TITLE: **PRICING THE IMPACT OF ADJUSTABLE FEATURES AND LOSS SHARING PROVISIONS OF EXCESS-OF-LOSS TREATIES**

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ABSTRACT: Many excess-of-loss reinsurance contracts contain non-proportional coinsurance clauses, where the ceding company is to pay a non-proportional share of losses without receiving a commensurate share of the reinsurance premium. Such clauses include aggregate deductibles, loss ratio caps or limited reinstatements, and loss corridor provisions. Quite frequently in the broker market, and less frequently in the direct market, excess-of-loss treaties will contain adjustable premium or commission features. These adjustable features include retrospective rating plans, profit commission or profit sharing plans, and sliding scale commission plans.

This paper compares two alternate approaches to pricing several relatively common treaty examples, the lognormal aggregate loss model and the Heckman-Meyers Collective Risk Model. These comparisons suggest that the lognormal model provides a satisfactory approximation to the theoretically more appropriate Collective Risk Model results when use of the latter more sophisticated procedure is not warranted due to resource limitations. Thus, application of the lognormal model can lead to significant efficiency gains in reinsurance price monitoring work and in pricing situations where limited information is available. Appendices summarize important excess-of-loss pricing methodologies and provide an expanded lognormal table.

ADJUSTABLE FEATURES AND LOSS SHARING PROVISIONS

Many excess-of-loss reinsurance contracts contain non-proportional coinsurance clauses, where the ceding company is to pay a non-proportional share of losses without receiving a commensurate share of the reinsurance premium. Such clauses include aggregate deductibles, loss ratio caps or limited reinstatements, and loss corridor provisions. Quite frequently in the broker market, and less frequently in the direct market, excess-of-loss treaties will contain adjustable premium or commission features. These adjustable features include retrospective rating plans, profit commission or profit sharing plans, and sliding scale commission plans. A relatively small number of excess-of-loss treaties will contain both adjustable premium or commission features and non-proportional coinsurance clauses.

This paper will compare two alternate approaches to pricing several relatively common examples, the lognormal model and the Heckman-Meyers Collective Risk Model. Our overall purpose is to determine if the lognormal model provides a suitable approximation for reinsurance price monitoring purposes and for pricing situations where limited information is available. If the lognormal model provides a satisfactory approximation to the Collective Risk Model results, significant efficiency gains would be achievable. A more sophisticated three parameter alternative to the lognormal is not tested under the presumption that the Collective Risk Model or an equivalent approach would be employed if the data and other resources would permit a more sophisticated approach. The bibliography contains several sources for those wishing to delve into reinsurance and excess pricing concepts in greater depth.

AGGREGATE LOSS DISTRIBUTIONS

In order to price the impact of adjustable features and non-proportional coinsurance clauses, it is necessary to estimate the aggregate loss distribution. Two methods of estimating this distribution are employed:

(a) The Lognormal Model If the aggregate loss random variable is viewed as the product of independent, identically distributed random variables, then the logarithm would be approximately normally distributed by the Central Limit Theorem. (The stringent condition that the factors be identically distributed may be relaxed.[1]) By definition, the aggregate loss random variable would be lognormally distributed. Standard formulas are employed based on the Patrik-John Collective Risk Model to estimate the aggregate mean and coefficient of variation based on assumed frequency and severity models.[2] An expanded lognormal table with excess pure premium ratios for coefficients of variation between .1 and 5 was programmed based on the formulas in Mr. Finger's well known paper.[3] Mr. Finger developed the lognormal model for severity applications, while we are testing it as an aggregate loss model. Appendix B summarizes the lognormal model and presents the expanded lognormal table. Parameter uncertainty can be modelled by subjectively weighting indications based on alternative parameter values.

(b) The Collective Risk Model Estimate parameters of frequency and severity distributions and judgmentally select parameters reflecting the degree of uncertainty in estimated frequency and severity means. If the shape of

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[1]Thomasian, A.J., **The Structure of Probability Theory with Applications**, McGraw-Hill, 1969, pp.239-241.

[2]Patrik, G.S., and John, R.T., "Pricing Excess-of-Loss Casualty Working Cover Reinsurance Treaties," 1980 CAS Discussion Paper Program, p.399.

[3]Finger, R. J., "Estimating Pure Premiums by Layer," PCAS LXIII (1976), p.34.

these distributions is also uncertain, one could assign subjective probabilities to several scenarios and compute a weighted average of the resulting cumulative probabilities and excess pure premium ratios. These quantities are computed using the Heckman-Meyers algorithm,[4] which uses piecewise-linear approximations of the cumulative severity distributions together with assumed frequency distributions to generate the characteristic functions of the severity and aggregate loss distributions. As the characteristic function uniquely determines a probability distribution, numerical methods are employed to evaluate the rather complicated formulas which accomplish this inverse transformation, yielding the aggregate loss cumulative probability distribution function and excess pure premium ratios needed to price the reinsurance conditions which are the focus of this paper. Technical details are summarized in Appendix C.

In Appendix D, we show that if the conditions for the ground-up occurrence count distribution to be Negative Binomial are satisfied, then the excess occurrence count distribution for an insured selected at random will be Negative Binomial. Based on this result, we derive the formula for calculating the excess occurrence count variance-to-mean ratio for an individual insured selected at random and show that this formula also applies to the class as a whole. This latter result is then used to demonstrate that if the proportion of occurrences exceeding the retention is small, then the excess occurrence count distribution for the class as a whole will be approximately Poisson. (Our proof is a direct application of the Gamma-Poisson model frequently

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[4]Heckman, P. E., and Meyers, G. G., "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," PCAS LXX (1983), p.22. The review of this paper and the alternate recursive procedure by Gary G. Venter is noteworthy.

encountered in the actuarial literature. We understand that these results have previously been established elsewhere, and note that Joseph Schumi has established these results using recursive relationships.[5])

In particular, we establish that

$$VMR_E = (1-p) + p(VMR_G),$$

where VMR_G and VMR_E are the variance-to-mean ratios for the ground-up and excess occurrence count distributions, respectively, and p is the probability that a claim will exceed the retention. If VMR_G is two or three, as in the ISO Increased Limits reviews, and p is less than .02, VMR_E will be close to unity. This implies that the excess occurrence count distribution for an insured selected at random and for the class as a whole will be approximately Poisson.

The Single Parameter Pareto (SPP) distribution is used to model occurrence severity. Mr. Philbrick's well known paper on this subject provides an excellent discussion of this distribution which is widely used in excess pricing.[6] Ms. Rytgaard recently presented a paper which compares alternative estimates of the SPP parameter and applies credibility theory to obtain more stable estimators of this parameter for portfolios of excess of loss treaties with similar characteristics.[7] In Appendix E, we summarize some of the key properties of the SPP distribution. In particular, we show that if ground-up loss occurrences excess of a particular attachment are

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[5]Schumi, J.R., "A Method to Calculate Aggregate Excess Loss Distributions," CAS Forum, Spring 1989 Edition, p. 195.

[6]Philbrick, S.W., "A Practical Guide to the Single Parameter Pareto Distribution," PCAS LXXII (1985), p.44. Noteworthy discussion by Kurt A. Reichle and John P. Yonkunas, p.85

[7]Rytgaard, M., "Estimation in the Pareto Distribution," Astin Colloquium XXI (1989), p.389.

distributed according to the SPP distribution with parameter q, excess occurrences are distributed according to the Shifted Pareto distribution (used by Insurance Services Office in Increased Limits pricing) with scale parameter equal to the attachment and shape parameter equal to q.

Theoretically, if the SPP is appropriate for loss occurrences excess of a particular attachment, it should be appropriate above all higher attachments and the parameter should remain constant. Fits to industry data have led us to conclude that the SPP parameter varies with the truncation point used in the fitting procedure. Moreover, if the truncation point used in the fitting procedure is less than 50% of the attachment for a particular pricing analysis, the errors become unacceptably large. Thus, we estimate development triangles of SPP parameter estimates for various truncation points and project ultimate values of this parameter by class of business and trucation point. In the examples discussed in this paper, we do not identify the class of business, because our intent is only to discuss actuarial methodology. Although we advocate that alternative two and three parameter distributions be tested when data permits, we believe the SPP distribution with these qualifications is a satisfactory severity model for reinsurance price monitoring work and in pricing situations where limited information is available.

EXAMPLES OF TREATIES WITH ADJUSTABLE FEATURES AND LOSS SHARING PROVISIONS

The remainder of this paper discusses the pricing of excess-of-loss treaties containing common types of non-proportional coinsurance clauses and adjustable premium or commission plans. This is accomplished through the examination of six hypothetical treaties, the key provisions of which are summarized on Page 1 of Appendices F through K, respectively. The analysis of each example involves two major steps. First, we calculate various parameters (such as the expected claim count, mean severity, and aggregate coefficient of variation) which underlie the distribution of losses in the reinsured layer. This allows us to obtain an appropriate set of excess pure premium ratios, either by reference to an appropriate lognormal table (via coefficient of variation matching) or by direct generation via the Heckman-Meyers Collective Risk Model. The second step involves the use of the set of excess pure premium ratios derived in the first step in order to determine the expected impact of the particular non-proportional coinsurance clause or adjustable feature being evaluated.

For the sake of clarity, excess pure premium ratios (which are called insurance charges in the examples) based on the lognormal assumption are initially used to analyze the six treaty examples. In the final section of the paper, a comparison is made to the results obtained when excess pure premium ratios generated by direct applications of the Collective Risk Model are employed.

Treaty I is an example of a contract containing an annual aggregate deductible provision. The calculation of the treaty's aggregate loss coefficient of variation (CV), which is displayed on Page 2 of Appendix F, is based on the theory and formulas presented previously in this paper as well as in Appendices A through E.

The computation of the impact of the aggregate deductible is shown on Page 3 of the appendix. The deductible amount is compared to the expected losses in the reinsured layer in order to obtain a corresponding entry ratio, which allows us to look up the appropriate insurance charge from the lognormal tables in Appendix B. (Linear interpolation is used to calculate excess pure premium ratios for CV and entry ratio combinations not explicitly listed in these tables.) Since the insurance charge (29.33% in this case) represents the expected proportion of aggregate losses above the deductible amount, it is easy to see the complement of this value (70.67%) is the expected percentage of treaty losses eliminated by the aggregate deductible. Thus, if a burning cost or similar study shows that the expected loss cost for the entire layer is 3.75% of subject premium, then the introduction of an aggregate deductible provision reduces this loss cost to 3.75% x [100% - 70.67%], or about 1.10% of subject premium. As shown in the appendix, this loss cost can easily be converted to an indicated treaty rate through the application of an appropriate expense, profit, and risk loading factor. It should be noted that the factor selected for this purpose should include a provision for risk commensurate with the degree of variability in layer losses after the application of deductible, which is higher than that for losses prior to the reflection of this provision.

Treaty II contains a limited reinstatement clause. The contract allows three free reinstatements of coverage during the treaty year, which means that the ceding company is covered for losses in the specified layer until those losses exceed four times the width of that layer. After that point, no coverage is provided. (This type of reinstatement clause should be contrasted with the kind which reinstates coverage after a certain amount of losses have occurred only if an additional premium is paid. This latter type is really a separate cover, rather than a form of coinsurance on the original treaty.)

The pricing of this treaty is summarized in Appendix G. As was done in the previous example, an entry ratio is calculated by dividing the dollar value of the limited reinstatement provision ($2,800,000 in this case) by the expected losses in the layer prior to all forms of coinsurance. The insurance charge corresponding to this entry ratio (2.37% in this example) is equivalent to the expected percentage of losses eliminated by the limited reinstatement clause. Combining this quantity with the treaty's 20% proportional coinsurance provision yields a 21.89% overall coinsurance percentage, which is then applied to the expected layer loss cost prior to all coinsurance in order to obtain an expected loss cost and an indicated rate for the treaty. As in the previous example, the loading to convert the expected loss cost to a rate includes a provision for risk which reflects the potential volatility in treaty losses after the limited reinstatement is taken into account. We note that this risk provision is somewhat lower than that for a similar treaty with no limited reinstatement clause, since this type of feature (along with most other kinds of mechanisms which place a cap on losses) tends to reduce loss variability.

<u>Treaty III</u> is an example containing a loss corridor provision. Under a loss corridor provision, the reinsurer pays all losses falling in the reinsured

layer up to a certain aggregate amount (called the lower bound of the loss corridor interval). Once this amount is reached the reinsurer stops paying all losses until the total losses in the layer exceed a second threshold amount (the upper bound of the loss corridor interval), after which the reinsurer resumes payment for all losses in the reinsured layer. The bounds of the loss corridor interval may be expressed in terms of dollar amounts, percentages of expected layer losses, or ratios to treaty premium.

In the example presented in Appendix H, the loss corridor bounds are stated as percentages of expected losses in the layer. This makes the analysis extremely straightforward, since these percentages are directly equivalent to the corresponding entry ratios. It is easy to see that the difference between the insurance charges at the lower and upper bounds, respectively, gives the expected percentage of layer losses eliminated by the loss corridor provision. The computation of the expected layer loss cost after coinsurance and the indicated treaty rate is analogous to the calculations presented in the first two examples. One should note, however, that unlike the previous examples there is no definite rule concerning the proper risk load to be included in the factor used to convert the loss cost into a rate. This is due to the fact that the loss corridor provision may either reduce or increase the variability of layer losses, depending on both the location and the size of the eliminated loss interval.

While the straightforwardness of the loss corridor analysis is not altered very much when the interval bounds are expressed in terms of dollars, the analysis does get complicated when the bounds are stated as ratios to treaty premium. This is due to the fact that the treaty premium is dependent on the treaty rate, which should already reflect the effect of the loss corridor. It is clear that the solution to this problem requires an iterative procedure in which the algorithm presented in Appendix H is repeated until the rate used to compute the loss corridor bounds (expressed as percentages of expected losses) equals the rate indication for the treaty with the loss corridor provision.

Having covered three common types of non-proportional loss sharing plans, we now turn to the analysis of accounts containing adjustable premium or commission plans.

Treaty IV is an example of an account with a one-year retrospective rating plan. Similar to the plans encountered in primary insurance, the adjusted treaty rate (and hence the adjusted premium) is based on the account's actual loss experience during the period subject to the plan. This rate is determined by loading the ratio of the treaty's actual losses to subject premium by a multiplicative loss conversion factor and/or an additive flat margin (to account for the reinsurer's expenses, risk, and profit). The computed rate is further subject to a maximum and a minimum as specified in the treaty. The main goal of this analysis is to determine the expected rate to be received on this treaty after all retrospective adjustments have been completed. This will enable us to assess the adequacy of the retro plan.

The calculation of the expected treaty rate for this example is outlined on Page 3 of Appendix I. As in the analysis of primary plans, the major step in this calculation is the determination of the effect of the retro plan's maximum and minimum rate on the expected layer loss cost to be charged to the reinsured. This is accomplished by dividing the loss costs which are consistent with the maximum and minimum rates, respectively, by the expected layer loss cost, in order to obtain entry ratios at these two points. These entry ratios enable us to look up the associated excess pure premium ratios, so that we may compute the insurance charge at the maximum and the insurance savings at the minimum. The difference of these latter two quantities is the net insurance charge. Applying the complement of the net insurance charge to the expected layer cost yields the adjusted expected layer cost, which is the losses expected to be charged to the reinsured. This latter quantity is loaded with the retro plan's loss conversion factor and any flat margin in order to obtain the expected treaty rate after retro adjustments. We note that the net insurance charge in this example is negative, indicating that the premium the reinsurer expects to lose because of the maximum rate provision is more than offset by the additional premium expected to be received due to the minimum provision.

In order to determine the degree of adequacy of the retro plan, the expected treaty rate after retro adjustments is compared to the equivalent treaty flat rate, which is the indicated treaty rate if the contract were flat rated. (To be comparable, the equivalent flat rate contains the same amount of risk load as that contained in the retro plan parameters.) As shown on the bottom of Page 3 of the appendix, the resulting ratio of 0.996 indicates a very slight redundancy in the retro plan.

