ARTICLES

RISK ALLOCATION IN CAPITAL MARKETS: PORTFOLIO INSURANCE, TACTICAL ASSET ALLOCATION AND COLLAR STRATEGIES

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ABSTRACT

The theory of risk exchange is applied on the allocation of financial risk in capital markets It is shown how the shape of individual payoff functions depends on risk tolerance and cautiousness. For the special case where the Neumann-Morgenstern utility functions of all individual investors belong to the HARA class and have non decreasing risk tolerance it is proved that generalized versions of "portfolio insurance", "tactical asset allocation" and "collars" are the only strategies occurring in price equilibrium.

KEYWORDS

Non linear risk sharing, price equilibrium; portfolio insurance.

1. INTRODUCTION

For quite a long time the MARKOWITZ (1952) approach and the Capital Asset Pricing Model (SHARPE, 1964; LINTNER, 1965) played a predominant role in financial economics In such a framework only linear risk allocations can occur. However, in 1973 BLACK and SCHOLES published their famous option pricing formula, which allows in particular for a replication of options by means of dynamic strategies. Options and their dynamic replication became increasingly popular. Nowadays, non linear investment strategies, such as portfolio insurance, tactical asset allocation and collars are widely used.

In actuarial science non linear risk allocations are a central issue in the reinsurance context. Already in 1960 Borch's theorem on Pareto efficient risk sharing was published. Later on, BUHLMANN (1984) proved the existence of a price density leading to a Pareto efficient risk allocation which is typically non linear. In BÜHLMANN (1980) and LIENHARD (1986) price densities were explicitly calculated for some special cases.

LELAND (1980) was the first who applied the actuarial results on non linear risk sharing in financial economics. By means of Borch's theorem he analysed the occurrence of portfolio insurance in the context of capital market equilibrium. MULLER (1990, 1991) applied Bühlmann's equilibrium model on the capital market and obtained some first results about the qualitative shape of risk allocations.

In this article total financial risk has to be allocated to n investors. Following the tradition of RUBINSTEIN (1976), BRENNAN (1979) and LELAND (1980) all investment decisions have to be made at one point of time. First, the main results of the theory of risk exchange are shortly summarized. Thereafter, different types of investment strategies such as portfolio insurance, tactical asset allocation and collars are explained in the context of risk exchange. It is shown how the type of investment strategy chosen by an individual investor depends on the risk tolerances of all investors and their sensitivity to wealth changes. Finally price equilibria are analysed in the special case where the Neumann-Morgenstern utility functions of all investors belong to the HARA class. Generalized versions of portfolio insurance, tactical asset allocation and collars are the only investment strategies which can occur.

2. THEORY OF RISK EXCHANGE

2.1. The model

As in RUBINSTEIN (1976), BRENNAN (1979) and LELAND (1980) trade takes only place at one point of time

There are n investors $i = 1, \ldots, n$ with the following characteristics:

1) All investors have the same planning horizon and the same expectations. In particular, their expectations with respect to total financial wealth (aggregate market value of all financial assets in an economy) at the end of the planning period are given by a random variable $\tilde{W}$.
2) Moreover, the value of investor i's ($i = 1, \ldots, n$) initial endowment at the end of the planning period can be represented by random variables $\tilde{X}_i$, s.t.

$$\tilde{X}_i \geq 0, \qquad \sum_{i=1}^{n} \tilde{X}_i = \tilde{W}, \text{ a.e.}^{1}.$$

3) Each investor $i = 1, \ldots, n$ evaluates his wealth at the end of the planning period by a Neumann-Morgenstern utility function

$$u_i : R \rightarrow R, \qquad i = 1, \ldots, n.$$

Hence, for the investors $i = 1, \ldots, n$ with the initial risk allocation

$$(\tilde{X}_1, \ldots, \tilde{X}_n)$$

[1] If initial endowments consist only of the market portfolio and a riskless asset, then

