AN APPLICATION OF RISK THEORY TO CONTROL SOLVENCY AND FINANCIAL STRENGTH

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BIOGRAPHY:

Dr. Bonsdorff is a mathematician in the insurance department of the Ministry of Social Affairs and Health of Finland (the insurance supervising authority in Finland and also responsible for private and social insurance legislation). Prior to in joining the ministry he was employed by Pohjola Insurance Company in 1981-1991 as a mathematician, the chief actuary of the group, a corporate planning manager and a research manager. He received the B.A. degree in 1966, M.A. degree in 1966, Lic. Phil. degree in 1968 in mathematics and Ph.D. degree in applied mathematics in 1980 at the University of Helsinki, and SHV-degree (insurance mathematician approved by the ministry of Social Affairs and Health) in 1984. He has written research papers on stochastic processes and on actuarial mathematics, and is one of the coauthors of the book by Pentikäinen et al. (1989). Since 1987 Dr. Bonsdorff has been a member of the board of the Finnish Actuarial Society and since 1991 an editor of the Scandinavian Actuarial Journal. Prior to his actuarial career he was employed by Paragon Ltd. (importer of Burroughs computers in Finland) as an OR-consultant and in the Department of Mathematics at the University of Helsinki as an assistant and acting associate professor of applied mathematics.

ABSTRACT:

Following mainly the ideas of the book by Pentikäinen et al. (1989) and taking into consideration some recent developments in the topic, the paper deals with solvency and financial strength, both from the point of view of management and regulatory solvency control, within a framework of a comprehensive stochastic model. The model aims at describing the insurance company and its operational environment as realistically as possible. The model has been implemented by means of simulation. As the model is introduced, the main factors considered are listed as follows: premium setting (as a tool to control the business), claims, outstanding claims (including run-off problems), investment returns (including change in value and investment income), business cycles, effects of inflation on all the factors mentioned above, dynamic managerial control. The operation of the model is illustrated by means of simulation examples and its implications for the control of solvency and financial strength are outlined.

AN APPLICATION OF RISK THEORY TO CONTROLLING SOLVENCY AND FINANCIAL STRENGTH

1. INTRODUCTION

The purpose of this paper is, following the ideas of Pentikäinen et al. (1989), to consider the solvency and financial strength of an insurer, from the point of views of both management and regulatory solvency control, under a framework of a stochastic model. Pentikäinen et al.(1989) presents a comprehensive stochastic model for insurance business. The model aims at describing the insurance company, mainly with respect to general insurance, and its operation environment as realistically as possible. The model has been implemented by means of simulation. The reader is also referred to the parallel British work, see Daykin et al.(1984), Daykin et al.(1987) and Daykin and Hey (1990).

The solvency situation and the financial strength of an insurer are affected by nearly all activities and decision-making processes of the insurer such as premium rating, evaluation of the reserve of outstanding claims and investment strategy. It is also affected by external factors such as changes in the underwriting and investment markets, inflation and international economic relations.

In Chapter 2 we present a general framework for the analysis. In Chapter 3 we consider some of the most important factors in greater detail. In Chapter 4 we present how these factors are integrated in the model of Pentikäinen et al. and give simulation examples. In the following, "model" refers to the mentioned model of Pentikäinen et al. The implications of the model to solvency control are considered in Chapter 5 and to financial strength in Chapter 6. The reader is referred to Ch. 7 of Pentikäinen et al. for the possibilities to extend the model to life insurance.

2. GENERAL FRAMEWORK

We present the development of the financial state of an insurer by the following basic equation

$$U(t) = U(t-1) + B(t) + I(t) - X(t) - E(t),$$

where $U(t)$ = solvency margin or net assets of a company, i.e. the difference of assets and liabilities in year t,

$B(t)$ = premium income in year t,

$I(t)$ = investment income in year t,

$X(t)$ = the aggregate amount of claims in year t,

$E(t)$ = expenses (in a wide sense, including, among other things, dividends).

The basic equation provides a year-by-year transition for the financial position. In the equation the premiums are earned premiums. Correspondingly, claims are incurred claims. Investment income consists of cash yield and change in value of assets. All the variables in the basic equation are stochastic.

We consider the claims process mainly in a traditional way. The claims process is divided into two parts: the number of claims and the size of individual claims. It is assumed that these induce a compound Poisson process. Moreover, the intensity of the number of claims is allowed to vary. In addition to this standard presentation we consider also problems concerning the evaluation of outstanding claims.

The random nature of investment income is of great importance, since the value of assets, in particular, may fluctuate considerably in practice.

In the model the premium level is not constant. The premiums may change, for instance because of price competition in the market. Let us emphasize this control-theoretical approach, differing from the point of view of the classical risk theory. In the model, as in reality, the insurer can control its financial position. One of the main tools in controlling the financial state is setting of premiums.

As well known, the assumption of a fixed premium level may lead to an unrealistic behavior of the solvency margin whereas a suitable control of premiums results in a stable, and realistic, development of the margin. For applications of control theory to solvency and related problems, the reader is referred to Martin-Löf (1983), Rantala (1984), Rantala (1988) and their references.

In Chapter 3 the mentioned key variables are considered in greater detail, as well as two important factors related to the basic equation: inflation and business cycles.

Inflation has an impact on all the key variables that determine the financial position of an insurer. Inflation is of importance because it affects all the central activities of the insurer but with different force and different time lags.

By business cycles we mean here a correlated fluctuation of the profit and loss figures of different companies in the same market.

