.3 An effectively defined capital requirement serves several purposes:

- provides a rainy day fund, so when bad things happen, there is money to cover them
- motivates a company to avoid undesirable levels of risk (from a policyholder perspective)
- promotes a risk measurement and management culture within a company, to the extent that the capital requirements are a function of actual economic risk
- provides a tool for supervisors to assume control of a failed or failing company
- alerts supervisors to emerging trends in the market
- ensures that the insurance portfolio of a troubled insurer can be transferred to another carrier with high certainty.

3.4 In developing capital requirements for insurers it is desirable to consider the concept not only of "target capital" (TC) but also "minimum capital" (MC). TC refers to the appropriate amount of capital to be held in consideration of the risks assumed by the insurer. MC serves as a final threshold requiring maximum supervisory measures in the event that it is breached. This Report focuses primarily on the issues surrounding the development of TC. Note that in this Report the WP uses the term "free surplus" (FS) to mean the financial statement excess of assets over liabilities and regulatory capital (TC) requirements.

3.1.1 Going Concern or Run-Off

3.5 Economic capital is what the firm judges it requires for ongoing operations and, for an insurance company, what it must hold in order to gain the necessary confidence of the marketplace, its policyholders, its investors and its supervisors. Economic capital can be considered to be the minimum amount of equity or investment to be maintained in the firm by its owners (shareholders) to ensure the ongoing operations of the firm. Since a firm's net income is often measured as a rate of return on investor equity, many firms are motivated to maintain actual capital as close as possible to economic capital in order to maximize return on equity.

3.6 The WP is concerned not with economic capital but with target regulatory capital (i.e., TC), the capital that a firm is required by its supervisor to hold as a condition of being granted a licence or to continue to conduct the business of insurance in a jurisdiction. The focus in discussing regulatory capital is often placed on the sufficiency of capital to support the winding up of a firm's affairs in the event of insolvency. From this point of view, regulatory capital is often thought of as providing for a successful run-off of the firm or a portfolio transfer. However, the firm before insolvency is a dynamic organization that is constantly changing. The capital that

would be needed in the event of insolvency depends on the company's business portfolio immediately preceding the event of insolvency. In this sense, a regulatory capital requirement is based on the ongoing dynamic insurer's business. Therefore, regulatory capital has aspects of both the going concern and run-off situations; it would be an error to characterize target regulatory capital as determined strictly on a going concern basis or strictly on a run-off basis.

3.7 Excessive capital requirements, while affording additional solvency protection, will also impede capital investment in insurers because the additional cost of capital may not be recoverable in product pricing. This either raises the cost of insurance to its buyers or prevents a market from existing.

3.1.2 Who and What is to be Protected

3.8 Providing protection to policyholders in the event of an insurer's failure is a traditional justification for a regulatory capital requirement. In some jurisdictions, protection may be provided for general creditors of the insurance company as well. Creditors' protection is not, however, a feature of many legal systems and will not be treated in depth in this report. Note that no consideration is given to the protection of the financial interests of the owners or shareholders of an insurer. In the case of a mutual insurance company whose owners are its policyholders, protection considerations apply only to these individuals as policyholders and not in respect of their roles as owners.

3.9 The type of protection to be provided to a policyholder in respect of an insurance or annuity contract will depend upon the terms of the contract and the nature of the insurance coverage.

3.10 Consider, for example, a typical short-term general (property and casualty) or group life or health insurance contract. If there were no incurred claims outstanding under the contract, the usual goal in an insurer's failure would be to provide insurance coverage for the remaining term of the policy. It is assumed that the insured would then be able to arrange for a continuation of insurance with another insurance company. This assumption is generally valid because these contracts normally do not contain guarantees with respect to renewability or the level of renewal premiums. If claims have been incurred under a policy by the time the insurer has failed, the goal in a company failure would be to provide sufficient funds to satisfy the outstanding claims.

3.11 Longer-term insurance policies often involve predetermined premiums that are level for extended periods during the lifetime of the contract. Under these contracts, the year-by-year cost of insurance is not the same as the amount provided in the level premium to meet this cost. This leads to the creation of active policy liabilities or reserves that are held by the company to meet future insurance costs. In some jurisdictions, some portion of this liability may be represented concretely by guaranteed cash surrender values.

3.12 Certain insurance contracts, particularly life and health policies, guarantee the continuing coverage or protection of the insured (preservation of insurability). Since an insured's condition may deteriorate over time, that individual might not be able to secure from another insurance company a continuation of insurance coverage in the event of failure of the primary insurer. For these contracts, in the event of insurer failure, supervisors or liquidators often seek to have these policies continued in force for their remaining terms.

3.1.3 Exit Strategy Under Failure

3.13 The method of liquidating a failed insurer is a principal consideration in determining regulatory capital. In many cases, the preferred method will be to have another insurer, or several insurers, assume the failed company's insurance portfolio. In this case, the primary goal in setting a regulatory capital requirement is to ensure there will be sufficient assets on hand in the company's estate so that another insurer will accept these assets as payment to assume the

business. In its work, the WP has assumed this is the course that would be followed in the event of an insurer's failure.

3.14   There may be circumstances under which the policy liabilities are not transferred to or assumed by another insurer. This may be more likely in the event of a failure of a general (property and casualty) insurer than of a life insurer. In this case, the liquidator's focus will be on the payment of incurred claims. The financial resources necessary to accomplish this will depend upon the organization established to run off this business. There can be considerable variation in the administrative costs of handling these claims. In setting capital requirements, a jurisdiction should not only provide for the amount of the claims on a failed insurer but it should also take into account the methods that would be used, and their associated costs, in settling these claims.

3.15   Many jurisdictions have consumer guarantee or compensation funds that protect policyholders in the event of failure of an insurance company. The coverage offered by the guarantee fund will usually have limits on benefits payable on a single contract. In some cases, these guarantee funds may be backed up by an organization that can assume the run-off of a failed insurer; this could have an important effect on the estimated costs of any future liquidation.

3.1.4   The Challenge of Insurer Solvency Assessment

3.16   Insurance contracts present unique challenges for solvency assessment. While insurers share a number of types of risk to which they are subject with other businesses, especially other financial institutions, their core risk is because of the fundamental nature of their business, the marketing and underwriting of risk. The types of risk to which insurers are subject, are detailed later in this report.

3.17   The proper assessment of underwriting risks usually requires the detailed examination of insurance product-specific and relevant industry data for both the frequency and severity of product events. The product events may involve the payment of specified amounts upon an event such as morbidity or death. They may also involve the reimbursement of specific types of costs whose amount will not be known until the insured service is actually provided (e.g., medical costs, property damage claims, etc.).

3.18   The assessment of underwriting risks for solvency purposes is challenging for several reasons:

- There is not a liquid market for many types of insurance contract liabilities.

- Insurable events can be subject to several types of assumptions (e.g., disability income claim payments require the estimation of the frequency and severity of claims as well as the rate of policyholder lapsation, among other assumptions).

- Appropriate assumptions may be dependent on the experience of the insurer underwriting that risk. Such experience may not be available in sufficient detail or volume to fully estimate all aspects of the assumption with credibility without referring to relevant industry data, where this is available. In addition, the risk is dependent on the manner in which the risk was sold. Sometimes, one contract may be sold to many customers via various distribution channels, other times each customer may get a uniquely defined contract.

- Due to the long-term nature of many insurance contracts, the time horizon for projecting the future contract cash flows can extend for several years or even decades into the future thus making the estimation of assumptions challenging.

- For several types of life insurance products, the benefits available to the policyholder are dependent in some manner on the performance of assets purchased by the insurer. Risk assessment must be able to model the manner in which the insurer carries out its asset/liability management responsibilities.

- Frequently, the assessment of underwriting risk requires the modelling of policyholder behaviour (e.g., premium payment lapsation, the exercising of policyholder options).

- The long-term nature of many insurance contracts requires that the uncertainty and extreme event components of underwriting risk be carefully considered.

- Significant risk dependencies within an insurer's risks need to be carefully considered in determining an appropriate solvency structure for insurers.

3.1.5 The Degree of Protection

3.19 The strength of a capital requirement can be thought of in terms of the probability that a company's assets backing liabilities, together with required capital, will be sufficient to satisfy all of its obligations to its policyholders. This probability represents a confidence level. It would be desirable to be able to calculate this probability once the amount of capital was known or to know how much protection is provided by current capital and surplus. Conversely, an approach to determining required capital would be to first choose this confidence level and then determine the amount of capital necessary to achieve it. A difficulty with this approach is that some risks are not quantifiable, either because of their qualitative nature or because sufficient data is not available to properly assess the risk. Nonetheless, this is a promising approach that the WP believes can yield good results. In adopting this approach, it is important for supervisors in each jurisdiction to decide on the confidence level they believe is appropriate for the insurance companies supervised. Two practical considerations involved in the introduction of a new confidence level may be that: 1) if the new requirements are substantially higher than the previous ones; an appropriate transition period may be needed and 2) for some extreme circumstances (e.g., a steep fall in the investment market) a clear and transparent mechanism may be needed for the temporary relaxation of the solvency rules in order to avoid widespread hardship on the entire industry.

3.20 It must be recognized that the confidence level must be less than one (1) or 100%. No finite amount of capital can provide an absolute guarantee that a company's policyholders will be protected in all circumstances. It is important to recognize that in any supervisory regime, no matter how strict, company failures will always be possible. This possibility cannot be eliminated through a high capital requirement.

3.1.6 Time Horizon

3.21 Financial statements, including reports on capital, are usually prepared by insurance companies at the end of each fiscal year or the end of each quarter year. Producing these statements is a considerable task that requires significant preparation time. Often there will be a delay of several months between the statement date and the actual receipt of the statement by the supervisory authority. The company management may also require some time to implement possible corrective actions. The supervisor, having many companies to oversee, may need several additional months to fully analyse a particular company's results. If this analysis shows a company's position to be weak, it will take additional time to formulate action plans and issue appropriate directions to the company. If it were necessary to remove a company's licence and "wind it up," the formalities of governmental and legal systems could introduce considerable delays before the supervisor's objectives are achieved. During the period until final action against a weak or insolvent company is taken, the company would continue to operate and conduct business, including the sale of new insurance and/or annuity contracts.

3.22 In formulating a capital requirement in a particular jurisdiction, a supervisor must take into account the time horizon between the date as of which company financial statements are prepared and the expected date by which a supervisor could take control of the insurer if this was deemed to be necessary. Since this time horizon depends upon local business practices, the supervisor's

resources, legislation and the legal system, this horizon will vary from one jurisdiction to another. However, it would be rare to assume this time horizon could be considerably shorter than one year.

Term of Assets and Obligations

3.23 This assessment time horizon should not be confused with the need to consider, in such an assessment, the full term of all of the assets and obligations of the insurer.

3.24 Regardless of the solvency assessment period time horizon (e.g., insurer's assets must be adequate within a 99% probability that the insurer will still be solvent in one year), the solvency assessment must reflect the full term of the assets and obligations of the insurer. These may extend for many years or decades beyond the end of the assessment period time horizon.

Period of Liquidation

3.25 Since supervisory intervention in a nearly bankrupt company still requires a period of time to run-off, rehabilitate or sell off the company, it is necessary to consider this additional period of time. The solvency assessment time horizon should not be shorter than the expected length of time between the technical point of insolvency to wind-up or restructuring of the distressed insurer.

3.26 This period may be different for an insurer with business that is likely to be simply run-off versus an insurer whose business will be sold or restructured as a going-concern entity.

Interaction with Confidence Level

3.27 If a certain fixed acceptable level of insolvency risk per year is assumed (expressed as a certain allowable annual probability of insolvency), then extending the time horizon should always result in the need for additional capital. Alternatively, a fixed amount of capital always provides a lower confidence level in solvency over a longer period (e.g., higher probability of insolvency over the longer time horizon).

Interaction with Modelling Behaviour

3.28 Extending the time horizon will generally increase the need to make explicit assumptions on future policyholder as well as management behaviour, since a longer time horizon will increase the probability that current behaviour will change. In particular, the longer the time horizon, the more reasonable it seems to allow for:

   a. future transfers of risk (e.g., by changing the reinsurance policy or transferring the portfolio to another party); for instance, because of its size, this other party may not ask for capital to cover the remaining volatility risk;
   b. future changes of the company's (re)investment strategy and/or internal risk management procedures, resulting in lower ALM risks and/or lower operational risks respectively;
   c.   future offsetting risks because of new business that shows "opposite" types of risk.

3.29 In general, using a longer time horizon requires increasing judgement to be applied in the projections (i.e., larger model errors).

Future Financial Condition Reports

3.30   A longer solvency assessment time horizon may be useful where the purpose is to provide insight into the future financial condition of the insurer under a variety of plausible adverse scenarios. Some supervisors require that a multi-period future financial condition report be prepared annually for presentation to the insurer's Board of Directors and a copy provided to the supervisor. Typically these reports are not publicly available because of the confidential nature of the information they contain.

3.1.7   Role of Accounting – The Need for a Total Balance Sheet Requirement

3.31   An insurer's capital is determined from its financial statements as the difference between the value of its assets and liabilities. The strength of that capital value is directly dependent on the relative strength of the methods and assumptions used to determine the asset and liability values. The use of inconsistent methods and assumptions in the determination of asset and liability values (or between components within the assets and liabilities) has the potential to significantly affect the relative strength of the capital positions of otherwise similar insurers. Applying a common set of capital requirements will likely produce different views of insurer strength for each accounting system used because of the different ways that accounting systems can define liability and asset values. These definitions may create a hidden surplus or deficit. In the view of the WP, capital requirements generated under these systems must appropriately recognize these hidden values.

3.32   Ignoring for the time being, the different possible types of capital or surplus (retained earnings), the amount of capital attributed to a particular insurance company will depend heavily on how its policy liabilities (actuarial reserves) are calculated. The methods used to determine these reserves vary considerably among jurisdictions. In certain jurisdictions, conservatism and financial strength are emphasized; one often hears mention of "hidden surplus" contained within these reserves. In others, the emphasis is placed upon the appropriate reporting of earned income and actuarial reserves are considerably less conservative than in the first case. This variability demonstrates that in choosing a capital requirement, or in comparing capital amounts between companies, it is necessary to take into account the methods and assumptions used to determine all the components of the balance sheet including actuarial reserves.

3.33   The WP is aware of the work currently being done by the International Accounting Standards Board (IASB) to bring about a uniform international accounting standard for financial institutions. As part of this project, the IAA is assisting the IASB in determining a standard approach to actuarial principles and methods for the determination of actuarial reserves in accordance with the new standard. Initially, the WP viewed its mandate as the determination of a standard capital requirement based on a standard accounting system. However, since the timing of the completion of the IASB project is uncertain and the date of its adoption by all jurisdictions is not clear at this time, the WP has selected a "total balance sheet" approach (more on this in section 4.3) as a common basis for establishing capital requirements.

3.2   Supplements to Capital

3.34   Capital requirements can be thought of as a defence tactic used to protect policyholders and depositors. However, it is not the only tactic in use by insurance companies and by supervisors. The other defensive tactics that are in place will influence the amount of capital required by an insurance company. In this section, we describe some of these factors and indicate how they could enter into the design of a capital requirement.

3.2.1 Corporate Governance

3.35 The primary defence in preserving a company's financial integrity is for the company to be well managed. There should be clear lines of responsibility and reporting and the company should have well-established and articulated operating rules and procedures. In summary, the company's corporate governance is an important factor in preserving its well-being and its solvency. In setting Pillar II surplus target levels, the quality of a company's corporate governance should be considered. If management or directors have less than optimal control of the company's affairs, a higher than normal capital target level might be required. If the supervisor has not communicated corporate governance standards to supervised institutions and the overall level of corporate governance in the jurisdiction is not thought to be strong, it would be appropriate to reflect this in the design of a capital requirement.

3.2.2 Risk Management

3.36 A risk management program in an insurance company is an organized program in which sources and volumes of risk are tracked and procedures are in place to track and report on this risk. Important features of risk management include risk limits and risk management policies established by the board of directors, regular reporting of risk at the appropriate level in the company, and oversight by risk officers who are independent of business unit heads.

3.37 Risk management can be viewed as the first line of defence in a company or as a way to prevent the emergence of situations that could imperil the company. Capital supplements risk management; capital is required to support the financial costs to the company of situations where risk management is not a sufficient deterrent.

3.38 If the supervisor has confidence that a company's risk management program is very sound and effective, it could be appropriate to reflect this in the calculation of required capital. This issue will be discussed in section 5.4 of this report.

3.2.3 Investment Policy and ALM

3.39 Since insurance companies usually pay policyholder benefits much later than the time at which premiums are received from policyholders, they must invest funds until these are required to pay claims. The investment income received from these assets is significant and is taken into account when premium rates are established. If investment income is insufficient or the value of invested assets declines significantly, an insurance company could experience significant difficulty.

3.40 Sound investment policies and a program of asset/liability management can significantly mitigate market, credit and mismatch risks. It would be appropriate for the design of a capital requirement to reflect the presence or absence of these risks and their effect on a company's risk profile.

3.41 Certain investment risks can be controlled through a program of hedging. This involves the use of derivative securities. Hedging could be recognized in the design of a capital requirement. However, the supervisor would also want to consider the insurer's hedging program, the availability of necessary financial instruments, the experience and abilities of company personnel engaged in this sophisticated activity and the company's ability and success in conducting the hedging program.

3.2.4 Stress Testing

3.42 Regular stress testing can provide significant insight for company management into the risks faced by an insurer. Such stress testing has been introduced under various names (e.g., DST, DCAT, DFCA, DFA, etc.) in several jurisdictions. The method involves the construction of a computer model of an insurance company and the projection of all cash flows under a variety of scenarios of possible future experience. It is possible to study the effects on the company of the

future emergence of adverse experience and to measure the effects of various management strategies to deal with this experience.

3.43   Stress testing is a supplement to risk management. It does not replace a capital requirement but complements it. In a number of implementations, the object of the exercise is to verify that the company will be able to satisfy its regulatory capital requirements under a variety of future adverse scenarios. The WP is aware that the IAIS has prepared a paper on stress testing.

3.2.5   Risk Sharing and Participating Business

3.44   Certain insurance policies, most often life insurance, are sold as participating or with-profits business. These products feature participation by the insured in the profits of the business line through a system of policyholder dividends or bonuses. Other policies, such as Universal Life, contain adjustable or non-guaranteed elements that also allow the insurance company to adjust policy values, benefits or premiums in accordance with its experience with respect to these business lines.

3.45   An insurance company's ability to pass unfavourable experience to its policyholders through the adjustment of dividends or policy values may be restricted. Restrictions can arise from a concept such as *policyholders' reasonable expectations (PRE)* whereby policyholders may develop an expectation that various adjustable policy elements will continue to be administered by the insurer in accordance with past practices. For example, insurers may be reluctant to pass on the effects of unfavourable experience to policyholders for marketing reasons. PRE may be affected if changes to dividends or policy values are introduced with a considerable delay since the experience first began to deteriorate. Restrictions may also arise from contractual limits for certain policy elements (e.g., premiums, mortality and expense charges, interest crediting rates).

3.46   When considering capital requirements, the argument is often made that if risk is shared with policyholders through participation or adjustment of policy values, then lower capital requirements are appropriate for risk elements arising from this business. This argument has validity. However, in designing a capital requirement, the supervisor should consider the amount of credit that can be granted for risk pass-through features. A principal consideration is how the insurance company actually implements participation or adjustments. Significant capital relief should only be provided if the insurer passes unfavourable financial experience on to its policyholders without significant delay. The case may be slightly different depending on whether target capital (TC) or minimum capital (MC) is being considered. When defining the TC, fewer possibilities to transfer risks to policyholders might be appropriate than in the case of MC, which triggers maximum supervisory measures.

3.2.6   Actuarial Peer Review

3.47   Actuarial policy liabilities usually constitute the single largest item on the balance sheet of an insurance company. Therefore, the financial soundness of a company will often depend upon the quality of the actuarial work that was done to determine these liabilities. Independent peer review of a company actuary's work (by an experienced reviewer) has been found in some jurisdictions to increase the quality of that work as well as the supervisor's confidence in the company's financial results. It has been used in these jurisdictions to enhance the supervisor's confidence in the company's financial results. These periodic actuarial peer reviews act in concert with capital requirements to enhance the protection of policyholders.

3.48   The actuarial profession can assist supervisors within the second pillar by providing independent peer review of the determination of policy liabilities, risk management, capital requirements, current financial position, future financial condition etc., where these entail the use of substantial judgement or discretion. Assistance can also be provided within the third pillar in the design of appropriate disclosure practices to serve the public interest.

3.2.7   Policyholder Protection Funds

3.49   Many jurisdictions have consumer protection or guarantee funds that (partially) compensate policyholders for losses incurred due to the failure of their insurance company. The question arises whether it would be appropriate to recognize the effect of these funds when designing a capital requirement. Recognition means that a company's required capital is reduced since policyholders can be compensated by the fund. If this were done, financial responsibility would shift from the insurance company to those who pay for the fund, perhaps the government, but most often the entire insurance industry in the jurisdiction. This introduces a moral hazard issue since, in this situation, company management might be tempted to rely on the guarantee fund and to accept more risk than is appropriate for the company. The WP suggests that it is unwise to recognize guarantee funds within a capital requirement.

3.2.8   Supervisory Approach

3.50   The WP notes the crucial role played by insurance supervisors in fostering and maintaining an active and healthy insurance market within their jurisdiction. While the WP expresses no preference for one supervisory approach over another, we recognize the integral role played by supervisors along with other mechanisms (including capital requirements) which provide protection to insurance consumers. To the extent that global supervisory approaches differ then it will be difficult to construct a truly global framework for insurer solvency assessment and their attendant capital requirements.

4.   Framework for Solvency Assessment

4.1   This section provides an introduction to the WP's suggested framework for capital requirements for insurance companies. The fundamental principles underlying the framework are described first. This section also considers various implementation issues that will influence a supervisor in designing a local requirement. Section 5 describes the nature of insurer risks and appropriate risk measures. Section 6 suggests standardized approaches to capital that can be applied uniformly to all insurance companies in a particular jurisdiction. Section 7 describes more advanced and company-specific approaches to capital. The final sections of this report address the unique nature of reinsurer risks as well as capital considerations that apply to the company as a whole after all of its risks and its business operations have been considered separately.

4.2   Since the framework is necessarily general, to allow for a variety of circumstances in various jurisdictions, several case studies have been included in the appendices to this report to illustrate the application of the framework. Several technical supplements that discuss certain ideas in much greater depth than would be appropriate in the body of the report are also included in the appendices.

4.1   The Three Pillars

4.3   The Basel Committee on Banking Supervision (BCBS) has articulated an approach to banking supervision (known as the Basel II proposal) involving three pillars: Capital, Supervision and Market Disclosure. The approach is well known and, because of its consistent structure, appears, in principle, to be adaptable to and suitable for the supervision of other financial institutions, including insurance companies. The WP agrees with this approach and it has been used in our work.

4.4   While the WP's task is to suggest a capital requirement framework for insurance companies, this might suggest that our work is mainly restricted to Pillar I. However, in considering the various risks that one would want to cover in a risk-based capital requirement, it has become apparent that there are several risks that are qualitative and not easily measured or quantified. Other risks are, in principle, quantifiable but not easily quantified since relevant data are not readily available and the appropriate models are not sufficiently developed at present. Since our approach to Pillar I is quantitative, these risks cannot be handled here. As does the BCBS, the WP suggests that these risks should be monitored by supervisors under Pillar II. While it might be appropriate for supervisors to increase the Pillar I capital requirements developed using our approach in consideration of these Pillar II risks, disclosure and corporate governance are also useful tools here as well. Suggestions concerning the treatment of these risks appear later in this report.

4.2   Fundamental Approach

4.5   The WP has assumed that the application by supervisors of the methods suggested herein would result in capital requirements that are consistent from one jurisdiction to another but are not necessarily identical. There are significant differences among jurisdictions in insurance products and markets, legal systems, accounting rules and population demographics that make it difficult to construct a universal capital requirement. Instead, the WP has sought to provide capital requirements that are both appropriate in individual jurisdictions and also consistent and comparable across jurisdictions.

4.6   To achieve our goal, we have sought to emphasize the basic principles that apply in each situation. It is only through an understanding of underlying principles that one can develop an appropriate treatment of various risks and aspects of the business of insurance.

4.7 Rules-based approaches to solvency assessment carry the advantage of simplicity of determination and of objectivity but can have the effect of encouraging insurers to "game the system" with respect to capital requirements, thus undermining the entire supervisory process. On the other hand, principles-based approaches focus on "doing the right thing." These approaches tend to require more subjective judgement in their preparation and a different approach to supervisory review.

4.8 The WP suggests that solvency assessment should be based on sound principles. Implementation of solvency assessment will require rules developed from these principles. Rules should be adaptable to current circumstances.

4.9 There is great value from having capital requirements that are internationally consistent. The number of multinational companies that operate in a variety of jurisdictions is increasing. Consistent or uniform capital requirements are desirable so that competing domestic and international carriers are subject to similar requirements and fair competition is maintained in all domestic markets. Unfortunately, multinational insurers are now subject to some combination of the requirements of their home jurisdictions as well as the requirements of each foreign jurisdiction in which they do business. For example, prudential supervision in the European Union (EU) is based on the so called home country principle, which means that legal entities are supervised by the home country supervisors for all their business; the supervision by the host state supervisors is restricted to some emergency situations only. Hence, a subsidiary of a multinational insurer is supervised by authorities of the state in which the subsidiary is domiciled, but a branch is supervised by the authorities of the state where the insurer is domiciled. Uniform international solvency standards would facilitate co-operation between the foreign and home supervisors of an international company and could enable the foreign supervisor to place significant reliance on the work of a company's home supervisor. In addition, fair competition and active insurance markets are encouraged when the requirements of the home and the foreign jurisdictions are consistent.

4.10 In particular, reinsurance is an international business. Both primary insurers and local supervisors require reassurance with respect to the financial strength of reinsurance companies who reinsure local business. A set of internationally consistent financial standards would greatly facilitate the understanding by all concerned of reinsurers' financial strength. It would also help to prevent the arbitraging of capital between (and within) insurers and reinsurers operating in different jurisdictions.

4.11 While standards should be internationally consistent, they must recognize important national characteristics of the insurance industry. There are significant differences among jurisdictions in product design and in claims experience as well as in financial markets, including the supply and quality of financial assets available for insurance company investment; these must be taken into account by any local capital requirement. The treatment of asset related risks (in particular, credit risk and market risk) will depend upon the supply of available assets, the depth of local financial markets and the existence of measures of asset quality (perhaps as measured by rating agencies). Credibility of claims experience for establishing premiums, policy liabilities and capital requirements will depend upon the availability of local data. It would normally not be sufficient to use data from other jurisdictions. Such data normally are collected through inter-company studies carried out by the local industry association or by the local actuarial profession. The supervisor is urged to encourage the local industry and actuarial profession to create or expand industry-wide experience studies as a basis for establishing national valuation and capital requirements. The IAA will continue to foster the development of common international approaches in this regard.

4.12   Although the WP bases its work on principles in order to ensure universal applicability, the WP is aware that readers of this report would appreciate an illustration of how the report can be applied to arrive at concrete and explicit capital requirements. To this end, we have provided three case studies, for life, general (property and casualty) and health insurance.   These case studies demonstrate how a capital requirement could be developed. The numbers contained in these case studies are for the purposes of illustration only and should not be taken as suggestions by the WP of explicit values that can be used in any local development of a capital requirement. The case studies are found in the appendices to this report.

4.3   Total Balance Sheet Approach

4.13   As described in the previous section, the application of a common set of capital requirements will likely produce different views of insurer strength for each accounting system used because of the different ways accounting systems can define liability and asset values. In the view of the WP, these definitions may create a hidden surplus or deficit which must be appropriately recognized for the purpose of solvency assessment.

4.14   The WP believes that a proper assessment of an insurer's true financial strength for solvency purposes requires appraisal of its total balance sheet on an integrated basis under a system that depends upon realistic values, consistent treatment of both assets and liabilities and does not generate a hidden surplus or deficit.

4.15   In addressing the solvency question, the WP has attempted to separate the issues of accounting from the questions of solvency. Accounting determines the financial progress from period to period. As such it gives greater emphasis to the annual profit and loss statement than does prudential regulation. While positive financial progress can be a very good "leading indicator" of future solvency, prudential regulation focuses on the balance sheet (i.e., the capacity of insurers to meet their obligations to pay the present and future claims to policyholders). In order to separate out the accounting issues, the WP believes that solvency would be better defined in terms of a "total balance sheet requirement" (i.e., the sum of both the liabilities and the solvency capital requirement). Using the total balance sheet requirement (TBS) allows solvency assessment to be relatively independent of the accounting system (although factor-based approaches will still require use of verifiable accounting values). One obtains the (solvency) capital requirement as the difference between the TBS requirement and the liability requirement determined on the basis of the accounting system. This implies that if the accounting rules for assets or liabilities differ, the requirements for capital may differ as well.

4.16   The WP understands that the IASB aims to develop an insurer financial reporting system whereby the total balance sheet is valued on an integrated basis using realistic values. The use of such a financial reporting system is intended to help readers of the financial statements to understand directly the elements of conservatism inherent in the financial statements. Equally, such an approach should enable insurer capital requirements to be better coordinated with the protection afforded within the policy liabilities (e.g., technical provisions or reserves).

4.17   However, since there is currently no international uniformity in accounting systems used by the insurance industry, the development of a global framework for insurer solvency assessment based on these current accounting systems is impossible. The WP has selected the TBS approach since it offers the most promise as the foundation for such a global framework. The TBS approach is relatively independent of the accounting system. Of course, standardized factor-based approaches will require use of verifiable accounting values and the degree of conservatism contained in these values should be taken into consideration as part of their determination.

4.4　Degree of Protection

4.18　As was discussed earlier in section 3.1.5 of this report, it is impossible for capital requirements, by themselves, to totally prevent failures. The establishment of extremely conservative capital requirements, well beyond economic capital levels, will have the impact of discouraging the deployment of insurer capital in the jurisdiction.

4.19　The WP considered the role of rating agencies in assessing insurers and the substantial volume of credit rating and default data available from these agencies. This data is helpful in identifying the rating classes that are indicative of insurers in difficulty. Further, the data is suggestive of the cumulative probability of default, over various time horizons, for different current ratings. The WP does not believe it is possible to directly link the solvency degree of protection to these ratings for a variety of reasons (e.g., different rating agency methodologies, different current credit ratings of insurers, etc.) but this information was helpful to the WP in forming its views on this matter.

4.20　As mentioned earlier in section 3.1.5, the degree of protection afforded by a set of capital requirements is dependent on the time horizon considered. In addition, as shown in section 4.5, the specific manner of applying the capital requirement risk measure may also affect the degree of protection chosen. The WP's recommendation for degree of protection is shown in section 4.5.

4.5　Time Horizon

4.21　As was discussed earlier in section 3.1.6 of this report, it is inevitable that there will be some time delay between the date the supervisor can take appropriate action with respect to an unacceptably weak or insolvent insurer and the date the published financial statements of the insurer are produced. During this period, it is likely that the company would continue operations. Therefore, a capital requirement must also provide for the company's business written during this time as well as being sufficient with respect to the existing business as of the statement calculation date.

Uncertainty

4.22　It is generally agreed that uncertainty risks (e.g., regarding the future levels of the best estimate mortality) must be considered for the full remaining term of the insurance contracts.

Volatility

4.23　On the other hand, some argue that volatility risks can be ignored in the long run, since these risks may be diversified away in the future (note that this is not universally true – some elements of volatility risk cannot be diversified away). It is argued that the greater danger to solvency from the volatility component of risk may result not from long run exposure but rather the ability to withstand short-term volatility, perhaps within a one-year time frame.

4.24　This suggests that the choice of the time horizon per risk should depend on the issue of whether the risk at hand could be considered as "systematic" or "diversifiable." However, the WP has strong doubts whether all types of risks can clearly be classified into one of these two categories in practice[1]. Many types of risks may have both systematic and diversifiable components. Moreover, this distinction may also depend on the size of the company and the character of the market(s) in which it operates.

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[1] The same doubts have been expressed in Section 5 of the IASB Draft Statements of Principles on Fair Value accounting.

Extreme Events

4.25   The solvency assessment time horizon should be long enough to capture the impact of extreme events, should they occur, and all associated ripple or tail correlation effects associated with the extreme events.

Recommendation

4.26   In consideration of the above elements of discussion, the WP is of the view that a reasonable period for the solvency assessment time horizon, for purposes of determining an insurer's current financial position (Pillar I capital requirements), is about one year. This assessment time horizon should not be confused with the need to consider, in such an assessment, the full term of the assets and obligations of the insurer.

4.27   The amount of required capital must be sufficient with a high level of confidence, such as 99%, to meet all obligations for the time horizon as well as the present value at the end of the time horizon of the remaining future obligations (e.g., best estimate value with a moderate level of confidence such as 75%).

4.28   Due to the long term and complex nature of some insurer risks, the insurer should consider valuing its risks for their lifetime using a series of consecutive one year tests with a very high level of confidence (say 99%) and reflecting management and policyholder behaviour (but no new business). Alternatively, this test can be conducted with a single equivalent, but lower (say 90% or 95%), level of confidence for the entire assessment time horizon. This lower level of confidence over a longer time horizon is consistent with the application of a series of consecutive higher level one-year measures.

4.29   The assessment of an insurer's future financial position under Pillar II according to various adverse scenarios might reasonably include projected future financial positions for five years for life insurance and two years for general insurance.

4.6   Standardized and Advanced Approaches

4.30   The WP has considered a variety of approaches for determining a capital requirement. The optimal approach would result in a requirement that is determined separately for each insurance company so as to produce a capital value most appropriate for that company. The result of this approach would be a calculation of the company's economic capital. However, this approach can be labour intensive and may require a degree of technical sophistication that may be beyond many companies' abilities and resources even though it directly aligns the management of a company's risks with its measurement process.

4.31   It is more practical to begin from the other end of the spectrum with a standardized approach. Under a standardized approach, capital would be determined using the same calculations for all companies in a jurisdiction. For each source of risk, a standardized measure of a company's exposure to that risk would be multiplied by a standardized factor determined for the jurisdiction as a whole. The factors would be calculated to reflect the circumstances of the jurisdiction. Since this approach is meant to determine a minimum value for capital for all companies licensed to conduct business, the factors would be expected to be fairly conservative. Nevertheless, the standardized approach should not be in clear contradiction with the principles of economic capital and it needs to be applied thoughtfully to ensure that the factors do not lead to inadequate risk measurement processes within the company. The various sources of risk and methods of determining the factors required for them are described later in this report. A complete discussion of the standardized approach is contained in section 6.

4.32   For stronger, more technically able companies with effective risk management programs, it may be appropriate to introduce alternate methods for determining the capital required with respect to specific risks. There is a wide variety of possible alternate methods. These range from the use of company-specific risk factors based upon company experience to alternate calculation methods and to the use of risk and cash flow projection internal models. In general, methods that are more tailored to the circumstances of an individual company would be expected to produce lower capital requirements than would be calculated using the standardized approach. This is due to the conservative bias in the standardized factors, required by the need for the standardized approach to apply to all companies. Low capital requirements could be acceptable to the supervisor if there was assurance that the resulting capital value was appropriate and the insurance company had in place very strong risk management and controls. Advanced methods, including internal models, are currently in use within the insurance capital regimes in Canada and Australia. They are also used under the current Basel Capital Accord to determine the capital for market risk in banks' trading books of assets. Advanced approaches are discussed in section 7 of this report.

4.7   Total Company Approach

4.33   The WP's risk-based approach to required capital treats each source of risk separately. Initial capital amounts are determined for each of them. However, the task does not end with these calculations. There are numerous reasons why the proper capital requirement for a company is not the simple sum of the requirements calculated in respect of each of the risks to which it is subject.

4.34   Adjustments to the simple sum of individual risk-based capital amounts may be required because of concentration of risks, diversification of risks, or dependencies among risks. The concept of correlation between risks is often introduced in connection with these elements.

4.35   The WP suggests that a company's total capital requirement should recognize the relationships among the various sources of risk that can affect its operations. Therefore, the simple sum of individual risk-based components should be adjusted appropriately. This topic is discussed more fully in sections 6-8.

4.8   Implementation Issues

4.36   There are many requirements for the introduction of a detailed risk-based solvency system for the supervision of insurance companies. Many of these, such as the legal framework, the accounting framework, and the business environment are outside the scope of the WP's charge. The WP understands that the IAIS has prepared and continues to develop guidance on these and related matters. There are, however, a number of practical implementation issues upon which the WP offers comment.

4.8.1   Data

4.37   Determining numerical factors in the standardized approach, or alternate methods in the advanced approaches, will be based upon extensive data covering the experience of the insurance industry in the local jurisdiction. Some of this data may have been collected by the supervisor through regular filings of required information. Other necessary data may have been collected by the industry trade associations. In many jurisdictions, the actuarial profession conducts regular inter-company studies of industry experience under insurance policies. Experience shows, however, that it may be necessary to conduct special-purpose data collection exercises for the purpose of calibrating a capital requirement. This can be a difficult though necessary undertaking that requires considerable planning.

4.8.2 Rating Agencies

4.38 Assessing credit risk with respect to specific assets usually involves consideration of the rating assigned to an asset by a leading rating agency. The WP recognizes that rating agencies may not operate in or cover the assets of all jurisdictions. If agency ratings are not available, the supervisor will require an effective local substitute.

4.8.3 Availability of Qualified Professionals

4.39 Determining a capital requirement, as well as assessing the results when this requirement is applied to the insurance industry, is a technically sophisticated matter requiring the skills of trained professionals, including actuaries. Supervisors may have the necessary personnel on their own staff. Supervisors may be able to recruit assistance from the local professional body or industry, or they could use the services of consultants, local or foreign. It is important that those who undertake to determine the details of a risk-based capital requirement have a sufficient knowledge of risk, statistics, finance and business.

4.9 Available Capital

4.40 A capital requirement is used to specify a minimum amount of capital that an insurance company must have. However, a statutory capital standard would be incomplete unless it specified what capital instruments a company could use to satisfy the requirement. This is the question of available capital.

4.41 The WP notes that the IAIS is currently developing guidance on available capital. Our understanding is that this guidance will use a tiered structure similar to that in the Basel Capital Accord. While the WP endorses convergence in this regard, careful consideration may need to be given to unique aspects of the insurance business which may require some modification of the banking approach.

5.   Insurer Risks

5.1   Risk Fundamentals

5.1   The overall management of an insurer includes:

- the design, pricing, marketing and underwriting of its insurance policies;
- the selection of assets backing the policies;
- the estimation of the size and volatility of the liabilities associated with those policies;
- the determination of the insurer's capital needs;
- claims management;
- the updating of all these elements over time as more data and other information becomes available or because the underlying risk processes change;
- an adequate/sound disclosure/communication process to key stakeholders (e.g., management, supervisors, policyholders and investors);
- future financial condition analysis which provides a prospective multi-scenario view of the company as a whole.

5.2   These steps in the overall management of an insurer are illustrated in the following diagram, similar to the one used by the Australian Institute of Actuaries to describe the "actuarial control cycle." The diagram illustrates that the operations of an insurer are bounded by the business environment in which it operates (e.g., legal, social, competitive, client, economic, governmental, tax, etc.) as well as the professionalism of all its employees.
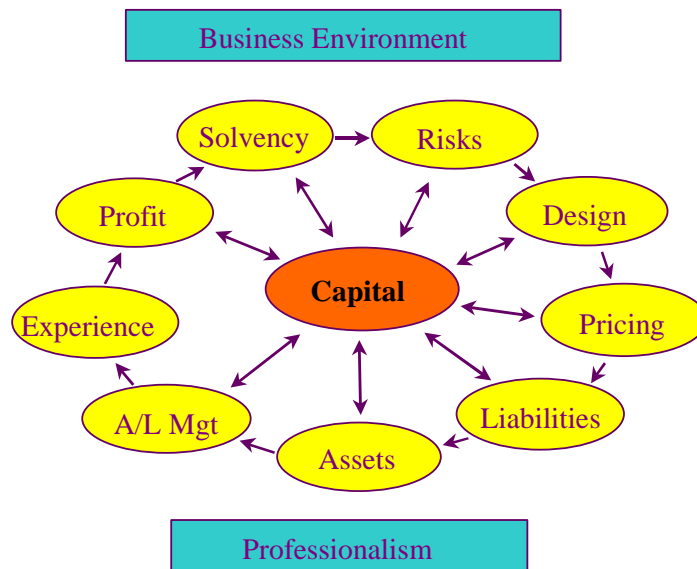
5.3   Risk is inherent in each of steps pictured in the diagram. The assessment of these risks is key to the operations of an insurer. Since actuaries specialize in the financial measurement and management of risk and contingent events, it is natural that actuaries can be of assistance in the assessment of risk, at many points of the "actuarial control cycle."

5.4   It is important to note the central role of capital in the above diagram. Capital represents an essential buffer to ensure that policyholder obligations can be met. The operations of an insurer, after the net effect of all their inherent risks, must yield a rate of return deemed reasonable by the providers of the insurer's capital.   If additional capital is required, beyond that needed for all of the appropriate risk factors at an adequate level of confidence (e.g., 99% confidence level), then (in an efficient market) less capital will be attracted to the insurance sector if the insurance products cannot be priced to recover the additional cost of capital. On the other hand, insufficient requirements, in comparison with that deemed necessary by modelling all of the appropriate risk factors at an adequate level of confidence, may result in inadequate pricing and will increase the exposure of the insurer, over time, to the risk of insolvency.

5.1.1   Definition of Risk

5.5   Throughout this report reference is made to risk. Because of its importance to this report, it is useful to understand clearly the definition of risk. There are many different definitions of risk but a useful one was published in 1995 by Standards Australia and Standards New Zealand when they released a Standard on Risk Management (ASNZS 4360:1995). Included in that Standard is the following definition of risk.

> *"Risk – the chance of something happening that will have an impact upon objectives. It is measured in terms of consequences and likelihood."*

5.6   This definition implies that risk may entail both upside as well as downside impacts. This concept is reinforced later in the Standard.

> *"Risk management is as much about identifying opportunities as avoiding or mitigating losses."*

5.7   Risk only has meaning in the context of a set of objectives or expected results. For example, we might expect the value of automobile insurance claims from a given portfolio of business to be a certain amount. In reality, the actual amount of claims may differ because of the presence of various risks. Appendix G contains a high-level layperson's example of the importance of risk to an insurer.

5.1.2   Introduction to Insurer Risk Types

5.8   While the WP recognizes that insurer risks include many internal dependencies that require an integrated approach to risk or solvency assessment, the WP suggests that insurer risks be categorized under four major headings:

- Underwriting
- Credit
- Market
- Operational

5.9   While each risk is listed as if it existed in isolation, independent of the other risks, there are situations, as occurs in life insurance, where products are specifically designed and then managed with the asset and liability risks modelled together in an integrated fashion. In addition, there are other important considerations for combining the impact of the various risks across the whole company. The "Sharma" report, commissioned by the EU insurance supervisors, studied a number of insurance company failures or near failures and concluded that the final cause for a

failure is always a realization of some concrete risk, but in most cases the real cause is much earlier and more abstract. The "Sharma" report introduced the concept of a "causal chain" of events leading to failure.

5.10   Specific insurance risks that are covered by the company through the insurance contracts it sells are specifically identified as underwriting risks. The other risks are generally present (to different degrees) in other financial institutions. For example, market risk is generally associated with changes in the values of invested assets.

5.11   The WP recommends that capital requirements against asset related risks (e.g., credit and market risks) need not be determined for free assets, those assets which are not supporting the liabilities or the capital requirements themselves. The imposition of capital requirements on these free assets discourages insurers from maintaining more capital than absolutely necessary in the insurer. The imposition of such requirements is therefore counterproductive in enhancing the protection of policyholders.

5.12   The WP notes that liquidity risk is frequently associated with the sale of assets, although the underlying cause of a liquidity situation may not be due solely to market risk. The WP recommends that liquidity risk be addressed within Pillar II rather than Pillar I capital requirements (see appendix D for additional commentary).

5.13   The WP uses the common definition of "operational risk" which includes non-underwriting risk losses internal to the insurer (over which the insurer may have significant control) as well as those that are caused by external non-underwriting risk events (i.e., "event" risks over which the insurer may have little control). The WP recommends that operational risk be eventually addressed, at least partially, within Pillar I capital requirements.

5.14   Each major category contains several more specific risks, which are described in more detail, later in this section.

5.1.3   Key Components of Risk

5.15   In modelling risk, actuaries pay special attention to the following key components of risk for each peril. The modelling tools described later will need to reflect the following components of risk resulting from each peril.

Volatility

5.16   Volatility is the risk of random fluctuations in either the frequency or severity of a contingent event, such as the risk that the rolling of one die will be different from its expected (or average) result of 3.5. This risk is "diversifiable," meaning that the volatility of the average claim amount declines as the block of independent insured risks (or the number of rolls of the die) increases.

5.17   In fully efficient markets, volatility would not be valued in the calculation of the fair value of a set of projected future cash flows. Only capital would be used to absorb the fluctuations arising from volatility risk. This efficient market pricing theory is based on an investor's point of view, whereby the risks in their own portfolio can be diversified. However, because of the relatively inefficient markets for valuing insurance risks, the volatility component of risk cannot be ignored, since policyholders usually cannot diversify that risk away. An insurer can go in to bankruptcy because of diversifiable risk and the policyholders should be protected against that risk.

Uncertainty

5.18   Uncertainty is the risk that the models used to estimate the claims or other relevant processes are misspecified or that the parameters within the models are misestimated. Uncertainty risk is non-diversifiable since it cannot be (relatively) reduced by increasing portfolio size.

5.19   Using the die example above, if the die actually has two 5's on it and no 6 (or a different number of sides), then the estimate of 3.5 based on a "normal" die has misspecified the expected value. Since insurance companies often have unique underwriting standards and market niches, they may be expected to have their own unique parameters. Thus, actual experience observed for one group may not be indicative of the future experience for another group and the experience of the whole population may not be appropriate for an individual company.

5.20   Included in uncertainty are three key elements:

1. The model itself may be incorrect (i.e., no parameters may exist that make the model an adequate description of reality). This is usually referred to as "model error" risk. This can occur when the distribution itself is misunderstood (such as the actual process may be lognormal and one assumes it is normally distributed) or when a key driver or relationship is wrong. However, this introduction of model error may be a deliberate choice in order to have a simpler, more usable model, with an acceptable error tolerance.

2. Even if the model of a cash-flow process is correct, and the underlying model is appropriate, the parameters need to be estimated. Parameter risk is the error in this estimation, which exists because

    ▪ the number of observations on which best estimates are based is limited because the observation period is too short
    ▪ the volatility of the observations makes estimation less certain
    ▪ the period over which the observations were made may not include certain calamitous events that, in fact, should be reflected in the parameters of the distribution
    ▪ the observations contain contaminated data.

3. In addition, the risk structure (i.e., parameters) can change over time or be uncertain for other reasons. This too needs to be considered in modelling the risks. Sometimes called structural risk, examples of this include a new court ruling that changes the interpretation of policy language, a new medical breakthrough (cure for cancer), or a new disease (AIDS). This risk is sometimes incorporated into the model through "structural" distributions of parameters.

5.21   For example, all of these uncertainty elements contribute to estimating the likelihood of an earthquake in the New Madrid area of the United States (St. Louis to Memphis along the Mississippi River). A significant uncertainty relates to whether such an earthquake is a 1 in 100-year event or a 1 in 1,000 or higher year event.

Extreme Events

5.22   Extreme events have also been described as high-impact, low-frequency events for the company as a whole. For any risk, one or more extreme events can cause fluctuations to be much greater than might be expected to arise from normal (modelled) fluctuations under items 1or 2 above. These are one-time shocks from the extreme, adverse tail of the probability distribution that are not adequately represented by extrapolation from more common events and for which it is usually difficult to specify a loss value, and thus an amount of capital to hold. For example, a contagious disease process may affect many persons simultaneously, nullifying the usual assumption of independence among persons; or, a rumour or dramatic public statement might lead to a severe liquidity shortfall scenario at an insurance company. Another possibility is that an event occurs

which has an extremely low probability of occurrence. Using the dice example again, there would be a very low chance that two dice end up leaning against each other with no clear result of the roll.

5.23   The risk of extreme events, beyond normal volatility of cash flows, needs special consideration since the resulting fluctuations may be so extreme as to require independent management strategies.

5.2   Types of Risks

5.2.1   Underwriting Risk

5.24   Insurance companies assume risk through the insurance contracts they underwrite. The risks within the underwriting risk category are associated with both the perils covered by the specific line of insurance (fire, death, motor accident, windstorm, earthquake, etc.) and with the specific processes associated with the conduct of the insurance business. The WP has chosen not to list all the specific hazards, but rather to focus on more generic risks that apply to all (or at least many) lines of insurance:

- Underwriting Process Risk- risk from exposure to financial losses related to the selection and approval of risks to be insured

- Pricing Risk- risk that the prices charged by the company for insurance contracts will be ultimately inadequate to support the future obligations arising from those contracts

- Product Design Risk- risk that the company faces risk exposure under its insurance contracts that were unanticipated in the design and pricing of the insurance contract

- Claims Risk (for each peril)- risk that many more claims occur than expected or that some claims that occur are much larger than expected claims resulting in unexpected losses. This includes both the risk that a claim may occur, as well as the risk that the claim might develop adversely after it occurs

- Economic Environment Risk- risk that social conditions will change in a manner that has an adverse effect on the company

- Net Retention Risk- risk that higher retention of insurance loss exposures results in losses due to catastrophic or concentrated claims experience

- Policyholder Behaviour Risk- risk that the insurance company's policyholders will act in ways that are unanticipated and have an adverse effect on the company

- Reserving Risk – risk that the provisions held in the insurer's financial statements for its policyholder obligations (also "claim liabilities," "loss reserves" or "technical provisions") will prove to be inadequate.

