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“Capital Tranching: A RAROC Approach to Assessing Reinsurance Cost Effectiveness”

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1. Introduction

In their short paper, the authors describe an elegant decision rule for evaluating the attractiveness of potential reinsurance transactions. In effect, they propose comparing the premium quoted by reinsurers for a particular reinsurance structure to the portion of its premiums the ceding company would need to allocate, given its cost of capital, to retain the risk. If the reinsurance premium is lower than the cedent’s indicated capital cost premium, then the reinsurance is a buy. Otherwise, the risk should be retained.

Before introducing their approach they set up a straw man in the form of what they refer to as the current “industry standard approach” or ISA, which they quickly and rightly demolish. Whether the ISA they describe is, in fact, in widespread use is debatable, but its defects for reinsurance decision-making and capital planning are serious, and the authors make a convincing case that their approach is superior.

However, tantalizing as the authors’ approach may be, the brevity of the paper, its reliance on a single example, and the lack of distinction in that example between the reinsurer’s quoted premium and the ceding company capital cost premium make it difficult to see how a ceding company would apply it in practice.

The aim of this discussion is to fill in key gaps in the paper in order to provide a clearer roadmap for the application of the proposed method.

2. Review of the paper’s example

The authors illustrate their approach using a simple example of an insurer with $500 million of gross loss exposure, which they break up into five stacked excess-of-loss layers (or tranches), each with limits of $100 million. Using certain simplifying assumptions, they determine the expected loss exposure in each layer as well as a price (i.e., premium) intended to reflect the cost of capital (which we will refer to as the “capital cost premium”) and the corresponding premium rate on line. They tabulate these results together with other statistics in their Table 3.

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The total capital cost premium across the five excess layers is $50 million. Given the total expected loss of $15 million, the implied total cost-of-capital risk load is $35 million. That risk load is spread across the five layers in proportion to the standard deviations of the losses in the five layers. Unfortunately, they do not explain how the total risk load is determined. They merely cite the use of the Kreps (1990) reinsurance premium formula that sums expected losses and their standard deviation times a "reluctance factor," which they fix at 42.48% for all layers to produce a total capital cost premium of $50 million.[1] They do not explain how a reluctance factor relates to capital or its cost.

This is where it would have been helpful to explain how an insurer seeking to apply the authors' method might determine its cost-of-capital risk load and the allocation of that risk load by layer. To remedy that shortcoming, we offer a method for doing so in Section 4 of this discussion.

Interestingly, the $35 million risk load is treated as the underwriting cost of capital of both the ceding company and reinsurers. The effect of that is to render the reinsurance decision a draw, which is odd in a paper promoting a new decision rule!

Moreover, $35 million seems far too low to be the cedent's capital cost for retaining the total risk net; it implies an implausibly low cost-of-capital rate, especially if the company is a U.S. taxpayer. The $35 million is much more plausible as an estimate of a reinsurer's cost of capital, or at least the capital cost the reinsurer will be allowed to recover in an efficient no-arbitrage market, which compensates participants only for undiversifiable risks. In sections 3 and 4 we describe and illustrate methods for estimating plausible reinsurer and ceding company capital costs and the implications for premiums.

If, as we believe, the ceding company faces a total capital cost premium of more than $50 million in the absence of reinsurance, compared to a total quoted reinsurance premium of $50 million, then according to the decision rule presented in the paper, the cedent should buy the reinsurance.[2]

In fact, the ceding company probably had to plan to buy the reinsurance from the start. If it is operating in an efficient market, it would not have been able to collect the full risk load needed to support its required capital in the absence of reinsurance. If it collects less than the full cost-of-capital risk load, then it can expect bottom-line losses if it retains the risk. Its only good alternative to not writing the business in the first place is to *plan* to access the reinsurance market.

2.1. RAROC vs. RORAC

In Section 3 the authors rightly observe that insurer capital is typically set as part of an annual planning cycle and, realistically, cannot be reduced over the short term. Therefore, a reinsurance transaction that reduces theoretically required capital would not normally result in the insurer being able to reduce actual capital. Instead, the capital would remain fixed, but being less exposed to loss, a lower return on the fixed capital would be entirely appropriate. The reinsurance purchase decision rule devised by the authors ensures that the reduction in the rate of return on capital is more than offset by the value of the risk transferred through the reinsurance. If not, the reinsurance is not purchased.

