Credibility Weighting for Reserving Using Bayesian MCMC

CLRS 2021

Poll Questions on CAS Exam Background

- Have you taken either Exam S or MAS I? (Yes or No)
- Have you taken Exam 4/c? (Yes or No)
- Have you taken Exam MAS II? (Yes or No)
- Have you taken Exam 5? (Yes or No)
- Have you taken Exam 7? (Yes or No)

Poll Questions on Work Background

- Have you worked in a reserving area? (Yes or No)
- Have you used a method beyond link ratios for reserving ?(Yes, No or NA)
- Have you used a GLM package to build models? (Yes or No)
- Have you used R packages? (Yes or No)
- Have you used the tidyverse R packages? (Yes or No)
- Have you used Rstudio? (Yes or No)
- Have you heard of the STAN Bayesian MCMC program? (Yes or No)
- Have you heard of the brms macro writer for STAN? (Yes or No)

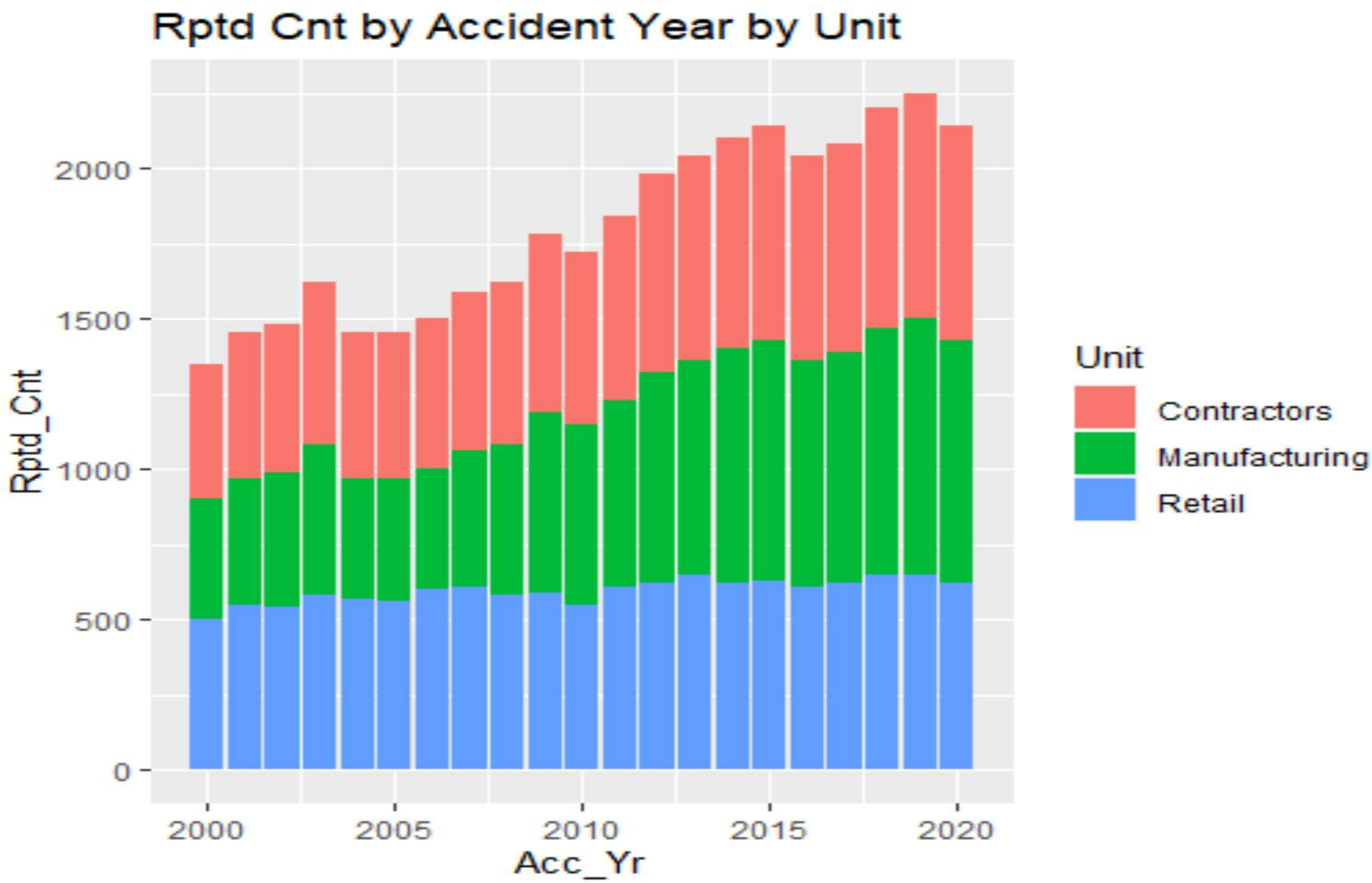
Goals of Presentation

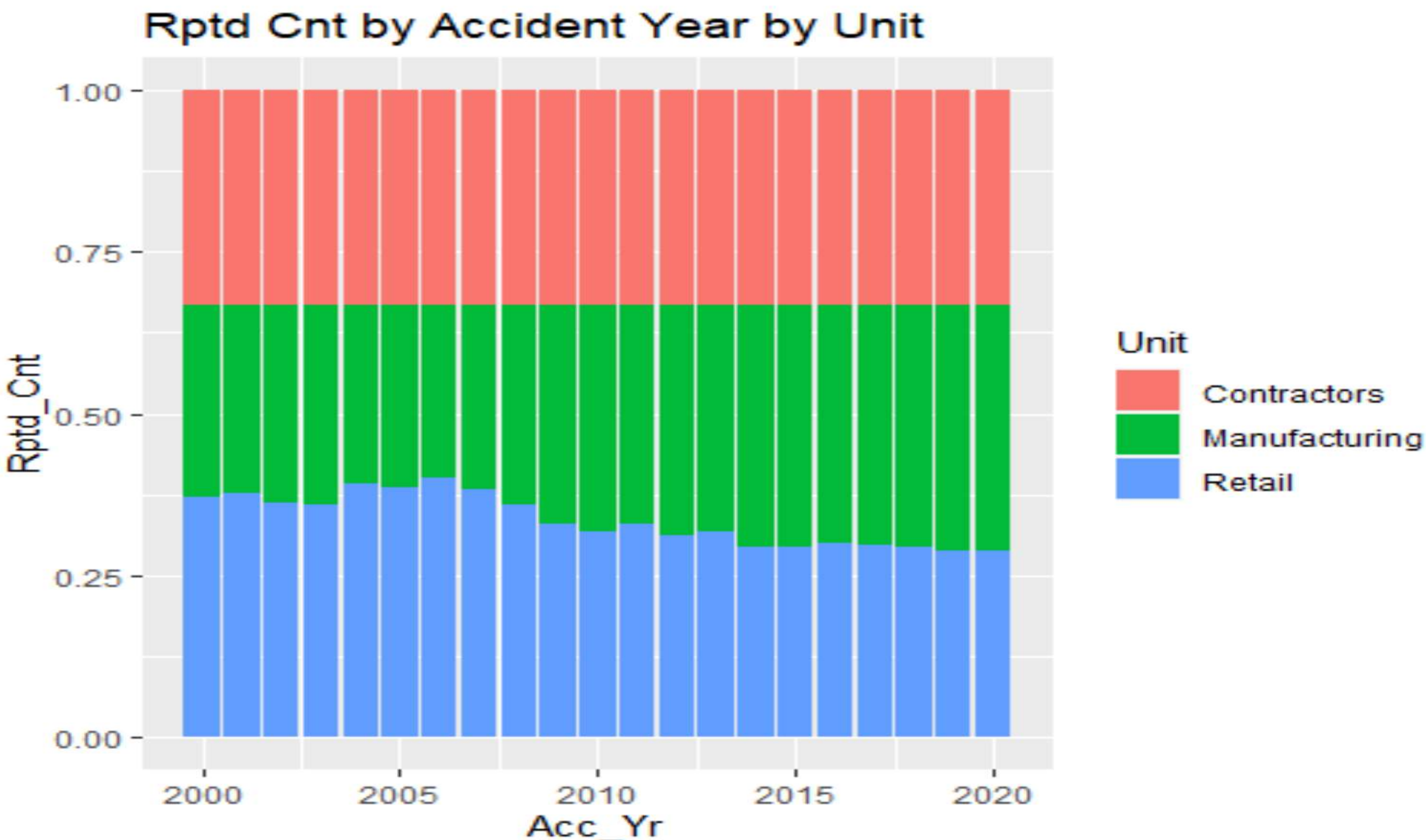
- Provide examples to motivate use of Bayesian MCMC in reserving
  - Multiple business units selling coverage in a given Annual Statement Line
    - Loss behavior is similar but not the same across units
    - Recognize differences without giving up stability of larger data set
  - Estimate reserves for a new, low volume line of business
    - Use data at hand to extent credible
    - Use knowledge of line behavior to set up credibility weighting
- Link Bayesian MCMC environment to Exam 4/c credibility concepts
  - Using prior parameter distribution option vs. conjugate priors
  - Group variables vs. least squares credibility

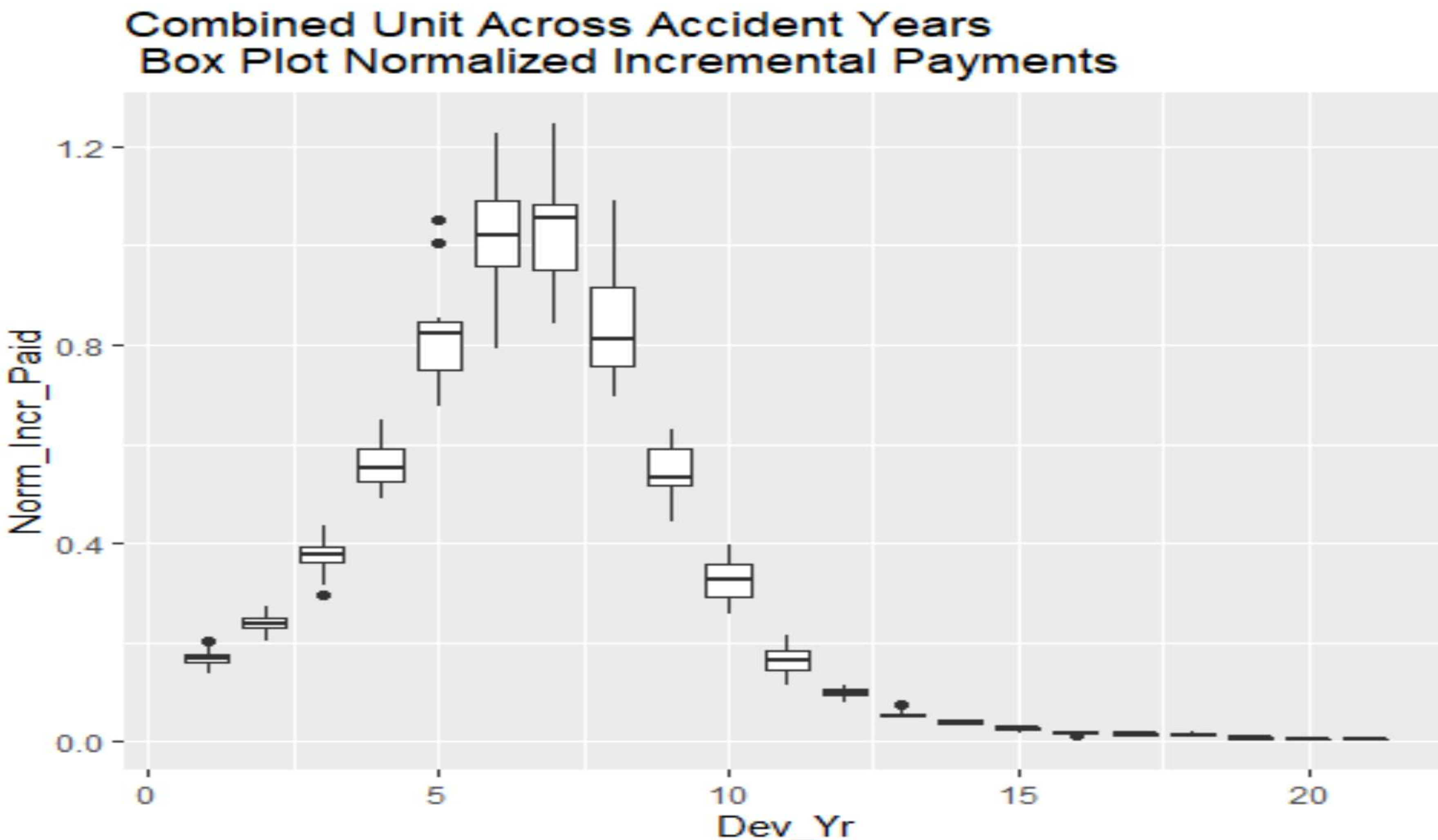
Slide Sequence

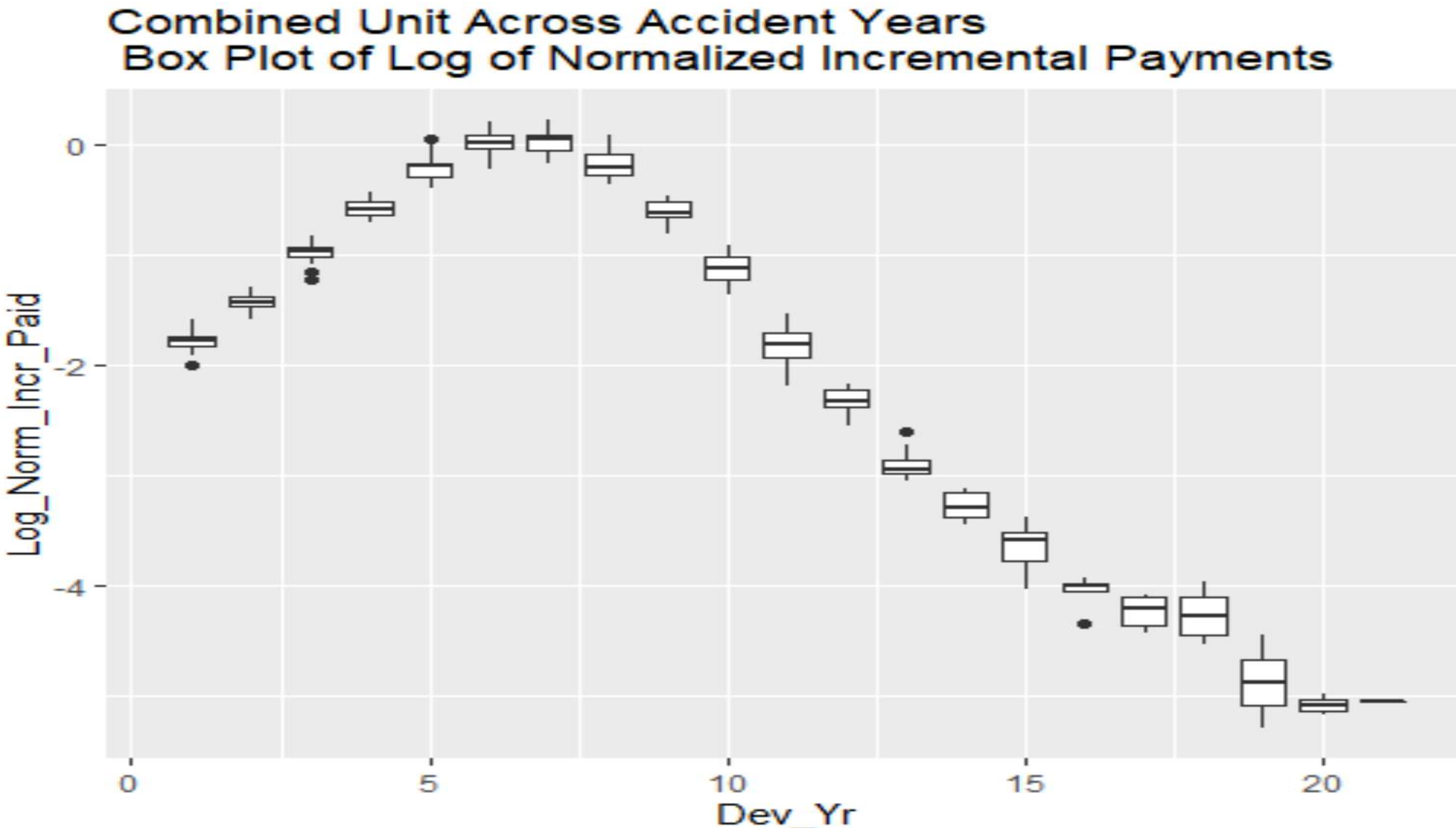
- Describe data set using graphs
- Results from reserve model with losses summed across business units
  - Show the fit in total
  - Motivate need to look at results by unit
- Results from two approaches to credibility weighting by unit
  - Start with combined unit model for prior distribution of population parameters
  - LASSO prior distribution for population parameters
- Outline process to model new line
- Details for models are in a separate handout

