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Bayesian Loss Development for Real People

David R. Clark, FCAS

Munich Re America – March 2021

Munich RE

Agenda

Munich RE

1. Business Context: Why we are doing this?
2. Basic Model: Combining Triangles
3. Extended Model: What about the tail?
4. Next Steps: Setting the Parameters

Business Context: Why are we doing this?

Munich RE

Business Context:

Goal is to improve estimation of loss development patterns for individual clients.

Including benchmark patterns helps stabilize this estimation.

- Avoid two extremes of relying solely on client data (variance) and using benchmark for everyone (bias).

4

Basic Model

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- Use conjugate distributions for simple implementation
  [*we are skipping the math for today*]
- Related to Chain Ladder method and applies to each age-to-age (ATA) factor

We will start with a blending example to build intuition.

5

Combining Triangles: Possible Even for Different Sizes

Munich RE

Company A
Triangle is complete for old years, but we did not get latest diagonal

	12	24	36	48	60	72	84	96
2011	22,010	44,059	58,548	68,746	76,157	81,650	85,778	
2012	22,010	44,059	58,548	68,746	76,157	81,650		
2013	22,010	44,059	58,548	68,746	76,157			
2014	22,010	44,059	58,548	68,746				
2015	22,010	44,059	58,548					
2016	22,010	44,059						
2017	22,010							
2018								
2011	2.002	1.329	1.174	1.108	1.072	1.051	na	
2012	2.002	1.329	1.174	1.108	1.072	na		
2013	2.002	1.329	1.174	1.108	na			
2014	2.002	1.329	1.174	na				
2015	2.002	1.329	na					
2016	2.002	na						
2017	na							
Col. 1	132,060	220,295	234,192	206,238	152,314	81,650		
Col. 2	264,354	292,740	274,984	228,471	163,300	85,778		
ATA	**2.002**	**1.329**	**1.174**	**1.108**	**1.072**	**1.051**		

Company B
We have latest diagonals, but not early evaluations on some years

	12	24	36	48	60	72	84	96
2011					81,011	84,250	86,778	89,000
2012				75,010	81,011	84,250	86,778	
2013			66,973	75,010	81,011	84,250		
2014		52,380	66,973	75,010	81,011			
2015	33,015	52,380	66,973	75,010				
2016	33,015	52,380	66,973					
2017	33,015	52,380						
2018	33,015							
2011	na	na	na	na	1.040	1.030	1.026	
2012	na	na	na	1.080	1.040	1.030		
2013	na	na	1.120	1.080	1.040			
2014	na	1.279	1.120	1.080				
2015	1.587	1.279	1.120					
2016	1.587	1.279						
2017	1.587							
Col. 1	99,045	157,140	200,919	225,030	243,033	168,500	86,778	
Col. 2	157,140	200,919	225,030	243,033	252,750	173,556	89,000	
ATA	**1.587**	**1.279**	**1.120**	**1.080**	**1.040**	**1.030**	**1.026**	

COMBINDED:

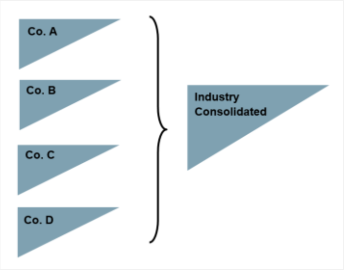
	12	24	36	48	60	72	84
Col. 1	231,105	377,435	435,111	431,268	395,347	250,150	86,778
Col. 2	421,494	493,659	500,014	471,504	416,050	259,334	89,000
ATA	**1.824**	**1.308**	**1.149**	**1.093**	**1.052**	**1.037**	**1.026**

Numbers for illustration only

6

Basic Model

Munich RE



An Industry consolidated triangle may be the source of a benchmark pattern.