In their example, the authors cite an insurer gross capital requirement of $500 million,[3] which is presumably the fixed capital they see as emerging from the insurer's planning cycle. We don't know that for sure, because, despite the title of the paper, they don't do any RAROC calculations. If they had, we suspect they might have started out with a 7% expected return (35/500) and then showed the effect on the

[1] On page 87 the authors state that the $50 million capital cost premium corresponds to a "capital cost rate" of 10%. This should not be construed as the cost of capital. Instead, it refers to the cost of capital premium divided by the limit, a ratio that is commonly referred to as the "rate on line."

[2] See page 89. The reinsurance buying decision should, of course, be made on a layer-by-layer basis, i.e., by comparing the capital cost premium and the quoted reinsurance premium for each layer. In Section 4 we illustrate that layer-by-layer comparison.

[3] See page 87. Note that by our calculations, the capital requirement is not $500 million, but we will save that discussion for Section 4.

rate of return of buying one or more of the reinsurances layers. For example, the risk load embedded in the first layer premium is \$9.26 million. Buying that layer reduces the retained risk load to \$25.74 million and the expected return on the fixed \$500 million of capital to 5.15% (25.74/500). The 5.15% is a "risk-adjusted return" in the RAROC framework. In contrast, in this context any RORAC calculations are theoretically interesting but practically meaningless in light of the short-term impossibility of adjusting the capital level.

The RAROC framework is not without issues. Investors are not necessarily pleased when they are told by a company that the expected return on their invested capital has been reduced from what they had been previously told, even if the reduction is justified by lower risk. Investors typically like to make the decisions about the risk in their portfolio themselves. If they had wanted mezzanine or bond-like risk and returns instead of equity risk and returns, they would have chosen that in the first place.

To address this reality, if the insurer can see during its planning process that it cannot collect sufficient premiums to pay the cost of \$500 million in capital, it will factor the reinsurance purchase decision into its capital planning. In this example, that means it will plan its capital with the expectation that it will buy all five reinsurance layers, in which the resulting capital need for this risk will be zero. While the insurer might be required to hold more than zero capital, it is inconceivable that the insurer would hold \$500 million.

The fact is that capital and reinsurance decisions are intertwined, and the optimal mix depends on their respective costs. In the remainder of this discussion, we address how to go about estimating those costs. In Section 3 we discuss catastrophe reinsurance pricing in an efficient market, and show why the \$35M risk load used in the example is plausible. In Section 4 we discuss the determination of insurer required capital, its cost, and how that cost can be attributed to reinsurance (or capital) tranche, both in general and as applied to the example in the paper.

3. Reinsurance market pricing

In this section, we review the pricing dynamics of the catastrophe reinsurance market as a whole and then examine the implications for the pricing of an individual reinsurance contract. Readers who are more interested in the ceding company perspective may wish to proceed directly to Section 4.

3.1. Pricing the reinsurance market portfolio

Let $x_1, x_2, x_3, \ldots, x_n$ be random variables representing the pre-tax dollar underwriting results at the end of the year on the n treaties[4] comprising the total catastrophe reinsurance market portfolio, where these random variables have respective standard deviations $\sigma(x_1), \sigma(x_2), \sigma(x_3) \ldots \sigma(x_n)$. Let $x_M = \sum_{i=1}^{n} x_i$ represent the total market dollar underwriting result.[5] That total market underwriting result x_M has a standard deviation of $\sigma(x_M) = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\operatorname{cov}(x_i, x_j)}$, where $\operatorname{cov}(x_i, x_j)$ refers to the covariance between treaties i and j. The covariance between treaty i and the total market is $\operatorname{cov}(x_i, x_M) = \sum_{j=1}^{n}\operatorname{cov}(x_i, x_j)$.

