Risk Modeling for Insurers: Real-World Assets and Risk-Neutral Liabilities

Gary Venter – ERM Symposium 2016

ERM Modeling Context

- Focus is modeling the risk to capital = assets – liabilities, especially on next year-end financial statement
- Modeling Assets – Liabilities requires modeling both
  - Assets at market value but liabilities not discounted
  - Reason for this is perhaps inflation, over longer periods correlated to interest rates –> don't discount
  - Net result is capital very sensitive to asset fluctuations especially as assets could be 3 - 5 times capital
  - Sometimes management wants to measure risk on "true economic" basis, i.e., discounting liabilities, but owners usually want to see risk to statement values
- Starts with sensitivity of assets to interest-rate changes, but tends to assume all shifts are parallel
- Never actually happens– better is empirical duration – observed changes in asset values in response to a selected yield change – like 3-year rate
- But interest rates only part of driving economics, so distribution of capital from ERM model including more drivers is a more comprehensive measure

Real-World vs. Risk-Neutral Scenarios

- Risk-neutral probabilities build in risk charges so mean is a price including risk loading, and thus are overly pessimistic if viewed as event outcomes
  - Paying risk-neutral mean to buy an asset builds in expected profit for the investor's risk
  - Paying someone risk-neutral mean to take a liability builds in some expected profit for them
  - Shown to be only way to provide consistent pricing for all buyers and sellers and subdivisions of risks, as long as right degree of pessimism is in scenarios
- Risk management for financial firms uses scenarios generated from risk-neutral probabilities to manage trading and hedging risk with high turnover, so most ESGs are risk neutral
- Insurance ERM doesn't focus on prices of hedges – more about distribution of results from fairly fixed portfolio of assets and liabilities – so needs real-world scenarios
- But can use risk-neutral liability probabilities for pricing per-risk and aggregate insurance deals ceded and assumed

Why Actuaries Should Do ALM

- At least since di Finnetti 60 years ago, actuaries have been quantifying risk to capital
  - To do now in ERM needs some asset modeling
  - Do analysis for current portfolio, but then it is a small further step to do it for hypothetical portfolios
  - Then someone might ask which is the best strategy in terms of risk and return, and you are doing ALM
- Financial quants approaching the same problem will bring the models that worked well in their QRM – risk-neutral, usually lognormal, models like BK2 and Libor Market Model
  - Will try to adapt these to real world but are fundamentally too heavy tailed and will give obviously too high 99th percentile, etc., discrediting modeling entirely
  - Also will have over-simplified liability models, if any
  - Plus likely to be stuck in paradigm of parallel yield-curve shifts or key-rate durations: only a few rates move
- With a little focus, actuaries can do better than this

My Take on ALM

- Corporate bonds often held to maturity as difficult to re-sell
- So-called credit spread is in fact more for this illiquidity and is especially larger than default spread, both proportionally and absolutely, for BB and BBB bonds.
- Holding to maturity thus captures liquidity premium but should be done with an eye towards liability cash needs
- Cash-flow matching bond coupons and maturity values with loss cash flows handles this well
- But should probably similarly match some portion of capital because of liability cash-flow uncertainty
- Still value fluctuations will flow to surplus but not to earnings under GAAP accounting
- Investors somewhat tolerant of this if clearly temporary
- Fluctuations reduced by picking bonds whose credit spreads move strongly inversely to interest rates
- Has been seen in financial sector bonds, for example

Statistical Conceptual Background

- "All models are wrong but some are useful"; George Box – or maybe Sergio Armani
- I call this the robust paradigm
- Another way to put it: The data was generated by a more complicated process than the model specifies
- This is contrary to assumptions behind statistical testing. Practical implication is to do out-of-sample testing – maybe with a hold-out sample
- Simpler models often do better on hold-out points than bigger models that look better on standard statistical tests
- Still "over-simplified models tend to be more wrong, but are sometimes more useful"; Me
- Basically you need to know how they are wrong, when to use them, and when not to

Levels of Wrong Models for Asset Risk Factors

- Talking about ESGs – simulation models that generate scenarios each with values for a number of risk drivers
  - Treasuries: Interest rates, yields for several maturities
  - Markets: Equities, corporate bonds, inflation, etc.
- 1. Pro forma analysis needing mainly a lot of numbers and a regulator to require them or someone to reference: Can use ESGs freely issued by actuarial societies, old papers...
- 2. Want some degree of believability such as nothing obviously impossible like easy arbitrage: most commercially available ESGs
- 3. Also want distributions of factors to look like history: There are degrees within this - volatiles by maturity, correlations, stochastic volatility – where we will look
- 4. Plus consistency with econometric macro models: Current leading edge. Macro models would need stochastic scenarios, plus add a lot of regressions

