



ENTERPRISE RISK
MANAGEMENT SYMPOSIUM

Enterprise Risk-Reward Optimization: Two Critical Approaches

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Goals for Today's Overview

- Risk modeling considerations
- Maximizing return on economic capital
- The mean-semivariance efficient frontier
- A mostly satisfied audience* (and a couple of confused looks)

**no guarantees of audience satisfaction are stated or implied*

Modeling Prerequisites

- Assume line of business (LOB) *distributions* of earnings have been modeled and vetted; include P&L and balance sheet modeling
- Each LOB distribution is a cumulative density function (CDF) for earnings over the upcoming year (Plan horizon)
- Models include financial statement details such as reserves, invested assets, GAAP/STAT earnings, capital, and distributable earnings

The Enterprise Risk-Reward Model

- Can use Excel: the model contains a sheet (tab) for each LOB
- Model contains financial statements in each LOB sheet: beginning of year balance sheet and stochastically simulated income statement and end of year balance sheet line items
- Rank correlations of earnings are reflected across LOBs through a non-parametric approach; we may randomly simulate each LOB's financial results and properly aggregate to the enterprise level

Model Output

- LOB level Plan year detail: simulated average assets, earnings, infusion (shortfall vs. targeted capital), etc.
- Enterprise detail: simulated company average assets, earnings, and *net* capital infusion need from LOBs
- distributable earnings (DE)
 - $DE = \text{statutory earnings} - \Delta \text{ targeted capital}$
 - Reflects capital needs; good proxy for “free cash flows”
 - *Negative DE is interpreted as a need for a capital infusion from the holding company/parent or “Corporate”*
 - DE distributions are developed at both the LOB level and enterprise level, reflecting cross-LOB correlations

Definitions

- Assume each LOB has *negative* DE at the 1st percentile level of its respective distribution for DE
- For a particular LOB “x” we have:
 - A_x = average asset level during forecast plan (includes planned surplus & any trapped capital)
 - I_x = | 1st percentile of DE |
- So A_x captures expected asset usage and I_x measures risk in terms of 1st %ile infusion

Sample DE and Infusion Results: 3 LOB Example

Simulation #	Simulated Annual Distributable Earnings				Scenario Infusions at:			
	LOB1	LOB2	LOB3	Enterprise	LOB1	LOB2	LOB3	Enterprise
1	75	119	59	253	0	0	0	0
2	2	15	61	78	0	0	0	0
3	3	119	225	347	0	0	0	0
4	68	114	225	407	0	0	0	0
5	26	-27	-26	-27	0	27	26	27
6	70	-7	-34	29	0	7	34	0
7	46	118	115	279	0	0	0	0
8	7	-9	40	38	0	9	0	0
9	41	115	209	365	0	0	0	0
10	40	112	148	300	0	0	0	0

*Correlation reflected across LOBs; enterprise DE is sum of LOB DE values in a given simulation

** Important: Enterprise infusion need exists when enterprise DE is negative; allows for offsets from a LOB with positive DE in a particular simulation

*** Enterprise infusion = $\min(0, |\text{enterprise DE}|)$

Simulation Results

- Average asset and infusion values for each LOB: A_1, A_2, A_3 & I_1, I_2, I_3
- *Distribution of DE_{ENT}* (CDF of enterprise DE)
- Assume: 1st percentile of DE_{ENT} is a negative value and find $I = |1^{\text{st}} \text{ percentile of } DE_{ENT}|$
- Typically $I \neq I_1 + I_2 + I_3$

Sample Numerical Results

- $A_1 = 500, A_2 = 1000, A_3 = 2400$
- $I_1 = 20, I_2 = 30, I_3 = 50$
- 1st percentile of enterprise DE is $I = 80$
- Note that $I = 80 < I_1 + I_2 + I_3 = 100$

Economic Capital

- Economic capital (EC): amount of capital sufficient to ensure:
 - business can continue to function under stress scenarios, specific percentile or CTE, and (possibly also...)
 - meet other company specific goals or constraints, e.g. rating agency or regulator target capital levels
- EC often regarded as “scarce” resource to be allocated through a risk-reward lens

A Risk Buffer

(or the Rainy Day Fund)

- Corporate will hold a risk *buffer* (\$**B**) to cover potential LOB *infusion* needs
- Recall, enterprise 1st percentile infusion = I :
 - $I = 80 < I_1 + I_2 + I_3 = 100$
 - Buffer mitigates this infusion risk
 - Company policy: $I \leq \text{buffer} \leq I_1 + I_2 + I_3$

