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ERM

*Symposium*

*Where Cutting Edge Theory Meets State of the Art Practice*

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# Hedging VA Guarantees

## From Defining Objectives to Optimizing a Program

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RGA Re

# Motivation

- Atilio Meucci's 2011 "Prayer"
  - Methodology for portfolio and risk allocation
- EBIG Conference in 2011
  - Many participants mentioned reinsurance as a potential solution for VA Guarantees
  - Availability of reinsurance

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# Introduction to Variable Annuities

- **What are VA's?**
- **Contract:** policyholder pays initial premium, receives promise of future payments starting at annuitization date
- **Accumulation and payout phases**
- During accumulation phase premium invested in mutual funds (equity market exposure and risk)
- Used in retirement planning

- **VA's selling points are:**
  - Tax advantages
  - Equity based guarantees: Living benefits and Death benefits

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# Introduction to DBs

- **Death benefit**: paid to beneficiary upon death of policyholder
- **Amount paid** may be the higher of account value and :
  - Return of premium
  - Ratchet
  - Roll-up
  - Combination of Ratchet and Roll-up
- **Cost** is percentage of account value, over the life of the policy
- **Protects beneficiary** from a down equity market at time of death of policyholder

# Introduction to DBs (2)

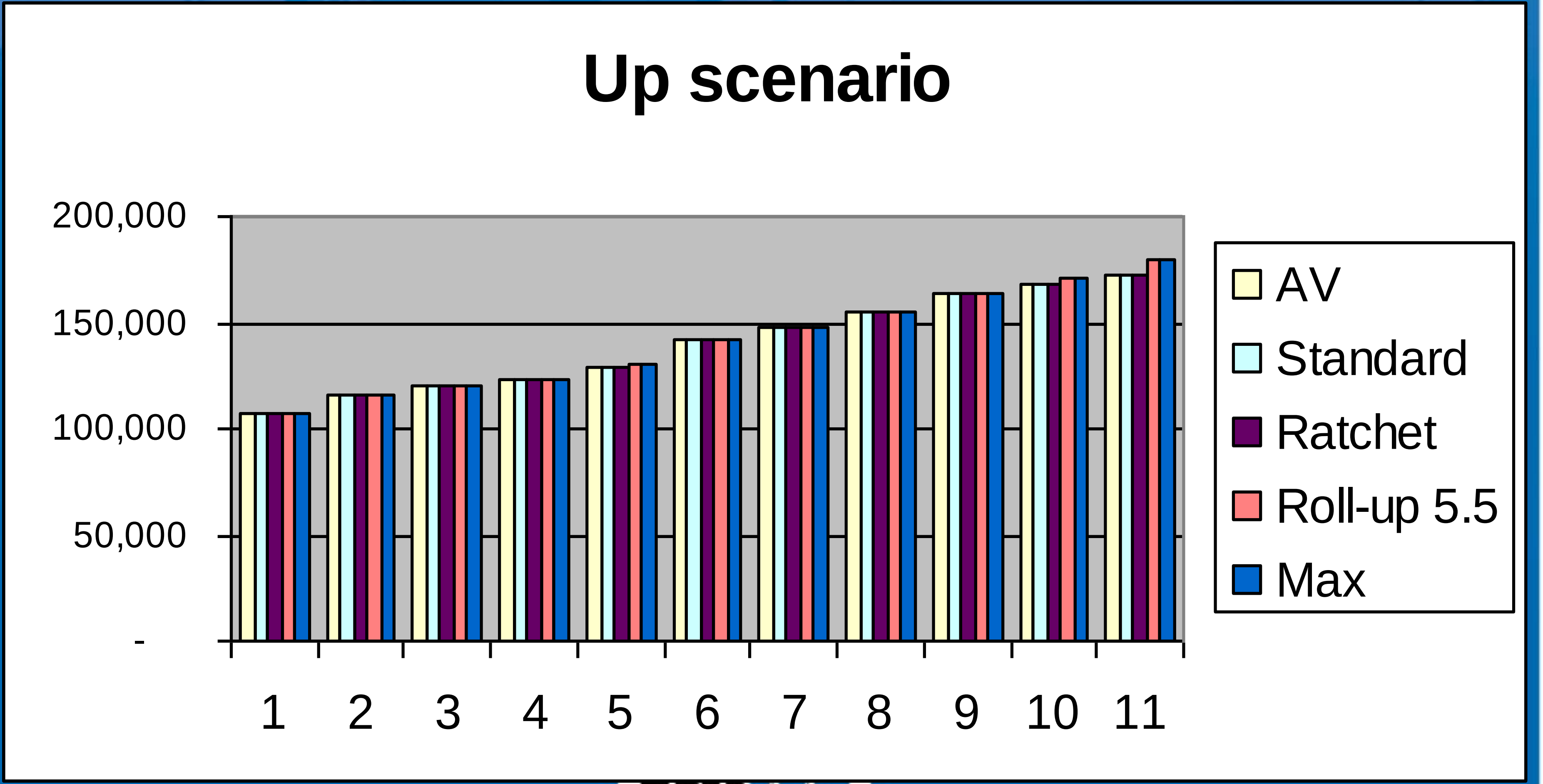
**Modeling of DBs**

- Annual or quarterly models
- Series of put options
- Probability of exercise: tied to survival/mortality
- Projections for 20, 30 or more years
- Path dependent
- Monte Carlo simulation of multiple assets at policy-level (computation intensive)
- Policyholder behavior (lapse, partial withdrawal)
- Impact of living benefits (and associated policyholder behavior) on DBs

# Relation of DB to Options

- The difference between DB and AV represents the payout of the option sold to the client
- Mortality and survival gives
  - the probability of exercise of the option
  - the expected time of exercise
- Time of Death is the actual time of exercise
- Some options are path dependent!

# In an up Scenario: DB is not worth much

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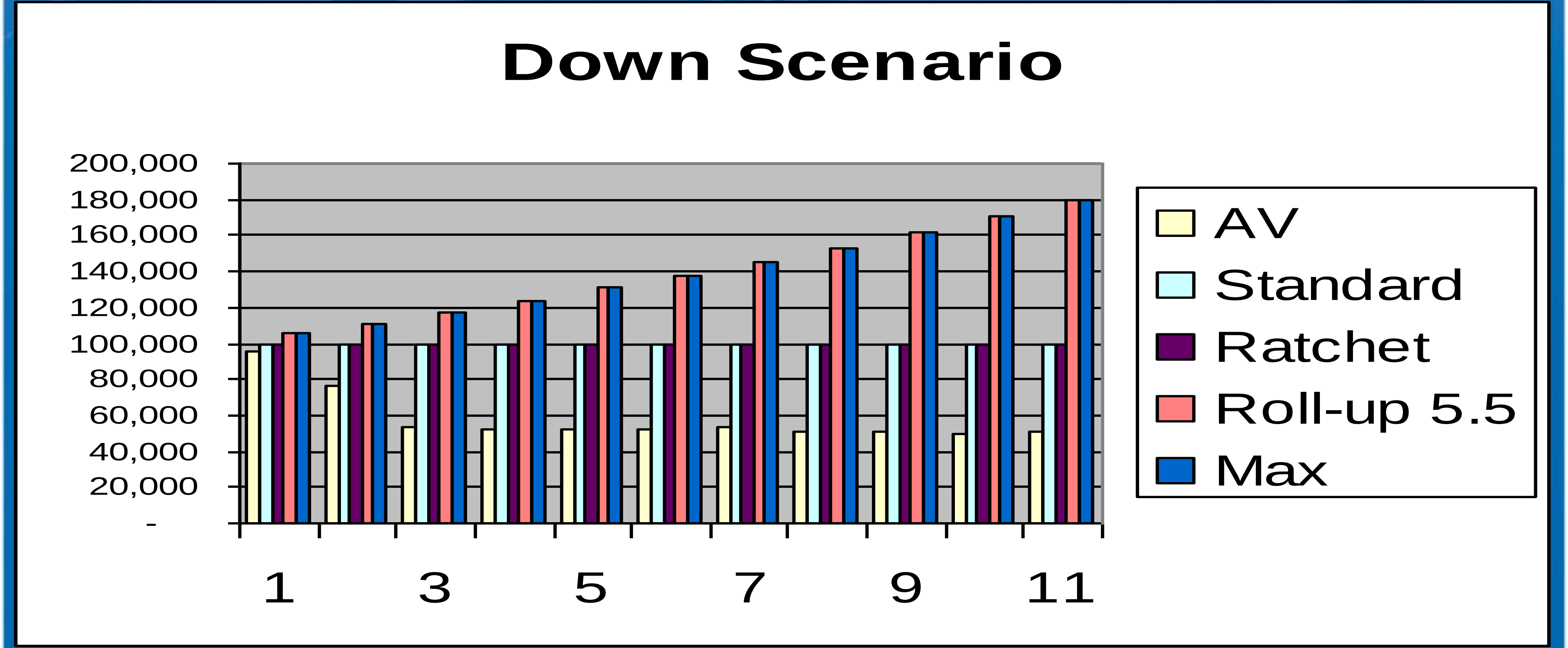
# Value of guarantees shown in down scenario