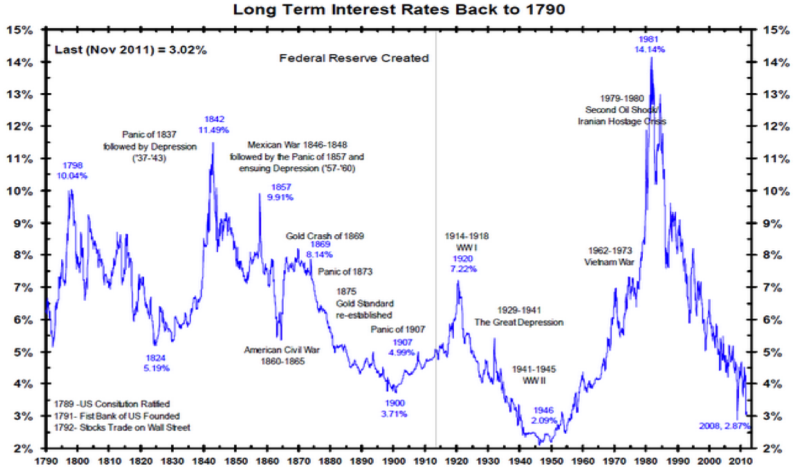
Review Some Asset Models

- Brief look now at four interest rate models – more later
  - 1. Pro forma analysis: AAA model is a fairly simple time-series model for a long and a short rate, with curves between them for other rates. Has arbitrage.
  - 2. Want some degree of believability : BK2 (Two-factor Black-Karasinski) model – normal mean-reverting time series for log of rate with reverting mean also stochastic. Lognormal works ok for risk-neutral rates but usually too skewed for real-world.
  - 3. Distributions of factors to look like history: Will look at CIR process – which has standard deviation of rates proportional to square-root of rate. Sum of three independent such gives realistic distribution of curve shapes but misses in other areas.
  - 4. A bit more realism Not tested here but discussed is a stochastic volatility model known as A23.
- Rate models have corporates and Treasuries. Then
  - Inflation Especially medical, which is a liability driver
  - Equity index: Not too hard to model as just one series

- ESG Empirical Tests
- ESG Formulas
- Risk Neutral Probabilities for Liabilities

What We Are Up Against

- Biggest problem for asset models is data properties not always easy to model and assumptions from stat texts do not always apply.
- Curves are steeper when short rates are lower
- Positive skewness and excess kurtosis, but both fairly modest for interest rates
- Fluctuation around temporary levels that only slowly revert to long-term mean
- Higher correlations for longer periods
- Volatilities change over time, with occasional jumps – not smooth process
- Yield curves upward sloping with downward sloping volatilities
- High correlations and autocorrelations of various series – more persistent than in standard models



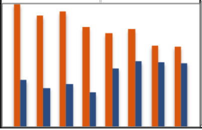
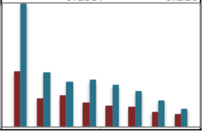
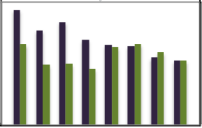
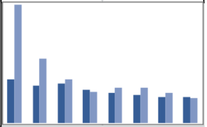
Source: What Drives The Bond Market?
Chicago CFA Handout by Bianco Research LLC
January 18, 2011

- 1940 looks a lot like 2010 – last 30 years was unusual
- Waves of maybe 50 – 70 years

Focus on a Few Key Facts

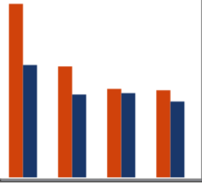
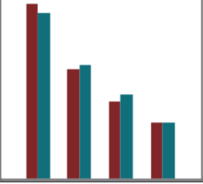
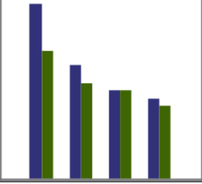
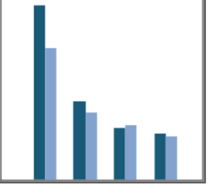
- Longer rates tend to be more stable historically
- But but offset by longer bond values being more sensitive to rate changes – definition of duration
- Volatility by maturity important for model to get right
- Another key risk issue is variety of shapes of yield curves
- Main characteristic is curve tends to flatten when short rate rises and gets steeper with lower short rates
- Relationship of risky and risk-free rates is tricky
- Usually spread negatively correlated to risk-free rate
- But not always – depends on inflation and economic activity and probably other things
- An example of why you would like to have an ESG fed by an econometric model
- Most models make spreads and rates independent but can target getting right correlation of risky and risk-free rates themselves instead of spreads