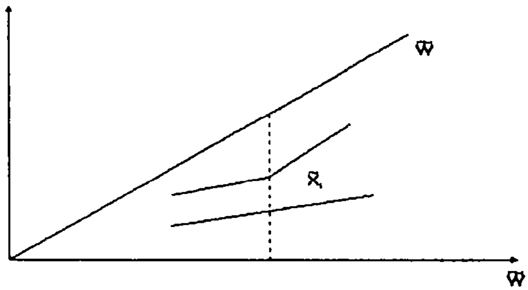
$$\tilde{X}_i = a_i + s_i \tilde{W}, \text{ a e } \quad i = 1, \quad , n, \quad \text{with} \quad \sum_{i=1}^{n} a_i = 0, \quad \sum_{i=1}^{n} s_i = 1$$

holds

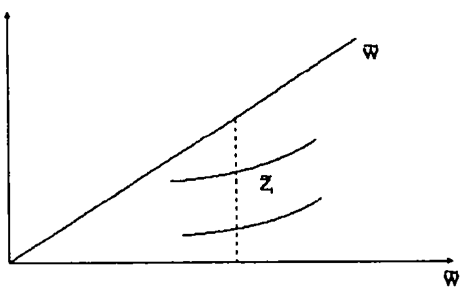
a reallocation of total financial risk

$$(\tilde{Z}_1, \ldots, \tilde{Z}_n) \qquad \text{with} \qquad \sum_{i=1}^{n} \tilde{Z}_i = \tilde{W}$$

has to be found.



Initial allocation of total financial wealth



Reallocation of total financial wealth

FIGURE 1

Obviously, this framework allows for the application of standard results in risk theory (e.g. BORCH (1960), BÜHLMANN (1980, 1984))

2.2. Theory of risk exchange: standard results

The following assumptions will be useful:

A.1. a) The random variable $\tilde{W}$ is given by the probability space $(R, \boldsymbol{B}, P)$, where $\boldsymbol{B}$ denotes the Borel-σ-algebra. There exist constants m, M with $0 < m < M < \infty$, such that

$$P[m \leq \tilde{W} \leq M] = 1 .$$

b) There exist measurable functions h_i, $i = 1, \ldots, n$ such that

$$\tilde{X}_i = h_i(\tilde{W}), \text{ a.e.}^2.$$

c) $E[\tilde{X}_i] > 0$.

A.2. The Neumann-Morgenstern utility functions u_i, $R \to R$, $i = 1, \ldots, n$ are twice differentiable and satisfy

$$u_i'(x) > 0, \quad u_i''(x) < 0 \quad \forall x.$$

A.3. The Neumann-Morgenstern utility functions u_i; $R \to R$, $i = 1, \ldots, n$ are three times continuously differentiable

Moreover, the following definitions are needed:

Definition 1: An n-tuple of random variables $(\tilde{Z}_1, \ldots, \tilde{Z}_n)$ is called a *feasible allocation* if it satisfies

$$\sum_{i=1}^{n} \tilde{Z}_i = \tilde{W}, \text{ a.e.}$$

Definition 2: A measurable function

$$\phi : [m, M] \to [0, \infty[$$

is called a *price density* if it satisfies

$$E[\phi(\tilde{W})] = 1.$$

Remark: Under a price density ϕ the value of a random variable $\tilde{Z}_i = f_i(\tilde{W})$ is given by

$$E[f_i(\tilde{W})\,\phi(\tilde{W})] = \int f_i(w)\,\phi(w)\,dP(w) \tag{1}$$

Definition 3: The tuple $\{\phi, (\tilde{Z}_1^*, \ldots, \tilde{Z}_n^*)\}$ consisting of a price density ϕ and a feasible allocation $(\tilde{Z}_1^*, \ldots, \tilde{Z}_n^*)$ is called a *price equilibrium* if for all $i = 1, \ldots, n$ $\tilde{Z}_i^*$ is the solution of

$$\max_{\tilde{Z}_i} E[u_i(\tilde{Z}_i)]$$

under

$$E[\tilde{Z}_i\,\phi(\tilde{W})] \leq E[\tilde{X}_i\,\phi(\tilde{W})].$$

[2] Assumption A 1 b is made for expository convenience As in BÜHLMANN (1984) one could define $\tilde{X}_1, \ldots, \tilde{X}_n$ as random variables on a common probability space (Ω, A, Π) Using the subsequent analysis one could show afterwards that $\tilde{W} = \sum_{i=1}^{n} \tilde{X}_i$ is a sufficient statistic for the problem under consideration

Definition 4: The feasible allocation $(\tilde{Z}_1^*, \ldots, \tilde{Z}_n^*)$ is called *Pareto efficient* if there exists no feasible allocation $(\tilde{Z}_1, \ldots, \tilde{Z}_n)$ satisfying

$$E[u_i(\tilde{Z}_i)] \geq E[u_i(\tilde{Z}_i^*)], \qquad i = 1, \ldots, n$$

with strict inequality for at least one $i \in \{1, \ldots, n\}$.