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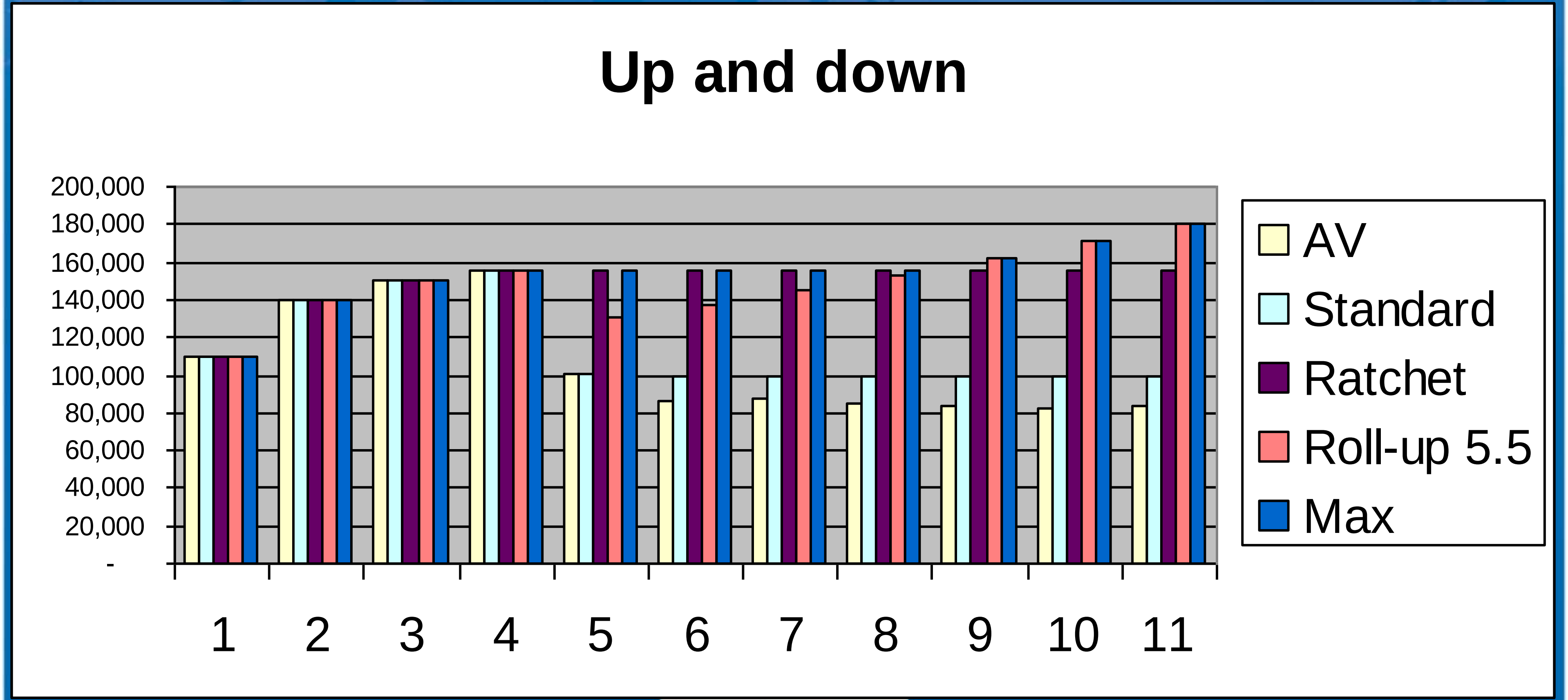
# Ratchet valuable in an up and down scenario

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# Introduction to LBs

- **Accumulation Benefit**
  - Increases benefit base to initial premium if market down
  - Exercisable ten years from issue
- **Income Benefit**
  - Rolls-up benefit base by x%
  - May be combined with a ratchet feature
  - Exercisable after ten or more years
- **Withdrawal benefit**
  - Guarantees minimum annual withdrawals as x% of initial premium for y years or for life of annuitant
  - Exercisable immediately

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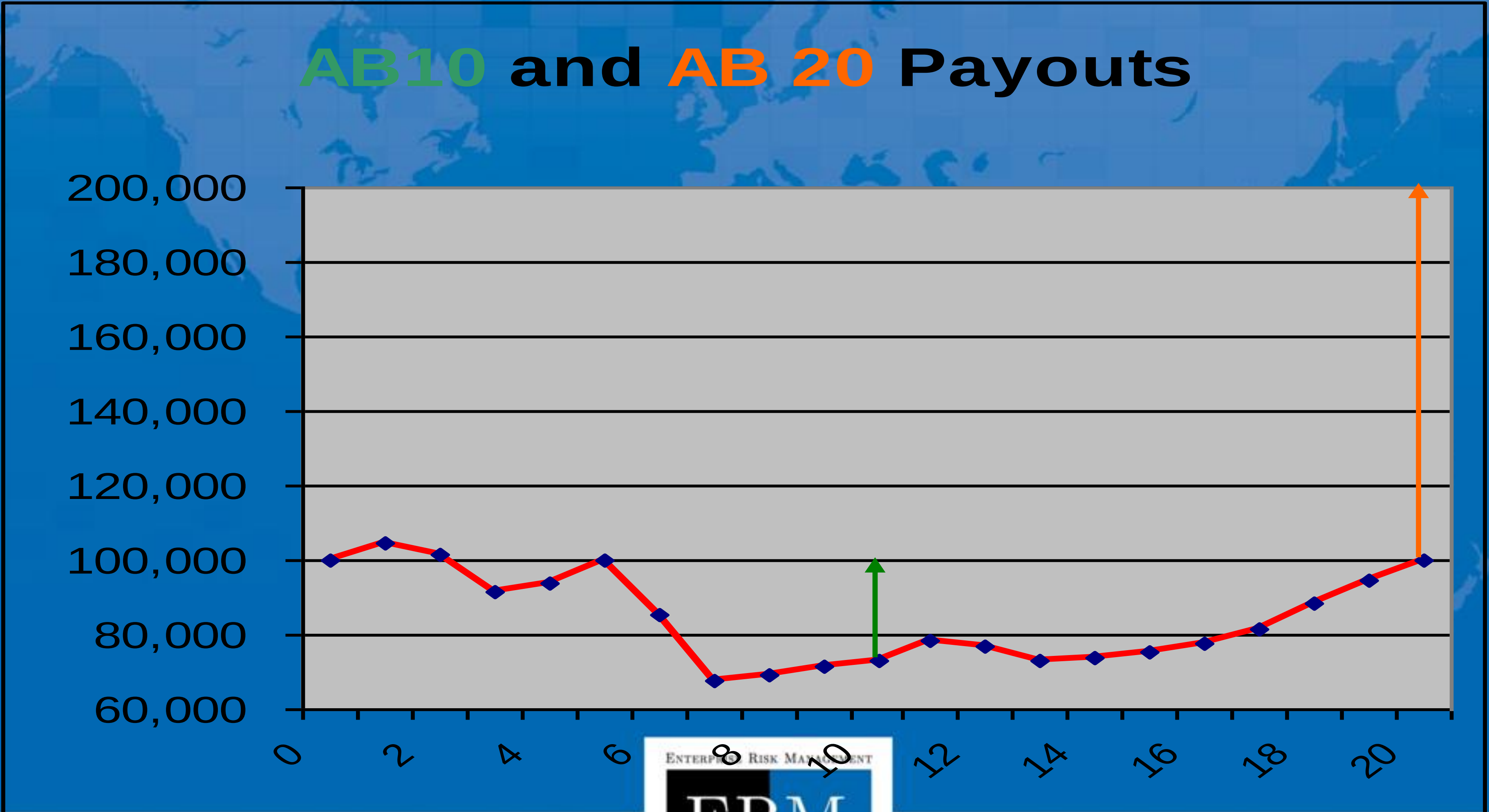
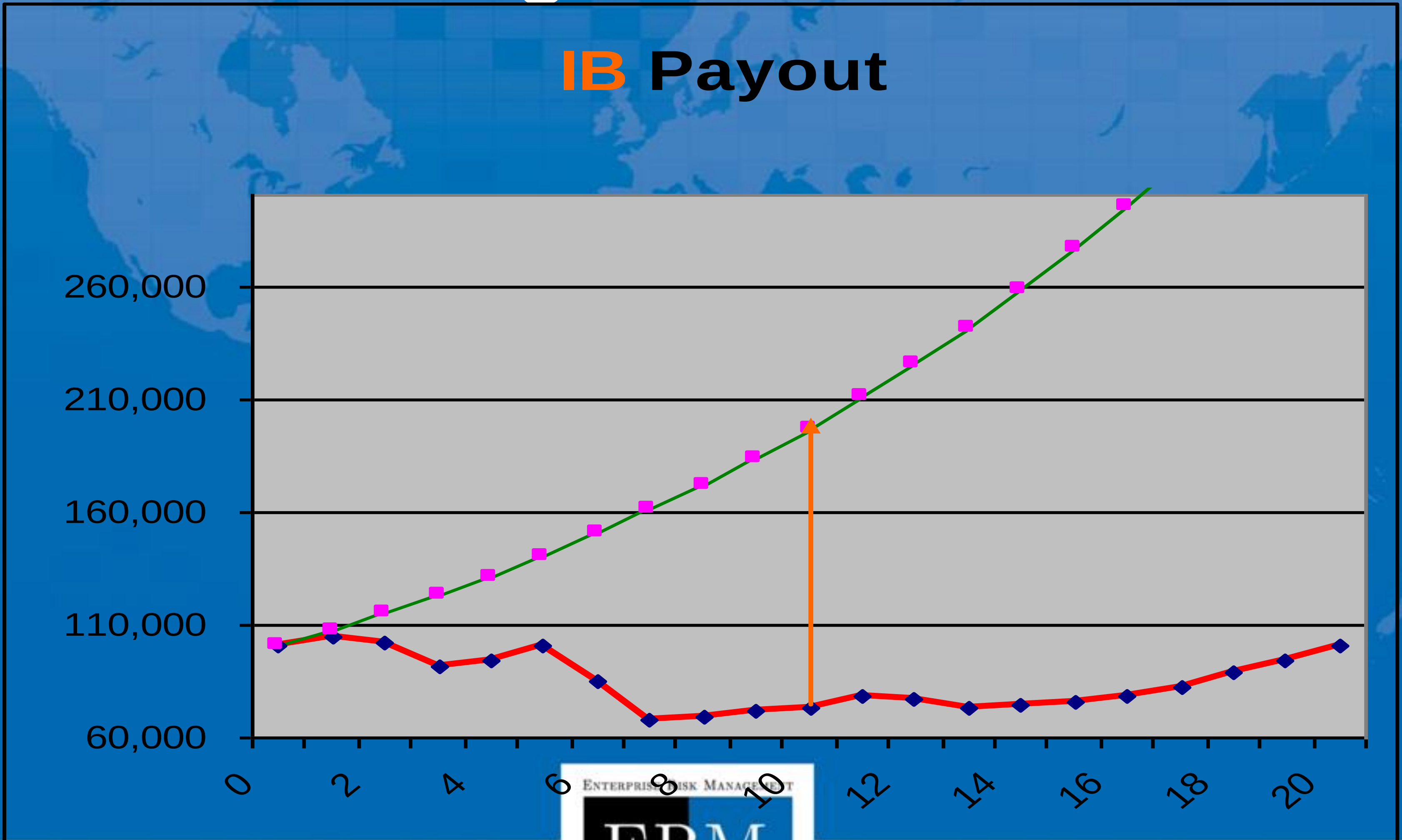
# Introduction to Living Benefits(2)