The expected total market underwriting result $E(x_M)$ can be expressed in terms of the market total pre-tax cost of equity capital roe_{PT}, the amount of equity capital C_M and the one-year risk-free rate r as follows:

$$E(x_M) = (roe_{PT} - r) \cdot C_M. \tag{3.1}$$

$E(x_M)$ can also be expressed in terms of the standard deviation of the total market result $\sigma(x_M)$ and the implied market reluctance factor MSD_M:

$$E(x_M) = MSD_M \cdot \sigma(x_M). \tag{3.2}$$

[4]Reinsurance contracts covering portfolios of insurance policies (or reinsurance contracts) are called "treaties." In a layered excess of loss reinsurance program, each layer is typically treated as a separate treaty. For purposes of this discussion the terms "treaty" and "layer" should be considered interchangeable.

[5]All underwriting results discussed here should be understood to be pre-tax even if not explicitly stated.

Our aim is to find an expression for MSD_M that is consistent with roe_{PT}, C_M and r.

Under the simplifying assumption that all premiums are collected at the beginning of the period and all claims are paid at the end of the period, the required aggregate market capital C_M is equal to the present value of total market losses at the α confidence level (a value-at-risk measure) net of total market premiums less expenses:[6]

$$C_M = v \cdot VaR_\alpha(L_M) - P_M, \tag{3.3}$$

where $v = \dfrac{1}{1+r}$ is the one-year risk-free discount factor, $VaR_\alpha(L_M)$ represents the α-percentile losses, and P_M represents premiums net of expenses. Assuming that P_M comprises the sum of the present values of expected losses and a cost-of-capital risk load, by expanding the premium term in Formula (3.3) we obtain:

$$C_M = v \cdot [VaR_\alpha(L_M) - E(L_M) - E(x_M)], \tag{3.4}$$

where $E(L_M)$ represents expected total market losses.

If we substitute the Formula (3.1) expression for $E(x_M)$ into Formula (3.4), after a bit of algebra we obtain Formula (3.5) for required capital:

$$\begin{aligned} C_M &= v \cdot [VaR_\alpha(L_M) - E(L_M) - (roe_{PT} - r) \cdot C_M], \\ &= \frac{VaR_\alpha(L_M) - E(L_M)}{1 + roe_{PT}}. \end{aligned} \tag{3.5}$$

$VaR_\alpha(L_M)$ can be expressed as $VaR_\alpha(L_M) = E(L_M) + NSD_\alpha \cdot \sigma(x_M)$, where NSD_α is the number of standard deviations that $VaR_\alpha(L_M)$ is away from $E(L_M)$. Then required capital can be expressed as

$$C_M = \frac{NSD_\alpha \cdot \sigma(x_M)}{1 + roe_{PT}}, \tag{3.6}$$

and $E(x_M)$ can be expressed as

$$E(x_M) = (roe_{PT} - r) \cdot \frac{NSD_\alpha \cdot \sigma(x_M)}{1 + roe_{PT}}. \tag{3.7}$$

[6]The choice of *VaR* as the basis of the equity capital determination is not critical to this discussion. Another basis such as *TVaR* could have been used without changing the general conclusions.

Equating the Formula (3.2) and Formula (3.7) expressions for $E(x_M)$ and dividing both sides by $E(x_M)$, we obtain the reluctance factor MSD_M for the total catastrophe reinsurance market portfolio:

$$MSD_M = NSD_\alpha \cdot \frac{roe_{PT} - r}{1 + roe_{PT}}. \tag{3.8}$$

To illustrate the application of Formula (3.8), let's assume $\alpha = 99.6\%$ (corresponding to a 250-year return time), $NSD_{99.6\%} = 5$ (corresponding to the difference between the 99.6th percentile and mean of our estimate of the global market annual aggregate catastrophe loss distribution, expressed as a ratio to the standard deviation of that distribution), $r = 3\%$, and $roe_{PT} = 20\%$, which assumes a target after-tax cost of capital of 15% and a "tax" rate of 25%.[7] Under those conditions the total market MSD_M is about 71%:

$$MSD_M = 5 \cdot \frac{0.20 - 0.03}{1 + 0.20} = 0.7083.$$

3.2. Pricing a reinsurance treaty

The expected underwriting result $E(x_i)$ on catastrophe reinsurance treaty i is given by

$$E(x_i) = MSD_i \cdot \sigma(x_i), \tag{3.9}$$

where MSD_i is the reluctance factor applicable to treaty i.