The standard results for this model can be summarized as follows (BÜHLMANN (1980, 1984)):

Theorem 1:

1) Under A.1., A 2., A.3.[3] there exists a price equilibrium $\{\phi, (\tilde{Z}_1^*, \ldots, \tilde{Z}_n^*)\}$.
2) Under A 1., A.2. each price equilibrium $\{\phi, (\tilde{Z}_1^*, \ldots, \tilde{Z}_n^*)\}$ has the following properties:
   a) The risk allocation $(\tilde{Z}_1^*, \ldots, \tilde{Z}_n^*)$ is Pareto efficient.
   b) There exist $\gamma_1, \ldots, \gamma_n \in (0, \infty)$ such that

(2) $$u_i'(\tilde{Z}_n^*) = \gamma_i \phi(\tilde{W})^4, \text{ a e} \qquad i = 1, \ldots, n.$$

As an immediate consequence one obtains:

Corollary 1: Under A.1., A.2. for each price equilibrium $\{\phi, (\tilde{Z}_1^*, \ldots, \tilde{Z}_n^*)\}$ there exist measurable functions f_i such that

(3) $$\tilde{Z}_i^* = f_i(\tilde{W}), \text{ a.e.}, \qquad i = 1, \ldots, n.$$

In the context of financial economics it is of particular interest to have some information about the shape of the functions $f_1, \ldots, f_n$ and ϕ. Some results on this issue are derived in the next section.

3. ANALYSIS OF PRICE EQUILIBRIA

3.1. Portfolio insurance, tactical asset allocation and collars

The term "portfolio insurance" is widely used for investment strategies where a reference portfolio is protected by a put option. Obviously such strategies lead to convex payoff functions. Therefore, LELAND (1980) introduced the term "general insurance policy" for convex payoff functions. In this article we use the following terminology:

Definition 5: An investment strategy leading to a twice differentiable payoff function

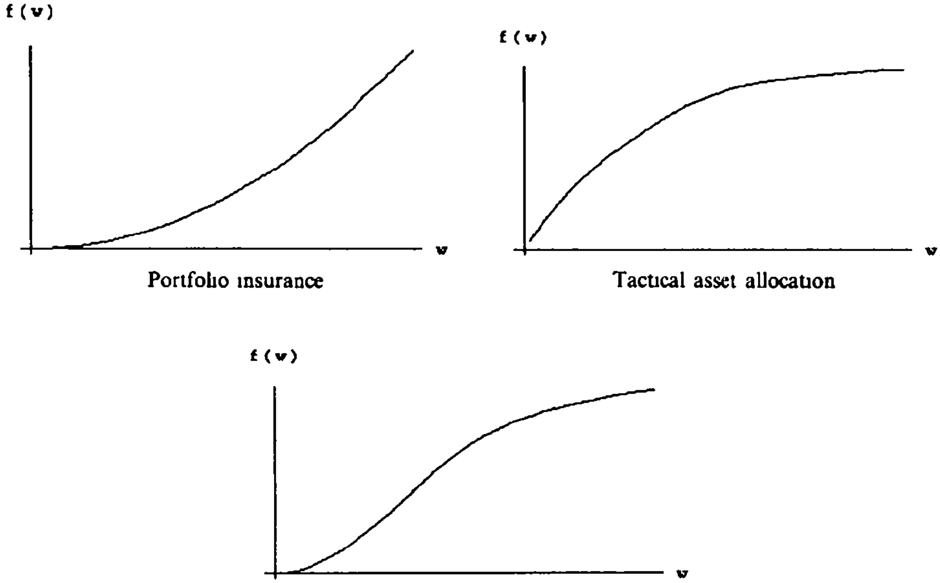
$$f \cdot [m, M] \to R$$

[3] Instead of A3 BÜHLMANN (1984) assumes that the functions $\rho_i(x) = -\frac{u_i''(x)}{u_i'(x)}$ satisfy a Lipschitz condition In E Chevallier's forthcoming thesis assumptions A 1, A 2, A 3 will be relaxed