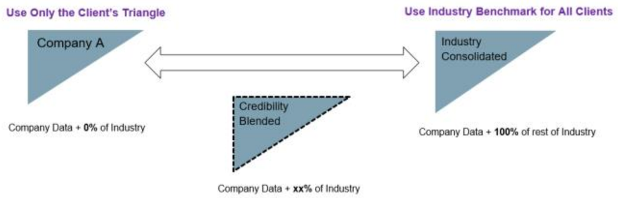
But it does not need to be a full triangle as we have seen: it can be a weighted average from selections for each company.

7

Basic Model

Munich RE

Credibility is like a compromise between two extremes (like variance/bias tradeoff).



8

Basic Model

Munich RE

Client Triangle

	1	2	3	4	5	6	Ultimate		
2012	20,000	20,000	20,000	20,000	20,000	20,000			
2013	10,000	10,000	14,926	14,833	14,833				
2014	23,073	32,945	31,747	31,676					
2015	10,000	10,000	10,000						
2016	24,858	25,054							
2017	10,304								
Col #1	87,931	72,945	66,673	34,833	20,000			A1	Client Data
Col #2	97,999	76,673	66,509	34,833	20,000			A2	
ATA	1.114	1.051	0.998	1.000	1.000			A3 = A2 / A1	
Benchmark Pattern									
Col #1	42,499	112,191	174,216	185,874	96,061	90,909		B1 = B2 / B3	Pseudo Data
Col #2	100,000	150,000	200,000	200,000	100,000	100,000		B2	
ATA	2.353	1.337	1.148	1.076	1.041	1.100		B3	
Credibility Weighted									
Col #1	130,430	185,136	240,889	220,707	116,061	90,909		C1 = A1 + B1	
Col #2	197,999	226,673	266,509	234,833	120,000	100,000		C2 = A2 + B2	
ATA	1.518	1.224	1.106	1.064	1.034	1.100		C3 = C2 / C1	

9

Basic Model

Munich RE

In the basic model, the actual client data is smoothed by supplementing it with "*pseudo data*" from the benchmark, which acts as ballast.

This is equivalent to a Bayesian credibility formula using a conjugate prior. Two alternative derivations can be found in the two papers below.

Clark, D.R. "Introduction to Bayesian Loss Development" *CAS Forum* 2016.
https://www.casact.org/pubs/forum/16sforum/Clark.pdf

Shi, Peng and Brian M. Hartman, "Credibility in Loss Reserving", *CAS Forum* 2014
http://www.casact.org/pubs/forum/14sumforumv2/Shi_Hartman.pdf

10

Basic Model

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The concept of pseudo data:

"Conjugate priors… have the desirable feature that prior information can be viewed as 'fictitious sample information' in that it is combined with the sample in exactly the same way that additional sample information would be combined.

"The only difference is that the prior information is 'observed' in the mind of the researcher, not in the real world."

- Bayesian Econometric Methods; Koop, Poirier & Tobias

PS: This is also what is done in ISO state advisory loss cost circulars.

11

Extended Model

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A limitation of the Basic Model:

- Each age-to-age (ATA) factor, or column of the triangle, is treated independently
- This means that we would use the benchmark "tail" even if ATA factors from the client were consistently less than the benchmark.

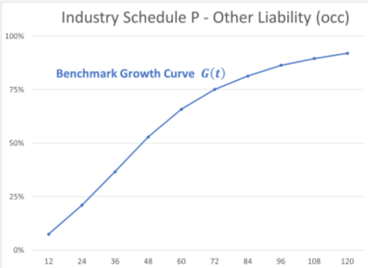
Shi & Hartman address this by introducing a correlation structure in the model.

An alternative is to first "nudge" the benchmark before applying the Basic Model.

12

Extended Model

Munich RE

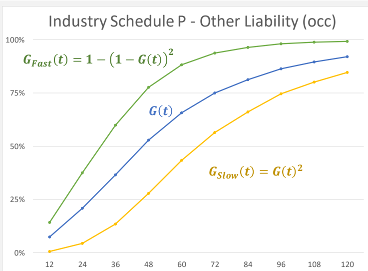


Actuaries select a benchmark development pattern (growth curve) for representative business segments.

The selected benchmarks may be based on data from various sources and judgmentally smoothed.