- Paid for with fees over the life of the policies
- Modeling issues:
  - Similar to DB modeling issues
  - Probability of exercise tied to survival
  - Policyholder behavior: if benefit in the money, policyholder may restrain making withdrawals which he would normally make in order to exercise option
  - Impact of LBs on DBs: additional fees taken out of account value and different persistency

# Living Benefits: AB10 and AB20



# Living Benefits: IB



# The Lifetime withdrawal benefit

- Guarantee a minimum level of annual withdrawals for the lifetime of the annuitant ( even if withdrawals reduce contract value to zero)
- Computation of the Benefit Base:

  Initial Premium

  Quarterly Ratchet

  7% Compounding Step-up
- MAW – maximum annual withdrawal percentages (for example:
  - 5% for age 59.5 to 69
  - 6% for age 70 to 79
  - 7% for age 80+

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# Death Benefits and Living Benefits summarized as options

- Essentially, guarantees are similar to put options sold on a basket of assets
- Policyholder Behavior is an essential additional factor
- Additional features bring strong path dependency

# Risk Identification

- Main risk to the company (direct writer/ reinsurer): financial loss due to insufficient fees/ large claims due to market fluctuations.
- Divide and conquer:
  - Equity level risk → Delta
  - Interest rate risk → Rho
  - Volatility risk → Vega
  - Other market risks ( Gamma, Cross-Greeks)
  - Model risk
  - Operational risk

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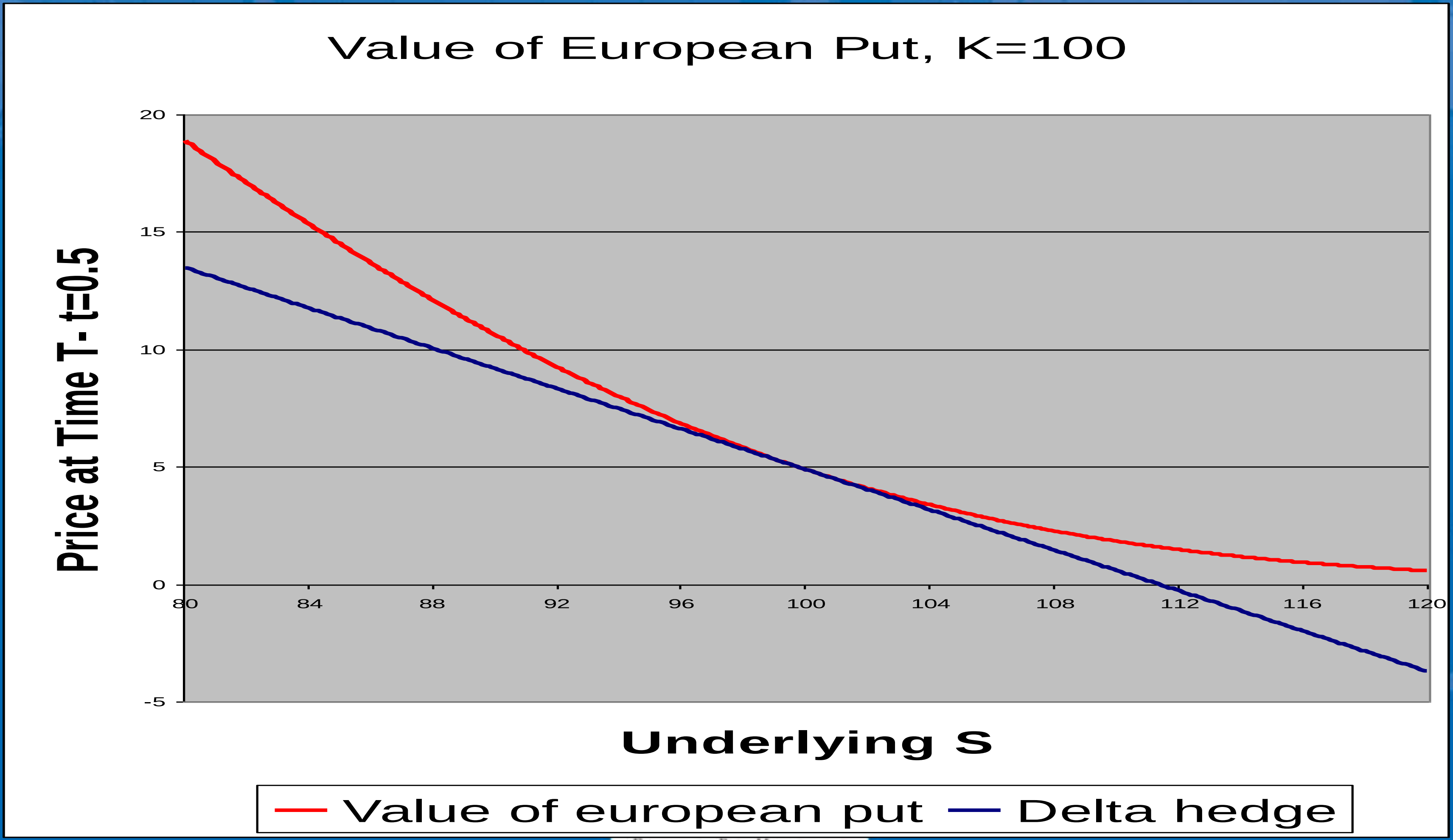
# The Greeks

- Delta $\Delta = \partial V / \partial S$ , sensitivity of the option with respect to the change in the underlying
- Gamma $\Gamma = \partial \Delta / \partial S$, sensitivity of Delta with respect to the change in the underlying
- Rho $\rho = \partial V / \partial r$, sensitivity of the option with respect to the change in the interest rate r
- Theta $\Theta = \partial V / \partial t$, sensitivity of the option with respect to the change in time t
- Vega $= \partial V / \partial \sigma$, sensitivity of the option with respect to the change in volatility $\sigma$

V here is the value of the option (PV of cost), S is the underlying (stock, AV for us)