Assuming that the underwriter chooses the retro plan parameters without regard to the degree of plan imbalance, Pages 4 and 5 of Appendix I show how the profit provisions built into the retro plan parameters can be adjusted to place the plan in balance. These pages also present examples of alternate retro plans which maintain the desired profit provisions but are in balance.

Treaty V contains a three-year profit commission plan, in which the profit commission ratio (to treaty premium) is computed via the following formula:

Profit Commission Ratio =

25% x [100% - (Actual 3-Year Treaty Loss Ratio) - (20% Reinsurer's Overhead Provision)]

Although the calculation of the expected profit commission ratio for the three-year period (1/1/90 - 12/31/92 in this case) may seem trivial (i.e., simply plug the three-year expected loss ratio into the formula), it is really not since a three-year loss ratio above 80% (the breakeven point) is implicitly capped at 80% to yield a 0% profit commission for the period. Hence, we must determine the effect of this capping on the expected loss ratio in order to estimate the expected commission. As in the previous examples, this involves the use of excess pure premium ratios for a lognormal distribution with an appropriate CV.

Page 2 of Appendix J displays the calculation of the CV for the distribution of one year's worth of aggregate losses in the reinsured layer. Since we are dealing with a three-year profit commission plan, we need to determine the CV appropriate for aggregate treaty losses for three years combined. This is accomplished on Page 3 of the appendix, using the formulas discussed in the first part of the paper and in the related appendices. In reviewing this exhibit, it should be assumed that the subject premium and expected layer cost given for 1990 are values based on ceding company projections and rating analyses, respectively, while the numbers shown for 1991 and 1992 are simply copied from 1990 since we presently do not have enough information to make independent projections for these years.

The calculation of the expected profit commission is shown on Pages 4 and 5 of the appendix. The expected treaty loss ratio of 48% is computed by reducing the expected loss cost for the entire layer by the 20% proportional coinsurance provision and then dividing the result by the rate the underwriter plans to charge for the account. By relating the 80% breakeven loss ratio to the expected loss ratio, we obtain an entry ratio from which the corresponding net insurance charge (NIC) is determined. Since the net insurance charge represents the percentage of expected losses eliminated from the profit commission formula by the implicit cap at the breakeven loss ratio, the expected profit commission ratio can be calculated via the following formula:

Expected Profit Commission Ratio =

(A) x [100% - ELR x (100% - NIC) - EXP],

where (A) = The proportion of profits to be paid to reinsured

ELR = Expected treaty loss ratio

NIC = Net insurance charge

EXP = Reinsurer's overhead provision

In the Appendix J exhibits, the expected profit commission based on the formula above is called the "Actuarial View", while that obtained by simply plugging the expected loss ratio into the profit commission formula is labelled the "Underwriting View". Page 6 of Appendix J explores the effect that the difference between these two quantities has on the profit that the reinsurer expects to realize on the treaty, as well as presents an alternative plan whose expected commission from an actuarial view matches the underwriter's expected percentage under the original plan.

Treaty VI contains another kind of adjustable commission provision known as a sliding scale plan. Like the profit commission in the previous example, the commission which is ultimately paid on this plan depends directly on the reinsured's actual experience as measured by the treaty loss ratio. The major difference between these two plans lies in the structure of the formula used to compute the adjustable commission. Whereas the profit commission formula is essentially a straight linear function of the treaty loss ratio (at least up to the breakeven point), the typical sliding scale plan is best described as a piecewise linear function of the loss ratio.

Under a typical sliding scale plan, a minimum commission ratio C_{min} is paid if the treaty loss ratio exceeds a certain fixed value (call it L_1). If the actual loss ratio is less than L_1 but greater than a second fixed value L_2, b_2 points of commission are added to C_{min} for each point by which the actual loss ratio falls short of L_1. Similarly, if the actual loss ratio is below L_2 but greater than some third value L_3, the commission ratio corresponding to L_2 is increased by b_3 points for each point of difference between L_2 and the actual treaty loss ratio. The commissions corresponding to actual loss ratios falling into successively lower intervals (i.e., $[L_i, L_{i-1}]$, where $i = 4,\ldots,n-1$) are calculated in a similar manner as those for loss ratios falling in the previous two intervals. Finally, if the loss ratio should fall below L_{n-1} (another fixed value specified in the plan), a maximum commission C_{max} is paid. It should be noted that the b_i's, which represent the commission slides on the various intervals, are generally less than unity. The sliding scale plan for Treaty VI (see the bottom of Appendix K, Page 1) is expressed in the format described above.

Since the typical sliding scale plan involves both a minimum and maximum commission as well as different commission slide percentages for the various loss ratio intervals, it is clear that the calculation of the expected commission ratio under such a plan requires more than simply looking up the commission which corresponds to the expected loss ratio. In Appendix L, we derive a concise formula for computing this expected commission, which can be expressed verbally as follows:

Expected Sliding Scale Commission Ratio =

$$C_{max} - \sum_{i=1}^{n} b_i \text{ \{Expected loss ratio points in the interval } L_i \text{ to } L_{i-1}\}$$

where: C_{max} is the maximum commission ratio,

b_i is the commission slide on the i^{th} loss ratio interval

(b_1 and b_n are defined to be 0).

Appendix L also shows that the above formula is equivalent to saying that the expected commission ratio equals the maximum commission ratio minus the expected points of commission lost over the entire range of possible loss ratios. This interpretation provides a good intuitive justification for the formula stated above.

We use this formula to calculate the expected commission ratio for the one-year plan given in Treaty VI, the details of which are provided on Page 3 of Appendix K. As this exhibit shows, in order to determine the expected number of loss ratio points falling in each interval specified in the plan, it is necessary to multiply the treaty expected loss ratio by the difference between the insurance charges corresponding to both end points of the given interval.

On the bottom of Page 3, we compare the expected sliding scale commission based on the above formula (the "Actuarial View") to that obtained by simply looking up the commission which corresponds to the expected loss ratio (the "Underwriting View"). As was done in the profit commission example, the remaining pages of Appendix K explore the effect that the difference between these two quantities has on profit provision built into the treaty rate, as well as provide an alternate sliding scale plan which yields an expected commission equal to the underwriter's estimate under the original plan.

The first three treaty examples presented in this section illustrate methods for pricing common types of non-proportional coinsurance provisions, while the latter three examples involve the analysis of treaties with adjustable premium or commission plans. We have not considered the case in which a treaty contains both a non-proportional coinsurance clause and an adjustable feature. In such a situation, we need to determine not only the effect that the non-proportional coinsurance clause has on expected treaty losses (which can be accomplished using the techniques discussed in the paper), but also the distribution of aggregate losses in the reinsured layer after the effect of the non-proportional coinsurance has been taken into account. The latter item is necessary in order to compute the expected impact of the adjustable premium or commission plan, since these plans generally operate on actual treaty experience after all coinsurance.

The calculation of the aggregate distribution after non-proportional coinsurance can be accomplished by making direct modifications to the aggregate loss distribution prior to coinsurance (eg, truncate it at the aggregate deductible amount or censor it at the loss ratio cap). The Collective Risk Model would be run again to compute the needed insurance charges, assuming that there will be one claim with a severity distribution equal to the aggregate loss distribution after all forms of non-proportional coinsurance. Another approach is to determine the effects that the non-proportional coinsurance feature has on both the occurrence count and the occurrence severity distributions which underlie the aggregate distribution. The adjusted count and severity distributions can then be combined in order to obtain an aggregate loss distribution which reflects the effects of the non-proportional coinsurance provision using either method discussed in this paper or the alternative recursive or simulation techniques.

We also have not considered the time value of money in the examples presented above, which is a legitimate underwriting consideration in evaluating alternative excess-of-loss reinsurance treaty proposals. The methods used in this paper can be used to develop aggregate loss distributions for the lines of business subject to the treaty prior to all forms of non-proportional coinsurance. The analysis then becomes a simulation problem. One would simulate annual losses before coinsurance for each line, apply payout patterns to estimate future loss payments by line, apply the non-proportional coinsurance provisions, and finally discount the future treaty losses. One would also need to estimate when future premium or commission adjustments would be made and when brokerage and other reinsurance expenses (including taxes) would be paid. The economic value of the proposed treaty would be the difference between discounted reinsurance premium and the sum of the discounted values of all expense items. This economic value should be adjusted for risk considerations, possibly through the selection of the interest rates used in the discounting procedure.[8]

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[8]Butsic, R.P., "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach," 1988 CAS Discussion Paper Program, p.147.

MODEL COMPARISONS

For the examples presented above, we compare the key item of interest (the adjusted rate or expected commissions) in the table below. The unadjusted rate is the loaded loss cost before all forms of coinsurance using the same expense and profit loading factor used to compute the adjusted rate. (In practice, the loadings for a treaty without coinsurance provisions or premium adjustments would generally not be considered appropriate for a treaty with such provisions.)

				Collective Risk Model			
Example	Unadjusted Rate	Item of Comparison	Lognormal Model	c=0 b=0	c=.05 b=.05	c=.05 b=.10	c=.10 b=.10
I	5.00%	Adjusted Treaty Rate	1.47%	1.58%	1.68%	1.73%	1.77%
II	25.00	Adjusted Treaty Rate	19.53	19.89	19.72	19.55	19.52
III	5.00	Adjusted Treaty Rate	4.02	3.67	3.71	3.74	3.73
IV	5.00	Expected Treaty Rate After Retro Adjustments	5.02	5.20	5.18	5.14	5.14
V		Expected Profit Commission	8.37	8.24	8.50	8.69	8.75
VI		Expected Sliding Scale Commission	31.04	30.31	30.90	31.22	31.33

The alternative indications for the Heckman-Meyers version of the Collective Risk Model reflect varying levels of parameter uncertainty. The contagion parameter is represented by c and represents the level of parameter uncertainty in the estimated frequency mean. The mixing parameter is represented by b and represents the level of parameter uncertainty in the estimated severity mean. Values of zero represent no parameter uncertainty, values of .05 represent a moderate level of parameter uncertainty, while values of .10 represent a relatively high level of parameter uncertainty. Please refer to Appendix C for futher technical details. The lognormal model was run under the same assumptions which were used to generate the Collective Risk Model results, but alternate scenarios were not considered in an effort to reflect parameter uncertainty in this procedure.

The comparisons above suggest that the lognormal model provides a satisfactory approximation to the theoretically more appropriate Collective Risk Model results when use of the more sophisticated procedure is not warranted due to resource limitations. Application of the lognormal model can lead to significant efficiency gains in reinsurance price monitoring work and in pricing situations where limited information is available.

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Computation of Aggregate Mean and Coefficient of Variation

(Patrik-John Version of Collective Risk Model)[2]

Let L represent the random variable of aggregate loss to be paid on a given contract for a particular coverage period.

$L = L_1 + L_2 + \ldots + L_K$,

where L_i represents the aggregate loss random variable for group i, i = 1,2,...,K.

The groupings may represent distinct groups of classes of insureds or coverages, similar insureds grouped by distinct policy limits, or the overall coverage time period split into sub-periods.

$L_i = X_{i1} + X_{i2} + \ldots + X_{iN_i}$,

where N_i is the random variable of the number of loss occurrences for group i and X_{ij} is the random variable of loss size of the jth loss for group i.

Let v represent the parameter vector containing all parameters necessary to specify the particular cumulative probability distribution functions (c.d.f.'s) for the L_i's, N_i's, and X_{ij}'s.

The following three assumptions guarantee that the total coverage has been split into independent, homogeneous coverage groups:

<u>Assumption 1</u> Given v, the L_i's are stochastically independent.

<u>Assumption 2</u> Given v, the X_{ij}'s are stochastically independent of the N_i's.

<u>Assumption 3</u> Given v and fixed i (i.e., a particular group), the X_{ij}'s are stochastically independent and identically distributed.

Let $F(x|v)$ represent the c.d.f. of L and let $F_i(x|v)$ represent the c.d.f. of L_i, i = 1,2,...,k.

Properties of Model with Known Parameters

(1) $F(x|v) = P(L \leq x|v) = F_1(x|v) * F_2(x|v) * \ldots * F_k(x|v)$,

where $F_i(x|v) = P(L_i \leq x|v)$ and * denotes the convolution operation.

That is, the c.d.f. of the aggregate loss L is the convolution of the aggregate loss c.d.f.'s for the individual groups.

(2) The cumulants of L given v are sums of the corresponding cumulants of the L_i's given v. This implies that

(a) $E(L|v) = \sum_i E(L_i|v)$ (The means are additive.)

(b) $Var(L|v) = \sum_i Var(L_i|v)$ (The variances are additive.)

(3) The aggregate loss c.d.f. of the i^{th} group, $F_i(x|v)$, can be expressed in the form

$F_i(x|v) = \sum_n P(N_i=n|v) \cdot G_i^{*n}(x|v)$,

where $G_i(x|v) = P(X_i \leq x|v)$ is the loss amount c.d.f. for the i^{th} group, and G_i^{*n} is the convolution of the n G_i's and represents the c.d.f. of the total amount of exactly n loss occurrences.

(4) The above properties imply that

(a) $E(L_i|v) = E(N_i|v) \cdot E(X_i|v)$

The mean aggregate loss for the i^{th} group is the product of the mean frequency and mean severity.

(b) $Var(L_i|v) = E(N_i|v) \cdot Var(X_i|v) + Var(N_i|v) \cdot E(X_i|v)^2$

The variance of the i^{th} group's aggregate loss is the sum of the mean frequency times the variance of severity and the variance of frequency times the square of the mean severity. Substitution into the formulas in (2) above yields the mean and variance of the aggregate loss distribution.

Collective Risk Model

We now delete the restriction that the parameter vector v is known. Assume that the set V of possible parameters is finite and known and that one can specify the subjective likelihood of each element v of V. The structure function U(v) is a discrete probability function which specifies the observer's uncertainty regarding the "best" parameter.

The unconditional c.d.f. F(x) of the aggregate loss L has the following properties:

(1) $F(x) = \sum_v F(x|v) \cdot U(v)$

The c.d.f. $F_i(x)$ of L_i is computed similarly.

(2) $E(L^m) = \sum_v E(L^m|v) \cdot U(v)$

The m^{th} moment of L_i about the origin is computed similarly.

(3) With v unknown, assumptions (1) - (3) above may no longer hold, for the uncertainty regarding v may simultaneously affect the model at all levels. With v unknown, only the first cumulant is additive:

$E(L) = \sum_i E(L_i)$,

but $Var(L) \neq \sum_i Var(L_i)$

However $Var(L) = E(L^2) - E(L)^2$

and $E(L^2) = \sum_v E(L^2|v) \cdot U(v) = \sum_v \{Var(L|v) + E(L|v)^2\} \cdot U(v)$

$Var(L|v)$ and $E(L|v)$ are evaluated using the formulas above for the model with known parameters.

The Patrik-John version of the Collective Risk Model uses the normal power approximation formula to estimate percentiles of the aggregate loss distribution. This requires formulas for the third moment of the aggregate loss distribution, which are developed analogously to the second moment formulas presented above.

The Lognormal Model [3]

If the aggregate loss random variable is viewed as the product of a large number of independent, identically distributed random variables, the logarithm would then be approximately normally distributed by the Central Limit Theorem. (The stringent condition that the factors be identically distributed may be relaxed.[1]) This implies that the aggregate loss random variable would be lognormally distributed.