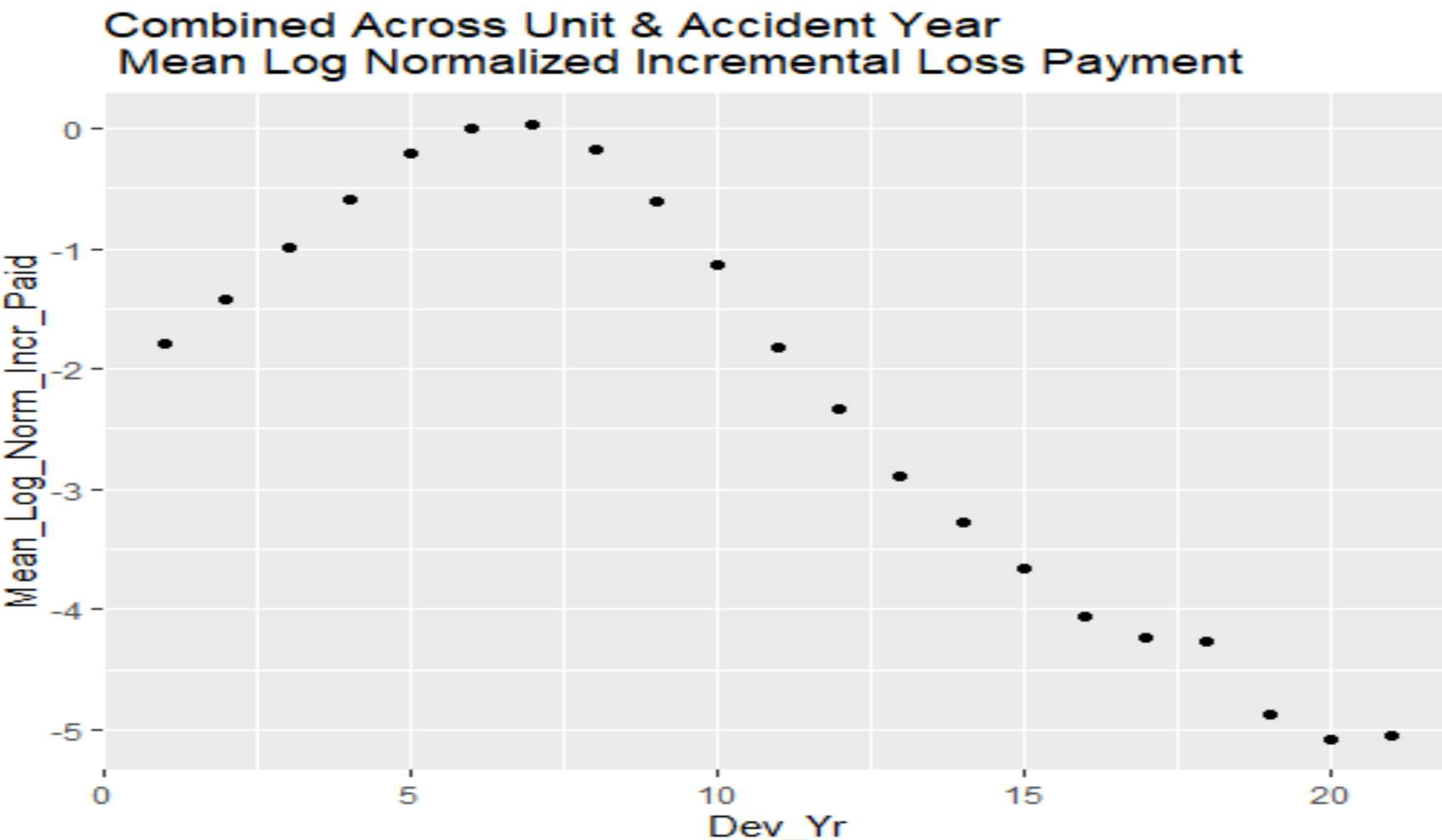
Describe the Reserve Data Set

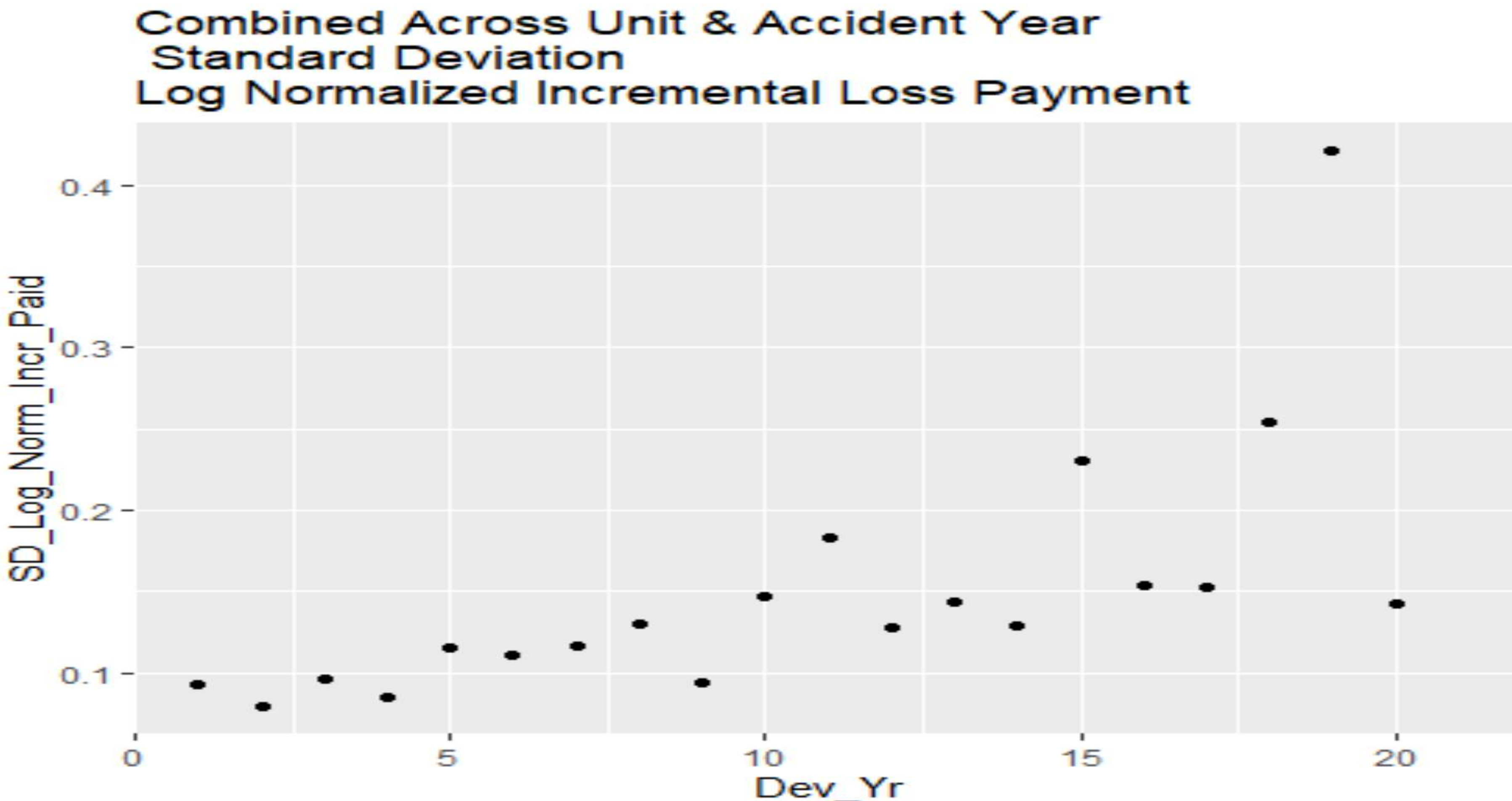

Rptd Cnt by Accident Year by Unit
Rptd_Cnt
0
500
1000
1500
2000
2000
2005
2010
2015
2020
Acc_Yr
Unit
Contractors
Manufacturing
Retail

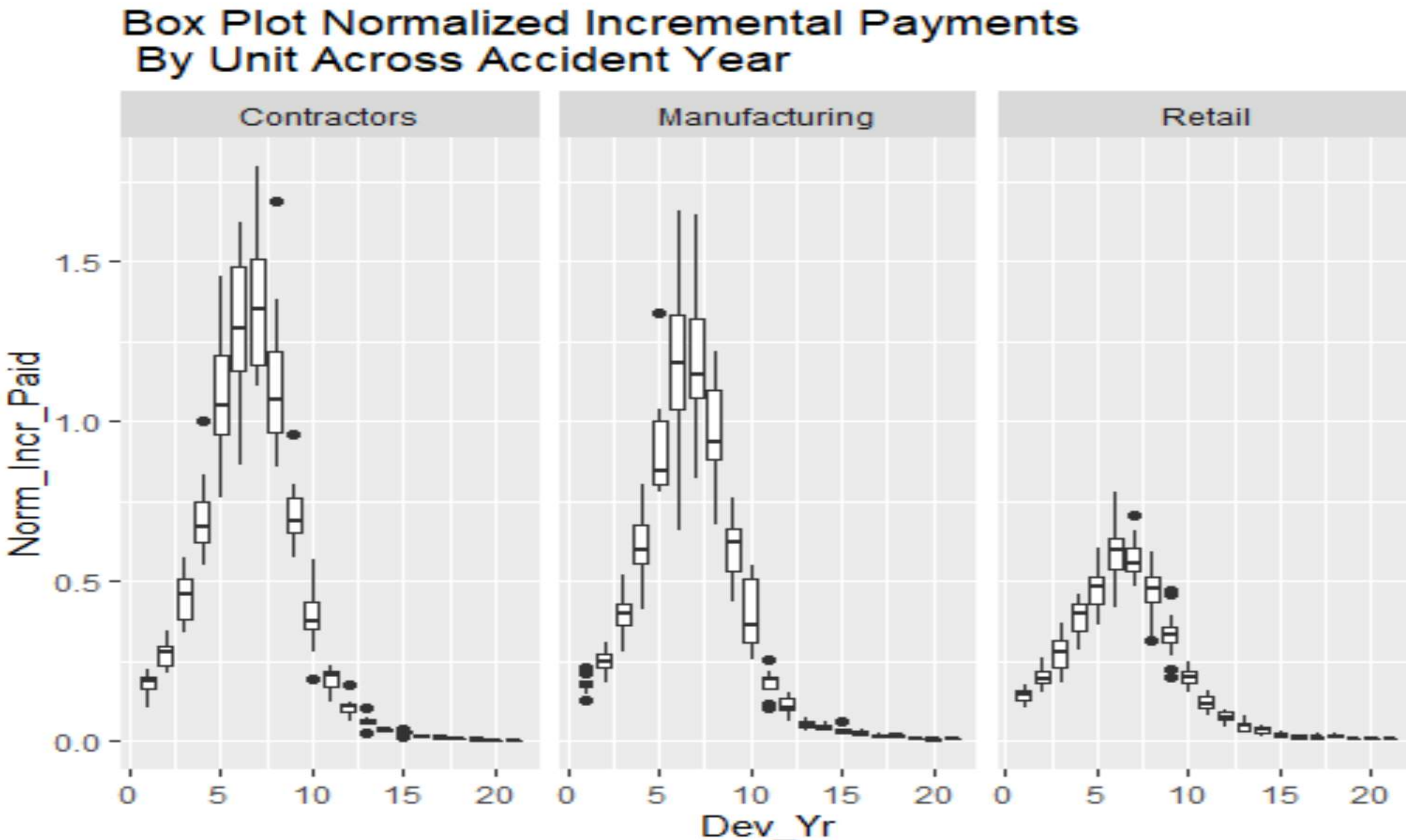

Rptd Cnt by Accident Year by Unit
1.00
0.75
0.50
0.25
0.00
Rptd_Cnt
2000
2005
2010
2015
2020
Acc_Yr
Unit
Contractors
Manufacturing
Retail

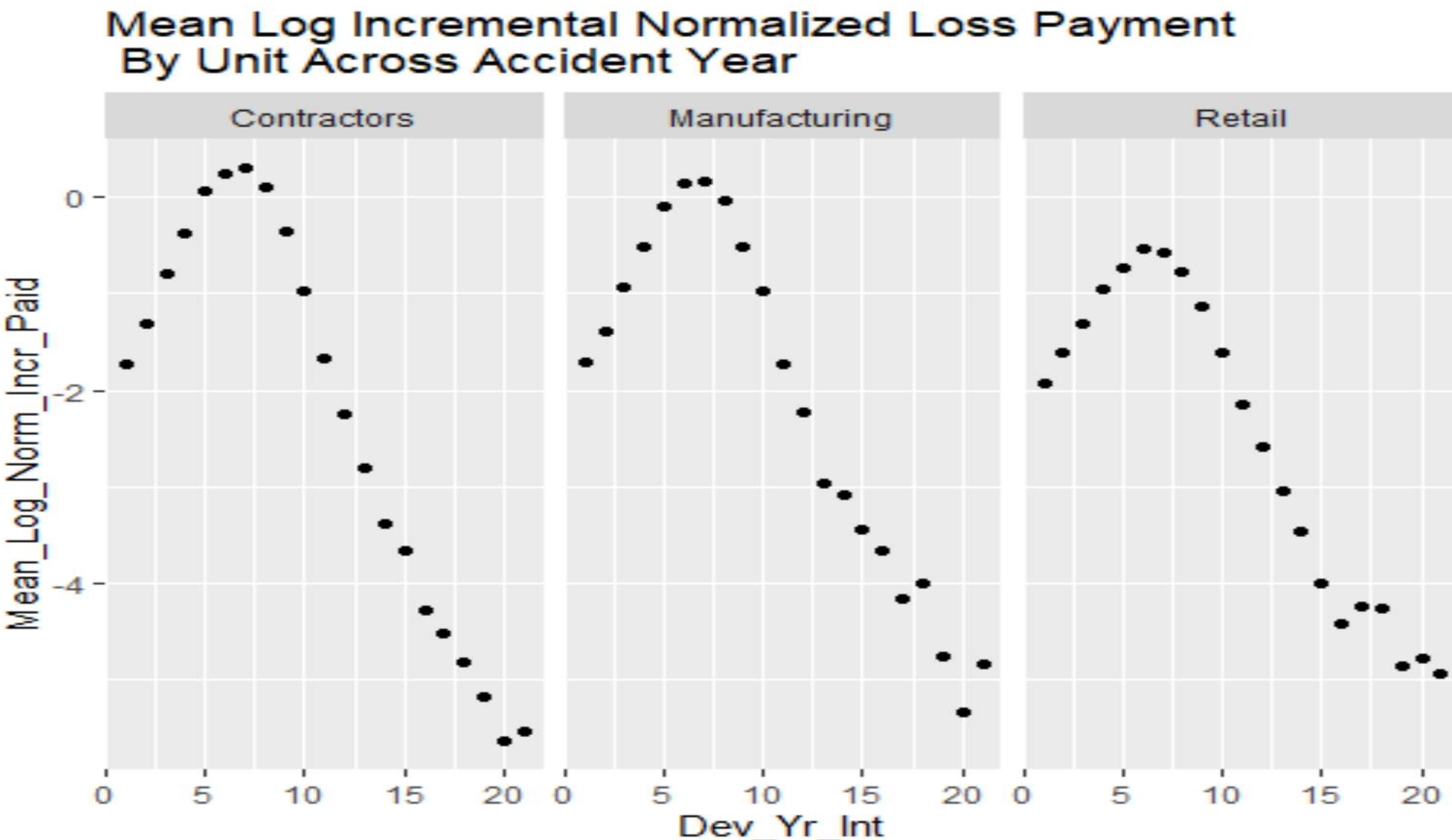

Combined Unit Across Accident Years
Box Plot Normalized Incremental Payments
1.2
0.8
0.4
0.0
Norm_Incr_Paid
0
5
10
15
20
Dev_Yr

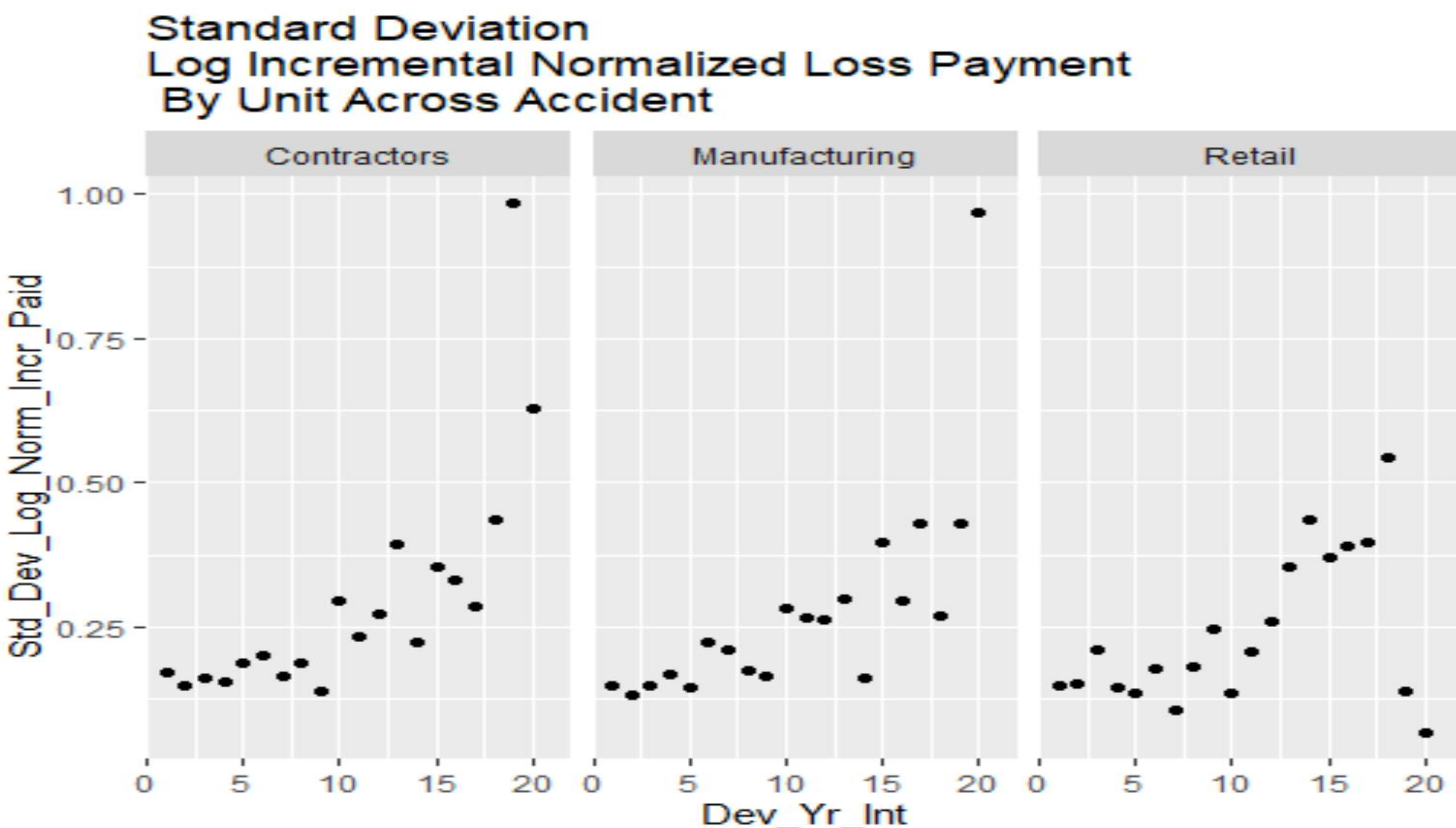

Combined Unit Across Accident Years
Box Plot of Log of Normalized Incremental Payments
Log_Norm_Incr_Paid
0
-2
-4
0
5
10
15
20
Dev_Yr