Extended Model

Munich RE



As a simplified method for setting a range around the benchmark, we can start by setting “fast” and “slow” patterns.

In this derivation, the benchmark will always be the exact midpoint between the fast and slow patterns.

Extended Model

Munich RE

We will assume that each client has a development pattern that is a weighted average of the “Fast” and “Slow” patterns around the benchmark.

If the weight for company *j* is exactly 50%/50%, then the benchmark pattern is used.

To start, we will constrain the weights to be between 0% and 100%. The parameter *p* is assumed to be a random variable from a beta distribution.

$$G(age|j) = p_j \cdot G_{Fast}(age) + (1 - p_j) \cdot G_{Slow}(age)$$

$$0 \le p_j \le 1$$

Extended Model

Munich RE

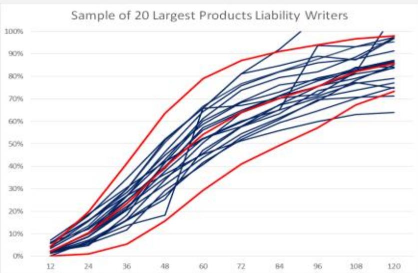
The form of the model is a linear combination of two “basis functions” (fast and slow).
- A simple form of ***Regression Spline***

The weight parameter *p* can be estimated various ways, along with its standard error.

Ideally, we damp this parameter close to .500 based on assigning a prior distribution (e.g., a Beta Distribution).
If we assume a prior uniformly distributed between 0 and 1, then the variance of hypothetical means = 1/12.

Extended Model

Munich RE



How well does this work?

Example here uses Products Liability payment patterns from Schedule P.

The fast and slow patterns reasonably bracket the range of patterns across companies.

Next Steps: Selecting the Prior Distribution

Munich RE

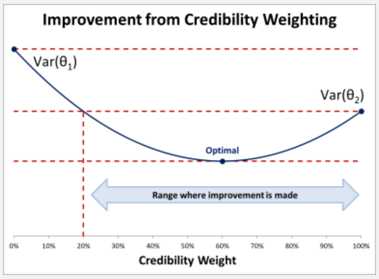
How do we set the spread around the benchmark parameter?

Subjective Bayes:
- Business expertise selects the range of possible values
  For example: how much faster or slower than average can a company settle its claims?

“Subjunctive Bayes” (Stephen Senn):
- Set prior parameters to get the credibility-weighted result that makes sense

Empirical Bayes:
- How much actual spread is there among the companies (or states)?
- Data Scientists call this cross validation

Next Steps: Selecting the Prior Distribution



Good News !

Even if we cannot estimate the optimal credibility perfectly, we can select a value that produces a blended estimate that is an improvement on either estimator alone.

We are just looking for a sensible weighted average.

19

Thank you!

Selected References

Clark, D.R. "*Introduction to Bayesian Loss Development*" CAS Forum 2016.
https://www.casact.org/pubs/forum/16sforum/Clark.pdf

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http://www.variancejournal.org/issues/11-01-02/95.pdf

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https://www.stat.washington.edu/peter/342/Senn.pdf

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http://www.casact.org/pubs/forum/14sumforumv2/Shi_Hartman.pdf

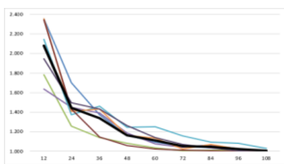
21

Strategies for Working with Loss Development Factors

Uri Korn, FCAS
Ratemaking, Product, and Modeling Seminar
March 16, 2021

Blending LDFs

- LDFs are volatile
- To reduce LDF volatility, leverage 2 related pieces of information
  1. Adjacent LDFs – Fit a curve
  2. Related LDFs - Blend with credibility



