ERM Enterprise Risk Management Symposium
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RED Session 3.1: Parameter Uncertainty

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Presenters:
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BYU BRIGHAM YOUNG UNIVERSITY

Parameter Uncertainty

Brian Hartman, Robert Richardson, and Rylan Bateman

2017 ERM Symposium

Project supported by the Joint Risk Management Section and Research Committee

Project Overview

- Literature Review
- Health Case Study
  - Predict diabetes improvement using regression
  - Variable selection through spike and slab prior
- Property/Casualty Case Study
  - Predict claim counts for 79K policyholders using Poisson and negative binomial regression
  - Show the impact of including parameter uncertainty in the regression parameters
- Life Case Study

Life Case Study

- Can incorporate parameter uncertainty into either the actual/tabular ratio or the mortality rates directly.
- Will show both the impact on the mortality rates and the expected present value of simple insurance products.

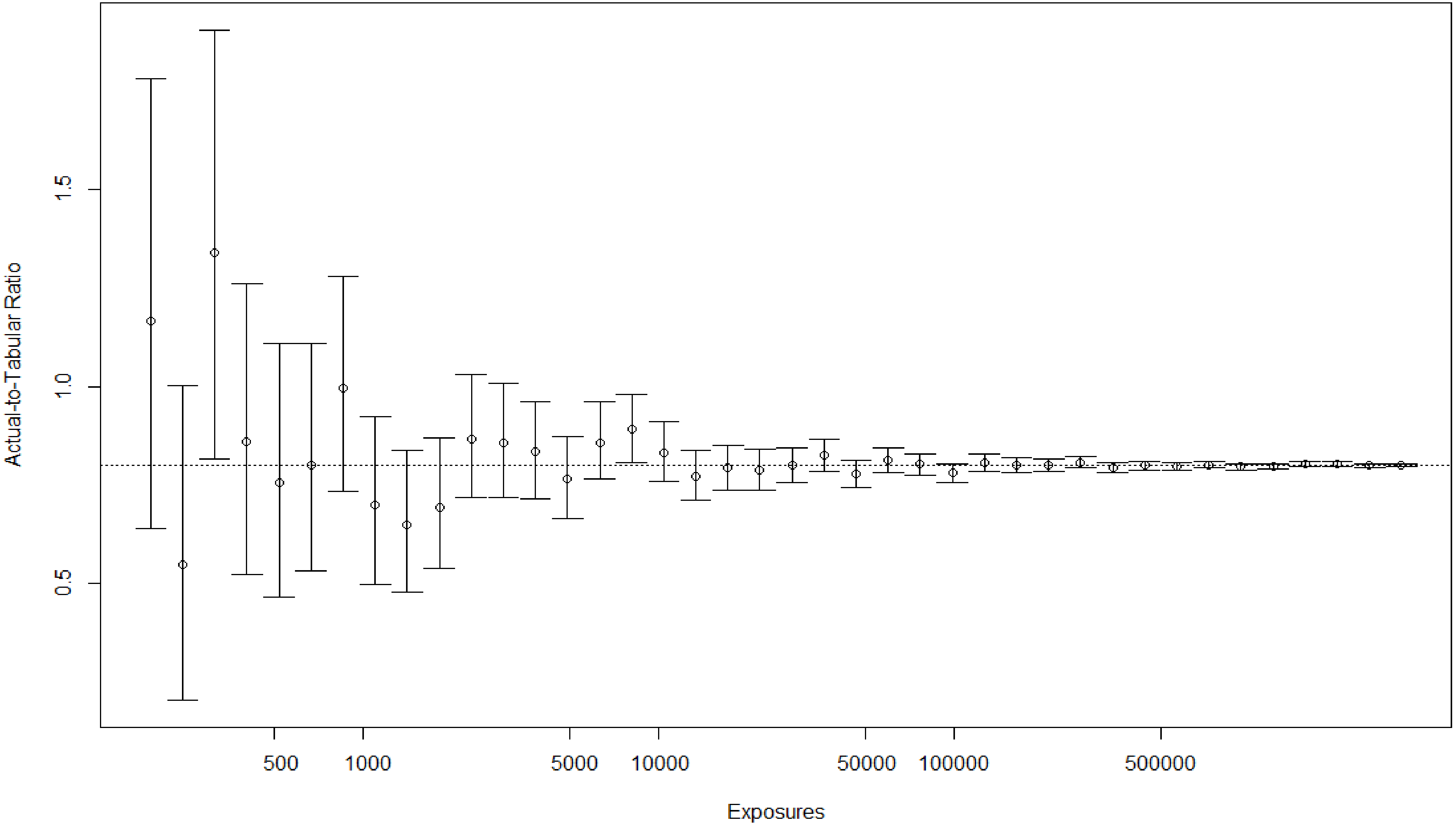
Actual/Tabular Model

$$Y_a \sim Bin(E_a, \theta T_a)$$

$$\theta \sim N(1, 1^2)$$

E_a is the number of exposures for age a, T_a is the tabular mortality rate.