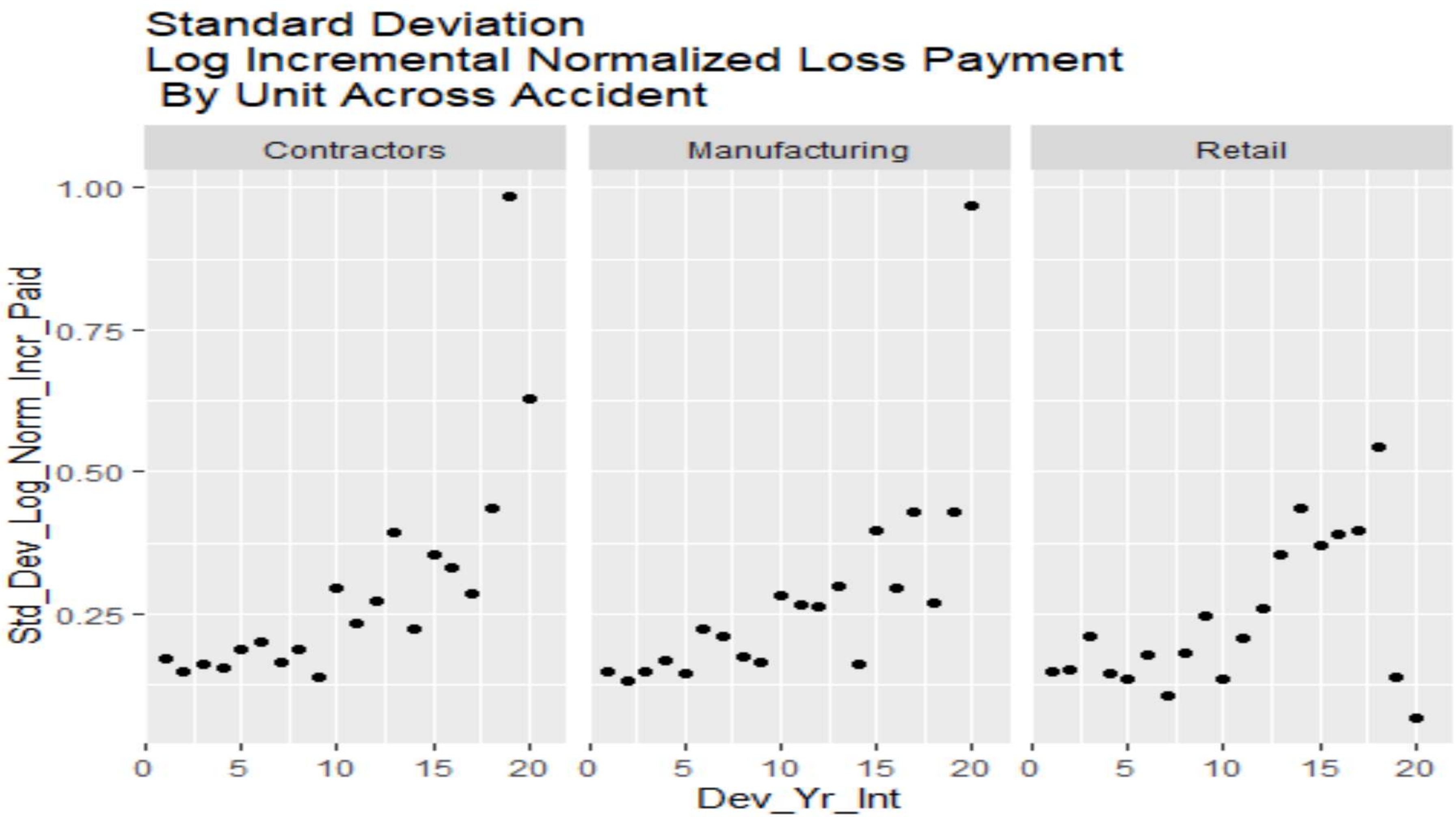

Combined Across Unit & Accident Year
Mean Log Normalized Incremental Loss Payment
Mean_Log_Norm_Incr_Paid
0
-1
-2
-3
-4
-5
0
5
10
15
20
Dev_Yr


Combined Across Unit & Accident Year
Standard Deviation
Log Normalized Incremental Loss Payment
SD_Log_Norm_Incr_Paid
0.1
0.2
0.3
0.4
0
5
10
15
20
Dev_Yr


Box Plot Normalized Incremental Payments
By Unit Across Accident Year
Contractors
Manufacturing
Retail
Norm_Incr_Paid
0.0
0.5
1.0
1.5
0
5
10
15
20
Dev_Yr


Mean Log Incremental Normalized Loss Payment
By Unit Across Accident Year
Contractors
Manufacturing
Retail
Mean_Log_Norm_Incr_Paid
0
-2
-4
0 5 10 15 20
Dev_Yr_Int


Standard Deviation
Log Incremental Normalized Loss Payment
By Unit Across Accident
Contractors
Manufacturing
Retail
1.00
0.75
0.50
0.25
Std_Dev_Log_Norm_Incr_Paid
0
5
10
15
20
Dev_Yr_Int


Standard Deviation
Log Incremental Normalized Loss Payment
By Unit Across Accident
Contractors
Manufacturing
Retail
Std_Dev_Log_Norm_Incr_Paid
1.00
0.75
0.50
0.25
0
5
10
15
20
Dev_Yr_Int

Combined Across Business Unit Model Results

Slide Sequence

- Explain model set up
- Show combined across unit results
- Show by unit results to demonstrate need to reflect by unit results

General Form for brms Models

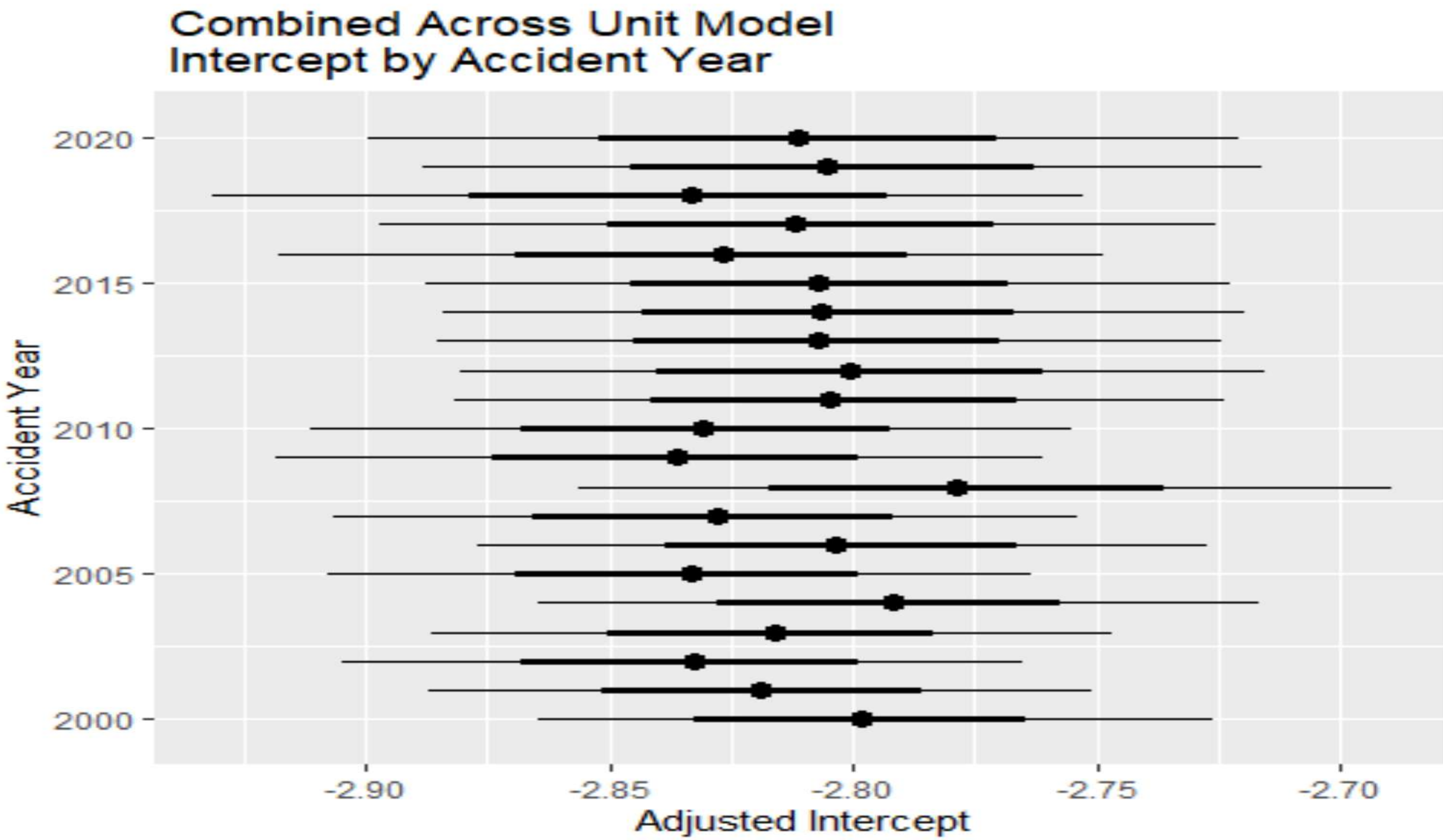
- Model formula: response ~ pterms + (gterms | group)
  - Response is item to be modeled
  - Pterms are population variables
  - Group is the variable identifying segments where a form of least squares credibility weighting is applied to gterms
- Additional instructions for the models in this presentation
  - Family is the distribution for the variable to be modeled
  - Data gives the name of the data set
  - Prior gives the prior distribution assumption formulas and sets up the credibility weighting between prior assumptions and data driven result

Model Form Selected for Combined Units

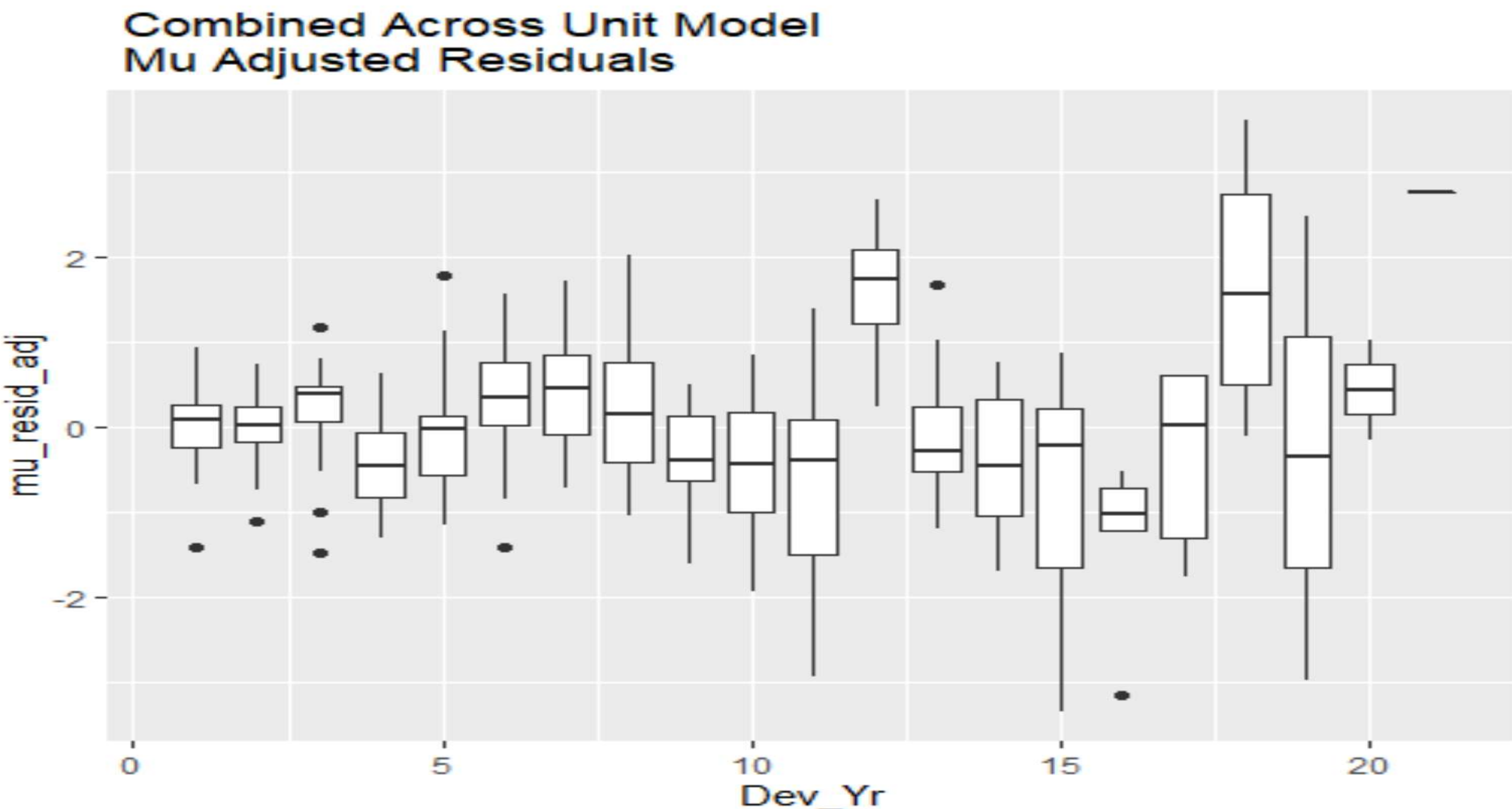
- Dependent variable: Incremental Payment/ Reported Counts DY 1
- Distribution model
  - Lognormal
  - Mu and sigma modeled separately and simultaneously
  - Identify link for mu and log link for sigma
- Linear form for mu
  - Parabolic for first 12 development years
  - Spline added for development years 13 plus
  - Categorical adjustment first 3 development years
  - Uniform trend factor by calendar year
  - Credibility adjusted by accident year adjustment to Intercept
- Linear form on transformed scale for sigma
  - Constant increase on transformed scale for first 14 years then constant
- Non-informative prior