23

The LDF Ninja

24

Part 1) LDF Curves

Inverse Power Curve

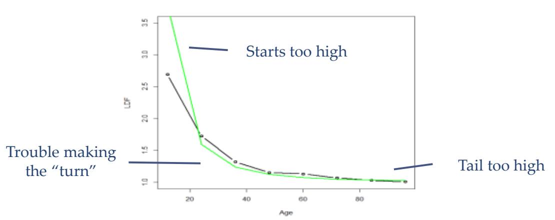
(Sherman 1984)

$$\log(LDF - 1) = A + B \times \log(age)^{*}$$

- Easy to implement
- But often poor fit to the data

* Using *age* instead of *1 / age*, since the regression equations are equivalent.
Also, ignoring the *c* parameter

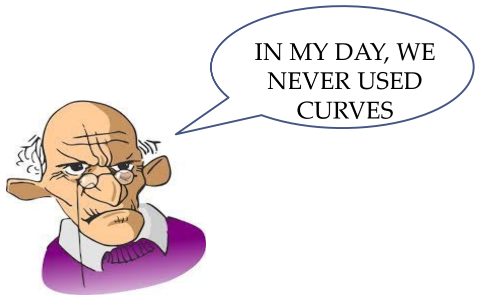
IPOC Fit



Problem with the IPOC

- Weights (assumed variances) aren't accurate
  - Tail LDFs are more volatile
  - High volatility at initial ages due to lack of volume (longer tailed lines)

28



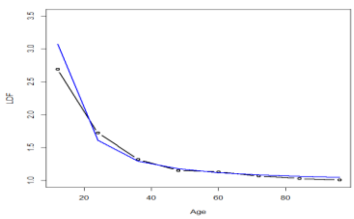
29

Double IPOC (DIPOC)

- Modify the weights of the Inverse Power Curve
- Weights are a function of age and loss volume
  - Use a weighted Gamma regression instead
- Fit a curve to the variance/weights by age
  1. Fit simultaneously with LDFs
  2. Or calculate directly from triangle beforehand

30

DIPOC Fit



Improved, but still flawed …

Smoothed IPOC (SMIPOC)

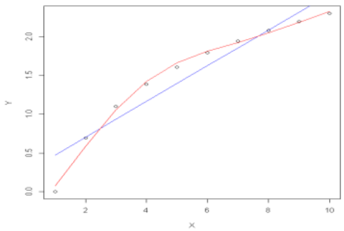
- Double IPOC with regression splines
    - (Concept borrowed from England & Verall 2001)
- Adds flexibility to the curve
- Can still be done in Excel

How do Splines work?

Original Variable	New Variable 1	New Variable 2
1	0.00	0.00
2	0.17	-0.11
3	0.32	-0.20
4	0.45	-0.25
5	0.54	-0.24
6	0.58	-0.16
7	0.57	0.01
8	0.51	0.23
9	0.44	0.49
10	0.34	0.77

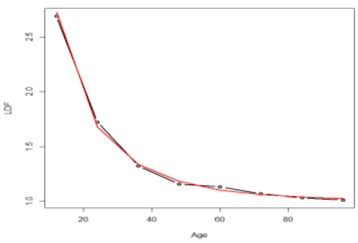
- Performs a special transformation on a variable (such as age)
- Run a regular regression on the new variables instead
- Enables a better fit to the data at the cost of additional variables

Splines Example



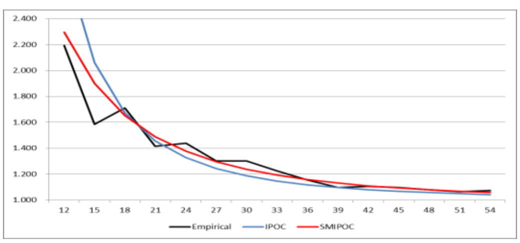
34

SMIPOC Fit



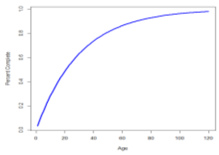
35

Real (Altered) Data Example



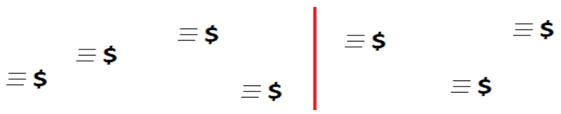
36

Inverse Power Distributions



- Model the percent completion distribution instead
  - Idea inspired by Clark 2003
- Use a similar inverse power function to define the CDF (and likelihood)