[4] See also BRENNAN/SOLANKI (1981)

is called

a) "Portfolio insurance" if $f''(w) > 0 \ \forall w \in [m, M]$.
b) "Tactical asset allocation" if $f''(w) < 0 \ w \in [m, M]$.
c) "Collar" if
$f''(w) > 0 \ \forall w \in [m, w_0)$ and
$f''(w) < 0 \ \forall w \in (w_0, M]$, where $m < w_0 < M$



Collar
FIGURE 2

Remarks:

1) Of course strategies with a continuous payoff function $f(w)$ can be approximated by buying and selling put and call options with different striking prices (see also LELAND (1980)).
2) The term "tactical asset allocation" is motivated by the widely used "buy low, sell high" strategies. The term "collar" is used for the popular investment policy where a reference portfolio is held, a put option is bought and a call option is sold.

3.2. Risk tolerance, cautiousness and properties of price equilibria

The following definition will be useful for the discussion of price equilibria.

Definition 6:

a) $\tau_i(x) = -\dfrac{u_i'(x)}{u_i''(x)}$ is called the *risk tolerance* of investor i.

b) $R_i(x) = \dfrac{d}{dx}\,\tau_i(x)$ is called the *cautiousness* of investor i[5].

Some well known characteristics of price equilibria can be formulated as follows:

Proposition 1: Under A.1., A.2. a price equilibrium $\{\phi, (f_1(\tilde{W}), \ldots, f_n(\tilde{W}))\}$ where ϕ and $f_1, \ldots, f_n$ are differentiable has the properties:

a) $\sum_{i=1}^{n} f_i'(w) = 1$,

b) $f_i'(w) = \dfrac{\tau_i(f_i(w))}{\sum_{j=1}^{n} \tau_j(f_j(w))} \in (0, 1)$,

c) $\phi(w) > 0, \qquad \phi'(w) < 0$,

d) $\dfrac{\phi'(w)}{\phi(w)} = -\dfrac{1}{\sum_{j=1}^{n} \tau_j(f_j(w))}$.

Proof: e.g. BÜHLMANN (1984, p 16-17) or HUANG/LITZENBERGER (1986).

In order to decide whether a payoff function f_i corresponds to portfolio insurance, tactical asset allocation or a collar strategy its second derivative $f_i''(w)$ has to be known. The notion of a "representative investor" will considerably simplify the analysis of $f_i''(w)$.

Definition 7: Given a price equilibrium $\{\phi, (f_1(\tilde{W}), \ldots, f_n(\tilde{W}))\}$ a function v_m with

$$v_m'(w) = \phi(w)$$

is called Neumann-Morgenstern utility function of the representative investor.

Remark: The representative investor is a fictious individual representing the market. Under the conditions of Proposition 1 and differentiability assumptions the risk tolerance $\tau_m(w)$ and the cautiousness $R_m(w)$ of the representative

[5] Hence, the cautiousness R_i is a measure for the sensitivity of the risk tolerance $\tau_i(x)$ with respect to wealth changes

investor are given by

$$(4) \qquad \tau_m(w) = -\frac{v'_m(w)}{v''_m(w)} = -\frac{\phi(w)}{\phi'(w)} = \Sigma_{j=1}^{n} \tau_j(f_j(w)),$$

$$(5) \qquad R_m(w) = \frac{d}{dw}\tau_m(w) = \frac{\Sigma_{j=1}^{n} R_j(f_j(w))\,\tau_j(f_j(w))}{\Sigma_{j=1}^{n} \tau_j(f_j(w))}.$$

Hence, the risk tolerance $\tau_m(w)$ is the sum of individual risk tolerances, whereas the cautiousness $R_m(w)$ is a weighted mean of individual cautiousness terms.