$E(x_i)$ can also be expressed in terms of $E(x_M)$ and ultimately in terms of the total market reluctance factor MSD_M:

$$\begin{aligned} E(x_i) &= \beta^a_{i,M} \cdot E(x_M), \\ &= \frac{\mathrm{cov}(x_i, x_M)}{\sigma^2(x_M)} \cdot E(x_M), \\ &= \rho(x_i, x_M) \cdot \sigma(x_i) \cdot MSD_M, \end{aligned} \tag{3.10}$$

[7]While many reinsurers operate in countries with no or low income tax rates, those low tax rates are offset to a large extent by higher expenses, including high labor costs, performance fees, and excise taxes. We have selected a blended tax and incremental expense rate of 25% for the global reinsurance market.

where $\beta^a_{i,M}$ is the "allocation beta" that distributes a portion of the total market expected result to treaty i based on its covariance with the total market, and $\rho(x_i, x_M)$ is the correlation coefficient between treaty i and the total market portfolio.[8]

Equating the right sides of Formulas (3.9) and (3.10), we obtain the following formula for the market-implied reluctance factor MSD_i for treaty i:

$$MSD_i \cdot \sigma(x_i) = \rho(x_i, x_M) \cdot \sigma(x_i) \cdot MSD_M,$$

$$MSD_i = \rho(x_i, x_M) \cdot MSD_M. \tag{3.11}$$

If the required total market $MSD_M = 0.7083$, as under the conditions described in Section 3.1, the required reluctance factor for treaty i is $MSD_i = \rho(x_i, x_M) \cdot (0.7083)$. Because $0 \le \rho(x_i, x_M) \le 1$ in realistic scenarios, if the market is in equilibrium, the required treaty MSD_i in the circumstances assumed in the illustration will always fall between 0 and 0.7083.

If the actual MSD_i observed in market transactions does not equal $\rho(x_i, x_M) \cdot MSD_M$, it may be that $\sigma(x_i)$ or the true cost of capital has been wrongly estimated, or there is disruption in the market leading to temporary disequilibrium.

3.3. Pricing the paper's example

The reluctance factor MSD_i of 42.48% used in the paper's example to price each of the five reinsurance layers implies that the correlation coefficient of each layer's underwriting result with total market result is an identical 0.60. Correlation coefficients in that range seem quite plausible, although that they would be identical across all five layers seems unlikely. We assume the authors selected a common reluctance factor for all layers merely to simplify their exposition.

[8]Venter (1991) discusses the benefits of allocating risk loads in proportion to covariance. One of the benefits is that the sum of individual risk loads across a portfolio always equals the risk load on the portfolio, irrespective of whether the individual components are independent or correlated. CAPM also assumes a covariance-based relationship between individual and market returns.

4. Insurer capital cost pricing

In this section we describe a way to determine the overall capital implications of the company's gross catastrophe exposure and its cost. Then we show the implications for capital costs and premiums by excess of loss layer, and compare the results to the quoted reinsurance premiums in the authors' example.

4.1. Pricing the insurer's total catastrophe exposure

Let $y_1, y_2, y_3, \ldots, y_m$ be random variables representing the pre-tax dollar results at the end of the year on the ceding company's total catastrophe underwriting portfolio subdivided into m excess of loss layers, where these random variables have respective standard deviations $\sigma(y_1), \sigma(y_2), \sigma(y_3) \ldots \sigma(y_m)$. Let $y_T = \sum_{i=1}^{m} y_i$ and $\sigma(y_T)$ represent, respectively, the company's total dollar catastrophe underwriting result and its standard deviation.

The total company expected underwriting result with respect to its catastrophe exposure can be expressed as:

$$E(y_T) = CSD_T \cdot \sigma(y_T), \tag{4.1}$$

where CSD_T is the reluctance factor for the total company catastrophe exposure implied by cost-of-capital pricing.