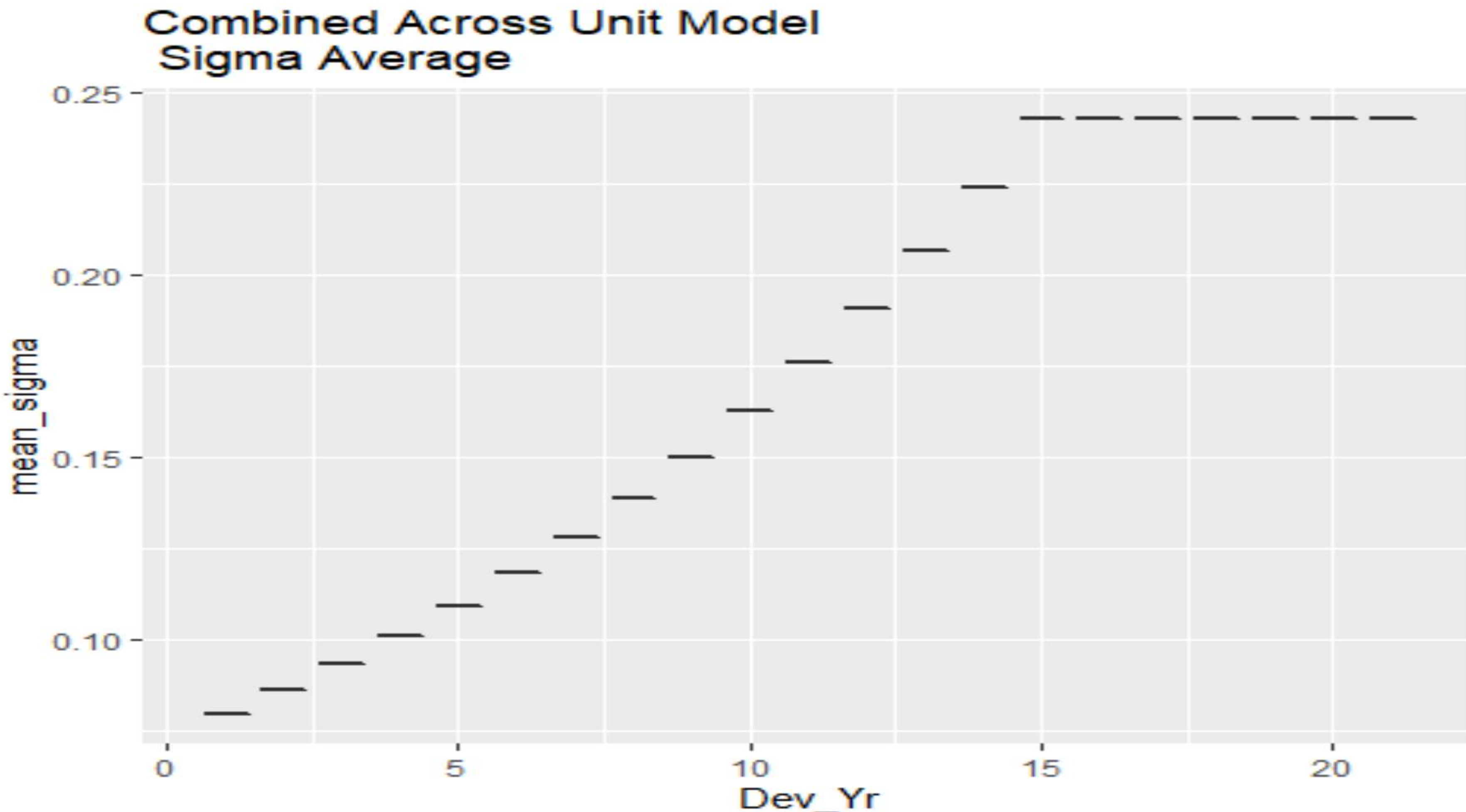
Introduction to Population Priors

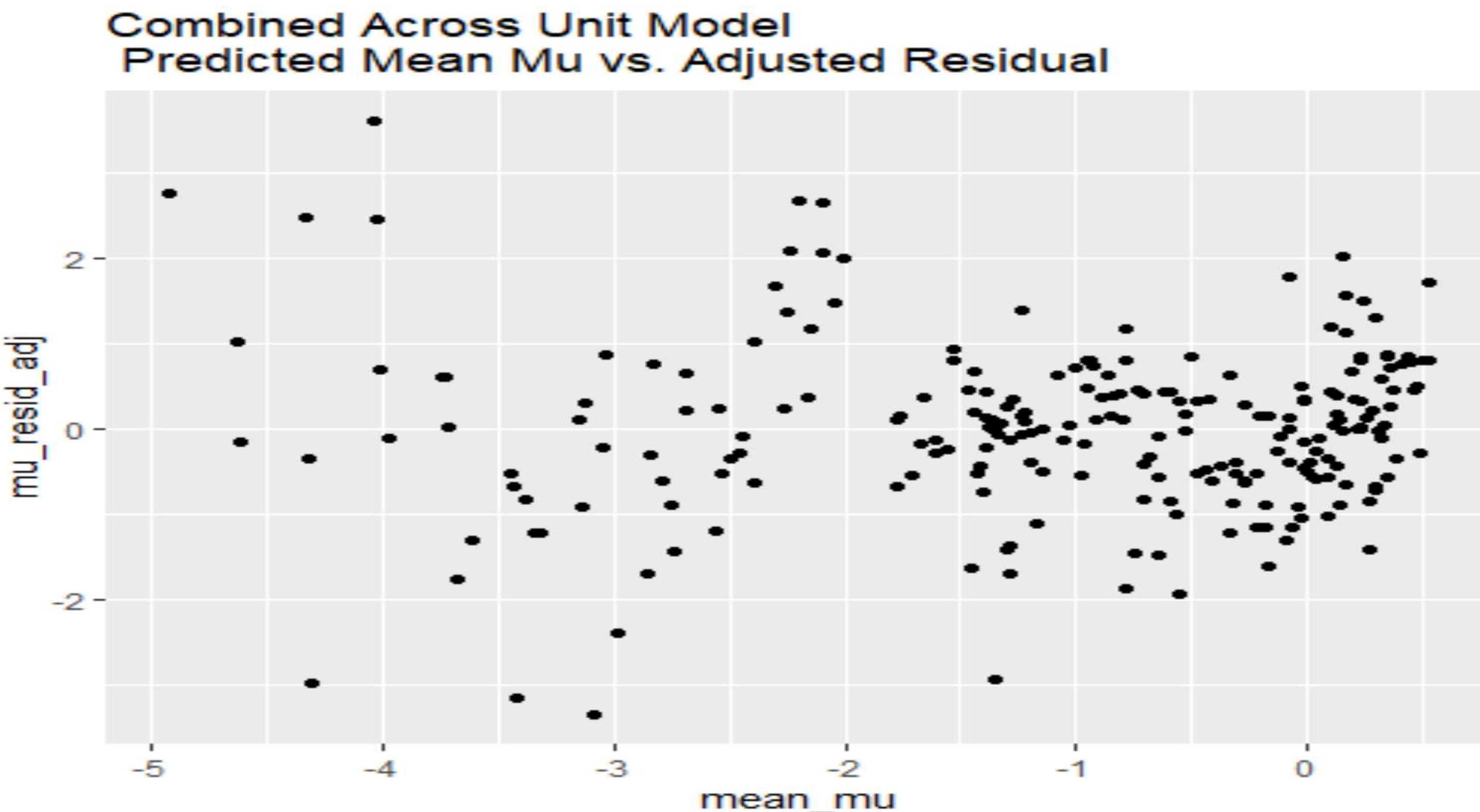
- Sample formula: set_prior("normal(0, 10)", class = "b", coef = "x1")
  - The starting assumption is that the beta for the variable “x1” has a Normal distribution with mu =0 and sigma = 10 (in this case)
  - Starting point for the Bayesian MCMC machinery
  - Means to induce credibility weighting
    - Prior knowledge
    - Information in the data set
  - Documents starting point for modeling exercise
    - Review for reasonability for reserving assumptions
    - Enables replication of estimates

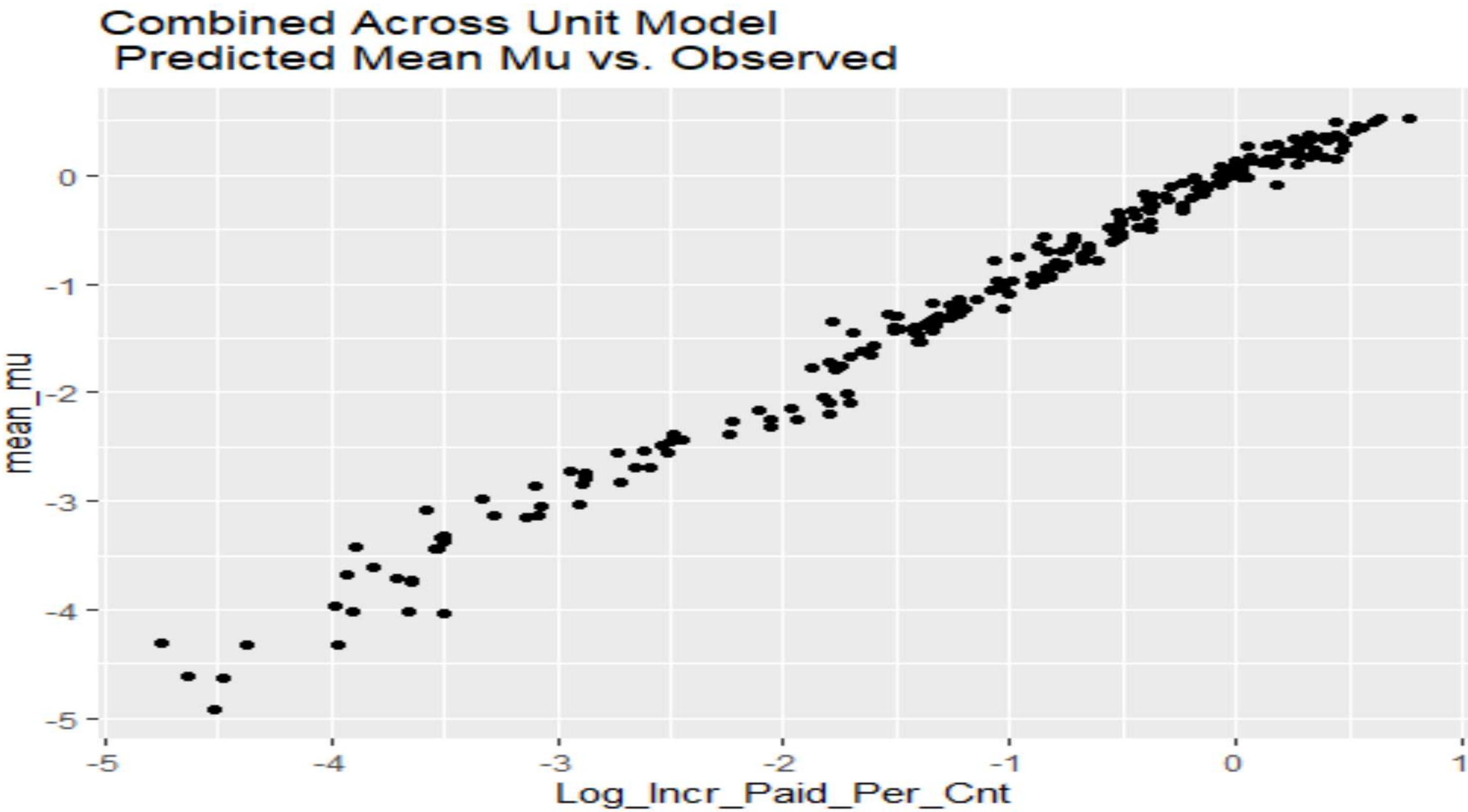


Combined Across Unit Model
Intercept by Accident Year
Accident Year
2020
2015
2010
2005
2000
-2.90
-2.85
-2.80
-2.75
-2.70
Adjusted Intercept

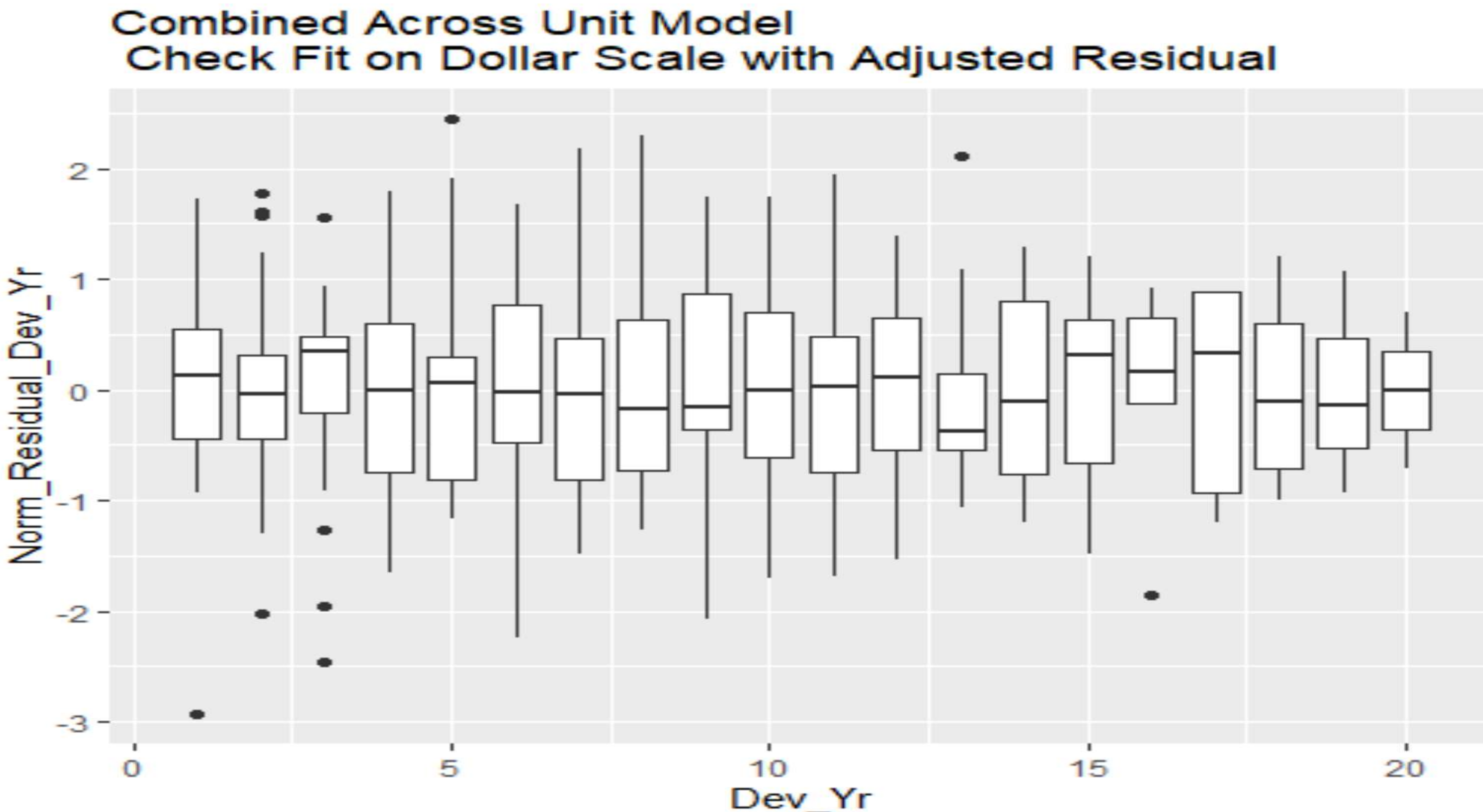

Combined Across Unit Model
Mu Adjusted Residuals
mu_resid_adj
2
0
-2
0
5
10
15
20
Dev_Yr

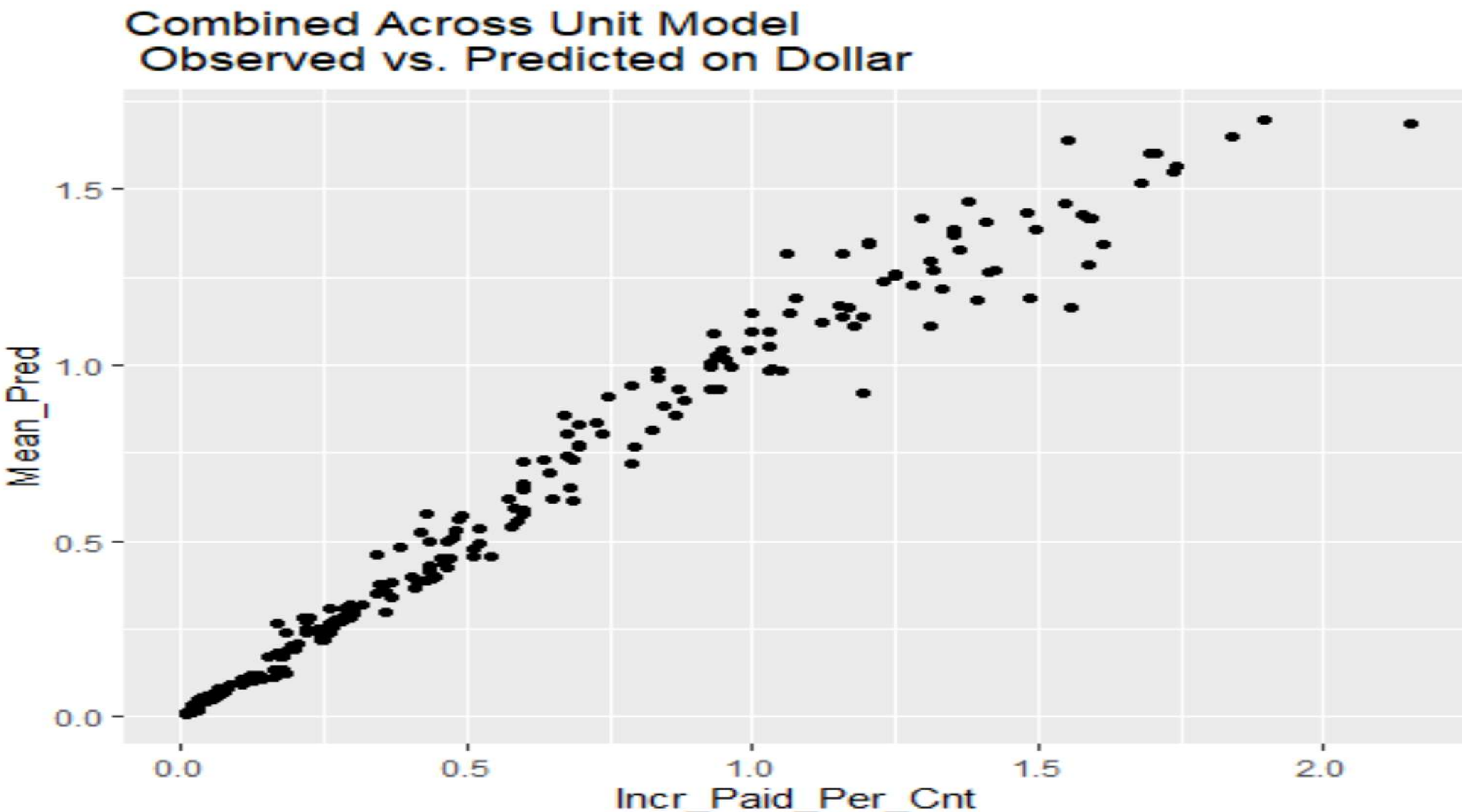

Combined Across Unit Model
Sigma Average
mean_sigma
0.25
0.20
0.15
0.10
0
5
10
15
20
Dev_Yr

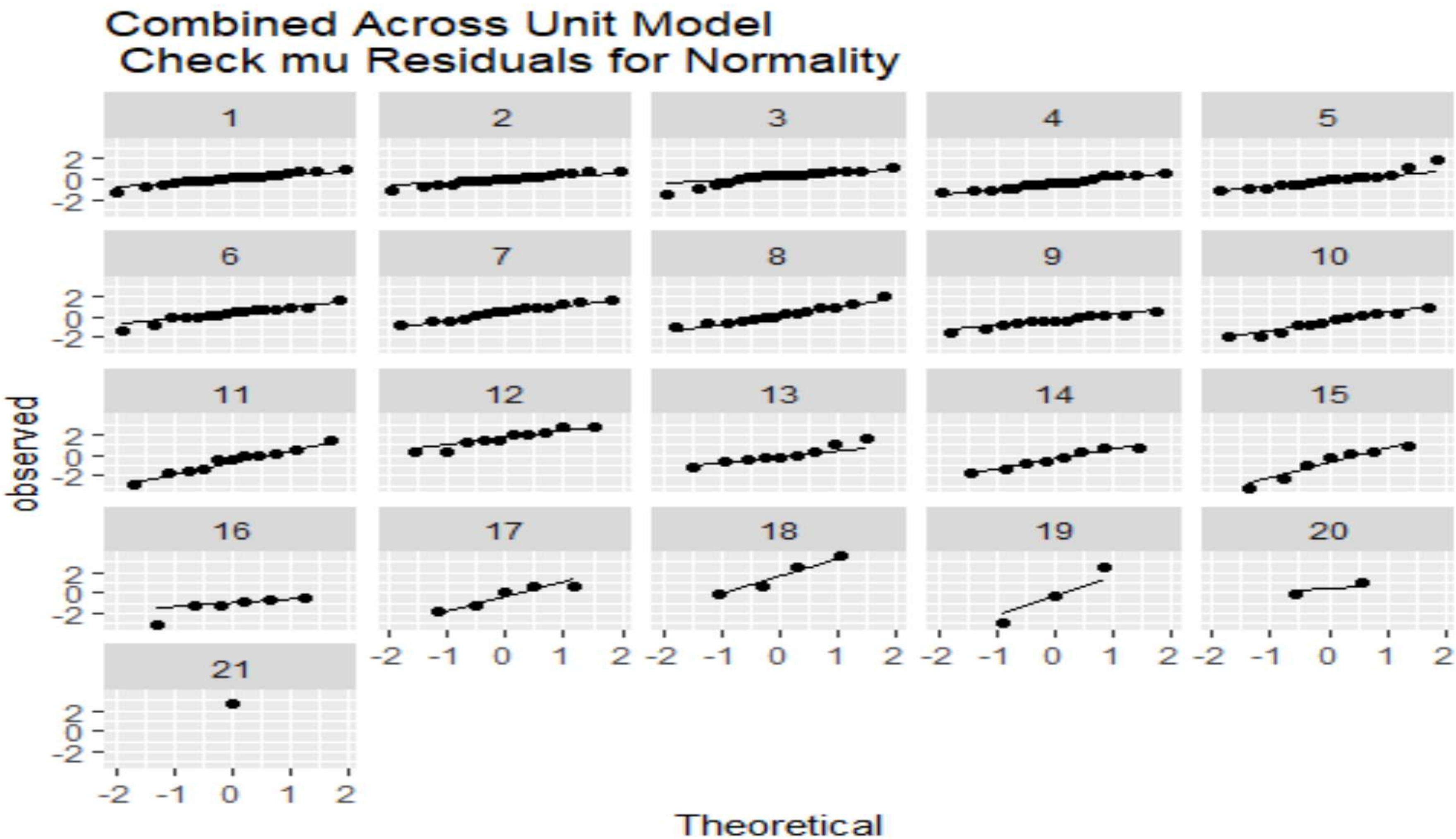

Combined Across Unit Model
Predicted Mean Mu vs. Adjusted Residual
mu_resid_adj
2
0
-2
-5
-4
-3
-2
-1
0
mean_mu