Now the result on the second derivatives of the payoff functions f_i and the price density ϕ can be formulated as follows:

Theorem 2: Under A 1., A.2., A.3. a price equilibrium $\{\phi, (f_1(\tilde{W}), \dots, f_n(\tilde{W}))\}$ where ϕ and $f_1, \dots, f_n$, are twice differentiable has the properties·

a) $\dfrac{f_i''(w)}{f_i'(w)} = \dfrac{1}{\tau_m(w)}\{R_i(f_i(w)) - R_m(w)\}, \qquad i = 1, \dots, n,$

b) $\dfrac{\phi''(w)}{\phi'(w)} = -\dfrac{1}{\tau_m(w)}\{1 + R_m(w)\},$

c) $\dfrac{d}{dw}\ln\left(\dfrac{f_i'(w)}{f_j'(w)}\right) = \dfrac{R_i(f_i(w)) - R_j(f_j(w))}{\tau_m(w)}, \qquad i, j = 1, \dots, n$

Comments:

1) In particular Proposition 1 and Theorem 2 contain the key result

$$(6) \qquad f_i'(w) = \frac{\tau_i(f_i(w))}{\tau_m(w)}, \qquad i = 1, \dots, n,$$

$$(7) \qquad \operatorname{sign}\{f_i''(w)\} = \operatorname{sign}\{R_i(f_i(w)) - R_m(w)\}, \qquad i = 1, \dots, n$$

In other words, the slope of the payoff function f_i is given by the ratio of the risk tolerances $\tau_i(f_i(w))$ and $\tau_m(w)$, whereas the curvature of f_i is related to the difference of the cautiousness terms $R_i(f_i(w))$ and $R_m(w)$.

2) Theorem 2.a) leads to the following criteria
   a) An investor $i \in \{1, \dots, n\}$ chooses *portfolio insurance* if and only if

$$(8) \qquad R_i(f_i(w)) > R_m(w) \qquad \forall w$$ [6]

   b) An investor $i \in \{1, \dots, n\}$ chooses *tactical asset allocation* if and only if

$$(9) \qquad R_i(f_i(w)) < R_m(w) \qquad \forall w.$$

[6] LELAND (1980) derived a similar result in a less formal context

c) If an investor $i \in \{1, \ldots, n\}$ chooses a *collar strategy* there exists $w_0 \in (m, M)$ such that
   1) $R_i(f_i(w_0)) = R_m(w_0)$,
   2) $R_i(f_i(w)) - R_m(w)$ is strictly decreasing in w_0.

3) An easy calculation shows that under A.2., A 3. $u_i'''(f_i(w)) > 0$, $i = 1, \ldots, n$ implies $R_m(w) > -1$.

Therefore, one concludes from Theorem 2b):

$$\phi''(w) > 0 \text{ if } u_i'''(f_i(w)) > 0,\ i = 1, \ldots, n. \tag{10}$$

Proof of Theorem 2:

a) Differentiation of the formula in Proposition 1b)

$$f_i'(w) = \frac{\tau_i(f_i(w))}{\tau_m(w)}$$

leads to

$$f_i''(w) = \frac{R_i(f_i(w))}{\tau_m(w)} f_i'(w) - \frac{\tau_i(f_i(w))\, R_m(w)}{\tau_m^2(w)} \tag{11}$$

or

$$\frac{f_i''(w)}{f_i'(w)} = \frac{1}{\tau_m(w)} \{R_i(f_i(W)) - R_m(w)\}. \tag{12}$$

b) Differentiation of the formula in Proposition 1d)

$$\frac{\phi(w)}{\phi'(w)} = -\tau_m(w)$$

leads to

$$\frac{\phi'^2(w) - \phi(w)\,\phi''(w)}{\phi'^2(w)} = -R_m(w) \tag{13}$$

or

$$1 + R_m(w) = -\tau_m(w) \frac{\phi''(w)}{\phi'(w)}. \tag{14}$$

c) From Proposition 1b) one obtains

$$\ln\left(\frac{f_i'(w)}{f_j'(w)}\right) = \ln(\tau_i(f_i(w))) - \ln(\tau_j(f_j(w))) \tag{15}$$

and

$$\frac{d}{dw} \ln\left(\frac{f_i'(w)}{f_j'(w)}\right) = \frac{R_i(f_i(w))}{\tau_i(f_i(w))} \frac{\tau_i(f_i(w))}{\tau_m(w)} - \frac{R_j(f_j(w))}{\tau_j(f_j(w))} \frac{\tau_j(f_j(w))}{\tau_m(w)}. \tag{16}$$

4. ANALYSIS OF PRICE EQUILIBRIA FOR THE HARA CLASS

In Section 3 general properties of price equilibria were derived. Now we look at the special case where the risk tolerance functions $\tau_i(x)$ are linear. Assumption A.2. and A.3. are replaced by the assumption:

A.4. The Neumann-Morgenstern utility functions are increasing, concave and satisfy

a) $\tau_i(x) = a_i + R_i x > 0$, with $R_i \geq 0$, $i = 1, \ldots, n$,
b) Not all R_j identical.