VOLATILITY BY MATURITY – HISTORICAL VS. RECENT

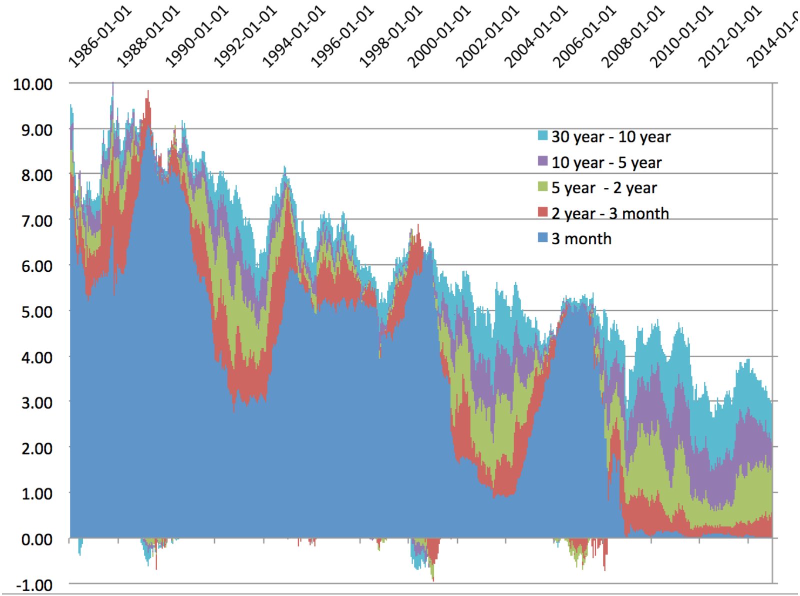
	Abs Vol	Since Jan 2009	Log Vol	Since Jan 2009	Abs Vol of 2/3 Power of Rates	Since Jan 2009	Abs Vol / Sqrt Rate	Since Jan 2009
3m	1.487	0.569	0.669	1.482	0.607	0.424	0.678	1.833
1y	1.345	0.472	0.335	0.657	0.498	0.316	0.579	0.987
2y	1.395	0.516	0.379	0.534	0.543	0.322	0.612	0.675
3y	1.209	0.411	0.288	0.560	0.449	0.294	0.512	0.475
5y	1.130	0.708	0.258	0.498	0.422	0.408	0.474	0.544
7y	1.179	0.788	0.242	0.430	0.416	0.424	0.438	0.545
10y	0.984	0.782	0.179	0.311	0.354	0.380	0.405	0.458
30y	0.972	0.775	0.153	0.216	0.338	0.336	0.395	0.390
Charts								

- With low short rates, short vol now lower, but vol of log rates higher – tried vol of 1/2 and 2/3 power of rates
- For 2/3 power, vol fairly level over time and maturity
- Will compare this to vol of simulated rates from models

Volatility by Maturity – A-Rated Corporates

Corporate Bond (A)								
	Abs Vol	Since Jan 2009	Log Vol	Since Jan 2009	Abs Vol of 2/3 Power of Rates	Since Jan 2009	Abs Vol / Sqrt Rate	Since Jan 2009
2Y	1.52	0.98	0.43	0.41	0.63	0.46	1.23	0.92
5Y	0.97	0.72	0.27	0.28	0.41	0.34	0.55	0.47
10Y	0.78	0.74	0.19	0.21	0.32	0.32	0.37	0.39
20Y	0.76	0.67	0.14	0.14	0.28	0.26	0.32	0.30
Charts								

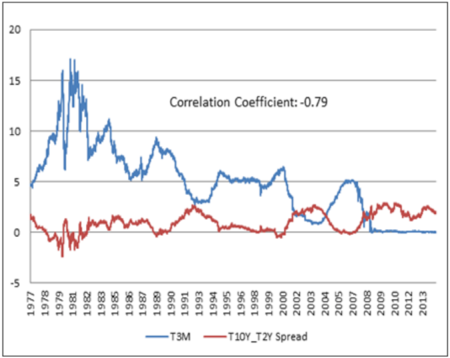
- For corporate bonds, log volatility is downward sloping across maturities but pretty constant over time