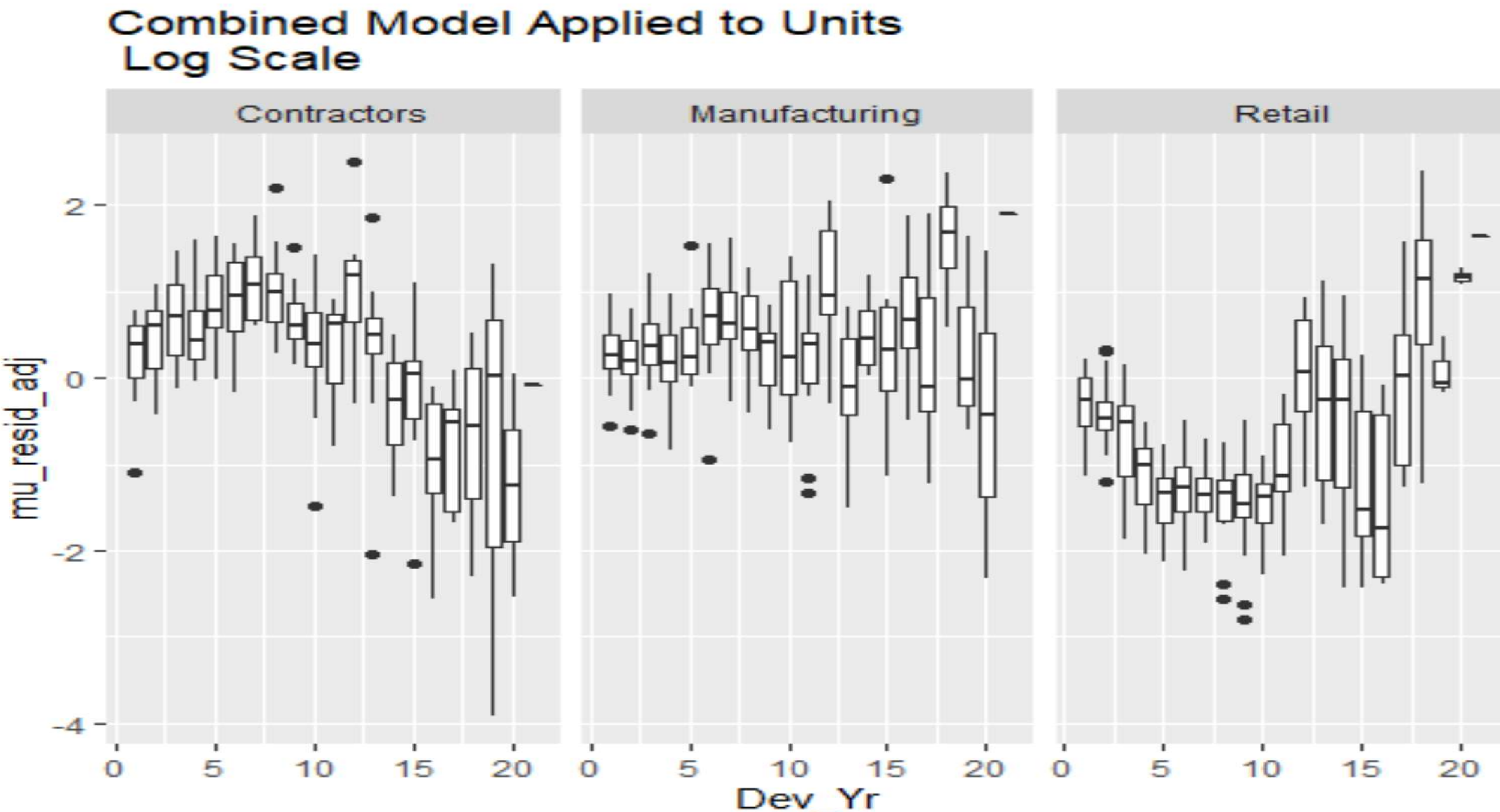


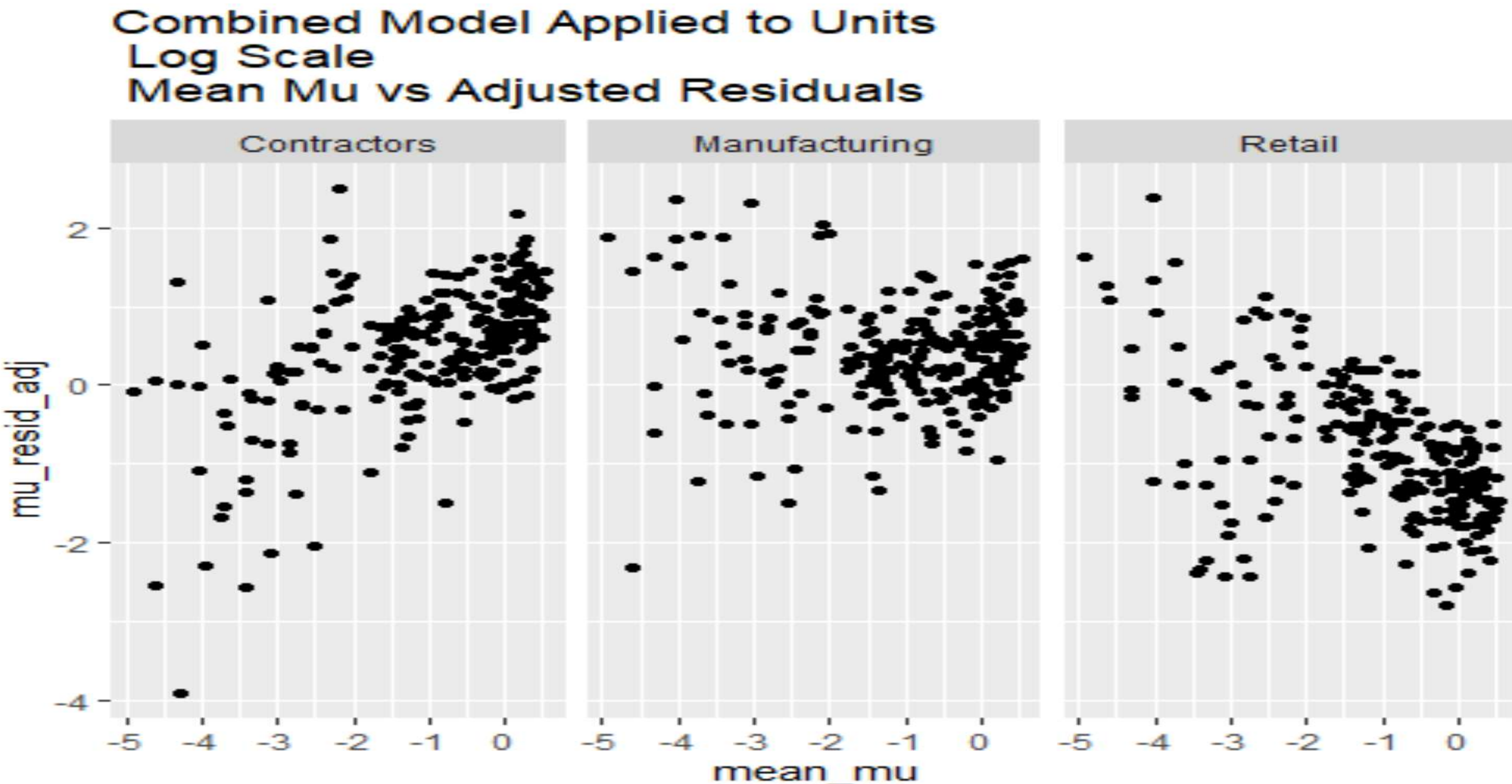
Combined Across Unit Model
Predicted Mean Mu vs. Observed
mean_mu
Log_Incr_Paid_Per_Cnt
0
-1
-2
-3
-4
-5
1

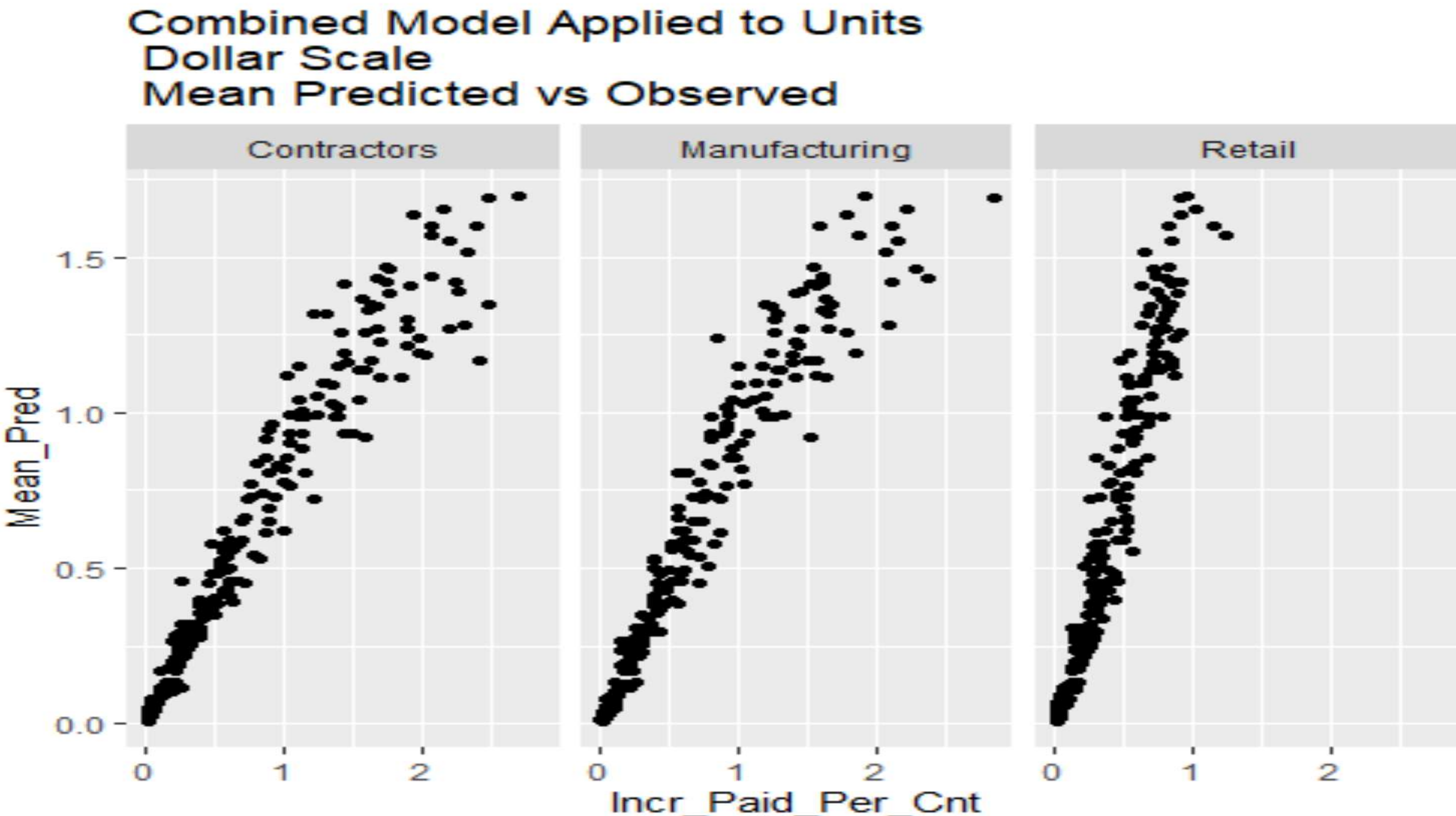

Combined Across Unit Model
Check Fit on Dollar Scale with Adjusted Residual
Norm_Residual_Dev_Yr
2
1
0
-1
-2
-3
0
5
10
15
20
Dev_Yr


Combined Across Unit Model
Observed vs. Predicted on Dollar
Mean_Pred
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
Incr_Paid_Per_Cnt


Combined Across Unit Model
Check mu Residuals for Normality
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
observed
Theoretical
-2
-1
0
1
2


Combined Model Applied to Units
Log Scale
Contractors
Manufacturing
Retail
mu_resid_adj
2
0
-2
-4
0
5
10
15
20
Dev_Yr


Combined Model Applied to Units
Log Scale
Mean Mu vs Adjusted Residuals
Contractors
Manufacturing
Retail
mu_resid_adj
2
0
-2
-4
-5
-4
-3
-2
-1
0
mean_mu


Combined Model Applied to Units
Dollar Scale
Mean Predicted vs Observed
Contractors
Manufacturing
Retail
Mean_Pred
1.5
1.0
0.5
0.0
0
1
2
Incr_Paid_Per_Cnt

By Unit Model Results With Credibility Weighting Using Combined Model Results for Population Variable Priors

Credibility Weighting Using Combined Model Slide Sequence

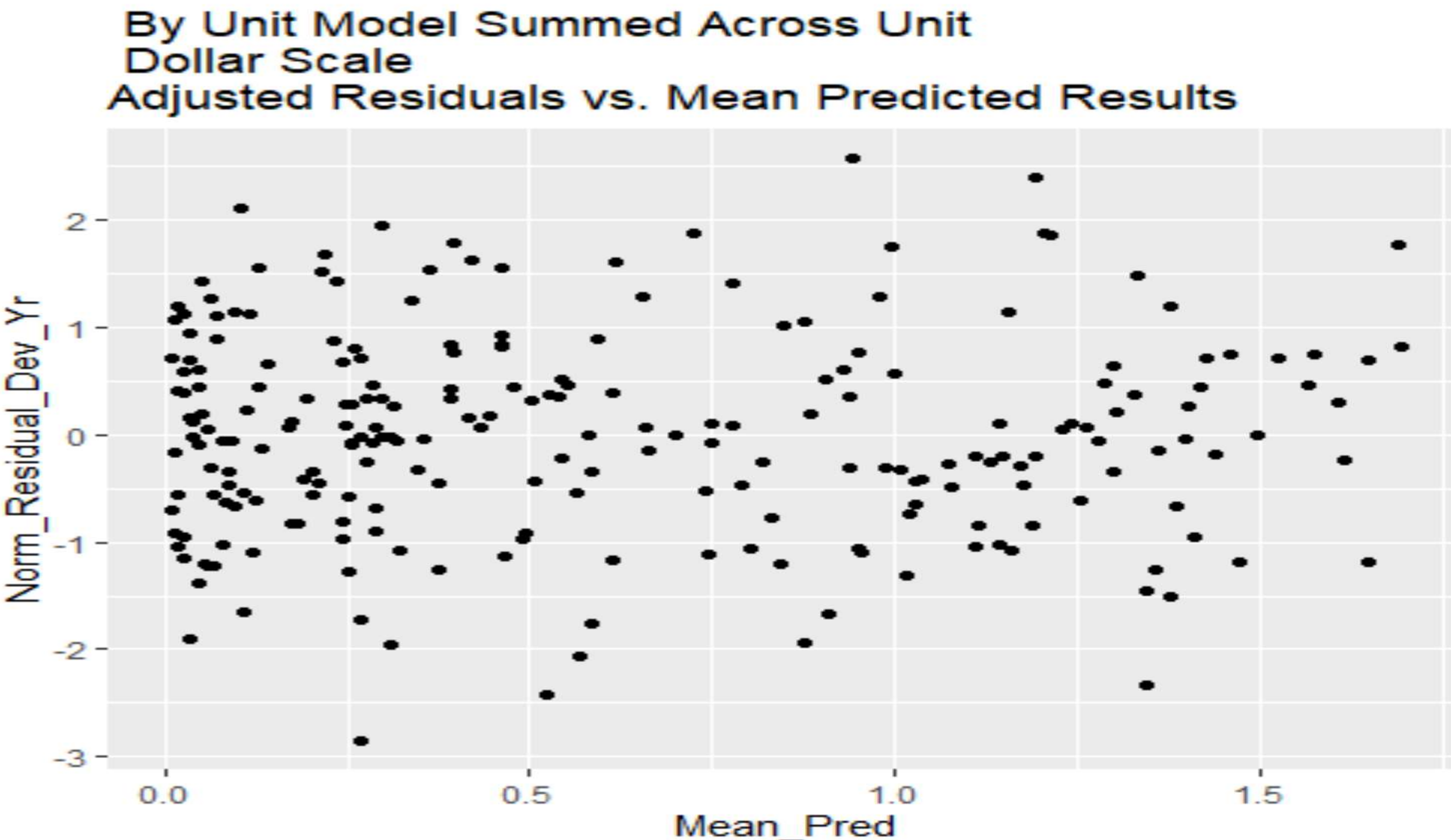
- Set up prior credibility weighting example
- Results for combined across unit view
- By unit view of results

