Reserve Risk using Ultimate Triangles

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Economic Capital Modeling

Which LOB’s reserve is more volatile, A or B

LOB A - Inc	12	24	36	48
2017	400	600	800	850
2018	300	500	700	
2019	450	700		
2020	500			

LOB B - Inc	12	24	36	48
2017	400	400	850	850
2018	200	1000	700	
2019	100	700		
2020	500			

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Does your answer Change?

LOB A - Inc	12	24	36	48
2017	400	600	800	850
2018	300	500	700	
2019	450	700		
2020	500			

LOB A - Ult	12	24	36	48
2017	400	600	800	850
2018	300	500	700	
2019	450	700		
2020	500			

LOB B - Inc	12	24	36	48
2017	400	400	850	850
2018	200	1000	700	
2019	100	700		
2020	500			

LOB B - Ult	12	24	36	48
2017	850	850	850	850
2018	750	750	750	
2019	950	950		
2020	1000			

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Agenda

- Reserve Risk – Intro
- Reserve Risk using ultimate triangles
- Advantages and Shortfalls
- Comparison of methods
- Quantitative Demonstration

Reserve Risk - Definitions

- Solvency II: Reserve Risk = Risk that the current reserves are insufficient to cover their run-off over a 12 month time horizon
- NAIC RBC: Reserve Risk = Risk that the company's recorded loss and loss adjustment expense reserves will develop adversely
- Mack: MSE = E[(Estimated Ult – True ult)^2 | Info as of now]

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Reserve Risk – Common Approaches

- Factor based (AM Best, PRA, BMA)
- Closed-form formula (Mack, Merz-Wuthrich, today's approach)
- Boot-strapping (Shapland)
- Monte-Carlo simulation

One year vs ultimate view

	12	24	36	48	60	Ult
2017	400	600	800	850		
2018	300	500	700			
2019	450	700				
2020	500					

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Rehman & Klugman (2010)

- Defines Ultimate Development Factors *g*
- Assumes Log-normal distribution
- Uses sample mean and variance as estimators

- Did not produce a result for the entire triangle

Rehman & Klugman (2010)

AY\Age	12	24	36	48
2017	$U_{2017,\,12}$	$U_{2017,\,24}$	$U_{2017,\,36}$	$U_{2017,\,48}$
2018	$U_{2018,\,12}$	$U_{2018,\,24}$	$U_{2018,\,36}$	
2019	$U_{2019,\,12}$	$U_{2019,\,24}$		
2020	$U_{2020,\,12}$			

AY\Age	12	24	36
2017	$\hat{g}_{2017,\,12}$	$\hat{g}_{2017,\,24}$	$\hat{g}_{2017,\,36}$
2018	$\hat{g}_{2018,\,12}$	$\hat{g}_{2018,\,24}$	
2019	$\hat{g}_{2019,\,12}$		

$$\hat{g}_{12} = \frac{(U_{2017,\,24} + U_{2018,\,24} + U_{2019,\,24})}{(U_{2017,\,12} + U_{2018,\,12} + U_{2019,\,12})}$$

Siegenthaler – Ultimate MSEP

$$\widehat{\mathrm{msep}}_{\sum_{i=0}^{I} U^i | \mathscr{F}_I}\left(\widehat{E}\left[\sum_{i=0}^{I} U^i \middle| \mathscr{F}_I\right]\right)$$

Process Variance

Parameter Error

$$= \sum_{i=1}^{I}\left\{\left[\sum_{k=I-i}^{I-1}\left(\prod_{j=I-i}^{k-1}\widehat{g}_j\right)\cdot\widehat{\sigma}_k^2\cdot\left(\prod_{l=k+1}^{I-1}\widehat{g}_l^2\right)\right]\cdot\widehat{U}_{i,I-i}+\left(1-\prod_{j=I-i}^{I-1}\widehat{g}_j\right)^2\cdot\widehat{U}_{i,I-i}^2\right\} \quad (1.2)$$

$$+\,2\sum_{1\le i<j\le I}\left(1-\prod_{k=I-i}^{I-1}\widehat{g}_k\right)\left(1-\prod_{k=I-j}^{I-1}\widehat{g}_k\right)\cdot\widehat{U}_{i,I-i}\cdot\widehat{U}_{j,I-j}.$$