The formulas in Appendix A for the model with known parameters are used to estimate the mean and variance of the aggregate loss distribution. It is assumed that the mean aggregate loss for each coverage of the excess-of-loss reinsurance contract has been estimated accurately using standard burning cost and/or exposure rating methods. A Single Parameter Pareto severity distribution is assumed for each coverage and is used to compute the mean and variance of the severity distribution (see Appendix E). The ratio of the mean aggregate loss to the mean severity is the mean number of loss occurrences for a given coverage. The variance of the excess frequency distribution is computed based on the assumptions and the formula developed in Appendix D. Thus, the mean and variance of the frequency and severity distributions for each coverage are specified and used to compute the variance of the aggregate loss distribution for each coverage. The sum of these variances for all of the coverages is the variance of the aggregate loss distribution for all coverages combined, since we assume independence of aggregate losses for the individual coverages.

The Coefficient of Variation (CV) of the aggregate loss distribution is the ratio of the standard deviation to the mean of L, based on the frequency and severity distributions specified by the vector of parameters v or based on empirical methods applied to burning cost analyses:

$$CV(L|v) = \frac{\{Var(L|v)\}^{\frac{1}{2}}}{E(L|v)}$$

For simplicity, let $M = E(L|v)$.

The Entry Ratio r is the ratio of the attachment A to the mean aggregate loss:

$$r = \frac{A}{M}$$

The Excess Pure Premium (XSP) for a particular attachment A is the expected aggregate losses excess of A:

$$XSP(A|v) = \int_A^{\infty} (L-A)dP(L|v),$$

where P is the c.d.f. of L, given the vector of parameters v. The Excess Pure Premium Ratio P_2 at entry ratio r is the ratio of the corresponding Excess Pure Premium to the mean aggregate loss:

$$P_2\ (r|v) = \frac{XSP(A|v)}{M}$$

Assume that the distribution of L is lognormal, given frequency and severity distributions specified by the vector of parameters v. If the parameters of this lognormal distribution are μ and σ^2, then

$$M = E(L|v) = e^{\mu+\frac{\sigma^2}{2}} \tag{1}$$

and $$CV = CV(L|v) = \left(e^{\sigma^2}-1\right)^{\frac{1}{2}} \tag{2}$$

The first moment distribution P_1 is also lognormally distributed, but with parameters $\mu+\sigma^2$ and σ^2. P_1 is defined by

$$P_1(r|v) = \frac{1}{M}\int_0^A L\cdot dP(L|v)$$

The first moment distribution represents the percentage of total aggregate losses corresponding to coverage periods where the aggregate loss is less than the entry ratio times expected losses. The Excess Pure Premium Ratio can be computed using

$$P_2(r|v) = \{1-P_1(r|v)\}-r\{1-P(r|v)\}$$

Given that M and CV have been estimated as described above, the parameters of the assumed lognormal aggregate loss distribution can be estimated from formulas (1) and (2) above:

$$\sigma^2 = \log_e(1+CV^2)$$

$$\mu = \log_e M - \frac{\sigma}{2}^2$$

As noted above, P_1 is also lognormally distributed with parameters $\mu' = \mu + \sigma^2$ and σ^2. The vector of parameters v determine M and CV through the formulas previously presented. While the Excess Pure Premium is a function of both M and CV, the Excess Pure Premium Ratio is solely a function of the CV. Thus, the Excess Pure Premium Ratios are computed using

$$P_2(r|CV) = \{1-P_1(r|CV)\}-r\{1-P(r|CV)\}$$

This formula was used to compute values for the expanded version of Mr. Finger's famous table which is displayed on Pages 5-7 of this Appendix.

The Excess Pure Premium for attachment A is given by

$$XSP(A|M,CV) = M \cdot P_2\,(r|CV), \text{ where } r = \frac{A}{M}$$

Parameter uncertainty may be reflected using the method described under the Collective Risk Model Section of Appendix A. For each element v of V, compute M and CV. Since U(v) = U(M,CV), the unconditional Excess Pure Premium for attachment A may be computed using

$$XSP(A) = \sum_{M} \sum_{CV} XSP(A|M,CV) \cdot U(M,CV)$$

For the sake of simplicity, we assign a probability of one to our most likely scenario for the examples in this paper.

Excess Pure Premium Ratios
Lognormal Model

Entry Ratio	Coefficient of Variation .1	.2	.3	.4	.5
0	1.000	1.000	1.000	1.000	1.000
.1	.900	.900	.900	.900	.900
.2	.800	.800	.800	.800	.800
.3	.700	.700	.700	.700	.700
.4	.600	.600	.600	.601	.603
.5	.500	.500	.501	.504	.510
.6	.400	.400	.404	.413	.426
.7	.300	.302	.313	.331	.351
.8	.200	.211	.234	.260	.286
.9	.107	.135	.168	.200	.232
1.0	.040	.079	.117	.153	.187
1.1	.009	.042	.079	.115	.150
1.2	.001	.021	.052	.086	.120
1.3	.000	.010	.034	.064	.096
1.4		.004	.022	.048	.077
1.5		.002	.014	.035	.062
1.6		.001	.009	.026	.049
1.7		.000	.005	.019	.040
1.8			.003	.014	.032
1.9			.002	.010	.026
2.0			.001	.008	.021
2.2			.000	.004	.014
2.4				.002	.009
2.6				.001	.006
2.8				.001	.004
3.0				.000	.003

The Coefficient of Variation is the ratio of the standard deviation to the mean of the assumed Lognormal distribution. The Entry Ratio is the ratio of the attachment to the mean. The Excess Pure Premium Ratios are ratios of excess pure premiums to the mean (i.e., ratios of expected excess losses to the total expected loss).

Excess Pure Premium Ratios
Lognormal Model

Coefficient of Variation

Entry Ratio	.6	.7	.8	.9	1.0
0	1.000	1.000	1.000	1.000	1.000
.1	.900	.900	.900	.900	.900
.2	.800	.800	.801	.802	.804
.3	.702	.704	.707	.710	.714
.4	.607	.613	.619	.627	.634
.5	.519	.530	.541	.552	.563
.6	.441	.456	.472	.487	.502
.7	.371	.392	.412	.430	.447
.8	.312	.336	.359	.381	.400
.9	.261	.289	.314	.337	.359
1.0	.218	.248	.275	.300	.323
1.1	.183	.213	.241	.267	.291
1.2	.153	.183	.212	.239	.263
1.3	.128	.158	.187	.213	.238
1.4	.107	.137	.165	.192	.216
1.5	.090	.118	.146	.172	.197
1.6	.076	.103	.130	.155	.180
1.7	.064	.089	.115	.140	.164
1.8	.054	.078	.103	.127	.150
1.9	.046	.068	.092	.115	.138
2.0	.039	.060	.082	.105	.127
2.2	.028	.046	.066	.087	.108
2.4	.020	.036	.053	.073	.092
2.6	.015	.028	.044	.061	.079
2.8	.011	.022	.036	.052	.068
3.0	.008	.017	.030	.044	.059
3.2	.006	.014	.025	.037	.052
3.4	.005	.011	.021	.032	.045
3.6	.004	.009	.017	.028	.040
3.8	.003	.007	.015	.024	.035
4.0	.002	.006	.012	.021	.031
5.0	.001	.001	.006	.011	.018
10.0	.000	.000	.000	.001	.002

The Coefficient of Variation is the ratio of the standard deviation to the mean of the assumed Lognormal distribution. The Entry Ratio is the ratio of the attachment to the mean. The Excess Pure Premium Ratios are ratios of excess pure premiums to the mean (i.e., ratios of expected excess losses to the total expected loss).

Excess Pure Premium Ratios
Lognormal Model

Coefficient of Variation

Entry Ratio	1.5	2.0	2.5	3.0	4.0	5.0
0	1.000	1.000	1.000	1.000	1.000	1.000
.1	.902	.905	.908	.911	.916	.921
.2	.813	.824	.834	.842	.855	.864
.3	.736	.756	.772	.786	.805	.820
.4	.670	.699	.721	.738	.764	.782
.5	.612	.649	.676	.697	.728	.750
.6	.562	.605	.637	.661	.697	.721
.7	.518	.567	.602	.630	.669	.696
.8	.479	.532	.571	.601	.644	.673
.9	.444	.502	.544	.575	.621	.652
1.0	.413	.474	.518	.552	.600	.633
1.2	.360	.426	.474	.511	.563	.599
1.4	.316	.386	.437	.476	.531	.570
1.6	.280	.352	.405	.445	.503	.544
1.8	.250	.323	.377	.418	.478	.521
2.0	.224	.297	.352	.394	.456	.500
2.2	.202	.275	.330	.373	.436	.481
2.4	.183	.255	.311	.354	.418	.464
2.6	.167	.238	.293	.337	.402	.448
2.8	.152	.222	.277	.321	.386	.433
3.0	.139	.208	.263	.307	.372	.419
3.5	.114	.179	.232	.275	.341	.389
4.0	.094	.155	.207	.250	.315	.364
4.5	.079	.136	.186	.228	.293	.342
5.0	.067	.120	.168	.209	.274	.322
5.5	.057	.107	.153	.193	.257	.305
6.0	.049	.096	.140	.179	.242	.290
6.5	.043	.087	.129	.167	.229	.276
7.0	.037	.079	.119	.156	.216	.264
7.5	.033	.072	.111	.146	.206	.252
8.0	.029	.066	.103	.137	.196	.242
10.0	.019	.047	.079	.110	.164	.208
20.0	.004	.015	.031	.049	.087	.120
30.0	.001	.007	.016	.029	.056	.083
50.0	.000	.002	.007	.013	.030	.049
100.0		.000	.000	.004	.012	.022

The Coefficient of Variation is the ratio of the standard deviation to the mean of the assumed Lognormal distribution. The Entry Ratio is the ratio of the attachment to the mean. The Excess Pure Premium Ratios are ratios of excess pure premiums to the mean (i.e., ratios of expected excess losses to the total expected loss).

Heckman-Meyers Version of Collective Risk Model [4]

We use the same notation as presented in Appendix A. Let N_i represent the number of loss occurrences for group i and let m_i represent the unconditional mean number of occurrences,

$$m_i = E(N_i)$$

Let C represent a random variable with E(C) = 1 and Var (C) = c. In this paper, C is assumed to be Gamma distributed. The parameter c is used to model parameter uncertainty in the frequency mean and is called the contagion parameter. Let X_{ij} represent the loss size of the j^{th} loss for group i. L_i is the aggregate loss of the i^{th} group:

$$L_i = X_{i1} + X_{i2} + \ldots + X_{iN_i}$$

Parameter uncertainty in the severity mean is modelled through a random variable B with E(1/B) = 1 and Var(1/B) = b. B is assumed to be Gamma distributed so 1/B is Inverse Gamma distributed. The parameter b is called the mixing parameter.

<u>The Algorithm</u>

(1) Select C at random from the assumed distribution.

(2) Select the number of loss occurrences N_i at random from a Poisson distribution with mean $C \cdot m_i$.

(3) Select B at random from the assumed distribution.

(4) Select the loss occurrence amounts $X_{i1}, X_{i2}, \ldots, X_{iN_i}$ at random from the assumed occurrence severity distribution.

(5) Compute the aggregate loss L_i as the sum of all loss occurrence amounts divided by the scaling parameter B.

Since C is assumed to be Gamma distributed, the frequency distribution generated by the above algorithm will be Negative Binomial. If the conditions in Appendix D are satisfied, the excess frequency distribution for each group will be approximately Poisson, and the excess frequency distribution for all groups combined will also be approximately Poisson due to the independence assumptions. The Negative Binomial frequency distribution is used to model uncertainty in the mean frequencies.

It is assumed that the shape of the severity distribution is known, and so the mixing parameter b models uncertainty in the severity means for the various groups. If uncertainty exists concerning the shape of the severity distribution, the approach to parameter uncertainty discussed in Appendix A may be applied through assignment of subjective probabilities to alternative scenarios concerning the shape parameter. In this paper, we assume a Single Parameter Pareto severity distribution, as discussed in Appendix E.

The examples in this paper are evaluated for the following combinations of b and c: $b = c = 0$, $b = c = .05$, $b = .10$ and $c = .05$, and $b = c = .10$. These combinations represent no parameter uncertainty, moderate parameter uncertainty, relatively high uncertainty concerning the mean severity but moderate uncertainty concerning the mean frequency, and relatively high parameter uncertainty.

Although many other combinations may be appropriate for particular circumstances, these values will be used in this paper to illustrate the impact of modelling parameter uncertainty.

The reader may presume that a simulation is performed by running the above algorithm a sufficiently large number of times for each group to generate an accurate estimate of its aggregate loss distribution. Once aggregate loss distributions for each group are obtained in this manner, the aggregate loss distributions for all groups combined can be estimated by conducting a second simulation as follows:

(1) For group i, select L_i at random from the aggregate loss distribution already estimated.

(2) Compute the aggregate loss L for all groups combined by summing the L_i's, $i=1,2,...,k$.

This second simulation is performed a sufficiently large number of times to generate an accurate estimate of the aggregate loss distribution for all groups combined. (Note that aggregate limits or deductibles may be applied to individual groups before the second simulation is performed.)

Instead of performing the above simulations, the Heckman-Meyers algorithm computes the aggregate loss distribution directly through application of the characteristic function method briefly summarized in this paper. The reader is referred to the paper and to the excellent review by Gary Venter for technical details.[4] The alternate recursive method which is discussed in Mr. Venter's review and in his recent CAS Forum contribution[9] is simpler and in some circumstances more accurate,[5] but in other circumstances it is less efficient than the characteristic function method and requires the structure function method discussed in Appendix A to model parameter uncertainty. A sample run of the model is presented on Page 4.

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[9]Venter, G.G., "Easier Algorithms for Aggregate Excess," CAS Forum, Fall 1989 Edition, p.19.

Collective Risk Model — Treaty IV

Line	Expected Loss	Claim Severity Distribution	Contagion Parameter	Claim Count Mean	Claim Count Std Dev
1	359995	class1.sev	0.0500 $= c_1$	5.154	2.546
2	90033	class2.sev	0.0500 $= c_2$	1.343	1.197

Mixing parameter 0.1000 $= b$
Aggregate mean 450028
Aggregate std dev 297472

Aggregate Loss Amount	Entry Ratio	Cumulative Probability	Excess Pure Premium	Excess Pure Premium Ratio
0.00	0.0000	0.0015	450028.21	1.0000
90005.64	0.2000	0.0572	362454.04	0.8054
180011.28	0.4000	0.1577	281663.70	0.6259
270016.93	0.6000	0.2988	212038.72	0.4712
360022.57	0.8000	0.4477	155661.12	0.3459
450028.21	1.0000	0.5832	112206.06	0.2493
540033.85	1.2000	0.6949	79912.14	0.1776
630039.49	1.4000	0.7811	56513.45	0.1256
720045.14	1.6000	0.8450	39840.27	0.0885
810050.78	1.8000	0.8911	28079.72	0.0624
900056.42	2.0000	0.9237	19828.73	0.0441

Collective Risk Model — Treaty IV

Line	Expected Loss	Claim Severity Distribution	Contagion Parameter	Claim Count Mean	Claim Count Std Dev
1	359995	class1.sev	0.1000 $= c_1$	5.154	2.795
2	90033	class2.sev	0.1000 $= c_2$	1.343	1.234

Mixing parameter 0.1000 $= b$
Aggregate mean 450028
Aggregate std dev 309940

Aggregate Loss Amount	Entry Ratio	Cumulative Probability	Excess Pure Premium	Excess Pure Premium Ratio
0.00	0.0000	0.0023	450028.21	1.0000
90005.64	0.2000	0.0667	363013.42	0.8066
180011.28	0.4000	0.1716	283301.51	0.6295
270016.93	0.6000	0.3120	214944.81	0.4776
360022.57	0.8000	0.4562	159564.03	0.3546
450028.21	1.0000	0.5861	116620.91	0.2591
540033.85	1.2000	0.6933	84374.46	0.1875
630039.49	1.4000	0.7767	60690.76	0.1349
720045.14	1.6000	0.8392	43547.09	0.0968
810050.78	1.8000	0.8850	31246.92	0.0694
900056.42	2.0000	0.9180	22462.93	0.0499

DERIVATION OF EXCESS OCCURRENCE COUNT VARIANCE TO MEAN RATIO

In this Appendix, we show that if the conditions for the ground-up occurrence count distribution to be Negative Binomial are satisfied, then the excess occurrence count distribution for an insured selected at random will be Negative Binomial. Based on this result, we derive the formula for calculating the excess occurrence count variance-to-mean ratio for an individual insured selected at random and show that this formula also applies to the class as a whole. This latter result is then used to demonstrate that if the proportion of occurrences exceeding the retention is small, then the excess occurrence count distribution for the class as a whole will be approximately Poisson. (Our proof is a direct application of the Gamma-Poisson model frequently encountered in the actuarial literature. We understand that these results have previously been established elsewhere, and note that Joseph Schumi has established these results using recursive relationships.[3])

Assume (1) An individual policy's distribution of ground-up occurrence counts over a given period of time is Poisson with parameter λ_i.