Inverse Power Distributions

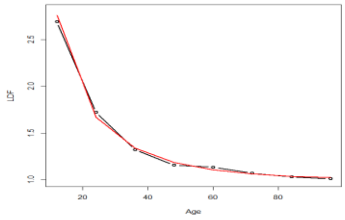


- Fit a distribution directly to the age of each dollar
- Similar to fitting ILFs, but is right truncated because of the unknown future

Smoothed Inverse Power Distribution (SMIPOD)

- Similar to before, use regression splines
- Better fit to data

SMIPOD Fit



- Only 3 parameters!

40

Part 2) Credibility

41

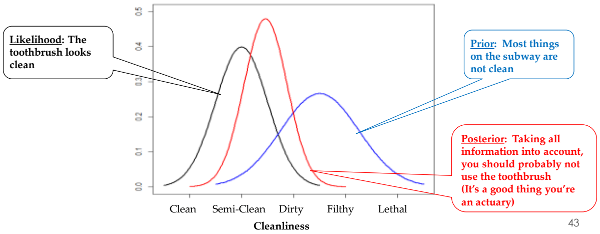
Credibility

- Best answer to the trade off between:
  - Fewer stable heterogeneous segments
  - Many volatile homogenous segments

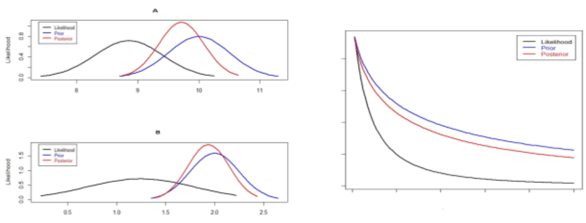
42

Bayesian Credibility

- You find a toothbrush on the subway!
- It looks semi-clean!
- Should you use it?

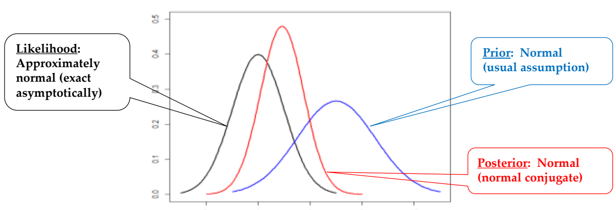


Bayesian Credibility on a Curve or Distribution



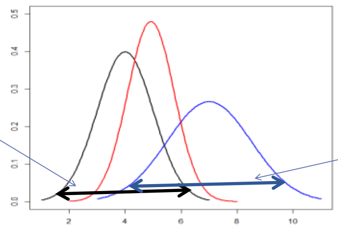
- Performs credibility weighting on the parameters simultaneously while fitting the curve/distribution

Implementing Bayesian Credibility in Excel

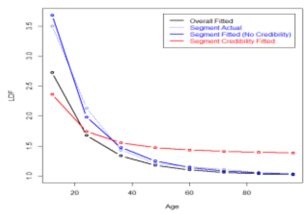


- Maximum Likelihood Estimation (e.g. via Solver) returns the mode of the distribution
- Same as the mean for the Normal distribution

Implementing Bayesian Credibility in Excel



The theory is fine but…



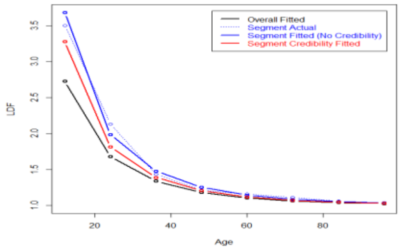
- (This only happens when credibility weighting multiple parameters)

Fixing the Credibility

- The prior should be calculated on the curve parameters
- What are the parameters?
  - **Intercept & Slope**
- But what if we…
  - **LDF1 & LDF2 ➔ Intercept & Slope ➔ Entire LDF Curve**
- Calculate the prior on the two predicted LDFs (even if the inversion wasn't performed)