5.25   Appendices A, B and C of this report provide detailed descriptions of the considerations involved in assessing underwriting risk in life, non-life and health insurers through case studies.

5.2.2   Credit Risk

5.26   Credit risk is the risk of default and change in the credit quality of issuers of securities (in the company's investment portfolio), counter-parties (e.g., on reinsurance contracts, derivative contracts or deposits given) and intermediaries, to whom the company has an exposure. Within this category, we include:

- Direct Default Risk - risk that a firm will not receive the cash flows or assets to which it is entitled because a party with which the firm has a bilateral contract defaults on one or more obligations

- Downgrade or Migration Risk - risk that changes in the possibility of a future default by an obligor will adversely affect the present value of the contract with the obligor today

- Indirect Credit or Spread Risk - risk due to market perception of increased risk (i.e., perhaps because of the business cycle or perceived credit worthiness in relation to other market participants)

- Settlement Risk - risk arising from the lag between the value and settlement dates of securities transactions

- Sovereign Risk - risk of exposure to losses due to the decreasing value of foreign assets or increasing value of obligations denominated in foreign currencies

- Concentration Risk - risk of increased exposure to losses due to concentration of investments in a geographical area or other economic sector

- Counterparty Risk - risk of changes in values of reinsurance, contingent assets and liabilities (i.e., such as swaps that are not otherwise reflected in the balance sheet).

5.27   The table below relates market and credit risks of an insurer to the business segments where they are manifest.

Insurer Market & credit risks (IR=Interest Rate risk; FX=Foreign Exchange risk)		Invested assets				Insurance contract liabilities		
		Fixed income	Equity	Other (real estate, mortgages)	Cash	Insurance contracts	Reinsur-ance receivables	Embedded options
Market risk	Change in value due to economic factors	IR + FX markets	Equity + FX markets	IR + FX + market conditions	FX risk	IR + FX + Equity Markets (Products with large investment element)	Not applicable	Low likelihood asymmetric payout events
Credit risk	Change in value due to default or expected default	Default, loss of low-grade bonds	Not applicable	Economic cycles affect returns and values	Small	Not applicable	Default of reinsurer	Not applicable
Dependencies should be considered. Example: dependency between market shocks and credit risk.								

5.28   Appendix E of this report provides a detailed description of the considerations involved in assessing credit risk for insurers.

5.2.3 Market Risk

5.29 Market risk arises from the level or volatility of market prices of assets. Market risk involves the exposure to movements in the level of financial variables such as stock prices, interest rates, exchange rates or commodity prices. It also includes the exposure of options to movements in the underlying asset price. Market risk also involves the exposure to other unanticipated movements in financial variables or to movements in the actual or implied volatility of asset prices and options. Within this category, are included

- Interest Rate Risk - risk of exposure to losses resulting from fluctuations in interest rates

- Equity and Property Risk - risk of exposure to losses resulting from fluctuation of market values of equities and other assets

- Currency Risk - risk that relative changes in currency values decrease values of foreign assets or increase the value of obligations denominated in foreign currencies

- Basis Risk - risk that yields on instruments of varying credit quality, liquidity, and maturity do not move together, thus exposing the company to market value variation that is independent of liability values

- Reinvestment Risk - risk that the returns on funds to be reinvested will fall below anticipated levels

- Concentration Risk - risk of increased exposure to losses due to concentration of investments in a geographical area or other economic sector

- Asset/Liability Mismatch Risk – to the extent that the timing or amount of the cash flows from the assets supporting the liabilities and the liability cash flows are different (or can draft apart) the insurer is subject to asset/liability mismatch risk

- Off-Balance Sheet Risk - risk of changes in values of contingent assets and liabilities such as swaps that are not otherwise reflected in the balance sheet.

5.30 Appendix D of this report provides a detailed description of the considerations involved in assessing market risk for insurers.

5.2.4 Operational Risk

5.31 The concept of operational risk has primarily emerged from the banking industry, and initially was defined in complementary terms, namely all risks other than market or credit. The Basel Committee on Banking Supervision (BCBS) has proposed a capital requirement for operational risk for banking institutions. In order to evaluate a capital requirement specific risks need to be identified and measured and this has led to the adoption of the definition that was initially developed by the British Banker's Association. Operational risk, for capital purposes, is defined as "the risk of loss resulting from inadequate or failed internal processes, people, systems or from external events".

5.32 The above definition is intended to include legal risks but exclude strategic, reputational and systemic risk.

5.33 In the banking industry thousands of transactions are processed each day. Therefore, the amount of data in respect of losses arising from operational failures is more abundant. This naturally lends itself to the development of frequency and severity models to evaluate the aggregate loss distribution and hence the capital requirement.

5.34 In the banking sector it is believed that credit accounts for 60% of all risk, operational risk is 30%, market risk is 5% and other risks represent the remaining 5%.

5.35 The Bank for International Settlements (BIS) has used Quantitative Impact Studies (QIS's) to gather operational risk experience. There appears to be insufficient data of the right type for a Pillar I requirement under Basel II at the present time.

5.36 The BIS will likely not increase the overall Pillar I target standard capital ratio of 8% to allow for operational risk since there is some belief that operational risk is already implicitly provided for in the setting of the 8% target standard ratio.

5.37 It appears likely that Basel II will require operational risk assessment within Pillar I. Banks will be offered the choice of "basic indicator", "standardized" and "advanced measurement approaches". Many banking supervisors require their banks to hold additional capital above Pillar I levels because of Pillar II issues.

5.38 Operational risk is also an important risk for insurers and it should be provided for in a multi-pillar supervisory framework. Operational risk has been recognized as an important risk for insurers as well as for banks (EU Supervisory "London Group" produced the Sharma Report that indicated management shortfalls led to many EU insurer failures).

5.39 However, because of the current general lack of sufficient insurer quantitative data (i.e., operational risk data gathering is less advanced than in the banks; nature of operational risk in insurers differs from that in banks because of the different nature of the businesses), there can be no experience-based explicit Pillar I requirement for insurers at this time. In the interim, a non-experience-based Pillar I requirement can be used but the WP recommends it be accompanied by incentives for companies to demonstrate sound operational risk management.

5.40 Due to the importance of operational risk in the causal chain of events leading to insolvency, the WP recommends that operational risk for insurers be addressed in Pillar I. It may be reasonable to offer a Basel II type of approach with a choice of a "basic indicator", "standardized" and "advanced measurement approach."

5.41 A challenge for insurers in assessing operational risk is to separate this risk from the loss experience data typically collected for the other underwriting, credit and market risks. For example, insurers will need to examine the portion of their "underwriting losses" that are really due to ineffective or faulty underwriting processes or client management.

5.42 It is recommended that insurance supervisors, the insurance industry and the actuarial profession work together to develop appropriate research to measure operational risk.

5.2.5 Liquidity Risk

5.43 Liquidity risk is inherent in the financial services industry. In an insurance context, liquidity risk is exposure to loss in the event that insufficient liquid assets will be available, from among the assets supporting the policy obligations, to meet the cash flow requirements of the policyholder obligations when they are due. In more general terms used in the financial industry, liquidity risk within insurance companies is called funding liquidity risk, as opposed to trading related liquidity risk that banking institutions face raising necessary cash to roll over their debt or to meet cash, margin or collateral requirements. An insurer should be aware of the potential liquidity risks associated with the early termination of insurance contracts. Losses due to liquidity risk can occur when a company has to borrow unexpectedly or sell assets for an unanticipated low price. The liquidity profile of a company is a function of both its assets and liabilities.

5.44   Life insurers often offer policyholders embedded options (e.g., settlement options) that have the potential to cause liquidity problems[2]. General insurers occasionally have to pay claim settlements earlier than expected, thereby being required to liquidate invested assets prematurely or at unfavourable terms.

5.45   There are different levels of liquidity management[3]:

- Day-to-day cash management, which is commonly a Treasury function within a company
- Ongoing cash flow management, which typically monitors cash needs for the next six to twenty-four months.
- Stress liquidity risk, which is focused on catastrophic risk.

5.46   It is important to recognize that stress liquidity risk management is distinct from asset/ liability management and capital management issues. It is therefore not generally covered by actuarial opinions and may not be included in normal measures of risk-based capital; rather, it is a separate and fundamental area of financial risk management.

Possible Sources of Liquidity Risk

5.47   Unexpected demand for liquidity may be triggered by,

- cash calls following major loss events
- a credit rating downgrade
- negative publicity, whether justified or not
- deterioration of the economy
- reports of problems of other companies in the same or similar lines of business
- extent of reliance on and performance of secured sources of funding and their terms (e.g., line of credit capacity and conditions)
- breadth of funding and accessibility/liquidity of capital markets (e.g., through catastrophe bonds).

5.48   Other random fluctuations in demand for liquidity and certain company-specific characteristics can amplify liquidity risk. However, these characteristics by themselves may or may not cause liquidity failure. Good liquidity management can significantly reduce that risk. Examples of company-specific characteristics that can contribute to liquidity risk exposure include:

- A single contract holder or a few contract holders who control large sums of money (policies or contracts). Institutional-type products are the biggest risk in this respect, although in retail lines, a small group of agents and/or brokers may control large blocks of business, and they pose a similar risk.

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[2] The most striking example of loss due to this risk is a "run-on-the-bank" event that causes an institution to fail. This type of event hit banks during the Depression when too many customers demanded to have their money paid immediately in cash, and the demand exceeded the cash reserves. An illustration of the liquidity risk problems which can occur in the insurance industry, occurred on July 30, 1999, when an American insurer's credit rating was downgraded by a major rating service company. In the days following the downgrade, many investors invoked the seven-day redemption clause in the short-term funding agreements issued by the insurer. The funding agreements suddenly behaved like short-term liabilities despite the fact that the assets supporting them were invested for longer terms. The company was unable to sell assets quickly enough to meet these requests and voluntarily sought state insurance department supervision. The cause of these problems was a mismatch between the term of the liabilities (due to the seven day redemption option) and their underlying assets, which, because of a downgrade, led to a liquidity crisis.

[3] Further reference material is available from the 2000 Report of the Life Liquidity Work Group of the American Academy of Actuaries to the NAIC's Life Liquidity Working Group. This report is available at the Academy website at www.actuary.org

- The size of the company may limit access to capital markets. If a company is too small, it may not have the funding choices available to larger companies. On the other hand, if a large company is forced to liquidate billions of dollars of assets at once, the marketplace may not be able to absorb the volume at fair value.

- Immediate demands for cash payments can be a risk if cash is in short supply. An unpredictable cash demand is a larger risk. If a funding agreement has a 7-day put option, the issuer has only one week to collect the cash needed to satisfy the obligation. A predictable cash demand is less of a risk. A well-managed company can structure its assets in such a way so that it has enough cash to cover the known obligations. GIC's with fully predictable pay-out dates and no surrender provision should have minimal liquidity risk in a well-managed company because the cash flows are predictable and planned for.

- Unpredictable deferred or deferrable demands on cash increase liquidity risk. However, the risk diminishes with longer deferral periods. For example, a cashable GIC contract may have a 90-day delay provision, which under normal circumstances gives the company a reasonable amount of time to access its liquidity sources.

- Insufficient ability to borrow short term through bank lines of credit, commercial paper, etc., increases the liquidity risk.

- Lack of diversity/fungibility in either the liability or the asset portfolio when analyzed by product, region, industry, creditor, etc. can create an over-concentration of illiquid assets, such as real estate or thinly traded securities, thus increasing the liquidity risk.

- Finally, liquidity risk can arise during a crisis in the capital markets. When market price moves become extreme, and their volatility increases dramatically, normal correlations break down. As investors begin to exhibit the same behaviour, assets can become non-tradable or illiquid.

5.49   In the case of a large U.S. life insurer that suffered a significant liquidity event, the event was triggered by a downgrade in its credit rating. The contributing factors to liquidity risk were large funding agreement contracts held by relatively few, sophisticated customers; these funding agreements contained 7-day put options. The ratings downgrade caused large amounts of GIC's to suddenly become cashable on very short notice.

5.50   The WP believes that liquidity issues in an insurer are typically triggered by difficult-to-predict events, frequently involving policyholder behaviour because of various operational risk events (e.g., ratings downgrade) and recommends that liquidity risk be subject to Pillar II rather than Pillar I assessment.

5.3   Risk Measures

5.51   Internal models produce probabilities of all possible outcomes of each component of the insurance company's risk that is included in the model. The sum of the outcomes of all risks combined is described as the "aggregate" outcome, usually measured as a "loss." The aggregate loss is described through a probability distribution, which measures the likelihood of all possible outcomes. A "risk measure" is a function of the probability distribution of losses. It is used to determine either the total capital requirement (based on the aggregate distribution of losses) or an indicated capital requirement for a component (based on the loss distribution of the component risk only).

5.52   The following diagram portrays a Normal (so named due to its distinctive shape) distribution of losses. This type of a distribution may reflect the statistical characteristics of some types of risks or be used as an approximation for other risks. This diagram displays the mean of the distribution as well as three types of risk measures, the standard deviation, Value at Risk (VaR – shown on this diagram at the 95th percentile) and Tail VaR.

5.53   Specifically the definitions of these risk measures are:
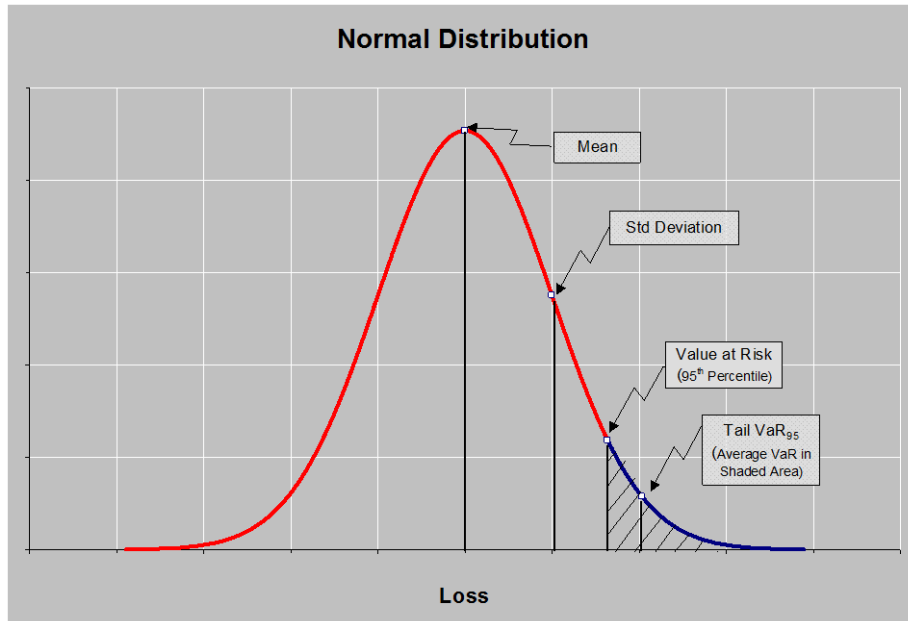
- Value-at-Risk (VaR) is a quantile of the distribution. For example, the 95th percentile of the distribution is the value for which there is a probability of exceedence of 5%.

- Standard Deviation of the distribution is a measure of the degree of uncertainty.

- Tail-Value-at-Risk (TVaR) is the quantile VaR plus the average exceedence of that quantile if such exceedence occurs. For example, the 95% TVaR is the arithmetic average of all VaR's from the 95th percentile on.

5.54   In the next diagram, a skewed distribution is shown. This distribution features a "fatter tail" than the Normal distribution. Risks subject to infrequent but sizeable losses (perhaps catastrophic losses) have "fat tail" distributions. Many insurance risks have skewed distributions. Note the impact of the skewness on the three risk measures. The advantages of using TVaR as a risk measure for solvency assessment purposes are clearly shown in these diagrams since it is the only one of the three risk measures to indicate the amount of catastrophic losses above a certain confidence level.

Skewed Distribution
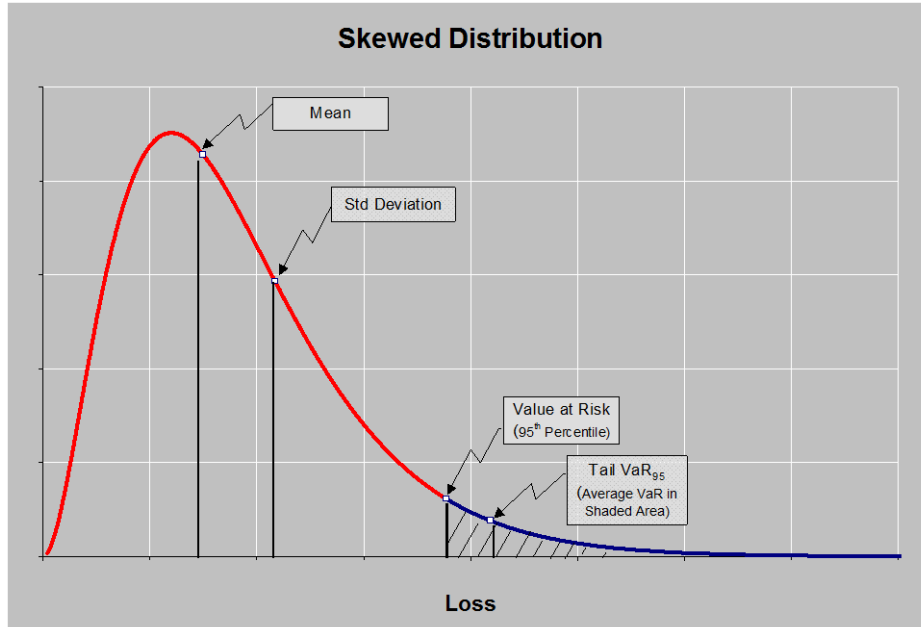
5.55 As is the practice in many areas of financial risk management, it is often useful to begin with the assumption that losses form multivariate lognormal distributions. In many cases this will be a more appropriate assumption than the multivariate normal assumption in an insurance context.

5.4 Risk Management, Mitigation and Transfer

5.56 An insurer can take a number of steps to lessen the risk associated with its business. These include the purchase of reinsurance, securitization of a portion of its asset or liability portfolio, hedging of financial guarantees using derivative instruments, the use of product design to pass risk on to the policyholder, as well as active risk management. To the extent that these measures effectively reduce a company's risk, they should be given appropriate recognition in the calculation of a company's required capital. The difficulty lies in properly assessing the actual degree of risk that has been transferred from the insurance company in these arrangements.

5.4.1 Reinsurance

5.57 Reinsurance is a common way for insurers to manage their risk. In the case of reinsurance in the normal course of business, or indemnification reinsurance, the insurer retains the risks inherent in the original policies sold, while the reinsurer and insurer separately agree to exchange certain specified payments. This has the impact of transferring a portion of the insurer's risk for those policies to the reinsurer. Indemnification reinsurance can be structured to permit the policyholder to retain varying degrees of risk (e.g., via deductibles, coinsurance, captive reinsurance, retrospective premium arrangements etc.). The presence of a reinsurance contract exposes the insurer to the risk of counter-party default.

5.58 Reinsurance can be used to reduce volatility, uncertainty and extreme event risk. For example, some types of insurance can be structured to directly insure against catastrophic events such as earthquakes or hurricanes. They succeed by limiting or "spreading" the risk due to one event through the use of reinsurance to limit their exposure.

5.59   Reinsurance contracts can contain a variety of financial arrangements that specify which party holds actuarial reserves for the business being reinsured. It is important to recognize that where the liabilities are held may not fully indicate which party has the risk. It is also important to recognize that certain forms of reinsurance, generally labelled as financial or finite reinsurance, are actually structured to provide financing by reinsurers to direct writers with only a minimal transfer of real risk.

5.60   In some jurisdictions, reinsurers are subject to regulation and supervision similar to that applied to direct writing companies. Also some jurisdictions require foreign reinsurers, though not directly regulated by the jurisdiction, to maintain sufficient funds locally to support the business they have assumed within the jurisdiction. In both these circumstances, the WP believes it is appropriate to grant credit within a capital requirement to an insurer that has passed on some of its risks through reinsurance. However, this granting of credit should be conditional upon verification that a real transfer of risk has taken place. In addition, the capital credit must recognize the counterparty risk being assumed by the direct writer.

5.4.2   Hedging

5.61   Hedging transactions result in a net reduction in risk as the insurer assumes an offsetting risk to one it currently holds. The insurer still retains the original risk but the offsetting hedging transaction results in a net reduction in risk for the insurer. It is important to note that the insurer assumes additional counter-party default risk as a result of the hedging transaction unless the hedge is a "natural" hedge. A natural hedge occurs when a company can offset risks in different lines of business. For example, writing both life insurance and life contingent annuities for similar groups of policyholders may help to provide a hedge against the impact of improving mortality.

5.62   Recognition of natural hedges introduced when an insurer writes complementary lines of business, can be introduced at the company-wide level once the various individual risk components of a capital requirement have been calculated.

5.63   Financial hedges involving the use of derivative instruments are used by some insurance companies to offset certain financial guarantees (with respect to interest rates or equity markets) that they have given to their policyholders. Before granting credit for financial hedging in a capital requirement, the supervisor should be comfortable that the company's hedging program is well formulated, is consistent with financial economic theory and effectively provides the desired hedge. The supervisor might also require assurance that financial markets offer a sufficient supply of the required derivative instruments and that the company's personnel executing the hedging strategy are competent and knowledgeable concerning financial economics. Financial hedges will usually be used only by more sophisticated companies. Credit for these programs within a capital regime will depend upon the demonstrated effectiveness of the program. This is likely to be possible only when internal models are being used to determine capital requirements for the risk that is being hedged. The WP does not propose to include an adjustment for hedging in the standardized factor-based approach, recognizing that the inability to adjust for hedges (or other market-driven risks) is an important shortcoming of standardized factor-based approaches.

5.4.3   Participating Insurance

5.64   Many insurance policies, particularly life insurance, are sold on a participating or with-profits basis. Under these contracts, the insurance company's experience with respect to this block of business is shared with policyholders through the payment of a policyholder dividend or a bonus; the dividend can take several forms including a cash payment, a credit towards the next premium or an additional amount of paid–up insurance. The argument is made that if a company's

experience with respect to participating business is unfavourable, then it can pass that bad experience on to its policyholders through a reduction in the bonus or dividends.

5.65   In addition, some companies offer policies that contain adjustable or non-guaranteed policy elements. These elements may include premiums, interest credited to the policyholder's account, or charges against the policy for mortality or expenses. Policies often contain limiting values that place constraints upon the company's ability to freely adjust values. For these policies, adjustments can only be made to future policy values. There are also policies, more in the nature of investment contracts, where the investment earnings credited to a policyholder are directly related to a financial index or the return on a designated portfolio of assets.

5.66   The WP suggests it would be appropriate to grant some credit within a capital requirement for risks that are passed through to, or shared with, policyholders under the various mechanisms described above. However, this credit should only be given when the supervisor is satisfied that the insurance company has in place a policy and practice of reducing the dividend or bonus scale or adjusting policy elements in its favour when it is subject to adverse experience. This satisfaction should be based upon explicit demonstration by the insurer of its policies and practices. Moreover, the supervisor should also be satisfied that the constraints placed upon insurers by the concept of "policyholders' reasonable expectations" with respect to participating policies, or the limits within adjustable policies, do not interfere with the company's ability to share unfavourable experience with policyholders consistent with its policies and past practices.

5.67   Since the determination of an unfavourable shift in experience may require a significant amount of time, and since reluctance has been observed on the part of many insurers to reduce bonus or dividend scales very quickly, it is not appropriate to allow complete credit for risk generated through participating or adjustable policies.

6.   Standardized Solvency Assessment

6.1   Introduction

6.1   This section outlines the key considerations that should be considered in designing and selecting a standardized solvency assessment approach for Pillar I insurer capital requirements. The determination of a specific set of requirements for a given jurisdiction should be developed in accordance with these considerations.

6.2   As stated earlier, the optimal approach to assessing insurer capital requirements would result in a requirement that is determined separately for each insurance company so as to produce a capital value most appropriate for that company. This approach can be complex, involve the extensive development of company-specific risk models and could require a degree of technical sophistication that may be beyond the abilities and resources of some insurers or jurisdictions.

6.3   Consequently, it may frequently be more practical to begin from the other end of the spectrum with a standardized approach. A family of standardized approaches is possible, ranging from the simplest and most objective approaches (e.g., set of risk factors common for a jurisdiction that could be multiplied by a company's relevant exposure amounts) to more complex formulaic approaches, which permit some use of an individual company's experience.

6.4   The standardized approach must be designed and calibrated to reflect the circumstances of the jurisdiction. In so doing, the key principles of insurer solvency assessment must be respected to the greatest extent possible. Since this approach is meant to determine a minimum value for capital for all companies licensed to conduct business, the factors would be expected to be fairly conservative. It is important to recognize that while a standardized approach may ease the burden of annual computation on each company, considerable research, analysis and fitting of the standardized approach selected will be required at the outset of the new approach by the jurisdiction itself and on an ongoing basis as new product and market risks evolve over time.

6.2   Range of Standardized Approaches

6.5   The design of a standardized approach begins with recognition that the risks assumed by an insurer have identifiable characteristics. Frequently, risks can be analyzed by their frequency and severity (and even in cases where claim incidence and cost cannot be separately estimated with any confidence, the alternative aggregate loss estimate is a proxy for "frequency x severity"). The combination of frequency and severity results in losses whose distribution (either probability or cumulative loss) is of interest in solvency assessment. In particular, the tail of the distribution is important for solvency purposes.

6.6   The most simple standardized approaches would apply a factor or scale of factors to an exposure amount. These factors would be designed to provide for the tail of the distribution. For example, mortality risk could be provided for by multiplying a factor or scale of factors by the exposed amount at risk. Of necessity, such a simple approach attempts to combine volatility, trend, level and catastrophic risk for all products for all companies into one factor or scale of factors.

6.7   Somewhat more complex standardized approaches identify many more components of insurer risk for separate determination of a capital requirement. Additional complexity can be added to allow for risk concentration and diversification. Once again, such approaches apply industry wide norms that will not reflect an individual company's specific circumstances. Of necessity, a conservative approach to factor setting will be required. While adding many more risk components to the standardized approach may help to better assess the dimensions of an insurer's solvency position, the increased complexity may result in spurious levels of perceived accuracy.

6.8   Some of the more complex standardized approaches begin to approach the accuracy of company-specific internal risk models in assessing risk. For example, some approaches would allow the insurer to input their expected frequency and/or severity data into a standardized computational model. Also input to the model would be jurisdiction specific parameters relating to the shape of the respective frequency and severity distributions. These inputs might be provided by the supervisor.

6.9   In building a standardized approach, appropriate recognition needs to be taken of risk dependencies. The simplest approach involves building a correlation matrix between risks. A conservative approach would allow for full correlation between risks. The company's aggregate risks would then be determined by adding together all of the individually determined capital requirements. In reality, there is frequently some degree of (but less than full) correlation between risks and the impact of correlation on the company's aggregate risks can be quite significant.

6.10   It should be remembered that for some risks, a factor-based approach will not work due to the uniqueness of the risks covered from company to company, the difficulty in defining a loss distribution or the importance of infrequent yet catastrophic losses. In these situations, other tools besides factors (or in addition to factors) will need to be implemented.

6.2.1   Development of a Standardized Factor-Based Approach

6.11   A standardized factor-based approach requires calculating the products of measures of risk exposures and specified factors. The results are summed with an adjustment to the sum to recognized dependencies, diversification, hedging, matching and other risk interactions. This allows for risk reduction methods to be recognized directly. Two approaches are described in the following paragraphs.

6.12   The first standardized approach for a set of risks can be described through a probability distribution of the amount of funds required to support the specified future liability associated with the set of risks. Setting the requirement at a level that provides a high probability of solvency (say 99%) requires determining the quantile (e.g., the 99th percentile) of the distribution of the amount of funds required. This quantile can always be described in terms of the mean and standard deviation of the distribution as $\mu + k\sigma$, where μ represents the mean or expected loss outcome and σ represents the standard deviation or volatility of loss outcomes. The quantity k is a factor that varies depending on the quantile chosen and the shape of the distribution of loss outcomes. For example, if the distribution is Normal (Gaussian), and the solvency standard is 95% then k is 1.64; if it is 99% then k is 2.33. If the distribution is different from Normal, then k is also different. The factor k may be increased to add a greater safety margin if the distribution has a heavier tail than the Normal or if there is additional uncertainty about the mean and variance. The factor k will vary by type of insurance company. Heavier tails will require larger values of k. For example, a life reinsurance company with only short term mortality risk coverages, will likely have a distribution that is not very different from Normal. On the other hand, a general (property/casualty) insurer may have a distribution that is much heavier in the tail due to the greater possibility of extremely large losses as a result of the characteristics of the individual underwriting risks or because of high correlations amongst risks. Thus for property/casualty insurers, k would be expected to be larger.

6.13   A second approach would be to approximate the distribution of the insurer's amount of funds with a specific distribution (e.g., lognormal) and calculate the measure of risk (such as TVaR) at the desired confidence level (say 99%) to determine the total balance sheet requirement.

6.14 Under the first standardized approach, the mean μ is considered to represent the best estimate liability while the $k\sigma$ represents the total capital requirement. This reflects the total balance sheet approach recommended in this report. Note that any amount of conservatism that is built implicitly or explicitly into the statutory or GAAP financial statements (when the reserve is higher than μ) should be recognized as "hidden" capital since it partly protects the company against adverse outcomes. Under the second approach, the observed μ, σ and any other parameters can be used to estimate the parameters of the specified distribution from which the risk measure is derived.

6.15 The discussion in the last paragraph can be applied at the level of product, risk type or line-of-business (LOB) level in the company. Under the first approach, if the LOB's are labelled using subscripts, then the total balance sheet requirement c_j for LOB$_j$ can be rewritten as $c_j = \mu_j + k_j\sigma_j$. Note that all three elements are specific to LOB$_j$. The factor k_j can then be made specific to LOB$_j$.

6.16 The capital requirement for LOB$_j$ is then $k_j\sigma_j$. If this is normalized by the expected losses, the capital requirement is $c_j = \mu_j k_j v_j$ where v_j is the coefficient of variation (CoV) or the ratio of the standard deviation to the mean. The capital requirement can be written as the product of three terms since:

1. μ_j representing the expected losses an "exposure" measure unique to the company and must be calculated by the company;

2. k_j is specific to the LOB and not the company, and can be prescribed by the supervisor; and

3. v_j depends on both the LOB and the size of the LOB for the company, and can be designed with the option of having a formula reflecting industry characteristics for the LOB and incorporating the company's size.

6.17 It should be noted that the exposure measure can be based on simple quantities such as premium volume or be based on more complex probability models reflecting frequency and severity. The capital requirement formula must reflect all future contractual obligations of the company (i.e., not only already incurred or outstanding claims but those that are expected in the future from existing contracts).

6.18 One of the challenges to be faced in developing any standardized approach is that the unintended consequence of its actual operation may be counterproductive to its intended effect. For example, a standardized approach that seeks to multiply gross premiums by a factor, while well intended, also has the effect that an insurer seeking greater financial soundness is actually penalized for increasing the profit margins in its premiums. In a similar fashion, a standardized approach which relies on statutory or GAAP (rather than best estimate) liabilities will inadvertently increase a company's capital requirement in the event that it selects a more conservative (i.e., not due to underlying experience) reserving basis. Both of these examples represent the difficulties in designing standardized approaches that remain faithful to the core principles for solvency assessment as laid out in this report.

6.19   The capital requirements of the LOB's need to be combined into a single capital requirement for the whole company. Simply adding them together will fail to recognize possible diversification between them. The formula,

$$c = \sqrt{\sum_j c_j^2 + \sum_{i \neq j} \rho_{i,j} c_i c_j}$$

allows for diversification of risks, where $\rho_{i,j}$ represents the "correlation" between LOB$_i$ and LOB$_j$. If the correlations are all equal to 1, then the formula is equivalent to summing all the capital requirements for the LOB's. If the LOB's are all mutually independent, then full diversification is possible and the correlations are all equal to 0 and the second term under the square root sign becomes zero. In practice the correlations between LOB's need to be estimated or prescribed. For example, if it is recognized that there is a strong correlation between two risk types (e.g., yields on bonds and yields on mortgages), then the supervisor could prescribe that a specific correlation could be set to 1 in order to have some additional conservatism. Similarly if the correlation between two LOB's is felt to be negative (e.g., annuity mortality and life insurance mortality), the supervisor could prescribe a correlation of 0 to be used so that additional conservatism be incorporated into the formula. In general, it is recommended that the correlation between all pairs of risk types be estimated.

6.20   This "correlation" need not be the standard linear correlation found in statistics textbooks. In particular, it could be a "tail correlation" to incorporate the possibility of simultaneous adverse outcomes in more than one LOB. It can also reflect the choice of "risk measure" used. If the risk measure places greater emphasis on adverse outcomes, then the correlation will be larger than otherwise. The appendices to this report include more technical material supporting the development of correlation formulas.

6.21   Under the second standardized approach each μ_i and σ_i can be used to calculate the mean, μ, and standard deviation, σ, of the amount of funds for the entire insurance company with the following formulas.

$$\mu = \sum_j \mu_j$$

$$\sigma = \sqrt{\sum_j \sigma_j^2 + \sum_{i \neq j} \rho_{i,j} \sigma_i \sigma_j}$$

μ and σ can be used to parameterize a particular (e.g., lognormal) distribution and the final capital requirement will be equal to the selected risk measure (e.g., TVaR(95)) minus the liabilities.

6.3   Underwriting Risk – Life Insurance

6.22   In assessing underwriting risk for an insurer, the basic principles for determining a standardized approach apply as laid out in the section above. However in "drilling down" into any layer of detail, there are distinctive characteristics of the various areas of life insurance that may require special consideration. This section describes some of the special considerations involved in developing standardized approaches for underwriting risk within the life insurance business.

6.23 Some of the special considerations of life insurance, which require consideration in the development of any standardized formula, include the following:

- heterogeneity of risk (even within established "classes" of insurance business)
- importance of mortality/morbidity, lapse and expense (underwriting) risks
- substantial effects of correlation between different underwriting risks
- long-term nature of the majority of the business
- significant role played by reinsurance (especially in relation to concentration of risk)
- difficulty in modelling policyholder behaviour for some products
- importance of adjustable product features in some products (e.g., participating or with-profits policies, etc.)

6.24 Any standardized approach for life insurance will need to take account of these characteristics and will require the classification of all life insurance business in each supervisory jurisdiction into defined product types – the level of detail in the definition effectively being in the control of the supervisor in the jurisdiction under consideration.

6.3.1 Mortality Risk

6.25 In this section an overview is provided of some of the standardized approaches that can be used to calculate capital for mortality risk. Several techniques for calculating the capital requirement for the key risk components of mortality for a life company are proposed below and, where possible, practical standardized measures will be recommended for their estimation. The mortality risk components are

- volatility
- catastrophe
- trend uncertainty
- level uncertainty

6.26 For illustrative purposes, the risk measure used is VaR with a degree of protection set at 99.5% for these approximations. As noted earlier in this report, the WP recommends the use of consistent measures of risk such as TVaR. However, in this example, there is only a small difference in the results between the VaR and TVaR measures with appropriately adjusted confidence levels. The degree of protection has been chosen solely for illustrative purposes. The WP recommendations for degree of protection and its relationship with the risk measurement time horizon are discussed earlier in this report.

6.27 In the models below, the expected mortality claim level or risk premium *RP* is:

$$RP = \mu = \sum_i q_i X_i$$

where q_i and X_i are the mortality rate and amount of insurance for the ith insured person. In general it is assumed that the number of deaths are Poisson distributed. The total claim level is Compound Poisson distributed.

6.28   This means that the standard deviation σ and the skewness γ of the distribution can be found in the following way:

$$\sigma = \sqrt{\sum_i q_i X_i^2}$$

$$\gamma = \frac{\sum_i q_i X_i^3}{\sigma^3}$$

Volatility

6.29   Traditional volatility risk is often calculated using a simulation model with many scenarios generated based on parameter input(s) into a Monte Carlo process. A good alternative is an analytical approach, the Normal Power approximation, using the first three moments of the Compound Poisson distribution. This approach will be less (computer) time-consuming than simulation models.

6.30   The capital at a 99.5% confidence level in the Normal Power approach is:
$$c_{volatility} = \sigma(2.58 + 0.94\gamma)$$

In this case, the value of k is $2.58 + 0.94\gamma$ at the 99.5% level of confidence.

6.31   Under typical circumstances this approach can be further simplified. With # as the number of insured risks and the average q_i is around 0.0025 the capital can be calculated as follows:
$$c_{volatility} = (\frac{77.4}{\sqrt{\#}} + \frac{942.7}{\#})\mu$$

Volatility example

6.32   Three portfolios are used to test these methods. The three portfolios have each their own characteristics.

Portfolio 1:   Typical distribution of sums assured, typical age distribution
Portfolio 2:   Distribution skewed to high sums assured
Portfolio 3:   Typical distribution but a rather small portfolio

Portfolio	Number insured(#)	Max insured/ average	Skewness
1	125,970	11.6	0.13
2	60,777	40.3	0.77
3	24,570	14.7	0.38

6.33   The results for the volatility capital are (% Risk Premium):

Portfolio	Simulation	Normal Power
1	22.7%	22.8%
2	69.9%	68.1%
3	57.2%	57.4%

6.34   Assuming that reinsurance caps the individual sum at risk at 1,000,000 the volatility gives the
        following results:

Portfolio	Simulation	Normal Power
1	22.9%	22.8%
2	37.9%	37.6%
3	56.6%	55.8%

6.35   These results indicate that the Normal Power approximation provides results that are highly
        accurate when compared with results based on simulation. Therefore, this approximation can be
        used in establishing a capital requirement for this component of mortality risk.

Catastrophe

6.36   Beyond "normal" random fluctuations (volatility) in mortality experience from one period of time
        to the next, extra capital is needed for extreme events that result in high positive deviations in the
        claim level. These events can be caused by

- severe epidemic (e.g., Spanish Flu in 1918)
- natural catastrophe (e.g., earthquake)
- terrorist attack (e.g., events of 9/11)

6.37   Due to a lack of data it is difficult to model this kind of risk and a very simple approach may be
        the most useful and appropriate. The capital for catastrophe risk can, for example, be based on
        portion of the expected number of deaths during one year. Based on the experience of the
        Spanish Flu epidemic, a doubling of one year's expected deaths may be appropriate.

Level Uncertainty

6.38   Level uncertainty is caused by the volatility observed in the past. This can make it difficult to
        estimate the "real" or "true" current average mortality. The same kind of model as in the
        volatility risk can be used to calculate this risk. However, the potential impact on the liability
        must be determined because level uncertainty involves the misestimation of the mortality
        assumption for all future years. This makes it difficult to find a simple factor approach.

6.39   One approach would be to "shock" the present value amount of the policy liabilities using best
        estimate mortality rates. To find this shock the same kind of approach can be used as for
        volatility (e.g., Normal Power).

Level Uncertainty Example

6.40   As an example, let us use Portfolio 1 with an assumption that the best estimate mortality
        assumption had been derived from three years of experience. Further, the number of insured
        persons were

- *Year (-3)   97,013*

- *Year (-2) 101,057*

- *Year (-1) 116,651*

For a total number of observations of 314,721. Therefore, based on the factor approach the
99.5% shock on the best estimate mortality rates is:

$$(\frac{77.4}{\sqrt{314721}} + \frac{942.7}{314721}) = 0.14$$

6.41   Therefore, the capital for level uncertainty can be based on liabilities calculated with 14% higher mortality rates (above best estimate, or BE, assumption) minus liabilities based on BE mortality rates. The impact of this on the liabilities will depend on duration, product and interest rate.

The effect of 10% higher mortality rates on single premium business is given by:

Duration/	Endowment		Pure endowment		Term	
Interest	4%	8%	4%	8%	4%	8%
5	0.07%	0.14%	0.73%	0.73%	9.70%	9.71%
10	0.19%	0.41%	1.14%	1.14%	9.49%	9.53%
20	0.44%	1.00%	1.54%	1.54%	9.33%	9.43%

6.42   These results indicate that the impact on a pure endowment is independent of the interest rate. The additional capital required for a 10% mortality change is simply the percentage in the table multiplied by the net single premium. On the other hand, a simple approach for term insurance can be to simply shock the liabilities by 10%.

Trend Uncertainty

6.43   Another mortality risk component is trend uncertainty, the difficulty in accurately assessing the future direction (e.g., improvement) of the mortality assumption in future years. With many product terms extending for the lifetime of the insured, this can be a considerable risk, especially for payout annuities. It is difficult to model mortality trend uncertainty in a simple way. The result depends on product, duration and interest rate. The graph below illustrates the value of trend uncertainty for a variety of products. The vertical axis indicates the value of trend uncertainty as a percentage of the underlying liability amount. The horizontal axis displays the remaining duration of the liability.

6.44 A simplified approach to provide for trend uncertainty could be to apply a factor multiplied by the present value amount of the liabilities (see following formula). The factor might be expressed as the lesser of α and β times the product duration n. Some sample values of α and β are also given in the table below.

$$c_{trend} = \min\{\alpha, \beta n\}(liability)$$

	α	β
Pure endowment	7%	0.35%
Endowment	3%	0.15%
Term	30%	1.50%

The uncertainty trend for a whole life annuity can be based on 4% of the liabilities (x>55). These calculations of trend uncertainty are based on a 99.5% confidence level.

6.45 The final capital requirement for mortality risks should provide for each of the components described in the preceding paragraphs, volatility, catastrophe, level and trend uncertainty. To the extent that the mortality experience is shared with the policyholders then corresponding credit should be granted in the capital requirements.

6.3.2 Lapse Risk

6.46 The risks posed to an insurer by an unanticipated rate of policy lapses, terminations or surrenders (collectively referred to here as 'lapse risk') are varied and complex. The treatment of lapse risk within a capital requirement will also vary from jurisdiction to jurisdiction. This variation is increased by the differences in how provision for lapses is or is not made within policy liabilities or actuarial reserves. It should be noted that in many jurisdictions, the valuation of liabilities is made using a modified net level premium approach that does not explicitly take lapses into account. The methodology used in other jurisdictions, particularly that based upon gross premiums, does make explicit recognition of the effect of lapses. The latter includes the valuation method being proposed in connection with the new international accounting standards being developed by the IASB.

6.47 There are two primary effects of unanticipated lapse rates. The first involves the payment of surrender or termination values. The relationship of the amount of a surrender payment to the value of the liability being held in respect of a particular policy is of great importance. When a policy lapses, the company pays the surrender value and 'receives' the actuarial reserve that is released by the policy's termination. If surrender values are lower than policy reserves, the company is at risk from lapse rates that are lower than expected, particularly if high lapse rates were anticipated in the pricing of a product. The case that surrender values exceed policy reserves results in higher lapse rates being unfavourable to the insurer. In some jurisdictions these risks are mitigated by regulations. A requirement that a company holds policy liabilities at least as large as surrender values provides partial protection against overly high lapse rates while minimum required surrender values reduce the likelihood that insurers will price their products using an assumption of high lapse rates. It is important to recognize that the relationship between the surrender value and the actuarial reserve is not fixed; it will generally vary with the duration of a particular policy.

6.48 The second primary effect of unanticipated lapse rates is that the insurer may not realise the expected recovery from future premiums of initial policy acquisition expenses. These acquisition expenses may be recognized implicitly in financial statements through the use of modified net level premium valuation methods. These implicit methods generally do not include any provision for unfavourable variations in lapse rates. Recovery of acquisition expenses may also be recognized explicitly through a reduction in policy liabilities or through introduction of a receivable asset. In this latter case, the adjustment to financial values is made subject to a form of

recoverability test. Under the second primary effect, the risk to insurers is generated by lapse rates that are greater than expected.

6.49   Unanticipated lapses can have other effects on the financial condition of an insurance company. For example, anti-selective lapse by healthier lives may lead to deterioration in a life insurer's mortality experience. This risk may be due to poor product design, an operational risk. In general, this risk is not treated for capital purposes as a lapse risk.

6.50   A capital requirement with respect to the first type of lapse risk requires the division of an insurance company's policies into two classes: 1) those policies for which actuarial liabilities L are greater than surrender values S, and 2) those policies for which $S>L$. The capital requirements would then be of the form $j(L-S)$ or $k(S-L)$ respectively, for appropriately chosen factors j and k. A capital requirement in respect of the second type of lapse risk could be of the form mU where m is an appropriately chosen factor and U is the present expected value of acquisition expenses recoverable from future premiums.

6.51   In the case that lapses are recognized explicitly in the valuation of actuarial liabilities, another approach to capital requirements in respect of the first type of lapse risk is available. This requires the division of policies into two classes: 1) those for which an increase in lapse rates results in an increase in policy liabilities, and 2) those for which policy liabilities increase when assumed lapses decrease. The capital requirement is of the form of the difference between a special valuation of policy liabilities and the normal valuation. For the special valuation, the lapse assumption is multiplied by a specified factor greater than one for policies in the first class and by a factor less than one for policies in the second class. As an example, in Canada, lapse rates are doubled for policies in the first class and reduced by one-half for those in the second class.

6.52   The last lapse case, which cannot be addressed in a factor-based approach are those products for which lapse risk does not act uniformly over the products life, such as lapses at early durations which may reduce the company's exposure to later risks for some policies and not for others.

6.3.3   Expense Risk

6.53   Operating expenses of an insurance company represent a considerable portion of an insurer's annual costs. The other major element of annual costs would include the change in policy liabilities (i.e., reserves or technical provisions) and policy benefits/claims. Solvency assessment of insurers should also consider the risks involved with the expenses of the company. It is important for an insurer to understand its expenses and their component parts for the purposes of proper product pricing, provisioning, solvency assessment, etc.

6.54   Most important in any analysis of insurer expenses is to obtain the split of expenses between acquisition and maintenance and also between fixed and variable. A table similar to the following should be developed.

Expenses	Fixed	Variable	
Acquisition	#	#	Total Acquisition
Maintenance	#	#	Total Maintenance
	Total Fixed	Total Variable	Total Expenses

6.55   Fixed expense are not those 'fixed per policy expenses' used in profit testing or embedded value calculations that are invariant to the size of the policy. The 'fixed expenses' are those expenses that do not vary in proportion to the volume of the total new and existing business at least over the short-term.

6.56   Especially important for assessing the adequacy of provisions and for solvency assessment is a proper determination of the split of expenses between acquisition and maintenance. This split is based on insurer judgement. If too many expenses are allocated to the acquisition category, then a forward looking view of the company's on-going maintenance expenses will be understated. This may result in the under-provisioning of such expenses in the liabilities and an overly optimistic view of the company's future financial condition.

Acquisition expense risk

6.57   New business consists of the sales of new policies. Although the value of future sales beyond the current year are not included, acquisition expense risk exists since acquisition expenses are partly fixed and the company may be subject to variances in new business volume.

6.58   Theoretically, a distribution could be fitted to model the past ratio of the actual and the planned sales volumes. The capital requirement could then be determined from the tail of that distribution.

6.59   A simple method that could be used to calculate the economic capital would be to calculate the capital as **Factor * Fixed Acquisition Expenses**. The fixed acquisition expenses are the fixed acquisition expenses in the subsequent year. The factor can be set at 100% and considers only a one-year time horizon.

Maintenance expense risk

6.60   Maintenance expense risk is due to:

*   Unexpected changes in the unit cost (assuming the portfolio runs off as expected) and
*   Unexpected changes in the volume of the portfolio.

6.61   It would be possible to run multiple projections for the existing business around which a distribution could be constructed in order to estimate the economic capital required for maintenance expense risk. However new business volumes and changes in business strategy have a significant impact on the expense structure of a company. Since in this phase new business beyond the valuation date is excluded we propose a simplified methodology for the calculation of the capital.

6.62   The first component is often related to the misestimation of inflation where this is expected to be a material risk factor. The methodology proposed is that the best estimate inflation assumptions are shocked by a factor (i.e., a 30% increase of inflation in the initial year decreasing linearly to the best-estimate assumption over 5-10 years; or perhaps a 1% increase over the lifetime of the business). The capital would then be calculated as the change in value of expense liability between the best estimate and shock scenario.

6.63   The second component is similar to that for acquisition expense risk and again a distribution could be fitted to model the past ratio of the actual versus the planned maintenance expenses. The basis for the solvency capital would be defined as the 0.5% tail of this distribution.

6.64   However, similar to the acquisition expense capital requirement, a simplified methodology for maintenance expenses may be more practical. A formula such as: **Factor * Fixed Maintenance Expenses** might be used. The factor would be based on expert judgement and reflect the company specific situation. A factor of the order of 75% might be reasonable and considers a three-year time horizon (i.e., assumes a 25% drop in the business volume and the inability of the company to adjust the fixed maintenance expenses over the period). The 75% factor assumes that the business does not have a material exposure to fluctuations in the equity markets that would impact the expected fee income. In this situation the factor is likely to be lower since equity risk already covers some of the maintenance expense business risk.

6.65   Alternatively, the policy liability including best estimate expense assumptions could be shocked to allow for both types of maintenance expense uncertainty (e.g., inflation and exposure to variable unit costs). The capital would then be calculated as the change in value of expense liability between the best estimate and shock scenario.

Alternate expense risk solvency calculation

6.66   The expense risk solvency calculation outlined above separated the risk for acquisition and maintenance expenses and involved determining the fixed and variable expenses. In some situations the classification of expenses into acquisition and maintenance / fixed and variable might be impractical or of limited benefit. In particular, this could be the situation in emerging markets where the experience is not stable and assumptions are based on short-term experience.

6.67   An alternate methodology for determining the expense risk capital requirement could involve looking at the expenses of a company in aggregate and simply estimating the economic capital as a **Factor * General Operating Expenses**. The factor would be based on expert judgement and reflect the company specific situation. General Operating Expenses would be the usual costs incurred in day-to-day operations. This would not include commission costs that are completely variable. A factor of the order of 100% might be appropriate.