# Put value (BS) and Delta

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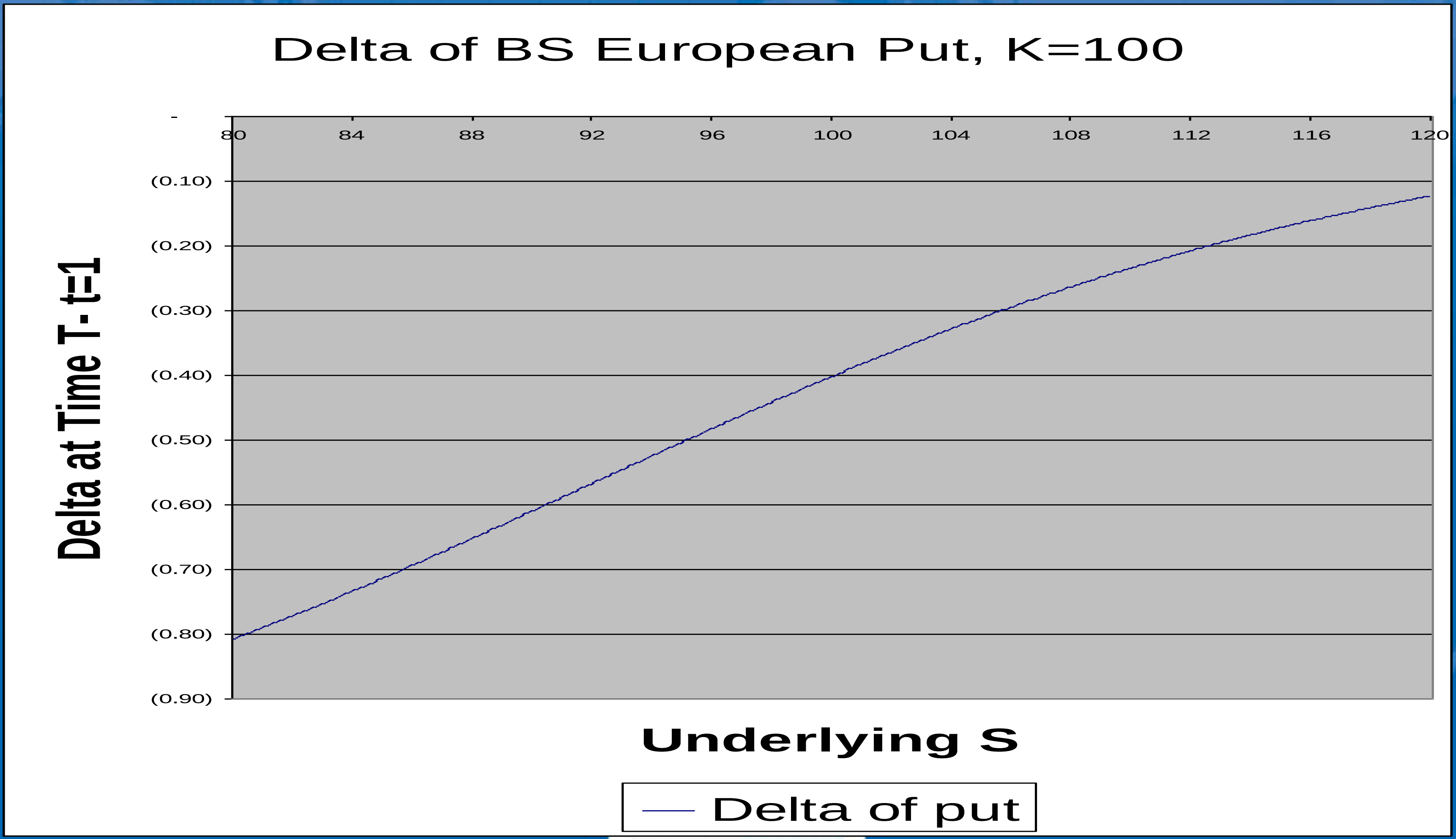
# Delta of put as a function of S

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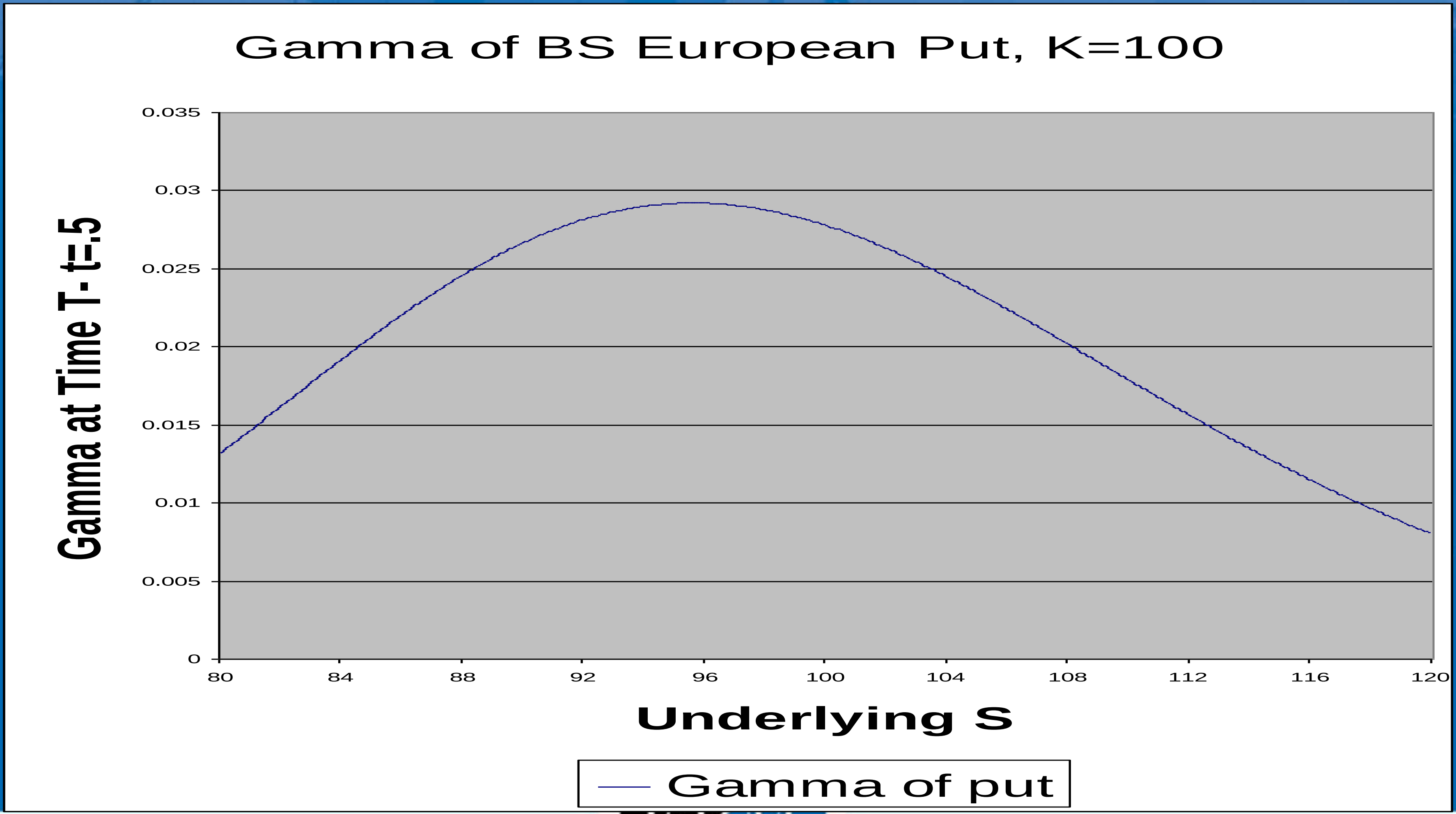
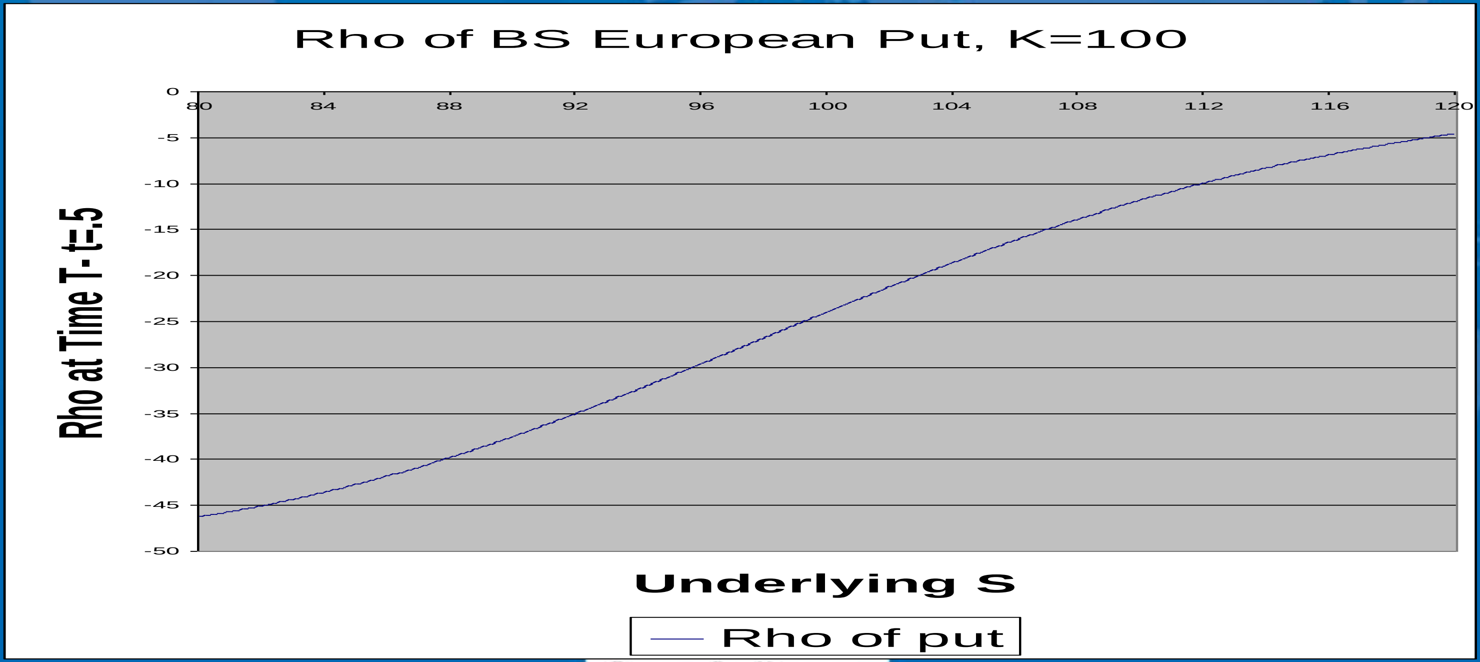
# Gamma of Put as function of S



# Graph of Rho ( r is constant?)

- Note in BS, Risk free rate is assumed constant, so rho is zero.
- The sensitivities would mark a transition form one BS world to another, with a different risk free rate.