(2) The policies in the given class are of identical size.

(3) The distribution of the individual policy expected occurrence counts (ie, the λ_i's) over the class is Gamma with parameters a,r.

(4) The probability of a given occurrence being an excess occurrence (ie, the probability that it exceeds a fixed retention R) is p. This probability is applicable to all policies.

Given (1) and (3) above, we know that the distribution of the observed ground-up occurrence counts for an individual policy selected at random is Negative Binomial with a mean $\mu_G = \frac{r}{a}$ and variance $\sigma_G^2 = \frac{r}{a}\left(\frac{a+1}{a}\right)$.[10]

This implies a variance-to-mean ratio $VMR_G = \frac{\sigma_G^2}{\mu_G} = \frac{a+1}{a} = 1 + \frac{1}{a}$.

Assuming that we know VMR_G (from the ISO Increased Limits Reviews or elsewhere), we can easily solve for a.

It follows from the assumptions of a Poisson process that if an individual policy's distribution of ground-up occurrence counts is Poisson with parameter λ_i, then the distribution of excess occurrence counts (claims above R) for the individual policy is also Poisson but with parameter $p\lambda_i$.[2]

The Gamma Distribution has the property that if λ has the distribution $\Gamma a,r$, then $p\lambda$ has a $\Gamma a/p,r$.[11] Hence, the distribution of the individual policy expected excess occurrence counts over the class is $\Gamma a/p,r$.

Thus, the distribution of observed excess occurrence counts for an individual policy selected at random from the class of policies is Negative Binomial with a mean $\mu_E = \frac{r}{[a/p]} = \frac{pr}{a}$ and variance $\sigma_E^2 = \frac{r}{[a/p]}\left[\frac{a/p+1}{a/p}\right] = \frac{pr}{a}\left[1+\frac{p}{a}\right]$.

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[10] Dropkin, L.B., "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records," PCAS XLVI, (1959), p.171.

[11] Hewitt, C.C., "Loss Ratio Distribution - A Model," PCAS LIV, (1967), p.76.

This implies a variance-to-mean ratio $VMR_E = \dfrac{\sigma_E^2}{\mu_E} = 1+\dfrac{p}{a}$,

which we can calculate since we already know a. Note that since $p<1$, $VMR_E<VMR_G$.

One can think of the group of policies covered by a particular excess reinsurance treaty as a statistical sample taken from the theoretically infinite population of all insureds belonging to the particular class.[12] Assuming that the sample is taken at random, the policies selected are independent of each other. From the above, each policy's excess occurrence count distribution has mean μ_E and variance σ_E^2. Given that n policies from the particular class are covered by the reinsurance treaty, the expected number of occurrences subject to the excess treaty is $n\mu_E$ and the variance of the number of occurrences subject to the treaty is $n\sigma_E^2$. This implies a variance-to-mean ratio of $\dfrac{n\sigma_E^2}{n\mu_E} = \dfrac{\sigma_E^2}{\mu_E} = VMR_E$ for the total number of occurrences subject to the treaty. Thus, the excess occurrence count variance-to-mean ratio for the entire group of policies covered by the reinsurance treaty equals the excess occurrence count variance-to-mean ratio for an individual policy selected at random from the class.

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[12] Lange, J.T., "Implications of Sampling Theory for Package Policy Ratemaking," PCAS LIII, (1966), p.286-287.

Given that we know VMR_G, a simple formula for calculating VMR_E can be easily derived using the following two relationships (which were proven above):

$$(1)\ VMR_G = 1 + \frac{1}{a}$$

$$(2)\ VMR_E = 1 + \frac{p}{a}$$

Solving equation (1) for a, we get

$$(3)\ a = \frac{1}{VMR_G - 1}$$

Substituting expression (3) into (2), we get

$$VMR_E = 1 + \frac{p}{\left[\frac{1}{VMR_G - 1}\right]} = 1 + p\ (VMR_G - 1) = (1-p) + pVMR_G$$

Based on the above formula, if VMR_G is two or three, as in the ISO Increased Limits reviews, and p is small (say less than .02), VMR_E will be close to unity. This implies that the excess occurrence count distribution for an insured selected at random and for the class as a whole will be approximately Poisson.

Single Parameter Pareto Severity Distribution [6]

General Properties of Model

Assume ground-up loss occurrences above the truncation point k are distributed according to the following cumulative distribution function:

$$F(w) = 1-\left(\frac{K}{W}\right)^q \qquad k>0,\ q>0,\ w\geq k$$

We "normalize" losses by setting $x = w/k$:

$$F(x) = 1-x^{-q}$$

and the density function is

$$f(x) = q\, x^{-(q+1)}$$

Note that

$$F(w) = 1-\left(\frac{k}{k+(w-k)}\right)^q$$

Let $y = w-k$ represent the occurrence size excess of the attachment k. Then

$$F(y) = 1-\left(\frac{k}{k+y}\right)^q, \text{ where } y\geq 0$$

Thus, occurrence losses excess of the attachment k are distributed according to the two parameter Shifted Pareto distribution, with scale parameter equal to the attachment and shape parameter equal to q.

Assume ground-up loss occurrences are censored at an upper limit equal to $k\cdot b$. Then

$$F(y) = 1-\left(\frac{k}{k+y}\right)^q \text{ if } 0\leq y<k(b-1)$$

$$\text{and } F(y) = 1 \qquad \text{if } y\geq k(b-1)$$

The mean excess occurrence is given by

$$E(y) = \frac{k(b^{1-q}-1)}{1-q} \text{ if } q \neq 1$$

and $E(y) = k \cdot \log_e b$ if $q=1$.

The variance of the excess occurrences is given by

$$\frac{Var(y)}{k^2} = \frac{q-2b^{2-q}}{q-2} - \left(\frac{q-b^{1-q}}{q-1}\right)^2 \qquad \text{if } q \neq 1,\ q \neq 2$$

$$\frac{Var(y)}{k^2} = 2b-1-(1+\log_e b)^2 \quad \text{if } q = 1$$

$$\frac{Var(y)}{k^2} = 1+2\cdot\log_e b - \left(\frac{2b-1}{b}\right)^2 \text{ if } q = 2$$

Maximum Likelihood Estimation of q

Assume we wish to compute the Maximum Likelihood Estimator (MLE) of q by fitting n loss occurrences above the truncation point k, W_1, W_2, ..., W_n. Let X_i represent the normalized losses, $X_i = W_i/k$. Assume m_j occurrences have been censored at limit C_j and let $b_j = C_j/k$, $j=1,2,...s$. Let $u=n-\sum_{j=1}^{s} m_j$ represent the number of uncensored occurrences. Then the MLE of q is given by

$$\hat{q} = \frac{u}{\sum_{i=1}^{u}\log_e X_i + \sum_{j=1}^{s} m_j\cdot\log_e b_j}$$

If no occurrences have been censored, the MLE of q is

$$\hat{q} = \frac{n}{\sum_{i=1}^{n}\log_e X_i}$$

If some occurrences have been censored, the losses need not be trended if the truncation point is sufficiently large, but q should be estimated separately for each year and a weighted average q should be calculated. If q is to be estimated by pooling the losses, they need to be adjusted by trend if some of the losses have been censored. If cases are developing, q should be estimated for each accident year or policy year at each evaluation and a triangulation approach should be used to project the ultimate estimate of q for losses excess of the truncation point used for the particular class of business.

Leveraged Impact of Inflation

Let m represent the number of loss occurrences above retention k at time 0, and assume the annual loss inflation factor between time 0 time n is 1+i. Based on the SPP distribution with parameter q, the projected number of loss occurrences excess of retention k at time n is

$$m\,(1+i)^{nq}$$

As long as inflation doesn't erode the real value of a retention to the point that the SPP distribution is no longer a satisfactory model above the retention, the parameter q and the average occurrence size in the layer of interest will remain constant over time. The leveraged impact of inflation over a fixed retention will be felt through the application of the adjustment factor

$$(1+i)^{nq}$$

to excess occurrence frequency. Thus, this factor may be thought of as measuring the leveraged impact of inflation above a fixed retention.

Change in Layer

Assume that one has credibly estimated expected losses in the layer from a to b and wishes to estimate expected losses in the layer from c to d, where the SPP distribution with parameter q is appropriate above the lower of the two retentions. The change in expected losses due to the change in reinsurance layer is given by

$$\text{Change in Layer} = \frac{c^{1-q}-d^{1-q}}{a^{1-q}-b^{1-q}} \quad \text{if } q \neq 1$$

$$\text{Change in Layer} = \frac{\log_e(d/c)}{\log_e(b/a)} \quad \text{if } q=1$$

(The layer limits need not be normalized in the above formulas.) The Change in Layer factor is applied to expected losses in the layer from a to b to estimate expected losses in the layer from c to d.

Treaty I

Summary of Key Contract Provisions

Treaty Period: 1/1/90 - 12/31/90

Layer Reinsured: $160,000 in excess of $40,000 per occurrence

Estimated Treaty Subject Premium: $12,000,000 for 1990, distributed as follows:

Class 1 - $9,000,000

Class 2 - 3,000,000

Expected Layer Loss Costs for the Entire Layer, Prior to All Forms of Coinsurance (Expressed as a Percent of Subject Premium):

Class 1 - 4.00%

Class 2 - 3.00%

Both Classes Combined - 3.75%

Loading to Convert Expected Layer Loss Cost After All Forms of Coinsurance into a Rate: 100/75

Proportional Coinsurance: None

Non-Proportional Coinsurance: Aggregate Deductible Equal to 3% of Subject Premium

DETERMINATION OF AGGREGATE LOSS DISTRIBUTION SPECIFICATION
PARAMETERS FOR NON-PROPORTIONAL LOSS SHARING PLANS

	CLASS OF BUSINESS				
	CLASS 1	CLASS 2	CLASS 3	CLASS 4	ALL CLASSES
NAMES OF INDIVIDUAL CLASSES OF BUSINESS ==>	CLASS 1	CLASS 2			COMBINED
(1) ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR TREATY PERIOD	9,000,000	3,000,000			12,000,000
(2) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO APPLICATION OF ALL FORMS OF COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	4.0000%	3.0000%			3.7500%
(3) EXPECTED LOSSES FOR ENTIRE REINSURED LAYER FOR TREATY PERIOD [(1)*(2)]	360,000	90,000	0	0	450,000
(4) PARETO Q-VALUES FOR SEVERITY DISTRIBUTIONS	0.900	0.950			
(5) MEAN CLAIM SIZE IN LAYER (EXCESS OF RETENTION) (BASED ON THE SELECTED PARETO Q) (μ_s)	69,848	67,039			69,267
(6) STANDARD DEVIATION OF EXCESS CLAIM SIZES IN LAYER (BASED ON THE SELECTED PARETO Q) (σ_s)	60,908	60,084			60,749
(7) EXPECTED NUMBER OF CLAIMS IN LAYER PRIOR TO THE APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (μ_c) [(3)/(5)]	5.154	1.343			6.497
(8) EXCESS CLAIM COUNT VARIANCE TO MEAN RATIO PRIOR TO APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (VMR_c)	1.032	1.067			1.039
(9) STANDARD DEVIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER $[\sigma_s^2 \cdot \mu_c + (\mu_c \cdot VMR_c) \cdot \mu_s^2]^{1/2}$	212,298	106,228	0	0	237,391
(10) COEFFICIENT OF VARIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER [(9)/(3)]	0.590	1.180			0.528

(11) SELECTED PARAMETERS NEEDED TO SPECIFY AGGREGATE LOSS DISTRIBUTION FOR ALL CLASSES COMBINED

(A) EXPECTED NUMBER OF CLAIMS 6.500

(B) COEFFICENT OF VARIATION FOR AGGREGATE LOSS DISTRIBUTION 0.528

(C) TECHNIQUE USED TO OBTAIN AGGREGATE DISTRIBUTION (EG. COLLECTIVE RISK MODEL, LOGNORMAL ASSUMPTION) LOGNORMAL ASSUMPTION

AGGREGATE DEDUCTIBLES

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(1)	ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR TREATY PERIOD	12,000,000
(2)	EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO THE APPLICATION OF ALL COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	3.7500%
(3)	COINSURANCE PERCENTAGE (CEDANT'S PARTICIPATION IN LAYER LOSSES NOT CORRESPONDING TO AN EXPLICIT SHARE OF THE REINSURANCE PREMIUM, EXCLUDING THE PRESUMED EFFECT OF THE AGGREGATE DEDUCTIBLE.)	0.00%
(4)	LOADING TO CONVERT EXPECTED LAYER LOSS COST AFTER ALL FORMS OF COINSURANCE INTO A LOADED RATE (EXPRESSED AS A MULTIPLICATIVE FACTOR TO BE APPLIED TO THE EXPECTED LAYER LOSS COST)	1.333 = 100/75
(5)	EXPECTED DOLLARS OF LOSS FOR ENTIRE LAYER PRIOR TO ALL COINSURANCE [(1)*(2)]	450,000
(6)	AGGREGATE DEDUCTIBLE AMOUNT IN DOLLARS APPLICABLE TO THE ENTIRE LAYER [3% x 12,000,000]	360,000
(7)	ENTRY RATIO CORRESPONDING TO AGGREGATE DEDUCTIBLE AMOUNT [(6)/(5)]	0.800
(8)	INSURANCE CHARGE AT ENTRY RATIO CORRESPONDING TO AGGREGATE DEDUCTIBLE AMOUNT	29.33%
(9)	EXPECTED PERCENTAGE OF TREATY LOSSES ELIMINATED BY AGGREGATE DEDUCTIBLE [100%-(8)]	70.67%
(10)	COMPOSITE COINSURANCE PERCENTAGE 100%-{[100%-(3)]*[100%-(9)]}	70.67%
(11)	EXPECTED LAYER LOSS COST FOR ENTIRE REINSURED PORTION OF LAYER, AFTER APPLICATION OF AGGREGATE DEDUCTIBLE (EXPRESSED AS A PERCENTAGE OF SUBJECT PREMIUM) (2)*[100%-(10)]	1.0998%
(12)	INDICATED TREATY RATE AFTER THE APPLICATION OF AGGREGATE DEDUCTIBLE AND ANY PROPORTIONAL COINSURANCE (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM) [(4)*(11)]	1.4664%

Treaty II

Summary of Key Contract Provisions

Treaty Period: 1/1/90 - 12/31/90

Layer Reinsured: $700,000 in excess of $300,000 per occurrence

Estimated Treaty Subject Premium: $6,000,000 for 1990,

distributed as follows:

Class 1 - $2,000,000

Class 2 - $2,000,000

Class 3 - $2,000,000

Expected Layer Loss Costs for the Entire Layer, Prior to All Forms of Coinsurance (Expressed as a Percent of Subject Premium):

Class 1 - 10.0%

Class 2 - 14.0%

Class 3 - 21.0%

All Classes Combined - 15.0%

Loading to Convert Expected Layer Loss Cost After All Forms of Coinsurance into a Rate: 100/60

Proportional Coinsurance: 20%

Non-Proportional Coinsurance: Three (3) full free reinstatements permitted under treaty.