6.68   The final capital requirement for expense risks should provide for each of the components described in the preceding paragraphs, acquisition, maintenance inflation and maintenance unit cost. To the extent that the expense experience is shared with the policyholders then corresponding credit should be granted in the capital requirements.

6.4   Underwriting Risk - Non-Life (General) Insurance

6.69   Some key idiosyncrasies of non-life (general) insurance, which require special consideration in the development of any standardized formula, include the following:

- heterogeneity of risk (even within established "classes" of insurance business)
- substantial effects of correlation between different underwriting risks
- difference between outstanding claims liabilities and liabilities because of unexpired risk inherent in unearned premiums
- annual renewal basis for the vast majority of the business
- significant role played by reinsurance (especially in relation to concentration of risk)
- difficulty in estimating separate claim incidence and severity in projecting experience for a minority of the business.

6.70   In summary, any standardized approach for non-life insurance will need to take account of these characteristics and will require the classification of all non-life insurance business in each supervisory jurisdiction into defined lines of business (LOB's) – the level of detail in the definition effectively being in the control of the supervisor in the jurisdiction under consideration.

6.71   The standardized approach will also require specification for each LOB of a LOB Coefficient of Variation (CoV), a LOB Size Factor (SF), and a LOB Confidence Factor (CF). In addition, a set of Correlation Coefficients (CC) will need to be specified for each pair of LOB's.

6.72   CoV's for outstanding claims liabilities would typically be expected to be in the range of 10% to 20% for short tail business and typically in the range 20% to 30% for long tail business. CoV's for unexpired risk liabilities would generally be expected to be between 25% and 75% higher than the CoV used in the same LOB for the outstanding claims liability.

6.73   LOB Size Factors would be specified to increase the level of capital required for smaller portfolios compared to medium or larger portfolios to reflect the increased impact of non-systematic risk in smaller portfolios.

6.74   For illustrative purposes, a standardized approach applicable to non-life insurance LOB's might reasonably be used

(a) Correlation coefficients between any pair of classes of business greater than or equal to 25%
(b) Correlation coefficients between any two long-tail classes of business greater than or equal to 50%.

Appropriate coefficients for each jurisdiction would need to be determined.

6.75   A simple illustration of these concepts is set out in the table below which assesses a total capital requirement of a hypothetical insurer to be $9,894 million where total expected losses (before diversification allowances) are $7,425 million.

6.76   A standardized approach of this kind also requires a genuine best estimate of the expected loss in each LOB from both unexpired risks and outstanding claims including incurred but not reported claims. This expected loss needs to be calculated net of reinsurance recoveries expected by the insurer other than for catastrophic losses for which capital requirements are modelled separately. It is recommended that the expected loss for each LOB be calculated either by adopting a frequency- and severity-based calculation based on actual exposures or, if data adequate to support such a calculation is not available, by using a projected loss ratio applied to premium earned. These calculations could be completed using either data supplied by the company or as specified by the supervisor, depending upon the requirements of the supervisor in each jurisdiction. It is important that these expected loss estimates not be made in an unduly conservative fashion so as not to compromise the integrity of the capital calculation methodology as a whole.

Illustration of Simple Non-Life Insurer
Standardized Factor Approach

Line of Business	Liability Type	Expected Loss ($M)	CoV %	Confi-dence Factor	Size Factor	Capital Required ($M)
Motor Car	Unexpired Risks	750.00	15.00%	2.50	1.0	1,031.25
Motor Car	Outstanding Claims	250.00	10.00%	2.50	1.0	312.50
Home	Unexpired Risks	500.00	18.00%	2.50	1.0	725.00
Home	Outstanding Claims	125.00	12.00%	2.50	1.0	162.50
Workers Compensation	Unexpired Risks	1,250.00	35.00%	2.50	1.0	2,343.75
Workers Compensation	Outstanding Claims	3,750.00	25.00%	2.50	1.0	6,093.75
Public Liability	Unexpired Risks	200.00	30.00%	2.50	1.0	350.00
Public Liability	Outstanding Claims	600.00	20.00%	2.50	1.0	900.00
Sub-Totals (before	Unexpired Risks	2,700.00				4,450.00
Diversification)	Outstanding Claims	4,725.00				7,468.75
	All Classes	7,425.00				11,918.75
Diversification Allowance	All Classes					-2,024.60
Totals	All Classes					9,894.15

Correlation Coefficients

	Motor Car	Home	Workers Comp.	Public Liability			
Motor Car	100.0%	50.0%	25.0%	25.0%			
Home	50.0%	100.0%	25.0%	25.0%			
Workers Compensation	25.0%	25.0%	100.0%	50.0%			
Public Liability	25.0%	25.0%	50.0%	100.0%			

6.77   The case study, described in appendix B, is more sophisticated than the simple standardized approach described above. It includes a table of calculated correlation factors that enables the effects of diversification of risk to be included in the model outputs. These factors may be prescribed or may be calculated from underlying data where available. Concentration of risk is identified and appropriate allowance for reinsurance may be included. However, the model does not attempt to accurately measure specific reinsurance effects and it is suggested that existing commercially available risk models be used to assist in this area.

6.78   In the model in appendix B, the input of data for each LOB enables the mean and variance of each insurer's aggregate loss distribution to be calculated. The loss component of the capital requirement, net of catastrophe reinsurance, is then assessed using a lognormal (i.e., skewed) assumption regarding distribution of losses and the TVaR at a "selected" level. The supervisor can select the TVaR level by taking account of the assessed market security requirement and judgement regarding the "goodness of fit" of the lognormal distribution, especially for some of the more heavily skewed risk distributions.

6.79   The case study model is simple enough to be represented easily in spreadsheet form and yet detailed enough to allow specific consideration of five of the six key issues listed at the beginning of this section (note that, in respect of the sixth issue, it is a relatively straightforward adjustment to replace the separate claim incidence and mean cost assumptions with an aggregate loss assumption for any selected LOB). The outputs from the model can be designed to be conservative, but enough detail remains in its representation of the business for the supervisor to ensure that any conservatism is not excessive.

6.80   The model includes "c" factors that take account of the size of the respective LOB's and "b" factors that essentially quantify the correlation effects. These b's and c's effectively combine to create the "k's", as well as allowance for the correlation effect in section 6.2.1.

6.81   Neither the standardized approach set out above nor the case study addresses either market risk or credit risk for the general insurer, since these risks are essentially common to all types of insurance.

6.82   The model does exhibit some of the necessary flexibility in that inputs may be largely governed by the supervisor or the individual insurer may well be responsible for the large majority of data for the model (and hence making strides towards the creation of an internal modelling approach). It is a member of the range of potential modelling approaches that may be use globally.

6.83   In order to extend the "family" of models into use in markets where more sophisticated and accurate data may be available, it is possible to augment the case study model with a number of features to improve flexibility and accuracy. These may include greater "tailoring" of the loss distribution curves to the characteristics of the business (e.g. through the use of a range of formulas or empirical data) and an increase in the range of factors used to model risk correlations). At the other end of the scale, to cater for a market in which actuarial advice is largely absent, it is possible to create a model with a reduced number of entirely prescriptive factors to be applied, for instance, to a broader definition of LOB, or to broad bands of business by size (or by a combination of both) similar to the standardized approach outlined above.

6.5   Underwriting Risk – Disability Income

6.84   The following paragraphs provide an illustration of the determination of standardized capital requirements for disability income products.

6.85   In the U.S. risk-based capital (RBC) formula for insurance companies, the most significant component for all health insurance products is the underwriting component. The factors for the

asset risk and operational risk are common for all of the insurance products. There is no interest rate risk component for any of the health products. The following description of the process used to determine the formula for the underwriting risk component for disability income (DI) is similar to the process used for the other health products.

6.86   Data and information was collected from all of the DI writers in the U.S. that were willing to contribute to the study. The data collected included incurred claims, earned premiums, policy reserves and tabular claim reserves for as many of the most recent ten years that were available. Interest adjusted loss ratios were calculated using changes in policy reserves that were not caused by a change in basis or reserve formula assumptions. The standard deviation and serial correlation of the loss ratios for each company and for all companies combined were calculated. This process was performed separately for each of the major forms of DI insurance sold in the U.S.

6.87   Other information collected in the study included: number of months after a loss ratio falls outside of an acceptable range until a premium rate change is implemented; the percentage of premium that is eventually changed where a rating action is indicated, expected loss ratio, expense ratio and profit ratio, all expressed as a percentage of earned premium.

6.88   A random walk, stochastic model was then built that, given a specified starting level of capital, calculated the operating gains and accumulated surplus for a five year measurement period. The model assumed a stationary population of in force business, where new sales equal terminations each year. The actual loss ratios collected in the survey were adjusted to reflect the difference between the actual loss ratio and its expected value given the premium rate changes generated by the model resulting from management actions. The time needed to implement a rating action is the "phase in factor" and is developed from the survey along with the upper and lower loss ratio limits that would cause a rating action to be initiated. The model adjusts the randomly generated loss ratio to reflect the indicated premium rate changes each year.

6.89   The model generates a loss ratio, or claim cost per $1 of premium, each year of the projection period which is the sum of three terms:

1.   **The previous year's loss ratio**. The model was run for a three year "seasoning period" prior to the beginning of the actual projection period, so there is a previous year's loss ratio even for the first year of the projection period.

2.   **A correlation deviate for the projection year**. This is based on a random normal distribution deviate with a standard deviation of the loss ratios collected with the adjustment discussed above and another adjustment to reflect the serial correlation calculated from the data.

3.   **A term to adjust the current year's loss ratio** to reflect changes in the premiums that would occur according to the rules for timing and amount of premium actions that would be initiated when loss ratios fall outside of specified limits.

6.90   50,000 scenarios were run for several test initial surplus amounts, calculating the resulting gain or loss and accumulated surplus amount per $1 of earned premium. The gain or loss is the sum of the $1 premium plus expected interest on reserves and accumulated surplus less the randomly generated claim cost, expected expenses and taxes. Ruin occurs when the resulting accumulated surplus falls below zero in any year of the projection period. By interpolation and successive iterations of the process the beginning surplus is found that results in a 5% probability of ruin. This is the RBC amount when expressed totally as a percent of premium.

6.91   A similar stochastic study determined that, if around 5% of claim reserves were added as starting surplus to the amount of claim reserves, the total fund would be adequate with a 95% confidence level. The final formula for the underwriting component adopted the 5% of claim reserves plus a percent of earned premiums, where the percent was reduced to reflect the amount shifted to the claim reserve.

6.92   The analysis was performed on both large and small blocks of business separately, resulting in larger earned premium factors for the small blocks. This was reflected by using a tiered formula, with a larger factor for the first $X of premium and a smaller factor for the excess over $X. As an example, the formula for individual non-cancellable disability income insurance is: 5% of tabular claim reserves plus 35% of the first $50 million of earned premium plus 15% of the earned premium in excess of $50 million. Several other types of DI insurance are specified, each having its own unique set of factors. Every U. S. insurance company that writes DI insurance must use this formula and set of factors to determine the underwriting component for DI in there RBC.

6.6   Credit Risk

6.93   Appendix E describes the sources of credit risk for an insurance entity. In summary, they are

- Direct Default Risk: risk that a firm will not receive the cash flows or assets to which it is entitled because a party with which the firm has a bilateral contract defaults on one or more obligations
- Downgrade or Migration Risk: risk that changes in the possibility of a future default by an obligor will adversely affect the present value of the contract with the obligor today
- Indirect Credit or Spread Risk: risk due to market perception of increased risk (i.e., perhaps due to business cycle or perceived credit worthiness in relation to other market participants)
- Settlement Risk: risk arising from the lag between the value and settlement dates of securities transactions
- Sovereign Risk: risk of exposure to losses due to the decreasing value of foreign assets or increase the value of obligations denominated in foreign currencies
- Concentration Risk: risk of increased exposure to losses due to concentration of investments in a geographical area or other economic sector
- Counterparty Risk: risk of changes in values of reinsurance, contingent assets and liabilities (i.e., such as swaps that are not otherwise reflected in the balance sheet).

6.94   From a supervisor's perspective, the main areas of focus in respect of credit risk are

- inordinate "peaks" of risk due to any number of factors
- reliability and consistency of any external or internal credit rating approaches.

6.95   Given that it is not possible to devise a simple capital framework to incorporate all credit risk factors into account in an accurate fashion, the suggested approach is one aimed at the major factors.

6.96   The approach is clarified by the separation of credit risk into "Type A" (or risk relating to actual assets held and the insurer's ability to manage its credit loss position) and "Type B" (or credit risk involved with future reinvested assets).

6.97   The time horizon is an important consideration for credit risk. The WP believes that one year is an appropriate limit for capital considerations. The capital requirements should be determined using a degree of confidence consistent with that chosen for other risks.

6.98   The Working Party (WP) recommends that the work of the BIS with respect to credit risk capital requirements for banks be also considered for use by insurers in capturing Type A credit risk. In considering the BIS approach, insurance supervisors will need to consider the appropriateness of the time horizon and confidence level assumptions implicit in the BIS approach. Also to be considered is the appropriate treatment of policyholder pass-through features.

6.99   By definition the development of standardized approaches for capturing Type B risks is fraught with difficulty. Where these risks are material in an insurer, the supervisor should encourage or even require the insurer to perform appropriate advanced approaches to modelling their Type B credit risk.

6.100   Standardized approaches to assessing Type B credit risk might include (from the simplest to the more sophisticated):

   a.   Where it is not possible to directly compute the present value of future liability cash flows, provision for Type B credit risk can be made approximately by applying a factor to the policy liabilities of long-term business. These factors would need to be tailored to the circumstances of an individual supervisor and their financial reporting structure for these liabilities.

   b.   Where it is possible to estimate the duration of long term business, provision for Type B risk can be made approximately by applying a credit risk spread to the duration (beyond that of the current assets) and the policy liabilities for long-term business.

   c.   Where it is possible to directly compute the present value of future liability cash flows, provision for Type B credit risk can be made directly through use of a credit risk spread.

6.7   Market Risk

6.101   The principal sources of market risk for insurers are

   •   Interest Rate Risk- risk of exposure to losses resulting from fluctuations in interest rates
   •   Equity and Property Risk- risk of exposure to losses resulting from fluctuation of market values of equities and other assets
   •   Currency Risk- risk that relative changes in currency values decrease values of foreign assets or increase the value of obligations denominated in foreign currencies
   •   Basis Risk- risk that yields on instruments of varying credit quality, liquidity, and maturity do not move together, thus exposing the company to market value variation independent of liability values
   •   Reinvestment Risk- risk that the returns on funds to be reinvested will fall below anticipated levels
   •   Concentration Risk- risk of increased exposure to losses due to concentration of investments in a geographical area or other economic sector
   •   Asset/Liability Mismatch Risk- to the extent that the timing or amount of the cash flows from the assets supporting the liabilities and the liability cash flows are different (or can draft apart) the insurer is subject to asset/liability mismatch risk.
   •   Off-Balance Sheet Risk- risk of changes in values of contingent assets and liabilities such as swaps that are not otherwise reflected in the balance sheet.

6.102   An in-depth discussion of insurer market risk appears in Appendix D.

6.103 Market risk can only be measured appropriately if the market value of assets and liabilities are measured adequately. Market values of assets can generally be deduced from listings in the various securities markets. Because of the lack of a real market for insurance liabilities, the market value of insurance liabilities can be approximated through evolving market/fair value techniques. The concept of the "replicating (asset) portfolio," may be a useful concept in measuring the market value of insurance liabilities.

6.104 In general, life and health insurers purchase assets to match their liabilities. Historically this has not been true for non-life insurers, who tend to manage separately the results from underwriting and investments. While all of the assets of an insurer are available to provide against adversity, it is common risk management practice for insurers to, implicitly or explicitly, allocate their assets for one of the following purposes:

- support insurance contract liabilities
- provide economic capital
- provide free surplus.

6.105 Sizeable portions of an insurer's liabilities can have durations (e.g., terms) comparable to readily available high quality liquid assets in the local market. In these situations it is possible to select assets whose cash flows can provide a very close match to the liability cash flows. In other words, a replicating portfolio of assets is available in the market. In this situation, market risk focuses on the volatility of the market value of the actual assets held and the market value of the replicating portfolio of assets and the ability of the insurer to manage that volatility. This type of market risk will be called **Type A** risk and it also includes the effect of volatility on the insurer's regulatory capital requirement for these risks and the assets representing that capital.

6.106 The long-term duration of some insurance (especially life insurance) liabilities requires the consideration of long-term rates of reinvestment since replicating portfolio assets of sufficient duration may not be currently offered in the market. Measuring market risk for these liabilities entails considerable uncertainty about the composition of the replicating portfolio and the manner of its reinvestment to mature the underlying cash flows. Lowered rates of reinvestment in the future are typically of concern. In addition, life insurance contracts may contain various complex, long-term options and/or guarantees for which replicating market positions may not currently exist (e.g., death and maturity guarantees on variable annuity products). These latter two types of market risk will be called **Type B** risk. Type B risk also includes the effect of volatility on the insurer's regulatory capital requirement for these risks and the assets representing that capital.

6.107 The assets and liabilities of an insurer are subject to Type A and possibly Type B risk. Shorter term insurance contracts without complex embedded options or guarantees are subject to Type A risk. Long-term insurance contracts and/or those containing complex embedded options or guarantees may be subject to both Type A and Type B market risk.

6.7.1 Standardized Approaches for Type A Risk

6.108 The essential ingredients required to assess Type A market risk are

- projected future asset and liability cash flows
- nature of embedded options
- time horizon
- confidence level
- current economic scenario
- series of possible future economic scenarios.

6.109 Approximations can be made with respect to these ingredients to simplify the Type A risk determination. The result is a range of standardized approaches from the most elemental to approaches that closely compare to the advanced approach.

6.110 One such approximation might use option adjusted durations to represent the price sensitivity of cash flows, the current market value of future cash flows and a set of investment return shocks. The shocks would need to be designed to reflect the time horizon and confidence level desired as well as the possible pattern of adverse scenarios. In this regard, it may be desirable to recognize the more active investment management conducted on closely managed blocks of business (i.e., when the active management holding period is less than the standard one-year time horizon).

6.111 Another approximation might require the grouping of future cash flows into various term "buckets" (BIS uses the term "maturity method"). The sum of the cash flows in these "buckets" would be multiplied by factors to produce the capital requirement. These factors would, in theory represent a combination of the above basic ingredients (i.e., time value of money from current economic scenario, adverse shock for desired confidence level and time horizon, etc.). This type of approach is currently used by the BIS in their standardized approach for banks.

6.112 A very simple approximation, which depends heavily on broad decisions about the industry's generalized exposure to Type A risk, is to simply multiply the balance sheet value of insurer assets and liabilities by a table of factors reflecting the presumed presence and size of Type A risk.

6.113 The relative merits of each type of approximation need to be viewed by the supervisor in light of local conditions, expertise and inherent industry risk. Objectivity and ease of calculation need to be balanced with greater accuracy, complexity and the overall impact of the method chosen on the management of market risk by insurers and the types of products that are offered in the market place.

6.114 To develop standardized approaches for market risks (or other risks for that matter) requires judgement and supervisory tradeoffs depending on the supervisor's choice of approximation and its method of application. Ideally, the conservatism inherent in a standardized approach should incent insurers to use (to the extent they are able) more advanced methods in the future.

6.115 One possible concern in designing more advanced approaches that allow judgement to be used by the insurer (e.g., if the degree of market risk is subject to the asset allocation practices of the insurer) is that the results will be less transparent since there may be opportunities for the insurer to self-select (to some extent) the resulting solvency requirement. It is important for the supervisor to consider in advance the possibilities and significance of such self-selection and to weigh this risk against the risk of accepting a factor approach which (through the use of broad or industry factors) may not recognize fully the risks of a specific insurer. For example, the concern surrounding asset allocation "games" can be addressed directly through a requirement that asset allocation for purposes of the capital requirement must coincide with the insurer's management of their business.

6.116 Particularly in life insurance, some market risk from the total asset portfolio may be transferred to policyholders. This is generally the case in Universal Life business and many forms of adjustable and "with profits" business. Clearly, such assets and the corresponding liabilities should be closely matched (ignoring the non-financial diversifiable risks that may affect these liabilities) and the degree of such sharing of market risk needs to be reflected in the chosen standardized approach.

6.117 The following subsections outline some important aspects in selecting a standardized approach for certain sources of market risk as well a possible treatment of dependencies.

6.7.2 Standardized Approaches for Type B Risk

6.118 By definition, the development of standardized approaches for capturing Type B risks is fraught with difficulty. Where these risks are material in an insurer, the supervisor should encourage or even require the insurer to perform appropriate advanced approaches to modelling their Type B market risk.

6.119 Standardized approaches to assessing Type B market risk might include

- For long-term interest guarantees in life insurance and annuity products, determining the present value of future liability cash flows on the presumption that long-term reinvestment returns revert to a conservative view of historical long-term averages.

- For complex options, deriving appropriately conservative factors based on rigorous stochastic modelling of industry-wide data to adequately capture the tail of the loss distribution for the confidence level required.

6.8 Operational Risk

6.120 For the reasons described earlier in this report, the WP believes it appropriate that an insurer's overall requirement contain a component for operational risk. However, the current shortage experience data in this area will require the determination of an appropriate level for this component of the overall capital requirement will be subject to judgement by the supervisor.

6.121 The approach adopted within Basel II by the banks may be worthy of consideration for insurers. The Basel II approach provides for a "standard", "basic indicator" and "advanced measurement approach" (AMA). The first two approaches are based on simple multiples of gross income. These simpler approaches are not risk-sensitive. Only the AMA allows the banks credit for various risk management techniques.

6.122 The WP suggests that a reasonable level at which to introduce an operational risk capital requirement may be in the range of 10-20% of the otherwise determined capital requirements. This amount may be actually calculated under a "standard" approach by applying a single percentage (or table of percentages) to one (or a combination) of readily determinable items such as:

- underwriting risk capital requirement
- credit risk capital requirement
- market risk capital requirement
- net earnings
- assets under management

6.123 Alternatively, under a "basic indicator approach" a set of adjustment factors or α's could be applied to allow the "standard" approach to be modified by major line of business.

6.9 Final Steps

6.124 The WP development of the standardized method has offered alternative approaches for developing factors based on risk exposures within each of the four major risk categories described above. These alternative approaches allow supervisors to choose the desired balance of simplicity and realism within each major risk category that is most appropriate for the given supervisory regime.

6.125 There are also alternative approaches to adjust for risk interdependencies (where not all of the risks can go bad all at the same time). These alternatives allow the supervisor to define the desired balance of simple measures and realistic capturing of aggregate company risk. The

general approach was described previously in paragraphs 6.19 & 6.20 and is also described in section 9.3.   Appendix I discusses the theory and value of copulas, which allow a supervisory framework to go beyond simple summing of risks and even beyond efforts to use the square root of the sum of squared risks.

6.126   It must also be remembered that standardized methods, by their nature, may not capture all types of risk accurately or at all. However, within the overall multi-pillar supervisory process all types of risk must be addressed. If risks are not adequately captured within Pillar I (e.g., perhaps liquidity, strategic, legal, etc. risks) then they need to be addressed within either, or both, of Pillar II and Pillar III.

7. Advanced Solvency Assessment

7.1 Introduction

7.1 The standardized approaches for determining capital requirements described in the previous section are intended to be applied uniformly by all companies of a fixed type, life, general or health, in a jurisdiction. Since they are meant to determine adequate capital for a wide variety of insurers, these methods are generally conservative. A jurisdiction may have large and technically sophisticated insurers for which these methods overstate required capital.

7.2 A jurisdiction may wish to allow certain companies to calculate required capital using methods that more directly reflect each company's particular exposure to risk. This chapter is devoted to a description of possible company-specific methods and to a discussion of issues a supervisor must deal with when allowing a company to apply these methods.

7.2 On Adopting Company-Specific Approaches

7.3 The notion of company specific approaches is found in several places in the Basel Accord. The earliest occurrence is contained in the section on the provision for market risk in a bank's trading book of assets. Here, banks may be allowed to use their own internal models, subject to specific requirements and conditions being satisfied. These requirements and conditions are discussed at length below. The proposed revision to the Accord, commonly referred to as Basel II, contains several additional examples of company specific approaches, generally labelled as advanced approaches, applied to credit and operational risks. The Working Party's suggestions represent an extension to insurance of the approach taken in the Basel Accord with respect to banks.

7.2.1 Similarities to and Differences from Basel II

7.4 However, there is a significant difference between banking and insurance. In the Accord, the standardized approach involves measures of a bank's exposure to risk that are based upon standard accounting conventions that do not involve any element of discretion or choice by the particular bank. This cannot be the case in insurance since many of the most common measures of exposure to risk for an insurer are related to (actuarial) policy liabilities. Actuarial liabilities, whether they are policy reserves for future claims or claim reserves for claims that have occurred but whose amount and time of payment are not known, involve estimates of uncertain future financial values. Although some of these liability amounts may be based upon standard assumptions that are set by law or regulation, the majority of calculations are based upon best estimates selected by a company's actuaries. The exercise of choice by a company in determining its liabilities is fundamental for insurance in a way that is not found in banking. The standardized approach suggested by the WP makes use of policy liabilities. It follows that the distinction between standardized and company-specific approaches is not the same in this report as it is in the Basel Accord.

7.5 The phrase *company-specific approach* means a method of determining an insurance company's capital requirement with respect to a particular source of risk that measures the intensity of the risk in relation to the company's own experience or the structure of its portfolio of business. By contrast, a *standardized approach* is one that is based upon standardized factors that measure the intensity of risk, applied to measures of the company's exposure to risk, or is based upon differing measures of the company's exposure to that risk (e.g. the difference between policy liabilities based upon the company's standard assumptions and those based upon specified variations in these assumptions).

7.2.2 Conditions for Approval

7.6   When a company calculates the component of required capital with respect to a specific source of risk by means of a company specific approach, it is to be expected that the result will be less than the value for that component that would result from a (more conservative) standardized approach. It is therefore necessary to consider the conditions under which the supervisor could be comfortable with this result. There are two sets of conditions: the first set involves the appropriateness and accuracy of the particular approach taken by the company while the second involves the actions of the company to manage and mitigate the particular risk.

7.7   The supervisor must have assurance that an insurer's company specific approach is appropriate and gives an accurate measure of required capital. This requires a detailed analysis of both the company's methodology and of the specific company data used in the calculation. The particular methodology used by a company will vary. It is particularly important that the supervisor examine it and have confidence that it is theoretically correct and properly implemented. In order to adequately evaluate the insurer's methodology, the supervisor must either have its own technical expertise or have access to independent outside experts. The supervisor must be satisfied with the integrity of the company's data that will be used in the calculation. Of major concern will be the data's sufficiency and credibility and the statistical methods used to organize and analyse the data. The qualifications of the insurer's personnel associated with this approach will also be of concern.

7.8   Required capital can be thought of as a second line of defence protecting an insurance company's solvency and its policyholders. The first line of defence is solid risk management. If trouble develops that cannot be prevented through management of a risk, then risk based capital should be available to cover the financial losses that emerge. It follows that in order for a supervisor to be content with a lower amount of required capital under a company specific approach, there must be some assurance that the particular source of risk is under control, its effects are well mitigated and there is a reduced need for the required capital. Therefore, in approving a company's use of an advanced or company specific approach, the supervisor should confirm that the company has in place appropriate risk management processes together with a satisfactory reporting structure.

7.3   Examples of Company Specific Approaches

7.9   There are a wide variety of company specific adjustments that could be introduced into a determination of an insurer's required capital. This report cannot possibly present more than a few of them. Our purpose here is to illustrate some company specific approaches that could be introduced and to point out some of the safeguards and conditions that a supervisor could require or impose before allowing an insurance company to adopt a company specific approach. A supervisor who has understood the approach and the reasoning outlined here should be in a position to adapt these examples to the specific products, insurance markets and legal systems in the particular jurisdiction.

7.3.1   Credit Risk in Basel II

7.10   Under Basel II, a bank's capital requirement for credit risk depends, in insurance terminology, on the frequency of asset defaults and upon the severity distribution of amount of loss give that default has occurred. The Basel II proposal offers two company specific approaches. In the first, a bank may make use of frequencies of default based upon its own asset quality ratings and frequencies of default while using standardized severity distributions. In the more advanced approach, the bank may also use its own severity distributions.

7.11   A bank would have to have experienced considerably more asset defaults in order to derive credible severity distributions than are required to derive credible frequencies. It is therefore to be expected that the more advanced approach would only be available to the largest and most technically sophisticated banks that have experienced extensive defaults, are able to thoroughly analyse their credit experience and have very sound risk management systems in place.

7.12   Banks originate many of their assets through their lending activities. These assets can usually only be assigned a quality or credit rating through use of a bank's own rating system. The determination of required capital for the credit risk makes use of credit ratings; the use of a bank's own rating system is regarded as an advanced approach under Basel II. While some insurers do invest in private placements, the bulk of their investments are in publicly traded assets that usually have been rated by a recognized rating agency. It follows that advanced approaches with respect to credit risk are generally more important for banks than for insurers. However, insurance supervisors should nonetheless pay attention to the ratings and attendant assumptions regarding the frequency and cost of asset defaults that are used by insurers.

7.3.2   Risk Pass-Through Products

7.13   A number of insurance products contain features under which the insurance company's experience (perhaps as measured by its financial results) under these policies is shared in whole or in part with policyholders. If this sharing mechanism is effective in reducing the risk to the insurer, it would be appropriate to recognize this in the calculation of required capital. The sharing mechanism is bound to depend upon the specific product design and the methods that the company employs to administer the business and operate a risk-sharing mechanism. Therefore, recognition given in the calculation of required capital to the reduction of risk to the company will be a company specific matter.

7.14   The type of products that are illustrated here are primarily those of life insurance companies; in particular, we consider participating or with-profits business, as well as certain types of Universal Life and other "new money" or adjustable products.

7.15   The supervisor's primary concern in allowing an insurer to reflect its risk sharing mechanisms in the determination of capital requirements is to ensure that the insurer will actually be able and willing to reflect unfavourable experience in policyholder dividends or bonus scales. The supervisor will want to examine, among other things, the insurer's dividend or bonus policy, its history in administering that policy in the past, the effects of any smoothing mechanism that may be in place, as well as the insurer's competitive position and the perceived effect on the part of company management and the supervisor that a reduction in dividends or bonus due to unfavourable experience would have on that position. The supervisor should take into consideration whether the concept of policyholders' reasonable expectations would inhibit or restrict the company's ability to pass on unfavourable experience to its policyholders.

7.16   Consider, the case of asset default risk for participating or with-profits business. An insurer might assert that the costs of asset defaults are passed on to or shared with policyholders and might then request a reduced capital requirement in respect of this risk. The supervisor would want to know that the assets supporting this line of business are clearly segmented from assets supporting other lines so that the assets to which a reduced requirement might apply and from which defaults might be generated are clearly identifiable. The supervisor would next examine the particular risk sharing mechanism (e.g. dividend scale, bonus scale, smoothing account, experience rating) used by the insurer in this instance. The degree of sharing of experience and the speed with which the mechanism effects that sharing would be reflected in the amount of capital relief that is granted.

7.17 In another product design, the investment income credited to a customer's account may be related to values of a recognized stock market index. The supervisor would want to examine the direct relationship between changes in the index and the amount of earnings (positive and negative) that are attributed to policyholders' accounts. Capital relief could be close to total when the correlation between changes in the index and interest credited to policyholders' accounts is close to one and less as this correlation decreases away from one. In order to gain approval for a reduced capital requirement, the company would have to demonstrate this correlation for historical data over, for example, the previous year.

7.18 It is important to distinguish between risk sharing mechanisms that are retrospective and those that operate only prospectively. The mechanisms described above are all retrospective in nature. They enable a company to share past experience with policyholders. Prospective mechanisms allow companies to adjust future premiums in anticipation of expected unfavourable experience. However, they do not provide any relief to a company that has already experienced significant losses. While they do not appreciably eliminate the need for capital, prospective risk-sharing mechanisms do put the company in a better position than it would have if these mechanisms were not in place. It should be noted that in the standardized approach, a capital requirement may be linked to the period for which future premium rates are guaranteed, with longer term guarantees requiring increased capital. Prospective risk-sharing mechanisms are in the nature of adjustments to future premiums and should be reflected in capital requirements as reductions in the length of premium rate guarantee periods.

7.3.3 Experience Rating

7.19 Retrospective experience rating is a feature of many large group insurance contracts. If an insurer has a binding undertaking from the policyholder to fully share in the insurer's experience with respect to the case, it could be appropriate to recognize this in the calculation of required capital. This would vary by the particular group insurance policy. A reduction in capital could only be granted if the contract wording legally binds the policyholder to pay for case deficits arising from unfavourable experience. Even then, an appropriate provision for counterparty risk will need to be made.

7.20 There are instances where insurers hold policyholders' funds on deposit with the understanding that the insurer can call upon these funds to make up for a case specific experience rating deficit. If the contact wording was sufficiently binding on the policyholder, it could be appropriate to recognize these deposits as offsets to or reductions in required capital. However, this recognition could only be granted on a policy-by policy basis since deposits attributable to one policy could not be used to offset unfavourable experience arising from another policy.

7.4 Internal Models

7.4.1 Introduction to Internal Models

7.21 The company specific approaches discussed above involve modifications to or adjustments of a standardized approach. They recognize the availability of company specific information or the nature and effect on company operations and risk of specific product designs or administrative methods. In this section, we consider methods for determining required capital that are not necessarily modifications of the standardized approach. They are, instead, alternate approaches and methods that recognize directly a company's specific circumstances.

7.22   These approaches are based upon computer models of a specific line of business or segment of a company's activity. They are usually stochastic in nature and directed to determining the amount of capital that will be sufficient to guarantee the success of that portion of the company's business to a high degree of probability. These models depend upon scenario generators that can produce a wide variety of scenarios that can affect the future course of the company's business.

7.23   In general, each company would construct its own model. The model reflects the company's specific product designs, its various administrative policies and procedures, and other practices including investment policy and claims settlement. It is the role of the supervisor to determine the level of probability that is to be tested as well as the length of the future period over which future model projections are carried out.

7.24   The supervisor must validate and approve the model. This approval extends to the scenario generator. In many cases, the primary stochastic element will be the performance of some set of economic indices such as interest rates or equity market averages over time. Here, the supervisor must be satisfied that the generator is consistent with the theory of financial economics and appropriate for use in the particular application. It is not to be expected that this type of scenario generator would vary significantly from one company to another in the same jurisdiction and operating in the same economic environment. In other models, particularly in the case of general (non-life) insurance, the scenario generator may be used to generate claims experience. In this case, the generator could well be specific to the company and the types of business it conducts. The supervisor would have to be satisfied that the generator captures the range of possible claims that the company could experience.

7.4.2   Uses of Internal Models

7.25   Internal models are currently in use under several capital regimes. These include:

- The Basel I capital requirement for banks allows for the use of internal models in setting the capital requirement in respect of market risk with respect to the block of assets held for trading. This is described in the paper:

    – *Basel Committee on Banking Supervision, 1996* Amendment to the Capital Accord to Incorporate Market Risks*, Basel Committee Publications no. 24. Basel, Switzerland. Online at* http://www.bis.org

- The Canadian capital requirement in respect of life insurance companies, MCCSR, allows the use of internal models for determination of the component of required capital for guarantees with respect to investment products known as segregated funds. The MCCSR is described in the paper:

    – *Office of the Superintendent of Financial Institutions Canada, 2003* Minimum Continuing Capital and Surplus Requirement*, Guideline A (Insurance), Ottawa, ON. Online at* http://www.osfi-bsif.gc.ca

- Conditions for use of a model are given in:

    – *Office of the Superintendent of Financial Institutions Canada, 2001* Use of Internal Models for Determining Required Capital for Segregated Fund Risks*, Instruction Guide (Insurance), Ottawa, ON. Online at* http://www.osfi-bsif.gc.ca

- The Australian capital requirement in respect of general (non-life) insurance companies permits the use of internal models. This is described in the paper:

    – *Australian Prudential Regulatory Authority,* Internal Model Based Method*, Guideline GGN 110.2, Sydney, NSW. Online at* http://www.apra.gov.au

7.4.3 Validation Criteria

7.26 The three instances described in the preceding section where internal models have been adopted for required capital calculations embody a similar approach. In particular, they contain similar criteria that supervisors impose before a company's model is approved for use. The essential criteria are described in the following paragraphs.

7.27 To be valid for use in the supervision of insurance, an internal model needs to be demonstrably capable of meeting a number of criteria in respect of prudence, comparability and consistency within the supervisor's jurisdiction.

7.28 *Prudential Requirements:* The insurer must demonstrate that the internal model operates within a risk management environment that is conceptually sound and supported by adequate resources. It also needs to be supported by appropriate audit and compliance procedures. A number of qualitative criteria follow from these minimum requirements:

- The insurer should have an independent internal risk management unit, responsible for the design and implementation of the risk-based capital model.
- The insurer's Board and senior management should be actively involved in the risk control process, which should be demonstrated as a key aspect of business management.
- The model should be closely integrated with the day-to-day management processes of the insurer.
- An independent review of the model should be carried out on a regular basis. (Amongst other considerations, it should be recognised that evolution of the modelling capabilities is to be encouraged)
- Operational risks should be considered

7.29 *Comparability and Consistency Requirements:* The model's output needs to fit closely with the supervisor's view of key minimum performance criteria, such as probability of default and other important measures of financial soundness. Quantitative criteria relating to these needs could include:

- A requirement for the model to calculate the capital needed to keep the annual probability of default below a certain level (or levels)
- An ability for calculating the likely spread of economic costs relating to a range of potential outcomes for the business, etc.

7.30 In addition the model should include the capability for specification of the key risk factors for the insurance business. These would include factors relating to both assets and liabilities including:

- Measurement of cash flows for both assets and liabilities
- The risk of changes in outstanding claims valuation due to changes in economic, environmental or experience-related factors
- The risk of changes to the adequacy of premium rates due to changes in economic, competitive or environmental factors
- Catastrophe concentration risk
- Expense risk; and
- The reinsurance security risk and risk of reinsurance cost variability

7.31 The model should include a facility to enable comparability of correlation effects between risk classes as well as a system of stress testing and other scenario-based examinations.

7.32 The model should be in a format to allow a reasonably straightforward detailed review by appropriately skilled representatives of the supervisor to enable a relatively "painless" approval procedure. Note that the preceding validation criteria should be viewed as minimum requirements and different jurisdictions may require stronger conditions for the validation of models.

7.4.4 Internal Models and the Valuation of Liabilities

7.33   The WP has indicated elsewhere in this report a preference for a Total Balance Sheet approach to the setting of capital requirements. This approach is particularly appropriate in situations where the present value of the insurer's future cash flows can be treated as a random variable whose distribution is derived using an internal model. In this case, policy liabilities can be determined at one point in the distribution and the sum of liabilities and required capital at another. Utilization of the same model, and indeed the same set of calculations, for the determination of policy liabilities also serves to satisfy the requirement expressed in the preceding section that an internal model should be closely integrated with the day-to-day management processes of the insurer and not be used solely for the calculation of required capital.

7.4.5 Internal Models and the Standardized Approach

7.34   There may exist situations in which a stochastic approach is the most natural approach to take in valuing a particular type of liability and determining the associated capital requirement. However, the insurance industry within a jurisdiction may not be at a state of technological readiness that would permit the introduction of internal models by all insurance companies. A possible solution would be to apply a generic model to data collected across the industry. The results of these calculations could be used to determine standardized factors that could be applied to various companies' measures of exposure to the particular risk. Since these factors would apply uniformly across the industry, they would be chosen with a conservative bias. In this way, an internal model could be used to develop a standardized approach for the particular risk. For the larger and more technically sophisticated insurance companies, the conservative bias of the factors would provide incentive to seek early approval for the use of their own internal models.

8.   Reinsurance

8.1   One of the most important risk management tools available to all types of insurers is reinsurance.

8.2   Reinsurance refers to insurance purchased by an insurer to provide protection against some or all of certain risks of the insurance policies issued by the insurer. In exchange for the assumption of these risks, the reinsurer receives payment in the form of reinsurance premiums or allowances from the direct writer of the business, the insurer. The insurer cedes either proportional amounts through quota share and surplus contracts, or losses exceeding a predefined threshold through non-proportional arrangements such as excess of loss or stop loss contracts.

8.1   Reasons for Purchasing Reinsurance

8.3   Reinsurance is purchased for different reasons. One can generally distinguish between two main objectives: one is the genuine transfer of risk with the goal of risk mitigation, the other one can be described as a risk transfer for the purposes of managing/spreading risk over time or achieving strategic business objectives.

8.4   Genuine risk transfer reasons primarily include

- Limiting large or catastrophic claims. Such coverage generally provides for the reinsurer to pay claims in excess of a certain limit, subject to a minimum number of claims and subject to a maximum amount of reinsurance per event. This coverage provides protection against concentrated claims arising from a single event (e.g., catastrophic events such as storms, earthquakes, or large loss events like plane crashes, loss of property, etc.).

- Limiting Total Claims. Some insurers, especially smaller ones, have need of stop-loss reinsurance to limit the aggregate amount of claims in a given year.

8.5   Strategic or financial objectives include

- Increasing new business capacity. One of the most common reasons[4] for the purchase of reinsurance is to enable an insurer to issue insurance policies with larger coverage limits or amounts than it would prudently issue on its own. If the insurer has no retention limit or it is set too high, the insurer runs the risk of insolvency if several large claims occur in a short period of time.

- Investment Risk Transfer. Insurers may reinsure a block of business to effect a transfer of investment risk from the insurer. This can occur because of the growth of interest-sensitive life and annuity products to either take advantage of reinsurer asset management capabilities or to avoid a large concentration of assets arising from a single product or annuity.

- Financial Results Management. Insurers can utilize the financial reporting impact of reinsurance agreements to optimize the insurer's earnings and surplus objectives and also to minimize taxes.

8.6   A mixture of both objectives can be achieved by reinsurance arrangements providing for:

- Gaining Product Expertise. Upon entering a new line of business, product or territory, an insurer may request the assistance of a reinsurer with existing experience in that market. In exchange for their advice, the reinsurer will participate via reinsurance in the future profitability of the business sold.

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[4] Tiller J.E. Jr., Tiller D.F., 1995, *Life, Health & Annuity Reinsurance, Second Edition*, ACTEX Publications Inc.

- Underwriting Advice. A benefit provided by reinsurers is their experience in underwriting. This can prove valuable during the design, pricing and underwriting of products, especially new, novel, large or complex ones.

- Divesting a Product Line. An insurer wishing to exit from a certain business, product or territory may choose to cede that business by means of an assumption reinsurance agreement or through indemnity reinsurance, thereby transferring a loss portfolio.

8.7   Finally, reinsurance is crucial to the viability of many, in particular smaller companies. Small or mono-line companies use reinsurance as a diversification and risk reduction tool, so that they can compete effectively against large diversified companies. Note that reinsurance protection is normally at the price of a reduction in the expected earnings of the cedant. This reduction of expected earnings reflects a "fair price" for the risk transfer. For many companies, buying reinsurance is a good economic decision despite the reduction in expected earnings. For them, the risk reduction from buying reinsurance may outweigh the economic cost of putting up the additional required capital.

8.2   Types of Reinsurance

8.8   Reinsurance covers typically have two different types: proportional or non-proportional. Proportional reinsurance covers are quota share of surplus covers, while non-proportional covers comprise excess of loss or stop loss contracts. Both types are often, mixed or aggregated.

8.9   A quota share cover assumes a contractually fixed percentage of each and every loss in exchange for the same percentage of premium income ceded by the insurer. This proportional mutual sharing of benefits as well as losses can be adjusted contractually through profit sharing such as sliding commissions and the like. In terms of risk mitigation, quota share reinsurance takes a fixed percentage of each and every loss, thereby simply "compressing" the risk profile. This means that the expected loss, or any percentile (such as the 99th percentile), reduces by the same percentage (i.e., the share ceded to the reinsurer). This is illustrated in the following exhibit.

Effect of Quota Share Reinsurance on Loss Distribution
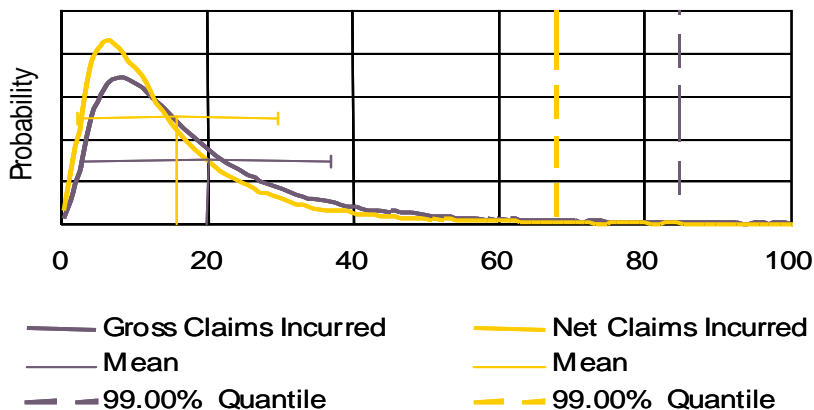
8.10   Excess-of-loss reinsurance provides that for each and every loss amount exceeding a predefined threshold, the so-called priority or attachment point, (e.g., US$ 1 mill.) is covered by the reinsurer, up to a certain limit. Under an excess of loss cover with priority P and cover limit L, the reinsurer assumes for each loss X incurred by the cedant:

$$\text{Max } (0 \; ; \; \text{Min}(X\text{-}P; \; L))$$

8.11   The effect of excess of loss reinsurance is that it truncates the loss distribution for the cedant at the priority, implying that any loss amount exceeding the priority will be assumed by the reinsurer, subject to not exceeding the limit per claim. This can be easily seen in the following exhibit, for an excess of loss treaty: the XL attaches in excess of 50 million. The net curve shows that there is zero probability of loss amounts higher than 50 million (unless losses exceed the limit) and therefore the distribution has a mass point at the priority. The gross loss distribution, however, extends well beyond 50 million with a 99th percentile at about 85 million.

Effect of Excess of Loss Reinsurance on Loss Distribution

	Gross Claims Incurred		Net Claims Incurred
	Mean		Mean
	99.00% Quantile		99.00% Quantile
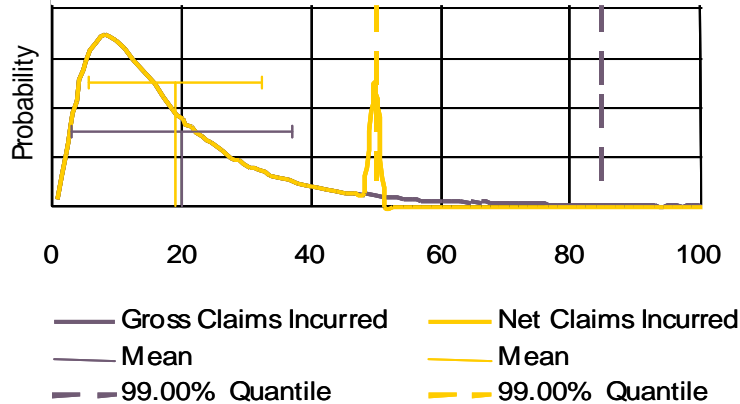
8.12   Excess of loss reinsurance applies on an individual claims basis rather than on a portfolio basis like proportional contracts. Repeat coverage for more than one claim is usually achieved through refilling the coverage by paying reinstatement premiums. Excess of loss coverage is provided on a per event basis covering one or multiple claims arising from one and the same event, or on a per risk basis where the coverage applies to one risk independent on the event affecting the insured risk.

8.13   Most importantly, the reduction in the expected loss for the excess of loss contract is minimal compared to the corresponding mean reduction of the quota share. However, the risk reduction of the excess of loss contract (e.g., measured at the 99th percentile) is vastly higher at approximately 35 million, while the quota share reduces risk only by approximately 15 million, thereby demonstrating the different transformation characteristics of proportional versus non-proportional reinsurance contracts.

8.3   Effect of Reinsurance on the Risk Profile

8.14   As outlined in the preceding section, any analysis of a company's risk profile (and therefore its capital requirement) is incomplete without proper treatments or recognition of its reinsurance arrangements. Reinsurance contracts typically have significant impacts on the company's aggregate risk profile, usually with the effect of reducing risk, and thus are important considerations for the capital requirement of an insurance company.

8.15   The following graph illustrates the effect of risk reduction through reinsurance on the company's results, the sum of premiums minus expenses and losses. While proportional reinsurance typically reduces the overall (nominal) risk in a linear way, non-proportional covers typically address the large losses, thereby reducing the company's net exposure to large loss/catastrophic events. Technically speaking, non-proportional reinsurance eliminates part or all of the volatility coming from the tail of the distribution.
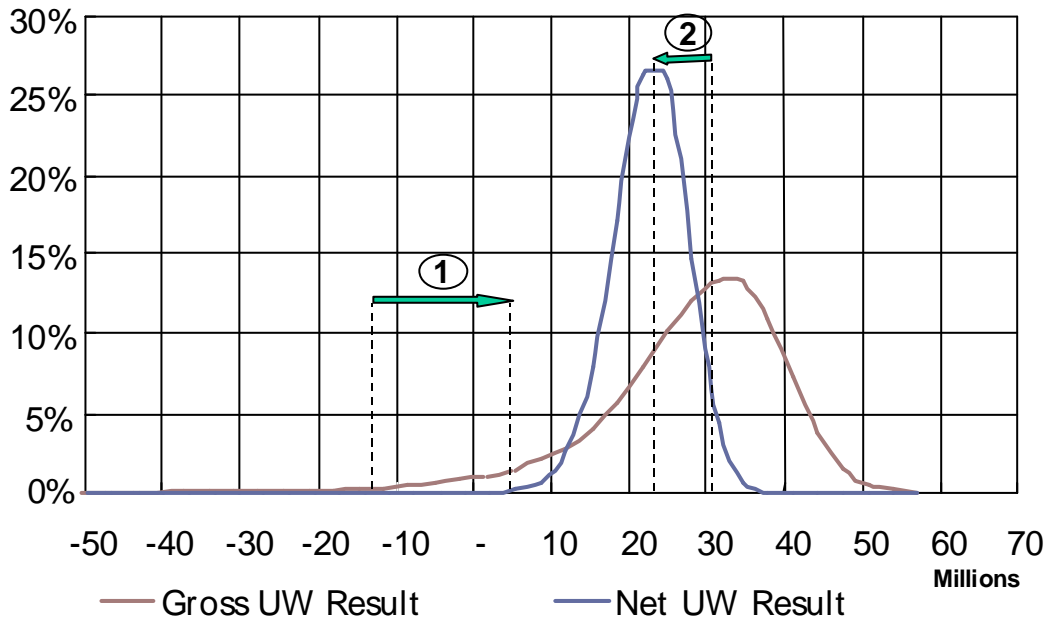
1: Reduction in TVaR, thereby reducing the extreme tail of the distribution
2: Sacrifice in earnings through premium due to reinsurers

8.16 The probability distribution in the above graph shows how the 99th percentile of the probability distribution of losses is shifted to the left, indicating a reduction in risk. Note in particular that the tail of the distribution is reduced materially, if not even eliminated.