Enterprise and LOB EC

- For a fixed choice of B (e.g. 90) define
 - Enterprise $EC = EC_{ENT} = A_1 + A_2 + A_3 + B = 500 + 1000 + 2400 + 90 = 3990$
 - Each LOB's proportionate *share of buffer B* is:
 $\alpha_j = I_j / \sum I_k$ for each $j = 1, 2, 3$
- Define EC for LOBs using the above:
 - $EC_1 = A_1 + \alpha_1 B = 500 + 20\% * 90 = 518$
 - $EC_2 = A_2 + \alpha_2 B = 1000 + 30\% * 90 = 1027$
 - $EC_3 = A_3 + \alpha_3 B = 2400 + 50\% * 90 = 2445$
 - So $EC_{ENT} = EC_1 + EC_2 + EC_3$

LOB DE as a Function of Allocated EC

- Often more risk, as measured by EC, can lead to greater expected reward (DE)
- A larger EC allocation to a LOB means a larger value for some values among A, α , or B
- Based on LOB Plan values for DE_i and EC_i we find approximate linear relations:
 - $DE_i \approx a_i * EC_i + b_i$ for each LOB $i=1,2,3$

Return on Economic Capital

- Return on EC (ROEC) = enterprise DE/enterprise EC = $(DE_1 + DE_2 + DE_3)/(EC_1 + EC_2 + EC_3)$
- Write this as $(DE_1 + DE_2 + DE_3)/(x_1 + x_2 + x_3)$
- Recall each DE_i in the numerator can be replaced as a linear function of its respective capital level, x_i
- So we may write ROEC = $(a_1x_1 + b_1 + a_2x_2 + b_2 + a_3x_3 + b_3) / (x_1 + x_2 + x_3)$

A ROEC Optimization

Maximize:

$$\text{ROEC} = (a_1x_1 + a_2x_2 + a_3x_3 + b) / (x_1 + x_2 + x_3)$$

Subject to:

$$m_1 \leq x_1 \leq M_1$$

$$m_2 \leq x_2 \leq M_2$$

$$m_3 \leq x_3 \leq M_3$$

$$x_1 + x_2 + x_3 \leq \text{EC}_{\text{MAX}}$$

- *A so-called “fractional-linear programming problem”*
- *A substitution transforms it into a linear programming problem*
- *An exact solution is found easily through matrix operations or free applications*

Investor Point of View

- Investors are not typically concerned with EC or its allocation
- Company earnings and its volatility drive company valuation (e.g., stock value)
- Harry Markowitz: “*Semivariance* seems more plausible than variance as a measure of risk, since it is concerned only with adverse deviations.”

Semivariance & Semicovariance

For a fixed benchmark (hurdle) return B:

$$\text{semivariance}_i \approx (1/T) \sum [\min (R_{it} - B, 0)]^2$$

$$\begin{aligned} SC_{ij} = \text{semicovariance}_{ij} &\approx \\ (1/T) \sum [\min (R_{it} - B, 0) * \min (R_{jt} - B, 0)] \end{aligned}$$

portfolio semivariance \approx

$$\sum_i \sum_j w_i w_j SC_{ij}$$

T = number of time periods for which returns observed

Periods denoted *t*=1, 2, ..., *T*

R_{it} = return for asset *i* during period *t* Note: “*Sc_{ij}*” is semivariance of asset *i*

Efficient Frontier

- Classic Version: For a benchmark/target **portfolio** return “B”, what allocation to a given set of **investable assets** produces the **portfolio** of minimum **variance** with expected return B?
- Levine Version: For a benchmark/target **company** return “B”, what allocation to a given set of **LOBs** produces the **product portfolio** of minimum **semivariance** with expected return B?

Stating the Problem (1 of 2)

- Must define what is meant by asset (product) *return*
- Candidates include ROEC, ROA, ROE, etc.
- Necessary assumptions and parameters:
 - Resource to allocate and allocation buckets
 - Expected return and semivariance for each “asset” (e.g., product line or LOB)
 - Semicovariances between pairs of assets

Stating the Problem (2 of 2)

Given a benchmark return B , find the allocation weights (w_1, w_2, \dots, w_N) to N “assets” with respective expected returns (r_1, r_2, \dots, r_N) which will:

Minimize: (appx.) portfolio semivariance $\sum_i \sum_j w_i w_j s_{ij}$

Subject to:

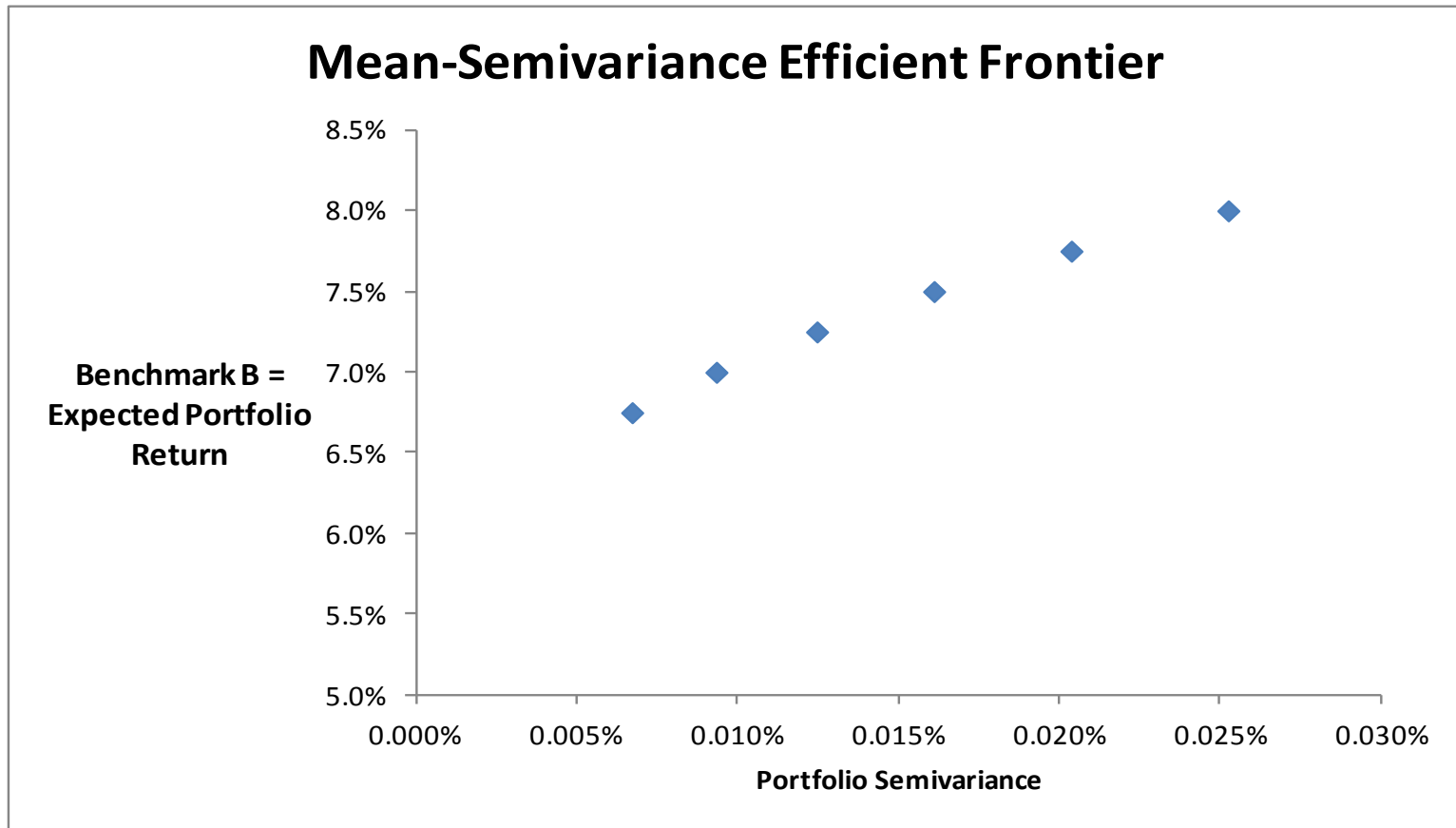
$\sum w_i = 1$ (the “budget constraint”)

$\sum w_i r_i = B$ (the return constraint)

Problem Solution

- The decision variables are $\{w_i\}$
- The objective function and constraints are functions of the $\{w_i\}$
- The constraints are *equality* constraints.
- Continuous first partial derivatives with respect to each of the weights exist for the objective function and the constraints
- Therefore, **Lagrange Multipliers** give solution!

MSV Efficient Frontier



The Impossible Frontier

- For many semicovariance matrices the optimal portfolio for a given benchmark return contains short positions (negative weights) for some assets
- Such allocations are typically impossible to implement (other than at long-short equity hedge funds)
- Numerical methods may be used to find (near?) optimal long-only portfolios

More in the Paper

- A (much!) slower build-up of concepts
- Step-by-step worked out examples and illustrative simulation output
- State of ERM and conflicting stakeholder expectations
- Link to financial planning process, product allocation, risk-adjusted performance measures, and compensation