Impact of Sample Size



Impact on SPIA

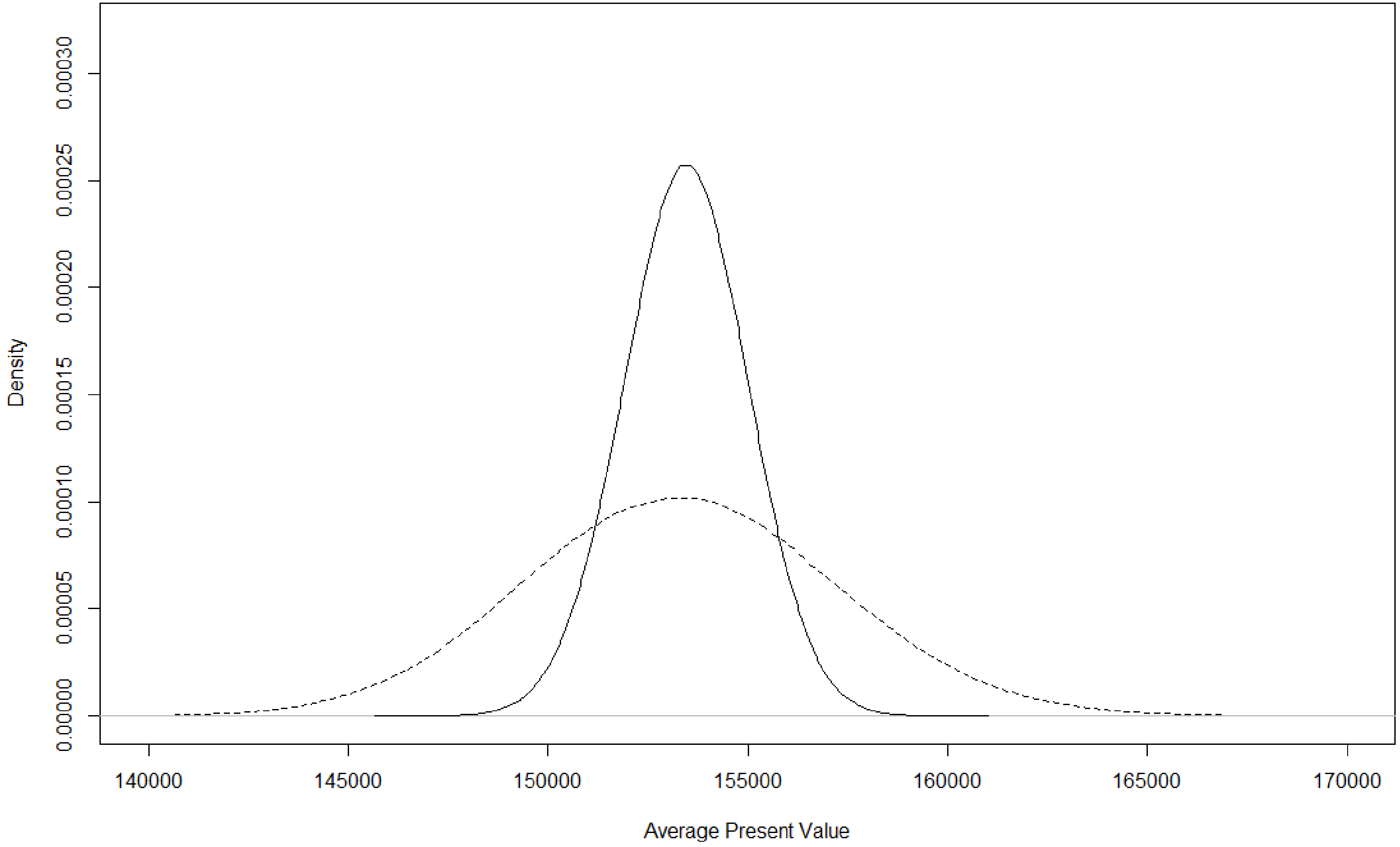


Figure shows present value of SPIA contracts both with (dashed line) and without (solid line) parameter uncertainty.

- $d = 0.03$
- Annual payment of 10,000
- 65-year-old female insured
- Portfolio of 1,000 contracts

Possible Extension

- Our current model is

$$Y_a \sim Bin(E_a, \theta T_a)$$
$$\theta \sim N(1, 1^2)$$