8.17 Reinsurance, in particular the non-proportional type, can greatly reduce, or even eliminate, the extreme tail of the cedant's loss distribution. This effect can be assessed mathematically if the TVaR risk measure is being used. On the other hand, the reduction in standard deviation (in the WP's opinion this is an inferior risk measure for a capital requirement) can be disproportionately less. If applied properly in a solvency or management context, reinsurance is a very efficient means of reducing risk (particularly if measured by TVaR) and therefore risk-bearing capital. Reinsurance can therefore be a useful alternative for (solvency) capital.

8.18 Given the above discussions, it is obvious that a proper recognition of reinsurance is a must to assess the risk reduction for the ceding company with implications for its capital requirements in order to ensure effective supervision of insurance enterprises in relation to solvency and capital requirement.

8.4 Challenges in Assessing the Impact of Reinsurance on a Company's Risk Profile

8.19 While proper treatments and recognition of reinsurance arrangements are necessary to assess the impact of the of a ceding company's risk profile, this is a difficult task for a number of reasons.

8.20   The first complexity comes from the tremendous diversity in the types of reinsurance contracts:

- typical reinsurance arrangements comprise both proportional and non-proportional covers
- some contracts have variable rating terms, such as sliding scales or loss corridors for a proportional reinsurance treaty, and reinstatements or contingent commissions for an excess-of-loss treaty
- some contracts cover just one line of business, others cover multiple lines of business and others cover single loss events only
- some contracts are on an aggregate basis, with aggregate deductibles and aggregate limits
- some financial type reinsurance contracts cover a hybrid of underwriting and financial risks.

8.21   The second complexity comes from the fact that *many reinsurance contracts do not bear a linear relationship with the underlying risks*. For instance, there is a leverage effect of claim inflation on the loss costs of excess-of-loss covers. In fact, the contracts transforming the overall risk into a "narrower" risk profile typically are exactly of this nature. The magnitude of the leverage effect depends on the sizes of the retention (attachment point, or priority) and the limit[5].

8.22   A properly structured reinsurance program can significantly reduce the cedant's risk exposure and capital requirement. However, *not all reinsurance warrants a reduction in the capital requirement*, in particular when it is inadequate. This introduces the third complexity of reinsurance:

- If improperly designed, a reinsurance program may be inefficient in reducing the total risk of the cedant.

- Some reinsurance contracts do not contain significant risk transfer and are mainly used for some specific accounting or tax effect. For instance, U.S. statutory accounting does not allow immediate recognition of the equity in unearned premium reserves; this created incentives for some companies to purchase proportional reinsurance treaties with ceding commissions as a surplus relief. As another example, U.S. statutory accounting does not allow for discounting of loss reserves; this created incentives for some companies to purchase loss portfolio transfers to indirectly achieve the effect of loss reserve discounting.

- Some reinsurance contracts may have credit risk exposures, that is, the loss recoverable may be non-collectable in the cases of contract dispute or reinsurance failure[6].

8.23   The fourth complexity lies in the fact that *the reinsurance recoverable may be highly correlated with the cedant's net risk exposures*. This correlation may go beyond simple linear correlation for excess-of-loss treaties.

8.5   Implications for Recognition of Reinsurance in a Future Solvency System

8.24   The recognition of reinsurance for solvency purposes must be closely linked to the ability of the company, supervisor or both to assess the impact of the reinsurance program of the risk profile. Given the diversity and complexity of reinsurance contracts, it is apparent that a simple factor-based approach is likely to be too crude to reflect the effect of reinsurance on capital requirements accurately. Therefore, standardized (e.g. factor-based) approaches should be used with caution since the proper treatment of reinsurance really requires a modelling approach. Similarly, if the gross risk profile is not, or is only very vaguely described, the proper recognition of reinsurance in terms of risk reduction is not possible.

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[5] While many European countries use an index clause to stabilize the impact of reduction in the risk profile, the index clause is not commonly used in the U.S.

[6] For a discussion on reinsurance credit risk, refer to section 5 and Appendix E

8.25   For the assessment of the reinsurance impact on the risk profile, there are two general methods of evaluating the amount of risk transfer in a reinsurance contract:

- judging the amount of risk transfer for a reinsurance contract by analyzing whether the cedant has transferred (reduced) risk, *on an enterprise-wide basis*
- judging the amount of risk transfer for a reinsurance contract by *focusing on a stand-alone single transaction* as defined in the contract.

8.26   According to the enterprise-wide approach, the impact of the entire reinsurance program on the risk profile needs to be evaluated. The assessment of individual contracts on a per risk or line of business basis, particularly of an excess of loss nature, is practically impossible. However, for proportional transactions, such as especially whole account quota shares, or stop loss, agreements can be evaluated on an approximate basis. For example, a whole account stop-loss limits the maximum downside of the underwriting result and thus, assuming the reinsurer performs, the maximum capital at risk can be quantified.

8.27   Except in the case of a stop-loss arrangement, a risk-based solvency assessment is impossible in the absence of reliable aggregate loss distribution data and exposure information. In this situation, a prudent supervisory approach would be to give no credit for the purchase of reinsurance.

8.28   This said, most companies have some exposure information for at least a few lines of business. Typically, property lines have at least exposure profiles and these can be used as a proxy. In addition, those profiles could be compared to industry data where available and blended to achieve a proxy for the company's risk profile. Combining the profile of several lines in an additive manner, thereby not allowing for diversification effects and introducing a level of conservatism to the proxy, an approximate total company profile could be derived.

8.29   Provided that an adequate internal enterprise-wide risk model is available however, one can evaluate the effect of all reinsurance contracts in a consistent manner. Basically, one can use the internal model to evaluate the total capital requirement on a gross basis (without reflecting reinsurance), and then on a net basis (net of reinsurance). Presumably, the internal model can reflect all the intricacies of the reinsurance contract terms. The enterprise-wide method is desirable from a total balance-sheet modelling perspective. However, it can be quite a challenge to model all parts of an enterprise and their interactions properly.

8.30   In summary, in the absence of an internal enterprise-wide model the risk reduction relative to an expected shortfall measure is virtually impossible to quantify reliably, with the possible exception of a stop-loss arrangement, and in such circumstances the reinsurance credit should be minimal or not given at all.

8.31   For the assessment of the risk transfer under a particular reinsurance contract, one can perform the risk modelling of the cash flows between two parties based on the contract terms, without referring to a full enterprise-wide model. This can be a much more pragmatic method when a satisfactory enterprise-wide model is not available. In practice, many companies have adequate partial risk models describing the risk profile for some of their segments. This is particularly the case for property coverage.

8.32   In those cases, the risk characteristics can be described by segmenting the underlying contracts into homogenous "risk buckets" describing the exposure of the underlying risks in terms of insured value, retention, policy limit and maximum loss/PML, to name a few. This data can then be used to derive a gross risk profile of the portfolio to be insured using frequency and severity/expected loss information. These gross loss distributions can be used to adequately apply proportional reinsurance transactions including loss-sensitive features as appropriate[7]. This said, the proper evaluation of the risk reducing impact of non-proportional reinsurance contracts

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[7]   Refer to the case study for non-life insurance for a general approach to practically apply such a routine.

is still not possible without either relatively complex mathematical transformations, which are typically beyond the of supervisory control mechanisms, or the use of simulations, which are standard routines for more complex risk modelling in internal models.

8.33   Ultimately, the most adequate assessment of the risk transfer capability of a reinsurance contract or a combination of several contracts, is through the description of risk using detailed loss and exposure data. From this information, possibly blended with industry data, the company can derive specified and validated loss distributions. These gross distributions can then be fed into routines transforming gross simulated loss samples into a net distribution by applying the relevant reinsurance terms to each figure and aggregating the transformed simulations into a net outcome. While this approach is obviously laborious and more time-consuming than the other approaches outlined above, it more adequately and reliably describes the risk reduction achieved by reinsurance.

8.34   While the approach described above can be applied for each "risk bucket" (e.g., line of business), it does not resolve the aggregation of the individual risk profiles into a total company profile as this process would need to consider dependencies between risks, like concentration or diversification. A discussion of this issue, and possible solutions, are presented in the following sections of this report.

8.35   In summary, the WP concludes that the possibility to adequately reflect the risk reducing impact of reinsurance crucially depends on the ability to reliably come up with a risk profile of the portfolio to be reinsured. The less information is available and the cruder the model is, the less adequately the impact of reinsurance can be assessed, and consequently, the less credit should be given. Conversely, detailed and consistent risk information enabling a company to describe its risk profile properly (e.g., such as in an internal model - even if it is only for part of its business), allows the evaluation of the impact of reinsurance and the corresponding credit for the purpose of solvency assessment, to be given proper consideration by the supervisor.

8.6   Reinsurance Credit Risk

8.36   Reinsurance arrangements often generate a long-term relationship between cedant and reinsurer. The reinsurer typically collects premium at contract inception and remunerates for losses falling under the policy as they are reported and paid. Obviously, the stability of the relationship crucially depends upon the financial strength of the reinsurer. Occasionally, reinsurance recoverables are not collectible as the reinsurer is either unable or unwilling to perform (i.e., when the reinsurer becomes insolvent or there is dispute regarding the coverage).

8.37   To recognize the credit risks on the reinsurance recoverable, a factor θ (say, $\theta = 70\%$) can be applied to the full amount of capital relief derived from having a reinsurance arrangement in place. The factor θ may vary depending upon

- the financial stability of the reinsurer (e.g., as expressed in quality rating)
- the amount of collateral being posted
- the nature of the reinsurance (i.e., short versus long tail)
- concentration risk (one reinsurer versus several)

8.38   The charge for reinsurance recoverable, θ, should be in line with the charge for bond defaults with similar default frequencies

8.39   As a consequence, the net capital requirement for the ceding company after reinsurance can be derived very simply and is equal to (assuming a linear ρ):

$$\rho(X) - (1-\theta)\, \rho(X_{\text{cede}}) = \rho(X_{\text{net}}) + \theta\, \rho(X_{\text{cede}}).$$

8.40   Where ρ denotes the risk measure applied (eg. Tail Value at Risk) and X denotes the aggregate loss.

9.   Total Company Requirement

9.1   Concentration

9.1   Concentration risk is the risk of having higher-than-normal relative risk exposure in a single risk. For example, investment of a high proportion of assets in a single economic sector might be considered concentration. In practice, concentration risk can result in a "penalty" in capital requirements; that is, more capital is required. Concentration is the opposite of diversification.

9.2   Diversification

9.2   Diversification reduces risk to the extent that less total relative capital is required when combining two risks. In practice, diversification benefits should be reflected in a capital formula to encourage insurers to have more diversified assets and liabilities.

9.3   Risk Dependencies

9.3   The risks an insurer faces often exhibit comovement or dependencies. This means that knowledge about results for one risk can be used to better predict the results of another risk. Dependence between two risks may be because there are known relationships between these two risks or simply because certain correlations or other relationships have been observed historically. Dependence can increase or decrease the capital required to support the combined two lines. If losses for one risk tend to increase as the losses for the other increases, there is a positive correlation, usually resulting in more capital required than if the two risks are mutually independent.   Similarly, if one tends to increase as the other decreases, the two risks form natural hedges and usually require less capital. If an insurer builds an internal model, it needs to reflect the nature of all significant dependencies. Similarly, with factor-based models, the formula used to combine risks needs to reflect all significant dependencies.

9.4   As in previous sections, we consider the total risk to which an insurance company is exposed (e.g., stemming from its insurance operations, investment activities, currency movements, etc.). Typically, the risks to which a company is exposed are not independent, but rather have some, sometimes minor and often difficult to observe, interaction. In many cases and unlike to financial markets, there is very little historical data to detect and quantify the real relationship between risk factors. Hence, it may not be possible to identify all sources of interaction and build them into an internal model or even estimate their correlations or related measures of interactions.

9.5   The comovement of risks faced by an insurer can be the result of two general types of dependencies: structural or empirical.   The structural comovements are due to known relationships, which can be accounted for in a modelling exercise; while empirical comovements are simply observed without any known (or capable of being modelled) relationships. Structural dependencies include situations where loss variables are driven by common variables. For example, economic factors like general economic inflation can drive costs in various lines of insurance in the same direction. Similarly, common events or "shocks" such as an automobile accident, can trigger several related claims (bodily injury, property damage, etc) simultaneously. Other common factors can drive losses in opposite directions. For example, improving mortality reduces costs for life insurance while increasing costs for life annuities.

9.6   In addition, the degree of dependency of insurance risks leading to comovement may increase in extreme outcomes of the risk. Actual examples in insurance include the catastrophes of September 11 affecting not only aviation insurance, but also property, business interruption, workers compensation, life, personal accident and several other lines of business. Similarly, it is easy and logical to imagine that major natural catastrophe, such as a California earthquake occurring on a weekday morning, would affect both property (catastrophe) insurance as well as workers compensation, lines of business that are typically regarded as largely independent (the company may also own property in a catastrophe area).

9.7   Structural dependencies can be modelled directly in internal models and reflected appropriately in factor-based formulas. This is illustrated in one of the case studies where the level of claims costs is determined by a common inflation risk factor. In this case, all claims will be larger if there is general inflation. The uncertainty about the level of claim amount (i.e., inflation) is the uncertain risk factor.

9.8   For many types of risks, particularly in property and liability areas, correlation in movements are observed, but may not be easily explained. In many cases, correlation may be understood by general reasoning, but may not be easily measured due to scarcity of data. This is especially the case for rare events, which may trigger various types of claims.

9.9   It is therefore necessary to find methods or models to describe dependencies both in the absence of reliable or scarce data as well as the increasing dependency in extreme events, i.e. in the tails of the probability distributions describing the risks. It may be possible to model dependencies directly if their nature is well understood. However, it is more likely necessary to construct dependency models that reflect observed and expected dependencies without formalizing the structure of those dependencies with cause-effect models. The theory of copulas provides a comprehensive modelling tool that can reflect dependencies in a very flexible way.

9.10   While structural dependencies are modelled directly in an internal model, empirical dependencies are most easily modelled using specific dependency models called copulas. Copulas describe the relationship between the quantiles of distributions of different risks. They can be selected in order to recognize so-called "tail dependencies" where dependencies only occur, or only appear, in extreme circumstances. Appendix I gives an overview of some technical aspects of dependency modelling using copulas.

Appendix A   Life Insurance Case Study

A.1   Introduction

1.1   This life insurance company case study has been prepared by the WP to illustrate some of the concepts discussed in this report. The main purpose of the case study is to describe calculations that a company might undertake in order to determine total solvency provisions for various risks, and to highlight some of the issues in these calculations.

1.2   The case study describes what might be considered an advanced approach, through the use of an internal company model to quantify the risks. Standardized approaches can be implemented as an approximation to the more advanced approaches, or as a minimum capital requirement, in the event that advanced internal modelling is not possible

1.3   With this objective in mind, the ultimate goal of this case study is to illustrate some of the concepts for advanced internal modelling, and to highlight some of the issues that standardized approaches must address in their formulation.

1.4   The case study has been designed with a focus on advanced models because of the complex nature of the life insurance business relative to shorter duration businesses such as banking or non-life insurance. Generally, the risks facing life insurers are of long duration, and tend to have complex interactions between them. In addition, there are a wide variety of products and management practices in different jurisdictions around the world. Thirdly, it is difficult to find simple common risk metrics can be used in a standardized approach. It would be overly simplistic, for example, to use sum assured as a base to apply factors, because this does not reflect important risk factors such as age or sex. A basis such as premiums might reflect these risk factors but also introduce the company's pricing philosophy in the local market into the equation, which would further complicate the factor development process. Another basis might be a prospective cash flow based valuation performed on a best estimate basis. This type of basis would again capture all the relevant risk factors, but introduces additional complexities such as what to do where the resulting liabilities are negative, or do not fit into the industry norms in some way (e.g., unusually large or small liabilities).

1.5   These characteristics make it difficult to create a simple standardized model that appropriately captures all risks across all jurisdictions. The starting point for developing a standardized approach in any particular jurisdiction would be to first understand how the risks behave using a more complex advanced model. This advanced model could then be simplified to arrive at a standardized approach.

1.6   There are two types of standardized models that could be conceived. The first could be described as a pure standardized approach, in which factors are developed that companies can apply to common exposure measures such as premiums, face amounts or liabilities.   The second can be described as an assumption-based approach, in which capital is determined by re-valuing the liabilities using specified assumptions. In some situations, one approach might be preferable to the other. For example, it is possible to measure mortality volatility risks on a pure factor basis, because the risk is short-term in nature. Longer duration risks, such as mortality level risk, might be more appropriately capitalized using an assumption-based approach, in which the regulator requires capital based on the difference between liabilities established using a specified assumption and that assumption used in deriving the liabilities.

1.7   The remainder of this case study focuses on advanced modelling approaches, with some discussion of the approaches that might be taken to derive a standardized approach.

A.1.1 Advanced Approach (Internal Models)

1.8　In this case study, the advanced approach is defined as the product of an internal model to quantify the various risks being considered. The general approach in this model considers each risk one by one, and quantifies the probability distribution of "liabilities", that result from each risk being considered. The solvency requirement for a particular risk is derived from this distribution, such that there is a high probability that the actual liabilities will prove to be less than the solvency requirement, in respect of that particular risk. The resulting solvency requirements for each risk are then aggregated into a total company requirement, taking into account the correlation between the various risks.

1.9　It is important to note that the case study focuses on the total solvency provisions, without regard for the allocation of these solvency provisions between liabilities and capital. The focus is on the quantification of the total provision for risk that is needed to establish a high confidence that the risk will be provided for, without worrying about the specific accounting implications.

1.10　The liabilities are defined in these calculations as the present value of future liability cash flows, discounted at the risk-free rate. All of the assumptions used in projecting the liability cash flows are the Company's best estimate of future experience, except for the assumption / risk that is being evaluated. For the risk being evaluated, the internal model varies the assumption and/or cash flows according to some underlying stochastic process, depending on the specific risk. This process generates scenarios in which the liabilities vary based only on the risk being measured. The probability distribution of liabilities is then tabulated, and the solvency provision is established.

1.11　Several specific points should be noted:

- The liabilities that are being modelled are defined on the basis of cash flows over a time horizon appropriate to the risk being modelled. With systematic (non-diversifiable) risks, such as misestimation of mortality parameters for example, the time horizon is the entire term of the liability. Non-systematic (diversifiable) risks, such as mortality volatility, are based on a 1-year horizon, which is the assumed length of time that a regulator requires to react to an adverse situation if necessary.

- The solvency provision for a particular risk is defined as the difference between the average liabilities that result under the worst 1% of scenarios, and the best estimate of liabilities. This is referred to as CTE (99) minus CTE (0), where CTE stands for "conditional tail expectation". For the risks covered in this case study, this is approximately equivalent to establishing capital at the 99.5th percentile of liability outcomes.

A.1.2 Standardized Approaches

1.12　Standardized approaches are less complex than the advanced internal model approaches. Such approaches can perhaps better be characterized as "assumption based" systems with respect to the establishment of liabilities (i.e., reserves), and as largely "standardized" systems with respect to the establishment of required capital.

1.13　In Canada, for example, policy liabilities are defined on the basis of the statement value of assets exactly needed to mature the liabilities with no resulting surplus, under adverse liability and economic scenarios. In projecting the liabilities, the actuary has some discretion, within bounds, of risk-adjusting the liability cash flows. Because the liability cash flows for life companies generally extend well beyond the duration of currently existing assets, further assumptions must be made about the reinvestment of future cash flows and assets to meet those obligations. The general intent is to model the actual reinvestment strategy followed by the Company, under various future economic scenarios. The actuary must perform this calculation for a certain number of prescribed scenarios, and can optionally perform additional scenarios. The final

liability that the actuary reports must equal statement value of assets needed to mature the liabilities under at least the highest of the prescribed economic scenarios.

1.14   Having established liabilities as above, the Canadian Company will report surplus as the assets in excess of liabilities. This surplus must exceed certain "Minimum Capital" requirements by a margin of 150%. The Minimum Capital is generally the result of applying factors to exposure bases. For example, capital for asset default risk is the result of factors applied to book value of assets, while the capital for mortality risk is the result of factors applied to the net amount at risk.

1.15   The Canadian approach, as for most jurisdictions, is focussed on the allocation between liabilities and capital. Conceptually, some risks are provided for in the liabilities and not in capital, whereas other risks are provided within capital and not in the liabilities. For example, provisions for the misestimation or deterioration of the mortality assumptions, is entirely within the liabilities. By contrast, provisions for volatility and catastrophe are entirely covered by capital, and not by the liabilities.

1.16   It is worth noting that any standardized approach will not fully capture the characteristics of the risks being evaluated, and may in fact produce misleading results.

A.1.3   Risks Covered by Case Study

1.17   Provisions are established in the case study for the following risks. These are described in additional detail in the following sections:

Mortality (Systematic Risks)
- Misestimation of the mean, i.e., the risk that the assumed best estimate mortality assumption in the liability calculation is not the true best estimate (statistical error)
- Trend, i.e., the risk that future mortality deteriorates (or improves) relative to the current date, in a manner different than we expect in our best estimate

Lapse (Systematic Risks)
- Misestimation of the mean, i.e., the statistical error associated with establishing the best estimate lapse assumptions

Non-Systematic Insurance Risks
- Mortality volatility risk
- Catastrophe risk (mortality)
- Lapse volatility risk

Market Risks
- Credit risk on assets supporting both liabilities and surplus
  Mismatch risk, or ALM risk, associated with the cash flow mismatches between liabilities and associated assets

1.18   Additional risks can also be considered, but were not in this case study to keep our sample company relatively simple. Of these risks, the most significant is the lapse risk caused by policyholder behavior and in particular, its interaction with changes in economic variables. This is an area where more experience is needed in extreme economic environments. It is also one risk that is very difficult to reflect appropriately on a purely factor-driven basis.

A.2   The Insurance Company

2.1   The company (i.e., the Company) constructed for this case study can be described as a medium sized insurance company that offers relatively simple term and whole life insurance products to its generally diverse customer base. It has experienced steady, but growing, sales over the past 15 years. In addition, the Company issues an immediate annuity product to retirees. All products are issued on a non-participating basis, and the Company has no equity-linked or interest-sensitive products, such as universal life. A simplified insurer has been used to illustrate the basic concepts.

2.2   Assets are managed at the segment level, with separate segments existing for the insurance products, annuity products and surplus. The Company generally invests in high grade fixed income securities to support the liabilities, but is more aggressive with its surplus, investing in common and preferred stocks in addition to fixed income securities.

2.3   The case study assumes the Company has various reinsurance arrangements in place, on one of its product lines (see the section entitled "Reinsurance Considerations"). First, the Company is considered on a gross of reinsurance basis.

The chart below summarizes some of the key features of this company.

Product Code	Type of Product	Number of Lives	Sum Assured or monthly payment
ALC 1001	Term to 100 insurance	56,971	3.6 Billion
ALC 1002	Non-par Whole Life	5,000	0.9 Billion
ALC 1003	Term to 100 insurance	94,560	9.0 Billion
ALC 1004	1 year renewable Term	7,463	1.4 Billion
ALC 1005	5 year renewable term	3,450	0.5 Billion
ALC 1007	Payout Annuities	250	1.5 million / month

2.4   On a Canadian GAAP basis, the balance sheet of the Company at December 31, 2002 can be summarized in the following table:

Balance Sheet at December 31, 2002
(Canadian GAAP Basis)

		Insurance	Annuity	Total	Surplus	TOTAL
Assets						
Cash and short term		89,304	21,347	110,651	19,116	129,767
Government Bonds		374,230	44,418	418,648	242,541	661,189
Corp Bonds	AAA	71,316	32,506	103,822	32,101	135,923
	AA	195,627	62,306	257,933	74,609	332,542
	A	101,963	57,284	159,247	46,460	205,707
	BBB	61,559	30,231	91,790	19,844	111,634
Total		893,999	248,092	1,142,091	434,671	1,576,762
Liabilities	Actuarial	887000	249000	1136000	0	1,136,000
	Other	24000	2600	26600	5750	32,350
		911,000	251,600	1,162,600	5,750	1,168,350
Surplus	Common Shares					250,000
	Retained					158,412
	Total					408,412
Total Liabilities plus surplus						1,576,762

A.3   Total Company Solvency Provisions

A.3.1   Summary of Total Company Provisions Using Internal Models

3.1   This section describes how an internal model might be used to quantify the various risks. First, the results of all of the calculations are summarized, then the details on each risk are explained. In each section, are presented some considerations in determining capital using standardized, or standardized, approaches.

3.2   The table below summarizes the solvency provisions generated by our internal model, for each risk/product type, and in aggregate at the Company level. Several points are worth mentioning or repeating. First, the figures in the table are the total solvency provision needed in addition to the best estimate liabilities. These figures are not estimates of the liabilities themselves, but estimates of the capital. Second, these are based on the present value of cash flows discounted at the risk-free rate, at the CTE 99 level. Third, not all risks have been quantified explicitly for each product segment. For market risk, the analysis was performed at the level at which the risk is managed, at the asset segment level. Finally, these capital provisions have been estimated separately for each type of risk and product. At the Company level, these separately determined provisions have been aggregated using methodologies described in section 3.1 "Risk Aggregation".

Total Solvency Provisions at December 31, 2002

	Systematic Insurance Risks			Non-systematic Insurance Risks			Market Risks		
	Mortality	Mortality	Lapse	Mortality	Mortality	Lapse			
	Level	Trend	Level	Volatility	Catastrophe	Volatility	Mismatch	Default	TOTAL
ALC 1001	43.1	50.1	28.9	3.4	6.2	3.5	-	-	73.7
ALC 1002	43.8	17.4	7.1	3.3	3.8	3.2	-	-	49.2
ALC 1003	105.7	163.6	103.3	9.5	35.1	10.9	-	-	227.5
ALC 1004	53.1	37.6	39.9	21.5	3.5	12.8	-	-	86.3
ALC 1005	8.6	5.8	3.9	3.9	4.4	2.1	-	-	14.8
Total Ins	-	-	-	-	-	-	335.7	-	335.7
ALC 1007	16.8	8.7	-	0.2	(0.1)	-	15.7	-	24.7
Surplus	-	-	-	-	-	-	-	-	-
TOTAL	178.8	265.8	152.8	29.7	53.0	26.1	351.4	11.5	511.6

3.3   In the table above, the total solvency provisions have been determined by assuming a matrix of correlations between types of risk and between products. The total solvency provisions result from the multiplication of these matrices.

A.3.2   Techniques to Aggregate Risk Provisions

3.4   There are several techniques that can be used to aggregate risks at the Company level. The most ideal solution, that is also the most difficult to achieve in practice, is to develop an internal model that reflects all correlations and dependencies between all risks and product types. The output of such a model would be total solvency provisions at the company level. With such a model, there is no need to make estimates or approximations about the manner in which risks are inter-related, rather there would be a need to develop approximate methods to reallocate the total capital requirements to the product/risk level. For most life companies, this approach is not feasible.

3.5   Another approach is to use copulas to aggregate the risks. As described elsewhere in this report, copulas are mathematical functions that describe the relationship between risks.

3.6   The approach taken in the case study is an analytic approximation, in which each risk and product is first modelled independently, and then simplified correlations between the risks are developed based on intuition, benchmarks and historical data where available. It was assumed that the economic capital for the combined distribution of all risks in Company could be approximated by the formula

$$EC_T = \sqrt{\sum \sum EC_i \times EC_j \times \rho_{ij}}$$

3.7   Where ρ_{ij} represents the correlation between risks i and j, and EC_i and EC_j represents the amount of capital that has been determined for risks i and j on a stand-alone basis.

3.8   The first set of assumptions made relates to the correlations between products, given a particular risk being evaluated. For example, if mortality volatility risk is evaluated at a high confidence level such as the 99th percentile or higher, the subjective assumption might be made that each product is 25% correlated with each other. In other words, the assumption is made that extreme levels of volatility have some effects on all product lines. At lower confidence levels (i.e., under normal operating conditions), a different assumption might be made, such as that the volatility risk is independent across product lines. The specific assumptions used are shown in the table below. Note that no assumption need be made for market risk (credit risk and mismatch risk) because these are modelled at the company or segment level. Note also that a simplifying

assumption is made that each product is correlated in the same way, for a given risk. For example, all products are 25% correlated with each other with respect to the mortality level uncertainty risk. If a more complicated assumption is desired (e.g., perhaps different products have different degrees of correlation), then it would be necessary to create a separate covariance matrix for each risk type.

Correlation Between Product Lines	
Risk	**Correlation**
Mortality level uncertainty	25%
Mortality trend uncertainty	100%
Mortality volatility	25%
Mortality catastrophe	100%
Lapse level uncertainty	50%
Lapse volatility	50%

3.9   Application of these correlations to the separately determined economic capital figures gives the combined capital measure for all products, given a particular risk type.

3.10   Next, we make an assumption about the degree to which the risks themselves are correlated. This is shown in the following table:

Risk Correlations								
	Mortality Level	Mortality Trend	Lapse Level	Mortality Volatility	Mortality Catastrophe	Lapse Volatility	Mis-match	Default
Mortality level	1	0	0	.25	0	0	0	0
Mortality trend	0	1	0	0	0	0	0	0
Lapse level	0	0	1	0	0	.25	0	0
Mortality volatility	.25	0	0	1	1	0	0	0
Mortality catastrophe	0	0	0	1	1	0	0	0
Lapse volatility	0	0	.25	0	0	1	0	0
Mismatch	0	0	0	0	0	0	1	0
Default	0	0	0	0	0	0	0	1

3.11   For example, it might be assumed that mortality volatility risk is weakly (25%) correlated with mortality level uncertainty risk, reflecting the fact that estimates of future mortality levels are at lest partially based on historical observed mortality, which is volatile.

3.12   The combined company level capital is then determined by applying this covariance table to the matrix of capital determined for each risk type.

A.4   Solvency Provisions for Mortality Risk

A.4.1   Mortality Level Risk (Misestimation of the Mean)

A.4.1.1 Internal Model

4.1   The Company derives best estimate mortality assumptions for each product segment in the portfolio. These best estimates are based on mortality studies, which are assumed to be derived from the same portfolio being evaluated. In practice, smaller portfolios would not rely entirely on their own experience data in establishing a mortality assumption. It is assumed that the mortality study has resulted in the creation of a mortality table that varies by age, duration and calendar year.

4.2   The mortality study is based on observations that, by nature, are volatile. The more volatile the observations, the higher the uncertainty in the underlying level of mortality assumption. Higher uncertainty can arise in smaller portfolios as well as in portfolios that are highly skewed in their distribution of insured amounts.

4.3   In doing a mortality study, it is presumed that the historical observations represent the best estimate level of mortality. It is possible however, that the observations are not a best estimate, but are somewhere in the tail of the true distribution. By assuming that the observations were actually at, say, the 95th percentile of the true distribution, the implied best estimate assumption can be solved for that could have resulted in such an observation. This can be done using an inverse Normal Power approximation [8]or as an approximation, by simulating the claims experience of the underlying portfolio for the same period of time as the length of the mortality study, and observing the 95th percentile of that distribution.

4.4   The approach taken in this case study is in fact to determine the mortality assumption that would be needed at several percentiles of confidence, using the Normal Power approximation. The liabilities are revalued under of each of these assumptions, keeping all other assumptions at the best estimate level. The table below illustrates the range in possible mortality assumptions that result. In all cases, the Company's best estimate of future mortality is 70% of the industry table, and what is being measured is the degree to which this best estimate could be wrong. From this table, it can be seen that the smaller the portfolio, the larger the range of possible outcomes for future mortality. In practice, a company might also partially rely on industry data using credibility theory. This adds an additional level of misestimation risk into the process that has not been considered here. It has been assumed that the Company sets mortality based solely on its own results.

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[8] van Broekhoven H. 2002, *Market Value of Liabilities Mortality Risk: A Practical Model*, North American Actuarial Journal

Mortality Assumptions at Various Confidence Levels			
	Group 1	**Group 2**	**Group 3,4,5**
Lives	56,791	5,000	103,000
μ/σ	.19	.39	.14
γ	.85	.76	1.35
Percentiles:			
5%	60%	52%	63%
15%	64%	58%	66%
25%	66%	63%	67%
35%	68%	66%	69%
45%	70%	69%	70%
50%	70%	71%	70%
55%	71%	72%	71%
65%	73%	75%	72%
75%	74%	79%	73%
85%	76%	84%	75%
95%	80%	92%	77%

4.5 For example, at the 50th percentile, the mortality assumption being tested is approximately 70% of the Industry table (71% in the case of ALC 1002). Note that the 50th percentile is not necessarily equal to the best estimate of 70% because of skewness in the portfolio. At the 95th percentile of confidence, the liabilities would be recalculated using 80% of the table for ALC 1001, and only 77% for ALC 1003.

4.6 The liabilities are revalued at these (and additional) percentiles and a range of possible liability results are derived. In the model, a statistical distribution of the liabilities is determined that best fit the liabilities at the percentiles that have been explicitly calculated. This is done so that the distribution of liabilities can be filled out and the results aggregated with other risks and/or product types if desired. It is not strictly necessary to do this, if all one was concerned about was the liability at a high confidence level for that particular product and risk. For example, the liabilities could have simply been revalued at the 99.5th percentile of assumptions, and the difference taken between this and the best estimate liability as the solvency capital for this risk. In effect, this is being done in our case study as well, except that the rest of the distribution is being filled out.

4.7   The table below shows the results of this process.

Liabilities at Various Percentiles – Level Uncertainty Risk
(Millions)

	ALC 1001	ALC 1002	ALC 1003	ALC 1004	ALC 1005	ALC 1007
Percentile:						
5.0	124.4	31.2	736.3	(267.1)	(27.8)	271.9
25.0	144.2	46.8	787.0	(241.6)	(24.0)	267.9
45.0	154.9	55.8	817.3	(228.4)	(21.9)	266.2
50.0	157.2	57.7	824.2	(225.8)	(21.4)	263.8
55.0	159.6	59.8	831.3	(223.0)	(20.9)	261.7
75.0	170.0	68.9	860.6	(211.1)	(19.0)	255.6
95.0	185.2	84.9	900.8	(191.5)	(15.8)	252.5
97.5	189.7	89.9	912.5	(186.1)	(14.8)	251.8
99.0	195.4	95.7	921.4	(179.2)	(13.7)	251.0
99.5	198.7	99.8	926.8	(174.9)	(13.2)	248.0
99.9	204.2	110.5	934.8	(167.1)	(12.1)	243.0
	-	-	-	-	-	-
σ	18.4	16.3	50.1	22.8	3.7	5.7
σ/μ	11.8%	28.1%	6.1%	-10.0%	-17.0%	2.3%
ratio: 99.9 / mean	130%	191%	114%	74%	56%	108%
CTE(99) - CTE(0)	43.1	43.8	105.7	53.1	8.6	16.8

4.8   Capital is determined as the CTE (99) less the CTE (0), or best estimate liabilities. For example, ALC 1001 has a best estimate liability of 157.2 million at an assumption of 70% of the mortality table. At the 99.9[th] percentile, however, which is 86% of the table, the corresponding liability is 204.2 million. Capital is based on the average of all liabilities in excess of the 99th percentile.

4.9   Two other points are worth making:

1.   The liabilities for two of the product lines are in fact negative. These two lines are the 1 year and 5 year renewable term liabilities, which are generally profitable at all ages and durations. As such, the premiums exceed the claim and expense amounts by a significant margin, and the corresponding present value of net liability cash flows is negative. If the company adopts a fair value, or cash flow based valuation system, then these negative reserves represent future profits that are being front-ended. In spite of this, the solvency capital is a positive number, because revaluing the liabilities under a more adverse mortality assumption results in a higher (i.e., less negative) liability.

2.   For the annuity line (ALC 1007), the liability amounts decrease with increasing percentiles. This is because the percentiles measure the mortality assumption, and not the corresponding liability amount. For example, the 99th percentile liability figures shown in the table correspond to mortality assumptions at the 99th percentile of possible assumptions (i.e., in excess of the mean), based on the Normal Power approach described above. For some product lines, this results in an increased liability, whereas for other product lines, this results in lower liabilities with higher mortality. Products such as this offer some natural hedging of this risk for the Company.

A.4.1.2 Considerations for Standardized Approaches

4.10   Level uncertainty risk is an example where an "assumption based" approach would be more appropriate than a pure factor-based approach. For example, one possible approach would involve quantifying the liabilities on two bases: (1) best estimate mortality; and (2) mortality at a more conservative level, where the specific assumption would be a multiple of the best estimate assumption (100% plus a number of standard deviations, based on the confidence level desired).

4.11   To apply this method, the regulator would prescribe only a confidence level requirement, such as CTE(99), which can reasonably be approximated as the 99.5^{th} percentile for most mortality risks. It would be up to each company to estimate the standard deviation of annual claims that can be expected from its specific portfolios, and to determine the more conservative mortality assumption to use.

4.12   A more restrictive approach might be to prescribe the additional mortality itself, rather than simply prescribing the confidence level. For example, the regulator might prescribe a solvency level of mortality as the best estimate plus a constant divided by the expectation of life. The constant to be added might vary from a low to a high range, depending on characteristics of the portfolio, such as credibility or homogeneity. Alternatively, the constant might not vary with the expectation of life, but simply be a flat additional percentage of mortality that applies to all ages. These constant factors would need to be sufficiently conservative to capitalize typical companies in the jurisdiction. In other words, in deriving the constant additional mortality, the regulator would be making an implicit assumption about the risk profile / standard deviation of portfolios in the jurisdiction, and testing the capital factors against this profile.

4.13   A "pure" factor-based approach would be the next step in this development, but would have to be developed with caution. Under this approach, the regulator would determine the capital using the assumption-based approach as above, but would express the capital as a percentage of an exposure base. The challenge is to develop an appropriate exposure base.

4.14   Consider this hypothetical example, in which the regulator has developed factors to be applied to "best estimate liabilities", being the present value of future liability cash flows using best estimate assumptions. These factors have been developed using "typical" portfolios in the jurisdiction. The table below illustrates both the assumption-based approach and the factor-based approach. In the assumption-based approach, the best estimate liabilities would be revalued using revised mortality assumptions as a percentage of best estimate mortality, where the factors vary based on the risk profile of the portfolio. In the factor-based approach, we apply the factors in the table below to the best estimate liability itself. Here, the factors would also have to vary by product type, or some other measure that captures the characteristics of the underlying cash flows (e.g., duration). For example, a portfolio of annual renewable term insurance (T1), whose risk profile was 0.05 standard deviations per average claim, could determine the total solvency provision under the assumption based approach by valuing the liability at 107% of best estimate mortality. Under the factor-based approach, total solvency provisions equal to 140% of the best estimate liability could be established.

Risk Profile	Assumption-Based	Factor-Based		
	% of Best Estimate Mortality	% of Best Estimate Liability		
σ/μ	All products	T-100	T-1	T-5
.00	100%	100%	100%	100%
.05	107%	107%	140%	109%
.10	114%	113%	175%	117%
.15	122%	120%	215%	125%
.20	130%	125%	250%	135%

4.15   For the Company, the assumption-based approach is essentially equal to the internal model approach, given that the assumption-based approach was set using the same approach as the internal model. The factor-based approach, however, produces dramatically different results, because the portfolios in the Company are different from the portfolios by which the factors were developed. This highlights the reason why the use of best estimate liability as an exposure base against which to apply factors could be problematic.

A.4.2   Mortality Trend Risk (Deterioration of the Mean)

A.4.2.1 Internal Model

4.16   An important part of the best estimate mortality is the trend. The significance of the trend assumption in establishing a liability is influenced by the remaining duration of the portfolio, (and any periods for which mortality is guaranteed, for example through reinsurance, or through guaranteed annuity payments).

4.17   The "best estimate trend" can be estimated based on observations in the past, sometimes including expert opinions. The resulting trend will of course be uncertain. This uncertainty can be split into two parts. The first part exists because of the fact that the observations from the past will have been volatile. This volatility (movement around a certain level) will also exist in the future. The second part of the uncertainty trend is caused by systematic changes in the trend, for example due to medical developments, new diseases (like AIDS), and environmental changes. This uncertainty will increase looking further in the future.

4.18   Mortality rates are highly correlated between various ages and genders. The development of mortality rates is correlated between ages and genders. The degree of correlation itself varies over time. For this reason, it would be highly speculative to put forward a model that tries to directly estimate future mortality rates, as this would require too many assumptions. Instead, we would ideally like to indirectly quantify trend uncertainty by revaluing the liabilities using historically observed trend assumption tables. Analyzing the impact on the liabilities of the several variants of the trends observed in the past can give us an idea of the uncertainty trend. The impact of the correlation between ages, gender will be automatically included in the analyses. Generational mortality tables with trend assumptions built in to their construction can be useful.

4.19   An important factor in these analyses is the duration of the trends being reviewed. For an insurance portfolio with a remaining duration of n years we have to analyse what can happen with the trend over a period of n years. To do this, we observe as many historical trends with the same duration as the data will allow. Each of these trends can be used to recalculate the liabilities.

4.20   It is quite possible that sufficient historical observations do not exist, and that if they do exist, they do not capture the spectrum of possible outcomes. In Canada, for example, such data would be quite limited.

4.21   For illustrative purposes in this case study, a range of possible trend outcomes is captured by speculating that the annual rate of mortality improvement is normally distributed with a mean and standard deviation of 0.50% improvement per year, which is broadly consistent with mortality improvements in the general population over the past decade. Further the years of mortality improvement has been limited to 40 years, and also the maximum and minimum improvement assumptions to 3% per year. Using this, a range of scenarios of future mortality assumptions can be derived for revaluing the liabilities. As with the level uncertainty approach, this provides a distribution of possible liability figures, in which only the future trend is varied. All other assumptions are at the best estimate level.

4.22   The capital needed can be based on a p% confidence interval from a Student-t distribution with n-1 degrees of freedom, if one is performing the calculation of capital based on revaluing the liability under n historically observed scenarios. Alternatively, if one is using a model to hypothesize future mortality improvement, then the capital can be established by revaluing the liability under a mortality improvement assumption that arises at various percentiles, and choosing the average of the largest 1% of liabilities, less the best estimate. It is important to recognize the subjective nature of either approach.

4.23   The table below illustrates the scenarios that were selected for testing. A particular scenario is assumed to apply to all products simultaneously, regardless of the effect of the assumption on the liability. For example, when we test a scenario that features a high rate of future mortality improvement, the effect is generally to produce lower liabilities for the insurance products, and higher liabilities for the annuity products. While one would normally expect an insurance portfolio to benefit from mortality improvements, this relationship can be reversed under certain reinsurance arrangements. This is illustrated further in the section on reinsurance.

Percentile	Annual Rate of Mortality Improvement
0.5%	1.77%
1.0%	1.66%
5.0%	1.32%
10.0%	1.14%
20.0%	0.92%
30.0%	0.76%
40.0%	0.63%
50.0%	0.50%
60.0%	0.37%
70.0%	0.24%
80.0%	0.08%
84.0%	0.00%
90.0%	-0.14%
95.0%	-0.32%
99.0%	-0.66%
99.5%	-0.76%

4.24   At the total company level, the risk exposure to trend uncertainty can be dampened or magnified, depending on the product mix of the company. An internal model can recognize these interactive effects, but this is difficult to achieve in a standardized approach, which might, for example, require that capital be determined separately for each product and the results simply added up. This type of approach might be overly conservative as it assumes that different mortality scenarios occur for different product types.

4.25   Note that from the above table, the overall average mortality improvement is 0.50% per year, for 40 years, regardless of the product type. This figure results from the relatively simplistic model that future mortality improvements are selected from a normal distribution, modified only for maximums and minimums. The 0.50% best estimate improvement itself is roughly consistent with observed mortality improvements, in aggregate, in some countries. As mortality has been generally improving over the past century, we do not have many observations in which negative trend, or deterioration, has occurred. Our model implicitly assumes that the observed levels of mortality improvement will continue into the future, which may not be true. A higher standard deviation has been selected to adjust for this so that at higher percentiles, we may get some deterioration that has not necessarily been observed in the past. Capital would then be established at this adverse assumption. For example, at the 99.5th percentile, we are effectively establishing capital on the assumption that mortality will deteriorate by 0.76% per year. This has the effect of increasing insurance liabilities, but decreasing annuity liabilities. If the company had a larger proportion of annuities than insurance, then it may be that the capital is effectively established on the assumption of mortality improvement of 1.77% per year, as opposed to a worsening. A company using an internal model approach should be careful to apply some judgement on the resulting scenarios, to ensure that the assumed mortality improvement / deterioration is not unreasonable.

4.26   The liabilities that result from this work is shown in the following table:

Percentile:	ALC1001	ALC1002	ALC1003	ALC1004	ALC1005	ALC1007	TOTAL
5.0	123.4	44.9	715.2	(249.4)	(25.2)	257.3	867.2
25.0	142.8	52.5	779.2	(235.6)	(23.1)	254.1	972.9
45.0	154.0	56.5	816.5	(227.7)	(21.8)	252.3	1,030.9
50.0	156.6	57.4	826.1	(225.9)	(21.6)	251.9	1,046.0
55.0	159.3	58.3	834.9	(224.2)	(21.3)	251.4	1,058.9
75.0	170.3	62.2	870.5	(216.5)	(20.0)	249.6	1,116.9
95.0	189.1	68.7	928.9	(202.7)	(17.9)	246.4	1,212.9
97.5	194.7	70.6	947.4	(198.1)	(17.2)	245.1	1,241.4
99.0	201.2	72.7	966.3	(193.0)	(16.5)	243.8	1,274.1
99.5	204.7	74.2	982.2	(189.9)	(16.0)	242.9	1,296.1
99.9	214.0	76.8	1,014.5	(182.2)	(15.0)	241.4	1,339.0
	-	-	-	-	-	-	-
μ	156.5	57.2	824.1	(226.0)	(21.5)	251.8	1,043.6
σ/μ	12.7%	12.5%	7.9%	-6.3%	-10.4%	1.3%	10.0%
ratio: 99.9 / mean	136.8%	134.2%	123.1%	80.6%	69.8%	95.9%	128.3%
CTE99 - CTE0	50.1	17.4	163.6	37.6	5.8	8.7	262.5

4.27   What is worth noting in this table is the total capital for this risk for the company is 262.5 million, when modelled in the aggregate assuming that the risk is 100% correlated by product, whereas the simple sum of capital requirements for each product (not shown in table) is $283.2 million. The annuity product, ALC 1007, has the effect of lowering capital requirements by approximately $20 million in aggregate, because it reacts favorably to adverse trend assumptions. This can be seen also by looking at the results by percentile. The insurance products (ALC 1001 to ALC 1005) all increase in liabilities at higher percentiles (i.e., at increasingly adverse mortality worsening), whereas the annuity liabilities decrease at higher percentiles. In the section on reinsurance, it will be shown that this effect is in fact amplified in a particular reinsurance situation. When modelling a mixed book of annuities and insurance products it may be wise to consider "non-parallel" shifts (i.e., using asset/liability – ALM – terminology) in mortality (e.g. by age or gender) to recognize the risk that mortality trend may not be perfectly hedged between the annuity and insurance books of business

A.4.2.2 Considerations for Standardized Approaches

4.28   There are several possible ways to develop standardized capital models for trend. One example of an "assumption-based" method is to establish a total solvency provision based on a conservative estimate of trend. The difference between the liability established using this conservative estimate, and the liability established using the true best estimate, could be considered the capital for trend uncertainty. For example, the solvency provision for an insurance portfolio might be based on an assumption of no future trend improvement, compared to a best estimate trend assumption of 0.50% improvement per year. Annuity lines would require a solvency provision based on future trends greater than the best estimate.

4.29   A simplified approach to provide for trend uncertainty could be to apply a factor multiplied by the present value amount of the liabilities (see following formula). The factor might be expressed as the lesser of α and β times the product duration n. Some sample values of α and β are also given in the table below.

$$c_{trend} = \min\{\alpha, \beta n\}(liability)$$

	α	β
Pure endowment	7%	0.35%
Endowment	3%	0.15%
Term	30%	1.50%

The uncertainty trend for a whole life annuity can be based on 4% of the liabilities (x>55). These calculations of trend uncertainty are based on a 99.5% confidence level.

A.4.3 Volatility

A.4.3.1 Internal Model

4.30   Volatility risk, or process risk, is the risk that cashflows will not occur as expected due to statistical fluctuations around the expected assumptions. In quantifying this risk, we assume that we have indeed selected the correct best estimate mortality assumptions and future trend assumptions, and that we are only concerned with volatility given those assumptions.