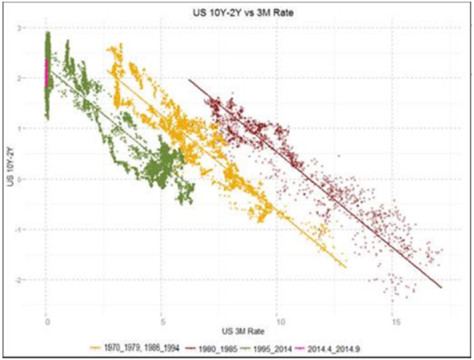


- Short rate getting high squeezes yield spreads

Curve Shape Regressions

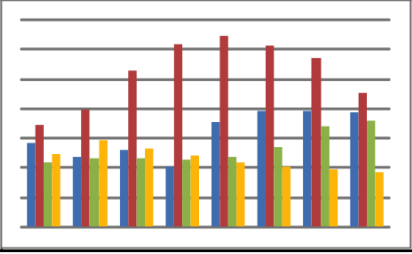
- Compare 3 month rate and spread between 2 year and 10 year rates by regressions – for different periods but same slope – intercepts may vary due to inflation..
- Used to test simulation output by fitting regressions to it with this slope, and comparing the residuals with ones from these lines, to test for distribution of shapes

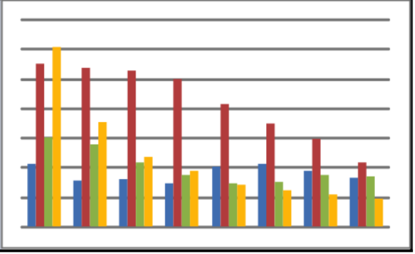




Test Volatilities – Blue Is Target – CIR3 OK

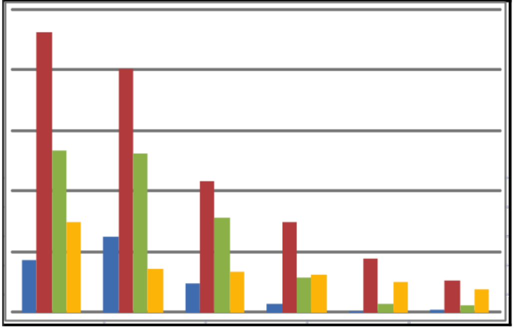
	Absolute Volatility				Volatility of R^2/3			
	2009+	E2BK	CIR3	AAA	2009+	E2BK	CIR3	AAA
3m	0.57	0.69	0.44	0.50	0.42	1.10	0.60	1.21
1y	0.47	0.80	0.46	0.59	0.32	1.08	0.56	0.71
2y	0.52	1.06	0.47	0.53	0.32	1.06	0.43	0.47
3y	0.41	1.24	0.46	0.49	0.29	1.00	0.35	0.38
5y	0.71	1.29	0.47	0.43	0.41	0.83	0.29	0.29
7y	0.79	1.22	0.54	0.41	0.42	0.70	0.30	0.25
10y	0.78	1.14	0.68	0.39	0.38	0.60	0.35	0.22
30y	0.77	0.91	0.72	0.37	0.34	0.44	0.34	0.19





Test Skewness – None So Great

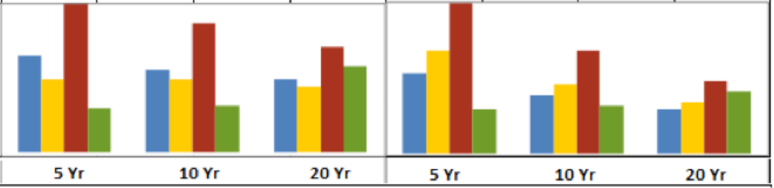
Skewness				
Tenor	2009+	E2BK	CIR3	AAA
3 MO	0.85	4.63	2.66	1.50
1 YR	1.24	4.01	2.61	0.71
3 YR	0.47	2.17	1.56	0.68
5 YR	0.14	1.48	0.56	0.62
10 YR	0.02	0.88	0.13	0.49
30 YR	0.05	0.51	0.12	0.37