Covariance between AYs

Siegenthaler – One year MSEP

Process Variance   Parameter Error

$$\widehat{\text{msep}}_{\sum_{i=0}^{I} \widehat{\text{CDR}}_i(I+1)|\mathscr{F}_I}(0) = \sum_{i=1}^{I} \left[\widehat{\sigma}^2_{I-i} \cdot \widehat{U}_{i,I-i} + (\widehat{g}_{I-i} - 1)^2 \cdot \widehat{U}^2_{i,I-i} \right] \tag{1.1}$$

$$+ 2 \sum_{1 \le i < j \le I} (\widehat{g}_{I-i} \cdot \widehat{g}_{I-j} - \widehat{g}_{I-i} - \widehat{g}_{I-j} + 1) \cdot \widehat{U}_{i,I-i} \cdot \widehat{U}_{j,I-j}.$$

Covariance between AYs

Siegenthaler Assumptions

- Ultimate Development Factors *g* are <u>unbiased</u>

- Ultimate loss has to be set using a method that Is consistent with “Linear Stochastic Reserving Method”
  - Chain Ladder, BF, Budget Loss Ratio are all examples of Linear Stochastic Reserving method
  - Implication: Diagonals are uncorrelated

- No assumption of independence between AY or development age

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Advantages

- Not dependent on the actuarial method
- Works well for lines with sparse claims activity in early years
- Fast
- Rewards accurate IBNR estimates
  - If ultimate losses are historically stable, this method shows low volatility as it ignores settlement activity

Does your answer Change?

LOB A - Inc	12	24	36	48
2017	400	600	800	850
2018	300	500	700	
2019	450	700		
2020	500			

LOB A - Ult	12	24	36	48
2017	400	600	800	850
2018	300	500	700	
2019	450	700		
2020	500			

LOB B - Inc	12	24	36	48
2017	400	400	850	850
2018	200	1000	700	
2019	100	700		
2020	500			

LOB B - Ult	12	24	36	48
2017	850	850	850	850
2018	750	750	750	
2019	950	950		
2020	1000			

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Pitfalls

- Need to construct historical ultimate triangles
- Assumption on Unbiased Ultimate
- Does not handle negative ultimate losses very well
- Negative covariance

Negative Covariance

Ult	12	24	36	48
2017	400	1200	800	850
2018	300	1000	700	
2019	450	1100		
2020	500			