4.31   Another important consideration is the time horizon, as described earlier. With the systematic risks, we wanted to provide for adverse liability cash flows for the entire term of the liability, because we cannot perform management or regulatory action to eliminate this risk. With diversifiable risks, however, such as volatility, we only project out for a 2 year time horizon, on the rationale that this risk can be managed. Our case study is therefore based on this 2 year time horizon with respect to volatility risk. As an illustration, we also show what the volatility capital would look like if we considered the time horizon to the full term of the liability. In aggregate, as one might expect, the resulting capital is larger when considering the larger time horizon, but the relationship between the 2 year and full term time horizons are not clear, and in some specific product cases, the 2 year horizon actually produces almost the same capital requirements as with the full term. This is because over the full term of the liability, time diversification is also occurring. Adverse mortality in the earlier years is ultimately followed by more favorable mortality in the later years, and partially offsets the adverse effect of the adverse mortality in the early years.

4.32   This case study has used a simulation approach, although analytic approaches are feasible to quantify volatility risk. Under the simulation approach, a Monte Carlo simulation of the portfolio was performed, with the intention of measuring either 2 years worth of claims or the present value of all liability cash flows to the full term of the liability (depending on the definition of volatility risk that we are exploring). The simulation is binomial, meaning that each person in the portfolio is simulated to live or die, based on an expected mortality equal to the best estimate assumption. The capital required is the difference between the claims (or liability) at $TVaR_{99\%}$ and the best estimate claims (or liability) over that same period.

A. Volatility Based on 2 Years Claims

4.33   The table below illustrates the results for the various products assuming that the volatility of claims is measured over a 2 year horizon. The effect of aggregating these capital requirements under two extreme assumptions is shown: the volatility risk is 100% correlated across each products; and the volatility risk is completely independent. It can be argued that the volatility risk is more likely to be uncorrelated, or only weakly correlated at extreme confidence levels, but for illustration purposes, both extremes are shown.

Claims Over 2 Year Horizon – Volatility Risk

	ALC1001	ALC1002	ALC1003	ALC1004	ALC1005	ALC1007	TOTAL correlated	TOTAL independent
Percentile:								
5.0	10.5	4.9	60.1	15.9	3.5	44.6	139.5	144.6
25.0	11.2	5.5	62.4	17.3	3.9	44.7	144.8	147.8
45.0	11.6	5.9	63.8	18.3	4.2	44.7	148.7	149.9
50.0	11.8	6.0	64.2	18.6	4.3	44.7	149.6	150.4
55.0	11.9	6.1	64.6	18.9	4.4	44.7	150.6	151.0
75.0	12.5	6.7	66.2	20.4	4.8	44.8	155.5	153.4
95.0	13.7	7.9	69.7	25.1	5.9	44.9	166.4	159.1
97.5	14.1	8.3	71.0	27.8	6.5	44.9	170.7	161.7
99.0	14.7	9.0	72.5	32.1	7.2	44.9	176.7	165.5
99.5	15.1	9.3	73.6	37.0	7.9	45.0	180.7	170.0
99.9	16.1	10.1	75.6	54.1	9.9	45.0	190.3	182.7
	-	-	-	-	-	-	-	-
μ	11.9	6.2	64.5	19.4	4.4	44.7	150.8	151.0
σ/μ	8.3%	14.9%	4.6%	19.1%	18.3%	0.2%	5.6%	3.3%
ratio: 99.9 / mean	135.5%	163.6%	117.2%	279.3%	222.3%	100.7%	126.2%	121.0%
CTE99 - CTE0	3.4	3.3	9.5	21.5	3.9	0.2	31.7	22.7

4.34 In aggregate, the capital is between 22.7 million and 31.7 million, depending on whether it is assumed that the volatility risk is correlated or not.

4.35 It is also worth noting that the capital as a percentage of expected claims is much higher for smaller or more skewed product distributions. For example, the largest capital requirements of $21.5 million (or almost 2 years worth of annual claims in this case), arises with product ALC 1004, which as the reader will recall, has only 7400 lives in the portfolio, and a wide range of sum assured in the portfolio. By contrast, product ALC 1003 has 95,000 lives and a more stable sum assured distribution, and the resulting capital requirements in this case are only 9.5 million, or 4 months of claims.

4.36 Related to this is the observation that for volatile products such as ALC 1004, the amount of claims increases in extreme measures at the tail of the distribution, relative to the other products. For example, the difference between claims at the 99.5th percentile versus the 99.9th percentile is an increase from $37 million to $54 million, which is a 50% increase. This type of jump is not seen in the other, more stable products.

4.37 Finally, under this approach, we attribute virtually no capital to the annuity lines (ALC 1007), as the impact of volatile mortality over a 2 year period on the monthly payments to annuitants (approximately $44 million over 2 years) is negligible.

B. Volatility Based on Present Value of Liability Cash Flows

4.38 The table below shows the capital that would result if we defined the capital based on the liability, or present value of future cash flows at the risk free rate. Generally, we see that the more volatile the product (for example, higher standard deviation of annual claims), the closer the capital requirements become regardless of the time horizon. ALC 1004 in particular, which we identified previously as the most volatile product, has virtually the same capital requirements regardless of the choice of definition. Large stable segments such as ALC 1003 would produce almost double the capital requirements, should the definition of capital be based on all liability cash flows. Also, under the full liability term definition, we do get capital requirements for the annuity product (ALC 1007), as volatility does affect the ultimate results in the long run. This is

perhaps appropriate for annuity type products, as it is arguable whether volatility risk for these products can be managed as easily as the insurance volatility risks.

Liabilities Over Full Term Horizon – Impact of Volatility Risk

Product Code	Capital based on 2 years claims	Capital based on all liability cash flows
ALC 1001	3.4	6.2
ALC 1002	3.3	5.4
ALC 1003	9.5	16.8
ALC 1004	21.5	23.9
ALC 1005	3.9	12.9
ALC 1007	0.2	7.6

A.4.3.2 Considerations for Standardized Approaches

4.39   Traditional volatility risk is often calculated using a simulation model. A good alternative is an analytical approach, such as the Normal Power approximation which uses the first 3 moments of the Compound Poisson distribution. Under this approach, the capital at a 99.5% confidence level in the Normal Power approach is:

$$C_{vol} = \sigma(2.58 + 0.94\gamma)$$

In other words, the capital would be a multiple of the standard deviation of annual death claims, with an adjustment for the skewness of the portfolio.

4.40   The table below compares these simplified standardized approach to the internal model results.

	Internal Model	Normal Power
ALC1001	3.4	3.2
ALC1002	3.3	3.2
ALC1003	9.5	9.1
ALC1004	21.5	30.9
ALC1005	3.9	3.7

As the table shows, the Normal Power method produces reasonably accurate approximations to the internal model, except for product ALC 1004, which as the reader will recall is a highly skewed portfolio with a large standard deviation.

A.4.4   Catastrophe Risk

4.41   This risk can be described as the risk that a catastrophe occurs that causes a one-time spike in mortality experience, with a corresponding impact on claims and/or liabilities. As there have not been many observed catastrophes that affect insured life populations in the past century or so, it is difficult, and perhaps spuriously accurate, to formulate a model that quantifies this. Any such model would be highly subjective, and we expect that the industry may start focussing on this item. Such a model would most likely be a frequency / severity model that assumes probabilities of various types of catastrophes that vary be severity in their impact. For example, there might be a very small probability of an epidemic such as the Spanish Flu of 1918, that caused a doubling of

infectious disease mortality in certain age groups, and a larger probability of a less severe epidemic or other incident.

4.42   We have not attempted to model this in this case study. Rather, we have taken a deterministic approach. Under this approach, we require that the company have enough capital to absorb a doubling of mortality in a 1-year period. (Our model specifically assumes that although the event that causes the doubling of mortality occurs in the first year, the actual mortality impact is spread over a 2-year period as 50% increases in the mortality rate in each of those 2 years).

4.43   Because there is interaction between the catastrophe and the volatility risk described above, we want to consider the impact of both of these risks occuring simultaneously. To that end, we quantified the volatility risk using the Monte Carlo simulation described in the volatility section, and assuming that the expected mortality was double our best estimate mortality in the first year. We then measure the claims over a 2 year period (or the liability, depending on our definition of capital), at the CTE (99) level as well as the best estimate level. The total solvency requirement for volatility and capital combined is the CTE (99) figure at this higher level of mortality, less the CTE (0) figure using our best estimate of mortality (i.e., before the catastrophe). We attribute the volatility component of this capital as based on our best estimate of mortality, and the catastrophic component is the incremental difference in CTE (99) at the higher mortality relative to the CTE (99) at the best estimate mortality.

A. Catastrophe Based on 2 Years Claims

4.44   When we define the capital to be based on claims over a 2 year period only, this approach effectively amounts to a requirement equal to 1 years worth of claims, less an adjustment for the interaction between normal volatility risk and catastrophe risk. As the table below illustrates, for large and stable portfolios, the catastrophe risk is significant relative to the volatility risk, whereas for small and skewed portfolios, the catastrophe risk is almost indistinguishable from normal volatility.

Claims Over 2-Year Horizon – Catastrophe and Volatility Risk

Capital	Risk Measure	Basis	ALC 1001	ALC 1002	ALC 1003	ALC 1004	ALC 1005	ALC 1007
Volatility	CTE 99 (Vol)	100% Expected	15.3	9.5	74.0	40.8	8.3	45.0
	CTE 0 (Vol)	100% Expected	11.9	6.2	64.5	19.4	4.4	44.7
	Capital for volatility		3.4	3.3	9.5	21.5	3.9	0.2
			-	-	-	-	-	-
Catastrophe	CTE 99 (Cat+Vol)	200% Expected	21.5	13.3	109.0	44.3	12.8	44.9
	CTE99(Vol)	100% Expected	15.3	9.5	74.0	40.8	8.3	45.0
	Capital for catastrophe		6.2	3.8	35.1	3.5	4.4	(0.1)
	TOTAL		9.6	7.2	44.6	24.9	8.3	0.1

B. Catastrophe Risk Based on Present Value of Liability Cash Flows

4.45 For information purposes, we show below the capital that would result under a doubling of mortality in the first year, as above, but where the capital is based on the present value of all future liability cash flows.

Liabilities Over Full Term Horizon – Impact of Catastrophe Risk

Product Code	Capital based on 2 year ca		Capital based on all liability cash flows	
ALC 1001		Term to 100 insurance	56,971	3.6 Billion
ALC 1002		Non-par Whole Life	5,000	0.9 Billion
ALC 1003		Term to 100 insurance	94,560	9.0 Billion
ALC 1004		1 year renewable Term	7,463	1.4 Billion
ALC 1005		5 year renewable term	3,450	0.5 Billion
ALC 1007		Payout Annuities	250	1.5 million / month

	Capital based on 2 years claims		Capital based on all liability cash flows	
Product Code	**Volatility**	**Catastrophe**	**Volatility**	**Catastrophe**
ALC 1001	3.4	6.2	6.2	5.2
ALC 1002	3.3	3.8	5.4	2.5
ALC 1003	9.5	35.1	16.8	25.4
ALC 1004	21.5	3.5	23.9	10.6
ALC 1005	3.9	4.4	12.9	4.5
ALC 1007	0.2	(0.1)	7.6	(2.6)

4.46 Under this definition, the relative magnitudes of catastrophe versus volatility have changed, with catastrophe not being as significant a component. Over time, the effects of the catastrophe become indistinguishable relative to volatility. In aggregate, the capital requirements are larger under this definition.

A.5   Solvency Provisions for Lapse risk

A.5.1   Lapse Level Risk (Misestimation of the Mean)

A.5.1.1 Internal Models

5.1   The lapse risk can be analyzed in a similar fashion to the mortality risk, although there are several other factors that need to be considered. In our case study, we have not dealt with these more complicated factors. It is a fair statement that significantly more work needs to be done by the actuarial profession in general to truly understand the lapse risk. Some of these factors include:

1.   A need to differentiate between those portfolios whose lapse rates are likely to show dependencies with other economic assumptions, from those portfolios that are not sensitive to economic conditions. Where the lapse rate does interact with other assumptions, the model should ideally reflect these dependencies. Such a model would be highly subjective, as there is little historical data to base this on. Even the form of the model would, at first, be speculative.

2.   The lapse assumption is highly dependent on the product itself, including the manner in which the product was sold, the competitive environment at the time of sale, the purpose of the product (eg, tax planning, insurance needs, etc). Even if the lapse assumption is based on large volumes of data, it is more difficult to apply those same lapse assumptions to portfolios other than the portfolio from which the lapse rates were derived. This increases the uncertainty around the lapse assumption significantly.

3.   The impact to the company of higher or lower than expected lapses can be positive or negative for different policy durations and product types. These relationships can change over time, not only with the natural aging of the policy, but also in the events that the other actuarial assumptions change in the future. This is further complicated by the potential impact of policyholder behavior.

4.   In addition to these, we also have the normal statistical error associated with estimating average rates from historical, volatile assumptions.

5.2   Our case study considers the last of these issues, the possibility that the best estimate lapse assumption, which is based on historical data for the company, is inaccurate due to statistical error.

5.3   To determine the statistical error in the lapse rates, we first analyze the lapse study that exists for the various product lines. These lapse studies give us, for each issue year within a product group, the actual lapse rates experienced by that cohort for several calendar years. From this, we determine our best estimate lapses as well as the standard deviations of those lapse rates. We make the assumption that the lapses are normally distributed, and we solve for lapse rates at alternate percentiles for each duration. For example, the best estimate lapse rate might be 10% in the first policy duration and grade to an ultimate lapse rate of 1% in 12 years. The corresponding lapse assumption at the 90th percentile might begin at 12.4% and grade to an ultimate of 1.2%. At the 10th percentile, the lapse assumption starts at 8.7% and grades to an ultimate of 0.8%. This effectively results in parallel shifts in lapse rate curves, although the degree of the shift varies by duration based on the standard deviations of the lapse rates.

5.4   Liabilities are recalculated using these various lapse assumptions, and from these deterministic scenarios, a distribution of liability amounts is fitted using statistical techniques. We do this so that we can fill out the distribution and combine with other risks if desired.

5.5 Of course, it is possible that the statistical error in the lapse rates is not always one-sided. In other words, it may be that the lapse rate for duration 1 might be overstated while the lapse rate for duration 7 is understated. By shocking the lapse rates in parallel by duration, we are assuming 100% correlation between the durations, which we *assume* will produce more conservative results than considering the lapse rates by duration as independent. We validate this assumption in our case study by performing some additional tests in which the lapse shocks do vary by duration. This may not always be appropriate, but in our case study, our approach turns out to be more conservative in the majority of scenarios tested.

Liabilities Under Lapse Misestimation Risk

Percentile	Lapse rates	ALC 1001	ALC 1002	ALC 1003	ALC 1004	ALC 1005	TOTAL Correlated	TOTAL Independent
5.0	Higher	138.1	49.2	742.5	(178.4)	(17.1)	965.3	951.0
25.0	Higher	148.7	52.3	787.6	(187.9)	(17.7)	1,006.1	999.7
45.0	Higher	154.6	54.1	812.6	(191.3)	(18.1)	1,028.4	1,026.2
50.0	Expected	155.9	54.5	818.1	(196.8)	(18.6)	1,033.7	1,032.2
55.0	Lower	157.2	54.9	824.0	(201.1)	(19.0)	1,039.2	1,038.6
75.0	Lower	163.2	56.5	847.0	(216.2)	(20.5)	1,061.8	1,064.6
95.0	Lower	173.9	59.1	884.7	(224.2)	(21.3)	1,097.5	1,105.6
97.5	Lower	177.4	59.9	895.9	(226.0)	(21.5)	1,107.0	1,118.3
99.0	Lower	181.3	60.7	910.3	(228.1)	(21.7)	1,119.7	1,133.8
99.5	Lower	183.8	61.3	917.0	(236.1)	(22.6)	1,126.7	1,143.1
99.9	Lower	188.9	62.4	933.4	(250.4)	(24.2)	1,147.4	1,160.7
s		10.9	3.0	43.3	14.9	1.6	40.0	47.2
s/m		7.0%	5.5%	5.3%	-6.6%	-7.2%	3.9%	4.6%
ratio: 99.9 / mean		121%	115%	114%	79%	79%	111.1%	112.6%
CTE(99) - CTE(0)		28.9	7.1	103.3	39.9	3.9	97.2	115.2

5.6 We observe several things from this table:

- The liabilities for products ALC 1001 to ALC 1003 all increase with decreasing lapse rates, whereas the liabilities for products ALC 1004 and ALC 1005 do the opposite. These latter products are highly profitable renewable term policies in which the premiums significantly exceed the claims and expenses at most or all durations. Lower lapse rates than expected for these products help the Company because it results in unexpected future profits. By contrast, the first three products have level premiums which are ultimately insufficient in and of themselves to pay for claims. The Company in these cases is better off with higher lapses in those later durations.

- Three of the products are exposed to lower lapse rates, and two of the products to higher lapse rates. When combining the capital from these different products, we must make an assumption about the degree to which they are correlated.

  - On the one extreme, we could take the view that the lapse risks for each product is completely independent. That is, we may have underestimated the lapse rates for one product but overstated them for another product. If we combine the risks using this assumption, we get total capital requirements of $115 million, which can be approximated by the square root of the sum of the squares of the individual capital requirements.

- We could also take the view that there is a systematic bias inherent in the lapse studies themselves, and that the lapse assumptions are therefore 100% correlated. For example, if we assume that all of the lapse studies for each product is done in the same corporate area using the same methodology, there may be a bias that causes the resulting lapse assumptions to be higher or lower than the true best estimate, for all products. If we aggregated the liabilities at the company level using this assumption, we would get aggregate capital requirements of $97 million. This is less than the capital that results from an assumption of independence between the products, because of synergies between the products. Higher liabilities arising from some product lines are offset partially by lower liabilities from other product lines in the same scenario.

- Finally, on the other extreme, we could assume 100% correlation between the liabilities themselves. This would mean that we pick adverse scenarios that vary by product. For products ALC 1001 to 1003, we would be setting capital assuming very low lapse rates, whereas with ALC 1004 and 1005, we would be assuming high lapse rates.

5.7 A final consideration is that we could account for the risks that we have not modelled either by selecting a higher confidence level to set capital, or perhaps to set the final capital levels using multiples of capital derived by considering the statistical error risk only. We have not done this in our case study, but is the type of approximation we would consider in a real situation.

A.5.1.2 Considerations for Standardized Approaches

5.8 There are two primary effects of unanticipated lapse rates. The first involves the payment of surrender or termination values. The relationship of the amount of a surrender payment to the value of the liability being held in respect of a particular policy is of great importance. When a policy lapses, the company pays the surrender value and 'receives' the actuarial reserve that is released by the policy's termination. If surrender values are lower than policy reserves, the company is at risk from lapse rates that are lower than expected, particularly if high lapse rates were anticipated in the pricing of a product. The case that surrender values exceed policy reserves results in higher lapse rates being unfavourable to the insurer. In some jurisdictions these risks are mitigated by regulations. A requirement that a company holds policy liabilities at least as large as surrender values provides partial protection against overly high lapse rates while minimum required surrender values reduce the likelihood that insurers will price their products using an assumption of high lapse rates. It is important to recognize that the relationship between the surrender value and the actuarial reserve is not fixed; it will generally vary with the duration of a particular policy.

5.9 The second primary effect of unanticipated lapse rates is that the insurer may not realise the expected recovery from future premiums of initial policy acquisition expenses. These acquisition expenses may be recognized implicitly in financial statements through the use of modified net level premium valuation methods. These implicit methods generally do not include any provision for unfavourable variations in lapse rates. Recovery of acquisition expenses may also be recognized explicitly through a reduction in policy liabilities or through introduction of a receivable asset. In this latter case, the adjustment to financial values is made subject to a form of recoverability test. Under the second primary effect, the risk to insurers is generated by lapse rates that are greater than expected.

5.10 Unanticipated lapses can have other effects on the financial condition of an insurance company. For example, anti-selective lapse by healthier lives may lead to deterioration in a life insurer's mortality experience. This risk may be exacerbated by poor product design, an operational risk. In general, this risk is not treated for capital purposes as a lapse risk.

5.11 In the case that lapses are recognized explicitly in the valuation of actuarial liabilities, an approach to capital requirements in respect of the first type of lapse risk is available. This requires the division of policies into two classes: 1) those for which an increase in lapse rates results in an increase in policy liabilities, and 2) those for which policy liabilities increase when assumed lapses decrease. The capital requirement is of the form of the difference between a special valuation of policy liabilities and the normal valuation. For the special valuation, the lapse assumption is multiplied by a specified factor greater than one for policies in the first class and by a factor less than one for policies in the second class. As an example, in Canada, lapse rates are doubled for policies in the first class and reduced by one-half for those in the second class.

A.5.2 Volatility

5.12 Analogous to mortality volatility, this risk provides for uncertainty in cash flows arising due to statistical fluctuation around the best estimate lapse assumptions. This component can also be defined on the basis of either the impact on cash flows over a short term horizon such as 2 years, or as the impact on the liability, or present value of cash flows, over the entire term of the liability. Although process risk generally can be considered diversifiable, it is more difficult for a company to manage its volatility due to lapses as opposed to mortality. For that reason, it may be appropriate to consider a longer time horizon.

5.13 In our case study, we define the capital for lapse volatility risk on the basis of the impact on the total liability, as opposed to a shorter term. If we were to measure on a shorter term horizon, we would establish virtually no capital, as the products in this company have little or no cash values, and the impact of adverse lapses on other cash flows over a short horizon is negligible.

5.14 The table below illustrates that even on the basis of the full term of the liability cash flows, the lapse volatility risk is relatively immaterial compared to the other risks:

Liabilities Over Full Term Horizon – Lapse Volatility Risk

	ALC1001	ALC1002	ALC1003	ALC1004	ALC1005
Percentile:					
5.0	154.7	53.0	814.6	(238.6)	(23.1)
25.0	156.0	54.1	818.4	(233.6)	(22.3)
45.0	156.7	54.7	820.8	(231.0)	(21.9)
50.0	156.9	54.9	821.3	(230.4)	(21.8)
55.0	157.1	55.0	821.8	(229.8)	(21.7)
75.0	157.8	55.6	824.0	(227.1)	(21.3)
95.0	159.1	56.8	828.1	(222.4)	(20.5)
97.5	159.5	57.2	829.5	(220.6)	(20.3)
99.0	160.0	57.6	830.7	(219.1)	(20.0)
99.5	160.2	57.9	831.7	(218.1)	(19.8)
99.9	160.9	58.6	834.1	(215.6)	(19.4)

A.6 Solvency Provisions for Expense Risk

6.1 A detailed understanding of the company's expense structure and expense drivers is a key element when determining the expense risk. In the calculation of the capital for expense risk we distinguish between acquisition and maintenance expense risk. Possible methodologies used to estimate the expense risk economic capital can range from simple to complex. However more

importantly whatever methodology is used the process focuses on understanding the underlying structure of company expenses

6.2   A few of the key risks facing an insurer include:

- Misclassification of expenses between acquisition and maintenance, with inappropriate liabilities being established for in-force policies
- Future changes in the product offerings of the company, leading to different cost structures in the future than current
- Unstable volumes of new business and in-force
- Inflation is different than expected

6.3   One simple method to calculate solvency capital for expense risk would be to calculate as a multiple of Fixed Acquisition Expenses plus Maintenance expenses. For example, one could require the company to have sufficient capital on hand to pay for one year of additional expenses, which might represent the length of time required for a regulator to settle a problematic situation.

6.4   Alternately, one could require that maintenance expenses be explicitly provided for by inflating the best estimate unit costs by a factor that varies based on the stability and accuracy of the company's expense studies. In this way, the total solvency provision would provide for the present value of best estimate maintenance expenses to mature the in-force policies, plus an additional provision that might range from , say 2.5% to 10% of this amount. Under this model, acquisition expenses would not be explicitly provided for as Pillar 1 capital, but could be covered under Pillar 2, in which a periodic review of the company's expense study would be performed.

6.5   Additionally, inflation could be covered by putting an explicit margin on the inflation assumption and revaluing the liabilities.

A.7   Solvency Provisions for Market Risk

A.7.1   Mismatch Risk (ALM Risk)

A.7.1.1 Internal Models

7.1   The mismatch risk considers the risk that the best estimate cash flows arising from the assets supporting the liabilities, do not match the best estimate liability cash flows, which results in required reinvestment, disinvestments or borrowing required by the insurer to satisfy liquidity needs. Because the future reinvestment environment is uncertain, this can result in additional gains or losses to the insurer based on the market values of the assets at those future points in time. In other words, this risk is ultimately that the market price of the assets changes unfavorably at a time when those assets need to be liquidated.

7.2   To quantify this risk using internal models, we perform the following calculation, under two sets of stochastic reinvestment scenarios:

- First, we project the best estimate asset and liability cash flows arising from the portfolio under the stochastic reinvestment scenarios being tested. This results in net cash flows being available for reinvestment or disinvestment in each future period. These cash flows are dealt with according to an assumed reinvestment strategy, that is based on the insurers actual strategy. Future reinvestment rates are modelled based a double mean reverting process., where future yield curves are modelled based on a random walk, but where the mean rate is based on a probability distribution.

- In each scenario, the insurer will be left with a certain amount of surplus or deficit at the end of the projection. We then solve for the amount of additional assets needed at the beginning of the projection, such that we end the projection with a zero balance.

- The assets required under the base scenario are also determined. The base scenario assumes that future reinvestment rates are the best estimate, or average, of the rates projected under each of the stochastic scenarios.

7.3   Two sets of stochastic scenarios of future reinvestment rates are tested.

- First, we generate future reinvestment rates from the current yield curve as of the current valuation date. A total solvency provision is determined as the difference between the assets required at $TVaR_{99\%}$. and the best estimate assets;

- The second tests are to generate future reinvestment rates from a shocked yield curve at the current valuation date. The shocked yield curve is derived from the current yield curve by applying the maximum shock that is likely to occur to the yield curve in a 1-year period with 99.5% confidence. A solvency provision is then determined as the difference between the assets required at a lower confidence level such as $TvaR_{75\%}$. and the best estimate assets.

7.4   The final provision is based on the greater of the two calculations. The intention of this calculation is to ensure that the Company has at least enough money to establish liabilities at a lower confidence level, such as CTE (75), having survived an adverse yield shift as might occur in a 1-year period, or to provide for longer term mismatch.

7.5   In our case study, the first test produces a larger figure. The table below summarizes the capital requirements that we derive. As a matter of interest, the scenarios that produce the largest liabilities are the low interest scenarios.

	Insurance	Annuity
Percentile		
5.00%	294.6	221.0
25.00%	406.0	226.3
45.00%	472.0	229.3
50.00%	489.2	230.4
55.00%	511.3	231.1
75.00%	577.0	236.5
95.00%	807.9	243.6
97.50%	836.8	244.9
99.00%	841.9	246.1
99.50%	842.7	246.6
99.90%	843.3	247.0
CTE		
0.00%	507.8	231.4
60.00%	657.1	238.9
80.00%	757.1	241.7
95.00%	838.9	245.6
99.00%	843.5	247.1
Total Capital / Margins:		
CTE 95	331.1	14.2
CTE 99	335.7	15.7

A.7.1.2 Considerations for Standardized Approaches

7.6 A simpler standardized approach is one that would not require the company to perform asset-liability modelling. We would instead require the company to measure various statistics about the degree of mismatch, and develop factors based on that. In developing these factors and the corresponding exposure measures, it is important to be aware of the limitations of each simplified approach, and perhaps introduce additional rules that deal with those limitations.

7.7 For example, one possible standard factor approach might consist of applying factors to the assets supporting a block of business, where the factors vary based on the difference in Macaulay duration of the assets and liabilities. It is well understood, however, that duration measures do not reflect the degree of cash-flow mismatch very well. It is certainly possible for a portfolio to have grossly mismatched asset and liability cash-flows, but with virtually equivalent asset and liability durations. If such a duration-based capital requirement were implemented, it might also be appropriate to require a minimum amount of capital to deal with this shortcoming. Adjustments might also have to be developed for unusual situations with respect to the exposure base.

7.8 A simpler approach is to assume that all portfolios of like characteristics are duration mis-matched to the same degree. A set of factors could then be developed that vary only with the characteristics of the liability portfolio, such as the length of guarantee periods remaining, the ability of policyholders to withdraw funds, etc.

A.7.2 Credit Risk

7.9 The case study also includes capital provisions for asset default risk. These provisions have been established using capital requirements from Basel II, which is a new accord being developed to provide more flexibility and risk sensitivity than exists in the original 1988 Basel Accord. The 1988 Basel Accord established credit risk as eight percent of risk weighted assets, where the risk weights are prescribed by type of asset. For example, all corporate bonds are given a 100% risk-weight in the 1988 Accord (regardless of credit rating), and OECD government bonds have a weighting of zero. Under Basel II, a bank will have a choice of three approaches for capital provisions:

- Standardized Approach: This is very similar to the original 1988 Basel Accord, except that the risk-weightings applied to each asset are based on a credit rating from an external rating agency.

- Foundation Internal Rating Based (IRB) Approach: Under this approach, a bank would develop its own risk weightings for each counterparty exposure, based on its own internal model. The risk weightings are achieved through a specified formula that takes into account the probability of default (from banks internal model), time to maturity and loss given default. The time to maturity is prescribed to 2.5 years, and the loss given default is 50% for all assets.

- Advanced Internal Rating Based (IRB) Approach: This approach is similar to the Foundation approach, except that the actual time to maturity of the assets is reflected, and the loss given default is also generated from the bank's internal model.

7.10 The case study is specifically based on application of the Basel II "Advanced IRB" Approach. Our internal model is used to generate probabilities of default, time to maturity and loss given default, for each of the assets in the portfolio. Application of the Basel II formulae results in capital provisions for these assets. Although designed as a banking application, we see no reason to recommend a different approach for insurance company solvency assessments in general, and for this case study in particular.

7.11   In the table below, the capital provisions using the Basel II Advanced approach are summarized. For illustrative purposes, the impact of using alternate regulatory models (i.e., the Basel II "Standardized" and "Foundation" approaches, as well as the original 1988 Accord) and several internal models is also shown.

Basel II (Advanced)

Assets		Par Value	Book Value	Required	As % Par
		(Exposure at Default)		Capital	Value
Cash and equivalents		129,767	129,767	-	0.0%
Bonds of OECD countries		654,903	661,189	-	0.0%
Subtotal		784,670	790,956	-	0.0%
Corporate Bonds	AAA	127,387	135,924	1,843	1.4%
	AA	325,341	332,544	5,815	1.8%
	A	204,578	205,706	5,730	2.8%
	BBB	105,003	111,635	7,419	7.1%
Subtotal		762,309	785,809	20,807	2.7%
Total		1,546,979	1,576,765	20,807	1.3%

7.12   For illustrative purposes, we also show the impact of using alternate regulatory models (i.e., the Basel II "Standardized" and "Foundation" approaches, as well as the original 1988 Accord) and several internal models. This analysis is done on the corporate bond portfolio only, as the other assets are assumed to have no default risk.

Capital for Asset Default Under Alternate Models

Assets	Par Value	Required	
	(Exposure at Default)	Capital	As % Par Value
Basel II (Advanced)	762,309	20,807	2.7%
Basel II (Standard)	762,309	23,827	3.1%
Basel II (Foundation)	762,309	11,485	1.5%
Basel 1988 Accord	762,309	60,985	8.0%
Internal Models			
Model (1)	762,309	12,197	1.6%
Model (2)	762,309	19,343	2.5%
Model (3)	762,309	26,229	3.4%

7.13   The Basel II (Advanced) model generates capital provisions of 2.7% of exposure at default (par value), based on the actual times to maturity of the assets in our portfolio, and based on our internal model estimates of probabilities of default. It is interesting to note that the Basel II (Standard) approach gives a slightly higher capitalization because of the use of prescribed risk weighting factors that vary only by credit rating. In the Basel II (Foundation) model, we generate significantly lower capital requirements than both the Standard and Advanced approaches, because we are allowed to reflect our own internal probabilities of default but are required to use an average time to maturity of 2.5 years. While this may be representative of a typical banking

book, most life insurance portfolios are of longer duration, which suggests that the Foundation approach may not be appropriate for life insurance asset portfolios. In the internal model results, we use three independent models. The first of our three internal models also generate capital requirements between 1.6% and 3.5% of exposure at default. The first model (the KMV model) attributes capital based on probabilities of default, loss given default, correlation between assets, and also the diversification of the portfolio. [9]The second and third models are described in the Institute of Actuaries of Australia paper by Martin Paino and Greg Martin, as being an adjusted Default model (DM) and an adjusted Mark to Market (MTM) model, shown as Model (2) and Model (3) respectively in the table above.

7.14 Capital requirements for asset default should only be based on those assets that support the liabilities and required capital of the company. Ideally, we would not establish provisions for assets that support the free surplus, for reasons explained in this Report. Ignoring asset defaults on free surplus assets would result in an iterative process to determine capital requirements. This is because the free surplus and the asset default capital requirements are inter-related. For simplicity, we have ignored this in the case study, and have simply shown asset default capital requirements for all assets in the company, regardless of whether the asset is considered free surplus.

7.15 The asset default requirements in the case study are considered "type A", which means that they provide for asset defaults on existing assets only. Because of the long term nature of life insurance, insurers must also be concerned with "type B" asset default risks, that is, asset defaults on future assets purchased by the insurer with future positive cash flows. We have provided for this in the case study by discounting liability cash flows at a risk-free rate. The spread between the risk-free rate and the expected returns of specific assets, however, reflect both asset default and liquidity risks, and so, we may be overly conservative in the case study by assuming that the entire spread represents an asset default provision.

A.8   Effects of Reinsurance on Internal Model

A.8.1 Effects on Insurance Risk

8.1 Our case study until now has been based on the assumption that there is no reinsurance in place. Suppose now that the company wanted to reinsure the mortality risk for one of its product segments, ALC 1001. The reader will recall that this product is a term to 100 product with approximately 56,000 lives in it, and approximately $150 million of liabilities as measured on a best estimate basis at the risk-free rate.

8.2 We are interested in the effects of various reinsurance structures on the mortality risk, both as regards the product ALC 1001 on a stand-alone basis, but also as it affects the total provisions for mortality risk.

8.3 We consider several different types of reinsurance arrangements. For the purposes of this analysis, we differentiate the reinsurance arrangements into 2 categories:

- Reinsurance that guarantees the future mortality cost for a portion of the risks, with reinsurance premiums guaranteed at the Company's expected mortality level for a period of time;

- Reinsurance that guarantees and lowers the future mortality cost for a portion of the risks. Reinsurance premiums in these cases are guaranteed at lower rates than the Company's expected mortality levels.

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[9] Martin G, Paino M., 2003, *Capital Reserving for Credit Risk for Insurers (Life and GI) and other Institutions*, Institute of Actuaries of Australia

Alternate Reinsurance Structures Applicable to Product ALC 1001

Reinsurance	Description	Amount of cession	Reinsurance premiums
1. Gross of reinsurance	No reinsurance	0	N/A
2. YRT Coinsurance at neutral reinsurance rates	45% of sum assured ceded on YRT basis	45% sum assured on each policy; roughly $2 billion in aggregate	YRT at 70% of Industry table, with annual adjustments equal to Company's expected trend
3. YRT Excess reinsurance, at neutral rates	Sum assured in excess of $50,000 ceded on YRT basis	Excess of sum assured over $50,000; roughly $2 billion in aggregate	As 2. Above
4. YRT coinsurance, neutral rates	90% of sum assured ceded on YRT basis	90% sum assured on each policy; roughly $3.2 billion in aggregate	As 2. Above
5. YRT Coinsurance at low rates	45% of sum assured ceded on YRT basis	As 2. Above	YRT at 70% of Industry table, with annual adjustments equal to Company's expected trend
6. YRT Excess at low rates	Sum assured in excess of $50,000 ceded on YRT basis	As 3. Above	YRT at 70% of Industry table, with annual adjustments equal to Company's expected trend
7. Quota Share	Reinsurer accepts 45% of all cashflows	45% of all cashflows	N/A

8.4   The results are shown in the table below. The following comments are noteworthy:

1.   The reinsurance is designed to cede away 45% of the risk, approximately (except in one case). As we can see in the table, the level and trend risks are indeed ceded away by roughly that amount, but the specifics depend on the structure of the reinsurance. For example, when we lock in premiums of 70% of the expected table, which is the same as the Company expected mortality cost, then we do indeed cede away approximately 45% of the risk. When we lock in more favorable rates, however, we see that the level and trend risks actually increase relative to the 70% premiums (The liability iteself decreases by $60 million because of the more favorable rates (not shown), but the capital relative to this figure increases). This is because we've changed our exposure to the risk. With these new terms, reinsurance becomes more expensive relative to the best estimate liabilities in favorable mortality improvement scenarios.

2. Obviously, if the goal is to control the volatility risk, then an excess retention structure is better. These structures reduce the volatility and catastrophe risk without materially impacting the level and trend risks. On this issue, though, it is important to note that many capital standardized systems do not differentiate between the types of reinsurance structures. In Canada, for example, the capital for volatiltiy and catastrophe would be the same for all of these reinsurance structures, even though clearly, the form of the structure affects the risk.

3. On the 90% coinsurance arrangement, we've ceded away over 90% of the level risk, but only 80% of the trend risk. In addition, the exposure has actually changed direction. The company is now better off if mortality worsens, because the company is only exposed to 10% of the actual mortality losses in current periods, and stands to gain on a reduction of future reinsurance premiums on the 90% that is ceded. A standardized system would only be able to capture such a dynamic if the assumptions themselves were carefully mandated, and not through simple use of factors.

Effect of Reinsurance on Mortality Capital
Product ALC 1001 Only

	Reinsurance	Ceded:	Reinsurance Premiums	Capital for Mortality Risks			
				Level	Trend	Volatility	Catastrophe
1	Gross	N/A	N/A	43.1	50.1	3.4	6.2
2	Coinsurance	45%	70% Table	20.9	20.3	1.8	3.4
3	Excess retention	>$50,000	70% Table	22.3	21.7	0.9	3.5
4	Coinsurance	90%	70% Table	2.2	9.2	0.3	0.6
5	Coinsurance	45%	45% Table	23.3	23.4	1.9	3.5
6	Excess retention	>$50,000	45% Table	23.6	25.2	0.9	3.6
7	Quota Share	45%	N/A	24.3	27.2	1.9	3.4

A.8.2 Counter-party Risk

8.5 Under the various reinsurance arrangements discussed above, the company would be subject to additional credit risk in the form of counter-party risk. This could be quantified by applying factors to the amount exposed to risk of default by the counter-party, i.e., the reinsurer.

8.6 One approach that could be taken is to base the probability of default on the credit rating of the reinsurer. For example, if the reinsurer in our case study were rated "A", we could assume probabilities of default consistent with any "A" rated asset as per Basel II. Using the Basel II (Foundation) factors, we would assign an annual probability of default of 0.7%.

8.7 The amount of risk exposed would reflect the amount of assets that the Company would lose should the reinsurer default. This would include any outstanding receivables from the reinsurer net of outstanding payables at a minimum, but may also include reserves ceded to the reinsurer which would have to be re-established on the balance sheet of the company. Reserves might be too conservative an estimate of the amount at risk, however, as the Company would potentially have the opportunity to obtain replacement coverage.

A.9 Conclusions

9.1 The case study highlights that one can conceptualize an advanced model for a life insurance company that in turn can be used to develop a standardized approach for those life risks that are well understood and for which there is ample historical data. One must exercise more care in developing a standardized approach for other life risks, to ensure that the impacts of policyholder behavior, complex options in the policies and the complex interactions between risks are reflected in an appropriate manner.

Appendix B   Non-Life (P&C) Insurance Case Study

B.1   Introduction

1.1   This non-life insurance company case study has been prepared by the WP to illustrate some of the concepts discussed in this report. The main purpose of the case study is to describe calculations that a company might undertake in order to determine total solvency provisions for various risks, and to highlight some of the issues in these calculations.

1.2   This non-life insurance company case study has been prepared by the WP to illustrate some of the concepts discussed in this report. The main purpose of the case study is to describe calculations that a company might undertake in order to determine total solvency provisions for various risks, and to highlight some of the issues in these calculations.

1.3   This case study begins by using a model of insurer aggregate losses to calculate the assets needed to support the insurer's liabilities. The model produces the distribution of the total loss arising from post calculation date exposures and unpaid claims liabilities arising from past exposures. From this distribution, we set the required assets equal to the Tail Value-at-Risk, evaluated at the 99% level ($TVaR_{99\%}$).

1.4   These assets can come from two sources. The first source is from the policyholders, after the provision for the various reserves and expenses (including reinsurance expenses) are removed. The second source is the investors, through either a direct contribution to capital or from retained earnings from prior years of operation.

1.5   In this case study, the risk-based capital charge is defined as:

$TVaR_{99\%}$ – Expected Net Losses on Current Business – Net Loss Reserve

1.6   The reserves are set at the expected value of future payments with no discounting for the time value of money. The size of the reserves to subtract from the assets deserves some discussion. The loss reserve could be set at the expected present value of future payments. If a more conservative estimate is desired, an insurer could remove the discount for the time value money, or even require a more conservative estimate. Ultimately, such a decision is left up to the insurance regulators.

1.7   This case study concentrates on underwriting risk and does not consider other sources of risk. A complete risk-based capital formula should also consider asset risk and well as the risk of premium deficiency, i.e. the risk that the market will not allow adequate premiums.

1.8   This case study illustrates two ways to calculate the insurance risk portion of the minimum capital requirement for a general insurance company. The first calculation will be a factor-driven formula where the parameters can be specified by either the regulator, or by the insurer – presumably with the regulator's approval. The second calculation will be derived from a more detailed model of the insurer's underwriting risk.

1.9   The working party proposes that the regulator prescribe a factor-based formula as a starting point for a risk-based capital analysis. Since it is a starting point, it should be subject to the operational constraints.

- Simplicity – The formula can be put on a spreadsheet. This may allow for some complexity in the formulas, as long as the objective of the formulas is clear.
- Input Availability – The inputs needed for the formula are either readily available, or can be reasonably estimated with the help of the appointed actuary.
- Conservative – When there is uncertainty in the values of the parameters, the parameters should be chosen to yield a conservative estimate of the required capital.

1.10   The working party proposes that, with the regulator's approval, an insurer may substitute its own internal model for the factor based formula. The internal model can be a minor change to the factor-based formula, or a completely different model. The regulator may want to set standards for internal models. A set of standards is proposed elsewhere in this report.

1.11   The case study will cover two different insurance companies each with three different reinsurance strategies.

B.2   The Insurance Companies

2.1   We illustrate the risk-based capital calculations on the hypothetical ABC Insurance Company and the XYZ Insurance Companies. Table 1 gives premium and loss reserve statistics for these insurance companies. Here are some additional details about these companies.

- The lines of insurance covered by these insurers are standard personal and commercial lines that are typically written by an insurer in the USA. In addition, there are separately identified catastrophe coverages.

- The distribution of losses was generated with the collective risk model. This model describes the losses in terms of the underlying claim severity and claim count distributions.

- The claim severity distributions for each insurance company are identical. The claim count distribution for the ABC Insurance Company has a mean that is ten times the mean of the claim count distribution for the XYZ Insurance Company for each line of insurance. As a consequence, expected loss for ABC is ten times that of XYZ for each line of insurance.

- Three different reinsurance strategies are considered. The first strategy is no reinsurance. The second strategy covers 95% of the losses in excess of $50 million ($5 million) of catastrophe losses for ABC (XYZ), but provides no coverage for the other lines. The third strategy adds a $1 million limit on the non-catastrophe lines.

Table 1

Statistics for the Sample Insurance Companies

Line of Insurance	ABC Insurance Company		XYZ Insurance Company	
	Direct Premium	Loss Reserve	Direct Premium	Loss Reserve
Auto Liability	430,000,000	403,110,711	43,000,000	40,311,071
Auto Physical Damage	325,000,000	19,455,630	32,500,000	1,945,563
Homeowners	475,000,000	162,578,183	47,500,000	16,257,818
Commercial Liability	130,000,000	352,190,005	13,000,000	35,219,001
Commercial Property	200,000,000	62,204,206	20,000,000	6,220,421
Total	1,560,000,000	999,538,735	156,000,000	99,953,873

B.3 The Loss Model Underlying the Factor Based Formula

3.1 In this case study, we give an example of a factor-driven risk-based capital formula. This formula is sensitive to:

1. The volume of business in each line of business;
2. The overall volatility of each line of insurance;
3. The reinsurance provisions; and
4. The correlation, or dependency structure, between each line of business.

3.2 The formula requires the insurer to input expected losses (and expected future payments for loss reserves) by line of insurance. Other parameters (specified below) can be determined by either the regulators or the insurers.

3.3 The formula is derived from a model that can be visualized as a computer simulation of the losses for each line of insurance. Using the parameters of the model, it calculates the first two moments of the aggregate loss distribution and then estimates the Tail Value-at-Risk at a selected level 99%, ($TVaR_{99\%}$), by assuming that the aggregate loss distribution is lognormal.

3.4 What follows is a more technical description of the model.

3.5 Simulation Algorithm Underlying Factor-Based Formula
1. For each line of insurance i, with uncertain claim payments, do the following:
    - Select a random number χ_i from a gamma distribution with mean 1 and variance c.
    - Select a random claim count K_i from a Poisson distribution with mean $\chi_i \cdot \lambda_i$ where λ_i is the expected claim count for line of insurance i.
    - For each i and for $k = 1,\ldots,K_i$, select a random claim size, Z_{ik}, from a lognormal distribution with mean μ_i and standard deviation σ_i.

2. Set $X_i = \sum_{k=1}^{K_h} Z_{ik}$ = Loss for line of insurance i.

3. Select a random number p, from a uniform $(0,1)$ distribution. For each line i, select β_i to be the pth percentile of a distribution with $E[\beta_i] = 1$ and $Var[\beta_i] = b_i$. This gives a multivariate distribution of the β_i's in which each coefficient of correlation, ρ_{ij} is equal to 1.

4. Set $X = \sum_i \beta_i \cdot X_i$ = Loss for the insurer.

3.6 Here are the formulas used to calculate the first two moments of X.

1. $E[X_i] = \lambda_i \mu_i$

2. $E[X] = \sum_i E[X_i]$

3. $Var[K_i] = \lambda_i + c_i \lambda_i^2$.

4. $Var[X_i] = \lambda_i \sigma_i^2 + \mu_i^2(\lambda_i + c_i \lambda_i^2)$

5. $Var[\beta_i X_i] = Cov[\beta_i X_i, \beta_i X_i]$
    $= (1+b_i)Var[X_i] + E[X_i]^2 b_i = (1+b_i)(\lambda_i \sigma_i^2 + \mu_i^2(\lambda_i + c_i \lambda_i^2)) + b_i \mu_i^2 \lambda_i^2$

6. For $i \neq j$ $Cov[\beta_i X_i, \beta_j X_j] = \lambda_i \mu_i \lambda_j \mu_j \rho_{ij} \sqrt{b_i b_j}$ (Note that we assume that $\rho_{ij} = 1$.)

7. $Var[X] = \sum_i \sum_j Cov[\beta_i X_i, \beta_j X_j]$

3.7 Given the mean and the variance of the insurer's aggregate loss distribution one can calculate $TVaR_\alpha(X)$ by the following steps. This description will make use of formulas for the lognormal distribution in Appendix A in the book, *Loss Models* by Klugman, Panjer and Willmot[10] (KPW).

1. Calculate the parameters of the lognormal distribution that has the same mean and variance of the insurer's aggregate loss distribution.

2. Calculate the Value-at-Risk at level α, $VaR_\alpha(X)$, (i.e, the αth percentile) of the lognormal distribution.

3. Calculate the limited expected value, $E[X^\wedge VaR_\alpha(X)]$ for the lognormal distribution.

4. Then $TVaR_\alpha(X) = VaR_\alpha(X) + \dfrac{E[X] - E[X \wedge VaR_\alpha(X)]}{1-\alpha}$

3.8 Using the Poisson distribution to model claim counts and the lognormal distribution to model claim severity are fairly standard assumptions in the actuarial theory of risk and we will not discuss these further. The role of the "b" and "c" parameters is not standard and thus it deserves some discussion.

3.9 Introductory treatments of insurance mathematics often make the assumption that there are n identical insurance policies each with independent and identically distributed loss random variables X_i. Let X be the sum of all the X_i's. Then the variance of the loss ratio, $X/E[X]$ is given by $Var[X_i]/(nE[X_i])$. This model implies that as n increases, the variance of the loss ratio decreases with the result that a very large insurance company can write insurance with minimal risk.

3.10 Let us now apply the same idea to a line of insurance defined by our model above.

$$Var\left[\frac{\beta_i X_i}{E[\beta_i X_i]}\right] = (1+b_i)\left(\frac{\mu_i^2 + \sigma_i^2}{\lambda_i} + c_i\right) + b_i$$

3.11 As λ_i increases, the variance of the loss ratio decreases, but it never decreases below $b_i + c_i + b_i c_i$. This means that, unlike the introductory result, an insurer will always be exposed to risk regardless of how many policies it writes in line i. This model better resembles the real insurance environment because a changing economic environment always makes the outcome of writing insurance uncertain.

3.12 Meyers, Klinker and Lalonde[11] (MKL) show how to estimate the b and c parameters from industry data. Making the assumption that the b and c parameters are the same for all insurers, they show how to estimate there parameters from the reported loss ratios of several insurers.

3.13 An experienced observer of insurer loss ratios by line of business should be able to develop some intuition about the magnitude of the b and c parameters. Note that loss ratios for large insurers are less volatile than smaller insurers. Note that the c parameters affect correlation between individual insurance policies within a line of business, while the b parameters affect correlations between lines of business. One can also form some intuition about the kind of events that drive insurer loss ratios across lines of business, such as inflation, and the degree to which these events are predictable.

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[10] Stuart Klugman, Harry Panjer and Gordon Willmot, *Loss Models: From Data to Decisions*, Wiley 1998.
[11] Glenn Meyers, Frederik Klinker and David Lalonde, "The Aggregation and Correlation of Insurance Exposure." To appear in the *CAS Forum*, Summer 2003.