	CLASS OF BUSINESS CLASS 1	CLASS 2	CLASS 3	CLASS 4	ALL CLASSES COMBINED
NAMES OF INDIVIDUAL CLASSES OF BUSINESS ==>	CLASS 1	CLASS 2	CLASS 3		
(1) ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR TREATY PERIOD	2,000,000	2,000,000	2,000,000		6,000,000
(2) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO APPLICATION OF ALL FORMS OF COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	10.0000%	14.0000%	21.0000%		15.0000%
(3) EXPECTED LOSSES FOR ENTIRE REINSURED LAYER FOR TREATY PERIOD [(1)*(2)]	200,000	280,000	420,000	0	900,000
(4) PARETO Q-VALUES FOR SEVERITY DISTRIBUTIONS	1.500	1.300	1.100		
(5) MEAN CLAIM SIZE IN LAYER (EXCESS OF RETENTION) (BASED ON THE SELECTED PARETO Q) (μ_s)	271,366	303,155	340,296		310,897
(6) STANDARD DEVIATION OF EXCESS CLAIM SIZES IN LAYER (BASED ON THE SELECTED PARETO Q) (σ_s)	246,592	257,600	266.584		260,265
(7) EXPECTED NUMBER OF CLAIMS IN LAYER PRIOR TO THE APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (μ_c) [(3)/(5)]	0.737	0.924	1.234		2.895
(8) EXCESS CLAIM COUNT VARIANCE TO MEAN RATIO PRIOR TO APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (VMR_c)	1.006	1.009	1.019		1.012
(9) STANDARD DEVIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER $[\sigma_s^2 \cdot \mu_c + (\mu_c \cdot VMR_c) \cdot \mu_s^2]^{1/2}$	315,301	383,323	483,065	0	692,606
(10) COEFFICIENT OF VARIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER [(9)/(3)]	1.577	1.369	1.150		0.770

(11) SELECTED PARAMETERS NEEDED TO SPECIFY AGGREGATE LOSS DISTRIBUTION FOR ALL CLASSES COMBINED

(A) EXPECTED NUMBER OF CLAIMS 2.900

(B) COEFFICENT OF VARIATION FOR AGGREGATE LOSS DISTRIBUTION 0.770

(C) TECHNIQUE USED TO OBTAIN AGGREGATE DISTRIBUTION (EG, COLLECTIVE RISK MODEL, LOGNORMAL ASSUMPTION) LOGNORMAL ASSUMPTION

LOSS RATIO CAPS/LIMITED REINSTATEMENTS

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(1) ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR TREATY PERIOD 6,000,000

(2) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO THE APPLICATION OF ALL COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENTAGE OF SUBJECT PREMIUM) 15.0000%

(3) COINSURANCE PERCENTAGE (CEDANT'S PARTICIPATION IN LAYER LOSSES NOT CORRESPONDING TO AN EXPLICIT SHARE OF THE REINSURANCE PREMIUM, EXCLUDING THE PRESUMED EFFECT OF THE LOSS RATIO CAP OR LIMITED REINSTATEMENT PROVISION) 20.00%

(4) EXPECTED DOLLARS OF LOSS FOR THE ENTIRE LAYER PRIOR TO THE APPLICATION OF ALL COINSURANCE [(1)*(2)] 900,000

(5) LOADING TO CONVERT EXPECTED LAYER LOSS COST AFTER ALL FORMS OF COINSURANCE INTO A LOADED RATE (EXPRESSED AS A MULTIPLICATIVE FACTOR TO BE APPLIED TO THE EXPECTED LAYER LOSS COST) 1.667 = 100/60

COMPLETE ITEM (6) IF THE TREATY LOSS RATIO CAP IS EXPRESSED AS A PERCENT OF TREATY EXPECTED LOSSES, OR ITEM (7) IF THE TREATY LOSS RATIO CAP IS EXPRESSED IN TERMS OF LIMITED REINSTATEMENTS. THEN HIT THE F9 KEY.

(6) TREATY LOSS RATIO CAP (EXPRESSED AS A PERCENT OF THE EXPECTED LOSSES FOR THE TREATY PRIOR TO THE APPLICATION OF THE CAP) ------------

(7) (A) NUMBER OF FREE REINSTATEMENTS ALLOWED UNDER TREATY 3

(B) LAYER RETENTION 300,000

(C) LAYER GROSS LIMIT 1,000,000

(D) LAYER WIDTH [(7C)-(7B)] 700,000

(E) EFFECTIVE AGGREGATE LIMIT FOR THE ENTIRE LAYER PRIOR TO ALL COINSURANCE (EXPRESSED IN DOLLARS) [1+(7A)]*(7D) 2,800,000

(F) EFFECTIVE TREATY LOSS RATIO CAP (EXPRESSED AS A PERCENT OF TREATY EXPECTED LOSSES) [(7E)/(4)] 311.11%

(8) ENTRY RATIO CORRESPONDING TO TREATY LOSS RATIO CAP [(6) OR (7F), EXPRESSED AS A DECIMAL] 3.111

(9) INSURANCE CHARGE AT ENTRY RATIO CORRESPONDING TO TREATY LOSS RATIO CAP. (THIS PERCENTAGE IS EQUIVALENT TO THE EXPECTED PERCENTAGE OF TREATY LOSSES ELIMINATED BY THE LOSS RATIO CAP OR LIMITED REINSTATEMENT PROVISION.) 2.37%

(10) COMPOSITE COINSURANCE PERCENTAGE 100%-[[100%-(3)]*[100%-(9)]] 21.89%

(11) EXPECTED LAYER COST FOR THE ENTIRE REINSURED PORTION OF LAYER, AFTER THE APPLICATION OF THE LOSS RATIO CAP OR LIMITED REINSTATEMENT PROVISION (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM) (2)*[100%-(10)] 11.7161%

(12) INDICATED TREATY RATE AFTER THE APPLICATION OF LOSS RATIO CAPS/LIMITED REINSTATEMENTS AND ANY PROPORTIONAL COINSURANCE (EXPRESSED AS A PERCENTAGE OF SUBJECT PREMIUM) [(5)*(11)] 19.5268%

Treaty III

Summary of Key Contract Provisions

Treaty Period: 1/1/90 - 12/31/90

Layer Reinsured: $400,000 in excess of $100,000 per occurrence

Estimated Treaty Subject Premium: $10,000,000 as follows:

distributed as follows:

Class 1 - $4,500,000

Class 2 - $4,500,000

Class 3 - $1,000,000

Expected Layer Loss Costs for the Entire Layer, Prior to All Forms of Coinsurance (Expressed as a Percent of Subject Premium):

Class 1 - 3.20%

Class 2 - 3.80%

Class 3 - 3.50%

All Classes Combined - 3.50%

Loading to Convert Expected Layer Loss Cost After All Forms of Coinsurance into a Rate: 100/70

Proportional Coinsurance: None

Non-Proportional Coinsurance: Loss Corridor - Reinsurer stops paying losses which fall in the reinsured layer when the ratio of actual losses in the layer to expected losses in the layer reaches 100%, but he resumes full payment of losses in the layer if this ratio goes above 200%.

	CLASS OF BUSINESS				
	CLASS 1	CLASS 2	CLASS 3	CLASS 4	ALL CLASSES
NAMES OF INDIVIDUAL CLASSES OF BUSINESS ==>	CLASS 1	CLASS 2	CLASS 3		COMBINED
(1) ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR TREATY PERIOD	4,500,000	4,500,000	1,000,000		10,000,000
(2) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO APPLICATION OF ALL FORMS OF COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	3.2000%	3.8000%	3.5000%		3.5000%
(3) EXPECTED LOSSES FOR ENTIRE REINSURED LAYER FOR TREATY PERIOD [(1)*(2)]	144,000	171,000	35,000	0	350,000
(4) PARETO Q-VALUES FOR SEVERITY DISTRIBUTIONS	1.000	1.250	1.050		
(5) MEAN CLAIM SIZE IN LAYER (EXCESS OF RETENTION) (BASED ON THE SELECTED PARETO Q) (μ_S)	160,944	132,504	154,638		145,133
(6) STANDARD DEVIATION OF EXCESS CLAIM SIZES IN LAYER (BASED ON THE SELECTED PARETO Q) (σ_S)	148,015	135,796	145,709		142,041
(7) EXPECTED NUMBER OF CLAIMS IN LAYER PRIOR TO THE APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (μ_C) [(3)/(5)]	0.895	1.291	0.226		2.412
(8) EXCESS CLAIM COUNT VARIANCE TO MEAN RATIO PRIOR TO APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (VMR_C) INPUT DIRECTLY ==>	1.012	1.024	1.029		1.020
(9) STANDARD DEVIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER $[\sigma_S^2 \cdot \mu_C + (\mu_C \cdot VMR_C) \cdot \sigma_S^2]^{1/2}$	207,499	216,795	101,856	0	316,908
(10) COEFFICIENT OF VARIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER [(9)/(3)]	1.441	1.268	2.910		0.905

(11) SELECTED PARAMETERS NEEDED TO SPECIFY AGGREGATE LOSS DISTRIBUTION FOR ALL CLASSES COMBINED

(A) EXPECTED NUMBER OF CLAIMS 2.400

(B) COEFFICENT OF VARIATION FOR AGGREGATE LOSS DISTRIBUTION 0.905

(C) TECHNIQUE USED TO OBTAIN AGGREGATE DISTRIBUTION (EG, COLLECTIVE RISK MODEL, LOGNORMAL ASSUMPTION) LOGNORMAL ASSUMPTION

LOSS CORRIDORS

(1)	ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR TREATY PERIOD	10,000,000
(2)	EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO THE APPLICATION OF ALL COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENTAGE OF SUBJECT PREMIUM)	3.5000%
(3)	COINSURANCE PERCENTAGE (CEDANT'S PARTICIPATION IN LAYER LOSSES NOT CORRESPONDING TO AN EXPLICIT SHARE OF THE REINSURANCE PREMIUM, EXCLUDING THE PRESUMED EFFECT LOSS CORRIDOR PROVISION)	0.00%
(4)	EXPECTED DOLLARS OF LOSS FOR THE ENTIRE LAYER PRIOR TO THE APPLICATION OF ALL COINSURANCE [(1)*(2)]	350,000
(5)	LOADING TO CONVERT EXPECTED LAYER LOSS COST AFTER ALL FORMS OF COINSURANCE INTO A LOADED RATE (EXPRESSED AS A MULTIPLICATIVE FACTOR TO BE APPLIED TO THE EXPECTED LAYER LOSS COST)	1.429 = 100/70
(6)	LOWER BOUND OF LOSS CORRIDOR INTERVAL (EXPRESSED AS AS A PERCENT OF EXPECTED LOSSES FOR THE TREATY PRIOR TO THE APPLICATION OF THE LOSS CORRIDOR PROVISION)	100.00%
(7)	UPPER BOUND OF LOSS CORRIDOR INTERVAL (EXPRESSED AS AS A PERCENT OF EXPECTED LOSSES FOR THE TREATY PRIOR TO THE APPLICATION OF THE LOSS CORRIDOR PROVISION)	200.00%
(8)	REINSURER'S PARTICIPATION PERCENTAGE IN LOSS CORRIDOR INTERVAL (IF ANY)	0.00%
(9)	ENTRY RATIO CORRESPONDING TO LOWER BOUND OF INTERVAL [(6) EXPRESSED AS A DECIMAL]	1.000
(10)	INSURANCE CHARGE AT ENTRY RATIO CORRESPONDING TO LOWER BOUND OF INTERVAL	30.11%
(11)	ENTRY RATIO CORRESPONDING TO UPPER BOUND OF INTERVAL [(7) EXPRESSED AS A DECIMAL]	2.000
(12)	INSURANCE CHARGE AT ENTRY RATIO CORRESPONDING TO UPPER BOUND OF INTERVAL	10.61%
(13)	PERCENTAGE OF EXPECTED TREATY LOSSES ELIMINATED BY LOSS CORRIDOR PROVISION [(10)-(12)]*[100%-(8)]	19.51%
(14)	COMPOSITE COINSURANCE PERCENTAGE 100%-{[100%-(3)]*[100%-(13)]}	19.51%
(15)	EXPECTED LOSS COST FOR ENTIRE REINSURED PORTION OF LAYER, AFTER APPLICATION OF LOSS CORRIDOR PROVISION (EXPRESSED AS A PERCENTAGE OF SUBJECT PREMIUM) (2)*[100%-(14)]	2.8173%
(16)	INDICATED TREATY RATE AFTER THE APPLICATION OF LOSS CORRIDOR PROVISION AND ANY PROPORTIONAL COINSURANCE (EXPRESSED AS A PERCENTAGE OF SUBJECT PREMIUM) [(5)*(15)]	4.0248%

Treaty IV

Summary of Key Contract Provisions

Treaty Period: 1/1/90 - 12/31/90

Layer Reinsured: \$160,000 in excess of \$40,000 per occurrence

Estimated Treaty Subject Premium: \$12,000,000 for 1990,

distributed as follows:

Class 1 - \$9,000,000

Class 2 - 3,000,000

Expected Layer Loss Costs for the Entire Layer, Prior to All Forms of Coinsurance (Expressed as a Percent of Subject Premium):

Class 1 - 4.00%

Class 2 - 3.00%

Both Classes Combined - 3.75%

Indicated Flat Treaty Rate Prior to the Application of All Forms of Coinsurance (Expressed as a Percent of Subject Premium): 5.00%

Proportional Coinsurance: None

Non-Proportional Coinsurance: None

Retrospective Rating Plan:

Adjustment Period - 1/1/90 through 12/31/90 (1 year)

Adjustment Formula -

Adjusted Treaty Premium = 100/75 x (Incurred Losses and ALAE in Layer), subject to a maximum of 10.00% of subject premium and a minimum of 3.00% of subject premium.