The required underwriting risk load in company premiums can be determined from the cost of the capital required to support the underwriting risk in the company's portfolio. The company's required equity capital C_T in the absence of reinsurance is equal to the present value of its losses at the α confidence level net of premiums less expenses:[9]

$$C_T = v \cdot VaR_\alpha(L_T) - P_T, \tag{4.2}$$

[9]For purposes of this discussion, we assume, like the authors, that the only source of enterprise risk is the underwriting risk under discussion. As in the catastrophe reinsurance market discussion, we assume that *VaR* is the basis of equity capital determination, but another basis such as *TVaR* could have been used without changing the general conclusions.

where $VaR_\alpha(L_T)$ represents the company's α-percentile losses and C_T represents its indicated capital cost premium net of expenses. The steps to derive the final formulas for the company's gross required capital C and target underwriting result $E(y_T)$ are similar to those described in section 3.1, and result in the following:

$$C_T = \frac{NSD_\alpha \cdot \sigma(y_T)}{1 + roe_{PT}}, \quad (4.3)$$

$$E(y_T) = (roe_{PT} - r) \cdot \frac{NSD_\alpha \cdot \sigma(y_T)}{1 + roe_{PT}}, \quad (4.4)$$

where $NSD_\alpha = \frac{VaR_\alpha - E(L_T)}{\sigma(y_T)}$ is determined from the company loss distribution.

If the company is pricing the catastrophe risk in its policies to cover its cost of capital,[10] the values of $E(y_T)$ given by Formulas (4.1) and (4.4) should be equal, which implies that

$$CSD_T \cdot \sigma(y_T) = (roe_{PT} - r) \cdot \frac{NSD_\alpha \cdot \sigma(y_T)}{1 + roe_{PT}},$$

$$CSD_T = NSD_\alpha \cdot \frac{roe_{PT} - r}{1 + roe_{PT}}. \quad (4.5)$$

Formula (4.5) yields the implied reluctance factor for the company's total catastrophe risk. To illustrate its application in the case of paper's example, let's again assume $\alpha = 99.6\%$ and $r = 3\%$. For the company, however, let's assume a higher pre-tax cost of capital $roe_{PT} = \frac{15\%}{1 - 35\%} = 23.08\%$ due to a higher tax rate.[11] The value of $NSD_{99.6\%}$ is also higher, at 6.678, corresponding to the difference between the 99.6th percentile ($500 million) and mean ($15 million) of the company's loss distribution, expressed as a ratio to the standard deviation of that distribution ($72.63 million). Under those conditions, the company's CSD_T is about 109%:

$$CSD_T = 6.678 \cdot \frac{0.2308 - 0.03}{1 + 0.2308} = 1.0895.$$

4.2. Pricing the company's exposure by layer

The expected underwriting result $E(y_i)$ of the company's exposure in layer i is given by

$$E(y_i) = CSD_i \cdot \sigma(y_i). \quad (4.6)$$

$E(y_i)$ can also be expressed in terms of $E(y_T)$ as

$$\begin{aligned} E(y_i) &= \beta^a_{i,T} \cdot E(y_T). \\ &= \frac{\operatorname{cov}(y_i, y_T)}{\sigma^2(y_T)} \cdot E(y_T), \\ &= \rho(y_i, y_T) \cdot \sigma(y_i) \cdot CSD_T, \end{aligned} \quad (4.7)$$

where, as described in Section 3.2, $\beta^a_{i,T}$ is the "allocation beta" that distributes a portion of the company's total expected underwriting result to layer i based on its covariance with the total company exposure, and $\rho(y_i, y_T)$ is the correlation coefficient between layer i and the total company portfolio. One of the advantages of using a covariance measure here is that the layer results sum to the total without the need for scaling to force a match.

Equating the right sides of Formulas (4.6) and (4.7), we obtain the following formula for the market-implied reluctance factor CSD_i for layer i:

$$CSD_i \cdot \sigma(y_i) = \rho(y_i, y_T) \cdot \sigma(y_i) \cdot CSD_T,$$

$$CSD_i = \rho(y_i, y_T) \cdot CSD_T. \quad (4.8)$$

If the required total company reluctance factor $CSD_T = 1.0895$, as under the conditions described in Section 4.1, the required CSD for layer i is $CSD_i = \rho(y_i, y_T) \cdot (1.0895)$. The correlation coefficients for layers 1 through 5 in the paper's example are given below, together with the implied values of the CSD for each layer:

[10] Note that the company may not be able to obtain premiums in the market that are high enough to cover its cost of capital. That inability does not change the calculation of the capital cost premium.