Note: In practice, using log(LDF – 1) works better

Fixed SMIPOC



49

Calculating the Prior Variance

- How do we calculate this prior variance?
  - (Equivalent to Between Variance and Z)
- Options:
  - Build a Bayesian model
  - Holdout/Cross validation
  - Buhlmann-Straub

50

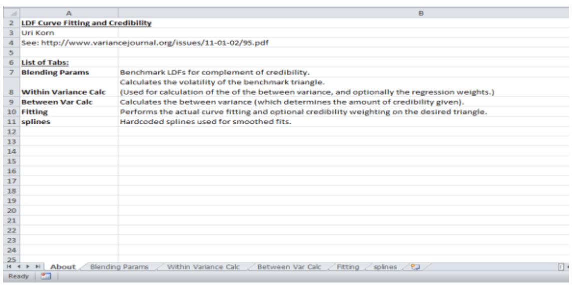
Buhlmann-Straub

- Remember: We are using LDF parameters
- Use the Buhlmann-Straub formulas on the actual LDFs as an approximation
- Fit a curve by age to smooth them out

51

Part 3) Excel Template

52



	A	B
2	LDF Curve Fitting and Credibility	
3	Uri Korn	
4	See: http://www.variancejournal.org/issues/11-01-02/95.pdf	
5		
6	List of Tabs:	
7	Blending Params	Benchmark LDFs for complement of credibility.
8	Within Variance Calc	Calculates the volatility of the benchmark triangle. (Used for calculation of the of the between variance, and optionally the regression weights.)
9	Between Var Calc	Calculates the between variance (which determines the amount of credibility given).
10	Fitting	Performs the actual curve fitting and optional credibility weighting on the desired triangle.
11	splines	Hardcoded splines used for smoothed fits.
12		
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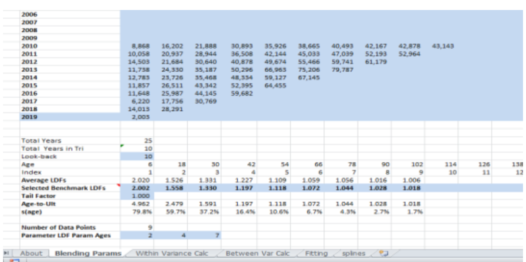
About | Blending Params | Within Variance Calc | Between Var Calc | Fitting | splines

Ready

53



2006												
2007												
2008												
2009												
2010	8,868	16,202	21,888	30,893	35,926	38,665	40,493	42,167	42,878	43,143		
2011	10,058	20,937	28,944	36,508	42,144	45,033	47,039	52,193	52,964			
2012	14,503	21,684	30,640	40,878	49,674	55,466	59,741	61,179				
2013	11,738	24,330	35,187	50,296	66,963	75,206	79,787					
2014	12,783	23,726	35,468	48,334	59,127	67,145						
2015	11,857	26,511	43,342	52,395	64,455							
2016	11,648	25,987	44,145	59,682								
2017	6,220	17,756	30,769									
2018	14,013	28,291										
2019	2,003											
Total Years	25											
Total Years in Tri	10											
Look-back	10											
Age	6	18	30	42	54	66	78	90	102	114	126	138
Index	1	2	3	4	5	6	7	8	9	10	11	12
Average LDFs	2.020	1.526	1.331	1.227	1.109	1.059	1.056	1.016	1.006			
Selected Benchmark LDFs	2.002	1.558	1.330	1.197	1.118	1.072	1.044	1.028	1.018			
Tail Factor	1.000											
Age-to-Ult	4.962	2.479	1.591	1.197	1.118	1.072	1.044	1.028	1.018			
s(age)	79.8%	59.7%	37.2%	16.4%	10.6%	6.7%	4.3%	2.7%	1.7%			
Number of Data Points	9											
Parameter LDF Param Ages	2	4	7									

About | Blending Params | Within Variance Calc | Between Var Calc | Fitting | splines