DETERMINATION OF AGGREGATE LOSS DISTRIBUTION SPECIFICATION
PARAMETERS FOR ADJUSTABLE PREMIUM OR COMMISSION PLANS

	CLASS OF BUSINESS				
	CLASS 1	CLASS 2	CLASS 3	CLASS 4	ALL CLASSES
NAMES OF INDIVIDUAL CLASSES OF BUSINESS ==>	CLASS 1	CLASS 2			COMBINED
(1) ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR TREATY PERIOD	9,000,000	3,000,000			12,000,000
(2) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO APPLICATION OF ALL FORMS OF COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	4.0000%	3.0000%			3.7500%
(3) EXPECTED LOSSES FOR ENTIRE REINSURED LAYER FOR TREATY PERIOD [(1)*(2)]	360,000	90,000	0	0	450,000
(4) PARETO Q-VALUES FOR SEVERITY DISTRIBUTIONS	0.900	0.950			
(5) MEAN CLAIM SIZE IN LAYER (EXCESS OF RETENTION) (BASED ON THE SELECTED PARETO Q) (μ_s)	69,848	67,039			69,267
(6) STANDARD DEVIATION OF EXCESS CLAIM SIZES IN LAYER (BASED ON THE SELECTED PARETO Q) (σ_s)	60,908	60,084			60,749
(7) EXPECTED NUMBER OF CLAIMS IN LAYER PRIOR TO THE APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (μ_c) [(3)/(5)]	5.154	1.343			6.497
(8) EXCESS CLAIM COUNT VARIANCE TO MEAN RATIO PRIOR TO APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (VMR_c)	1.032	1.067			1.039
(9) STANDARD DEVIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER $[\sigma_s^2 \cdot \mu_c + (\mu_c \cdot VMR_c) \cdot \mu_s^2]^{1/2}$	212,298	106,228	0	0	237,391
(10) COEFFICIENT OF VARIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER [(9)/(3)]	0.590	1.180			0.528

(11) SELECTED PARAMETERS NEEDED TO SPECIFY AGGREGATE LOSS DISTRIBUTION FOR ALL CLASSES COMBINED

(A) EXPECTED NUMBER OF CLAIMS 6.500

(B) COEFFICENT OF VARIATION FOR AGGREGATE LOSS DISTRIBUTION 0.528

(C) TECHNIQUE USED TO OBTAIN AGGREGATE DISTRIBUTION (EG. COLLECTIVE RISK MODEL, LOGNORMAL ASSUMPTION) LOGNORMAL ASSUMPTION

ADJUSTABLE PREMIUMS (RETROSPECTIVE RATING)

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(1) ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR RETROSPECTIVE RATING PERIOD	12,000,000
(2) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO THE EFFECTS OF RETRO PLAN (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	3.7500%
(3) COINSURANCE PERCENTAGE (CEDANT'S PARTICIPATION IN LAYER LOSSES NOT CORRESPONDING TO AN EXPLICIT SHARE OF THE REINSURANCE PREMIUM, EXCLUDING THE EFFECTS OF NON-PROPORTIONAL LOSS SHARING PLANS.)	0.00%
(4) PERCENT REDUCTION IN LAYER LOSSES DUE TO NON-PROPORTIONAL LOSS SHARING PROVISIONS ONLY	0.00%
(5) EXPECTED LOSS COST FOR ENTIRE REINSURED PORTION OF LAYER PRIOR TO THE EFFECTS OF RETRO PLAN (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM) (2)*[100%-(3)]*[100%-(4)]	3.7500%
(6) MAXIMUM RATE (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	10.0000%
(7) MINIMUM RATE (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	3.0000%
(8) MULTIPLICATIVE LOSS LOAD (LOSS CONVERSION FACTOR) (ENTER UNITY IF NO LOSS CONVERSION FACTOR IN PLAN)	133.333% =100/75
(9) ADDITIVE LOSS LOAD (FLAT MARGIN)	0.0000%
(10) LOADED EXPECTED LAYER COST [(5)*(8)]+(9)	5.0000%
(11) ENTRY RATIO CORRESPONDING TO MAXIMUM RATE [(6)-(9)]/[(10)-(9)]	2.000
(12) INSURANCE CHARGE AT MAXIMUM (EXCESS LOSS PERCENTAGE CORRESPONDING TO MAXIMUM ENTRY RATIO)	2.60%
(13) ENTRY RATIO CORRESPONDING TO MINIMUM RATE [(7)-(9)]/[(10)-(9)]	0.600
(14) INSURANCE CHARGE AT MINIMUM (EXCESS LOSS PERCENTAGE CORRESPONDING TO MINIMUM ENTRY RATIO)	43.02%
(15) INSURANCE SAVINGS AT MINIMUM [(100%*(13))+(14)-100%]	3.02%
(16) NET INSURANCE CHARGE [(12)-(15)]	-0.42%
(17) ADJUSTED EXPECTED LAYER LOSS COST (EXPECTED VALUE OF LOSSES LIMITED BY RETRO PLAN MAXIMUM AND MINIMUM) (5)*[100%-(16)]	3.7656%
(18) (A) EQUIVALENT TREATY FLAT RATE (INDICATED TREATY RATE IF CONTRACT WERE FLAT RATED) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	5.0000%
(B) EXPECTED TREATY RATE AFTER RETRO ADJUSTMENTS (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM) [(8)*(17)]+(9)	5.0208%
(C) RETRO PLAN OFF-BALANCE FACTOR [(18A)/(18B)] (A FACTOR GREATER THAN 1.000 INDICATES A PLAN INADEQUACY: WHILE A LESS FACTOR THAN 1.000 INDICATES A PLAN REDUNDANCY)	0.996

PROFIT ANALYSIS - RETROSPECTIVE RATING

COMPLETE ITEMS (19)-(21) ONLY IF THE RETRO PLAN IS NOT IN BALANCE. (IE. THE RETRO PLAN OFF-BALANCE FACTOR DOES NOT EQUAL 1.000)

(19) (A) PROFIT PROVISION BUILT INTO LOSS CONVERSION FACTOR (ITEM 8) (EXPRESSED AS A PERCENT OF ADJUSTED PREMIUM, EXCLUSIVE OF FLAT MARGIN) 5.00%

(B) PROFIT PROVISION INCLUDED IN FLAT MARGIN (ITEM 9) (EXPRESSED AS AN ADDITIVE LOADING TO THE ADJUSTED TREATY RATE) --------------

(20) PROFIT PROVISIONS WHICH REFLECT EFFECT OF RETRO PLAN MAX AND MIN

ASSUME THAT THE UNDERWRITER DID NOT CONSIDER THE EFFECT OF THE RETRO PLAN'S MAXIMUM AND MINIMUM RATE PROVISIONS (IE. THE ASSOCIATED NET INSURANCE CHARGE) IN HIS SELECTION OF A LOSS CONVERSION FACTOR AND/OR FLAT MARGIN FOR THE PLAN. IF THE UNDERWRITER DOES NOT AMEND HIS LOSS CONVERSION FACTOR OR FLAT MARGIN TO ACCOUNT FOR THE NET INSURANCE CHARGE, HE IS IN EFFECT CHANGING HIS EXPECTED PROFIT PROVISIONS IN ITEM 19. THIS IS DUE TO THE FACT THAT A PORTION OF THESE PROFIT PROVISIONS WILL BE ABSORBED BY THE PLAN'S IMBALANCE. SINCE THIS ITEM IS NOT EXPLICITLY REFLECTED ELSEWHERE IN THE RETRO FORMULA THROUGH THE INCLUSION OF A NET INSURANCE CHARGE. THE AMENDED PROFIT PROVISIONS ARE AS FOLLOWS:

(A) ASSUMING THAT THE EFFECT OF THE PLAN'S IMBALANCE IS TO BE FULLY ABSORBED BY THE PROFIT PROVISION INCLUDED IN THE LOSS CONVERSION FACTOR, THE AMENDED PROFIT PROVISION IN THE LOSS CONVERSION FACTOR IS: (SEE DERIVATION OF FORMULA BELOW) 5.31%

$$(19A) - \left\{ \left[\frac{100\%}{(8)}\right] \times \left[\frac{(16)}{100\% - (16)}\right] \right\}$$

(B) ASSUMING THAT THE EFFECT OF THE PLAN'S IMBALANCE IS IS TO BE FULLY ABSORBED BY THE PROFIT PROVISION INCLUDED IN THE FLAT MARGIN, THE AMENDED PROFIT PROVISION IN THE FLAT MARGIN IS: N/A
(19B)-[(5)*(8)*(16)]

DERIVATION OF FORMULA GIVEN IN (19B):

RETROSPECTIVE PREMIUM UNDER THE ORIGINAL PLAN IS CALCULATED AS FOLLOWS: LCF x TREATY LOSSES + M,
WHERE: LCF = ORIGINAL LOSS CONVERSION FACTOR
M = FLAT MARGIN.

LCF CONTAINS AN EXPENSE PROVISION E AND A PROFIT PROVISION P, BUT NO LOADING FOR THE NET INSURANCE CHARGE. MATHEMATICALLY,

$$LCF = \frac{100\%}{100\% - E - P}$$

LET LCF$'$ BE THE LOSS CONVERSION FACTOR WHICH INCLUDES THE NET INSURANCE CHARGE NIC AND THE AMENDED PROFIT PROVISION P$'$. MATHEMATICALLY,

$$LCF' = \frac{100\%}{[100\% - NIC][100\% - E - P']}$$

IF LCF = LCF$'$, THEN P $\neq$ P$'$. SOLVING FOR P$'$ GIVES FORMULA IN (19B).

PROFIT ANALYSIS - RETROSPECTIVE RATING (CONTINUED)

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(21) GIVEN THAT THE UNDERWRITER DID NOT CONSIDER THE EFFECT OF THE RETRO PLAN'S MAXIMUM AND MINIMUM RATE PROVISIONS (IE. THE ASSOCIATED NET INSURANCE CHARGE) IN HIS SELECTION OF A LOSS CONVERSION FACTOR AND/OR FLAT MARGIN FOR THE PLAN. HERE ARE SAMPLE SETS OF RETRO PLAN PARAMETERS WHICH REFLECT THE DESIRED PROFIT PROVISIONS IN ITEM 19 AND PLACE THE PLAN IN BALANCE.

PLAN A:

ASSUMING THAT THE NET INSURANCE CHARGE IS TO BE FULLY REFLECTED IN THE LOSS CONVERSION FACTOR. ONE SET OF RETRO PLAN PARAMETERS IS:

LOSS CONVERSION FACTOR (LCF*)	132.781%	$(8)/[100\% - (16)]$
FLAT MARGIN: (M)	0.0000%	(ITEM 9)
MAXIMUM RATE:	9.9586%	$[(6)-M] \times [LCF^*/(8)] + M$
MINIMUM RATE:	2.9876%	$[(7)-M] \times [LCF^*/(8)] + M$

PLAN B:

ASSUMING THAT THE NET INSURANCE CHARGE IS TO BE FULLY REFLECTED IN THE FLAT MARGIN. ONE SET OF RETRO PLAN PARAMETERS IS:

LOSS CONVERSION FACTOR: (LCF)	133.333%	(ITEM 8)
FLAT MARGIN: (M*)	-0.0208%	$M + [(5) \times LCF \times (16)]$
MAXIMUM RATE:	9.9792%	$(6) - M + M^*$
MINIMUM RATE:	2.9792%	$(7) - M + M^*$

PLEASE NOTE THAT MANY OTHER BALANCED PLANS WHICH REFLECT THE DESIRED PROFIT PROVISIONS ARE POSSIBLE. MOST OF THESE REQUIRE THE USER TO COMPLETE THE 18 ITEMS SHOWN ON APPENDIX I, PAGE 3 ITERATIVELY USING DIFFERENT SETS OF PLAN PARAMETERS UNTIL AN ACCEPTABLE SET WHICH PRODUCES MINIMAL PLAN OFF-BALANCE IS FOUND.

Treaty V

Summary of Key Contract Provisions

Treaty Period: 1/1/90 - 12/31/90

Layer Reinsured: $700,000 in excess of $300,000 per occurrence

Estimated Treaty Subject Premium: $6,000,000 for 1990, distributed as follows:

Class 1 - $2,000,000

Class 2 - $2,000,000

Class 3 - $2,000,000

Expected Layer Loss Costs for the Entire Layer, Prior to All Forms of Coinsurance (Expressed as a Percent of Subject Premium):

Class 1 - 10.0%

Class 2 - 14.0%

Class 3 - 21.0%

All Classes Combined - 15.0%

Indicated Treaty Rate Prior to the Application of All Forms of Coinsurance, but Reflecting a Provision for the Underwriter's Expectation of Profit Commission to be Paid (Expressed as a Percent of Subject Premium): 25.0%

Proportional Coinsurance: 20%

Non-Proportional Coinsurance: None

Profit Commission Plan: Adjustment Period - 1/1/90 through 12/31/92 (3 years)

Reinsurer to pay cedant 25% of the amount by which treaty premiums during the Adjustment Period exceed incurred losses, ALAE, and a 20% provision for the reinsurer's overhead expense.

	CLASS OF BUSINESS				
	CLASS 1	CLASS 2	CLASS 3	CLASS 4	ALL CLASSES
NAMES OF INDIVIDUAL CLASSES OF BUSINESS ==>	CLASS 1	CLASS 2	CLASS 3		COMBINED
(1) ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR TREATY PERIOD	2,000,000	2,000,000	2,000,000		6,000,000
(2) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO APPLICATION OF ALL FORMS OF COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	10.0000%	14.0000%	21.0000%		15.0000%
(3) EXPECTED LOSSES FOR ENTIRE REINSURED LAYER FOR TREATY PERIOD [(1)*(2)]	200,000	280,000	420,000	0	900,000
(4) PARETO Q-VALUES FOR SEVERITY DISTRIBUTIONS	1.500	1.300	1.100		
(5) MEAN CLAIM SIZE IN LAYER (EXCESS OF RETENTION) (BASED ON THE SELECTED PARETO Q) (μ_s)	271,366	303,155	340,296		310,897
(6) STANDARD DEVIATION OF EXCESS CLAIM SIZES IN LAYER (BASED ON THE SELECTED PARETO Q) (σ_s)	246,592	257,600	266,584		260,265
(7) EXPECTED NUMBER OF CLAIMS IN LAYER PRIOR TO THE APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (μ_c) $[(3)/(5)]$	0.737	0.924	1.234		2.895
(8) EXCESS CLAIM COUNT VARIANCE TO MEAN RATIO PRIOR TO APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (VMR_c)	1.006	1.009	1.019		1.012
(9) STANDARD DEVIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER $[\sigma_s^2 \cdot \mu_c + (\mu_c \cdot VMR_c) \cdot \mu_s^2]^{1/2}$	315,301	383,323	483,065	0	692,606
(10) COEFFICIENT OF VARIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER $[(9)/(3)]$	1.577	1.369	1.150		0.770

DETERMINATION OF AGGREGATE LOSS DISTRIBUTION SPECIFICATION
PARAMETERS FOR ADJUSTABLE PREMIUM OR COMMISSION PLANS

	ADJUSTMENT PERIOD YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	TOTAL ADJUSTMENT PERIOD
DATES OF INDIVIDUAL CONTRACT YEARS IN ADJUSTMENT PERIOD ==>	1/90-12/90	1/91-12/91	1/92-12/92			
(1) ACTUAL OR ESTIMATED SUBJECT PREMIUMS FOR ALL CLASSES COMBINED	6,000,000	6,000,000	6,000,000			18,000,000
(2) (A) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO THE APPLICATION OF ALL FORMS OF COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	15.0000%	15.0000%	15.0000%			15.0000%
(B) PERCENT REDUCTION IN LAYER LOSSES DUE TO NON-PROPORTIONAL LOSS SHARING PROVISIONS. (IGNORE ALL PROPORTIONAL FORMS OF COINSURANCE.)	0.00%	0.00%	0.00%			0.00%
(C) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER AFTER THE APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS ONLY (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM) (2A)*[100%-(2B)]	15.0000%	15.0000%	15.0000%	0.0000%	0.0000%	15.0000%
(3) EXPECTED LOSSES FOR ENTIRE REINSURED LAYER AFTER THE EFFECT OF ALL NON-PROPORTIONAL COINSURANCE PROVISIONS (1)*(2C)	900,000	900,000	900,000	0	0	2,700,000
(4) PARETO Q-VALUE FOR SEVERITY DISTRIBUTION (ENTER A VALUE ONLY FOR MONOLINE CONTRACTS WHICH DO NOT HAVE ANY NON-PROPORTIONAL LOSS SHARING PROVISIONS)						
(5) MEAN CLAIM SIZE IN LAYER (EXCESS OF RETENTION) (μ_S) (COPIED FROM APPENDIX J, PAGE 2)	310,897					
(6) STANDARD DEVIATION OF EXCESS CLAIM SIZES IN LAYER (σ_S) (COPIED FROM APPENDIX J, PAGE 2)	260,265					
(7) EXPECTED NUMBER OF CLAIMS IN LAYER [(3)/(5)] (μ_c)	2.895	2.895	2.895	0.000	0.000	8.685
(8) EXCESS CLAIM COUNT VARIANCE TO MEAN RATIO AFTER THE APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (IF ANY): (VMR_c)	1.012	1.012	1.012			1.012
(9) STANDARD DEVIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER $[\sigma_S^2 \cdot \mu_c + (\mu_c \cdot VMR_c) \cdot \mu_S^2]^{1/2}$	692,606	692,606	692,606			1,199,629
(10) COEFFICIENT OF VARIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER [(9)/(3)]	0.770	0.770	0.770			0.444