[11] We assume the ceding company is a U.S. taxpayer facing an income tax rate of 35%.

$$\rho(y_1, y_T) = 0.9002 \quad \Rightarrow \quad CSD_1 = (0.9002) \cdot (1.0895) = 0.9808.$$

$$\rho(y_2, y_T) = 0.9415 \quad \Rightarrow \quad CSD_2 = (0.9415) \cdot (1.0895) = 1.0258.$$

$$\rho(y_3, y_T) = 0.9322 \quad \Rightarrow \quad CSD_3 = (0.9322) \cdot (1.0895) = 1.0156.$$

$$\rho(y_4, y_T) = 0.8556 \quad \Rightarrow \quad CSD_4 = (0.8556) \cdot (1.0895) = 0.9322.$$

$$\rho(y_5, y_T) = 0.6711 \quad \Rightarrow \quad CSD_5 = (0.6711) \cdot (1.0895) = 0.7312.$$

4.3. Pricing the paper's example (all amounts in millions)

Company in total

The total dollar underwriting result $E(y_T)$ that is consistent with the company's cost of capital is given by Formula (4.1) as

$$E(y_T) = CSD_T \cdot \sigma(y_T) = (1.0895) \cdot (\$72.63) = \$79.13.$$

The company's total capital cost premium net of expenses $P_T = \frac{\$15 + \$79.13}{1.03} = \$91.39$. The value of that premium at the end of the year is $\$15 + \$79.13 = \$94.13$, which is the amount available to pay claims.

According to Formula (4.3), the company's required capital in the absence of reinsurance is given by

$$C_T = \frac{NSD_\alpha \cdot \sigma(y_T)}{1 + roe_{PT}} = \frac{(6.678) \cdot (72.63)}{1.2308} = \$394.07.$$

The value of capital with accumulated investment income at the end of the year is $(\$394.04) \cdot (1.03) = \$405.89$, which is available, in addition to premiums with accumulated interest, to pay claims at the $VaR_\alpha(L_T)$ level.

Company by layer

According to Formula (4.6) the dollar underwriting gain required for layer i is given by $E(y_i) = CSD_i \cdot \sigma(y_i)$, which for layers 1 through 5 produces the following results:

$$E(y_1) = CSD_1 \cdot \sigma(y_1) = (0.9808) \cdot (\$21.79) = \$21.37.$$

$$E(y_2) = CSD_2 \cdot \sigma(y_2) = (1.0258) \cdot (\$19.60) = \$20.11.$$

$$E(y_3) = CSD_3 \cdot \sigma(y_3) = (1.0156) \cdot (\$17.06) = \$17.33.$$

$$E(y_4) = CSD_4 \cdot \sigma(y_4) = (0.9322) \cdot (\$14.00) = \$13.05.$$

$$E(y_5) = CSD_5 \cdot \sigma(y_5) = (0.7312) \cdot (\$9.95) = \$7.28.$$

The sum of the five layers' expected results matches the company total required gain of \$79.13 within a penny.

The capital cost premium P_i for layer i is given by

$$P_i = \frac{E(L_i) + E(y_i)}{1.03}, \tag{4.9}$$

which implies the following capital cost premiums for layers 1 through 5:

$$P_1 = \frac{E(L_1) + E(y_1)}{1.03} = \frac{\$5 + \$21.37}{1.03} = \$25.60.$$

$$P_2 = \frac{E(L_2) + E(y_2)}{1.03} = \frac{\$4 + \$20.11}{1.03} = \$23.41.$$

$$P_3 = \frac{E(L_3) + E(y_3)}{1.03} = \frac{\$3 + \$17.33}{1.03} = \$19.74.$$

$$P_4 = \frac{E(L_4) + E(y_4)}{1.03} = \frac{\$2 + \$13.05}{1.03} = \$14.61.$$

$$P_5 = \frac{E(L_5) + E(y_5)}{1.03} = \frac{\$1 + \$7.28}{1.03} = \$8.04.$$

The sum of the five layer capital cost premiums is \$91.40, which also matches the company total capital cost premiums within a penny.