Remarks:

1) Assumption A.4. allows for all Neumann-Morgenstern utility functions which belong to the HARA class and have a non negative cautiousness[7].
2) In the case where all R_j are identical the risk allocation is linear and a detailed analysis can be found in BÜHLMANN (1980) and LIENHARD (1986).
3) For $R_i > 0$ the Neumann-Morgenstern utility function u_i is only defined on the interval $\left(-\frac{a_i}{R_i}, \infty\right)$. Therefore, assumption A.2 is not satisfied and existence of a price equilibrium is not guaranteed by Theorem 1 However, it can be easily verified that Proposition 1 and Theorem 2 are still valid if Assumption A 2 and A 3. are replaced by A.4.

By restricting the analysis to the HARA class one obtains:

Lemma 1: Under A.1., A.4. a price equilibrium $\{\phi, (f_1(\tilde{W}), \ldots, f_n(\tilde{W}))\}$ where ϕ and $f_1, \ldots, f_n$ are differentiable[8] has the property:

$$R_m(w) \text{ is strictly increasing.}$$

Proof: A.4 and (5) lead to

$$R_m(w)\,\tau_m(w) = \sum_{j=1}^{n} R_j \tau_j(f_j(w)), \tag{17}$$

$$R'_m(w)\,\tau_m(w) + R_m^2(w) = \sum_{j=1}^{n} R_j^2 f_j'(w)^{9}. \tag{18}$$

[7] A negative cautiousness would lead to problems with satiation and an unrealistic investment behaviour (see ARROW (1965))

[8] If $\phi, f_1, \ldots, f_n$ are differentiable, then due to Proposition 1b), 1d) and A 4 they are also twice differentiable

[9] From the derivation of formula (18) it becomes obvious that Lemma 1 depends crucially on the assumption that each investor i has a constant cautiousness R_i (A 4 a)

Moreover, Proposition 1b), (5) and (18) imply

$$(19) \qquad R'_m(w) = \frac{\sum_{j=1}^n R_j^2 \tau_j(f_j(w))}{\tau_m^2(w)} - \frac{R_m(w)\sum_{j=1}^n R_j \tau_j(f_j(w))}{\tau_m^2(w)}$$

and the strict monotonicity of $R_m(w)$ follows from

$$\begin{aligned}
&\sum_{j=1}^{n} (R_j - R_m(w))\, R_j \tau_j(f_j(w)) \\
&\quad = \sum_{R_j > R_m(w)} (R_j - R_m(w))\, R_j \tau_j(f_j(w)) + \\
&\quad + \sum_{R_j < R_m(w)} (R_j - R_m(w))\, R_j \tau_j(f_j(w)) > \\
&\quad > \sum_{j=1}^{n} (R_j - R_m(w))\, R_m(w)\, \tau_j(f_j(w)) = 0 .
\end{aligned}$$

Lemma 1 leads to the main result of this section

Theorem 3: Under A.1., A.4. a price equilibrium $\{\phi, (f_1(\tilde{W}), \ldots, f_n(\tilde{W}))\}$ where ϕ and $f_1, \ldots, f_n$ are differentiable has the properties:

a) The only investment strategies chosen by investors $i = 1, \ldots, n$ are
   - portfolio insurance,
   - tactical asset allocation,
   - collar strategy.

b) Investors $i \in \{1, \ldots, n\}$ with $R_i = \max_{j=1,\ldots,n} R_j$ choose portfolio insurance[10]

c) Investors $i \in \{1, \ldots, n\}$ with $R_i = \min_{j=1,\ldots,n} R_j$ choose tactical asset allocation[10]

Proof: Formula (5) implies

$$(20) \qquad \min_{j=1,\ldots,n} R_j < R_m(w) < \max_{j=1,\ldots,n} R_j \qquad \forall w \in [m, M].$$

Now, a), b) and c) follow immediately from Theorem 2a) and Lemma 1.
Some additional information about price equilibria in the HARA case is provided by the next result.