Risky Volatilities – Extended Models – So-So Tests

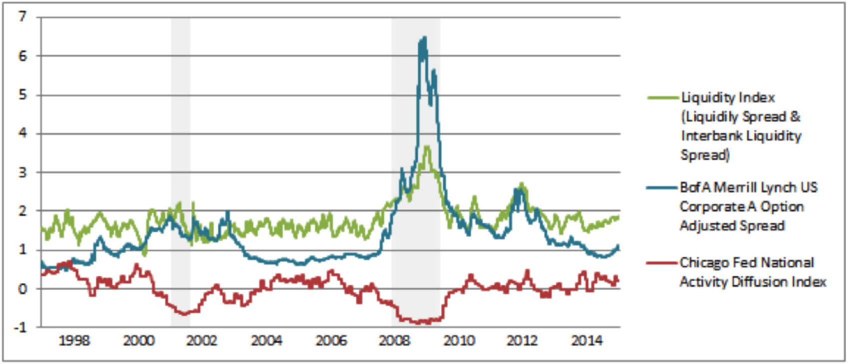
Volatility of Risky Rates

	Volatility (%)				Log Volatility			
	Historical 1996-2014	Historical 2009-2014	JLT	CIR3	Historical 1996-2014	Historical 2009-2014	JLT	CIR3
5 YR	0.99	0.74	1.52	0.45	0.22	0.28	0.41	0.12
10 YR	0.84	0.74	1.31	0.47	0.16	0.19	0.28	0.13
20 YR	0.74	0.67	1.07	0.88	0.12	0.14	0.2	0.17



- Vendor too high at 5-year and CIR equally too low – CIR a little better at longer rates
- Risky bonds majority of portfolio but modeling not so impressive

Drivers of Credit Spreads Not In Most ESGs



- Market liquidity index and economic activity index both related to spread
- A23 model discussed below does better on risky rates but more complicated

Curve Shape Tests

- Did similar regressions for 10-year to 30-year spread, and change in slope between the two spreads
- CIR best but not enough volatility, so not enough variety of shapes, in 10-30 spread

	Model	Intercept	Beta	Res_SD	Skew
Spread 10Y-2Y	Historical	2.11	-0.39	0.39	-0.45
	E2BK	2.74	-0.39	0.70	0.61
	CIR3	2.74	-0.39	0.46	0.06
	AAA	1.92	-0.39	0.16	0.31
Spread 30Y-10Y	Historical	0.96	-0.14	0.16	-0.11
	E2BK	0.75	-0.14	0.35	-0.54
	CIR3	0.75	-0.14	0.08	-0.08
	AAA	0.70	-0.14	0.06	0.31
Curvature	Historical	-1.16	0.25	0.38	0.54
	E2BK	-1.99	0.25	0.77	-0.85
	CIR3	-1.99	0.25	0.38	-0.07
	AAA	-1.22	0.25	0.10	-0.31

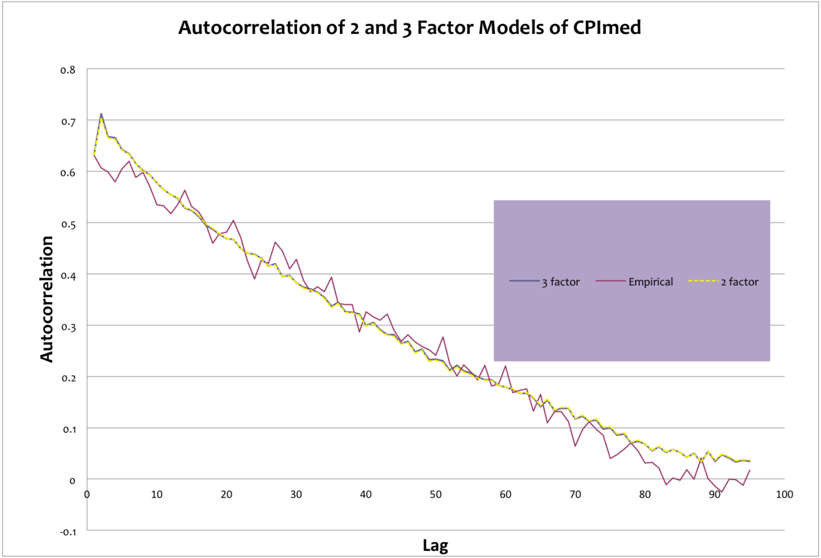
Medical CPI – Connecting Economy & Liabilities

- A key statistical property of medical inflation is autocorrelations – about 70 monthly lags have autocorrelation above 10%
- That means inflation is a sticky series – it tends to be high for a long time and low for a long time
- Since losses pay over decades and inflation accumulates, that will create extreme scenarios for loss development – some quite high and some low.
- It is also a modeling issue – important to get right
- An AR-1 process won't do this, but sum of two independent AR-1s works well
- Fitting by simulated method of moments (SMM) can match specific features of a series like autocorrelation better than efficient methods like MLE and MCMC
  - You simulate a long series with trial parameters, measure the statistical properties you care about, and seek parameters that match target properties