3.14   Simple analyses of industry accident year loss ratios by line of business can provide a rough quantification of $b_i + c_i + b_ic_i$. As an example, let's suppose that one estimates that the standard deviation of the loss ratio (actual loss divided by expected loss) for a line of business can be no smaller than 20% regardless of the size of the insurer. This would tell us that $b_i + c_i + b_ic_i$ is equal to $0.2^2 = 0.04$. Suppose further that we estimate the standard deviation of inflationary effects to be 5%. This means that $b_i = 0.05^2 = 0.0025$. Then $0.04 = 0.05^2 + c_i + 0.05^2 \cdot c_i$ which implies that $c_i = 0.0374$.

3.15   The intuitive ideas expressed in the above two paragraphs are formalized in the estimation procedure provided in MKL.

B.4   Calculating the Risk-Based Capital with a Factor Based Formula

4.1   To use the above model to calculate the risk-based capital the regulators, in consultation with the insurers, must determine the following parameters, before the application of the reinsurance, of the loss model for each line of insurance for both current business and unsettled claims for past business.

- The expected value of the lognormal claim severity distribution
- The coefficient of variation, CV_i, of the lognormal claim severity distribution
- The b_i and c_i parameters

4.2   The parameters used in this case study are given in Table 2 below.

Table 2
Model Parameters for Factor-Based Formula

Line Name$_i$	Mean$_i$	CVi	c_i	b_i
Auto Liability	6,000	7	0.02	0.003
AL – Reserve	18,000	4	0.02	0.003
Auto Phys Dam	1,500	2	0.01	0.002
APD – Reserve	1,500	2	0.01	0.002
Homeowners	4,000	5	0.04	0.010
HO – Reserve	5,000	4	0.04	0.010
Business Liability	16,000	16	0.03	0.003
BL – Reserve	65,000	10	0.03	0.003
Business Property	20,000	12	0.04	0.010
BP – Reserve	20,000	12	0.04	0.010

4.3   Using formulas in Appendix A of KPW, the insurer then calculates the parameters μ_i and σ_i after the application of reinsurance. The μ_i's and the σ_i's for no reinsurance, and for reinsurance covering the excess over $1 million per claim are given in Table 3.

Table 3
Moments of the Claim Severity Distributions

Line Name	No Reinsurance		Excess Reinsurance over $1 Million	
	μ_i	σ_i	μ_i	σ_i
Auto Liability	6,000	42,000	5,844	27,821
AL – Reserve	18,000	72,000	17,522	52,604
Auto Phys Dam	1,500	3,000	1,500	3,000
APD – Reserve	1,500	3,000	1,500	3,000
Homeowners	4,000	20,000	3,975	16,929
HO – Reserve	5,000	20,000	4,980	17,889
Business Liability	16,000	256,000	13,169	63,119
BL – Reserve	65,000	650,000	47,082	134,818
Business Property	20,000	240,000	16,825	70,720
BP – Reserve	20,000	240,000	16,825	70,720

4.4   The next step is for the insurer to provide estimates of the expected claim counts, λ_i, for each line of insurance. These estimates are derived by dividing the expected claim severity, μ_i, into the insurer's estimate of expected losses by line of insurance. These insurer estimates are based on its volume of business in each line. Table 4 contains the λ_i's used in this case study. These λ_i's were determined by dividing the μ_i's in Table 3 into the insurer estimates of its expected losses by line when there is no reinsurance.

Table 4
Expected Claim Counts

Line Name	ABC Insurance Company		XYZ Insurance Company	
	Expected Loss	λ_i	Expected Loss	λ_i
Auto Liability	350,000,000	58,333.33	35,000,000	5,833.33
AL - Reserve	403,110,711	22,395.04	40,311,071	2,239.50
Auto Phys Dam	250,000,000	166,666.67	25,000,000	16,666.67
APD - Reserve	19,455,630	12,970.42	1,945,563	1,297.04
Homeowners	350,000,000	87,500.00	35,000,000	8,750.00
HO - Reserve	162,578,183	32,515.64	16,257,818	3,251.56
Business Liability	100,000,000	6,250.00	10,000,000	625.00
BL - Reserve	352,190,005	5,418.31	35,219,001	541.83
Business Property	150,000,000	7,500.00	15,000,000	750.00
BP - Reserve	62,204,206	3,110.21	6,220,421	311.02

4.5   Tables 2, 3 and 4 above give all the information necessary to calculate the mean and variance (or standard deviation) of the aggregate loss distributions for each insurer and reinsurance strategy using the formulas provided in the previous section. The results of these calculations are given in Table 5.

Table 5
Moments of the Aggregate Loss Distributions

	ABC Insurance Company		XYZ Insurance Company	
Reinsurance	**None**	**XS $1 Million**	**None**	**XS $1 Million**
$E[X]$	2,199,538,735	2,028,476,777	219,953,873	202,847,678
$StDev[X]$	209,192,020	186,362,345	27,654,067	19,462,856

4.6　　It is worth noting that while the expected losses for ABC are exactly ten times the corresponding expected losses for XYZ, the standard deviations for ABC are less than ten times the corresponding standard deviations for XYZ.

4.7　　Now that we have the means and variances of the aggregate loss distributions we turn to calculating the risk based capital. Following the formulas outlined in the previous section we calculate the $TVaR_{99\%}$ for each insurer and reinsurance strategy.

4.8　　As noted above, the $TVaR_{99\%}$ was calculated by approximating the aggregate loss distributions with a lognormal distribution with the same first two moments. The working party did not feel that this was appropriate when the insurer was exposed to catastrophic risk. Thus the formula determines the final risk-based capital for the underwriting risk by adding a catastrophe probable maximum loss to the $TVaR_{99\%}$. In this case study we used the 99th percentile of a catastrophe loss distribution generated by the catastrophe model maintained by Applied Insurance Research. Thus the formula for the risk-based capital is given by:

$$TVaR_{99\%} - \begin{matrix}\text{Expected Net Loss} \\ \text{On Current Business}\end{matrix} - \text{Net Loss Reserve} + \text{Catastrophe PML}$$

4.9　　The final risk-based capital calculations for the various reinsurance strategies are included in Table 6.

Table 6
Risk-Based Capital from Factor Based Formula

	ABC Insurance Company			XYZ Insurance Company		
Reinsurance	**None**	**Cat Only**	**All Lines**	**None**	**Cat Only**	**All Lines**
$TVaR_{99\%}$	2,821,018,276	2,821,018,276	2,580,135,062	304,943,284	304,943,284	260,723,343
Expected Loss	1,200,000,000	1,200,000,000	1,147,246,365	120,000,000	120,000,000	114,724,636
Reserve	999,538,735	999,538,735	881,230,412	99,953,873	99,953,873	88,123,041
Cat PML	143,000,000	65,000,000	65,000,000	14,300,000	6,500,000	6,500,000
Capital	764,479,541	686,479,541	616,658,285	99,289,411	91,489,411	64,375,665

4.10　　While this factor based formula does involve a number of equations, it can be implemented on a fairly compact spreadsheet. The necessary mathematical manipulations are doable by a recently trained actuary.

B.5 Calculating the Risk-Based Capital with an Internal Risk Management Model

5.1 It should be clear that there are several alternatives to the model underlying the factor based risk-based capital formula. The working party believes that a model underlying a prescribed risk-based capital should be deliberately conservative. The working party proposal allows the insurer to use its own model for risk-based capital calculations, subject to standards for risk-based capital formulas. This section gives an example of such a model.

5.2 The model described here is applied to the ABC and XYZ insurance companies. It differs from the model used in the factor-based formula in the following respects.

- The choices of the claim severity distributions were not conservative. It uses claim severity distributions that were derived from its own analysis of claim severity.

- The structure of the model is richer. Random multipliers applied to the claim count distributions across lines allow for a relaxation of the conservative assumption that $\rho_{ij} = 1$ for all lines of business i and j.

- The model calculates the aggregate loss distribution directly, rather than approximate the aggregate loss distribution with the first two moments.

- Determining the needed assets for the insurer by adding the catastrophe probable maximum loss to the $TVAR_{99\%}$ is in essence, adding "worst case scenarios." The catastrophe model was incorporated directly into the internal risk-management model.

5.3 Additional details on the construction of this model are given by MKL. Table 7 gives the risk-based capital charge derived from the internal risk management model for the ABC and XYZ Insurance Companies for the various reinsurance strategies.

Table 7
Risk-Based Capital from Internal Risk Management Model

Reinsurance	ABC Insurance Company			XYZ Insurance Company		
	None	Cat Only	All Lines	None	Cat Only	All Lines
$TVaR_{99\%}$	2,665,306,927	2,649,246,793	2,431,822,820	305,543,931	304,931,938	245,968,540
Expected Loss	1,215,000,000	1,212,045,992	1,158,671,051	121,500,000	121,204,599	115,867,105
Reserve	999,538,735	999,538,735	879,134,113	99,953,873	99,953,873	87,913,411
Capital	450,768,192	437,662,066	394,017,656	84,090,057	83,773,466	42,188,024

B.6 Provisions for Adverse Deviations in Reserves

6.1 The working party also considered methods for including a provision for adverse deviation (PAD) in the reserves. In this section we give an example of how this might work with the factor based formula.

6.2 Rather than book the reserves for losses at their expected value, this example first calculates the PAD as the tail value-at-risk at the 75% level minus the expected loss for the reserve in each line of insurance. The PAD calculation is also done for the expected loss in current business.

6.3   Next the PAD is calculated for the insurer in total. Because of diversification, this PAD is less than the sum of the PADs for each line of business. The each line of business PAD is adjusted proportionally so that the line of business PADs sum to the overall PAD.

6.4   Note that the total assets for the insurer remain the same, and the expected losses remain the same. The PAD's simply shift a portion of the capital over to the insurer's liabilities.

6.5   The results of these calculations for the ABC and XYZ Insurance Companies are included in Table 8.

Table 8

Risk-Based Capital from Factor Based Formula with Reserve PADs

	ABC Insurance Company			XYZ Insurance Company		
Reinsurance	**None**	**Cat Only**	**All Lines**	**None**	**Cat Only**	**All Lines**
$TVaR_{99\%}$	2,821,018,276	2,821,018,276	2,580,135,062	304,943,284	304,943,284	260,723,343
Expected Loss + PAD@75%	1,343,215,450	1,343,215,450	1,282,664,387	137,436,601	137,436,601	128,555,154
Reserve + PAD@75%	1,129,887,753	1,129,887,753	989,316,751	118,954,857	118,954,857	99,750,020
Cat PML	143,000,000	65,000,000	65,000,000	14,300,000	6,500,000	6,500,000
Capital	490,915,073	412,915,073	373,153,923	62,851,825	55,051,825	38,918,169

Appendix C   Health Insurance Case Study

C.1   Introduction

1.1   This health insurance company case study has been prepared by the WP to illustrate some of the concepts discussed in this report. The main purpose of the case study is to describe calculations that a company might undertake in order to determine total solvency provisions for various risks, and to highlight some of the issues in these calculations.

1.2   This case study should be regarded as a general example in which typical health insurance issues are discussed, with a focus on medical insurance. As there are major differences between coverages, policy conditions and legislation of health insurance arrangements in different countries, it is not possible to cover all existing arrangements here.

1.3   This Appendix contains methodology for modeling risks in medical insurance and related products. A categorization of the risks is made into volatility, uncertainty and extreme event elements, as outlined in the main report.

1.4   In the next section some special features of medical insurance are discussed. The third section comments on the model structure while the fourth section discusses the separate risk categories in more detail, illustrated by case studies. In the fifth section a separate case study for medical inflation is shown. Finally the sixth section discusses methodologies for standardized approaches and aggregation.

C.2   Special Features of Medical Insurance

2.1   In this section some of the special features of medical insurance are discussed. Due to these special features the models for Life and P&C insurance risks may not always be sufficiently equipped to deal with medical insurance.

C.2..1   Medical Inflation

2.2   Medical expenses generally show a tendency to increase more than general inflation. There are several explanations for this phenomenon.

2.3   Developments in the field of medical technology can lead to increases of the overall expense level in health care. For example, some technological developments have lowered the expenses involved in the treatment of specific illnesses and have provided better outcomes for the persons with those illnesses. Some new technologies have dramatically improved the detection rate of certain illnesses, which then leads to rapid increases in the overall costs of treating those illnesses and usually much improved success rates. Other new technologies have greatly increased the cost of treatment of certain illnesses usually with significantly better outcomes (often the patient living considerably much longer). Finally there are a number of new technologies that have significantly reduced the risk of poor outcomes from certain treatments thus enabling these treatments to be provided to a much wider group of patients.

2.4   In the future, new detection technologies, while relatively cheap, are likely to be very widely demanded. Also, the further individualization and increased sophistication of medical interventions is unlikely to reduce the overall rate of growth in the expense levels of health care. The resulting longer life expectancies that are obtained from new medical technologies will also enable some people who benefit from these technologies to consume even more medical resources over their lifetime.

2.5   Improved information gathering, recording and reporting leads and will lead to efficiency gains, which enable many more conditions to be detected and/or treated external to the labor intensive hospital setting. But many of these conditions would not have been detected and/or treated prior to these technology changes so the efficiency gains eventually show up as long-term improvements in population health status and, to some extent, longevity - outcomes, which are hard to measure or relate back to the technologies in the short term.

2.6   As a result of efficiency gains in other sectors of the economy the cost of labor increases in all sectors including the health sector. As there is little efficiency gain in the hospital sector which can reduce the number of staff required the overall expense level of this sector of health care can be expected to become relatively more expensive. The hospital sector in particular is relatively labor-intensive so there are generally less efficiency gains which can be realized through automation than there are in other sectors of the economy. The skill sets of hospital labor are also being constantly upgraded, which is putting further pressure on labor costs.

C.2..2 Political Risk

2.7   It is common for democratically elected Governments to make promises in respect of the supply of health care services. In endeavoring to reduce their health expenditure these Governments will then often try to reduce the price of health care by controlling the supply of health care services through various rationing techniques including through the control of prices health care professionals can obtain for their services. These constraints do tend to reduce expenditure in the short to medium term but in the longer term the eventual constraints on supply of quality services create their own political risks.

2.8   A further method of reducing Government health expenditure is to regulate both the market for private health insurance and the extent of the services covered by private health insurance. So Governments often dictate policy conditions and premium rates of medical insurance so that it may not be possible to fully adjust rates and conditions to the level commercially desired.

2.9   In the field of disability insurance, incidence rates and periods of disablement may also be influenced by Government. Sometimes incidence and periods of disablement are influenced by Government mandated benefit levels or Government mandated underwriting requirements.

C.3   Modeling Structure

3.1   The modeling structure includes a 'best estimate' and various types of risk. The best estimate is the expected claims liability that will result for the insurer. Due to various types of risks, the best estimate will in reality almost never materialise, but a higher or lower claims liability will occur.

The best estimate is discussed in the first subsection, risk types are discussed in the second subsection.

C. 3.1 Best Estimate

3.2   The best estimate is the expected liability under the in-force contract. We distinguish between the best estimate in the first period and in the periods thereafter. The term is expressed as a "period" because medical insurance modeling is in some cases done annually, and in others quarterly or monthly.

3.3   The best estimate (BE) in the first period is determined as:

BE first period = $n*d*l$

with

n : average number of insured in the in force portfolio during the period;

d: discount factor applied to reflect that claims occur on average in the middle of the period. In projections over short periods or in low interest rate environments d is usually excluded from the equation.

l: expected incurred claims liability per insured.

3.4   In this formula the impact of lapses is ignored, which leads to a more conservative estimate of the liability. In the longer term, lapses may result in an antiselection effect against the insurer (i.e. higher lapse rate for insureds who are less likely to claim). However the effect of antiselection is limited in the short term, especially when premiums have been received in advance for the entire contract period.

3.5   The approach described above can be used for products where premiums are periodically adjustable so that a best estimate projection is only needed for the first period. In the case of products with multi-period guarantees, or where there are conditions that restrict the insurer's ability to increase premium rates to reflect increasing claim costs, a more sophisticated model approach is required. Such an approach takes into account the development of the expected incurred claims amount and the impact of lapses over longer periods.

C.3.2   Risk Types

3.6   Three types of risks are distinguished which will cause the actual liability to deviate from the best estimate.

3.7   *Volatility Risk*: the risk that the actual frequency and severity of claims differs from the best estimate in the particular period under consideration, but the expected liability for the average insured in the whole population is correctly estimated. When projections are performed for monthly or quarterly periods then it is also important to consider the seasonal effects on the volatility of claim rates.

3.8   *Uncertainty Risk*: the risk that the expected liability per insured is incorrectly estimated at present or it is correctly estimated at present but changes over time. Usually for multi-period models it is important to include at least the first order changes over time. These will be related to the change in the demographics of the insured population (for example the change in the average hospital utilization rates at older ages is greater than at younger ages) and the expected medical inflation rate for that insured population (this also tends to be higher for middle aged and older populations than younger persons). For some types of contracts, there can also be a moral hazard risk: the risk of individual insured persons deliberately selecting against the insurer. This can particularly happen in cafeteria arrangements or in any insurance arrangement when a number of choices are available to insured individuals.

3.9   *Extreme event/Calamity risk*: the risk of large one-off accumulation of claims outside the normal experience pattern.

C.4 Modeling Techniques

4.1   In this section the modeling of volatility, short term uncertainty and extreme event risk is discussed. A separate, more extensive case study for long term uncertainty caused by medical inflation is contained in section 5. For the short term risk model, the time period chosen is one year.

C.4.1 Volatility

4.2   The volatility risk is determined only for the first period of projection[12]. To determine volatility risk, the parameters driving frequency and severity of claims are assumed to be fixed and given. The remaining risk is the risk that the claims volume is different from its expected level due to randomly occurring deviations.

4.3   The volatility risk can be modeled with a probability distribution of the frequency and severity of the individual claims.

4.4   We define:

N : the number of claims;

X_i : the claim size of the i-th claim, with $i = 1,2,...,N$;

$S = \Sigma\, X_i$, the total claims volume.

4.5   Furthermore we assume that:

The incidences of claims are mutually independent (i.e., there is no single cause leading to claims by different insureds).

The claim severities X_i are also independent and all have the same probability distribution.

4.6   Although these assumptions do not completely reflect reality, they work sufficiently well for a portfolio of reasonable size. Dependence between insureds caused by overall circumstances affecting the whole population is not reflected in the volatility risk. For example, if the costs of surgery increase as a result of new technologies invented, this will simultaneously lead to a higher claim severity for all insureds. However, this type of dependence will be captured in the uncertainty risk. Hence for the volatility risk calculation it is assumed that given the general cost level of surgery, the costs arising from individual claims are independent of each other.

4.7   An exception occurs when several people are involved in the same accident. In this case the incidences of their individual claims are not independent of each other. However, considering that accidents normally only involve a limited number of people, for a portfolio of several hundred insureds or more the impact of this type of dependency is very small. For accidents which can affect a large number of people, for example epidemics, industrial accidents or terrorist attacks, a separate 'calamity' risk charge needs to be added.

4.8   We are now interested in fitting a probability distribution for the total loss S. This is done as follows. Firstly, expectation, variance and skewness of S are estimated. Then the normal power or translated gamma distribution is fitted to the estimated moments. As a result the expectation, variance and skewness of the estimated probability distribution of S are equal to the estimated moments. Both distributions can be used and generally will give outcomes in the same range. Also, when the number of independent insureds is large, the normal distribution will also give a good approximation due to the Central limit Theorem. For very low values of the skewness, the translated gamma distribution can cause computational difficulties.

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[12] See the section on 'Time Horizon' in the main report.

4.9   For the expectation, variance and skewness of S, we have:

$E[S] = E[N]E[X_i]$ (1)

$Var[S] = E[N]E[X_i^2]$ (2)

$\gamma[S] = E[N]E[X_i^3]/ Var[S]^{3/2}$ (3)

E[S] is the best estimate of the liabilities. As can be seen from the formulae above, the estimates of expected value, variance and skewness of S are found by estimating *E[N]*, *E[X_i]*, *E[X_i^2]* and *E[X_i^3]*. This can be done in the following way: *E[N]*, the expected number of claims is estimated as:

E[N] = number of insureds * observed average claims frequency   (4)

E[X_i] is estimated as the average of the observed claim amounts, with a possible adjustment if claims in the next year are expected to be higher on average than in previous years. Similarly, *E[X_i^2]* is estimated as the average quadratic claim amount, and *E[X_i^3]* as the average third power of the observed claim amounts.

4.10   By using the average observed claims frequency for the portfolio as a whole we ignore the heterogeneity that is most likely present in the portfolio. For example, claim frequency increases with age of the insured. It can be proven that by ignoring heterogeneity a stop-loss safe estimation of the aggregate loss distribution is obtained, meaning the estimate contains some conservatism especially in the right tail of the distribution (see 'Stochastic ordering' by Kaas, Goovaerts et al).

C.4.2   Case Study Volatility

4.11   In this case study it is shown how a distribution function for the volatility risk of an arbitrary portfolio was estimated. The portfolio consists of 130,000 policyholders with an average claim frequency of 5% per year. The mean annual claim size is $4,125, which is simply the observed average in the latest year, with a possible loading for claims inflation/indexation. The input data used for the calculation are shown in table 1.

4.12   Table 1: portfolio and claims information

Number of policies:	130,000
Average claim frequency per insured	5%
Expected aggregate number of claims	6,500
Claim severity distribution:	
Mean ($):	4,125
Variance:	70,074,170
Third central moment:	9.28072E+12

4.13   Table 2 shows the first three moments (mean, standard deviation and skewness) of the aggregate losses *S*, calculated according to formulae 1,2 and 3.

Table 2: moments of the aggregate losses S according to formulae 1,2,3

Mean	26,811,351
Variance	455,482,101,858
Skewness	0.20

4.14 With the estimated moments of S given as input we will now fit three types of probability distribution: the translated gamma, the normal power and the normal distribution. This gives the following results:

4.15 Table 3: 99% upper limit of aggregate losses S with respect to volatility risk

Translated gamma distribution	28,478,025
Normal distribution	28,381,385
Normal power distribution	28,478,771

4.16 The 99% point of the distribution function of the total liabilities is given below, meaning the total liability will be equal to or lower than the amount shown with a probability of 99%. As can be seen, the three approaches produce very similar results due the large number of (assumed) independent risks.

C.4.3 Uncertainty

4.17 Uncertainty risk can be split into the uncertainty risk in the first coming period and all periods thereafter. This split is useful to make as the short term uncertainty is of a different nature and more commonly present than the long term uncertainty in medical insurance products. The nature of the product and the possibility to adjust premium rates determine whether uncertainty risk beyond the first year needs to be considered. If rates can be adjusted periodically, this obviously reduces the long term uncertainty risk. But in many markets there is also the political factor that governs the extent to which premium rates can be adjusted. Usually the more politicized the rate setting process the greater the risk to the insurer of not being able to adjust premium rates to fully compensate for past errors in the estimation of future liabilities.

In the next subsection, modeling approaches for the first year/short term uncertainty risk are discussed. The long term uncertainty risk for medical insurance is constituted by medical inflation risk and this is discussed in section 5.

C.4.4 Short Term Uncertainty Risk

4.18 The short term uncertainty risk in medical insurance can be treated in the same way as is done for P&C products, as described in the P&C case study. Using an approach that is based on loss ratios, the earned premium component of the loss ratio should be determined as the earned premium which is allocated to the calendar year under consideration. This earned premium can significantly differ from the written premium received minus expense allowances in the case that there is an ageing reserve. An ageing reserve allows the written premium to stay level over time or increase less while utilization increases due to the ageing of the insured population. In the presence of an ageing reserve the earned premium equals the written premium minus expenses allowances minus (plus) any addition (subtraction) from the ageing reserve.

4.19 The framework outlined in the P&C case study can be applied to medical insurance as follows:

4.20 There is a single line 'medical insurance' for which parameters b_i, c_i, λ_i, μ_i and σ_i need to be estimated. As we are looking at a single line only, the index i can be omitted so we will use b, c, λ, μ and σ instead.

λ: expected claim count (number of in-force policies * expected claim frequency);
μ, σ: parameters of the individual claim severity distribution.
b,c: parameters reflecting systemic risk in claims frequency and severity.

4.21   Following the approach outlined in the P&C case study, *b* and *c* can be estimated as follows:

The variance of the annual loss ratio of an imaginary, infinitely large portfolio equals:

$$\text{Var(loss ratio)} = (b + c + bc)*E[\text{loss ratio}]^2 \tag{5}$$

4.22   As an approximation for the loss ratio of an infinitely large portfolio, one can use the industry-wide loss ratio for medical insurance. If for example, the expected loss ratio equals 60% and the standard deviation of the industry-wide loss ratio equals 20% then we have:

$$\text{Var(loss ratio)} = (b + c + bc)*E[\text{loss ratio}]^2 \qquad \text{hence}$$

$$0.2^2 = (b + c + bc)*0.6^2 \tag{6}$$

4.23   Also, *b* is the sole parameter indicating the variability of inflationary effects. As *b* is defined as the variance of a random variable β (as mentioned in the P&C case study) with expectation 1, we have:

1+ Medical Inflation[t+1] = β *(1+ average medical inflation).

Therefore,

Variance (1+ Medical inflation[t+1])=

Var[β]* (1+average medical inflation)^2 .

Also,

Variance (1+ Medical inflation[t+1]) = Variance (Annual Medical Inflation])
and   *b = Var[β].*

It follows that:

b = Variance (Annual Medical Inflation]/ (1+average medical inflation)^2. $\qquad$ (7)

4.24   From equations *(6)* and *(7)* we can now derive the values of *b* and *c*. The P&C case study outlines the approach to determine the capital requirement based on the *b* and *c* parameters, as part of a standardized or advanced approach, and as a stand-alone line of business or as part of a P&C company.

4.25   We can also find the capital for the uncertainty risk alone by assuming independence of the volatility and the uncertainty component, as follows:

(Capital uncertainty)^2 =

(Capital uncertainty and volatility combined)^2 -/- (Capital for volatility)^2 .

4.26   As in many jurisdictions medical insurance is underwritten by Life insurance companies rather than P&C companies, incorporation into a capital requirement for a Life company also needs to be considered. This will be addressed further in section 6.

C.4.5 Extreme Event Risk

4.27 The determination of the effect of an extreme event/calamity on a health insurance product will depend largely on the type of product, the type of calamity and the country that it is written in. A calamity, involving a large number of persons becoming ill or disabled for some period of time could have a major effect on a disability income product but very little effect on a hospital insurance product, written by the same insurer on the same group of lives.

4.28 Not all calamities require extensive hospitalisations of large numbers of people. Even if large numbers of people did require lengthy hospitalisation as a result of a calamity it is unlikely that the capacity of the local hospital system would cope. Often Governments will quickly react to calamities by providing additional facilities and support perhaps using defence force medical facilities or decommissioned hospitals. In these circumstances the cost is usually born by the taxpayer. Also in a state organised health insurance system if Government support was not forthcoming in the event of a calamity then all insurers or a group of insurers operating in the geographic area of the calamity would be likely to have financial difficulties together and the Government would not wish to see the market fail due to the eventual effect on Government outlays so capital adequacy rules would tend to be relaxed and/or regulatory measures introduced to ensure the rest of the industry assisted as necessary.

4.29 Reinsurance also can play a part in reducing the financial effect of a calamity. The extent of reinsurance support on an insurance product will also depend on the product and the country the product is written in. For products written in countries where there is little political inference generally insurers will have obtained catastrophe reinsurance to cover the effects of calamities. In some countries where there are taxation incentives to individuals or employees to be covered by medical and hospital expense insurance the Government may not even permit insurers to reinsure risk out of the local industry because of the taxation implications. In these environments there are often internal reinsurance arrangements or legislation compelling financially sound insurers to "prop-up" those that are not so sound financially.

4.30 As normally no or only very scarce data are available to calibrate extreme event risk, a pragmatic approach needs to be taken to determine a capital requirement. One can argue that the same causes that underlie the extreme event risk for mortality also apply to accident and medical insurance claims. Circumstances that cause increased mortality can cause increased medical and disability claims to the same extent. Hence, in line with the mortality approach, the combined capital for extreme event and the volatility and first year uncertainty risk can be determined by assuming claim frequency will double under these extreme circumstances while claim severity remains unchanged.

C.5 Case Study Medical Inflation

5.1 In this section a case study for the risk of long term medical inflation is discussed. The case study contains the following elements:

5.1 Medical inflation
5.2 Portfolio composition
5.3 Expected individual claim size by age/ age cohort
5.4 Rating structure
5.5 Simulation
5.6 Results

C.5.1 Medical Inflation

5.2 On the basis of historical data, future medical inflation can be modeled using statistical and econometric modeling techniques.

5.3 The model applied in this case study is an autoregressive time series of the second order:

$INF(t) = c_0 + c_1 INF(t-1) + c_2 INF(t-2) + random\ error(t)$ with:

$INF(t)$: medical inflation in year t;

$c_0,\ c_1,\ c_2$: model parameters.

random error (t): random, unexplained annual change of medical inflation rate.

5.4 The second order structure of the model implies there is a direct dependence between the inflation in a certain year and that in the two preceding years. As a result we have:

- Autocorrelation between successive observations: if the medical inflation is above (below) average in a certain year, it is likely to be above (below) average as well in the next year;

- Cyclicality: the second order of the model allows the possibility that there is a cyclical pattern in the observed inflation rates: possibly periods of several years with inflation rates above average are succeeded by several years with inflation rates below average.

5.5 It should be noted that it is assumed in this model that medical inflation is equal for all ages. When medical inflation is higher for higher ages than it is for lower ages, the inflation for a portfolio in run-off, with an increasing average age, will be higher than the overall medical inflation for the entire population.

C.5.2 Rating Structure

5.6 The current rating structure and the possibility to changes rates in accordance with experience should be taken into account. Rate adjustments may be limited due to government restrictions or market movements. The model allows for rate adjustments equal to the minimum of:

The annual medical inflation rate for the insured population as a whole;

A maximum allowed annual rate increase.

5.7 These restrictions are given purely by way of example however different types of premium restrictions may be in force in various jurisdictions.

C.5.3 Portfolio Profile

5.8 The age distribution of the portfolio is given at the beginning of the projection period. No future new business is included in the current model setup. However for as far as the rating structure for new business is the same as for existing business, the model can easily be extended to allow for this. If a different rating structure is introduced for new business, one would have to allow for this by building a second model with the new rating characteristics. Expected lapse rates are assumed to be age dependent, decreasing with the age of the insured.

### C.5.4 Claim Size by Age/Age Cohort

5.9 In the case study, the annual medical expenses increase exponentially with age, by way of an example. The values used can be replaced by any age-dependent estimate of the annual liability per insured, to represent expected medical expenses by age (group) in the present year. The effects of medical inflation are not yet taken into account in this stage of the calculation.

C.5.5   Running the Model Simulation

5.10   On the basis of the input described above model simulations are performed with which future medical inflation rates are simulated. Annual premium adjustments follow as a function of the inflation rates.

5.11   Volatility in lapses or incidences of claims of individual policyholders are not modeled by way of stochastic simulation. As the model projects over a very long period, e.g. 30 years or more, the effect of randomness of individual incidences of lapses and/or claims on policyholder level will be negligible. The uncertainty in the level of medical expenses in future years is a far more influential factor in determining the total liability.

5.12   Randomness in individual claim incidences is included in the volatility risk model (see section 4), but only in the first period.

C.5.6   Results

5.13   The simulation produces output in the following form:

5.14   A set of premium and claims cash flows in every future year that is included in the model for every run of the simulation.

5.15   A present value of claims and premiums for every scenario, based on a fixed discount rate or yield curve.

5.16   An estimate for the probability distribution of the present value of claims, the present value of premiums and the present value of claims minus premiums.

5.17   The present values of claims, premiums and their differences are expressed as a multiple of the risk premium for the portfolio as a whole at the inception date. The estimated probability distributions are shown graphically below.

Density of Net Liability by Age Cohort
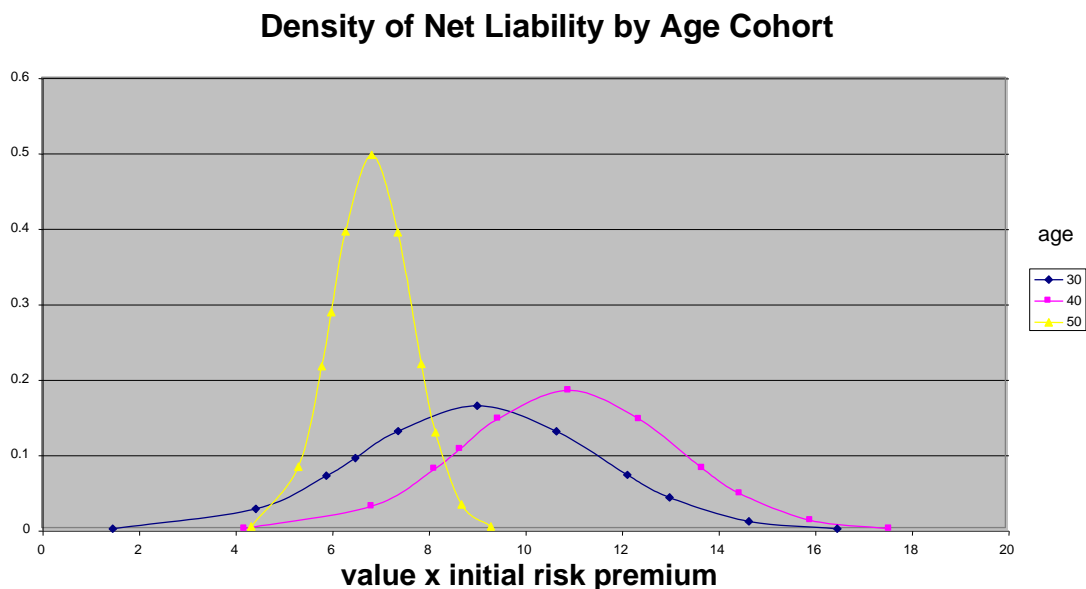
5.18   In the graph, the density functions of the distribution of liabilities are shown for three different ages: 30, 40 and 50. The values are expressed as a multiple of the risk premium for the individual at the inception date of the projection. For example for the 40-year old insured the expected present value of the net liability (claims minus premiums) is approximately equal to 11 times the

annual risk premium for the insured at the inception date. A capital requirement can be determined as a percentile or TVaR (e.g. 99% or 99.5%) of the distribution. The medical inflation risk affects all policyholders simultaneously therefore there is no diversification between policyholders. As a result, a capital requirement, i.e. VaR/TVaR value, for a portfolio can be determined as the sum of the capital requirements for each individual policyholder or portfolio cohort.

C.6   Standardized Approach and Aggregation

6.1   In this section, a possible standardized approach for long-term medical inflation is discussed, as well as linking the different components discussed into an overall capital requirement.

C.6.1   Standardized Approach for Long Term Medical Inflation

6.2   Although it is very difficult to determine a universally valid standardized approach for medical inflation, one could proceed as follows. Starting from the recommendations in chapter 7 of the main report, three factors are distinguished in establishing a capital requirement under a standardized approach.

1. μ : company specific expected losses;
2. k: specific to the line of business, prescribed by the regulator;
3. v: company specific factor.

   1.   As an exposure measure μ, the total risk premium for the portfolio in any given year can be used.

   2.   The second factor k can be determined by the regulator, as the ratio of the present value of future claims liabilities in a worst case, over the current total risk premium of the portfolio. The simulation model as described in the previous sections can be used to determine this factor based on a number of general portfolio characteristics such as the average (and possibly spread of) remaining term of policies until expiration. Note that this factor only reflects the future claims liability, and not the premiums still to be received. This distinction is made because medical inflation is a phenomenon which affects all companies, while the possibilities of premium adjustments may vary by company.

   3.   The third factor v is a company specific factor reflecting the rating of a particular company. This factor should reflect the adequacy of current rates as well as the possibility of adjusting rates in case of unexpectedly high future medical inflation. In the most extreme situations, v will be 100% if no premiums can be charged at all in future years or 0 if future risk premiums can be charged to fully cover the worst case claims liability at all times.

6.3   If premiums can be adjusted without limitation but are currently inadequate, some additional capital will be required for the period that premiums will be inadequate, as the management of a company may decide not to raise premiums directly.

6.4   Where rate increases are subject to approval by a government body at the time they are submitted and this is based on judgement and political factors rather than a rigorous numerical rule, it is almost impossible to derive a factor reflecting this practice.

C.6.2   Aggregation of Capital Requirements for the Various Risk Components

6.5   In this case study, capital requirements have been determined for:

1.   Short term volatility and uncertainty risk combined;

2. Short term extreme event (calamity) risk;
3. Long term medical inflation risk.

6.6 In order to derive an overall capital requirement for all of these risks combined, the following observations are made:

6.7 Short term volatility/uncertainty and extreme event risk can be regarded as more or less independent of each other. Extreme event risk, such as the outbreak of epidemics, is caused by unexpected one-off events which are generally unrelated to other developments leading to increased claims experience within the course of one year.

6.8 In the main report it is recommended that a capital requirement be determined as the maximum of two measures, one related to the first year, and one to all future years. Applying this recommendation to the health case study, the minimum overall capital requirement is found by taking the maximum of:

1. The capital required for category 1 and 2 combined at a very high confidence level;
2. The capital required for category 3 at a fairly high confidence level.

6.9 Capital requirements for category 1 have been determined using the approach for the P&C case study. However, as in many jurisdictions health insurance is underwritten by Life companies, consideration also needs to be given to correlations with other risks that life insurers are facing.

6.10 In section 7.2. of the main report, it is described how capital requirements for separate components of risks can be combined into a single overall capital requirement. This requires the determination of correlations between different types of risk. Although these correlations can best be chosen individually for each country or jurisdiction, it can be stated in general that:

6.11 Medical inflation tends to be generally higher than general inflation. The aggregate rate might be reasonably constant over time but it will vary between in-hospital[13] and out-of-hospital services and also vary substantially between various types of out-of-hospital services (for example medical, pharmaceutical[14], dental and optical).

6.12 Medical inflation tends to be correlated with economic prosperity, as medical inflation is driven by technological development. Hence the extraordinary growth in the overall cost of pharmaceuticals and surgical implants.

6.13 As a result, medical inflation is most likely negatively correlated with mortality trend in the long term.

6.14 In the short term, mortality and sickness, and therefore health claims, can be positively correlated for some health insurance portfolios in some countries.

6.15 Mortality and morbidity calamity are highly correlated as they are the result of the same or similar causes.

6.16 Short term and long term type risks generally have low correlation to each other due to the fact they are manifest in different periods of time, hence driven by different causes.

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[13] Some components of hospital cost tend to have much higher inflation rates than other components. For example the overall costs of surgical implants had explosive growth in the last decade of the twentieth century.
[14] In many countries pharmaceutical costs have an extraordinary high inflation rate.

Appendix D   Market Risk

D.1.   Definition of Market Risk

1.1   Market risk results from the volatility and uncertainty risk inherent in the market value of future cash flows from insurer assets and liabilities. Market risk is thus driven by exposure to movements in the level of financial variables. These include: stock prices, interest rates, exchange rates or commodity prices and the exposure of options in either the assets or liabilities to movements in underlying pricing variables such as movements in the actual or implied volatility of prices and options.

1.2   A related risk is liquidity risk, the risk that various events will require the insurer to attempt to liquidate various asset holdings prematurely on short notice and under unfavourable terms. A trigger for liquidity risk could be market risk, but other operational and policyholder behavior risk factors could be the trigger. The Working Party suggests that liquidity risk is better placed within Pillar 2 actions of the supervisor than to require a Pillar 1 requirement.

1.3   In addition to the volatility of market risk affecting the net market value of the insurer's assets, market risk may also affect the liabilities (and net surplus position) as follows:

   1. Changing asset yields will affect the market value of the liabilities through their effect on the rate(s) used to explicitly or implicitly discount the liability cash flows.

   2. Changing asset returns (yields) may affect the amount and/or timing of future liability cash flows. Policyholders may be entitled to some form of profit sharing related to actual and/or historical asset returns. In this respect, the different types of 'interest' profit sharing within the global insurance market might be categorised into the following three groups:

      A. Profit sharing that is fully based on objective indicators of the performance of the capital market, e.g. an indicator of the actual interest rate level that is calculated and published periodically by a government agency, or a stock market index. The company may or may not actually be holding these asset referenced benchmarks to back the liabilities.

      B. Profit sharing that is somehow related to the actual performance of the company ('performance-linked'), particularly with respect to the company's investments. Note: This type includes the systems where the management is entitled to 'declare the bonus rate'.

      C. Profit sharing that is related to the actual performance of the assets that are 'locked-in' at the policy holders discretion, i.e. policyholders themselves are, at least partially, responsible for the way their premiums are invested. Note: The typical example of this type of profit sharing in Life insurance is the profit sharing that is (implicitly) offered with Unit Linked/Universal Life (UL) products in Europe or variable (separate account) products in the US.

1.4   All three types of profit sharing may also include certain types of guarantees offered by the insurer, e.g. a bonus rate that will never be negative or a minimum level of the maturity benefit.

1.5   Changes in asset returns in the external market may affect the amount and/or timing of future liability cash flows by inducing policyholders to "arbitrage" the external returns with those available in the policy be either surrendering or paying additional premiums. (Note, this policyholder behavior may not always appear "rational" due to differing tax implications and liquidity/risk preferences of the policyholder.)

1.6   The following definition of market risk for insurers is proposed:

Market risks relate to the volatility of the market values of assets and liabilities due to future changes of asset prices(/yields/returns). In this respect, the following should be taken into account[15]:

- *Market risk applies to all assets and liabilities*
- *Market risk must recognize the profit sharing linkages between the asset cash flows and the liability cash flows (e.g. liability cash flows are based on asset performance)*
- *Market risk includes the effect of changed policyholder behavior on the liability cash flows due to changes in market yields and conditions.*

D.2   Types of Market Risk

2.1   The principal sources of market risk are:

- **Interest Rate Risk**- risk of exposure to losses resulting from fluctuations in interest rates.
- **Equity and Property Risk**- risk of exposure to losses resulting from fluctuation of market values of equities and other assets.
- **Currency Risk**- risk that relative changes in currency values decrease values of foreign assets or increase the value of obligations denominated in foreign currencies.
- **Basis Risk**- risk that yields on instruments of varying credit quality, liquidity, and maturity do not move together, thus exposing the company to market value variation that is independent of liability values.
- **Reinvestment Risk**- risk that the returns on funds to be reinvested will fall below anticipated levels.
- **Concentration Risk**- risk of increased exposure to losses due to concentration of investments in a geographical area or other economic sector.
- **Asset/Liability Mismatch Risk**- to the extent that the timing or amount of the cash flows from the assets supporting the liabilities and the liability cash flows are different (or can drift apart) the insurer is subject to asset/liability mismatch risk.
- **Off-Balance Sheet Risk**- risk of changes in values of contingent assets and liabilities such as swaps that are not otherwise reflected in the balance sheet

2.2   Market risk can only be measured appropriately if the market value of assets, as well as the market value of the liabilities, is measured adequately. Market values of assets can generally be deduced from listings in the various securities markets. Due to the lack of a real market for insurance liabilities, the market value of insurance liabilities can be approximated through evolving market/fair value techniques. The concept of the 'replicating (asset) portfolio', defined in section 6.2, is a useful concept in measuring the market value of insurance liabilities.

2.3   In general, life and health insurers purchase assets to match their liabilities. Historically this has not been true for non-life insurers who tend to manage separately the results from underwriting and investments. While all of the assets of an insurer are available to provide against adversity, it is common risk management practice for insurers to implicitly or explicitly allocate their assets for one of the following purposes:

- To support insurance contract liabilities
- To represent economic capital
- To represent free surplus

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[15] This also includes the situation where policy benefits, e.g. pensions within Life insurance, are indexed to adjust for price or wage inflation (either 'unconditionally' or 'conditionally' depending on the available capital). In that case there is inflation risk. Note: Inflation risks related to Health and Non-Life insurance benefits or future internal expenses are ignored here, since they are considered as special types of 'trend risks' and 'operational risks' respectively.

2.4 Sizeable portions of an insurer's liabilities can have durations comparable to readily available high quality liquid assets in the local market. In these situations it is possible to select assets whose cash flows can provide a very close match to the liability cash flows. In other words, a replicating portfolio of assets is available in the market. In this situation, market risk focuses on the volatility of the market value of the actual assets held and the market value of the replicating portfolio of assets and the ability of the insurer to manage that volatility. This type of market risk will be called **Type A** risk and it also includes the effect of volatility on an insurer's stand alone surplus or economic capital assets.

2.5 The long-term duration of some insurance (especially life insurance) liabilities requires the consideration of long term rates of reinvestment since replicating portfolio assets of sufficient duration may not be currently offered in the market. Measuring market risk for these liabilities entails considerable uncertainty about the composition of the replicating portfolio and the manner of its reinvestment to mature the underlying cash flows. Lowered rates of reinvestment in the future are typically of concern. In addition, life insurance contracts may contain various complex, long term options and/or guarantees for which replicating market positions may not currently exist (e.g., death and maturity guarantees on variable annuity products). These latter two types of market risk will be called **Type B** risk.

2.6 The assets and liabilities of an insurer are subject to Type A and possibly Type B risk. Shorter term insurance contracts without complex to value embedded options or guarantees are subject to Type A risk. Long-term insurance contracts and/or those containing complex embedded options or guarantees may be subject to both Type A and Type B market risk.

D.3   Time Horizon

3.1 In contrast to market risk for banks, where the risk measurement time horizon is generally defined in terms of days or weeks, insurer market risk is more appropriately determined using a time horizon of one year. One year recognizes the generally less active trading environment of insurers with respect to their asset and liability cash flows. One year reflects a conservative view of the time required by a supervisor to assume control of the affairs of a weakened insurer. One year reflects a conservative view of the time required for an insurer to rebalance a mismatched portfolio of assets and liabilities (i.e., presuming replicating portfolio assets are available). Failure to rebalance such a portfolio within one year is more appropriately the subject of Pillar 2 type supervisory measures.

3.2 When the market risk of liabilities is compared with the market risk of the assets used to support them, the net market risk for these liabilities can be measured. This net asset/liability mismatch position is generally subject to specific asset/liability management (ALM) policies and procedures of the insurer. Type A risk is diversifiable to the extent that another manager could immediately eliminate the mismatch risk through rebalancing the portfolio.

3.3 The Type B market risk for cash flows which extend beyond the term of currently available replicating portfolio assets requires consideration of future reinvestment decisions and reinvestment rates in the future. To a certain extent, market risk for these liabilities involves systematic (undiversifiable) risk due to the limited availability of (parts of) the replicating asset portfolio or, at least, uncertainty about its composition. In theory, these risks must always be assessed for the full remaining term of the liabilities. The best fitting replicating portfolio assets must be reinvested in accordance with the insurer's policies and practices with respect to investments so as to provide for the lengthy future cash flows. The requirement of a full term time horizon is considered necessary due to the considerable uncertainties involved in providing for future cash flows beyond the term of currently available replicating portfolio assets.

D.4   Confidence Level

4.1   The market risk capital requirements should be determined in a manner consistent with the overall goal for the confidence level of Pillar 1 capital requirements. For example, they could be determined for all risks such that there is a very high (e.g., 99% CTE) confidence level that the assets of the insurer would be sufficient in one year's time to provide for the policy liabilities determined one year later at a moderate (e.g., 75% CTE) level. In addition, a second condition may also be imposed, such as, if the present value amount of the policy liabilities determined at time zero for all future durations at a fairly high (e.g., 90 or 95% CTE) confidence level is greater, then this amount should be held.

D.5   Advanced Approach – Type A Risks

5.1   This section outlines the advanced approach to be used in determining a Pillar 1 capital requirement for Type A market risks. Type A risk may be present in any of the cash flow generating assets and liabilities of an insurer.

5.2   The most advanced approach for determining Type A risks would involve the use of risk models by the insurer. These models would need to satisfy the requirements of the supervisor as suggested elsewhere in the WP report. The market value of assets or liabilities with future cash flows can usually be determined with reference to the financial markets for similar or identical instruments. Similarly the volatility of their market value can also be deduced. For future liability cash flows, especially insurance contract cash flows, their market values and market value volatility can be approximated through evolving market/fair value techniques. The concept of the 'replicating (asset) portfolio', defined in section 6.2, is also a useful concept in measuring the market value of insurance liabilities.

5.3   Market risk should include provision for both specific risk (e.g., perhaps as implied by the credit spread inherent in the yield of securities offered by the issuer) and general market risk (e.g., general sensitivity to future rates of return).

5.4   Market risk can be determined by modelling cash flows over a broad range of economic scenarios using stochastic modelling for the time horizon specified and the confidence level desired. The time horizon for this modelling would be one year at a high (e.g., 99% CTE) confidence level.

5.5   In situations where the insurer has a block of insurance contracts which exhibit only Type A market risk, the insurer may choose to conduct integrated modelling of the projected future cash flows resulting from the insurance contracts and their matching assets. Such modelling must reflect the actual asset allocation, reinvestment policies and practices of the insurer for that business. At the end of the one year time horizon, the reinvested matching assets must be sufficient to mature the then remaining liabilities with a prudent level of confidence (e.g., 75% CTE).

D.5.1   Practical Approximations

5.6   These might be considered by supervisors depending on their local circumstances and the appropriateness of the approximation.

- Allow for the use of a deterministic liability basis at the end of the one year horizon (rather than a multi-scenario or stochastic model approach at the 75% CTE level).
- Replace the stochastic modelling during the one year horizon with a series of deterministic scenarios designed to stress test economic scenario shocks at the 99% CTE level. Stochastic modelling of the resultant shocked portfolio after one year would then be required at the 75% CTE level.

D.6   Advanced Approach – Type B Risks

6.1   This section outlines the advanced approach to be used in determining a Pillar 1 capital requirement for Type B market risks. Type B risk may be present in any of the cash flow generating assets and liabilities of an insurer.