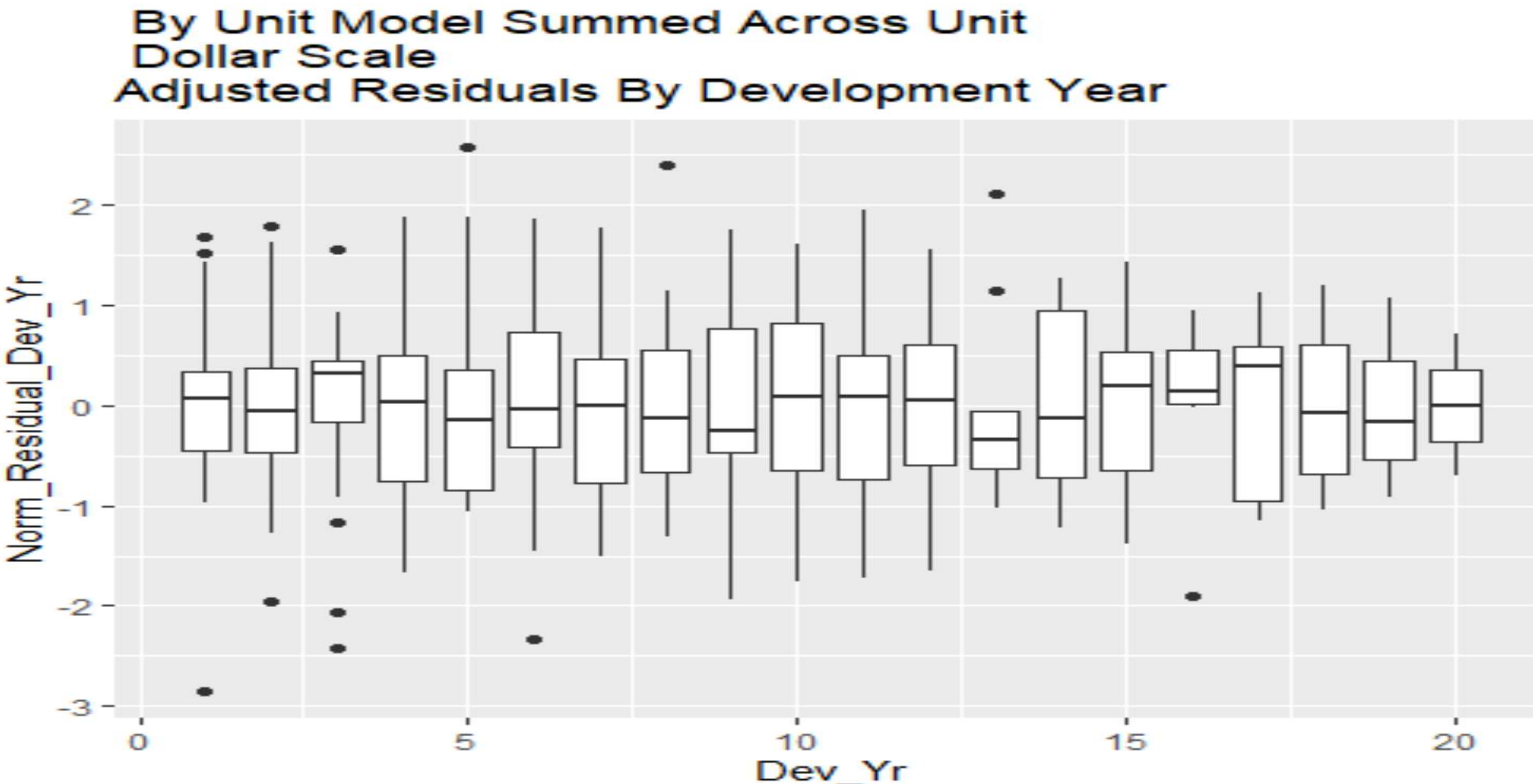
Model Using Combined Model Information In Priors

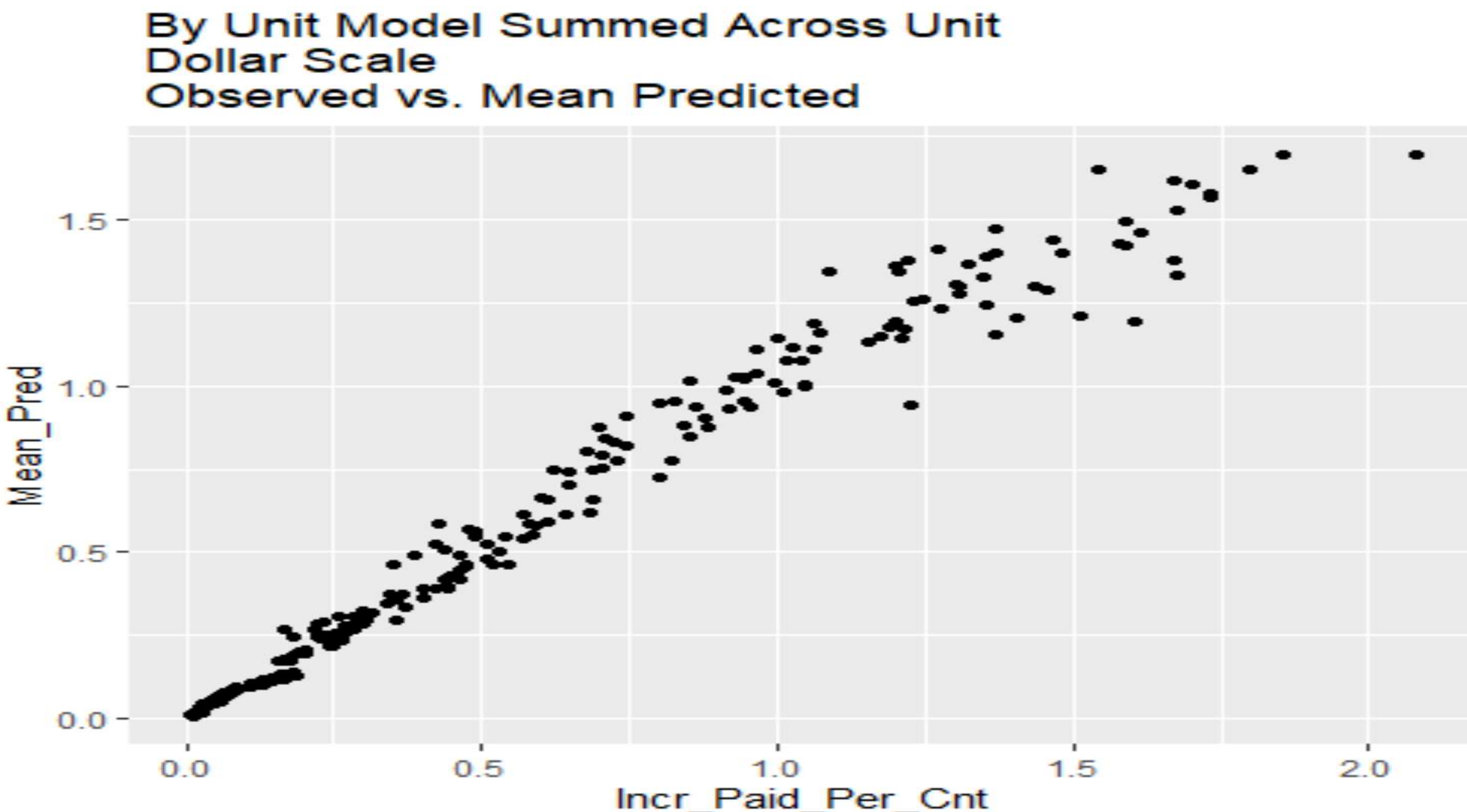
- Dependent variable: Incremental Payment/ Reported Counts DY 1
- Distribution model
  - Lognormal
  - Mu and sigma modeled separately and simultaneously
  - Identify link for mu and exponential link for sigma
- Linear form for mu
  - Parabolic for first 12 development years and interaction with Unit
  - Spline added for development years 13 plus and interaction with Unit
  - Categorical adjustment first 3 development years and interaction with Unit
  - Uniform trend factor by calendar year
  - Credibility adjusted by accident year adjustment to Intercept
- Linear form on transformed scale for sigma
  - Constant increase on transformed scale for first 14 years then constant
- Population priors use information from combined unit model

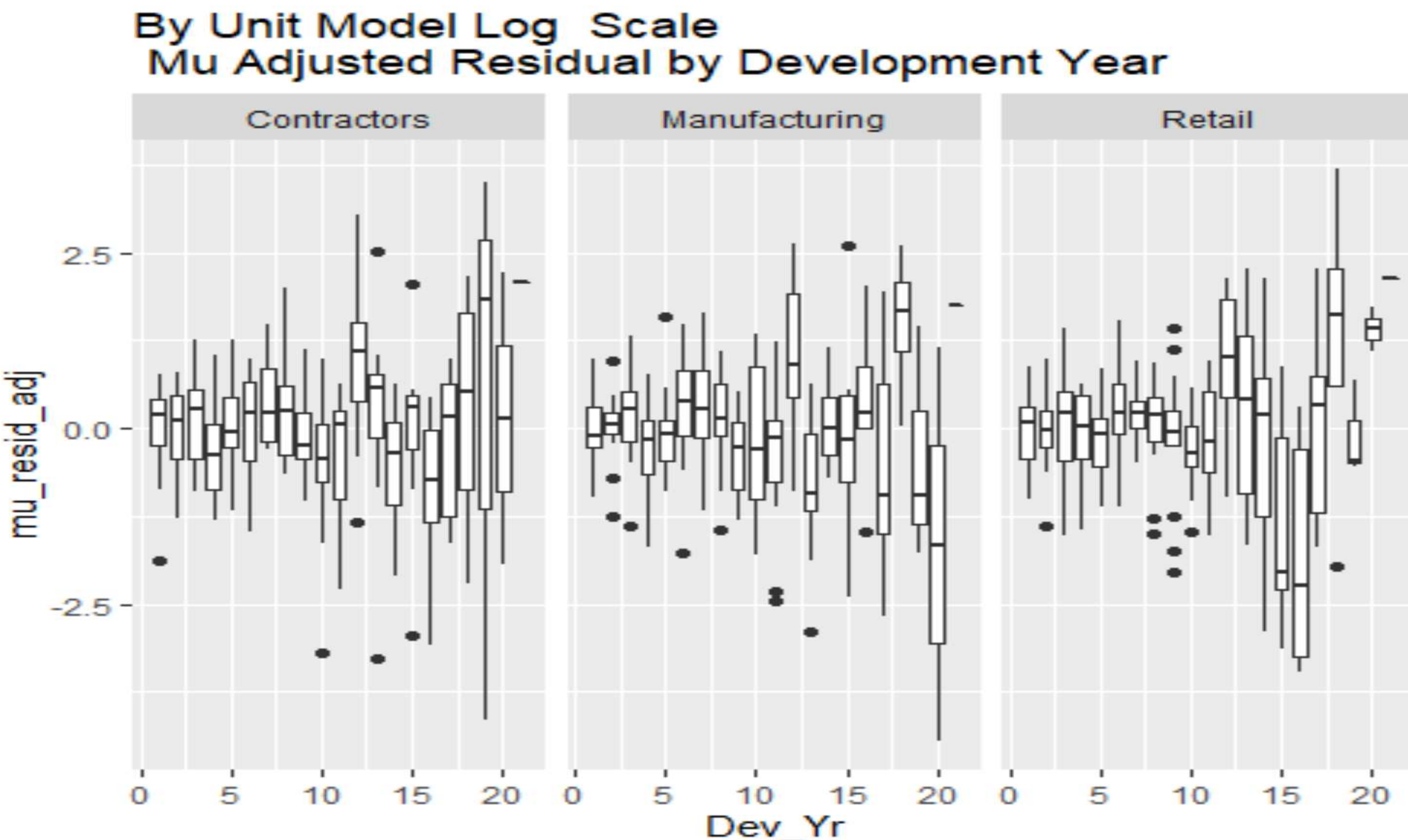
Prior Population Example

- Select one variable: Dev_Yr_12_Cap
- Prior population instructions used:
  - prior(normal(1,.5),class=b, coef=Dev_Yr_12_Cap)
  - prior(normal(0,.5),class=b, coef=Dev_Yr_12_Cap:UnitManufacturing)
  - prior(normal(0,.5),class=b, coef=Dev_Yr_12_Cap:UnitRetail)
- Use results from modeling on combined data for the non-interaction beta as starting point (sigma is judgement adjusted)
- Used 0 as the starting mean assumption for the interaction terms (sigma is judgement selected)

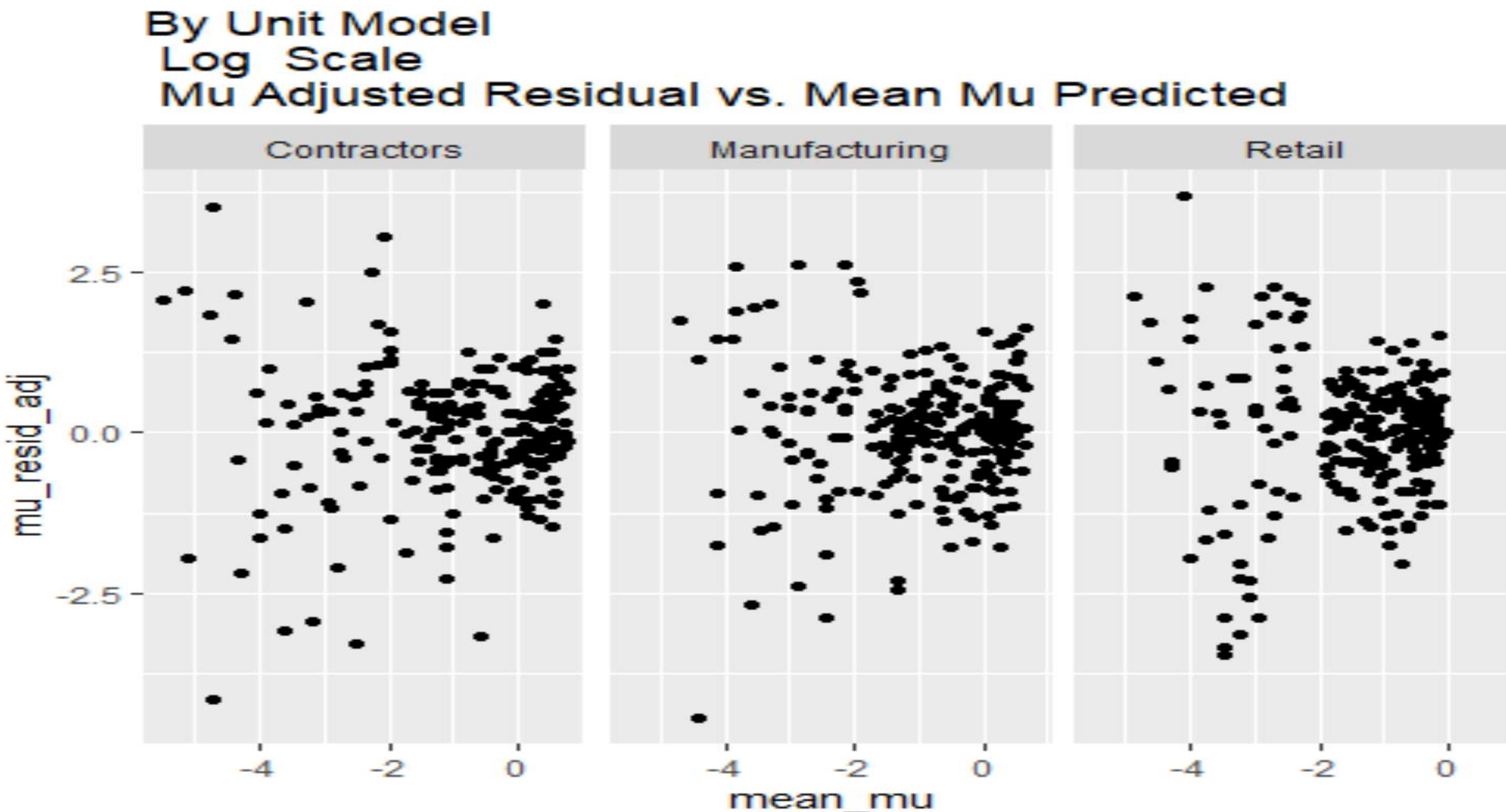

By Unit Model Summed Across Unit
Dollar Scale
Adjusted Residuals vs. Mean Predicted Results
Norm_Residual_Dev_Yr
2
1
0
-1
-2
-3
0.0
0.5
1.0
1.5
Mean_Pred


By Unit Model Summed Across Unit
Dollar Scale
Adjusted Residuals By Development Year
Norm_Residual_Dev_Yr
2
1
0
-1
-2
-3
0
5
10
15
20
Dev_Yr


By Unit Model Summed Across Unit
Dollar Scale
Observed vs. Mean Predicted
Mean_Pred
0.0
0.5
1.0
1.5
0.0
0.5
1.0
1.5
2.0
Incr_Paid_Per_Cnt



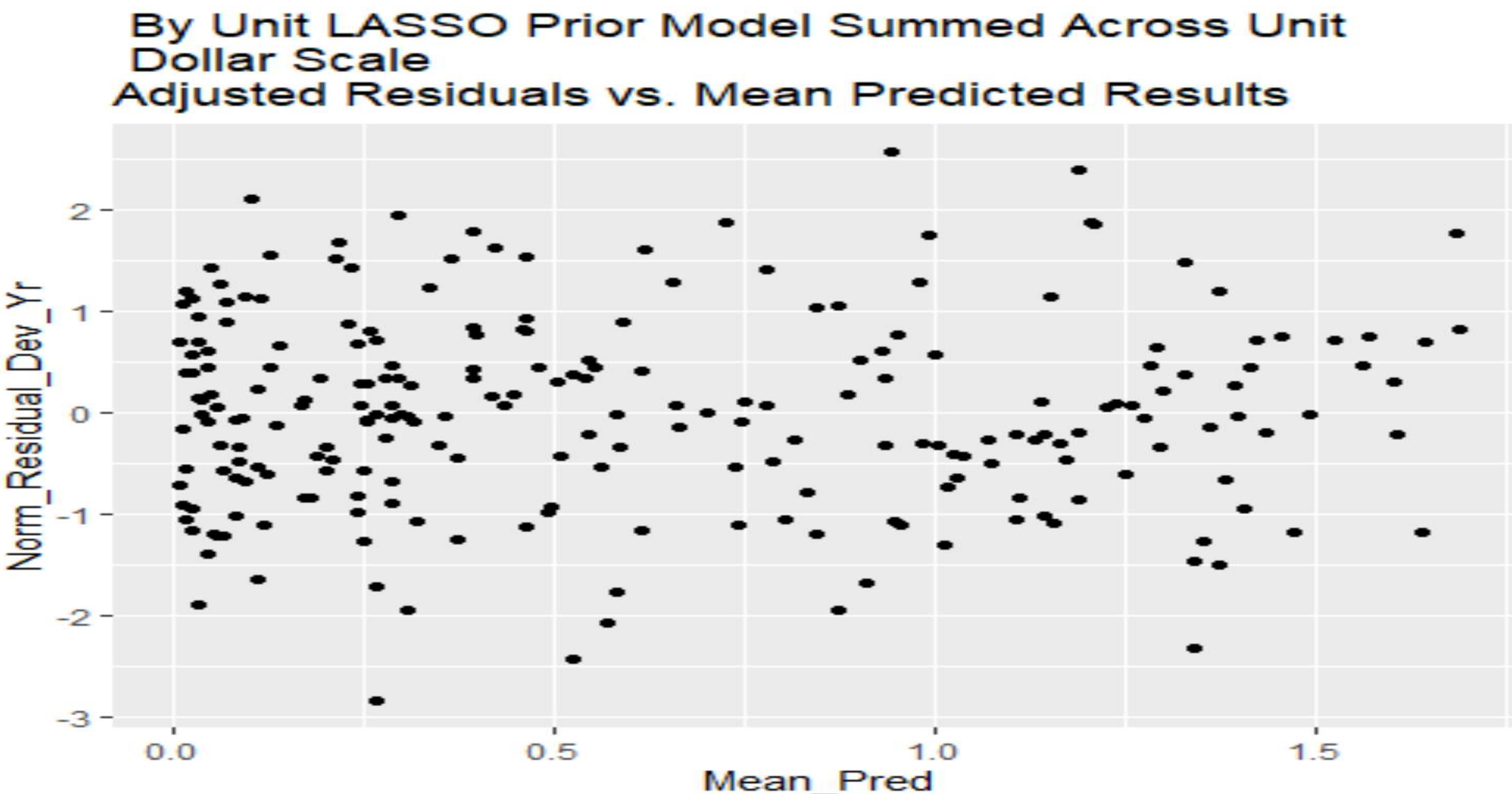
By Unit Model Log Scale
Mu Adjusted Residual by Development Year
Contractors
Manufacturing
Retail
mu_resid_adj
2.5
0.0
-2.5
0
5
10
15
20
Dev_Yr