- To allow θ to vary by a subgroup, say gender and age, the model can be adjusted as follows:

$$Y_{ag} \sim Bin(E_{ag}, \theta_{ag} T_{ag})$$
$$\theta_{ag} \sim N\left(\mu + \tau_g + \nu_a, 0.02^2\right)$$
$$\mu \sim N(1,1^2)$$
$$\tau_g \sim N(0,0.1^2)$$
$$\nu_a \sim N(0,0.1^2)$$

Modeling the mortality rates directly

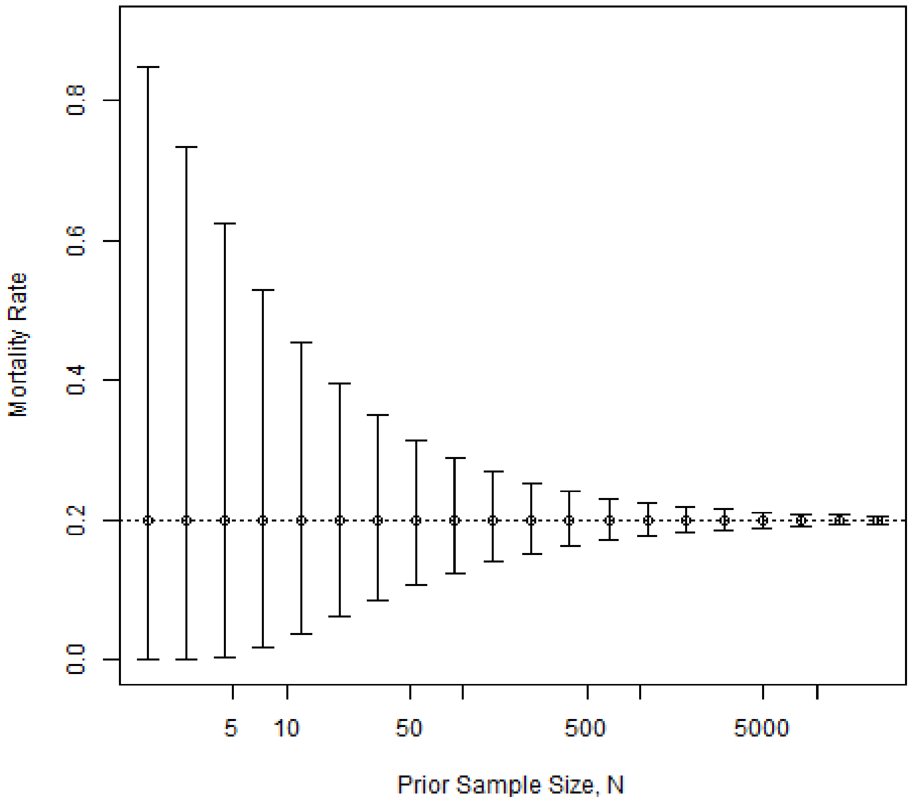
$$q_a \sim Beta\left(Nq_a^{ind}, N(1 - q_a^{ind})\right)$$
$$q_a | E, X \sim Beta(\alpha + X, \beta + E - X)$$