[10] See also MÜLLER (1990)

Proposition 2: Under the assumption of Theorem 3 one obtains

a) $$f_i'(w) = \frac{a_i + R_i f_i(w)}{a + \sum_{j=1}^{n} R_j f_j(w)}, \quad \text{where} \quad a = \sum_{j=1}^{n} a_j,$$

b) $$\frac{d}{dw} \ln \left(\frac{f_i'(w)}{f_j'(w)} \right) = \frac{R_i - R_j}{\tau_m(w)}.$$

Proof: Special case of Proposition 1b) and Theorem 2c).

Comments:

1) In particular Proposition 2b) implies

(21) $$\frac{d}{dw} \left(\frac{f_i'(w)}{f_j'(w)} \right) \gtreqless 0 \Leftrightarrow R_i \gtreqless R_j \qquad i, j = 1, \ldots, n.$$

2) For sufficiently large values of w one can show $f_i(w) > 0$ for $i = 1, \ldots, n$ and Proposition 2 leads to the following inequalities

a') $$f_i'(w) > \frac{a_i + R_i f_i(m)}{a + w \ R_{\max}} \quad \text{with} \quad a = \sum_{j=1}^{n} a_j, \quad R_{\max} = \max_{j=1,\ldots,n} R_j,$$

b') $$\frac{d}{dw} \ln \left(\frac{f_i'(w)}{f_j'(w)} \right) > \frac{R_i - R_j}{a + w \cdot R_{\max}}, \quad \text{if} \quad R_i > R_j.$$

Finally, an example illustrates some typical properties of a price equilibrium in the HARA case.

Example:

— The random variable $\tilde{W}$ representing total financial wealth is uniformly distributed over [0.3, 20].

— There are $n = 3$ investors with risk tolerance functions

$$\tau_1(x) = 20 \cdot x,$$
$$\tau_2(x) = 2.5 \cdot x,$$
$$\tau_3(x) = x$$

and an initial risk allocation

$$(\tilde{X}_1, \tilde{X}_2, \tilde{X}_3) = (0.16 \cdot \tilde{W}, 0.35 \cdot \tilde{W}, 0.49 \cdot \tilde{W}).$$

The price equilibrium $\{\phi, (f_1(W), f_2(\tilde{W}), f_3(\tilde{W}))\}$ is illustrated in Figure 3.

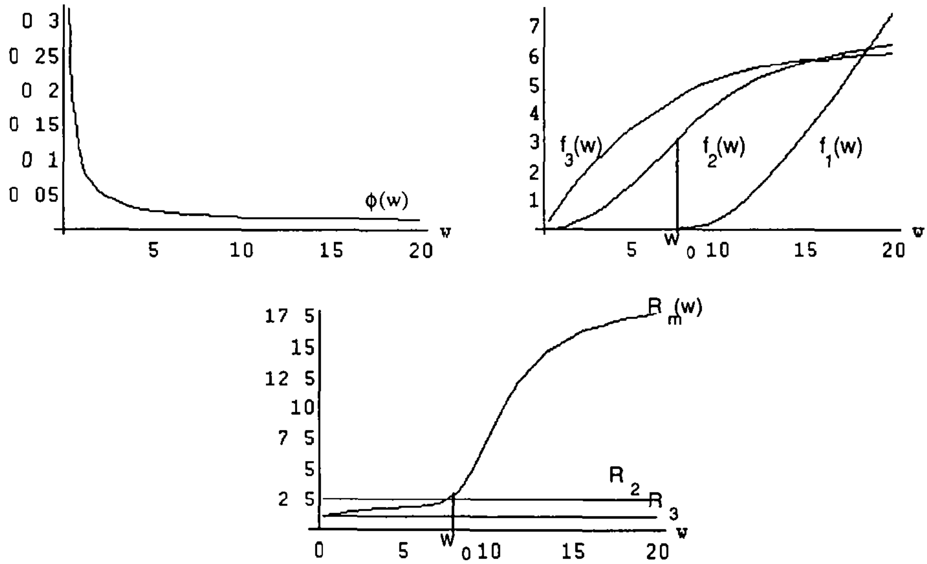


FIGURE 3

5. CONCLUSIONS

In this article the theory of risk exchange was applied to the allocation of financial risk Special emphasis was put on the shape of the payoff functions in price equilibrium. Under general conditions the role of risk tolerance and cautiousness was analysed. The notion of a representative investor was very useful for the interpretation of the results. Finally, in the HARA case a full characterization of all equilibrium payoff functions was possible.

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