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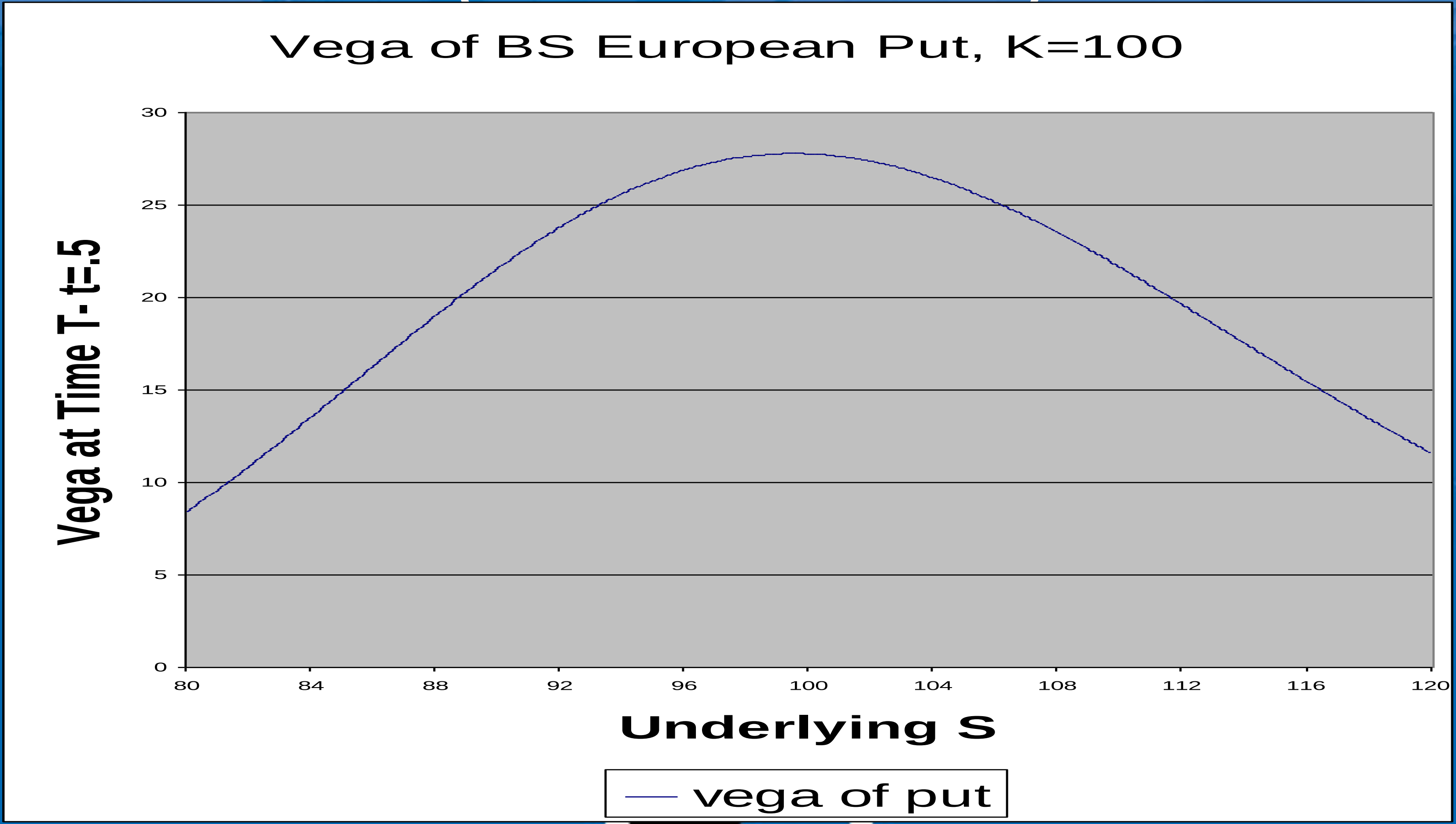
# Vega as function of S
# (σ is constant ?)

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# Black- Scholes Framework

- Key Assumptions
  - Underlying follows lognormal random walk (can be relaxed)
  - Risk free interest rate is constant (can be a known function of time)
  - No dividends on the underlying (may have continuous dividends)
  - Delta hedging done continuously
  - No transaction fees
  - No arbitrage

- Stochastic process:

  $dS = \mu S\,dt + \sigma S\,dX$

- Leads to PDE:

  $\Theta + \frac{1}{2}\sigma^2 S^2 \Gamma + rS\Delta - rV = 0$

- Which has as solution the Price of Call Options:

$C = S\,N(d_1) - K\exp(-r(T-t))N(d_2)$

where N is the normal distribution,

$d_1 = (\log(S/K) + (r + \frac{1}{2}\sigma^2)(T-t))/\sigma\sqrt{(T-t)}$

$d_2 = (\log(S/K) + (r - \frac{1}{2}\sigma^2)(T-t))/\sigma\sqrt{(T-t)}$

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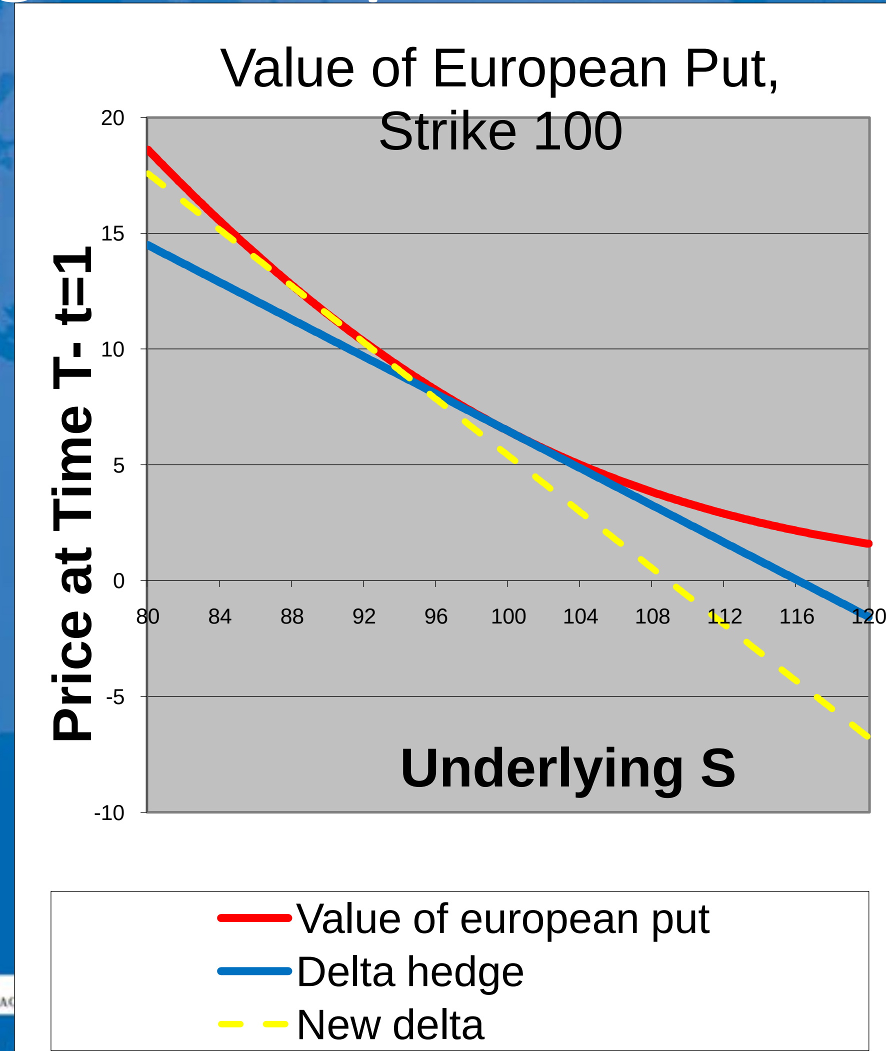
# Hedging VA Guarantees

- Take positions to neutralize the change in liabilities due to movements of:
  - Equity (delta hedging)
  - Interest (rho hedging)
  - Volatility (vega hedging)
- Common Hedging Instruments:
  - Index Futures, Interest rate swaps, Options

# Delta Hedging: Concept

- Risk is that underlying funds (proxy: S&P 500 index) decline, values of guarantees increase (red line put)
- Delta: slope of tangent
- To hedge, we short S futures (blue line): match slopes
- If S declines, Delta becomes more negative (yellow dotted line), so we have to short more, i.e. "sell low"
- "Sell low, buy high" to mitigate risk
- The trading losses are also mitigated (up to a certain level of volatility) by the time decay of the option