where

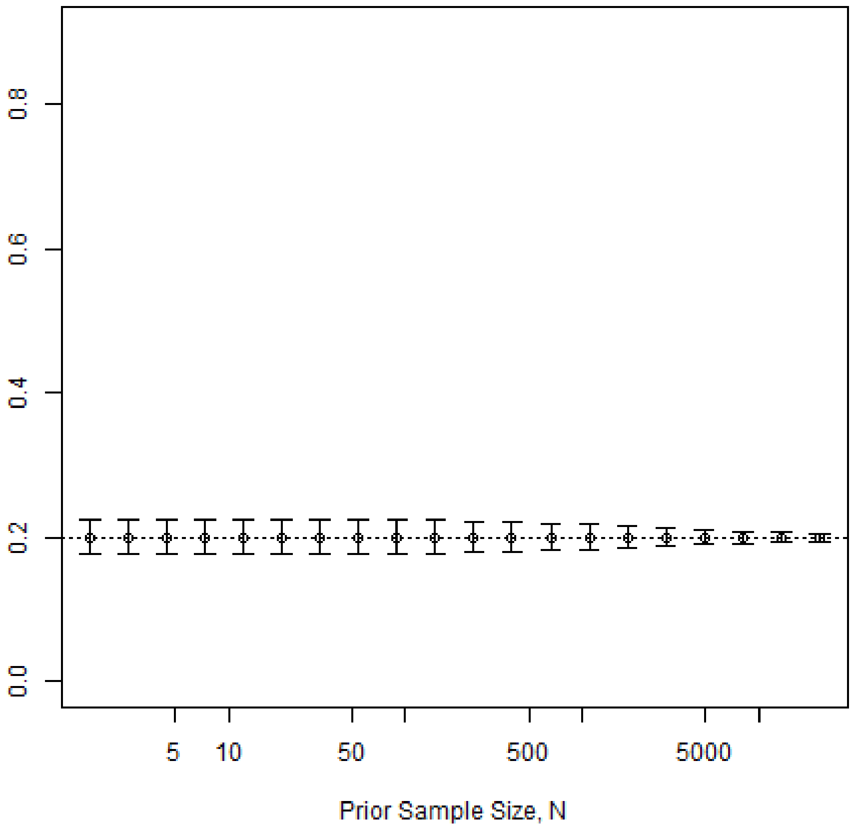
- q_a is the company mortality rate for age a
- q_a^{ind} is the industry mortality rate for age a
- N is the prior sample size
- E is the number of exposures in your data
- X is the number of deaths for age a.

