NOTE ON ACTUARIAL MANAGEMENT IN INFLATIONARY CONDITIONS

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This note is an attempt to put the problems referring to the reserves for outstanding claims into a simple understandable form in order to facilitate the discussion of the difficult questions. In that purpose I have taken up some of Harald Bohman's ideas of the subject *). I find it convenient to start with the simplest case where the liability consists of index-regulated payments at fixed epochs. My presentation is restricted to reserves of incurred and reported claims.

1. *Loss reserve of index-regulated payments*

Expected value of liability of paying a total sum of S in the money unit of $t = 0$ according to a cumulated weight function $F(t)$ by the time scale t, for which we have $F(0) = 0$ and $F(\infty) = 1$. The function can also be interpreted as a distribution function (see below).

The function $F(t)$ can be a step function with the steps f_t, which means that the payment at t is $s_t = S \cdot f_t$, but for the simplicity of the formulas we assume $F(t)$ continuous with existing $F'(t)$.

The calculation of the liability is made according to a basic intensity rate of interest of δ and to a basic inflation "intensity" rate of ρ.

The net value V_t in the fixed money unit of $t = 0$ is according to the basic assumptions equal to (for $S = 1$)

$$V_t = \int_t^\infty \exp - (\delta - \rho)\,(u - t)dF(u),$$

satisfying the differential equation

$$V'_t = V_t(\delta - \rho) - F'(t).$$

* Harald Bohman, "Insurance business and inflation" to be published in S.A.J.

Let us now assume that the real inflation rate has not been ρ but ρ^* in the time interval $(0, t)$. The necessary amount of the reserve in the applied money unit of current purchasing power will then be

$$V_t^* = V_t \exp \rho^* t$$

which reserve satisfies the differential equation (for $S = 1$).

$$V_t^{*\prime} = V_t^*(\delta - \rho + \rho^*) - F'(t) \exp \rho^* t. \qquad (1a)$$

Interpreted for an accounting period this equation signifies the fact that in the money unit of current purchasing power the loss reserve at the end of the accounting period will be equal to the loss reserve at the beginning of the period increased by the observed inflation rate ρ^*

+ the calculated interest amount according to the basic rate
— the calculated inflation amount according to the basic rate
— the amount of payment in the money unit of current purchasing power.

Since the prospective reserve

$$V_t^* = \exp \rho^* t \int_t^\infty (\exp - (\delta - \rho)(u - t)) dF(u) \qquad (1b)$$

satisfies the equation (1a) with $V_0^* = V_0$ it will be equal to the retrospective reserve.

In case the actual rate of interest δ^* surpasses the basic rate δ by less than the difference between the actual inflation rate ρ^* and the basic rate ρ, there will be a deficit.

1.2 *Fluctuation reserves*

In order to meet temporary losses on account of increasing liabilities by inflation (see above) fluctuation reserves are needed. Further, the rate of interest δ is object of systematic and random variations which influence the market values of the assets. To meet such variations of the asset values bank companies as well as insurance companies need contingency reserves, which can be called valuere-gulating funds.

Since the normal rate of interest uses to be positively correlated to the inflation rate a rising trend of inflation may in addition necessitate a higher level in the fixed money unit of the regulating funds in order to meet the deterioration of the bond values and the values of other nominal assets.

2. *Applications of the model to the reserve of outstanding claims of non-life insurance*

The reserve for the outstanding claims is the sum of the discounts of the expected future payments of the outstanding claims. Besides inflation, there are regularly during the settlement period possibilities that the estimates of the different claims amounts might be changed. The claim reserve shall be an estimate upon known facts regarding the claim in question. These facts might change during the settlement period and on such occasions the estimate for the claim reserve must be changed. Such changes will be called "run-off result" according to the terminology used by Harald Bohman.

If $F(\tau)$ is the probability that the claim is settled before τ, the conditioned probability at t of the claim becoming settled before τ is for $\tau > t$ equal to $(F(\tau) - F(t)) : (1 - F(t))$. The distribution function F is dependent on the branch of non-life insurance and on the expected size of the claim amount. The distribution function will for small amounts increase quickly from 0 to 1, and the influence of inflation will be relatively small. For large claims, e.g. on liabilities by damages of persons, which can give rise to index-regulated annuities of disability life and of life annuities of surviving individuals, the distribution function will be slowly increasing and the value 1 is attained first after 5-10 years. The influence of inflation will then be of great importance. Although amounts will be paid before the definite settlement to compensate loss of income and also e.g. losses of hospital care, the essential part of the losses will often refer to the time of definite settlement. The model could also be refined by introducing the concept of partial settlement.

Given the claim amount S and the distribution function F with respect to the duration until settlement the loss reserve is defined by a modification of the equation (1a) and the solution (1b). We will primarily think of the claim amount S as a fixed sum in the fixed money unit of $t = 0$. The model will then correspond to a claim amount of a fixed but index-regulated sum. If the settlement takes place at t the sum $S \cdot \exp \rho^* t$ will be paid in the money unit of current purchasing power, and the reserved amount V_t^* will become available.

The inflation rate should refer to an index of the actual claim costs.

As explained above the amount S and the distribution function F are subject to regular re-estimations. As long as the estimations S and $F(t)$ are applicable, we write the differential equation as follow

$$V_t^{*\prime} + (S - V_t^{*\prime})(F'(t) : (1 - F(t)) \cdot \exp \rho^* t = \delta V_t^* + (\rho^* - \rho) V_t^*. \tag{2a}$$

In this application the equation expresses that the claim costs in the money unit of current purchasing power according to the left member (where the increase of the reserve can be both positive and negative) shall be covered by the calculated rate of interest plus the additional amount corresponding to the difference between the observed and the calculated inflation rate.

The equation (2a) is satisfied by the solution

$$V_t^* = S \cdot (\exp \rho_t^*) \int_t^\infty (\exp - (\delta - \rho)(u - t)) dF(u) : (1 - F(t)). \tag{2b}$$

$F'(t) : (1 - F(t))$ denotes the conditional probability of the settlement taking place in the small time interval dt if it has not taken place before t.

If the sum S is to be paid when death occurs we have $F(t) = 1 - l_{x+t} : l_x$ and $F'(t) : (1 - F(t)) = \mu_{x+t}$.

Profit or loss appears in reference to the equation (2a)

a) when the difference between the actual and calculated rates of interest exceeds or is below the difference between the actual and the calculated inflation rates,
b) when the actual payments are below or exceed the expected payments by settlement,

and further at the end of the period

c) if the estimate S of future payments are changed by new estimation or/and if the distribution function F is changed by new estimation,
d) if the basic rates of interest and inflation are changed.

The difference between the prospective reserve with actual estimations of S and F and the retrospective simultaneous reserve, containing the preceding estimations of S and the distribution function F, will give the "run-off result" according to c).

Profit and losses according to a), b) and c) are expressed in the money unit of current purchasing power. The variations will increase in the same progression as the inflation, and consequently also the need of equilization funds to meet the different kinds of systematic and random variations. A critical situation will soon appear if the investments don't give sufficient means for increasing not only the loss reserve but also the equalization funds in pace with the inflation.