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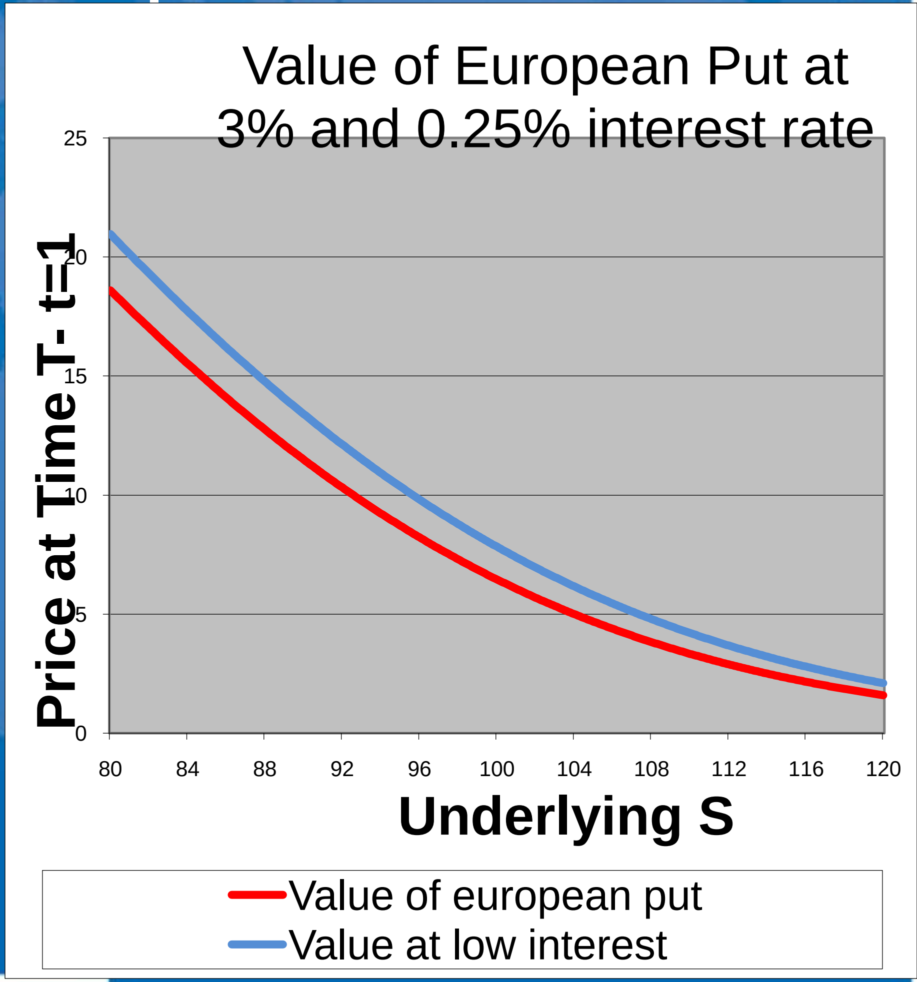
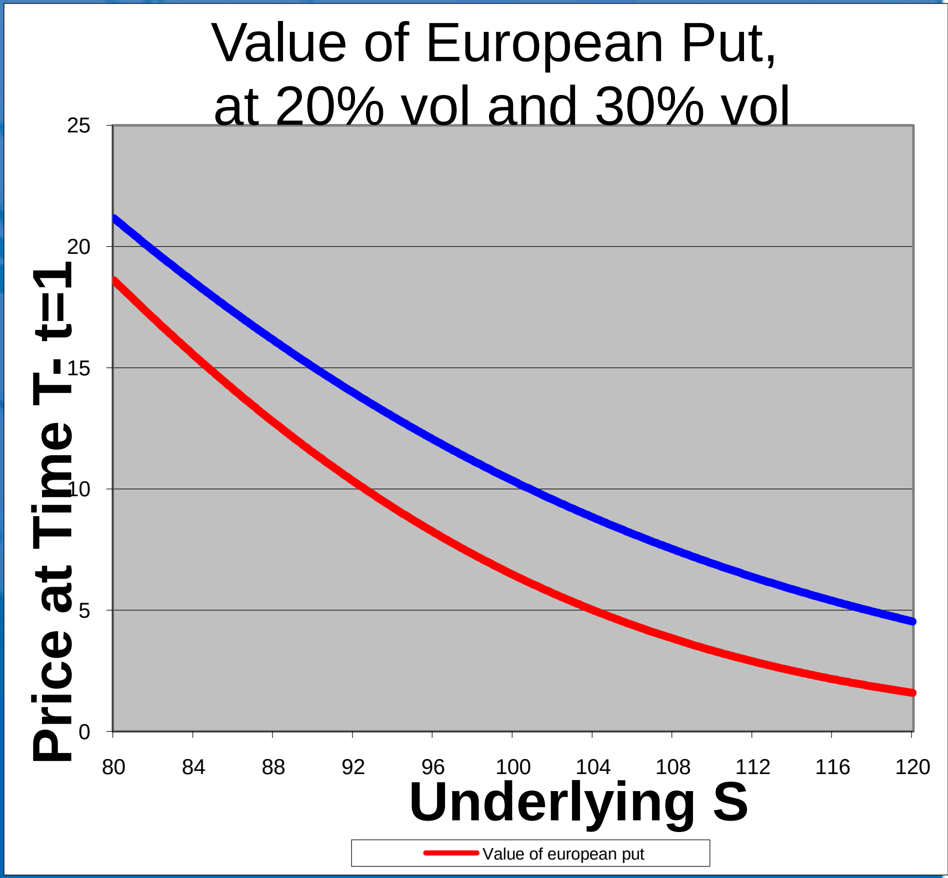
# Rho Concept

- Risk is that as interest rates decrease the value of the option will increase
- Rho is sensitivity of value of the option with respect to interest change
- When equities are down due to slowing economy, flight to treasuries, interest rates come down
- Hedge Rho, by entering into swaps (long term receive fixed)
- As interest rates come down, more Rho, more swaps are needed



# Vega Concept

- Risk is that volatility will go up and increase the value of the guarantees
- Vega is rate of increase of option with Volatility
- In a High Vol environment, we will experience more trades to rebalance the Delta position, and losses on “Buy high, sell low”
- Mitigate by hedging Vega

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# Hedging Objectives

- Need to be defined:
  - Economic, Accounting, Tail?
  - The answer may vary by company
- Once this defined
  - Define thresholds for  Greek exposures
  - Define derivatives usage limits
  - Define liquidity  and capital constraints

# Case Study: Model Specification

- Using a state space model, We model the joint distribution of the yield curve with index return and implied volatility, which are the key risk factors for VA product.

- Some Key Assumptions:

| | |
|---|---|
| Interest rate: | Mean-reverting process, |
| | Affine Term Structure, |
| Equity Index: | Stochastic Volatility, |
| | Index Return Jump, |
| | Flight to Quality Effect |
| Implied Volatility: | Jones Equation, Leverage Effect |

# Case study: Model Estimation

- The model is estimated utilizing swap term structure, equity index return and VIX index level back to May of 1994.
- The model is estimated using a Markov Chain Monte Carlo (MCMC) Bayesian Approach.

"Learn" about the unobserved latent factors using the observed interest rate data and S&P 500 Index Returns.

Allows flexible amount of prior information to be updated with the large panel of observed data.

Integrate out the parameters in order to reflect parameter uncertainty in the forecasted scenarios.

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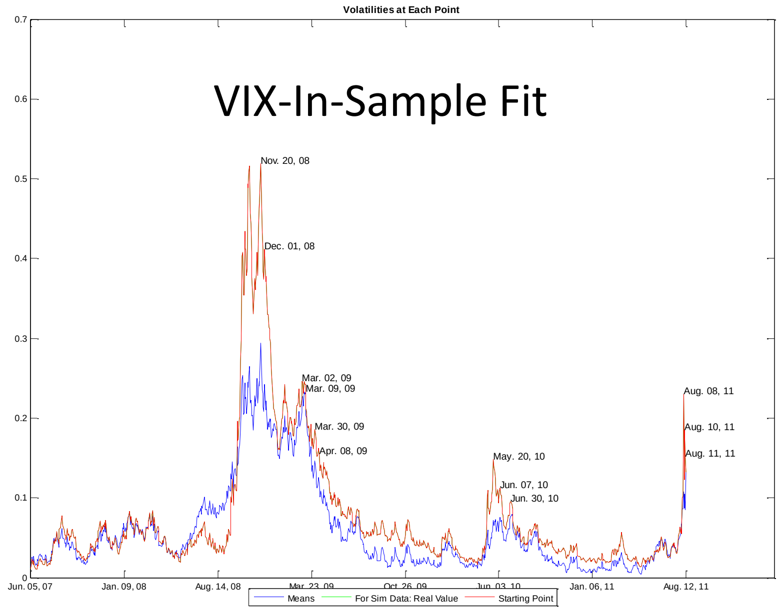
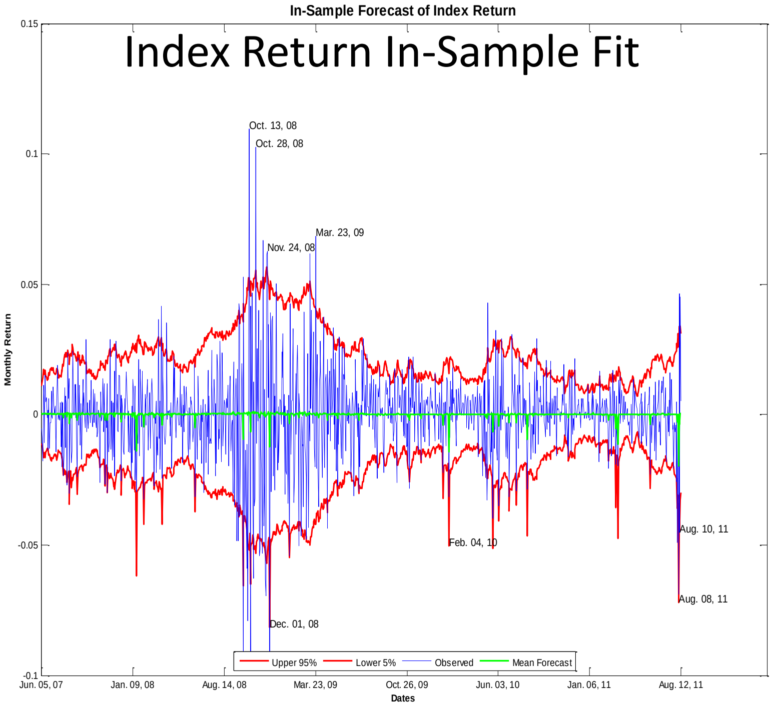
# In Sample Fit For Index Return and Volatility