Pros and Cons of SMM

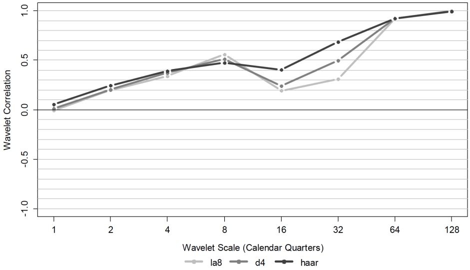
- Not efficient in statistical sense – other estimators may have lower variance
- May be more robust – less sensitive to unsual history
    - "Efficient (estimation) may pay close attention to economically uninteresting but statistically well-measured moments." A Cross-Sectional Test of an Investment-Based Asset Pricing Model, John H. Cochrane, Journal of Political Economy, 1996
    - Comparing Multifactor Models of the Term Structure, Michael W. Brandt, David A. Chapman: "... the successes and failures of alternative models are much more transparent using economic moments... In contrast, when models are estimated (by efficient methods), it is much more difficult to trace a model rejection to a particular feature of the data. In fact, the feature of the data responsible for the rejection may be in some obscure higher-order dimension that is of little interest to an economic researcher."

SMM Fits of Two and Three AR-1s to Medical CPI



3-Month T Bill Rate

Vs. Medical Care Inflation



mo wavelet.png

- Wavelets decompose series into periodic functions
- Correlation higher for longer periods well established
- Can pick a time frame of importance and force correlation to be right for that period at least
- For instance correlate two CPI factors with two of the CIR factors using correlated random draws that still left all CIR factors and both CPI factors independent

Or Use Error Correction Model to Get Interest Rates and Inflation to Converge

- Error correction can model short-term change in inflation as original model plus a factor times short-term change in interest rate and a factor times the difference between current interest and inflation
- Let C be inflation rate and I be interest – say 5-year rate, assuming usually $C = 1.2I$. Then could add error correction to model, with that part like:

$$\Delta C_i = \rho \Delta I_i + \alpha(1.2 I_{i-1} - C_{i-1}) + \epsilon_i$$

- Here ρ is the short term correlation. Assuming the long-term correlation is 100%, this moves inflation towards 1.2 * interest rate, at rate of convergence α.

- ESG Empirical Tests
- **ESG Formulas**
- Risk Neutral Probabilities for Liabilities

What Exactly Is the CIR Model?

$$dr(t) \quad = \quad \kappa \left[\theta - r(t)\right] dt + \eta \sqrt{r(t)} dB_r(t)$$

- The short rate at t is $r(t)$, reverting to mean θ at speed κ with volatility $\eta\sqrt{r(t)}$. $dB_r(t)$ is the instantaneous change in a Brownian motion process, which over time t has changes normally distributed in variance t.
- Square-root process cannot go negative: if $r(t)$ is zero, volatility = zero and change $dr(t)$ is $\kappa\theta > 0$.
- Get the real-world yield curve as the expected payoff discounted along a risk-neutral process that increases the drift (dt portion) by the market price of risk
- Yield rate on a T-year bond is $a(T) + b(T)r$, ("affine"): r is the short rate, a and b are functions of the parameters, the market price of risk and T, but not r.
- Need sum of 3 independent CIRs to get realistic variety of yield curves. Add a 4th for risky spread.

Discussion of CIR

- $a(T), b(T)$ a bit complex, but Excel can simulate yields
  - Pick a small interval of time s to represent dt – simulate Brownian motion as a random normal draw with variance s – easier with data tables
  - Another enhancement shown by Brigo-Mercurio is that you can adjust beginning yield curve – and so ending average yield curve – by adding a constant by maturity to every simulation – used in tests above
- With very low short rates, yield curve comes from market prices of risk (one for each process and one for spread) – CIR works better with higher short rates
- Alternatives
  - Stochastic volatility models, discussed in more detail below, can perform better in tests
  - Probably need a programming language like R or Matlab to do them
  - Have "almost closed-form" yield curves – that is, requiring standard canned numerical routines