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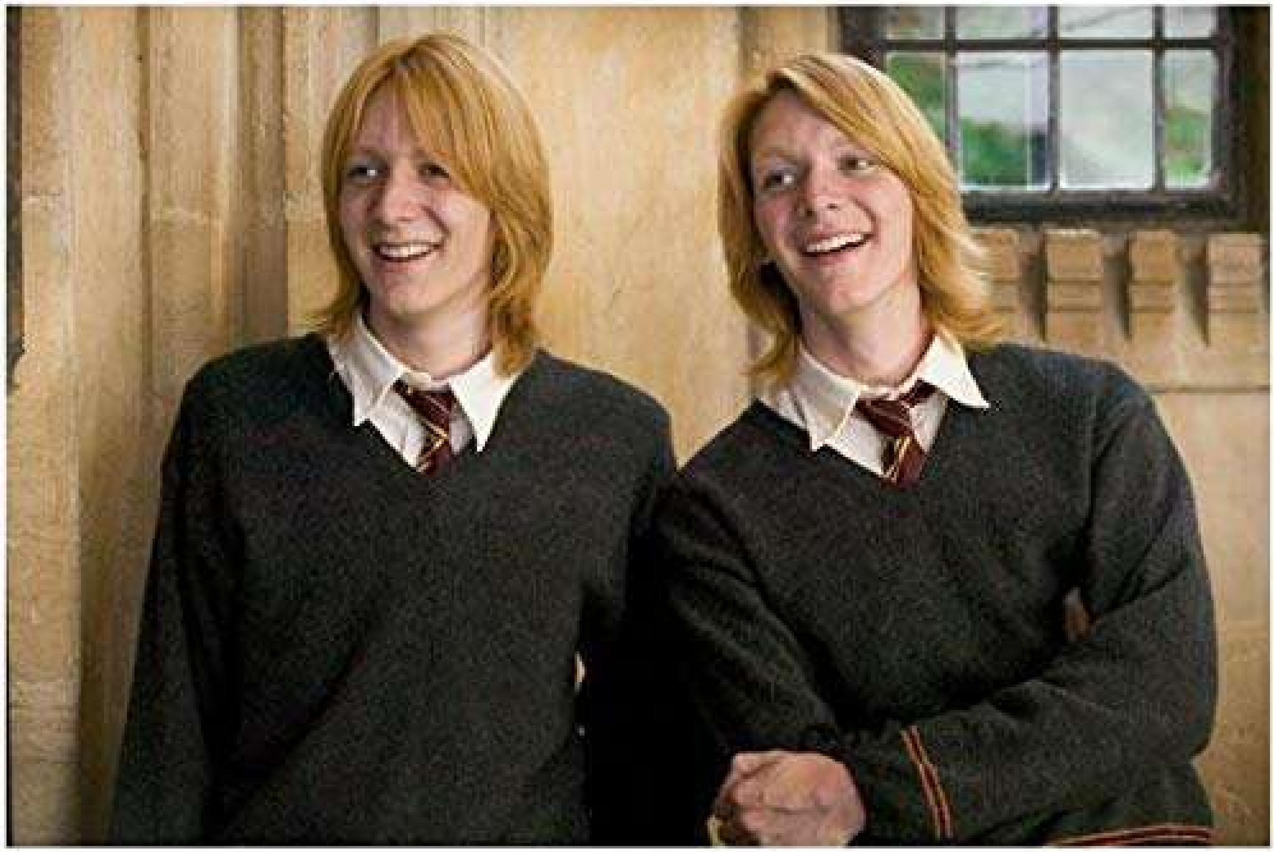
Comparing with Mack and M-W

	Siegenthaler	Mack and Merz-Wuthrich
Triangle Used	Ultimate Triangle	Paid or Incurred Triangle
Development Factor	$\widehat{g}_j = \frac{\sum_{i=0}^{I-j-1} \widehat{U}_{i,j+1}}{\sum_{i=0}^{I-j-1} \widehat{U}_{i,j}}$	$\hat{f}_j^I = \frac{\sum_{i=0}^{I-j-1} C_{i,j+1}}{\sum_{i=0}^{I-j-1} C_{i,j}}$
σ	$\widehat{\sigma}_j^2 = \frac{1}{I-j-1} \sum_{i=0}^{I-j-1} \widehat{U}_{i,j} \left(\frac{\widehat{U}_{i,j+1}}{\widehat{U}_{i,j}} - \widehat{g}_j \right)^2$	$\hat{\sigma}_j^2 = \frac{1}{I-j-1} \sum_{i=0}^{I-j-1} C_{i,j} \left(\frac{C_{i,j+1}}{C_{i,j}} - \hat{f}_j \right)^2$

Comparing with Mack and M-W

- Moreover, without parameter error and assuming paid or incurred development factors are fixed, Siegenthaler formula reduces to the famous Mack Formula

$$\mathrm{msep}^{\mathrm{Mack}}_{\sum_{i=0}^{I} U^i | \mathscr{F}_I}\left(\widehat{E}\left[\sum_{i=0}^{I} U^i \middle| \mathscr{F}_I\right]\right) = \sum_{i=1}^{I}\left(\widehat{U}_{i,I-i}\right)^2 \sum_{j=I-i}^{I-1} \frac{\widehat{s}_j^2}{\left(\widehat{f}_j^{(I)}\right)^2}\left[\frac{1}{C_{i,I-i}\cdot\prod_{l=I-i}^{j-1}\widehat{f}_l^{(I)}}\right]$$



Comparison of methods - Quantitative

- Based on data supplied on Siegenthaler (2018) paper

	One Year Standard Error	Total Run-off Standard Error
Siegenthaler	12,025	15,228
Mack/M-W	11,203	13,457

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References

- Rehman, Z and Klugman, S.A. (2010) *Quantifying Uncertainty in Reserve Estimates*.
- Siegenthaler, F. (2018) *One-year and Total Run-off Reserve Risk Estimators Based on Historical ultimate Estimates*.

Conclusions and Questions