Impact of the prior sample size, N

q_a^{ind}=0.02



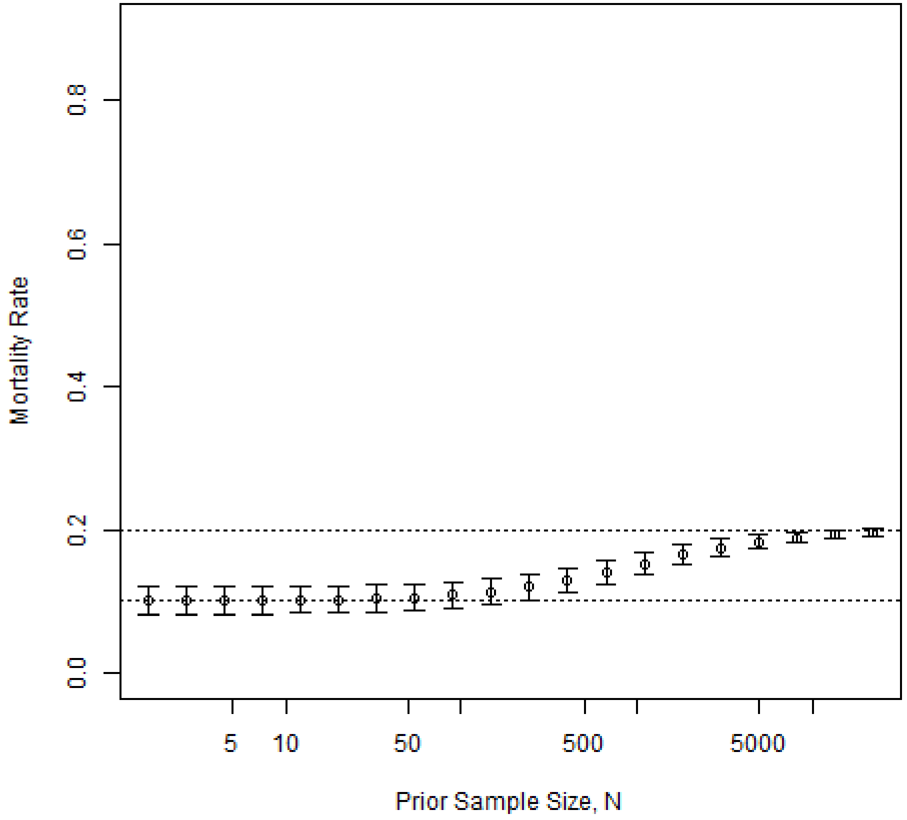
With no company data



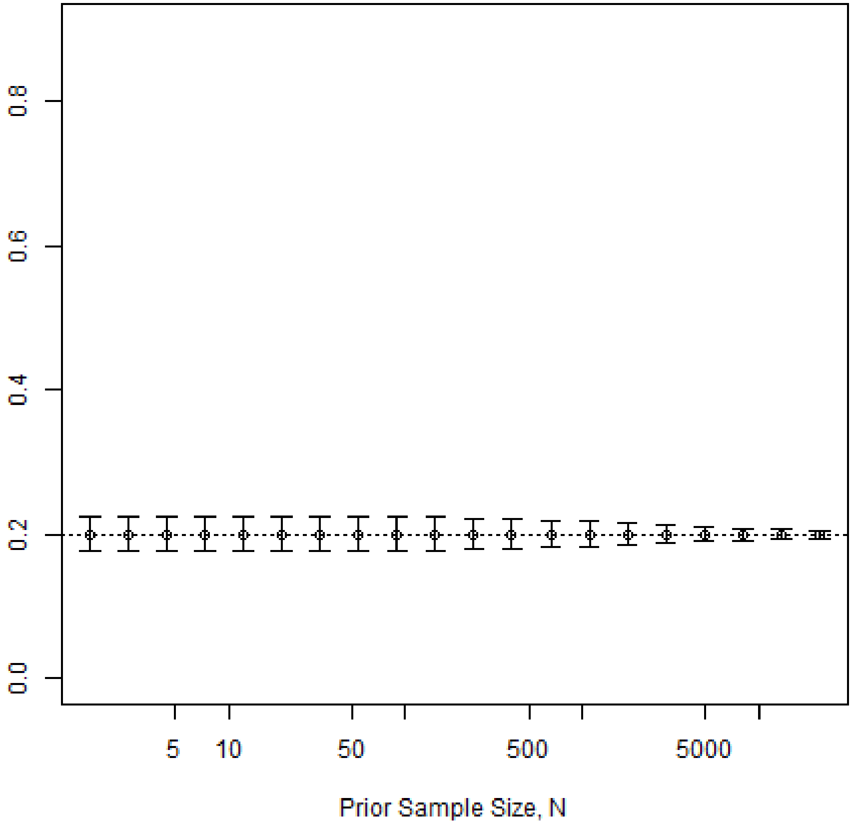
With 5,000 company exposures and 1,000 company deaths