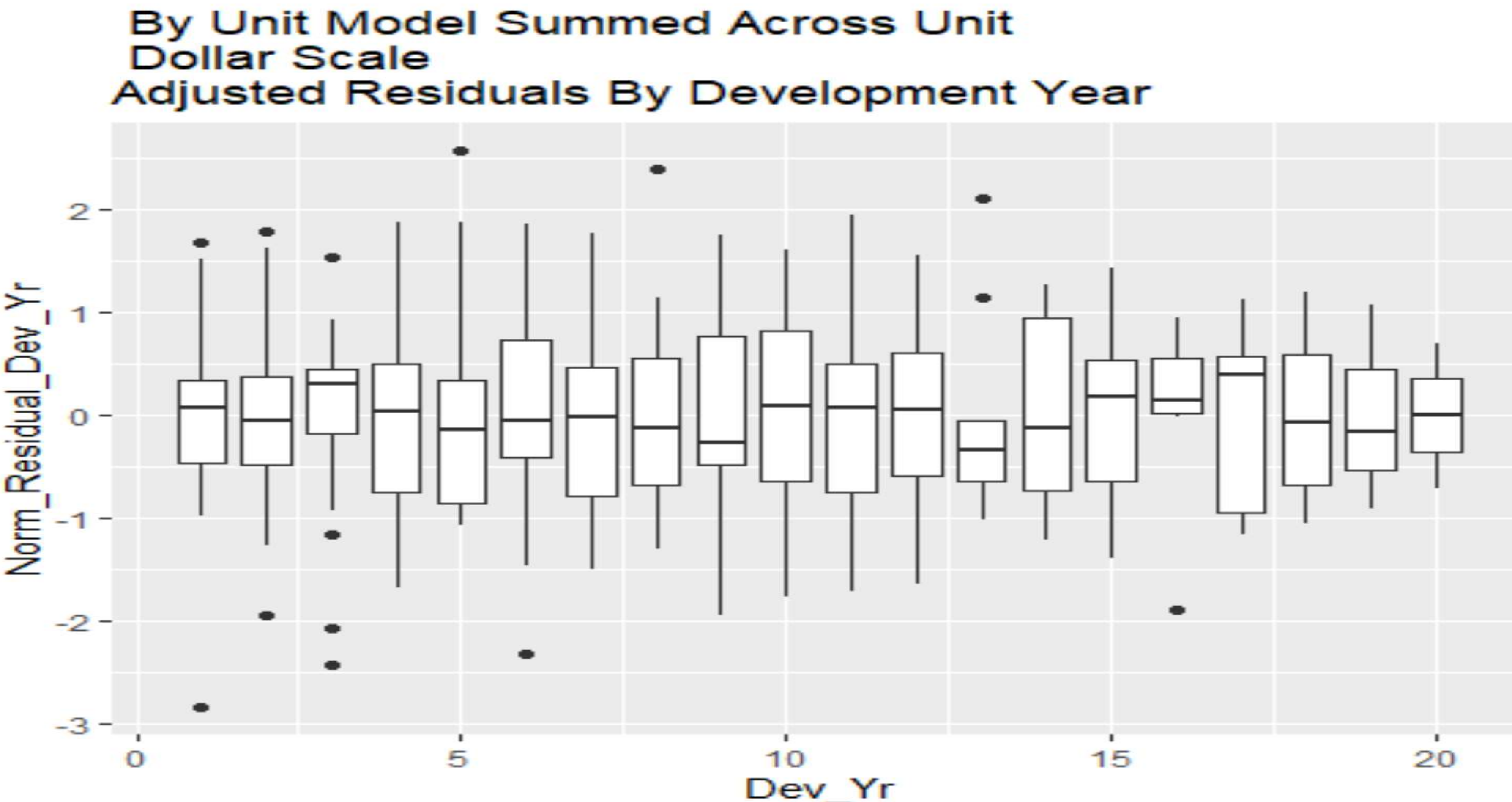

By Unit Model
Log Scale
Mu Adjusted Residual vs. Mean Mu Predicted
Contractors
Manufacturing
Retail
2.5
0.0
-2.5
mu_resid_adj
-4
-2
0
mean_mu

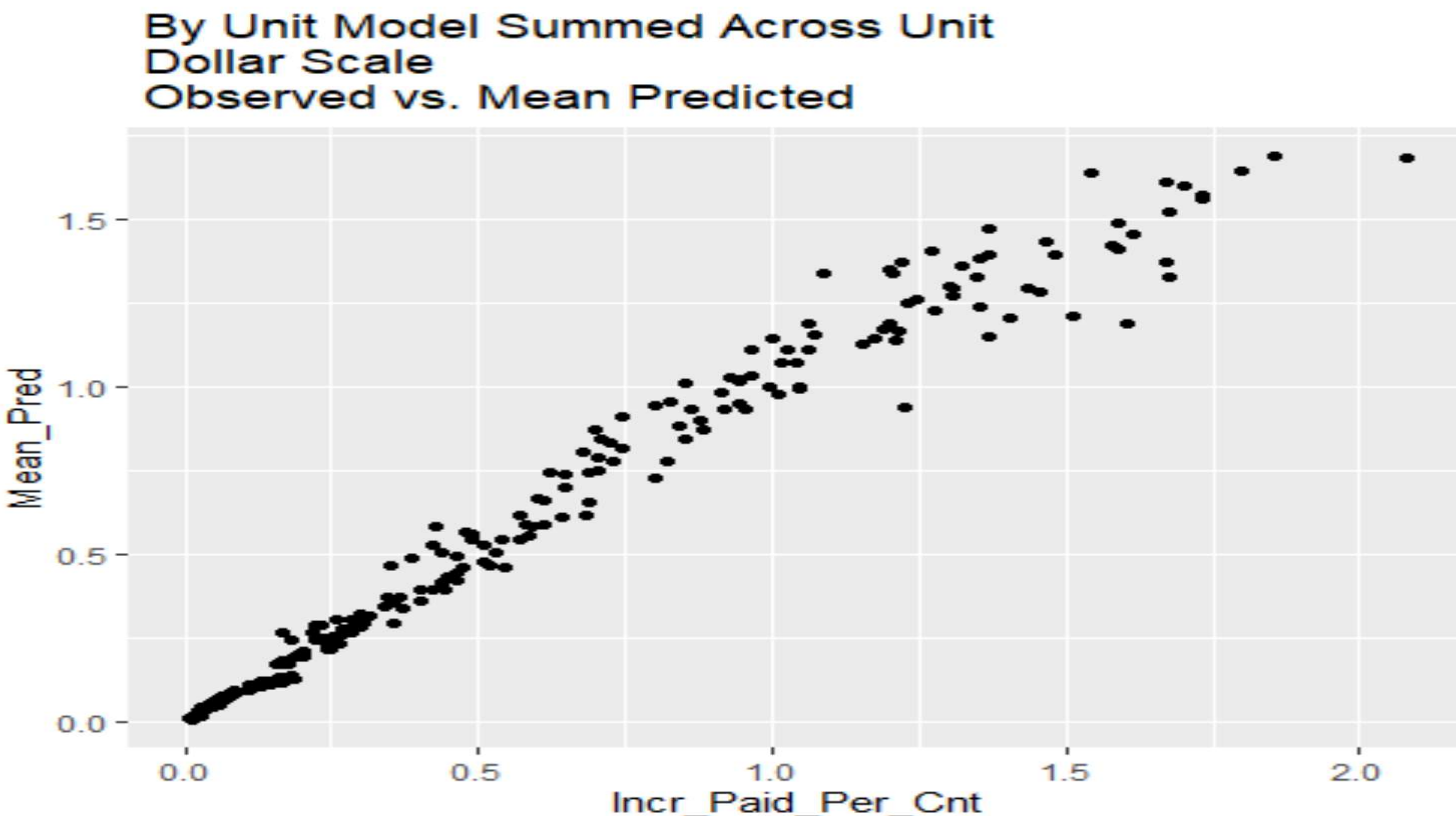
By Unit Model Results With Credibility Weighting Using LASSO Population Variable Priors

Model Using LASSO Priors

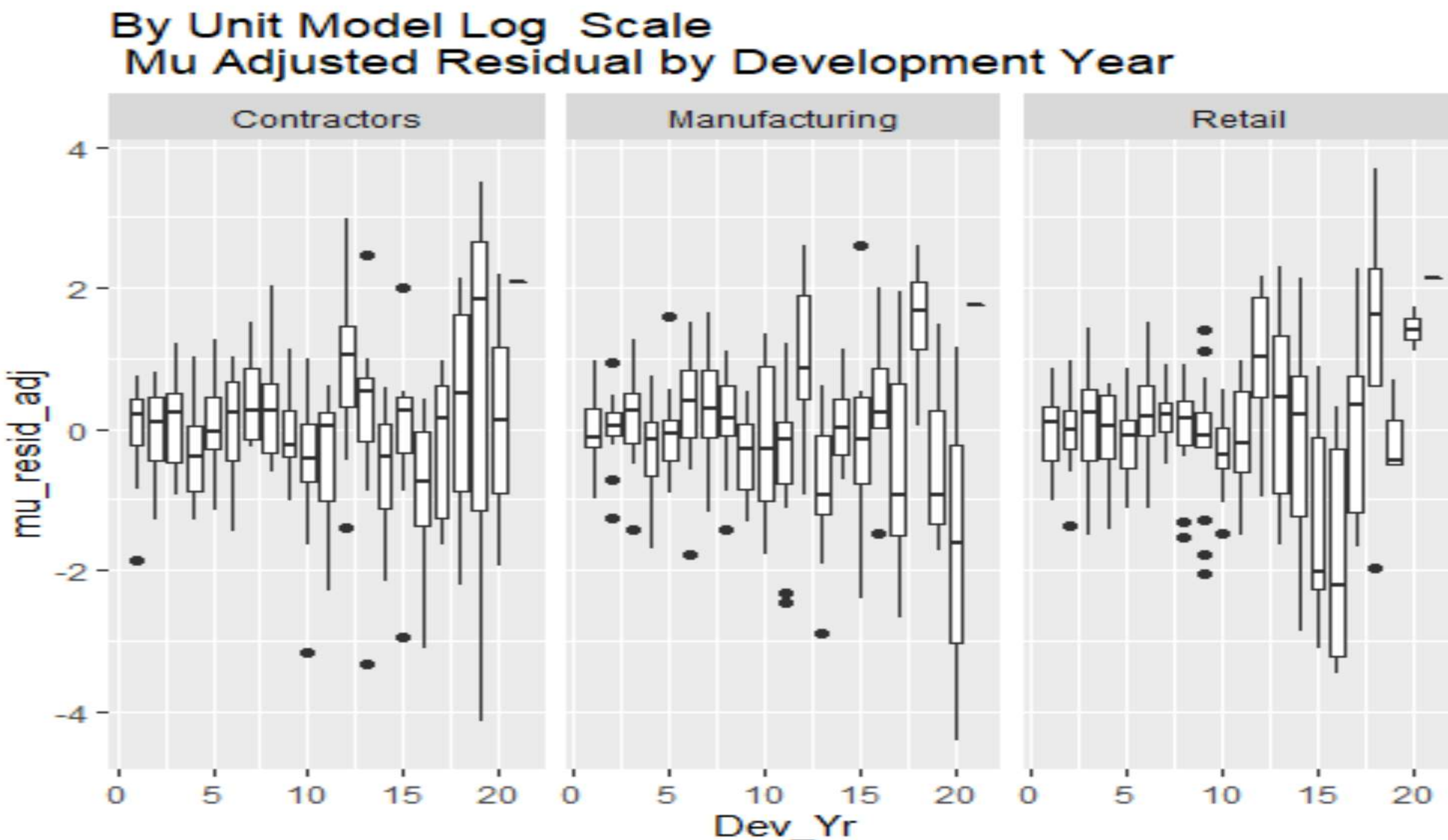
- Dependent variable: Incremental Payment/ Reported Counts DY 1
- Distribution model
  - Lognormal
  - Mu and sigma modeled separately and simultaneously
  - Identify link for mu and exponential link for sigma
- Linear form for mu
  - Parabolic for first 12 development years and interaction with Unit
  - Spline added for development years 13 plus and interaction with Unit
  - Categorical adjustment first 3 development years and interaction with Unit
  - Uniform trend factor by calendar year
  - Credibility adjusted by accident year adjustment to Intercept
- Linear form on transformed scale for sigma
  - Constant increase on transformed scale for first 14 years then constant
- Population priors use LASSO: prior(lasso(df = 1, scale = 10))

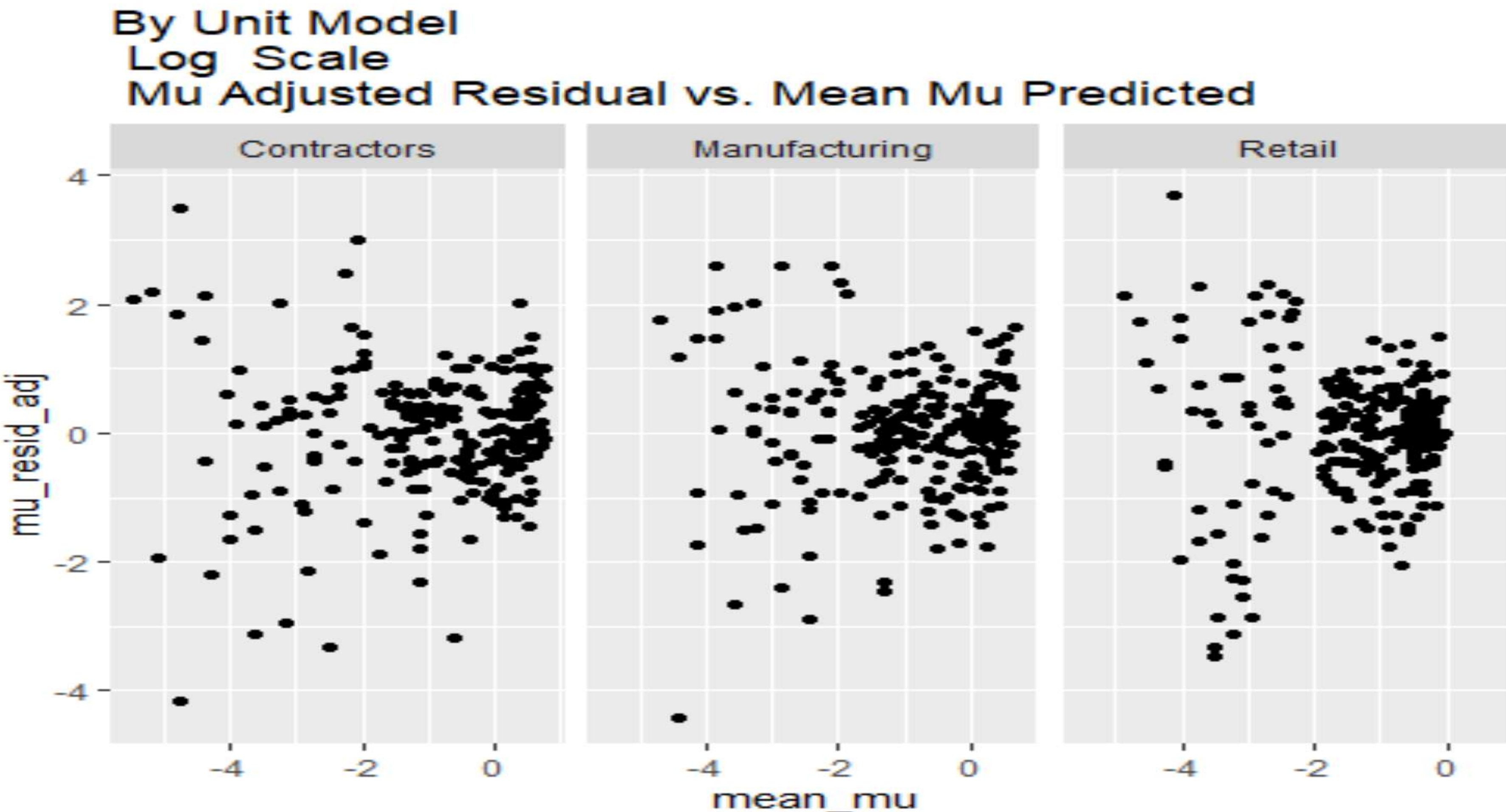

By Unit LASSO Prior Model Summed Across Unit
Dollar Scale
Adjusted Residuals vs. Mean Predicted Results
Norm_Residual_Dev_Yr
Mean_Pred
2
1
0
-1
-2
-3
0.0
0.5
1.0
1.5


By Unit Model Summed Across Unit
Dollar Scale
Adjusted Residuals By Development Year
Norm_Residual_Dev_Yr
2
1
0
−1
−2
−3
0
5
10
15
20
Dev_Yr