Comparing these capital cost premiums by layer with the quoted reinsurance premiums reported in the paper, we see that the quoted reinsurance premium is lower than the company's capital cost premium in every layer:

Layer 1: \$14.26 vs. \$25.60.
Layer 2: \$12.32 vs. \$23.41.
Layer 3: \$10.25 vs. \$19.74.
Layer 4: \$7.95 vs. \$14.61.
Layer 5: \$5.23 vs. \$8.04.

The company's conclusion should therefore be to buy every reinsurance layer rather than to retain the exposure. As mentioned in Section 2, it is possible and even likely that the insurer, especially if it is small and not well diversified, cannot charge premiums in the marketplace as high as its indicated capital cost premiums. In that case, the insurer will be highly motivated to access the reinsurance market in order to transfer its risk at a lower cost than it faces by keeping it (and to plan its capital accordingly).

4.4. Company required underwriting returns by layer

As the discussion in the previous section shows, it is not necessary to identify the insurer's capital requirement by layer to evaluate the reinsurance decision. However, we will do so now in order to illustrate and underscore one of the authors' key points, namely, that risk-adjusted underwriting returns are not necessarily the same across all layers.

The insurer's required pre-tax required underwriting return on equity capital $ROE_{PT,i}$ for layer i is given by

$$ROE_{PT,i} = \frac{E(y_i)}{C_i}. \quad (4.10)$$

Using Formula (4.2) to obtain the required capital C_i by layer together with the corresponding target underwriting gain $E(y_i)$ determined earlier in this section, we obtain the following required pre-tax underwriting returns on equity capital by layer:[12]

$$C_1 = \frac{\$100.00}{1.03} - \$25.60 = \$71.48$$

$$\Rightarrow \quad ROE_{PT,1} = \frac{\$21.37}{\$71.48} = 29.90\%.$$

$$C_2 = \frac{\$100.00}{1.03} - \$23.40 = \$73.69$$

$$\Rightarrow \quad ROE_{PT,2} = \frac{\$20.11}{\$73.69} = 27.29\%.$$

$$C_3 = \frac{\$100.00}{1.03} - \$19.74 = \$77.36$$

$$\Rightarrow \quad ROE_{PT,3} = \frac{\$17.33}{\$77.36} = 22.40\%.$$

$$C_4 = \frac{\$100.00}{1.03} - \$14.61 = \$82.48$$

$$\Rightarrow \quad ROE_{PT,4} = \frac{\$13.05}{\$82.48} = 15.82\%.$$

$$C_5 = \frac{\$100.00}{1.03} - \$8.041 = \$89.05$$

$$\Rightarrow \quad ROE_{PT,5} = \frac{\$7.28}{\$89.05} = 8.18\%.$$

We see that the implied underwriting returns on equity are very different by layer. The returns display a pattern of declining pre-tax underwriting ROEs as we rise through the program from the first layer through the fifth. Interestingly, while here we are observing returns on equity capital, that pattern of declining returns is similar to that observed in a corporate capital structure as we move from the pure equity part of the structure through mezzanine capital and finally to senior debt. That required underwriting returns likewise vary according to the risk presented by the underwriting exposure, i.e., they display RAROC characteristics, is a key point made by the authors, and our attempt to formulate a realistic illustration supports the authors' contention.

5. Conclusion

To summarize this discussion, the authors are to be applauded for identifying an excellent decision rule for reinsurance purchasing, which is significantly better than what they called the industry standard approach. Their capital-tranching approach also helps to highlight the extent to which reinsurance, espe-

[12]Formula (4.2) is expressed in terms that refer to the company total level *T*, but it is equally applicable at the layer level, in which case the subscript *T* is replaced with *i*.

cially catastrophe reinsurance, can and should be an integral part of capital planning. Unfortunately, their paper did not include clear guidance about how, in practice, to calculate the reinsurance-capital tradeoff. The aim of this discussion has been to remedy that shortcoming by providing a more comprehensive roadmap for the actual application of the approach described in the paper for reinsurance decision-making and capital planning.

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