(11) SELECTED PARAMETERS NEEDED TO SPECIFY AGGREGATE LOSS DISTRIBUTION

(A) EXPECTED NUMBER OF CLAIMS 8.700

(B) COEFFICENT OF VARIATION FOR AGGREGATE LOSS DISTRIBUTION 0.444

(C) TECHNIQUE USED TO OBTAIN AGGREGATE DISTRIBUTION (EG. COLLECTIVE RISK MODEL, LOGNORMAL ASSUMPTION) LOGNORMAL ASSUMPTION

PROFIT COMMISSIONS

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(1)	ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR COMMISSION ADJUSTMENT PERIOD	18,000,000
(2)	EXPECTED LAYER LOSS COST FOR ENTIRE LAYER (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	15.0000%
(3)	COINSURANCE PERCENTAGE (CEDANT'S PARTICIPATION IN LAYER LOSSES NOT CORRESPONDING TO AN EXPLICIT SHARE OF THE REINSURANCE PREMIUM, EXCLUDING THE EFFECTS OF NON-PROPORTIONAL LOSS SHARING PLANS.)	20.00%
(4)	PERCENT REDUCTION IN LAYER LOSSES DUE TO NON-PROPORTIONAL LOSS SHARING PROVISIONS ONLY	0.00%
(5)	TREATY RATE BASED ON UNDERWRITER'S ESTIMATE OF IMPACT OF PROFIT COMMISSION PROVISION (I.E., THE RATE WHICH INCLUDES A PROVISION FOR THE COMMISSION WHICH RESULTS WHEN THE ELR (ITEM 6) IS PLUGGED INTO THE FORMULA BELOW.) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	25.0000%
(6)	EXPECTED TREATY LOSS & ALAE RATIO (ELR) {(2)*[100%-(3)]*[100%-(4)]}/(5)	48.00%

PROFIT COMMISSION FORMULA IS IN THE FORM:
(P) = (A) * [100% - TREATY LOSS & ALAE RATIO - (B)],
SUBJECT TO A MAXIMUM OF (C)
WHERE: (P) = PROFIT COMMISSION RATIO:
(A) = PROPORTION OF PROFITS TO BE PAID TO CEDANT:
(B) = REINSURER'S OVERHEAD PROVISION (EXP):
(C) = MAXIMUM PROFIT COMMISSION (IF APPLICABLE)

(7)	PROPORTION OF PROFITS TO BE PAID TO CEDANT [ITEM (A)]	25.00%
(8)	REINSURER'S OVERHEAD PROVISION [ITEM (B)] (EXPRESSED AS A PERCENTAGE OF TREATY PREMIUM)	20.00%
(9)	MAXIMUM PROFIT COMMISSION, IF DIFFERENT FROM THAT CORRESPONDING TO A ZERO LOSS & ALAE RATIO [ITEM (C)] (EXPRESSED AS A PERCENTAGE OF TREATY PREMIUM)	------------
(10)	UNDERWRITER'S ESTIMATE OF EXPECTED PROFIT COMMISSION RATIO (EXPRESSED AS A PERCENTAGE OF TREATY PREMIUM) (7)*[100% - (6) - (8)], OR (A)*[100% - ELR - EXP]	8.00%

PROFIT COMMISSIONS (CONTINUED)

(11) BREAKEVEN LOSS & ALAE RATIO FOR PROFIT COMMISSION PURPOSES [100%-(8)]	80.00%
(12) ENTRY RATIO CORRESPONDING TO BREAKEVEN POINT [(11)/(6)]	1.667
(13) INSURANCE CHARGE AT BREAKEVEN POINT (EXCESS LOSS PERCENTAGE CORRESPONDING TO BREAKEVEN ENTRY RATIO)	3.09%
(14) LOSS & ALAE RATIO CORRESPONDING TO THE MAXIMUM PROFIT COMMISSION AMOUNT [100%-(8)-[(9)/(7)]]	0.00%
(15) ENTRY RATIO CORRESPONDING TO MAXIMUM PROFIT LOSS & ALAE RATIO [(14)/(6)]	0.000
(16) INSURANCE CHARGE AT MAXIMUM PROFIT LOSS & ALAE RATIO (EXCESS LOSS PERCENTAGE CORRESPONDING TO MAXIMUM PROFIT LOSS & ALAE RATIO)	--------------
(17) INSURANCE SAVINGS AT MAXIMUM PROFIT LOSS & ALAE RATIO [(100%*(15)]+(16)-100%]	0.00%
(18) NET INSURANCE CHARGE (NIC) [(13)-(17)]	3.09%
(19) ACTUARIAL ESTIMATE OF EXPECTED PROFIT COMMISSION RATIO (EXPRESSED AS A PERCENTAGE OF TREATY PREMIUM) (7)*[100%-[(6)*[100%-(18)]]-(8)], OR (A)*[100% - ELR*[100%-NIC] - EXP]	8.37%
(20) AMOUNT BY WHICH THE ACTUARIAL ESTIMATE OF THE EXPECTED PROFIT COMMISSION RATIO EXCEEDS THE UNDERWRITING ESTIMATE [(19)-(10)]	0.37%

PROFIT ANALYSIS - PROFIT COMMISSION PLANS

COMPLETE ITEMS (21)-(23) ONLY IF THE ACTUARIAL AND UNDERWRITING ESTIMATES OF THE EXPECTED PROFIT COMMISSION RATIO DIFFER (AS INDICATED BY A NON-ZERO RESULT FOR ITEM 20).

(21) REINSURER'S PROFIT PROVISION BUILT INTO TREATY RATE (EXPRESSED AS A PERCENT OF TREATY PREMIUM)------------ 5.00%

(22) AMENDED REINSURER'S PROFIT PROVISION IF NO CHANGE IN TREATY RATE OR PROFIT COMMISSION FORMULA

ASSUME THAT THE UNDERWRITER DOES NOT AMEND THE TREATY RATE AND/OR THE PROFIT COMMISSION FORMULA TO CORRECT FOR THE DIFFERENCE BETWEEN THE ACTUARIAL AND UNDERWRITER'S EXPECTED PROFIT COMMISSION RATIOS IN ITEM 20. BY NOT MAKING THESE CHANGES, THE UNDERWRITER IS IN EFFECT CHANGING HIS ANTICIPATED PROFIT MARGIN GIVEN IN ITEM 21, SINCE A PORTION OF THE PROFIT MARGIN WILL BY ABSORBED 3Y THE AMOUNT BY WHICH THE ACTUARIAL ESTIMATE OF THE PROFIT COMMISSION PLAN'S RESULTS EXCEEDS THE UNDERWRITER'S ESTIMATE. UNDER THIS SCENARIO, THE AMENDED REINSURER'S PROFIT PROVISION IS: [(21)-(20)] 4.63%

(23) ALTERNATE PROFIT COMMISSION FORMULA WHICH REFLECTS THE REINSURER'S DESIRED PROFIT PROVISION

ASSUME THAT THE UNDERWRITER DOES NOT WANT TO CHANGE THE TREATY RATE SPECIFIED IN ITEM 5, BUT INSTEAD WANTS TO CHANGE THE PROFIT COMMISSION FORMULA SO THAT THE EXPECTED PROFIT COMMISSION RATIO FROM AN ACTUARIAL POINT OF VIEW WILL EQUAL THE UNDERWRITER'S EXPECTED RATIO UNDER THE ORIGINAL PLAN (ITEM 10). ASSUMING THAT THE REINSURER WILL ACHIEVE THE DESIRED PROFIT PROVISION (ITEM 21) IF THE UNDERWRITER'S EXPECTED PROFIT COMMISSION AMOUNT UNDER THE ORIGINAL PLAN IS PAID, THEN THE NEW PLAN SHOULD PRODUCE NO CHANGES IN THE REINSURER'S EXPECTED PROFIT MARGIN. IF THE ORIGINAL PLAN HAS NO SPECIFIED MAXIMUM PROFIT COMMISSION RATIO (OTHER THAN THAT YIELDED WHEN A ZERO LOSS & ALAE RATIO IS PLUGGED INTO THE FORMULA), ONE WAY TO ACHIEVE THE DESIRED RESULTS IS TO CHANGE THE PROFIT SHARING PROPORTION (ITEM 7) IN THE PROFIT COMMISSION FORMULA.

THE NEW PLAN IS AS FOLLOWS:
(P) = (A) * [100% - TREATY LOSS & ALAE RATIO - (B)],
WHERE: (A) = 23.89% (A) = (7) x [(10)/(19)]
(B) = 20.00%

MANY OTHER PROFIT COMMISSION PLANS EXIST WHICH YIELD THE DESIRED REINSURER'S PROFIT PROVISION FROM AN ACTUARIAL POINT OF VIEW. MOST OF THESE REQUIRE THE USER TO COMPLETE THE 20 ITEMS ON PAGES 4-5 OF APPENDIX J ITERATIVELY USING DIFFERENT SETS OF PLAN PARAMETERS AND/OR TREATY RATES UNTIL AN ACCEPTABLE COMBINATION WHICH PRODUCES THE DESIRED PROFIT PROVISION TO THE REINSURER IS FOUND.

Treaty VI

Summary of Key Contract Provisions

Treaty Period: 1/1/90 - 12/31/90

Layer Reinsured: $900,000 in excess of $100,000 per occurrence

Estimated Treaty Subject Premium: $25,000,000

Expected Layer Loss Cost for the Entire Layer, Prior to All Forms of Coinsurance (Expressed as a Percent of Subject Premium): 10.0%

Indicated Treaty Rate Prior to the Application of All Forms of Coinsurance, but Reflecting a Provision for the Underwriter's Expectation of Sliding Scale Commission to be Paid (Expressed as a Percent of Subject Premium): 20.0%

Proportional Coinsurance: None

Non-Proportional Coinsurance: None

Sliding Scale Commission Plan:

Adjustment Period - 1/1/90 through 12/31/90 (1 year)

Plan - Minimum Commission of 20% at a 65% loss ratio.

Commission increases by 0.5% for each 1% decline in loss ratio for loss ratios between 55% and 65%.

Commission increases by 0.75% for each 1% decline in loss ratio for loss ratios between 35% and 55%.

Maximum Commission of 40% at a 35% loss ratio.

	ADJUSTMENT PERIOD YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	TOTAL ADJUSTMENT PERIOD
DATES OF INDIVIDUAL CONTRACT YEARS IN ADJUSTMENT PERIOD ==>	1/90-12/90					
(1) ACTUAL OR ESTIMATED SUBJECT PREMIUMS FOR ALL CLASSES COMBINED	25,000,000					25,000,000
(2) (A) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER PRIOR TO THE APPLICATION OF ALL FORMS OF COINSURANCE (LAYER BURNING COST) (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)	10.0000%					10.0000%
(B) PERCENT REDUCTION IN LAYER LOSSES DUE TO NON-PROPORTIONAL LOSS SHARING PROVISIONS. (IGNORE ALL PROPORTIONAL FORMS OF COINSURANCE.)	0.00%					0.00%
(C) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER AFTER THE APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS ONLY (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM) (2A)*[100%-(2B)]	10.0000%	0.0000%	0.0000%	0.0000%	0.0000%	10.0000%
(3) EXPECTED LOSSES FOR ENTIRE REINSURED LAYER AFTER THE EFFECT OF ALL NON-PROPORTIONAL COINSURANCE PROVISIONS (1)*(2C)	2,500,000	0	0	0	0	2,500,000
(4) PARETO Q-VALUE FOR SEVERITY DISTRIBUTION	1.000					
(5) MEAN CLAIM SIZE IN LAYER (EXCESS OF RETENTION) (μ_s) (BASED ON THE SELECTED Q)	230,259					
(6) STANDARD DEVIATION OF EXCESS CLAIM SIZES IN LAYER (σ_s) (BASED ON THE SELECTED Q)	284,481					
(7) EXPECTED NUMBER OF CLAIMS IN LAYER [(3)/(5)] (μ_c)	10.857	0.000	0.000	0.000	0.000	10.857
(8) EXCESS CLAIM COUNT VARIANCE TO MEAN RATIO AFTER THE APPLICATION OF NON-PROPORTIONAL LOSS SHARING PROVISIONS (IF ANY): (VMR_c)	1.029					1.029
(9) STANDARD DEVIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER $[\sigma_s^2 \cdot \mu_c + (\mu_c \cdot VMR_c) \cdot \mu_s^2]^{1/2}$	1,212,856	0	0	0	0	1,212,856
(10) COEFFICIENT OF VARIATION OF DISTRIBUTION OF AGGREGATE LOSSES IN LAYER [(9)/(3)]	0.485					0.485

(11) SELECTED PARAMETERS NEEDED TO SPECIFY AGGREGATE LOSS DISTRIBUTION

(A) EXPECTED NUMBER OF CLAIMS 10.850 (B) COEFFICENT OF VARIATION FOR AGGREGATE LOSS DISTRIBUTION 0.485

(C) TECHNIQUE USED TO OBTAIN AGGREGATE DISTRIBUTION (EG. COLLECTIVE RISK MODEL, LOGNORMAL ASSUMPTION) LOGNORMAL ASSUMPTION

SLIDING SCALE COMMISSIONS

(1) ACTUAL OR ESTIMATED SUBJECT PREMIUM FOR COMMISSION ADJUSTMENT PERIOD — 25,000,000

(2) EXPECTED LAYER LOSS COST FOR ENTIRE LAYER (EXPRESSED AS A PERCENT OF SUBJECT PREMIUM) — 10.0000%

(3) COINSURANCE PERCENTAGE (CEDANT'S PARTICIPATION IN LAYER LOSSES NOT CORRESPONDING TO AN EXPLICIT SHARE OF THE REINSURANCE PREMIUM, EXCLUDING THE EFFECTS OF NON-PROPORTIONAL LOSS SHARING PLANS.) — 0.00%

(4) PERCENT REDUCTION IN LAYER LOSSES DUE TO NON-PROPORTIONAL LOSS SHARING PROVISIONS ONLY — 0.00%

(5) TREATY RATE BASED ON UNDERWRITER'S ESTIMATE OF IMPACT OF SLIDING SCALE COMMISSION PROVISION — 20.0000%
(I.E., THE RATE WHICH INCLUDES A PROVISION FOR THE COMMISSION THAT CORRESPONDS TO THE ELR IN ITEM 6)
(EXPRESSED AS A PERCENT OF SUBJECT PREMIUM)

(6) EXPECTED TREATY LOSS & ALAE RATIO (ELR) — 50.00%
{(2)*[100%-(3)]*[100%-(4)]}/(5)

IN THE REMAINDER OF THIS SPREADSHEET, ALL COMMISSION AND LOSS & ALAE RATIOS ARE EXPRESSED AS PERCENTAGES OF TREATY PREMIUM.

(7) MINIMUM COMMISSION 20.00% CORRESPONDING LOSS & ALAE RATIO 65.00%

(8) THE DETAILS OF THE SLIDING SCALE COMMISSION PLAN ARE SUMMARIZED IN COLUMNS (A) THROUGH (E). VALUES USED IN THE CALCULATION OF THE EXPECTED SLIDING SCALE COMMISSION ARE GIVEN IN COLUMNS (F) THROUGH (I).