54

Segment LDFs	6	18	30	42	54	66	78	90	102	114	126
1	1.639	1.452	1.390	1.189	1.078	1.040	1.062	1.024	1.000		
2	1.785	1.260	1.140	1.081	1.036	1.006	1.013	1.013	1.000		
3	2.147	1.374	1.461	1.247	1.252	1.155	1.090	1.081	1.028		
4	2.357	1.409	1.433	1.165	1.137	1.024	1.070	1.024	1.001		
5	4.381	2.785	1.501	1.527	1.115	1.109	1.134	0.987	1.000		
6	2.346	1.433	1.143	1.058	1.025	1.009	1.005	1.002	1.000		
7	1.945	1.498	1.435	1.263	1.142	1.067	1.034	1.003	1.013		
8	3.136	1.892	1.112	1.235	1.003	1.980	1.482	1.001	1.002		
9	6.058	2.076	1.388	1.277	1.136	1.067	1.070	1.008	1.120		
10	1.638	1.240	1.102	1.035	1.003	1.003	1.000	1.000	1.000		
Cumulative Losses											
1	20,990	30,524	40,056	48,970	49,477	43,871	30,365	22,752	12,117		
2	42,243	64,141	70,071	64,699	57,571	46,786	33,737	22,282	10,298		
3	5,514	11,582	13,940	14,246	13,236	11,137	9,754	3,849	1,435		
4	12,273	21,154	23,926	28,630	24,818	20,246	14,976	9,683	4,739		
5	2,303	8,801	21,510	25,562	31,254	23,934	18,189	10,199	3,680		
6	11,611	20,296	22,635	18,118	15,177	11,556	8,450	5,309	2,599		
7	7,890	12,653	15,466	18,379	18,401	16,987	12,610	7,283	3,660		
8	2,908	5,284	6,060	2,361	753	743	697	736	487		
9	1,733	9,775	16,816	16,142	13,391	11,647	7,037	3,786	861		
10	12,072	17,369	19,100	17,972	15,822	12,778	9,390	6,230	2,966		
Avg Segment LDFs	2.030	1.464	1.289	1.187	1.088	1.048	1.055	1.014	1.005		
Empirical log(LDF - 1)	0.029	(0.767)	(1.242)	(1.675)	(2.427)	(3.026)	(2.895)	(4.246)	(5.371)		
Fitted Within Var	0.747	2.490	4.359	6.303	8.303	10.346	12.425	14.535	16.673	18.835	21.019

About | Blending Params | Within Variance Calc | Between Var Calc | Fitting | splines

Segment Triangle

Year \ Age	6	18	30	42	54	66	78	90	102	114	126	138
2010	343	1,071	1,824	3,099	3,983	4,871	5,056	5,181	5,299	5,346		
2011	247	937	2,192	3,156	3,697	4,000	4,206	4,255	4,413			
2012	333	866	1,511	2,124	2,547	3,089	3,071	3,265				
2013	243	652	1,081	2,070	2,915	3,264	3,440					
2014	322	715	2,078	2,800	3,231	3,361						
2015	276	675	1,248	1,627	2,365							
2016	387	1,244	2,401	3,313								
2017	283	508	1,089									
2018	363	815										
2019	349											

Average Claim Size (used for IPOD credibilities) 2,500

Age to Age

Year	6-18	18-30	30-42	42-54	54-66	66-78	78-90	90-102	102-114	114-126	126-138	138-150
2010	3.122	1.703	1.699	1.285	1.173	1.082	1.025	1.023	1.009			
2011	3.794	2.339	1.440	1.168	1.107	1.031	1.012	1.037				
2012	2.601	1.745	1.406	1.199	1.213	0.994	1.063					
2013	2.683	1.658	1.915	1.408	1.120	1.054						
2014	2.220	2.906	1.347	1.154	1.040							
2015	2.446	1.849	1.304	1.454								
2016	3.214	1.930	1.380									
2017	1.795	2.144										
2018	2.245											
2019												