6.2   The most advanced approach for determining Type B risks would involve the use of risk models by the insurer. These models would need to satisfy the requirements of the supervisor as suggested in this report. Many of the same modelling requirements outlined for Type A risks are also applicable to Type B risks.

6.3   The appropriate time horizon for measuring this type of market risk is the entire duration of the (longer and containing complex options)) liability cash flows. The general market risk component can best be measured at an advanced level through modelling of the insurer's actual reinvestment policies and practices. Separate provision need also be made for specific risk inherent in the asset and liability cash flows. Specific risk results from an adverse movement in the price of an individual security owing to factors related to the individual issuer. The confidence level chosen will be the greater of 2 options:

-   A very high (e.g., 99% CTE) confidence level that the assets will be sufficient in one year's time to provide for the policy liability cash flows determined at a moderate (e.g., 75% CTE) level at that time.
-   A fairly high (e.g., 90 or 95% CTE) confidence level that the assets will be sufficient to provide for all future policy liability cash flows.

The following sub-sections describe in considerable detail the level of sophisication needed for the advanced approach.

D.6.1   Modelling Process

6.4   The modelling process begins with an identification of the assets and liabilities to be modelled. In particular, the process for generating their future cash flows under varying economic scenarios must be understood (i.e., the impact of embedded options). For this to be possible, the primary risk factors affecting market risk must be identified (e.g., interest rates, equity returns, property values, inflation etc.), and defined for their impact on policyholder and company behaviors/strategies. This must then all be modelled as part of an integrated set of economic scenarios. If the market risk for the liabilities is to be determined separately from the actual assets used to support them, then the concept of a replicating portfolio of assets will need to be employed. The combined asset and liability future cash flows will need to be modelled in an integrated manner to allow for a) asset/liability linkages, b) pass-through of risks to policyholders, c) reinvestment strategy and practices and d) impact of economic scenarios on policyholder behavior. The range of scenarios tested (e.g via deterministic or stochastic modelling) will enable the market risk for Type B risks to be determined.

6.5   The modelling process to determe the market risk of insurers may differ from that employed by the banks in a number of ways. Some of these differences are shown in the table that follows.

Traditional Banking	Traditional Insurance
▪ Recipe approach	▪ Global in scope
▪ Detailed single risk (silo) models	▪ Generalized, multiple risks
▪ Risk neutral (pricing)	▪ Real world (cash flow)
▪ Preference for analytic forms	▪ Preference for "moving parts"
▪ Variance reduction	▪ Monte Carlo simulation
▪ Accurate (within narrow scope)	▪ No objective market benchmark
▪ Calibrates to market (volatility)	▪ Complex calibration & estimation

D.6.2 Replicating Portfolios

6.6   The general approach to determining market risk requires the modelling of the reinvestment of the relevant cash flows in accordance with the insurer's investment policies and practices over an appropriate time horizon, using a range of economic scenarios to a high degree of confidence.

6.7   For cash flows whose duration does not extend beyond the replicating portfolio horizon (i.e., the longest duration, publicly available, debt instruments), the appropriate time horizon (as stated above) for modelling investment management behavior is one year. This is the Type A aspect of market risk.

6.8   For cash flows whose duration extends beyond the replicating portfolio horizon (primarily some types of insurance liabilities), the appropriate time horizon for modelling investment management behavior is the entire duration of those future cash flows. These cash flows are subject to both Type A and B aspects of market risk.

6.9   The difference between market risk determinations for general market interest rate risk for two sets of future cash flows, one slightly shorter than the replicating portfolio horizon and the other slightly longer, will be minimized the more accurately the investment practices of the insurer can be modelled.

6.10 In principle, the replicating portfolio generates cash flows that 'replicate' (i.e. coincide with) the annual liability cash flows in each individual future year. Therefore, the replicating portfolio provides a perfect 'hedge' against the liability risks.

6.11 Obviously, this is a theoretical concept. Liability cash flows are subject to several types of risks (e.g. mortality risks) that cannot be hedged by financial instruments. Therefore, the following definition of the replicating portfolio is proposed:

The replicating portfolio (only) replicates the liability cash flows that are ('risk'-) adjusted for the systematic non-financial risks, while volatility due to diversifiable non-financial risks (e.g. volatility risk as a consequence of mortality) is fully ignored.

6.12 Consequently, the replicating portfolio should provide a <u>full</u> hedge against the <u>financial</u> risks that may affect future insurance liability cash flows before the replicating portfolio horizon.

D.6.3 Embedded Options

6.13 The replicating portfolio (i.e., the asset portfolio used to represent the future cash flows, should include specific financial instruments that provide a full hedge against (financial) 'embedded options' like minimum investment return guarantees related to profit sharing (if offered by the insurer).

6.14 Guarantees <u>always</u> offer additional value to the policyholders, since they indicate, implicitly or explicitly, that certain risks are transferred to the insurer. Therefore, they always increase the market value of the liabilities. Theoretically, the market value of these guarantees is equal to the market value of the financial instruments that are necessary to hedge these guarantees.

6.15 As these instruments are generally specific types of options or, if the guarantees also apply to future premiums, swaptions, their market value can generally be approximated by applying calibrated Black-Scholes types of option-price formulas; see e.g. Bouwknegt and Pelsser (2002) regarding annual minimum investment return guarantees for traditional Dutch regular premium business with profits, and Nonnenmacher and Russ (1997) for rather complex minimum investment return guarantees in German UL-business[16]. If so, it will also be possible to measure the sensitivity of these market values to changes in asset yields. Therefore, including these

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[16] Alternatively, a so-called deflator approach may be useful (see e.g. Jarvis *et al.*, 2001). However, this methodology is still very much under development.

instruments in the replicating portfolio allows for the sensitivity of the total market value of the replicating portfolio with the sensitivity of the future cash flow stream being measured.

6.16   One final consideration/note is that many contracts also contain embedded options which can be exercised by the insurance company. These options will then, obviously, always reduce the market value of the liability.

D.6.4 Incompleteness of the Capital Market

6.17   Unfortunately, investment return guarantees in life insurance products are often complex. As a consequence, financial instruments to hedge the corresponding risks are generally not amply available. These instruments may even be non-existent in practice. Nevertheless, it may still be possible to approximate their market values by applying option-pricing theory. Alternatively, their market values may be approximated through stochastic simulation using a combination of currently available financial instruments.

6.18   In some cases, insurers have only expressed the intention, not the guarantee, to cover certain risks or to provide a certain minimum level of profit sharing. For example, some life insurance benefits are 'conditionally' indexed for price or wage inflation. Some performance-linked with profits business may offer positive bonus rates if the financial condition of the company, as assessed by management, allows for the extra pay-outs. Such embedded options have a positive value to policyholders. In some cases policyholders' expectations in this regard may be granted in court even if the conditions for their granting are not satisfied[17].

6.19   Some life insurance liabilities may extend more than 30, possibly even 80, years into the future. This is much longer than the longest term of fixed-interest securities purchasable in the capital market (generally somewhere between 20 and 30 years for mature and developed investment markets). In these cases the insurer faces non-avoidable (systematic) reinvestment risks in the long term (i.e., Type B aspect of market risk). The present value of these liability cash flows far into the future can always be determined through modelling of the reinvestment policies and practices of the insurer in to the future using currently available financial instruments.

D.6.5 Economic Scenarios

6.20   In developing appropriate economic scenarios the following desirable characteristics of the constructed scenarios are noteworthy:

Interest rates
- Nominal yields must remain positive and not increase indefinitely
- Are subject to mean reversion but the reversion target is not constant
- Rate volatility decreases with maturity
- Higher volatility occurs with higher rates
- High correlation between maturities
- Distinctive yield curve shapes

Equity returns
- Negative skewness
- Fat tails over short periods
- Volatility clustering
- Exogenous shocks
- Markov property; only the current state is important
- Market correlations increase under extreme conditions

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[17] The Dutch insurance supervisor (PVK) has described such insurance liabilities recently as 'soft' liabilities. A possible way to handle them may be to assess them in a less quantitative and more qualitative way within the second pillar of supervision (the 'supervisory review process').

- Price appreciation versus dividend income

Inflation
- Non-persistence of extremely high or low (negative) inflation
- Realized may equal expected plus exogenous shock
- Mean reversion but target does not appear to be constant
- Volatility clustering
- Various forms of inflation
- Relationship to other economic factors

D.6.6 Discount Rates

6.21 The market value of a replicating portfolio can be determined by discounting their cash flows using appropriate discount rates. Assuming these cash flows correspond with the liability cash flows that are adjusted for the systematic non-financial risks, and ignoring the diversifiable non-financial risks as advocated before in section 3, the discount rates can be set equal to the actual risk-free spot yields. This approach implicitly provides for the specific risk of the issuer of the cash flows and allows the liquidity premia preferences in market yields to emerge over the life of the cash flows. Readers of this report should note that widespread market discussion on the proper allowance for these two aspects of market yields is on-going.

6.22 The use of different spot yields by different insurance companies should be avoided. It may be prudent for national insurance supervisors to prescribe the levels of the risk-free spotyields to be used for discounting the replicating asset cash flows within the process of determining solvency requirements. Of course, this requires an adequate procedure for estimating periodically the actual risk-free spotyield curve. For this, several methods are available. We mention the specifications that were proposed by Nelson and Siegel (1987) and Svensson (1994, 1995). For instance, the Nelson-Siegel approach implies estimating the following (non-linear) specification:

$$r_t^{spot} = \beta_0 + (\beta_1 + \beta_2) * \frac{1 - \exp(-\frac{t}{\tau})}{\frac{t}{\tau}} - \beta_2 * \exp(-\frac{t}{\tau})$$

6.23 The parameters to be estimated are β_0, β_1, β_2 and τ. Nice characteristics of this specification are:
- the specification is reasonably parsimonious
- the spotyield for the very short duration is equal to $\beta_0 + \beta_1$
- the estimated spotyields for the long term converge to β_0.

6.24 Alternatively, the so-called splines methodology is also broadly applied in practice, particularly by central banks and asset management departments of banks and (larger) insurance companies. See Anderson & Sleath (2001) for a recent comparison and assessment of the Nelson-Siegel-type and splines-type methods.

D.7 Standardized Approaches – Type A Risks

7.1 This section outlines standardized approaches to be used in determining a Pillar 1 capital requirement for Type A market risks. Type A risk may be present in any of the cash flow generating assets and liabilities of an insurer.

7.2 Market risk should include provision for both specific risk (e.g., perhaps as implied by the credit spread inherent in the yield of securities offered by the issuer) and general market risk. (e.g., general sensitivity to future rates of return).

7.3   As described earlier in section 5, Type A risk can be determined using an advanced approach by modelling cash flows over a broad range of economic scenarios using stochastic modelling with a one year time horizon and a high (e.g., 99% CTE) confidence level. This section outlines some standardized approaches which may be considered as approximations for measuring market risk. The appropriateness of these approximations will be highly dependent on local country circumstances and the specific risk profile of the insurer.

D.7.1   Methodology

7.4   The essential ingredients required to assess Type A market risk are,

- Projected future cash flows
- Nature of embedded options
- Time horizon
- Confidence level
- Current economic scenario
- Series of adverse scenarios

7.5   Approximations can be made with respect to these ingredients to simplify Type A risk determination. The result is a range of standardized approaches from the most elemental to approaches which closely compare to the advanced approach.

7.6   One such approximation might use option adjusted durations to represent the price sensitivity of cash flows, the current market value of future cash flows and a set of investment return shocks. The shocks would need to be designed to reflect the time horizon and confidence level desired as well as the possible pattern of adverse scenarios. In this regard, it may be desireable to recognize the more active investment management conducted on closely managed blocks of business (i.e., when the active management holding period is less than the standard one year time horizon).

7.7   Another approximation might require the grouping of future cash flows into various term "buckets" (BIS uses the term "maturity method"). The sum of the cash flows in these "buckets" would be multiplied by factors to produce the capital requirement. These factors would, in theory represent a combination of the above basic ingredients (i.e., time value of money from current economic scenario, adverse shock for desired confidence level and time horizon etc.). This type of approach is currently used by the BIS in their standardized approach for banks.

7.8   A very simple approximation (which depends heavily on broad decisions about the industry's generalized exposure to Type A risk) is to simply multiply the balance sheet value of insurer assets and liabilities by a table of factors reflecting the presumed presence and size of Type A risk.

7.9   The relative merits of each type of approximation need to be viewed by the supervisor in light of local conditions, expertise and inherent industry risk. Objectivity and ease of calculation need to be balanced with greater accuracy, complexity and the overall impact of the method chosen on the management of market risk by insurers and the types of products that are offered in the market place.

7.10   To develop standardized approaches for market risks (or other risk for that matter) requires judgement and supervisory tradeoffs depending on the supervisors choice of approximation and its method of application. Ideally, the conservatism inherent in a standardized approach should incent insurers, as they are able, to use more advanced methods in the future. One possible concern in designing approaches which allow judgement to be used by the insurer (e.g., if the degree of market risk is subject to the asset allocation practices of the insurer) is that the results will be less transparent since there may be opportunities for the insurer to 'manipulate' the resulting solvency requirement. It is important for the supervisor to consider in advance the possibilities and significance of such self-selection. For example, the concern surrounding asset

allocation "games" can be addressed directly through a requirement that asset allocation for purposes of the capital requirement must coincide with the insurer's management of their business.

7.11   Particularly in life insurance, some market risk from the total asset portfolio may be transferred to policyholders. This is generally the case in Universal Life business and many forms of adjustable and "with profits" business. Clearly, such assets and the corresponding liabilities should be closely matched (ignoring the non-financial diversifiable risks that may affect these liabilities) and the degree of such sharing of market risk needs to be reflected in the chosen standardized approach.

7.12   The following sub-sections outline some important aspects in selecting a standardized approach for certain sources of market risk as well a possible treatment of dependencies.

D.7.2   Fixed Interest Securities and Liabilities

7.13   The risk of fixed-income investments depends on some properties of these investments. The relevant properties are duration (the sensitivity against an interest rate increase) and rating (which matters for assessing the credit risk). Aside from ordinary bonds, there are mortgage-backed and asset-backed securities, which behave similarly, except for prepayments in a period of falling interest rates. Bonds denominated in a foreign currency are affected by the foreign exchange (FX) risk.

7.14   The market risk for fixed income investments is dominated by the risk of increasing interest rates. When the relevant interest rates (or the whole yield curve) increase by one percent, the value of a bond (portfolio) decreases by the amount of duration times 1%. The duration can either be exactly computed from the cash flows of the bond and the current zero-coupon yield curve, or it can be approximately assessed as 80% of the mean time to maturity.
Example: a bond with a time to maturity of 10 years will lose about 8% of its value when the interest rate level increases by 1%.

7.15   The classical standardized approach to calculating a mismatch position is to employ a Macaulay duration analysis. This approach has a number of drawbacks. We mention three of them:

   a. The duration approach as described is based on a first-order Taylor approximation of the interest sensitivity of the present value. This approximation is not very good for larger interest changes. A better approximation is possible by including the second-order term, i.e. the so-called convexity.
   b. More importantly, the duration approach assumes a parallel shift of the spot yield curve, while non-parallel shifts are equally possible, and possibly even more 'dangerous' for the company. Non-parallel shifts can be taken into account by applying the approach for some duration bands individually and summing the results. Such an alternative approach can also be considered as an approach that allows for correlations between the changes of the 'average' spot yields per duration band that are less than one.
   c. Still requires a fair degree of complex modelling by the company.

7.16   To assess market risk, requires the probability distribution of interest rate changes over a time horizon. This can be done through a statistical analysis of empirical economic data. The variance of interest rate changes is slightly higher for short maturities than for long ones, and higher for currencies with a high interest rate level than for low-interest currencies.

As a very rough estimate for major currencies, the standard deviation of yearly interest rate changes is of about 1.25%. Thus we obtain an approximate standard deviation of the yearly value change:

$$\sigma_{bond} \approx \frac{meanTimeToMaturity}{100} \times assetValue$$

7.17 If this approximation is used instead of a detailed statistical calculation, it should be loaded with a prudent factor. The formula can be applied to whole portfolios, not only individual bonds, as the risk reduction due to diversification between bonds is rather small. When aggregating bond portfolios, we add σ_{bond} values rather than σ_{bond}^2 values (which would apply if bond returns were independent).

7.18 Changes in bond yields may be caused by changes in the underlying risk-free rates, changes in the spreads that reflect the liquidity risk and credit risk of the asset, or changes in both components simultaneously. In section 3.4 we already suggested that the replicating asset portfolio (i.e. the liability cash flows) should be valued by discounting its cash flows on the basis of the risk-free spot yields. Consequently, changes in spreads only affect the market value of the actual assets available, while changes in risk-free rates affect both the market value of the assets available and the market value of the replicating portfolio (liabilities). It may be more logical to consider changes of spreads as typical forms of credit risk.

7.19 If a bond is denominated in a foreign currency, the volatility of the corresponding FX rate has to be accounted for. A typical yearly standard deviation for returns of a freely floating foreign exchange rate is around 10%. Thus we obtain

$$\sigma_{FX} \approx 0.1 \cdot assetValue$$

7.20 Again, this may be either used with a conservative factor or replaced by a statistical assessment.

7.21 The two risk components σ_{bond} and σ_{FX} can be combined as described in Section 3.4.1:

$$\sigma_{foreignBond} \approx \sqrt{\sigma_{bond}^2 + \sigma_{FX}^2} \approx \sqrt{\frac{meanTimeToMaturity^2}{100^2} + 0.01}$$

7.22 In this case, assuming zero correlation between the factors, is conservative. K. Froot ("Currency hedging over long horizons") and others have shown that foreign assets tend to have a lower variance than the formula indicates, especially in the long run.

7.23 The formulas for $\sigma_{foreignBond}$ and σ_{bond} can be used as pieces in a large, multivariate normal model. They describe the risk of stand-alone bond portfolios. However, in the case of asset-liability matching, the true risk may be smaller. If the times to maturity match the expected times of claim payments, and the assets in foreign currencies match foreign liabilities, the total value of assets and liabilities may become more immune against market fluctuations. Of course, an insurance company has some invested surplus capital in excess of the expected liabilities, which has the full market risk. Exact calculations are only possible with a full ALM model.

D.7.3 Equity and Property

7.24 Equity and property positions are subject to Type A market risk when these assets are used to fund similarly performing policyholder liabilities (e.g., unit linked funds with no material guarantees) or represent free surplus. In these situations, market risk results from short term volatility in the market value of the underlying assets. The longest time horizon to be considered in this case (as discussed earlier) is one year. Shorter time horizons based on local products or conditions might be considered by local supervisors.

7.25 The variance of equity returns has been analyzed in numerous studies. The volatility (= annual standard deviation) is higher than for bonds. Even for the best diversified portfolios as

represented by index-tracking portfolios, the standard deviation of yearly returns may easily be 20% of the asset value. For individual equities of reasonable quality, it may be about 30%. Some individual equity titles may have distinctly higher risks. These risks have to be quantified, based on empirical data.

7.26   If equity is denominated in foreign currency, the standard deviation is

$$\sigma_{foreignEquity} \approx \sqrt{\sigma_{equity}^2 + \sigma_{FX}^2}$$

7.27   As discussed for fixed income, this is a conservative formula. Foreign equity investments often have a standard deviation of returns lower than this, mainly in the long run.

7.28   When aggregating equity investments of different currency zones, we should add their standard deviations, assuming total dependence, rather that adding the squares (assuming independence). This conservative assumption may be refined by a detailed analysis of correlations between equity indices of different countries.

7.29   Real estate investments can exactly be treated as equity. Real estate indices take the role of equity indices. The diversification between different countries may be slightly stronger than the analogous diversification effect for equity.

7.30   Real estate prices tend to increase when mortgages are becoming cheap, i.e. when interest rates fall.

D.7.4   Derivatives and Embedded Options

7.31   In sections 3.1 and 3.2 we already stressed the need to value embedded options explicitly. In particular, their value should be set equal to the actual market value of the assets needed to hedge these options. However, these assets may not be actually available. Therefore, in that case, special attention is needed for possible mismatches between the options that are embedded in liabilities and the derivative assets that are intended to cover them. The solvency requirement defined for this should be equal to a conservative estimate of the possible change of the difference between their market values. While these market values should always take the full remaining terms of the contracts into account, the mismatch buffer only needs to cover the possible change of its difference within the limited time period under consideration (one year).

7.32   Generally, calculating a mismatch provision for embedded options will not be an easy task. If it is possible to get a reasonable approximation of their actual market value, i.e. the market value of the replicating asset portfolio, by applying a (calibrated) Black-Scholes type of formula, it will generally also be possible to get a reasonably conservative estimate of its possible change. Such formulas generally have two types of parameters, namely the risk-free rate(s) and the implied volatility. For complex options, the market value of the embedded options can only be reasonably estimated by running stochastic simulations.

D.7.5 Other Types of Assets

7.33 Asset portfolios may contain many other types of assets. Some of them may even be off-balance sheet items. Typical examples are investments in private equity, commodities and all kind of derivatives that are not intended to hedge options embedded in the liabilities. As with equity and property investments, the relating market risks are (generally) 'asset-only' risks. Therefore, the corresponding mismatch provision can be calculated similar to the way it is calculated for equity and property investments.

7.34 Some of these assets may only be available 'over-the-counter' and are hence, illiquid. In this situation, both their actual market values and the possible change of these values within the limited time period (one year) have to be estimated conservatively.

D.7.6 Currency Risk

7.35 Currency risk is important if not all assets and liabilities are denominated in the same currency. A solvency requirement for currency risk can be defined in a similar way as for equity and property risks (i.e., by setting it equal to the actual market value of the assets denominated in foreign currency times a conservative estimate of the potential change of value within the first next year). The 'potential change' factor can include the effects of both the potential change of the yields (/prices) and the potential change of the currency.

D.7.7 Dependencies

7.36 Dependencies between asset market prices/yields of different asset types, particularly fixed-interest, equity and property (but excluding derivatives) are generally low. Correlations between prices/yields/returns of assets in local currency and those of assets that are denominated in foreign currency may be anything between –1 and +1, depending on the global and local economic conditions, the type of asset and the specifics of the assets (industry). It may therefore be reasonable to assume zero correlation between all these asset types in a factor-based approach. Consequently, the total solvency requirement for market risks can be set equal to the square root of the sum of squared requirements for these individual asset classes.

7.37 Of course, market prices of derivatives, including those that hedge options that are embedded in the liabilities are closely linked to the market prices of the underlying assets. Therefore, as mentioned before in section 5.2.3, it is very important to have consistency between the approaches for the 'leading' assets and the derivatives. In particular, if the approach for leading assets is based on an assumed change of the price/yield, the same change should be assumed in determining the change of the value of the derivatives. The resulting solvency requirement can be aggregated into the total requirement by simply adding it to the total defined in the foregoing paragraph.

7.38 Correlations within individual categories are generally high. Implicitly, this is taken into account by defining and summing different solvency requirements for different asset categories, instead of defining and summing them for individual assets. Any 'extra' correlations due to possible concentration within categories, e.g. many investments in shares of the IT-industry, can be 'penalised' by adding solvency requirements for concentration risks.

7.39 However, within the category of fixed-interest securities, special attention is needed for correlations between spot yields for different durations (maturities), if the factor-based approach for fixed-interest securities is applied to individual duration bands independently (see also section 5.2.1). In that case a choice has to be made for the way the corresponding solvency requirements are combined into one requirement for all fixed-interest securities (better: $S^{(fix)}$). This issue is

closely linked to the correlations issue. Spot yields for different durations are generally highly, but not perfectly, correlated. Therefore, the actual spot yield curve may also show non-parallel shifts. The following approach per duration band allows for such shifts:

1. Select a number of (modified) duration bands, e.g. 0-2 years, 2-5 years, 5-8 years, 8-12 years, 12-16 years, 16-24 years, and more than 24 years, with corresponding 'median' durations $dur^{(i)}$ ($dur^{(1)} = 1$, $dur^{(2)} = 3.5$, $dur^{(3)} = 6.5$, …, $dur^{(7)} =$ say 28) and corresponding actual (risk-free) spot yields ($r^{(1)}$, $r^{(2)}$, …..) according to the actual risk-free spot yield curve.

2. Define 'maximum' potential absolute changes of spot yields that may occur within the first next year, for each of the individual spot yields individually ($\Delta r^{(1)}$, $\Delta r^{(2)}$, …..). Preferably, these are based on an analysis of historical changes for each of the spot yields individually.

3. Allocate the cash flows of the available fixed-interest securities and liabilities respectively to the different duration bands, calculate the actual market values as well as their balance per duration band ($S^{(fix)(1)}$, $S^{(fix)(2)}$,) and define the solvency requirement for each duration band i as
    $$Solv^{(fix)(i)} = ABS \{S^{(fix)(i)} * dur^{(i)} * \Delta r^{(i)}\}.$$

4. Finally, define the total solvency requirement for fixed interest securities (balanced with the liabilities) as the sum of the requirements for the individual duration bands:
    $$Solv^{(fix)} = \sum_i Solv^{(fix)(i)}$$

7.40   This way, implicitly, it is assumed that each of the individual spot yields may either rise or fall within the next year. In this respect zero correlation between individual spot yields is assumed. Therefore, the final outcome of this approach may be higher than the outcome based on a rise or fall of all spot yields at the same time (by ($\Delta r^{(1)}$, $\Delta r^{(2)}$, …..) or ($-\Delta r^{(1)}$, $-\Delta r^{(2)}$, …..) respectively), as it allows for non-parallel shifts. However, by simply summing the resulting individual solvency requirements we implicitly assume correlations to be equal to one.

7.41   Finally, this approach can be considered as a mix of duration matching and cash flow matching. The more different duration bands are distinguished, the more it will stimulate insurers to do actual cash flow matching.

D.8   Standardized Approaches – Type B Risks

8.1   By definition the development of standardized approaches for capturing Type B risks is fraught with difficulty. Where these risks are material in an insurer, the supervisor should encourage or even require the insurer to perform appropriate advanced approaches to modelling their Type B market risk.

8.2   Standardized approaches to assessing Type B market risk might include:

1. For long term interest guarantees in life insurance and annuity products the present value of future liability cash flows must be determined on the presumption that long term reinvestment returns revert to a conservative view of historical long term averages.
2. For complex options, appropriately conservative factors must be derived based on rigorous stochastic modelling of industry wide data to adequately capture the tail of the loss distribution for the confidence level required.

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APPENDIX E   Credit Risk

E.1   Definition of Credit Risk

1.1   Credit risk is the inability or unwillingness of a counterparty to fully meet its on and/or off-balance sheet contractual financial obligations. The counterparty could be an issuer, a debtor, a borrower, a broker, a policyholder, a reinsurer or a guarantor

1.2   Credit risk has been traditionally associated with assets. However, it can exist with respect to any set of projected future cash flows. Credit risk is therefore also important in assessing the true relief provided by a counterparty to an insurance transaction, such as reinsurance or a party to whom the insurer has outsourced some of its work functions. Credit risk might even be considered to exist in regard to the projected future cash flows resulting from the policyholder obligations. This latter aspect of credit risk is quite controversial as it suggests the value of policyholder obligations diminishes as the credit risk of the insurer declines. The WP recommends that insurer capital requirements for credit risk do not reflect the potential ability of the insurer to default on it own cash flows.

1.3   Credit risk can be reflected in the present value of a set of cash flows either implicitly via a credit risk spread incorporated in the discount rate or via explicit modelling of the cash flows themselves.

1.4   The market value of a stream of projected future cash flows (e.g., a bond) reflects the current market view (among many things) of the credit risk of the provider of the cash flows. Such a view might reflect a variety of market knowledge of the bond issuer such as credit ratings provided by various agencies. Necessarily, such a view will likely reflect the current financial position of the issuer as well as the current economic environment. Such a view will consider the possibility of the issuer slipping in its ratings (i.e., ability to pay) as well as the probability of default (PD) and the amount of loss given that default occurs (LGD).

1.5   The Bank for International Settlements (BIS) defines the capital requirements for banks. In particular, its April 2003 consultative document entitled "The New Basel Capital Accord" contains extensive materials related to the determination of credit risk capital requirements, including both standardized and advanced approaches. The WP recommends that similar approaches be used for insurers. The WP recommends that the BIS approach may require some modification to address insurer specific issues. These modifications are noted throughout this portion of the WP report.

E.2   Types of Credit Risk

2.1   The principal sources of credit risk are:

- Direct Default Risk: risk that a firm will not receive the cash flows or assets to which it is entitled because a party with which the firm has a bilateral contract defaults on one or more obligations.

- Downgrade or Migration Risk: risk that changes in the possibility of a future default by an obligor will adversely affect the present value of the contract with the obligor today.

- Indirect Credit or Spread Risk: risk due to market perception of increased risk (i.e., perhaps due to business cycle or perceived credit worthiness in relation to other market participants).

- Settlement Risk: risk arising from the lag between the value and settlement dates of securities transactions.

- Sovereign Risk: risk of exposure to losses due to the decreasing value of foreign assets or increase the value of obligations denominated in foreign currencies.

- Concentration Risk: risk of increased exposure to losses due to concentration of investments in a geographical area or other economic sector.

- Counterparty Risk: risk of changes in values of reinsurance, contingent assets and liabilities (i.e., such as swaps that are not otherwise reflected in the balance sheet).

2.2　In general, life and health insurers purchase assets to support their liabilities. Historically this has not been true for non-life insurers where there has been a tendency for insurers to manage separately the results from underwriting and investments. While all of the assets of an insurer are available to provide against adversity, it is common risk management practice for insurers to implicitly or explicitly allocate their assets for one of the following purposes:

- support insurance contract liabilities
- represent economic capital
- represent free surplus

2.3　The allocation of assets to support specific policy liabilities is especially important for those insurance products whose performance depends directly on the performance of the underlying assets. In situations where the asset performance (including the impact of credit risk) is shared directly or indirectly with the policyholder, then appropriate credit can be taken in the determination of the credit risk capital requirement. Such credit must take into account policyholders' reasonable expectations in this regard as well as the insurer's practices in sharing such experience with policyholders.

2.4　Sizeable portions of an insurer's liabilities can have durations comparable to readily available high quality liquid assets in the local market. In these situations it is possible to select assets whose cash flows can provide a very close match to the liability cash flows. In other words, a replicating portfolio of assets is available in the market. In this situation, credit risk focuses on the actual assets held and the ability of the insurer to manage its credit loss position within the replicating portfolio horizon. This type of credit risk will be called **Type A** risk.

2.5　The long-term duration of some insurance (especially life insurance) liabilities requires the consideration of long term reinvestment of existing assets since a replicating portfolio assets of sufficient duration may not be currently offered in the market. For this type of business appropriate account must be taken not only of credit risk in current assets (Type A credit risk) but also the credit risk involved with future reinvested assets as well. This latter aspect of credit risk will be called **Type B** risk. Assessing Type B credit risk entails considerable uncertainty about the composition of the replicating portfolio and the manner of its reinvestment to mature the underlying cash flows. The length of the reinvestment period may extend through several economic periods.

E.3　Key Drivers of Credit Risk

3.1　Some of the key drivers of credit risk include[18]

- Credit quality – Credit quality of an investment or an enterprise refers to the probability that the issuer will meet all contractual obligations. This assessment normally occurs at both the initial investment and at each renewal point. One of the common measurements used in assessing credit quality is the rating assigned to the issuer. A variety of ratings agencies

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[18] Canadian Institute of Actuaries, 2003 *Report of the CIA Sub-Committee on Credit Risk*
Online at http://www.actuaries.ca/publications/2003/203087e.pdf (English), http://www.actuaries.ca/publications/2003/203087f.pdf (French)

provide these assessments to the public, giving the investor a perceived level of confidence in the issuer's ability to make good on the repayment schedules to which it is committed.

- Maturity – The longer the term to maturity of an investment, the longer even a high quality issuer has to potentially deteriorate

- Concentration by industry – Conditions that trigger credit events have a tendency to impact on the entire economy simultaneously. Within this general characteristic, however, the impact of economic development often varies between sectors of the economy. Within a sector, however, there tends to be uniformity between the entities participating in that sector. Degrees of separation within a sector will exist, but these are on a smaller scale than those that normally occur between sectors.

- Concentration by geography – Credit risk has been shown to carry a large degree of contagion. Periods of relatively few credit events are followed by periods where default experience is extremely high. Similarly, economically depressed regions tend to produce high levels of default experience in comparison with more prosperous areas. That these regions can and do change over time creates a challenge to the process of credit risk analysis

- Size of expected loss - The size of loss due to a credit event can vary widely, from loss of some or all of the return on an investment to loss of some, or all, of the inherent principal. Losses can also occur from a delay in the timing of a scheduled payment, causing either a loss of return during the deferral period, a reduction in available reinvestment rate during the deferral period, or both. When a scheduled payment is delayed for any reason, there is also the potential for an associated loss if the payment were needed to match a scheduled outflow. The investor would then be required to make good on its obligation by borrowing or selling other assets. They might need to delay payment of their own scheduled obligation, possibly incurring a penalty.

E.4   Controls and Hedging Strategies

4.1   Important in the management of credit risk are a combination of sound underwriting practices and appropriate lending limits within the insurer.

4.2   A broad definition of hedging strategies used to offset credit risk would include

- letters of credit
- contingency deposits
- securitization of mortgages (Mortgage Backed Securities)
- securitization of other assets (Asset Backed Securities)
- credit derivatives
  - *credit default swaps*
  - *total return swaps*
  - *collateralized debt obligations (CDOs)*
  - *credit-linked notes*
  - *credit spread options*
  - *basket derivatives*

4.3   Investment performance features of some insurance products also permit some, or all (policyholder reasonable expectations may at issue), of the credit losses for assets deemed to be used to support the policyholder obligations of specific blocks of insurance products.

E.5   General Modelling Approaches

5.1   There are a number of generalized approaches that are used to model credit risk[19]. A few of them will be summarized in the following paragraphs.

5.2   In default models, the rates of default and recovery are modelled explicitly. Present values are taken using the risk-free interest rate curve, and different cash flows under assumptions of default or non-default are valued using probabilities.

5.3   For example, assume a $100 cash flow is expected in one year from XYZ Corporation. Their probability of default is known to be p, and recovery on default is expected to be R. The risk-free one-year rate is i. Then the current value of the cash flow is

$$100 \, (1 - p) / (1 + i) + 100 \, p \, R / (1 + i)$$

5.4   Estimates of R are very difficult, and so it is usually set to a constant around 40% to 50%, based on experience. Even most stochastic models take this approach. Values of p can be found for given credit ratings from the various credit rating agencies, and the combination of p and R can be compared to the spread of the corporation's bonds for reasonableness.

5.5   In default models there are two states considered, either in default or not in default. Credit migration models consider not only the risk of default, but also the risk that an investment will lose (or gain) value due to changes in the corporation's credit rating. For example, if you hold a bond rated AA and it is downgraded to A, the bond will lose value, since it will be less desirable to potential buyers. Central to all credit migration models is a matrix of values known as a transition matrix. The matrix contains the probability that a bond will change from its current credit rating to another credit rating.

5.6   Asset models were developed in the 1970's by Merton. The general concept is that a firm will go into default if the value of its assets becomes less than the value of its debts, and so the firm's debt can be modelled as an option against its assets. The basic approach developed by Merton has been considerably expanded since its initial introduction.

5.7   An asset model can be combined with a model of correlations between obligors to produce a portfolio-level risk management model. For example, correlations between different obligors' underlying asset values are sometimes estimated by reference to correlations between stock prices. This approach underlies a number of commercially available credit risk models.

E.6   Degree of Protection

6.1   The credit risk capital requirements should be determined in a manner consistent with the overall goal for the degree of protection (confidence level) inherent in Pillar I capital requirements.

E.7   Time Horizon

7.1   Consistent with the time horizon for other insurer risks, credit risk should generally be determined using a time horizon of one year. One year recognizes the generally less active trading environment of insurers with respect to their asset and liability cash flows. One year reflects a conservative view of the time required by a supervisor to assume control of the affairs of a weakened insurer. One year reflects a conservative view of the time required for an insurer to address the credit risk in its assets. Failure to actively manage credit risk within such a portfolio within one year is more appropriately the subject of Pillar II type supervisory measures.

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[19] Recommended reading includes a paper from the Australian Institute of Actuaries, 2003 *Capital Reserving for Credit Risk for Insurers (Life & GI) and Other Institutions*. Online at http://www.actuaries.asn.au

7.2   This assessment time horizon should not be confused with the need to consider, in such an assessment, the full term of all of the assets and obligations of the insurer. For example, Type B credit risk requires consideration of future reinvestment decisions and future economic scenarios for the full term of the obligations of the insurer. Credit risk for these liabilities involves systematic (undiversifiable) risk due to the limited availability of (parts of) the replicating asset portfolio or, at least, uncertainty about its composition. In theory, these risks must always be assessed for the full remaining term of the liabilities. The best fitting replicating portfolio assets must be reinvested in accordance with the insurer's policies and practices with respect to investments so as to provide for the lengthy future cash flows. The requirement of a full term time horizon is considered necessary due to the considerable uncertainties involved in providing for future cash flows beyond the term of currently available replicating portfolio assets.

E.8   Advanced Approach – Type A Risks

8.1   The BIS has developed considerable experience with respect to credit risk capital requirements in the banking sector. The WP believes that a similar approach should also be considered for use by insurers in capturing Type A credit risk.

8.2   In considering the applicability of the BIS approach, insurance supervisors will need to consider the appropriateness of several elements in the BIS approach. For example,

- Degree of protection – the WP recommends consistency throughout the Pillar I requirements
- Time horizon – the WP recommends consistency throughout the Pillar I requirements
- Diversification – the WP recommends insurers reflect the diversification in their portfolios
- Correlation – the WP recommends that consideration be given to allowing insurers to reflect their own asset correlations
- Cycles –the WP believes the use of "current" versus "through the cycle" distributions for the frequency and severity of default – the WP believes this issue requires further study
- Migration – the WP supports the use of credit migration techniques (non-absorbing hitting probabilities for rating migration events) in the framework

A General Approach

8.3   The following paragraphs outline a general approach to the modelling of Type A credit risk.

8.4   Corporate bonds involve credit risk. The value of such a bond shrinks if the rating of the issuing company falls. This is the downgrade risk or, more generally formulated, the credit spread risk. The most extreme case is default. Expected default probabilities are available from rating agencies.

8.5   There are several commercially available software products to assist in the modelling of credit risk. Some of these focus on default modelling only while others also include credit spread modelling as well. One product explicitly models transition probabilities between ratings, where the lowest level, the default, is an absorbing state.

8.6   A supervisory credit risk assessment guideline should be designed in way not to demand the use of commercial software packages or services. The proposed guideline should provide a simple formula that is compatible to the multivariate normal framework of the base-line approach. One such formula is proposed here.

8.7   In this simple credit risk model, a bond is essentially characterized by its mean time to payment T and the current yearly default probability p_1 of the issuer. For rating BBB, for example, a typical yearly default probability is $p_1 = 0.2\%$. For assessing credit risk, we neglect the many

small cash flows due to coupon payments and assume just one large cash flow, the principal payment, at time now $+ T$. The theoretical value of such a bond is

$$assetValue \approx (1 - p_T) \cdot principal$$

where p_T is the default probability for the whole time period T. For the same period, the variance of the value change due to credit risk can be computed as

$$\sigma_{credit}^2(T) \approx p_T \cdot assetValue^2$$

8.8   The distribution function is binomial and has a form that is very different from normal. Another conservative assumption is that the corporate bond has a zero value after a default whereas, in reality, a small part of the face value may be recovered.

8.9   Our time horizon is 1 year, so we need the return variance $\sigma_{credit}^2 = \sigma_{credit}^2(1 year)$ due to credit risk, rather than just $\sigma_{credit}^2(T)$. This variance σ_{credit}^2 is affected not only by defaults but also by fluctuations in the rating of the issuer during the maturity period. A default-only model underestimates the variance. Rating fluctuations have to be included to arrive at estimated standard deviations that are large enough.

8.10   The model just assumes two things:

(1)   There is a rating scale on which the rating fluctuation can be described as a Brownian motion in a sufficiently good approximation.
(2)   There is a minimum value on this scale that corresponds to a default and serves as an absorbing state of the Brownian motion.

8.11   All the rest of the model can be derived from these two assumptions. The theoretical hitting probability of the absorbing state within a time interval T is

$$p_T = 1 - \Phi\left(\frac{const}{\sqrt{T}}\right)$$

where $\Phi(.) = 2N(.) - 1$ and $N(.)$ is the cumulative standard normal distribution with unit variance. The constant depends on the initial rating, but does not matter here.

8.12   Now we can relate default probabilities for different time intervals:

$$p_T = 1 - \Phi\left[\sqrt{\frac{1 year}{T}}\Phi^{-1}(1 - p_1)\right]$$

where Φ^{-1} is the inverse function of Φ, with $\Phi^{-1}[\Phi(p)] = p$. Given an annual default probability p_1, this formula allows computing the default probability p_T of the same issuer over a time interval of size T, including the rating fluctuation effect.

8.13   The same model leads to an approximation formula for the yearly variance of returns due to credit risk:

$$\sigma^2_{credit} \approx \frac{1\,year}{T}\,p_T = \frac{1\,year}{T}\left\{1 - \Phi\left[\sqrt{\frac{1\,year}{T}}\Phi^{-1}(1-p_1)\right]\right\}$$

or, as a numerical approximation in closed form,

$$\sigma^2_{credit} \approx \frac{1\,year}{T}\cdot e^{\frac{1}{b}\left\{c^2 - \left[\sqrt{\frac{1\,year}{T}}\left(\sqrt{c^2 - b\log p_1} - c\right) + c\right]^2\right\}}$$

with $b = 2.37$ and $c = 0.85$. This formula quantifies the credit risk of a bond as a function of its mean time to payment, T, and the issuer's current annual default probability, p_1.

8.14   As an example, we regard two BBB bonds. One has a remaining maturity of $T = 1$ year, the other bond has a maturity of $T = 5$ years. The issuer has a current annual default probability of $p_1 = 0.002 = 0.2\%$. Using the formula above, the 1-year bond has a credit risk of $\sigma_{credit} \approx \sqrt{0.002}\cdot assetValue \approx 4.5\%\cdot assetValue$. For the 5-year bond, we obtain a credit risk of $\sigma_{credit} \approx 18.3\%\cdot assetValue$. This higher value reflects the additional risk due to expected rating fluctuations over the 4 last years of the 5-year maturity period.

8.15   For a portfolio of different corporate bonds, there are diversification effects, which are limited by the fact that defaults may be correlated, depending on the geographical or economic proximity of the different issuers. In general, default frequencies also depend on worldwide economic cycles. In economically difficult times, many companies are subject to simultaneous downgrading or even default. There may be chain reactions in case of defaults. Statistics show that annual default frequencies exhibit a level of volatility distinctly higher than expected in a purely stochastic, Poisson-like world.

8.16   A simple, conservative model for the diversification is proposed. The credit risks $\sigma_{credit,i}$ of all bonds are computed with the formula presented above. The maximum risk, for full dependence, is

$$\sigma^2_{credit,\max} = \left(\sum_i \sigma_{credit,i}\right)^2$$

8.17   In case of no dependence, we have

$$\sigma^2_{credit} = \sum_i \sigma^2_{credit,i}$$

8.18   We conservatively assume that the best diversification can be approximated by correlation coefficients of 0.5. The resulting credit risk of a bond portfolio is

$$\sigma^2_{credit} = \frac{\alpha}{2} \sum_i \sigma^2_{credit,i} + \left(1 - \frac{\alpha}{2}\right)\left(\sum_i \sigma_{credit,i}\right)^2$$

where α is the estimated degree of diversification. Bonds from the same issuer have $\alpha = 0$, and an optimally diversified bond portfolio has $\alpha = 1$.

8.19   Sophisticated software products explicitly model the dependencies between defaults. Chain reactions in case of defaults may lead to a fat tail of the true overall credit risk.

8.20   Eventually, we combine market and credit risk of a fixed-income portfolio:

$$\sigma_{fixedIncome} \approx \sqrt{\sigma^2_{bond,market} + \sigma^2_{credit}}$$

assuming independence between bond markets on one hand and downgrades and defaults on the other hand. This assumption has to be checked and perhaps replaced be the more conservative assumption of a slightly positive correlation.

E.9   Advanced Approach – Type B Risks

9.1   Type B credit risk is inherent in insurance products of long duration (i.e. beyond the duration of current assets or replicating portfolio assets). Type A credit risk provisioning (e.g., as per the Basel Accord) only provides for the credit risk inherent in currently held assets.

9.2   If, in valuing the insurer's assets and liabilities in accordance with a total balance sheet approach, the future policy liability cash flows are present valued using investment returns which are net of credit risk, then the present value amount of the policy liabilities so determined will include a provision for credit risk for the entire term of the liabilities.

9.3   The present value amount of this credit risk provision can be estimated through determination of the credit spread inherent in future investment returns. Care must be exercised to avoid double-counting the credit risk provision for Type A credit risk in both the liabilities and via direct reference to the current assets. Care must also be exercised that an appropriate provision for Type B credit risk has been made. If the credit spread assumed in the future simply reflects expected losses or simply the current position in the credit cycle, then it may be insufficient for solvency purposes.

E.10   Standardized Approaches – Type A Risks

10.1   The Working Party (WP) recommends that the work of the BIS with respect to credit risk capital requirements for banks be also considered for use by insurers in capturing Type A credit risk. In considering the BIS approach, insurance supervisors will need to consider the appropriateness of the time horizon and confidence level assumptions implicit in the BIS approach. Also to be considered is the appropriate treatment of policyholder pass-through features.

E.11   Standardized Approaches – Type B Risks

11.1   By definition the development of standardized approaches for capturing Type B risks is fraught with difficulty. Where these risks are material in an insurer, the supervisor should encourage or even require the insurer to perform appropriate advanced approaches to modelling their Type B credit risk.

11.2   Standardized approaches to assessing Type B market risk might include (from the simplest to the more sophisticated):

1. Where it is not possible to directly compute the present value of future liability cash flows, provision for Type B credit risk can be made approximately by applying a factor to the policy liabilities of long-term business. These factors would need to be tailored to the circumstances of an individual supervisor and their financial reporting structure for these liabilities.

2. Where it is possible to estimate the duration of long term business, provision for Type B risk can be made approximately by applying a credit risk spread to the duration (beyond that of the current assets) and the policy liabilities for long-term business.

3. Where it is possible to directly compute the present value of future liability cash flows, provision for Type B credit risk can be made directly through use of a credit risk spread.

APPENDIX F   Lessons from Insurer Failures

F.1   Lessons from Recent Insurer Failures

1.1   There have been a number of high profile insurer failures in recent years. Before attempting to identify the characteristics of a workable international risk-based solvency approach, it is worthwhile to examine the reasons behind the failures, to the extent possible at this stage. This will help focus the needs of the risk-based measures more closely**.** The WP also notes the Sharma Report, available from the Conference of European Supervisors which provides an excellent summary of the lessons learned from European insurer failures.

F.1.1   HIH Insurance (Australia)

Background:

1.2   The HIH Insurance Group (HIH) was declared provisionally insolvent in March 2001. Following investigation by the provisional liquidator, insolvency was confirmed in August of the same year. The estimate of the deficit in assets to support the outstanding liabilities is still uncertain, but the shortfall, estimated to be between A$3.5 billion and $5.3 billion in August 2001, appears to be firming up at a figure towards the middle of that range

1.3   A Royal Commission was established to look into the reasons behind the failure. In April of 2003, the Royal Commissioner presented his report on the reasons behind the collapse and on recommendations to minimise the chance of future similar occurrences. His report focused on a failure by management to provide sufficiently for outstanding liabilities as the key reason for failure, compounded by "blind faith" in the leadership and an aggressive approach to growth. However, the report and the testimony from witnesses to the Commission provides a more detailed account, as follows.

Relevant Issues:

1.4   Firstly, HIH had a unique business spread. Its portfolios were predominately longer tailed and more risky than the market norm, despite more recent attempts to increase the shorter tailed business focus. The accompanying high volatility of outcomes accentuated the risk of failure.

1.5   HIH had been founded and led for many years by a strong-minded, goal-driven CEO. It emerged from the Royal Commission's investigations that, whereas in the earlier years of the company's history, this approach was very successful, the CEO's approach had contributed to more recent problems.

1.6   In 1999, HIH completed the take-over of another major Australian based general insurer, FAI Insurance. It has emerged that the price paid for FAI was substantially greater than the net asset value. Indeed, in hindsight there are strong indications that FAI was technically insolvent, at the time of take-over. No due diligence was performed as part of the take-over procedure.

1.7   The evidence presented to the Royal Commission on claim reserving and management practices, raised questions as to the level of objectivity applied and the level of prudence. HIH management argued that risk margins in setting outstanding claim reserves were made unnecessary by the company's outwards reinsurance program. This was shown to be a false security.   Similarly, underwriting, risk pricing and premium setting practices came into question and in some areas were found to be deficient.

1.8   A number of connections between the company's external auditor and HIH were highlighted by the Commission. As an example, the company's CFO at the time of failure was previously a senior partner at the auditor as were several members of its board, including its Chairman. These connections caused questions to be asked about the level of independence, and hence effectiveness, of the auditor.

1.9   HIH was the major client of the external actuary who examined the long tailed claims portfolios. Fees from his work for the company represented the majority of his annual income. This put pressure on any statement regarding the actuary's level of independence.

1.10   The Royal Commission unearthed a substantial dearth of data for actuarial, accounting and underwriting studies. Any such lack of reliable data would inevitably have led to an increase in the level of subjectivity for key decisions.

1.11   In the final years of the company's life, it entered into "financial reinsurance" deals that demonstrably did not include a transfer of risk (because of the existence of "side letters" that precluded any claims), and hence were effectively loans. Whilst it could be argued that these deals did no more than delay the inevitable, they appear to have at least increased the size of the ultimate deficit.