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)
LOSS & ALAE RATIO INTERVAL		PERCENTAGE INCREASE IN COMMISSION RATIO PER 1% DECREASE IN LOSS & ALAE RATIO	CORRESPONDING COMMISSION RATIO INTERVAL		ENTRY RATIO CORRESPONDING TO LOWER BOUND LOSS RATIO IN COLUMN (A)	INSURANCE CHARGE CORRESPONDING TO LOWER BOUND ENTRY RATIO IN COLUMN (F)	EXPECTED LOSS RATIO POINTS IN INTERVAL	EXPECTED REDUCTIONS FROM MAXIMUM COMMISSION RATE (C)*(H)
LOWER BOUND	UPPER BOUND		LOWER BOUND	UPPER BOUND				
65.00% AND ABOVE		0.00%	20.00%	20.00%	1.300	9.12%	4.56%	0.00%
55.00%	65.00%	0.50%	25.00%	20.00%	1.100	14.47%	2.68%	1.34%
35.00%	55.00%	0.75%	40.00%	25.00%	0.700	34.80%	10.16%	7.62%
0.00%	35.00%	0.00%	40.00%	40.00%	0.000	100.00%	32.60%	0.00%
						TOTAL	50.00%	8.96%

(9) EXPECTED CEDING COMMISSION RATIO FROM AN UNDERWRITER'S POINT OF VIEW [COMMISSION RATIO CORRESPONDING TO THE TREATY ELR (ITEM 6), GIVEN THE PLAN ABOVE.] — 28.75%

(10) EXPECTED COMMISSION RATIO FROM AN ACTUARIAL POINT OF VIEW [MAXIMUM COMMISSION RATIO - (TOTAL 9I)] — 31.04%

(11) AMOUNT BY WHICH THE ACTUARIAL ESTIMATE OF THE EXPECTED COMMISSION RATIO EXCEEDS THE UNDERWRITING ESTIMATE [(10)-(9)] — 2.29%

PROFIT ANALYSIS - SLIDING SCALE COMMISSION PLANS

COMPLETE ITEMS (12)-(14) ONLY IF THE ACTUARIAL AND UNDERWRITING ESTIMATES OF THE EXPECTE COMMISSION RATIO DIFFER (AS INDICATED BY A NON-ZERO RESULT FOR ITEM 11).

(12) REINSURER'S PROFIT PROVISION BUILT INTO THE TREATY RATE (EXPRESSED AS A PERCENTAGE OF TREATY PREMIUM) 5.00%

(13) AMENDED REINSURER'S PROFIT PROVISION IF NO CHANGE IN TREATY RATE OR SLIDING SCALE COMMISSION PLAN

ASSUME THAT THE UNDERWRITER DOES NOT AMEND THE TREATY RATE AND/OR THE SLIDING SCALE COMMISSION PLAN TO CORRECT FOR THE DIFFERENCE BETWEEN THE ACTUARIAL AND UNDERWRITER'S EXPECTED COMMISSION AMOUNTS IN ITEM 11. BY NOT MAKING THESE CHANGES, THE UNDERWRITER IS IN EFFECT CHANGING HIS ANTICIPATED PROFIT MARGIN GIVEN IN ITEM 12, SINCE A PORTION OF THE PROFIT MARGIN WILL BE ABSORBED BY THE AMOUNT BY WHICH THE ACTUARIAL ESTIMATE OF THE SLIDING SCALE COMMISSION PLAN'S RESULTS EXCEEDS THE UNDERWRITER'S ESTIMATE. UNDER THIS SCENARIO, THE AMENDED PROFIT PROVISION IS: 2.71%
[(12)-(11)]

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(14) ALTERNATE SLIDING SCALE COMMISSION FORMULA WHICH REFLECTS REINSURER'S DESIRED PROFIT PROVISION

ASSUME THAT THE UNDERWRITER DOES NOT WANT TO CHANGE THE TREATY RATE SPECIFIED IN ITEM 5, BUT INSTEAD WANTS TO CHANGE THE SLIDING SCALE COMMISSION PLAN SO THAT THE EXPECTED COMMISSION RATIO FROM AN ACTUARIAL POINT OF VIEW WILL EQUAL THE UNDERWRITER'S EXPECTED COMMISSION RATIO UNDER THE ORIGINAL PLAN (ITEM 9). ASSUMING THAT THE REINSURER WILL ACHIEVE THE DESIRED PROFIT PROVISION (ITEM 12) IF THE UNDERWRITER'S EXPECTED COMMISSION UNDER THE ORIGINAL PLAN IS PAID, THEN THE NEW PLAN SHOULD PRODUCE NO CHANGES IN THE REINSURER'S EXPECTED PROFIT MARGIN. A SIMPLE WAY TO ACHIEVE THE DESIRED RESULTS IS TO SUBTRACT THE DIFFERENCE BETWEEN THE ACTUARIAL AND UNDERWRITER'S ESTIMATES OF THE EXPECTED COMMISSION RATIO (A CONSTANT AMOUNT) FROM THE COMMISSION TO BE PAID AT EACH POSSIBLE LOSS & ALAE RATIO UNDER THE ORIGINAL PLAN.

HENCE, THE NEW SLIDING SCALE COMMISSION PLAN IS AS FOLLOWS:

LOSS & ALAE RATIO INTERVAL		PERCENTAGE INCREASE IN COMMISSION RATIO PER 1% DECREASE IN LOSS & ALAE RATIO	CORRESPONDING COMMISSION RATIO INTERVAL	
LOWER BOUND	UPPER BOUND		LOWER BOUND	UPPER BOUND
65.00% AND ABOVE		0.00%	17.71%	17.71%
55.00%	65.00%	0.50%	22.71%	17.71%
35.00%	55.00%	0.75%	37.71%	22.71%
0.00%	35.00%	0.00%	37.71%	37.71%

MANY OTHER SLIDING SCALE COMMISSION PLANS EXIST WHICH YIELD THE DESIRED REINSURER'S PROFIT PROVISION FROM AN ACTUARIAL POINT OF VIEW. MOST OF THESE REQUIRE THE USER TO COMPLETE THE 11 ITEMS ON PAGE 3 OF APPENDIX K ITERATIVELY USING DIFFERENT SETS OF PLAN PARAMETERS (EG, MINIMUM COMMISSION/CORRESPONDING LOSS & ALAE RATIO, LOSS & ALAE RATIO INTERVALS/CORRESPONDING COMMISSION SLIDES FOR EACH INTERVAL) AND/OR TREATY RATES UNTIL AN ACCEPTABLE COMBINATION WHICH PRODUCES THE DESIRED PROFIT PROVISION TO THE REINSURER IS FOUND.

DERIVATION OF A FORMULA FOR CALCULATING THE EXPECTED CEDING COMMISSION UNDER A PIECEWISE LINEAR SLIDING SCALE COMMISSION PLAN

Let $L_1, L_2, L_3, \ldots, L_n$ be a series of loss ratios such that $L_1 > L_2 > \ldots > L_n = 0$. This sequence divides the range of possible loss ratios into n consecutive intervals, starting with the first interval $[L_1, \infty)$ followed by the intervals $[L_i, L_{i-1}]$ where $i = 2, 3, \ldots, n$.

Using the above notation, the typical linear sliding scale commission plan can be expressed as follows:

The minimum commission ratio (to treaty premium) C_{min} is paid if the treaty loss ratio falls in the first interval.

The ceding commission ratio increases by b_2 points for each 1% decline in the loss ratio in the second interval.

The ceding commission ratio increases by b_3 points for each 1% decline in the loss ratio in the third interval.

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The ceding commission ratio increases by b_{n-1} points for each 1% decline in the loss ratio in the (n-1)-st interval.

The maximum ceding commission C_{max} is paid if the treaty loss ratio falls in the n-th interval.

Let $f(L_i)$ equal the ceding commission ratio corresponding to an L_i loss ratio, $i = 1, 2, \ldots, n$.

If L is a random variable representing the observed treaty loss ratio, then the sliding scale commission plan described above may be expressed as a continuous piecewise linear function of L in the following form:

(1)

$$C = f(L) = \begin{cases} f(L_1) = C_{min} & \text{if } L \geqslant L_1 \\ f(L_2)+b_2(L_2-L) & \text{if } L_2 \leqslant L < L_1 \\ f(L_3)+b_3(L_3-L) & \text{if } L_3 \leqslant L < L_2 \\ \vdots & \\ f(L_n) = C_{max} & \text{if } 0=L_n \leqslant L < L_{n-1} \end{cases}$$

For convenience, let $a_i = f(L_i)+b_i L_i$ ($i=1,2,\ldots,n$). Also, define both b_1 and b_n to be 0, since there is no commission slide on either the first or last loss ratio intervals. Using this notation, we can rewrite (1) as follows:

(2)

$$f(L) = \begin{cases} f(L_1) & \text{if } L \geqslant L_1 \\ a_2-b_2L & \text{if } L_2 \leqslant L < L_1 \\ a_3-b_3L & \text{if } L_3 \leqslant L < L_2 \\ \vdots & \\ a_n-b_nL & \text{if } 0=L_n \leqslant L < L_{n-1} \end{cases}$$

Let $p(L)$ be the probability density function of L.

Then, the expected ceding commission ratio $E(C)$ may be expressed as follows:

$$(3)\quad E(C)=\int_L f(L)p(L)dL=\int_{L_1}^{\infty} f(L_1)p(L)dL+\sum_{i=2}^{n}\left[\int_{L_{i-1}}^{L_i}(a_i-b_iL)\; p(L)dL\right]$$

$$= \quad f(L_1)\int_{L_1}^{\infty} p(L)dL + \sum_{i=2}^{n}\, [a_i \int_{L_{i-1}}^{L_i} p(L)dL]$$

$$-\sum_{i=2}^{n}\, [b_i \int_{L_{i-1}}^{L_i} Lp(L)dL]$$

Let $M = E(L)$ = Expected treaty loss ratio,

$P(L)$ be the cumulative distribution function of L,

$P_1(L)$ be the first moment distribution function of L.

By definition, $P_1(L_i)=\frac{1}{M}\int_{0}^{L_i} Lp(L)dL$.

Thus, we may rewrite (3) as

$$(4)\quad E(C)=f(L_1)[1-P(L_1)] + \sum_{i=2}^{n} a_i\; [P(L_{i-1}) - P(L_i)]$$

$$-M\sum_{i=2}^{n} b_i\; [P_1(L_{i-1})-P_1(L_i)].$$

Now define $P_2(L)$ to be the excess pure premium ratio at loss ratio L.

$P_2(L)$ may be expressed in terms of $P(L)$ and $P_1(L)$ as follows:

(5) $P_2(L)=[1-P_1(L)]-\frac{L}{M}[1-P(L)]$.

Solving (5) for $P_1(L)$ yields

(6) $P_1(L) = [1-P_2(L)]-\frac{L}{M}[1-P(L)]$.

Substituting (6) into (4), we get

$$(7)\quad E(C) = f(L_1)[1-P(L_1)] + \sum_{i=2}^{n} a_i \left[P(L_{i-1})-P(L_i)\right]$$

$$- M\sum_{i=2}^{n} b_i\left[\left(\left\{1-P_2(L_{i-1})\right\} - \frac{L_{i-1}}{M}\left\{1-P(L_{i-1})\right\}\right)\right.$$

$$\left. - \left(\left\{1-P_2(L_i)\right\} - \frac{L_i}{M}\left\{1-P(L_i)\right\}\right)\right]$$

$$= f(L_1)[1-P(L_1)] + \sum_{i=2}^{n} a_i\, [P(L_{i-1}) - P(L_i)]$$

$$-\sum_{i=2}^{n} [Mb_i P_2(L_i) - Mb_i P_2(L_{i-1}) + b_i L_i - b_i L_{i-1} + b_i L_{i-1} P(L_{i-1})$$

$$- b_i L_i P(L_i)]$$

$$= f(L_1)\,[1-P(L_1)] + \sum_{i=2}^{n} a_i\, [P(L_{i-1}) - P(L_i)]$$

$$-\sum_{i=2}^{n} b_i\, [L_{i-1}P(L_{i-1}) - L_i P(L_i)] + \sum_{i=2}^{n} b_i (L_{i-1} - L_i)$$

$$- M\sum_{i=2}^{n} b_i\, [P_2(L_i) - P_2(L_{i-1})].$$

Consider the expression

$$(8)\quad f(L_1)\,[1-P(L_1)] + \sum_{i=2}^{n} a_i\, [P(L_{i-1}) - P(L_i)]$$

$$-\sum_{i=2}^{n} b_i\, [L_{i-1}P(L_{i-1}) - L_i P(L_i)],$$

which is the first 3 terms of the righthand side of (7) above.

Letting $a_i = f(L_i) + b_i L_i$ ($i=2,3,\ldots,n$), (8) becomes

$$(9)\ f(L_1)[1-P(L_1)] + \sum_{i=2}^{n} [f(L_i) + b_i L_i]\,[P(L_{i-1}) - P(L_i)]$$

$$- \sum_{i=2}^{n} b_i\,[L_{i-1}P(L_{i-1}) - L_i P(L_i)]$$

$$= f(L_1) - f(L_1)P(L_1) + \sum_{i=2}^{n} f(L_i)P(L_{i-1}) - \sum_{i=2}^{n} f(L_i)P(L_i)$$

$$+ \sum_{i=2}^{n} b_i L_i P(L_{i-1}) - \sum_{i=2}^{n} b_i L_i P(L_i) - \sum_{i=2}^{n} b_i L_{i-1} P(L_{i-1}) + \sum_{i=2}^{n} b_i L_i P(L_i)$$

$$= f(L_1) - \sum_{i=1}^{n} f(L_i)P(L_i) + \sum_{i=2}^{n} f(L_i)P(L_{i-1})$$

$$+ \sum_{i=2}^{n} b_i L_i P(L_{i-1}) - \sum_{i=2}^{n} b_i L_{i-1} P(L_{i-1})$$

$$= f(L_1) - \sum_{i=1}^{n} f(L_i)P(L_i) + \sum_{i=2}^{n} [f(L_i) + b_i (L_i - L_{i-1})]P(L_{i-1}).$$

From the definition of f given in (1), $f(L_{i-1}) = f(L_i) + b_i (L_i - L_{i-1})$ for $i=2,3,\ldots,n$. Thus, (9) becomes

$$(10)\ f(L_1) - \sum_{i=1}^{n} f(L_i)P(L_i) + \sum_{i=2}^{n} f(L_{i-1})P(L_{i-1})$$

$$= f(L_1) - \sum_{i=1}^{n} f(L_i)P(L_i) + \sum_{i=1}^{n-1} f(L_i)P(L_i)$$

$$= f(L_1) - f(L_n)P(L_n) = f(L_1),\ \text{since } P(L_n) = P(0) = 0.$$

Therefore, we may substitute $f(L_1)$ for the first 3 terms in the rightmost expression of equation (7) to obtain

$$(11)\ E(C) = f(L_1) + \sum_{i=2}^{n} b_i\,(L_{i-1} - L_i) - M \sum_{i=2}^{n} b_i\,[P_2(L_i) - P_2(L_{i-1})].$$

From the definition of f given in (1), it is evident that $C_{max} = f(L_n) = f(L_1) + \sum_{i=2}^{n} b_i\,(L_{i-1} - L_i)$. Also, since $b_1 = 0$, the rightmost summation in equation (11) can be set to begin at $i=1$. (We are implicitly defining $P_2(L_0) = P_2(\infty) = 1$.) Hence, we may rewrite (11) as

$$(12)\ E(C) = C_{max} - M \sum_{i=1}^{n} b_i\,[P_2(L_i) - P_2(L_{i-1})].$$

Equation (12) provides a convenient formula for calculating the expected ceding commission ratio under a piecewise linear sliding scale plan, since one only needs a description of the plan, the expected treaty loss ratio M, and the appropriate table of excess pure premium ratios (ie, the $P_2(L_i)$'s) in order to use it.

Based on the definitions given for M and P_2 above, it follows that the expression $M[P_2(L_i) - P_2(L_{i-1})]$ represents the expected number of loss ratio points falling in the interval from L_i to L_{i-1}. Hence equation (12) may be expressed verbally as follows:

$$(13)\quad E(C) = C_{max} - \sum_{i=1}^{n} b_i \left\{ \begin{array}{l} \text{Expected loss ratio points in the} \\ \text{interval from } L_i \text{ to } L_{i-1} \end{array} \right\}$$

where: $E(C)$ is the expected commission ratio,

C_{max} is the maximum commission ratio,

b_i is the commission slide on the i-th loss ratio interval.

Since the product of b_i and the expected number of loss ratio points in the i-th interval represents the expected number of commission points lost in that interval, one sees from (13) that the expected ceding commission ratio equals the maximum commission ratio minus the expected points of commission lost over the entire range of possible loss ratios. This provides an intuitive justification of the formula given in (12) above.