Incr Losses

2010	343	728	753	1,275	884	688	185	125	118	47	-
2011	247	690	1,255	964	531	393	126	49	158	-	-
2012	333	533	645	613	423	542	(18)	194	-	-	-
2013	243	409	429	989	845	349	176	-	-	-	-
2014	322	393	1,363	722	431	130	-	-	-	-	-
2015	276	399	573	379	738	-	-	-	-	-	-
2016	387	857	1,157	912	-	-	-	-	-	-	-
2017	283	225	581	-	-	-	-	-	-	-	-
2018	363	452	-	-	-	-	-	-	-	-	-
2019	349	-	-	-	-	-	-	-	-	-	-

About | Blending Params | Within Variance Calc | Between Var Calc | Fitting | splines

Total Years	25								
Total Years in Tri	10								
Look-back	10								
Age	6	18	30	42	54	66	78	90	102
Index	1	2	3	4	5	6	7	8	9
Empirical LDF	**2.675**	**2.013**	**1.475**	**1.259**	**1.128**	**1.044**	**1.030**	**1.029**	**1.009**
Tot Cuml Losses	2,797	6,668	12,335	14,876	16,363	15,104	12,333	9,436	5,299

Fit All Curves

It is recommended to close all other slow calculating workbooks before running

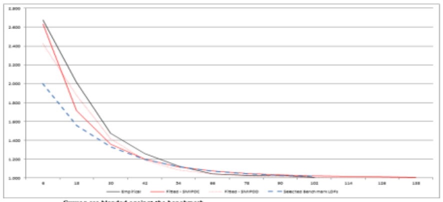
Do Blending	TRUE	*Select "TRUE" to perform credibility weighting*
Benchmark Weights by Age	TRUE	*Recommended when performing credibility weighting Although occasionally can give questionable results*

DIPOC	Intercept	Slope	CoV Intercept	CoV Slope					
Params	2.467	-1.025	1.000	0.010					
Fitted LDF	**2.878**	**1.689**	**1.361**	**1.255**	**1.197**	**1.161**	**1.135**	**1.117**	**1.103**
CoV Param	0.976	2.320	3.973	6.119	8.900	12.467	16.999	22.702	29.828
CoV	0.584	0.898	1.131	1.587	2.200	3.208	4.840	7.391	12.958
loglik	-0.843	-1.083	-0.432	-0.217	0.013	0.456	0.173	-0.561	-0.440
Credibility Data Points	-0.496	-1.365							
Complement of Cred	-0.594	-1.626							
loglik prior	-0.324	-0.942							
Total loglik	(7,504.30)								

SMIPOC	Intercept	Slope 1	Slope 2	CoV Intercept	CoV Slope				
Params	0.707	-5.431	-6.305	1.000	0.010				
spline seq 1	0.349	0.495	0.532	0.544	0.545	0.541	0.534	0.525	0.515
spline seq 2	(0.267)	(0.262)	(0.194)	(0.101)	(0.020)	0.057	0.130	0.199	0.264
Fitted LDF	**2.634**	**1.717**	**1.359**	**1.200**	**1.119**	**1.075**	**1.049**	**1.033**	**1.023**
CoV Param	0.976	2.320	3.973	6.119	8.900	12.467	16.999	22.702	29.828

About | Blending Params | Within Variance Calc | Between Var Calc | Fitting | splines

Summary	6	18	30	42	54	66	78	90	102	114	126
Empirical	2.675	2.013	1.475	1.259	1.128	1.044	1.030	1.029	1.009		
SMPOC	2.634	1.717	1.359	1.200	1.119	1.075	1.049	1.033	1.023	1.017	1.012
SMPOD	2.426	1.877	1.418	1.190	1.088	1.042	1.021	1.011	1.006	1.003	1.002
Complement of Cred	2.002	1.558	1.330	1.197	1.118	1.072	1.044	1.028	1.018	-	-



Curves are blended against the benchmark

About | Blending Params | Within Variance Calc | Between Var Calc | Fitting | splines

58

Thank You!

For more details, refer to:
http://www.variancejournal.org/issues/11-01-02/95.pdf

59