1.12   The ambitious nature of the management approach created a strong "top line" (i.e. written, which may have increased the pressure on reserve adequacy and detracted from the need to protect "bottom line" results (i.e. net profit).

1.13   Although the Royal Commission clearly absolved the supervisor, APRA, from direct blame for the collapse, the report did highlight a number of areas where APRA's access to relevant data and other information was lacking and which, the Commissioner argued, caused a delayed response.

1.14   Governance was found to have been wanting across a number of aspects of HIH's operations, not the least being overseas subsidiaries and the underwriting of new lines of business.

1.15   The Australian supervisory requirements for general insurers have been renewed, and substantially upgraded with effect from 1 July 2002. Although already planned prior to the HIH failure, it can be argued that the final model was guided by an interpretation of the reasons behind the collapse in an attempt to prevent a repeat situation. In addition, the Royal Commissioner's report included 61 recommendations, many of which have already been acted on by the Australian Federal Government.

F.1.2   Independent Insurance (UK)
Background:

1.16   Independent Insurance Plc was a UK based general insurer that specialised in general and public liability business transacted via intermediaries and as well as personal lines. The Company also participated in the London market by accepting lines on larger risks. Premium income was approximately £830m in 2000, however the company had expanded significantly in the year experiencing 64% growth in premiums. Approximately 75% of the written premium was in the hands of intermediaries at the end of 2000.

1.17   Independent Insurance was unusual in that since its floatation in 1993 on the stock exchange it had included an actuary's opinion on the reserving adequacy in its published accounts. This is not a requirement in the UK and to date only a few companies have followed this practice.

1.18   In the late 1990's there have been changes in the legal environment that have led to increased costs in settling liability claims. This has impacted the entire liability insurance market.

1.19 In May 2001 the Company's actuarial advisers advised Independent's board that it could not form an accurate actuarial assessment of the insurer's reserves after discovery of claims that had not been entered into the company's accounting systems. The Company was placed in provisional liquidation in June 2001.

1.20 The Serious Fraud Squad is investigating the circumstances surrounding the failure of the Company and therefore it will be some time before the full facts are revealed.

1.21 From press comment it appears that there were some very significant reinsurance contracts that were entered into by the Chief Executive without the full knowledge of the board.

Reasons
- Rapid growth
- Insufficient reserves
- Failure to price adequately
- Legal and claims environment changes
- Ineffective corporate governance

Warning Signs
- Dominant Chief Executive
- Negative cash flow
- Unidentifiable competitive advantage

F.1.3 Equitable Life Assurance Society (UK) ("the Society")

Background:

1.22 Equitable Life is a mutual insurance company with assets in excess of £25bn that has been trading since the eighteenth century and ceased accepting new business in December 2000. Between 1957 and 1988 most of the Society's new pensions policies included the right to use the fund built up to buy a pension on guaranteed terms ("GAR"). In 1978 legislation introduced Open Market Options ("OMOs") for new retirement annuity contracts. These options gave the policyholder the right to purchase an annuity in the open market.

1.23 When interest rates are high the policyholders can buy the annuity from the open market or the Society and when interest rates are low they can buy annuities from the Society using their GAR option.

1.24 The Society believed at that time that these policies provided a minimum guaranteed level of cash benefit and a minimum guaranteed level of annuity to protect policyholders against very low or very high interest rates. It believed that, in times of normal interest rates, bonus rates could be adjusted to avoid either of these guarantees causing significant cost to the with-profits fund. The bonus consisted of regular bonuses and a final bonus when the annuity was taken.

1.25 In 1988 the Society ceased offering GARs, however the existing GAR policyholders had the right to invest new premiums under their existing contracts (Open-ended option).

1.26 In late 1993 annuity rates fell below those guaranteed in most GAR policies. The Society declared final bonuses so that the value of total benefits, including the value of the guaranteed annuity, was broadly equal to each policy's notional share of the with profits fund ("asset share"). With lower annuity rates, the option to take a pension at the guaranteed annuity rate had significant value. If a policyholder chose not to take a GAR option, preferring the flexibility of an alternative option, then the benefits were of lower value. The Society believed that asset share should be delivered whichever option was selected. This led to a lower rate of final bonus for policyholders taking the GAR option than for those not using the GAR option.

1.27   In September 1998 a number of complaints were made to the Pensions Ombudsman as some policyholders believed that the Society's stance was unlawful.

1.28   The High Court held that the Society's Board had exercised its discretion as to final bonuses in a legally permissible manner. The Court of Appeal determined by a majority that it was not lawful to differentiate in this way within the group of GAR policyholders. It decided that GAR policyholders should receive the same proportionate final bonus irrespective of the form of the benefits taken (i.e. OMO or GAR option). The Court did not, however, decide that it was unacceptable for the Society to differentiate between GAR and Non-GAR policyholders in this respect. This allowed any cost of the GAR options to be "ring-fenced" to those policyholders with GAR policies. The Society appealed the decision to the House of Lords.

1.29   The House of Lords' decision took matters beyond this by saying that the Society could not apply a different bonus policy to GAR and non-GAR policyholders.

1.30   Equitable's solvency position and the decisions facing the prudential regulator FSA changed dramatically after the House of Lords' judgement. FSA then had to decide whether to close Equitable to new business or to allow them to try to sell the company as a going concern. The prudential regulator's primary objective was to protect existing policyholders' interests by ensuring that Equitable remained solvent and able to meet their liabilities. FSA took the view that Equitable's strategy of seeking a buyer was likely to result in the best outcome for policyholders. Equitable said, and FSA accepted, that a sale could result in Equitable acquiring sufficient funding to repay the seven months of bonus withheld in response to the House of Lords' judgement, and possibly to make a goodwill payment to existing policyholders on top of that. That position could only be achieved - if at all - through a sale.

1.31   However, the Board was not able to find a purchaser and on 8th December 2000 the Society was closed to new business. Nevertheless a report from the Parliamentary Ombudsman in July 2003 ruled out any prospect of compensation on the basis of regulatory failure.

1.32   People who had Equitable Life with-profits policies in force on 8 February 2002, when Equitable's Scheme of Arrangement came into effect, are covered by the terms of that scheme and are therefore unable to pursue complaints about misselling.

1.33   An initial adjudication by the Financial Ombudsman service in May 2003 found complainants had been given negligent and misleading advice. Equitable Life appealed and the ombudsman is now considering a final decision in the light of comments on a legal opinion on how to approach redress.

1.34   A key decision due in the last quarter of 2003 is the Financial Ombudsman's ruling on five lead mis-selling cases dealing with people who bought policies between September 1998 and July 2000 when the house of Lords decision was announced.

Equitable's Reputation

1.35   The Society had an enviable track record of offering a cost efficient service to its members. No commissions were paid to intermediaries and the administration capabilities were seen as amongst the best in the industry. As the Society did not pay commissions to intermediaries it is possible that very few intermediaries made any searching comparisons between the Society and its peers, and therefore the Society was able to adopt policies and practices which were not prevalent in the industry.

Equitable's Bonus Philosophy

1.36   The Society is unique amongst mutual insurers in that it did not maintain a free reserve. The philosophy was that each generation of policyholders should get its own asset share and neither inherit from the past or give to the future. This stance led to higher bonus levels in periods of high investment returns and this helped the sales force generate high volumes of new business, and this subsequently led to low costs of administration.

Equitable's Business Mix

1.37   The Society benefited from the legislation in the 1970's that encouraged saving for retirement and the majority of the Society's business relates to this type of business. Given its market positioning many of its members were self-employed and in the professions. As the contracts were designed to be flexible for the self-employed who tend to have variable earnings these contracts allow for variable premiums and therefore these policyholders have the open ended option to invest new premiums which benefit from the GAR. Approximately 25% of the assets are in respect of the GAR policyholders.

Industry Issues

1.38   Many intermediaries and insurance companies have had to pay compensation to policyholders because of alleged mis-selling of pensions contracts where individuals were encouraged to leave their occupational schemes even though this was not in the policyholder's best interests.

Reasons:
- Concentrated in pensions business
- High proportion of contracts with open ended options
- Low level of surplus (in line with philosophy)
- Court's view different to Directors
- No documented method of charging for guarantees and options (i.e. differential bonus policy from when contracts were introduced)
- Industry issues (pensions mis-selling)

F.1.4   Nissan Mutual Life (Japan)
Background:

1.39   The Ministry of Finance ordered suspension of business according to Insurance Business Law in April 1997. It was the first failure of insurance company in Japan after the World War II. Liabilities in excess of assets were Y322.2 billion.

Possible Reasons:
- single premium (or prepaid premium) annuity with too high guaranteed rates
- bad debt caused by loans to realty business
- high risk investment
- collapse of "bubble" economy (crash of stock, property and real estate markets)
- continuation of extraordinary low interest rate policy
- a large amount of negative interest rate spread

1.40   The Life Insurance Association of Japan established a new company, (i.e. Aoba Life Insurance Company), and transferred insurance contracts en bloc to the company. Then the guaranteed interest rate was lowered. Aoba Life received financial aid of Y200 billion from The Life Insurance Industry's Fund for Policyholder Protection. In November 1999, Aoba Life was sold to a subsidiary company of Althemis, France. After this case, disclosure of solvency margin to the public became required.

F.1.5   Taisei Fire and Marine (Japan)
Background:

1.41   The Taisei Fire & Marine filed for protection under the special corporation rehabilitation law for insurers to the Tokyo District Court in November 2001 and their property was preserved intact. Liabilities in excess of assets were Y94.5 billion.

Possible Reasons:
- a large amount of reinsurance claims to be paid particularly including claims arising from the 11 September 2001 terrorist attacks in the United States
- reinsurance arrangement was entrusted to an agent in the U.S.
- reinsurance contract does not transfer the risk
- management does not grasp the risk of the reinsurance contract
- insufficient risk management

1.42   The Taisei is to merge with the Sompo Japan Insurance Inc.; the second largest general insurance company in Japan, in December 2002 after it sold off the reinsurance business. The Taisei received financial aid of Y5.3 billion from the Non-life Insurance Policyholders Protection Corporation of Japan.

F.1.6   Common Threads

1.43   It would be too simplistic to dismiss the similarities between the various case studies as being related to "out on a limb" decisions by key personnel not covered adequately by internal risk control practices.

1.44   Perhaps a more helpful analysis would be to identify the lack of key information as a means of precipitating the type of badly founded decisions that appear to have led to most, if not, all of our examples of company failure.

APPENDIX G   Introduction to Insurance Risk

G.1   Insurance Risk Example

1.1   The insurance business is difficult to assimilate for anyone not involved in its intricacies on a day-to-day basis. Many aspects of the business are counter-intuitive, even to those well versed in the broader commercial business markets.

1.2   The following example uses the analogy of the rolling of dice to help explain the uncertainty of outcomes for all insurance contracts, and the rationale for the need for capital support that this engenders for the business.

G.1.1   Insurance Basics

1.3   A number of features are common to all insurance transactions:

- Outcomes of risks from individual policies are unknown when underwritten

- However, when many similar risks are underwritten, expected results of total portfolio become more predictable

- Claims processes are driven by:
    - *Frequency (or probability) of a claim event occurring; and*
    - *Severity (of size) of a claim if it occurs*

- Risks inherent in different classes of insurance vary:
    - *High frequency / low severity (e.g., motor and health) – outcomes easy to predict reliably*
    - *Low frequency / high severity (e.g., earthquake and hail) – outcomes hard to predict reliably*

G.1.2   The Need for Capital

1.4   For an insurance company, capital is essentially needed to cover the risk of business outcomes being greater than those predicted (i.e. largely the cost of claims to be settled in the future relating to business already underwritten, but also assets being held to support those claims and the relevant future operational costs).

- Premiums charged generally pay for expected losses (50% Probability) plus expenses of operation

- Insurers must have capital so as to be able to fund unexpected losses (when claims exceed expected levels)

- Profit margin in premium charged generally provides the return on capital needed when unexpected losses arise

- Provides support in face of adverse unexpected outcomes from insurance activities, investment performance and operations

- Finances growth and capital expenditure

- Provides security to policyholders that claims will be paid

- Can be defined as = Total Assets – Total Liabilities

G.1.3 Capital Management

1.5 There is a "healthy tension" between policyholders' needs and shareholders' (stakeholders') needs that creates a balanced position when determining the appropriate capital support needs for the business.

The Balance of Capital:
- Policyholders and Supervisors will always like to see more capital
  – Better Security
  – Better Credit Ratings attract business

- Shareholders will generally like to see less capital
  – Enables better RoE
  – But less capital = higher risk

1.6 Here is a good point to introduce our example. It helps someone uninitiated in the intricacies of insurance contracts and risk management to understand how the "right" amount of capital is determined by a company's Board and senior management.

G.1.4 The Unbiased Die Example

1.7 We shall use the random outcomes of throwing an unbiased die to illustrate the uncertainty of outcomes from insurance contracts, and how insurers deal with the risks to their business that this entails.

G.2 Reserving for Claims
Illustration

2.1 Assume we roll a unbiased die 100 times to represent the results of underwriting 100 policies

If 1 is result, insurer pays a claim of $1

If 2 is result, insurer pays a claim of $2

-

etc.

-

If 5 is result, insurer pays a claim of $5.

Illustration

2.2 What is the likelihood that total claims will be greater than $250?

2.3 The higher the amount reserved the greater the probability that there are sufficient funds to pay all claims.

Levels of Reserving - IBNR known as "incurred but not reported or claim amount (Before the Die is thrown)

Probability of Sufficiency PoS	Level	$ needed
50%	Central estimate	$250
75%	Illustrative Supervisor's Minimum Requirement	$262
90%	Illustrative Company Standard	$272

2.4   Hence, at outset we have a liability of $272

2.5   This amount is greater than the amount with which the Insurance Supervisor would see as an absolute minimum for safely managing the business, and consistent with the company's view of the "appropriate appetite for risk".

G.3   Premium and Profit

3.1   Retaining our "die" example, we now illustrate the concepts of premium and profit by introducing a cost for each of our 100 throws.

Illustration - Premium & Profit

3.2   For simplicity, assume there are no expenses.
        Suppose insurer charges $3 per throw.
        Hence total premium = $300
        "Expected" profit = $300 - $250 = $50
        (A lower profit will occur 50% of the time and higher profit will occur 50% of the time)

        Is this the profit that can be reported as earned?

Levels of Reserving
(After 50 throws)

        Suppose after 50 throws we have:

RESULT	FREQUENCY	CLAIMS $
1	7	7
2	7	14
3	7	21
4	7	28
5	7	35
6	15	0
	50	105

3.3   Reserve will now be = Actual Claims + IBNR
        for remaining 50 throws = 105 + 272 x 50 / 100 = 241

        "IBNR" stands for "Incurred but not Reported" and reflects the unknown outcome of claims relating to policies (or throws of the die) for which we have already received a "premium".

Profit Reported

3.4   Premium = $300

Actual claims are $105 compared $125 expected

Hence profit after first 50 throws

=   300 – (105 + 272/2)

=   59

This profit has three components

25   Expected Profit ((300-250)/2)

20   "Unexpected" Profit (125-105)

<u>14</u>   Release of Risk Margin (50% of (300 - 272))

59

3.5   So we have demonstrably done better than expected. No uncertainty remains about the outcomes of the 50 throws we have made, so we can safely recognise the profit relating to those throws broken down into the three types in the above table.

Levels of Reserving

(After 100 throws)

3.6   Suppose after 100 throws we have:

RESULT	FREQUENCY	CLAIMS $
1	10	10
2	10	20
3	20	60
4	20	80
5	18	90
6	22	0
	100	260

3.7   Reserve will now be   = Actual Claims

= 260

Levels of Reserving

3.8   Premium earned for 100 throws = $300

Actual claims are $260 compared to $250 expected

Hence profit from 100 throws

=   300 - 260

=   40

Profit/(loss) from 100 throws was

50   Expected Profit

<u>(10)</u>   "Unexpected" Loss (250-260)

40

3.9   Note that the result of the second 50 throws was a loss of 19 as

40 - 59 = (19)

3.10   So, because we have presumably already used the profit from the first 50 throws, we must now draw on our capital to support the loss from the second 50 throws. (If we were being prudent, of course, some of our profit from the first 50 throws may have bolstered our capital).

G.4   Capital Requirement

4.1   Even if we were being prudent, we could not guarantee that we would be solvent after either the first 50 or first 100 throws, or whenever unless we had an extra "cushion" of capital to support our business. (What if the second 50 throws had come first? What if we had the same outcome for the second 50 throws for all of the 100 throws?!)

4.2   Assume that the Minimum Capital Requirement (MCR) the Supervisor requires in addition to the outstanding claims liability in this case is $100.

4.3   Hence possible range of funds insurer needs at outset is:

PoS	Liability	+	MCR	=	Total Funds Needed
75%	262+		100	=	362
90%	272 +		100	=	372

Illustration - Capital Needed

4.4   What capital does insurer need to have in addition to the premium charged to be able to operate?

PoS	Total Funds Charged	-	Premium Needed Needed	=	Capital
75%	362	-	300	=	62
90%	372	-	300	=	72

4.5   Note that this reflects the **minimum** capital support position.

Illustration Profit (Loss)& Returns on capital (RoC)

PoS	CAPITAL LEVEL	WORST RESULT	BEST RESULT	EXPECTED RESULT	EXPECTED RoC
75%	62	(200)	300	50	80%=50/62
90%	72	(200)	300	50	69%=50/72

Illustration Risk of Ruin

4.6   But what is wrong with these scenarios? If claims exceed $372 the insurer will fail! Hence, the insurer needs reinsurance to prevent this outcome.

4.7   "Hence the insurer probably needs more capital, since the likelihood of failure will appear too great to a prudent Board of Directors. An alternative that may appear more efficient is the use of reinsurance. We shall extend our example to include an illustration of the value of reinsurance in reducing the risk to the insurer"

G.5   Reinsurance

5.1   Illustration- Reinsurance

- If claims exceed $362 an insurer operating at 75% PoS will be bankrupt ($372 for 90% PoS).

- The Supervisor will want safeguards in place to prevent this, so reinsurance must be purchased.

- If reinsurer agrees to pay all claims in excess of $362 for a cost of $38, or all claims over $372 for $36, what is the result?

Illustration- Reinsurance Impact

PoS	CAPITAL LEVEL	EXPECTED PROFIT BEFORE REINSURANCE	REINSURANCE COST	EXPECTED PROFIT AFTER REINSURANCE	EXPECTED RoC
75%	62	50	38	12	19%
90%	72	50	36	14	19%

5.2   It will be noted that the return on capital is now much lower than in our "un-reinsured" illustration. However, it is still better than the return would have been if we had increased the capital support to the substantially increased level that would have effectively nullified the risk of failure (without the reinsurance)

G.6   Summary

6.1   This simple example shows that,

- Reserving Levels
- Capital Requirements
- Premiums Charged
- Projected Profit
- Expected Return on Capital; and
- Reinsurance needs
- All are INTERLINKED in their impact on an insurer's overall financial position.

6.2   The example also demonstrates how the risks inherent in insurance business create a distinctive set of management decisions related to the balance between risk and return on invested capital.

APPENDIX H   Analytic Methods

H.1   Developing a Base-Line Model

1.1   This Appendix deals initially with analytic methods for developing the base-line model, the multivariate Normal distribution, as well as risk measure.   It then goes on to deal with non-Normal risks for which analytic approximations to risk measures are developed. These are used for developing factor-based formulas that are good approximations to results using an internal model.

1.2   Since, for internal models, the distribution of the outcome X may be quite complicated, it is useful to develop a "base-line" model of the distribution of the outcome, recognizing that approximations are involved.   The cumulant generating function of X is

$$\varphi_X(t) = \ln E\!\left(e^{tX}\right)$$

1.3   The cumulant generating function for each distribution is unique and characterizes the distribution.   It can be written as a series expansion as

$$\varphi_X(t) = \mu t + \sigma^2 \frac{t^2}{2} + \kappa_3 \frac{t^3}{3!} + \kappa_4 \frac{t^4}{4!} + \cdots$$

1.4   Where μ is the mean of the distribution, σ^2 is its variance and $\kappa_3, \kappa_4, \cdots$ are the higher cumulants of the distribution. The Normal distribution has cumulant generating function

$$\varphi_X(t) = \mu t + \sigma^2 \frac{t^2}{2}$$

1.5   With all higher cumulants equal to zero. Hence, the Normal distribution can be viewed as a first-order approximation to the "true" distribution.

1.6   Applying this idea to all the risk components, as well as at to the aggregate risk, results in the multivariate Normal distribution serving as the first-order approximation or base-line model.

1.7   The error of the approximation can be measured by examining the size of the higher cumulants or by other methods. One such method is to obtain upper bounds on the error of key quantities such as risk measures when the mean and variance are fixed but the higher cumulants are unknown. There is well-developed theory for finding these upper bounds. It is not anticipated that such bound would be used in practice. However, they are useful for a supervisor in evaluating the maximum possible error in adopting a relatively simple model as a baseline model.

1.8   If $X_1, X_2, …, X_n$ have a multivariate Normal distribution (or the Normal model is used as a first approximation), the model is completely specified by its mean vector and its covariance matrix:

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{1n}\sigma_1\sigma_n & \rho_{2n}\sigma_2\sigma_n & \cdots & \sigma_n^2 \end{pmatrix},$$

1.9   Where $\rho_{i,j}$ is the correlation between the *i*-th and the *j*-th risk components, and σ_j is the standard deviation of the *j*-th component.

1.10   The standard deviation of the aggregate distribution is $\sigma = \sqrt{\sum_{i,j=1}^{n} \rho_{ij}\sigma_i\sigma_j}$.

1.11   Thus, within the Normal distribution framework, by specifying the correlations of all pairs of component risks and the means and standard deviations of each, the aggregate distribution can be fully specified.

1.12   In practice, two major sources of error need to be recognized. First, when the Normal model is used as a base-line model, the "true" distribution, errors can occur. The true probability distributions associated with particular risks may be quite different from the Normal distribution. Although the Normal distribution is used extensively in financial theory, it is often found the observed extreme events suggest a tail of the distribution that is heavier than that of the Normal distribution. Heavier tails are also observed for losses for many insurance lines, especially in the property-liability areas. Typical risk measures, such as standard deviation or VaR can seriously underestimate the true risk the true model is significantly different from the Normal distribution. Second, when the marginal distributions of the various risks are combined into a multivariate distribution, the linear correlation used in the Normal distribution may not be well suited to combining interactions in the extreme tails of the distribution, since normal correlation describes the degree of linearity of the relationship between two risks over the entire range of the distributions, and does not focus mainly on the tails, which is the area of interest for supervisors?

1.13   A supervisory framework can recognize the errors described in the previous paragraph in a number of ways:

   a)   Requiring a multiple (e.g. 150%) of the capital indicated by using a specific model. This provides a cushion for "model error."
   b)   Incorporating directly some conservative elements into assumptions, parameters, and correlations in the base-line model.

H.2   Base-Line Capital Requirement Framework

2.1   The base-line risk measure "standard deviation" is closely related to other concepts in the case of normally distributed risks. One such concept is the Value-at-Risk (VaR) that corresponds to a quantile that is away from the mean by a fixed multiple of the standard deviations. For instance, the 99th percentile corresponds to 2.33 times standard deviation in addition to the mean as a total balance sheet requirement.

2.2   When the standard deviation is used as the risk measure and the indicated capital requirement is a multiple of the standard deviation

$$C_j = k\sigma_j,$$

2.3   The capital requirement of the aggregate risk can be written as

$$C = \sqrt{\sum_{i,j=1}^{n} \rho_{ij} C_i C_j} \ .$$

2.4   This formula provides a base-line formula capital requirement. It requires calculation of the indicated capital requirement for each component risk and combining them using the above formula which incorporates the linear correlation coefficient as a measure of association between the component risks. It is noteworthy that for the Normal distribution, the above formula also holds if TailVaR is used as a risk in place of standard deviation.

2.5   In practice, insurance risks and investment risks often depart form the multivariate Normal assumptions, and the baseline risk-measures become less effective. Common criticisms of risk measures based on the Normal distribution include:

    a.   they may fail to differentiate between upside and downside for risks with skewed and fat-tailed distributions;

    b.   they may fail to reflect non-linear correlations (e.g. higher tail correlations); and

    c.   they may violate some of the "consistency" rules for a coherent risk measure.[20]

2.6   To address some of these issues, there have developed analytic tools that can overcome the drawbacks of the baseline risk measures for non-Normal distributions, while still retaining the baseline for Normally distributed risks.

2.7   An example of one such coherent risk measure that extends the standard deviation for non-Normal risks is the Wang Transform. For a risk with a loss distribution $F(x)$, the Wang transform $F^*(x)=\Phi[\Phi^{-1}(F(x))-\lambda]$ gives a transformed distribution, where Φ is the standard normal cumulative distribution function. The Wang transform of a Normal distribution with mean μ and standard deviation (volatility) σ is another Normal distribution but with the mean replaced by $\mu+\lambda\sigma$ and the standard deviation, σ, unchanged. In this case, the mean of the transformed distribution $\mu+\lambda\sigma$ is the risk measure, or required capital.

H.3   Analytic Approximations

3.1   In order to develop factor-based formulas for capital requirements that reflect the individual characteristics of an insurance company, one needs to develop "exposure" quantities, measuring the level of risk-exposure of the company to any risk type. Thus, one can consider the capital C as a function of the exposure levels of each of the component risks. In practice these exposure measures need to be defined. Simple proxies for exposures can include amounts-at-risk, premiums, or reserves, among others.

3.2   Thus one can write

$$C(e_1,e_2,...) = \int x \cdot dg[F(x)]$$

where e_j is the exposure measure for the j-th risk component. Note that we can rewrite the loss as

$$X_j = e_j \cdot Y_j$$

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[20] See Artzner, Ph 1991 Application of coherent risk measures to capital requirements in insurance. NAAJ 3, Nov 2,11-25*t*

where Y_j is a new standardized loss variable.

$$C(\lambda e_1, \lambda e_2, \ldots) = \lambda C(e_1, e_2, \ldots).$$

3.3   This is not generally true. It is true for some types of risks but the basis of insurance is LLN, where the very idea is that the homogeneity property is NOT satisfied (e.g. if the amounts at risk in fire insurance increase due to increase in the number of policies). The expression above should be modified accordingly. Consequently, all subsequent considerations (except as approximations) hold only for cases, where homogeneity property is satisfied.

3.4   This is easily justified by considering a change of currency. From this, it follows that

$$\sum_j \frac{\partial C}{\partial e_j} \cdot e_j = C.$$

H.4   Linear Approximation

4.1   One can then write a simple series expansion for the capital function C. For a specific company in terms of the capital function for a base-line representative company with exposures $e_1^0, e_2^0, \ldots$. The mix of risks of the base-line representative company will be referred to as the target point or target mix.

4.2   In practice most capital functions will be highly non-linear functions of the exposure variables and will likely exist as complex computer models rather than closed form analytic expressions. Since the capital function may be difficult and expensive to compute, it makes sense to have analytic expressions which approximate the capital in a neighbourhood of a target point $e_1^0, e_2^0, \ldots$. A simple Taylor expansion about this point yields

$$C(e_1, e_2, \ldots, e_n) = C(e_1^0, e_2^0, \ldots) + \sum_j \frac{\partial C}{\partial e_j^0} \cdot (e_j - e_j^0) + \sec ond \ \& \ higher \ order \ terms,$$

$$= [C(e_1^0, e_2^0, \ldots) - \sum_j \frac{\partial C}{\partial e_j^0} \cdot e_j^0] + \sum_j \frac{\partial C}{\partial e_j^0} \cdot e_j + \sec ond \ \& \ higher \ order \ terms.$$

4.3   However, the homogeneity of the capital results in

$$C(e_1, e_2, \ldots) = \sum_j \frac{\partial C}{\partial e_j^0} \cdot e_j + \sec cond \ \& \ higher \ order \ terms.$$

4.4   This means that if the actual mix of risks is close enough to the representative mix, the capital requirement is approximated by a factor-based formula where the factors are derived from the derivatives of the capital function at the target risk mix. Note that the factors depend on the mix of risks but not the scale of the risks at the target point.

H.5   Quadratic Approximation

5.1   If the linear approximation described above is not good enough it is possible to develop a convenient quadratic approximation to C^2. If we define the matrix $r_{i,j}$ at the target risk mix by

$$r_{i,j} = \frac{1}{2}\frac{\partial^2 C^2}{\partial e_i^o \partial e_j^o}$$

5.2   then a Taylor expansion of the function of C^2 shows that

$$C^2(e_1, e_2, ..., e_n) = \sum_{i,j} r_{i,j} e_i e_j + third\ \&\ higher\ order\ terms.$$

5.3   This result clearly shows that, the approximation $C = \sqrt{\sum_{i,j} r_{i,j} e_i e_j}$ should be valid in a neighbourhood of the target risk mix. This is in the same spirit as the base-line capital formula suggested above.

5.4   A paper[21] shows that for a standard deviation or TVaR risk measure on multivariate Normal risks the quadratic approximation is exact. In this case the $r_{i,j}$ terms are the linear correlation coefficients.

H.6   Higher Order Approximation

6.1   The approximation process generalizes to arbitrarty m in ths sense that if we look at a Taylor expansion of C^m we find that the first m terms of the expansion collpase down to an expression of the form

$$C^m(e_1, e_2, ..., e_n) = \frac{1}{m!}\sum_{i_1 i_2 ... i_m} \frac{\partial^m C^m}{\partial e_{i_1}^0 \partial e_{i_2}^0 ... \partial e_{i_m}^0} e_{i_1} e_{i_2} ... e_{i_m} + terms\ of\ order\ m+1\ \&\ higher.$$

H.7   Factor-Based Assessment

7.1   The methods described in the last section show how the base-line approximation can be improved. The base-line approximation is based on multivariate Normal distribution using standard deviation or TVaR as a risk measure. The results of quadratic approximation above allow for any risk measure and any distribution. One needs to obtain the second order derivatives of the square of the capital requirement C for the representative company yielding the generalizations of the linear correlations. Once this is done for the industry, the calculations for each company are analagous to those under the base-line approximation.

7.2   Clearly, higher order approximation is also possible. However, at this point it is not known how much gain there will be in going beyond quadratic approximation.

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[21] H. Panjer, "Measurement of risk, solvency requirements and allocation of capital within financial conglomerates" Institute of Insurance and Pension Research, University of Waterloo, 2002.

# Appendix I   Copulas

I.1   Introduction

1.1   Suppose that the overall risk X of the company can be described as, $X = \sum_{j=1}^{n} X_j$ (i.e., X can be decomposed into risk components X_j).

1.2   In the sequel, we assume that we can have adequate information and can describe the risks (i.e., we have models for the individual risks or risk components X_j). We now need to address the issue of combining these risks in order to obtain an appropriate model for X.

1.3   The model for X is completely specified if we assume a multivariate Normal setting in which each component has a univariate normal distribution and all dependencies are expressed through correlations. However, insurance claim data immediately show shortcomings of this assumption as,

- loss distributions are usually skewed and heavy tailed (i.e., the downside risk due to large losses is substantial,
- dependency between risks usually increases in the tails (i.e., various lines of business may look almost independent in "normal" situations, but they are strongly correlated in the tails – as occurred with September 11, 2001).

1.4   Notice that in a multivariate Normal setting, the X_j's are asymptotically independent if the linear correlations are less than one[22]. Therefore, it is advisable to model dependencies in the above setting in a different way. To this end, copulas provide one feasible framework.[23]

1.5   The following paragraphs briefly provide a mathematical overview, which is also given in more detail in Appendix H. More importantly, we describe in this Appendix more intuitively how copulas work and why they are an alternative approach to describing dependencies.

1.6   An n-dimensional copula is an n-dimensional distribution function with uniform marginal distributions. The dependence structure between $X_1,\ldots,X_n$ is described by C if the distribution function F of $X_1,\ldots,X_n$ is given by

$$F(x_1,\ldots x_n) = C(F_1(x_1),\ldots,F_n(x_n))$$

where F_j denotes the marginal distribution function of X_j. In other words, the joint distribution of the quantiles of $X_1,\ldots,X_n$ is given by the function C.

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[22] See page 19 of P. Embrechts, F. Lindskog, A. McNeil, *Modelling Dependence with Copulas and Applications to Risk Management*, Sept. 2001, www.risklab.ch/Papers.html#MTLindskog .

[23] For a comprehensive introduction and discussions of copulas, we refer to the papers Embrechts *et al (op cit)* and P. Embrechts, A. McNeil, D. Straumann, *Correlation and Dependence in Risk Management: Properties and Pitfalls*, RiskLab Research papers, Dept. Math. ETH Zürich, Aug. 1999, www.risklab.ch/Papers.html#Pitfalls which also serve as the main references for this section.

1.7   To illustrate the concept of copulas, four graphs have been prepared. Graph I.1 shows an
      example of two random variables X_1, X_2 which each have a marginal uniform distribution. The
      simulated joint samples are scattered across the plot showing no pattern and thus the outcome of
      the one variable seems to have no connection to the outcome of the other variable. In this case,
      the two variables are mutually independent.

Graph I.1: Scatterplot of Two Independent Variables
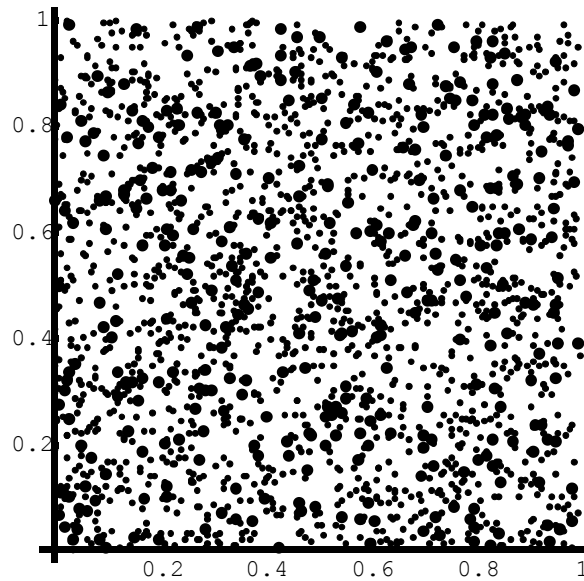
1.8   The other extreme of the joint outcome of two uniform variables would be that the outcome of
      X_1 predetermines the outcome of X_2. For example in Graph I.2, $X_1 = X_2$. In this case, the
      two random variables exhibit complete dependency.

**Graph I.2: Scatterplot of Two Perfectly Correlated Variables**

172
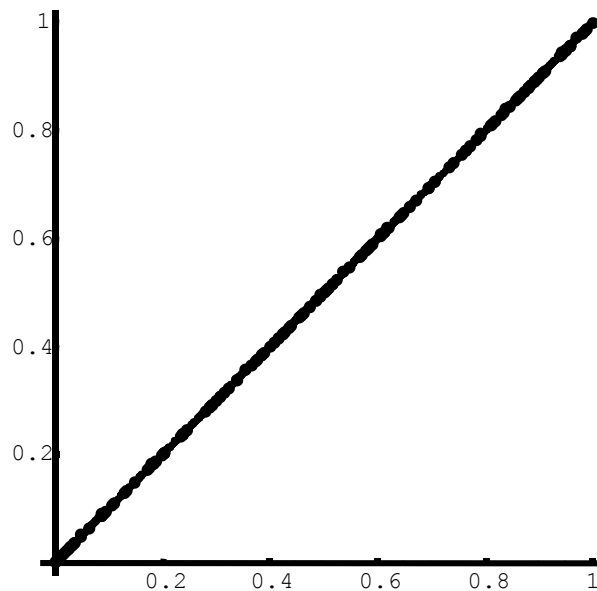
1.9   Finally, the most interesting case and more typical situation is when there is some dependency between the variables. The outcome of the two variables may appear at first glance to be uncorrelated. This is illustrated in Graph I.3. It would appear that the outcomes are approximately uniformly distributed over the square. However, on close examination of the more extreme cases where both variables are close to 1 or both are close to 0, the outcomes appear to be more dense (i.e., more clustered). This suggests that if X_1 is close to 1, it implies that X_2 is also more likely to be close to close to 1 as well.

Graph I.3: Scatterplot of Two Variables That Exhibit Correlation in Both Tails
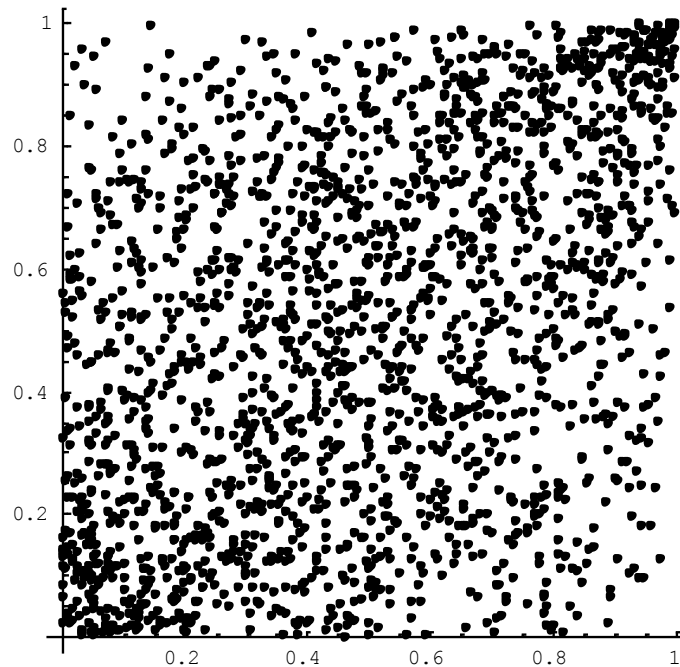
1.10   Graph I.4 shows the application of a copula to two risks X_1, X_2 that are more representative of real data than Graph I.3. In this graph the axes are now in terms of real monetary values.

Graph I.4: Scatterplot of Outcomes of Two Lines of Insurance.
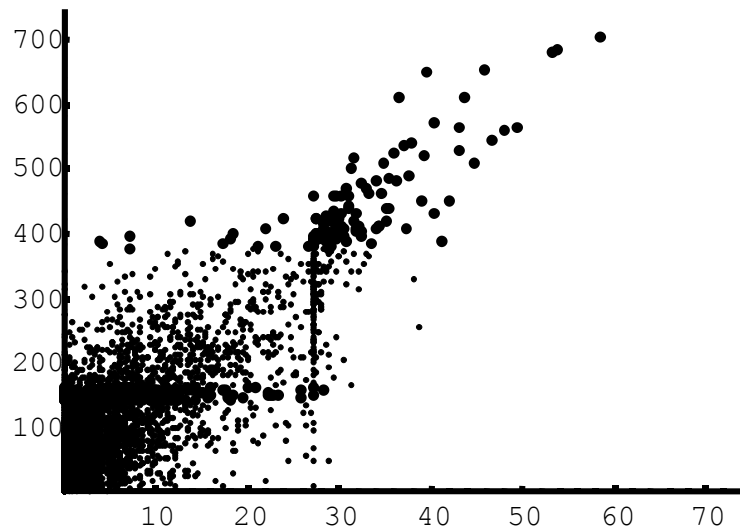
1.11   It can be seen from Graph I.4 that the nature of dependency between the two risks is different for smaller outcomes (as depicted in the lower left region) from that for large outcomes (as depicted in the upper right). In fact, the interdependency of the two risks when one of the outcomes is small is relatively low. However, when the outcome of one of the variables becomes larger, the other is more likely to also be larger, indicating an increasing co-movement. This example therefore shows clearly that the two risks have a dependency in the right-hand tail.

1.12   More technical background on copulas is given in subsequent sections of this appendix and in the references already cited.

I.2   Theoretical Background

2.1   In order to capture stochastic dependencies between insurance risks, the traditional concept of linear correlation is insufficient. In this technical appendix we introduce some of the mathematical framework of copulas which can be used to model dependencies on a deeper level. In this way, one can for instance take into account that many insurance risks seem to be almost independent in "normal" situations but heavily dependent in the extreme.

2.2   A copula is a function that associates the quantiles of one random variable to the quantiles of another random variable.

2.3　**Definition:** A n-dimensional copula is a distribution function $C:[0,1]^n \to [0,1]$ with uniform marginal distributions. The dependence structure between $X_1, \ldots, X_n$ is described by C if the distribution function F of $X_1, \ldots, X_n$ is given by

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n))$$

where F_j denotes the marginal distribution function of X_j. In other words, the joint distribution of the quantiles of $X_1, \ldots, X_n$ is given by C.

2.4　Suppose now that the dependence between $X_1, \ldots, X_n$ can be described by a copula C and that each X_j can be adequately represented by a model (i.e., we know the marginal distribution functions F_j). Furthermore, we assume that we have an algorithm to simulate independent random vectors $(u_1^k, \ldots, u_n^k)$, $k = 1, 2, \ldots$ from C. Then $F_1^{-1}(u_1^k) + \cdots + F_n^{-1}(u_n^k)$ are independent random samples of X and in this way we have obtained a model for X.

2.5　**Definition:** The *upper* and *lower tail dependence* between two random variables is respectively

$$\lambda_u(X_1, X_2) = \limsup_{u \to 1} P(X_1 > F_1^{-1}(u) \mid X_2 > F_2^{-1}(u))$$
and
$$\lambda_L(X_1, X_2) = \limsup_{u \to 0} P(X_1 \leq F_1^{-1}(u) \mid X_2 \leq F_2^{-1}(u)),$$

2.6　The tail dependencies can be determined *directly* from the copula for X_1 and X_2.

2.7　Since copulas describe the dependence between variables on the level of quantiles, the following property holds:

2.8　**Property:** Suppose that C is a copula for $X_1, \ldots, X_n$. If $\varphi_1, \ldots, \varphi_n$ are non-decreasing functions, then C is also a copula for $\varphi_1(X_1), \ldots, \varphi_n(X_n)$.

2.9　This property has the following practical applications:

- Insurances, government agencies, brokers etc have access to sensitive claims data which they do not want to or may not be allowed to make public use of. However, after transforming the data by an increasing function, the data is not back traceable, i.e. has lost substantial sensitivity, but it contains still the same information for estimating copulas. Copulas are thus a potential tool to make otherwise sensitive data available to public use with out violating confidentiality.

- A reinsurance structure in a certain line of business typically is a non-decreasing function of the underlying losses. Hence, the copula for the gross losses can reliably be assumed to be the same for the net losses.

I.3   Some Parameterised Families of Copulas

Gauss Copulas

3.1   Let Φ denote the distribution function of the standard normal distribution and Φ_R^n the n-variate standard normal distribution function with correlation matrix R. The n-dimensional Gaussian copula with correlation matrix R is given by

$$C_R^{\text{Gauss}}(u_1,\ldots,u_n) = \Phi_R^n(\Phi^{-1}(u_1),\ldots,\Phi^{-1}(u_n)).$$

3.2   If $X_1,\ldots,X_n$ are multivariate normally distributed with correlation matrix R, then their copula is C_R^{Gauss}.

3.3   It is important to note that Gauss copulas are not suitable to model the tail of X. Indeed, if the correlation $R_{ij} \neq 1$, then the tail dependencies between X_i and X_j are zero[24].

T-Copula

3.4   In order to overcome this shortcoming of Gaussian copulas, t-copulas could be used. In the same way as the Gaussian copulas, they are parameterized by a "correlation matrix" but there is one additional parameter to control the tail dependencies. The limiting case $\nu = \infty$ is the corresponding Gaussian copula.

3.5   Suppose $Y_1,\ldots,Y_n$ are multivariate normally distributed with correlation matrix R and S is a random variable with χ_ν^2-distribution. Let $t_{\nu,R}^n$ denote the distribution function of $\sqrt{\nu/S}\cdot(Y_1,\ldots,Y_n)$ and t_ν the distribution function of $\sqrt{\nu/S}\cdot Y_1$, i.e., the equal margins of $t_{\nu,R}^n$. Then the *t-copula* with parameters ν,R is given by

$$C_{\nu,R}^{\text{t}}(u_1,\ldots,u_n) = t_{\nu,R}^n(t_\nu^{-1}(u_1),\ldots,t_\nu^{-1}(u_n))$$

3.6   The tail dependencies for the copula $C_{\nu,R}^t$ are

$$\lambda_U(X_i,X_j) = \lambda_L(X_i,X_j) = 2 - 2t_{\nu+1}\left(\sqrt{\nu+1}\sqrt{1-R_{ij}}\,/\,\sqrt{1+R_{ij}}\right).$$

3.7   In order to aggregate models for $X_1,\ldots,X_n$ with a t-copula, we need an algorithm to generate independent samples $(u_1,\ldots,u_n)$ of $t_{\nu,R}^n$. A feasible algorithm is:

- Find the Cholesky[25] decomposition A of R.
- Simulate n independent random numbers $z_1,\ldots,z_n$ from the standard normal distribution
- Simulate a random number s from χ_ν^2 independent of $z_1,\ldots,z_n$
- Set $\mathbf{x} = \sqrt{\nu/s}\cdot A\mathbf{z}$
- Set $u_j = t_\nu(x_j)$, $j = 1,\ldots n$

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[24] See page 19 of P. Embrechts, F. Lindskog, A. McNeil, *Modelling Dependence with Copulas and Applications to Risk Management*, Sept. 2001, www.risklab.ch/Papers.html#MTLindskog .

[25] See W. Press, S. Teukolsky, W. Vetterling, B. Flannery, *Numerical Recipes in C,* 2nd Edition, Cambridge University Press, 1992 for an algorithm for Cholesky decomposition.

3.8   This algorithm to generate random samples of the t-copula is fast.

        Comonotonic Copula:

3.9   The comonotonic copula ensures that risks always move in the same direction. This is a kind of "worst case" for insurers. As such, the results provide an upper bound on the capital requirement since quantiles (VaR) and TailVaR risk measures are additive. In the special case of the multivariate Normal distribution, the results correspond to assuming a correlation of 1 between risks.

3.10   In general, any dependency at a single point in the multivariate distribution can be described as a linear combination of the comonotonic copula and the independent copula (obtained by multiplying marginal distributions together).

Glossary

Coefficient of variation	The ratio of the standard deviation to the mean of a distribution.
Coherent	A risk measure satisfying the following four axioms is called coherent (note that other risk measures not satisfying one or more of these axioms may have useful properties as well). ▪ Subadditivity - Capital for two risks is not larger than sum of capital for each risk separately. ▪ Positive homogeneity - Capital is invariant under scale transformations (doubling the risk doubles the capital). ▪ Translation invariance - Capital is invariant under location transformations (adding a certain risk increases the capital with this certain amount). ▪ Monotonicity - Capital is larger for larger risks.
Comonotonic	Two random variables, X and Y, are said to be comonotonic if there exists another variable, Z, and increasing real-valued functions, u and v, such that $X = u(Z)$, $Y = v(Z)$. When the outcomes of insurers A and B are comonotonic; that is, they always move up or down together, then it is believed that the required capital for the combined company should equal the sum of the required capitals for the two individual companies.
Copula	A copula is a function that associates the distribution function of one random variable to the distribution function of another random variable. Using copulas to model dependencies on a deeper level, one can for instance take into account that many insurance risks seem to be almost independent in "normal" situations but heavily dependent in the extreme.
Credit risk	Credit risk is the risk of default and change in the credit quality of issuers of securities, counter-parties and intermediaries, to whom the company has an exposure.
Diversifiable risk	A risk is diversifiable when the volatility of the average claim amount declines as the block of combined insurer risks increases.
Economic capital	Economic capital is what the firm judges it requires for ongoing operations and, for an insurance company, what it must hold in order to gain the necessary confidence of the marketplace, its policyholders, its investors and its supervisors.
Liquidity risk	Liquidity risk is exposure to loss in the event that insufficient liquid assets will be available, from among the assets supporting the policy obligations, to meet the cash flow requirements of the policyholder obligations when they are due or assets may be available, but only at excessive cost.
Market risk	Market risk arises from the level or volatility of market prices of assets. Market risk involves the following: - exposure to movements in the level of financial variables - exposure of options to movements in the underlying asset price - exposure to other unanticipated movements in financial variables - exposure to movements in the actual or implied volatility of asset prices and options
Non-diversifiable risk	A risk is non-diversifiable when it cannot be (relatively) reduced by increasing portfolio size.

Operational risk	Operational risk is defined as the risk of loss resulting from inadequate or failed internal processes, people, systems or from external events.
Quantile	A α-quantile of a random variable X is any value x such that $\Pr(X \leq x) = α$. For example, the 95th percentile of the distribution is the value for which there is a probability of exceedence of 5%. Value-at-Risk (VaR) is a quantile of the distribution.
Risk	Risk is the chance of something happening that will have an impact upon objectives. It is measured in terms of consequences and likelihood.
Systematic risk	Also called non-diversifiable risk
TVaR	Tail-Value-at-Risk (TVaR or TailVaR) is the quantile VaR plus the average exceedence of that quantile if such exceedence occurs. Alternatively, TVaR at level p is the arithmetic average of all VaR's from level p on. It is sometimes also called Conditional Tail Expectation (CTE) or Expected Shortfall.
Time horizon	Time horizon is a period over which a risk is measured. Assuming a certain fixed acceptable level of insolvency risk per year, extending the time horizon should always result in a higher capital need.
Total balance sheet	Total balance sheet requirement is the sum of both the liabilities and solvency capital requirement upon realistic values. Using the total balance sheet requirement allows solvency assessment to be relatively independent of the accounting system.
Type A risk	Type A credit risk is the credit risk relating to actual assets held.
	Type A market risk is the market risk relating to the volatility of the market value of the actual assets held and the market value of the replicating portfolio of assets.
Type B risk	Type B credit risk is the credit risk involved with future reinvested assets.
	Type B market risk is the market risk involved with future reinvestment assets and long term options and/or guarantees.
Underwriting risk	Underwriting is the specific insurance risk arising from the underwriting of insurance contracts. The risks within the underwriting risk category are associated with both the perils covered by the specific line of insurance and with the specific processes associated with the conduct of the insurance business.
Volatility risk	Volatility is the risk of random fluctuations in either the frequency or severity of a contingent event.