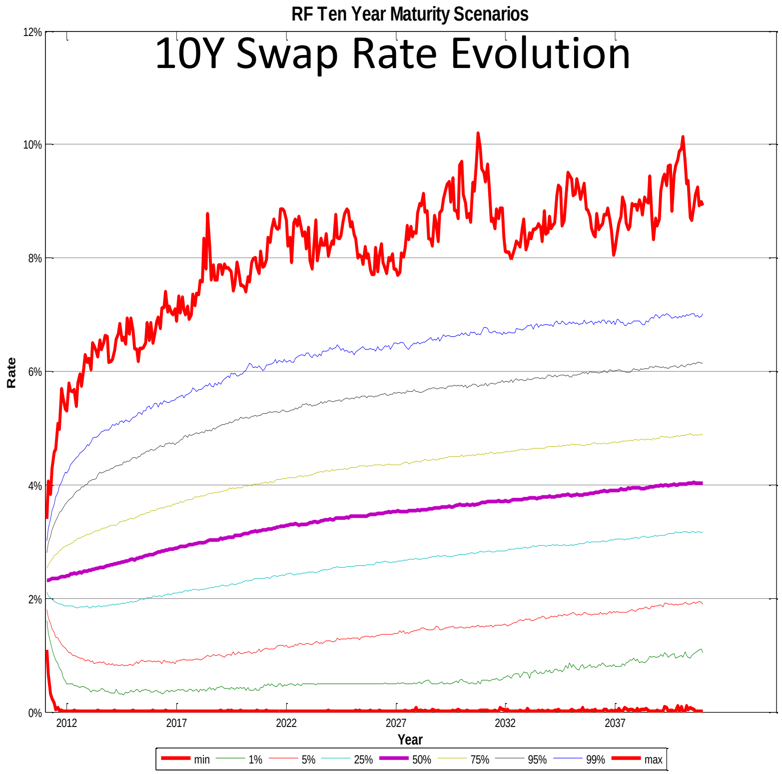


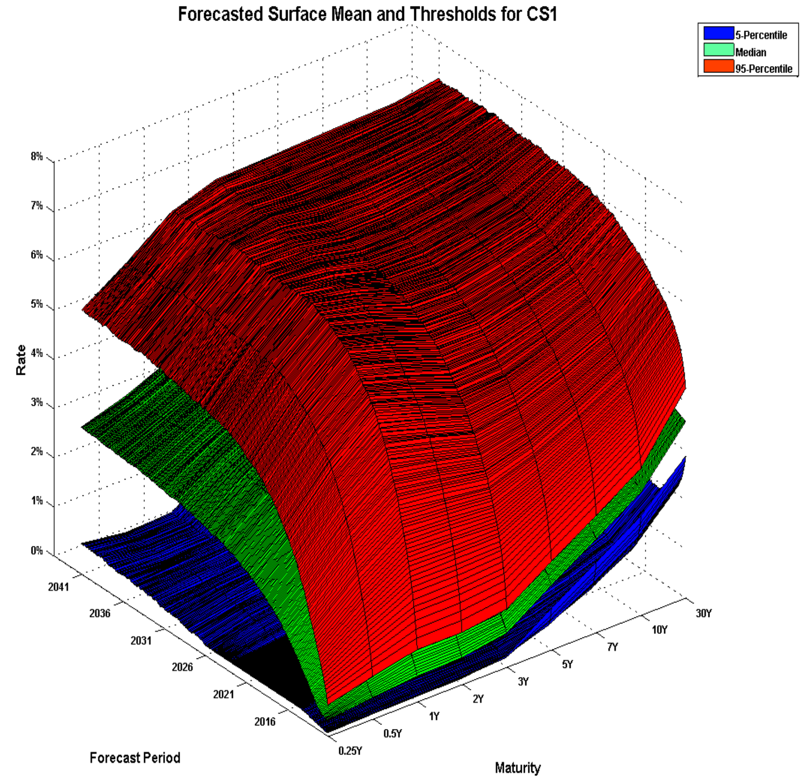

Make sure the model capture the dynamic of historical market behavior

# Out of Sample Forecasting: Swap Term Structure

10Y Swap Rate Evolution




Forecasting Interest Rate Surface

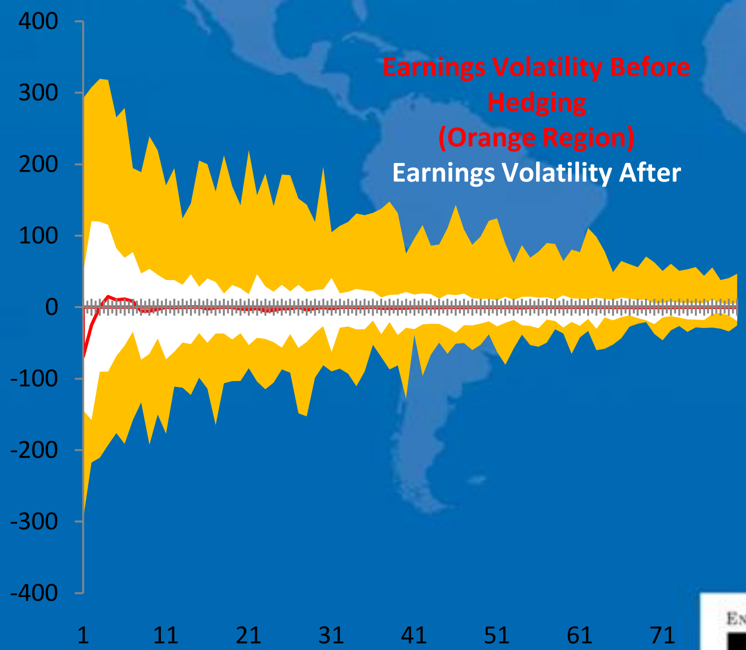


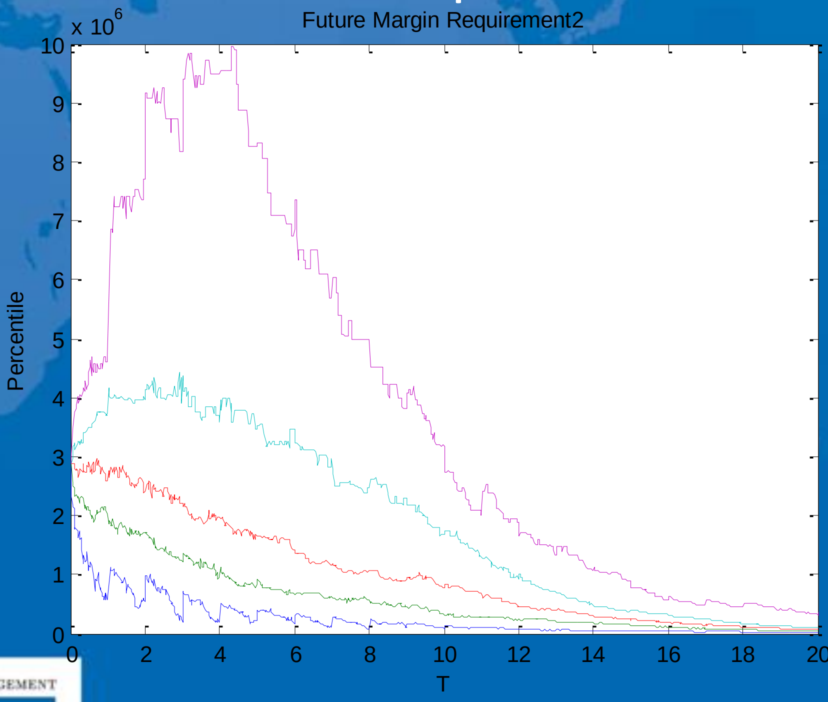
# Scenarios Generation

- Use Monte Carlo method to generates 10,000 of possible outcomes from the model, to describe the distribution of market in the futures.
- Evaluate both Hedging Instruments (Asset) and Liability under each time point and each scenario.
- On each “run”, implement the hedging strategy, calculate the amount of derivatives we need, tracking market value, Greeks, liquidity requirement and any other important measure for the hedging position.

# Hedging Strategy Testing

- Monte Carlo Simulation helps us to address not only average scenarios of the out come, but also tail risk of the portfolio.