Stochastic Volatility Affine Interest Rate Models

- Chen model:

$$\begin{aligned} dv(t) &= \mu\left[\overline{v} - v(t)\right] dt + \eta\sqrt{v(t)}dB_v(t) \\ d\theta(t) &= \nu\left[\overline{\theta} - \theta(t)\right] dt + \zeta\sqrt{\theta(t)}dB_\theta(t) \\ dr(t) &= \kappa\left[\theta(t) - r(t)\right] dt + \sqrt{v(t)}dB_r(t). \end{aligned}$$

- The short rate at t is $r(t)$, reverting to temporary mean $\theta(t)$ at rate κ with volatility $\sqrt{v(t)}$
  - $\theta(t)$ reverts to its mean $\overline{\theta}$ as a square-root process
  - $v(t)$, volatility of r(t), square-root process, reverting to $\overline{v}$
  - Sort of closed-form yield curves – like hypergeometric
- More general $A_2(3)$ model has two correlated processes Y_1, Y_2 instead of θ, v, then $\theta(t)$ and $v(t)$ are linear combinations of those processes
- Correlation between processes allows better fits
- Can also make credit spread, like to A bond, a linear combination of Y_1, Y_2

Background and Extensions of Stochastic Volatility Affine Models

- Categorization of affine models can be found in Dai and Singleton (2000). Specification analysis of affine term structure models. Journal of Finance 55 (5)
- A major limitation of affine models turns out to be having just a single market price of risk for each factor.
- There have been a few extensions but the most successful appears to be the semi-affine models, which provide a lot of parameters for the price of risk.
- Peter Feldhütter, 2008.Can Affine Models Match the Moments in Bond Yields? – documents this and finds that semi-affine $A_2(3)$ is one model that fits data well.
- Incorporating credit spreads in affine models: Duffie and Singleton (1999) Modeling term structures of defaultable bonds. Review of Financial Studies12:4
- This seems to work better in the semi-affine case

- ESG Empirical Tests
- ESG Formulas
- Risk Neutral Probabilities for Liabilities

Risk-Neutral Models for Liabilities

- Two levels of use
  - Pricing business unit aggregate risk based on company risk targets – assumed here to be given exogenously
  - Pricing individual deal risk based on business unit risk targets
- Getting business unit targets
  - Transforms of unit aggregate distributions
- Pricing deals
  - Co-ordinated transforms of frequency and severity distributions

Risk-Neutral Probabilities to Business Units

- Starting point is probability transform for company
  - In typical simulation application, every scenario is given an alternative probability with higher probability to the adverse scenarios. Then the transformed mean is higher than the actual mean, by the target profit.
  - Usually every business unit will have a simulated loss in that scenario and the probability assigned to that scenario applies to every unit's loss in the scenario
  - Gives consistent risk pricing to every unit
- Most risk measures give risk-neutral transforms
  - Take T = $TVaR_{0.99}$. That is a transformed mean where all the probability goes to the top 1% of scenarios.
  - If the price is mean plus 0.05T, can get that by reweighting probabilities. Again every business unit loss in the scenario gets the scenario weight.
  - A loading of k * standard deviation is a transformed mean if k < CV. Standard deviation allocated to unit by unit standard deviation times correlation of unit with the company.

Continuous Transforms with Tail Emphasis

- Some continuous transforms give tail emphasis but more smoothly than TVaR
- Two are the Exponential and Normal (= Wang) transforms, each transforming the CDF F to F* and having a parameter k, respectively:

$$
\begin{aligned}
F^*(x) &= [e^{kF(x)} - 1]/[e^k - 1] \\
F^*(x) &= \Phi[\Phi^{-1}(F(x)) - k]
\end{aligned}
$$

  - Here Φ is the standard normal CDF. PDFs can be calculated by differencing
- Loading of a*T as a transformed distribution can be done by computing b = a/(1 – r – a), where r is mean / T. Then decrease every probability not in the tail by dividing by 1+b. Finally increase the probabilities in the tail by a factor of 100b + 1/(1+b).

Three Line Example

- Sometimes ERM actuaries approximate business units with a gamma distribution, but as long as the skewness > the gamma skewness of 2CV, a shifted gamma can match three moments.
  - X – s is gamma in a, b. X has mean s + ab, variance ab^2 and $skew^2 = 4/a$, which can be used to get the parameters from the moments.

	Auto	Liability	Property
a	16	4	1.2345679
b	37,500	100,000	162,000
s	2,400,000	1,600,000	800,000
mean	3,000,000	2,000,000	1,000,000
sd	150,000	200,000	180,000
cv	0.05	0.1	0.18
skw	0.5	1	1.8

- The company needs 10% of expected losses, or 600,000 as profit. We allocate to line by exponential and Wang transforms and the standard deviation.
- Expenses are 34% of the 10,000,000 premium.