Impact of the prior sample size, N

q_a^{ind}=0.02

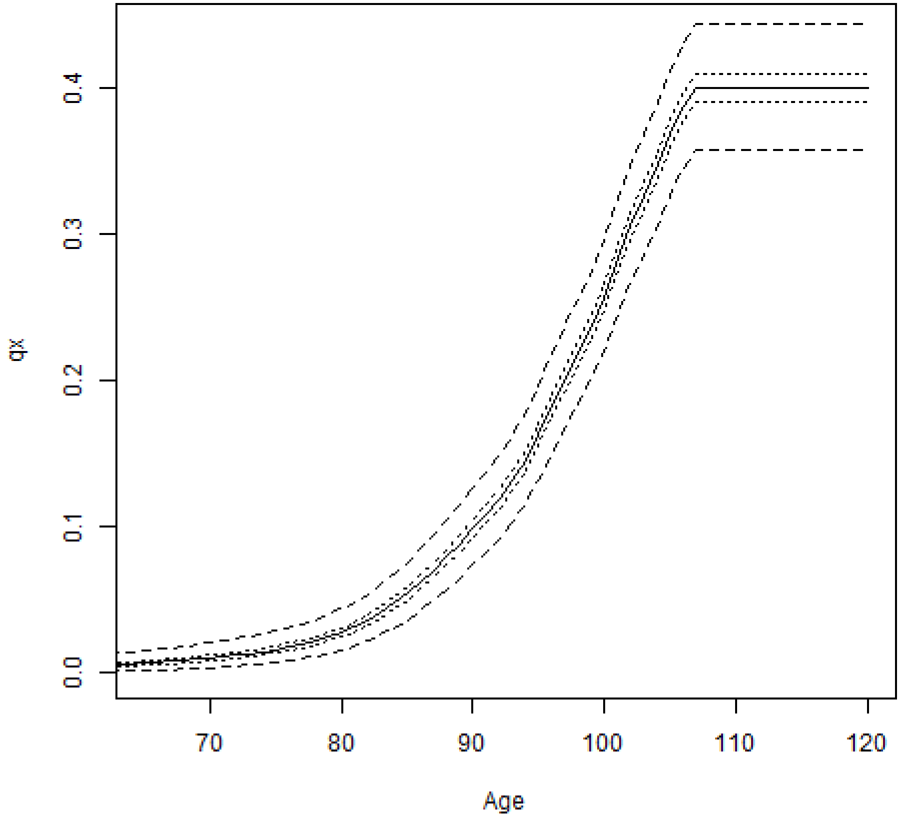


With 5,000 company exposures and 500 company deaths

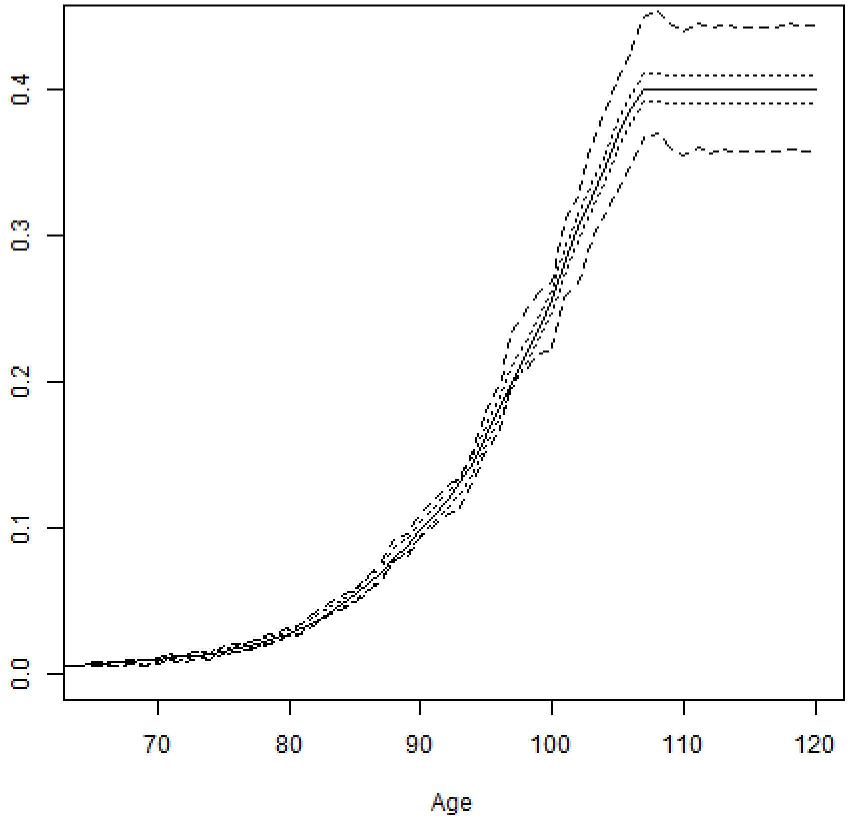


With 5,000 company exposures and 1,000 company deaths

Estimating the entire mortality curve



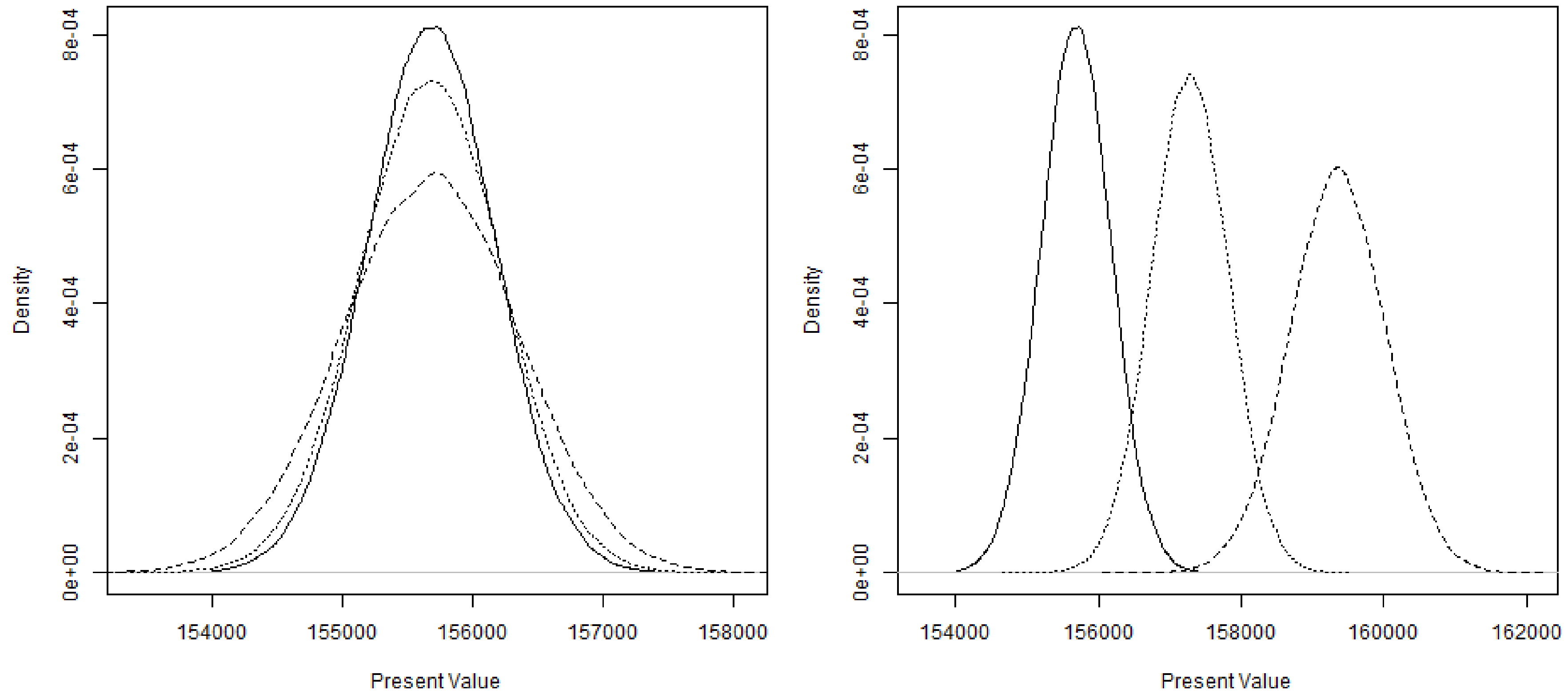
No company data



Cohort of 10,000 lives

Solid line: Table rate
Dotted: $N = 500$
Dashed: $N = 10{,}000$

Impact on SPIA



Company data follows industry table

Company data better than industry table

10,000 SPIA issued to age 65 female

Solid line: No Parameter Uncertainty

Dotted: $N = 500$

Dashed: $N = 10{,}000$

Tail Quantiles

	75%	90%	95%	99%	99.5%	99.9%
No Parameter Uncertainty	156011.5	156309.3	156487.8	156828.9	156944.6	157182.7
Actual-to-Tabular	157364.6	158117.2	158571.7	159410.9	159718.1	160333.2
Individual, N=500	156134.6	156543.7	156784.8	157230.1	157400.4	157739.5
Individual, N=10,000	156048.6	156374.6	156569.8	156940.2	157076.4	157349.6

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