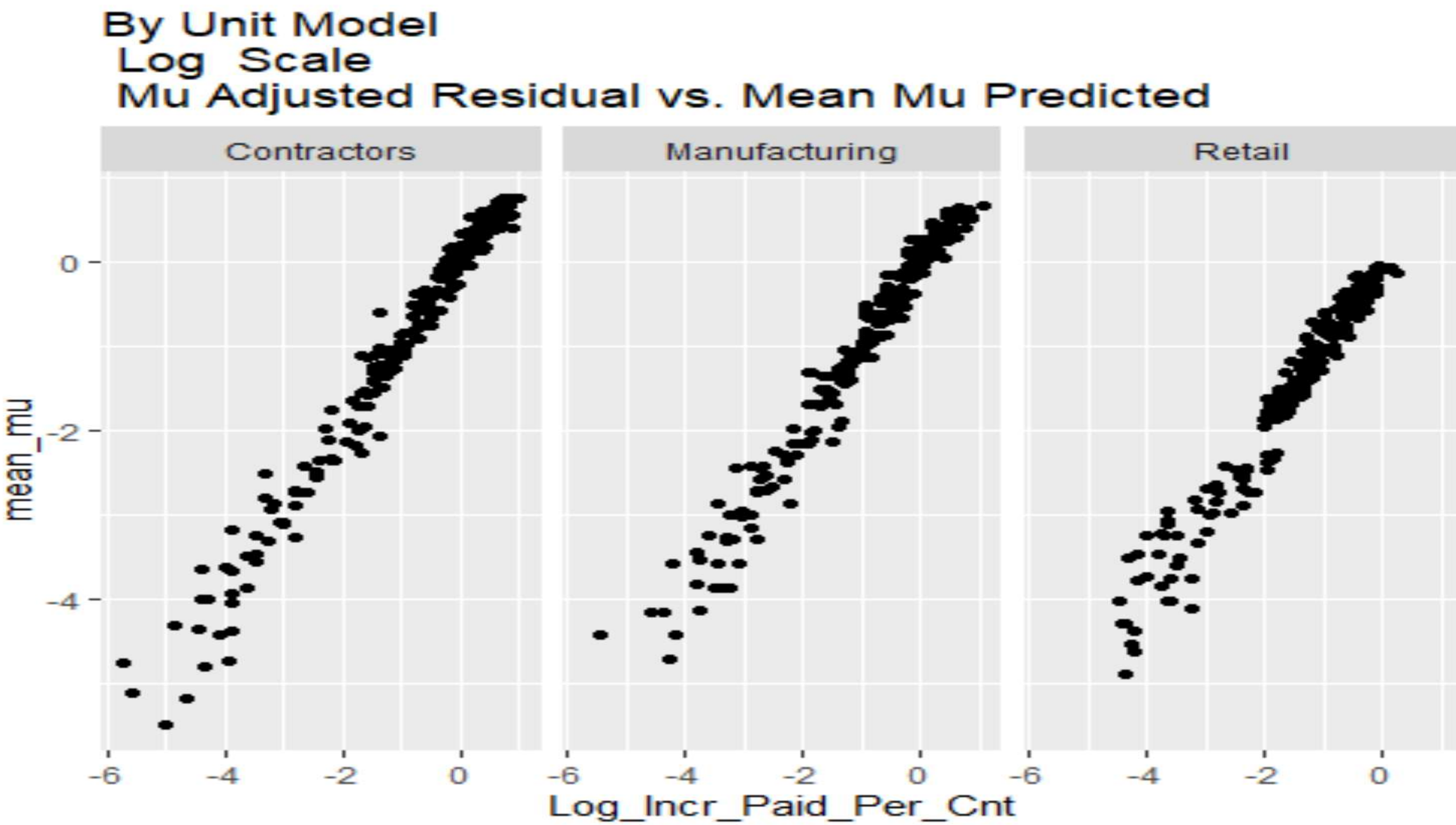


By Unit Model Summed Across Unit
Dollar Scale
Observed vs. Mean Predicted
Mean_Pred
0.0
0.5
1.0
1.5
0.0
0.5
1.0
1.5
2.0
Incr_Paid_Per_Cnt

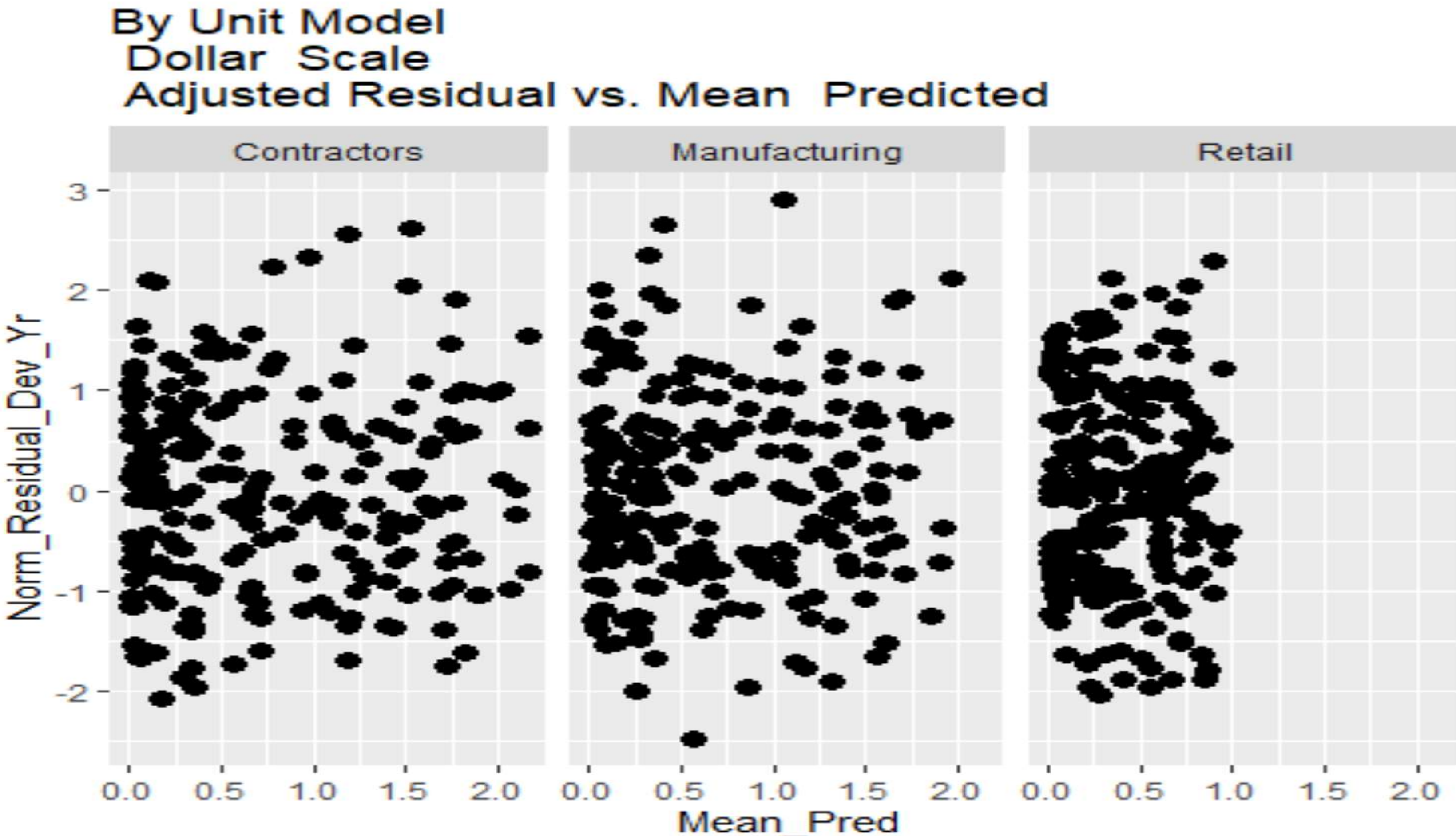

By Unit Model Log Scale
Mu Adjusted Residual by Development Year
Contractors
Manufacturing
Retail
mu_resid_adj
4
2
0
-2
-4
0
5
10
15
20
Dev_Yr

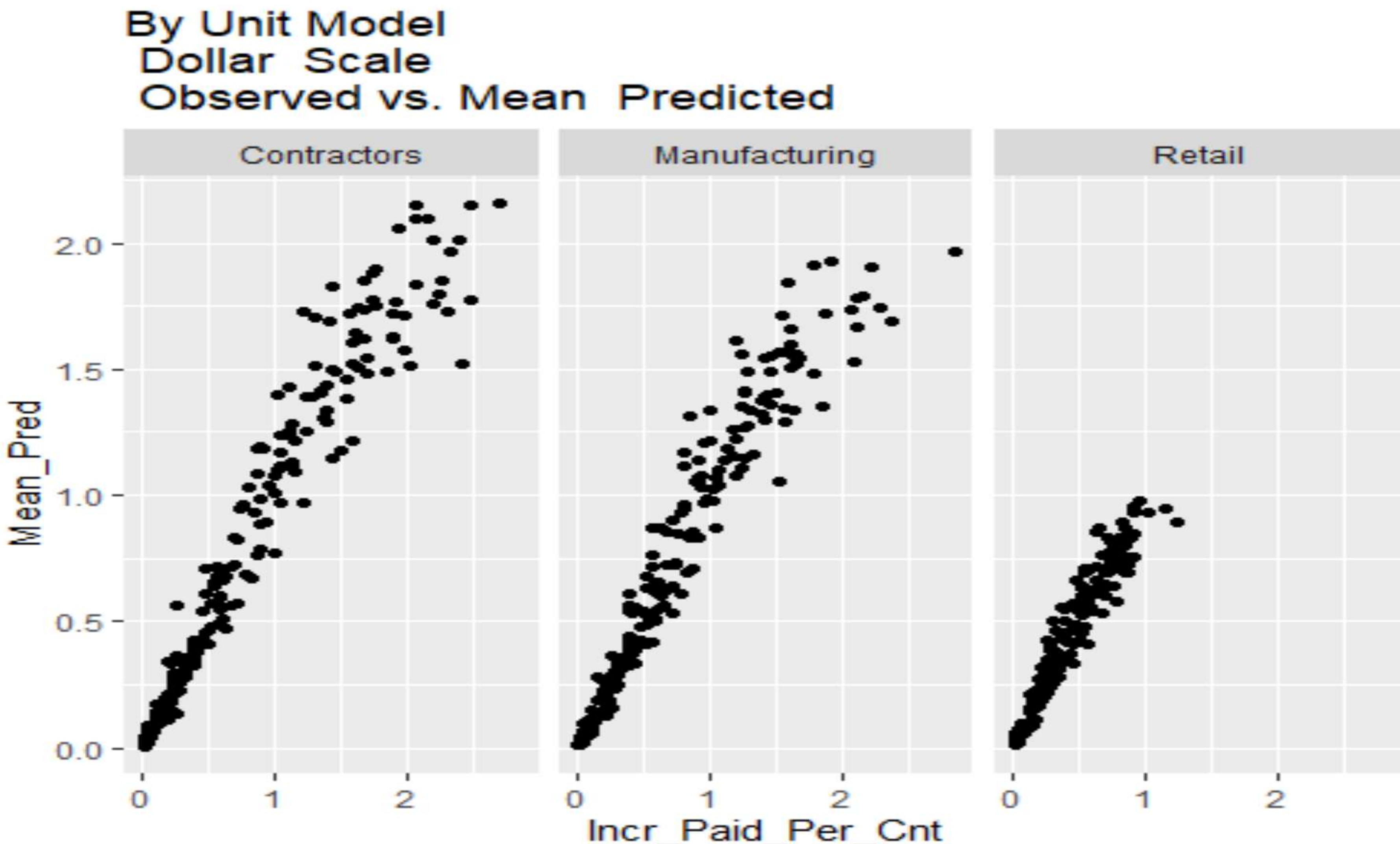


By Unit Model
Log Scale
Mu Adjusted Residual vs. Mean Mu Predicted
Contractors
Manufacturing
Retail
mu_resid_adj
mean_mu
4
2
0
-2
-4



By Unit Model
Log Scale
Mu Adjusted Residual vs. Mean Mu Predicted
Contractors
Manufacturing
Retail
mean_mu
0
-2
-4
-6
-4
-2
0
Log_Incr_Paid_Per_Cnt


By Unit Model
Dollar Scale
Adjusted Residual vs. Mean Predicted
Contractors
Manufacturing
Retail
Norm_Residual_Dev_Yr
Mean_Pred
3
2
1
0
-1
-2
0.0
0.5
1.0
1.5
2.0


By Unit Model
Dollar Scale
Observed vs. Mean Predicted
Contractors
Manufacturing
Retail
Mean_Pred
2.0
1.5
1.0
0.5
0.0
0
1
2
Incr_Paid_Per_Cnt

Poll Questions on Multiple Business Units

- Is this a situation you have come across in practice in reserving? (Yes, No, NA)
- Is the approach outlined something you would consider? (Yes, No, Maybe)
- Should the CAS provide case studies showing how to apply this technique including the code required to create the simulated data sets and run the modeling packages?(Yes, No)

New Line of Business Outline

Reserving New Line Work Flow Outline

- Information for development curve beyond data for new line
  - Industry sources
  - In house information from underwriters & claims
- Set up priors for Bayesian MCMC
  - Experiment to find plausible curve starting point
- Example
  - Assume lognormal distribution
  - Graph two options
  - First step in setting up plausible priors

Example Options for Development Curve

Option 1

- Mu parameters

mu	Intercept	-1
	first_degree	0.2
	2nd degree	-0.05

- Sigma parameters

sigma	Intercept	-0.5
	first_degree	0.1
	2nd degree	0

Option 2

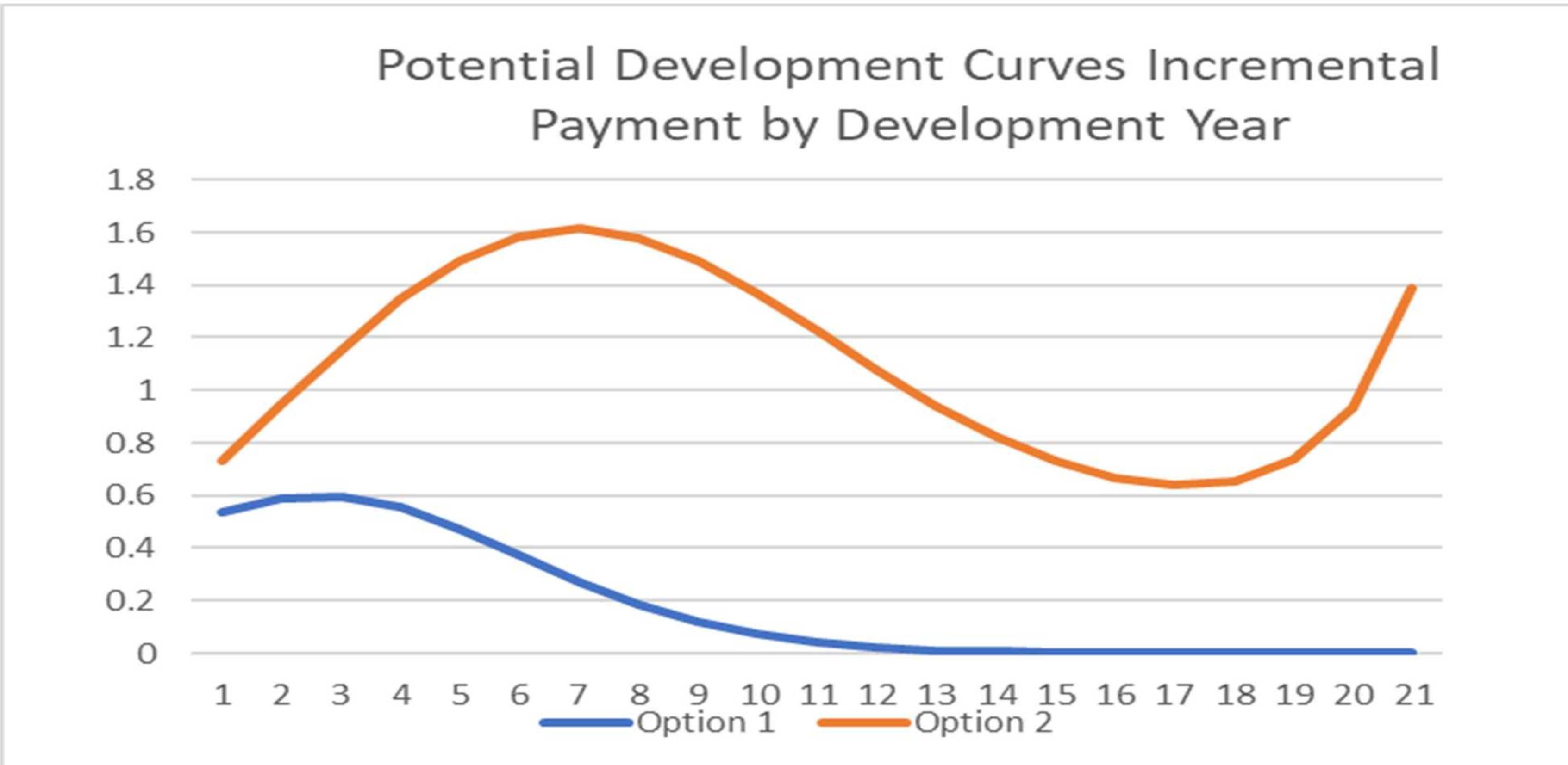
- Mu parameters

mu	Intercept	-0.8
	first_degree	0.3
	2nd degree	-0.03

- Sigma parameters

sigma	Intercept	-0.5
	first_degree	0.09
	2nd degree	0

Compare Development Curves



Example Defining One Parameter Prior

- Assume you have only two choices: Option 1 or Option 2
- Option 2 is implausible: payments should end eventually
- First part of defining one prior for mu
  - First degree beta prior using Dev_Yr
  - prior(normal(.2, ?),class=b, coef=Dev_Yr)
  - Standard deviation for that beta is yet to be determined
    - Simulate with different standard deviations to see range of curves
    - Select using knowledge of the line and simulation results

Poll Questions on New Business Unit

- Is this a situation you have come across in practice in reserving? (Yes, No, NA)
- Is the approach outlined something you would consider? (Yes, No, Maybe)
- Should the CAS provide case studies showing how to apply this technique including the code required to create the simulated data sets and run the modeling packages?(Yes, No)

Conclusion

- Bayesian MCMC expands regression type models
  - Include credibility concepts to temper results
  - Informed use of priors limits the scope of parameter values to evaluate making modeling subsets of data or small data sets practical
  - Non-linear models are possible (we did not cover that in this example)
  - Can model complete distribution parameters
  - Can measure correlation (we did not cover that in this example)
- Credibility weighting choices in Bayesian MCMC document your starting beliefs on reserve behavior
- Prior parameters allow you to link results from one analysis to the next