Loads by Line

- 10,000 simulations for each line sorted by total.
- Parameter found by Goal Seek for each transform so that risk-neutral mean = 1.1 times actual mean.
- Loaded losses = transformed mean shown for each line for each method where transformed probability for the scenario is used for each line's losses in that scenario. Also load as portion of loss shown.

Loaded Loss	Auto	Liability	Property	Total
epntl	3,111,601	2,253,577	1,234,821	6,600,000
wang	3,099,189	2,228,432	1,272,379	6,600,000
st dev	3,141,934	2,252,633	1,205,432	6,600,000
corrl	48.1%	64.7%	58.5%	100%
epntl	0.037	0.127	0.235	0.100
wang	0.033	0.114	0.272	0.100
st dev	0.047	0.126	0.205	0.100

- Wang transform hits tail risk hardest, std dev least
- Exponential target combineds are 0.976, 0.926, 0.874

Risk-Neutral Unit Probability Miscellany

- Such transforms give marginal risk pricing – the change in price for a change in unit volume is the derivative of the risk price for the company with respect to the volume of the business unit.
- The standard deviation risk-neutral mean for unit X_i is given by the probability shift below, using the overall shift below it and gives the price below that:

$$
\begin{aligned}
F^*(x_i) &= F(x_i)[1 + k[E(X|X_i) - EX]/std(X)] \\
F^*(x) &= F(x)[1 + k(x - EX)/std(X)] \\
E^*(X_i) &= E[X_i] + k * cov(X_i, X)/std(X) \\
E^*(X_i) &= E[X_i] + k * corr(X_i, X)std(X_i)
\end{aligned}
$$

  - Need k < CV for this to be a probability transform, but the correlation allocation would still work otherwise.
  - If not a transform there are deals that could be done that would guarantee a no-risk profit to the reinsurer, mostly involving low-loss results.

Options Pricing Interpretation

- 25 years ago Merton & Perold suggested that the cost to a company of a business unit is the value of the put option it is giving the unit – the right the unit has to have the company pay its financial losses, which they price as the risk-neutral mean of premiums minus losses and expenses given that difference is negative.
- The benefit from the unit can be similarly valued as the call option the company has to take the profits of the unit, given that they are positive. For price P the value of the unit is the value of the call minus the put:

$$E^*(P - X_i)^+ - E^*(X_i - P)^+ \quad = \quad P - E^*(X_i)$$

  - The unit is thus adding value at any price greater than the risk-neutral mean

Risk-Neutral Pricing within Individual Lines

- At the level of an individual business line, with a frequency and severity distribution, coordinated transforms of frequency and severity distributions can be used to price specific deals – like layers, etc.
- The coordinated approach can be traced to Thomas Møller's paper "Stochastic orders in dynamic reinsurance markets" from the 2003 ASTIN Colloquium, available on the colloquium site ASTIN 2003.
- To apply, start with a severity transform to increase the mean – possibly just by changing the parameters.
- Some probabilities will increase and some will decrease. Find the greatest decrease – probably near 0. Increase the mean frequency to offset that.
- That way no possible layer will get a negative loading.
- To get consistent pricing, it is key to use the same transformed parameters for all deals.

Pareto Example

- Take for example a Pareto severity with mean b/(a–1), where you want to increase the severity mean by decreasing the shape parameter a.
- The density limit at zero is a/b, so increase expected frequency by whatever percentage a decreases.
- For example take $a = 1\frac{1}{5}$, which would be for a fairly heavy-tailed liability line.
- Suppose a decreases to $1\frac{5}{27}$. The mean severity increases to 5.4b from 5b, or by 8%. The frequency mean is increased by $1\frac{1}{5}/1\frac{5}{27}$, or by 81/80 = 1.0125.
- The transformed mean is thus higher by 9.35%, or by a factor of $3^7/2000$.
- Assuming this is the right pricing level for the line, then all layers can be priced using these frequency and severity parameters.

Møller's Transforms

- Møller proposed a general methodology for generating severity transforms and gave two pdf examples, which he called – the minimum martingale transform and the minimum entropy martingale transform, which are given, respectively by:

$$
\begin{aligned}
f^*(x) &= f(x)[1 - k + kx/EX] \\
f^*(x) &= f(x)e^{kx}/Ee^{kX}
\end{aligned}
$$

- Looking at x near zero shows that the respective frequency loadings are $1/(1-k)$ and Ee^{kX}.