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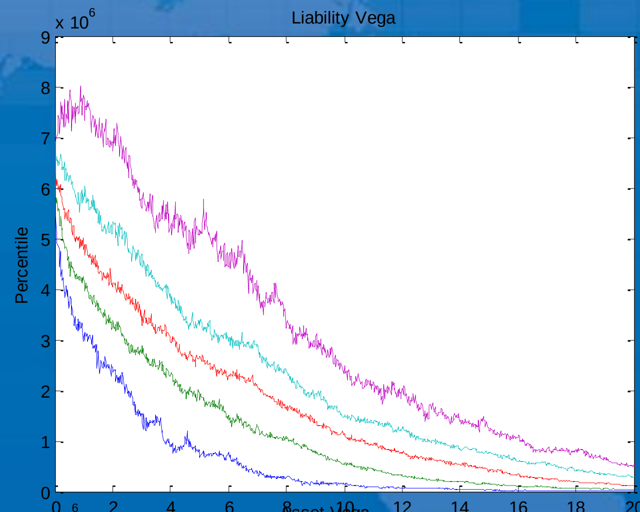
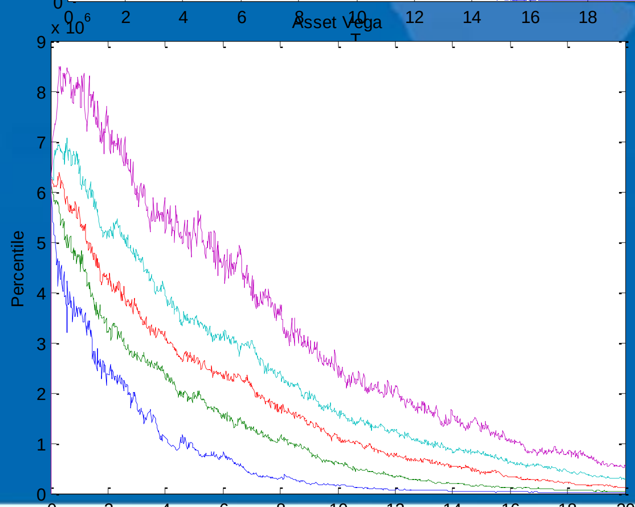
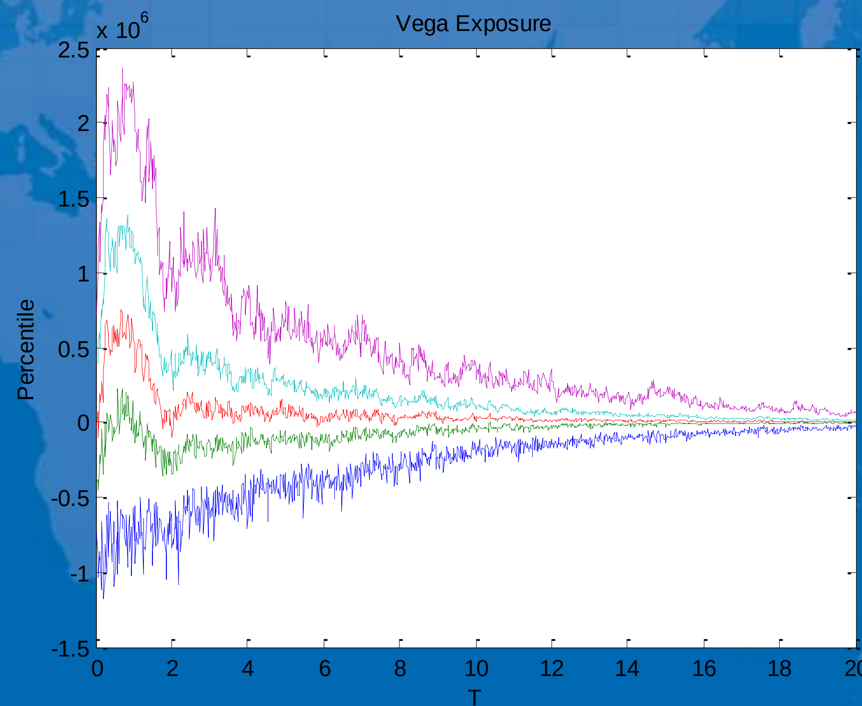
# Hedging Strategy Testing

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# Hedging Strategy: Innovation and Optimization

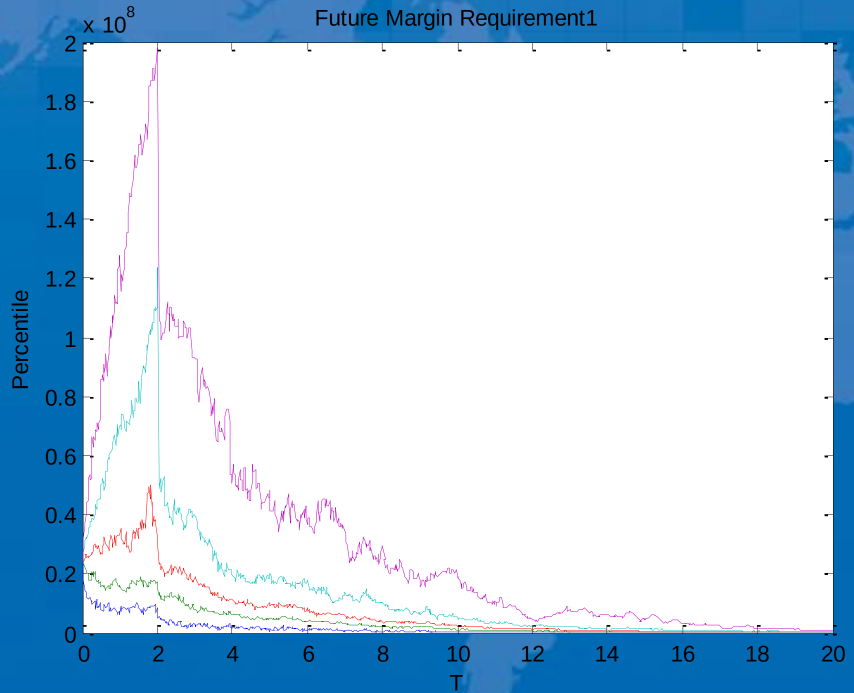
To standardize our optimization procedure, we have to define:

A. Target Function: f(x) (The economic output we want to optimize)
   Total P&L, Total Liquidity requirement…

B. Constraint: g(x) (Define the resource we allocate to this project)
   Derivatives Position should be less than…
   5% Value at Risk should not exceed…

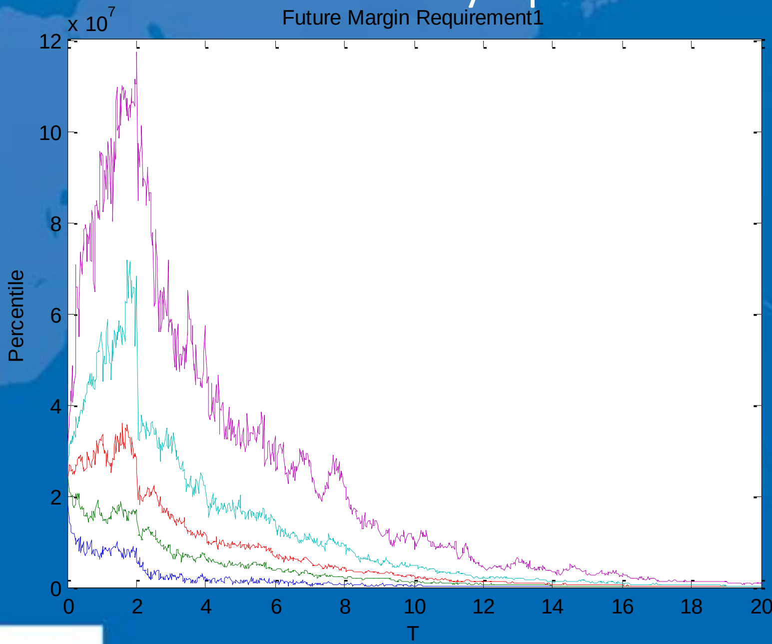
C. Variable: x (The hedging strategy we define)
   Threshold and Hedge Effectiveness,
   Different Hedging instruments,
   Specific Strategy Given certain market condition.

# Hedging Strategy: Innovation and Optimization
# An Example




Basic Strategy: 90% Hedge Effectiveness for Delta

Improved Strategy: 90% Hedge Effectiveness for Delta + Unwind Out of Money Option

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# Hedging Strategy: Innovation and Optimization An Example

- Find out Economic Interpretation for Improved strategy

# Model Risk

- Taleb's Black Swan Definition: A highly improbable event, with great impact, explainable after the fact
- Name explanation: before discovery of Australia all swans known (to Europeans) were white

# Example of Black Swans

- Positives
  - Discovery of Americas by Columbus
  - Invention of the Internet
- Negatives
  - Pandemics
  - Losses due to a catastrophic earthquake and/or tsunami
  - Financial crisis of 2008

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# The Thanksgiving paradox

- Turkeys are fed everyday by humans
- They would expect, based on past performance, to continue to be fed
- Something else happens just before Thanksgiving

# Modeling Black Swan issue

- (One) Key Idea: The (log)normal distribution does not accurately describe the evolution of stocks: we are using the wrong probability distribution
- Recent example: May 2010 “Flash-Crash”

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# What could happen to hedging? (a few scenarios)

- Markets could freeze (or be halted)
  - Can not trade, experience a gap
  - Continuous rebalancing assumption is key
- Dealer that holds your derivatives may go bankrupt
  - Example: Lehman Brothers in 2008
- Dealer may change collateral requirements or spreads in crisis

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# What to do about it? (a few thoughts)

- Use more than one model  to value risk (think ranges, not values)
- Use different hedging instruments
- Measure counterparty risk
- Diversify counterparty risk
- Measure and manage collateral
- Monitor liquidity of markets you are in

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# Other Risks to Consider

- Lapse and longevity assumptions
- Basis risk: performance of the underlying mutual funds relative to indices they are mapped to
- Correlations and Higher Order Greeks

# Operational Risks

- Risks related to trading operations
  - Person
  - System
- Risks related to periodic valuation of liabilities and Greeks (whether in-house or outsourced)

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# Governance

- Oversight needed
  - For definition and execution of strategy
  - For maintaining the program within the limits agreed on
- Multi-departmental effort: Need to include not just Risk Management, but also Investments, Valuation, Pricing, Treasury
- DUP and ISDAs

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# Concluding remarks

- Measuring, Managing and Mitigating risks associated with VA guarantees:
  – Is not easy
  – Is intellectually challenging and interesting
  – Can help the company write a profitable product

# Disclaimer

- The views and methodologies presented herein reflect the authors opinions and do not necessarily reflect those of RGA