3.1. PREMIUMS

Let us recall that in the classical risk theory premiums are assumed to be constant. In reality, however, the rating of premiums depends on claims experience, on the market and on the strategy of the company, among others. This is a very complicated process and not easily adapted to mathematical modelling.

It is clear that it is impossible to find any one formula that will cover all the alternatives in premium setting. However, it is useful to view the problem in all its complexity and to try to find rules - even if only approximate - for the anticipated behavior of the insurer. In the following we classify some approaches to premium setting.

A simple procedure is to derive premiums from the insurer's own claims statistics by using a suitable formula. This approach can be formalized as follows

(1) $B = f(X)$,

where f is a decision-making procedure based on the past claims experience X of the insurance company.

A more general procedure is also to take into account the current financial position, U, of the company. For instance, if the company's financial position is strong, one or two poor claims ratios may not yet necessitate an increase of rates. The generalized formula would be

(2) $B = f(X,U)$.

A further generalization is to bring into the decision-making process the prevailing market situation, symbolized by

(3) $B = f(X,U,M)$,

where M is the "market", for example, in terms of the current level of premiums.

This approach requires finding ways of evaluating the consequences of possible deviations of an insurer's premiums from the market level. We shall return to this question later.

In order to focus on the general idea of the tariff rule, we give a very simplified example. We consider first the case where the market effect is not taken into account. Let the premiums of an insurer $B_i(t)$ be controlled as follows

$$(4) \qquad B_i(t) = B_0(t) + a(U_0 - U(t-2)),$$

where $B_0(t)$ is a "basic" premium level, for instance the expected

amount of claims in year t, U_0 is a prefixed target level of the solvency margin, and a is a controlling coefficient, $a > 0$. Thus the premiums are controlled by the solvency margins with a two-year time lag. The idea is that if the solvency margin exceeds the target value U_0, premiums are lowered and vice versa.

Consider now the case where the market effect is taken into account. It is natural to assume that an insurer's premiums level should be close to the market level. The market effect is introduced into the model by defining the insurer's premium level as a weighted average of the market premium level and the premium level omitting the market effect, in formula

$$(5) \qquad B(t) = (1 - c)B_i(t) + cB_m(t),$$

where c is a weight factor $0 < c < 1$,

$B(t)$ = the insurer's premium in year t,

$B_i(t)$ = the premium in year t omitting the market effect, see (4),

$B_m(t)$ = the market premium in year t.

If this weighted average deviates from the market price level, the model uses a simple price elasticity formula to take into account the effect on the insurer's market share.

In the simulations, to which we will return later in greater detail, the market premium level $B_m(t)$ is derived and simulated, as if the market were a very big insurer, in a similar way as the premium level omitting the market effect $B_i(t)$.

In the simulations, the market has an effect on the insurer but not vice versa. This kind of situation may be true in a very big market but in a smaller market where a limited amount of leading insurers are competing, also the insurers affect the market. For example, if some insurer reduces premium prices in order to get a bigger market share, this probably leads to reactions by the others. Consequently, it arises a need to construct interactions between the insurers in the model (cf.Taylor, 1988) . The complexity of the real interactions makes the modelling difficult. However, we find that this is a promising topic for a further study.

3.2. CLAIMS

As mentioned, we present the claims process as composed of two parts: the number of claims and the size of individual claims. It is assumed that the claims process is a compound Poisson process with a varying intensity of the number of claims. Since the modelling of the claims process is much discussed in actuarial literature (see e.g. Beard, Pentikäinen and Pesonen, 1984), we will consider claims process here only from the point of view of the paid claims and the reserve for the outstanding claims.

The claims that have been incurred by the end of the accounting year but have not yet been settled - possibly not even reported - are called outstanding claims. Since usually a substantial part of the outstanding claims is unknown when the balance sheet is compiled, their total has to be estimated. This estimate is the reserve for the outstanding claims. The reserve for the outstanding claims is, to a considerable degree, subject to errors. Among other things, this is due to the fact that it may take many years, in some insurance lines even decades, until all the claims incurred in any one year will be settled.

An incorrect outstanding claims reserve is misleading in many ways and can have fatal consequences. For instance, an underestimation of the outstanding claims can lead to unprofitable premium rates. Underestimation of the outstanding claims also implies an overestimation of the solvency situation, which can delay corrective action by the management.

In considering outstanding claims it is appropriate to treat paid claims as a distinct variable. This variable is important also as part of cash flow.

The estimated incurred claims in year t, $\hat{X}(t)$ can be presented as

$$\hat{X}(t) = X_p(t) + C(t) - C(t-1),$$

where $X_p(t)$ are claims paid in year t, and C(t) is the reserve for outstanding claims in year t.

In the following we shall briefly consider a model for the paid claims process $X_p(t)$ as well as related reserving problems, presented in Pentikäinen et al. The reserving rules discount the future claims taking into account the investment income, on the one hand, and the expected future claims inflation, on the other.

The paid claims can be presented as follows

$$X_p(t) = \sum_{u \le t} X(u,t)$$

where X(u,t) is the aggregate amount of claims incurred in year u and paid during year t. A standard risk theory model is used for X(u,t). It is assumed that the ratios of the expectations of X(u,u+i) and X(u) do not depend on u, i.e.

$$\frac{EX(u,u+i)}{EX(u)} = r_{u,i} = r_i.$$

The reserving rules are based on the known run-off distribution of the claims, which we denoted by $r_i, i = 0,1,2,\ldots,\Sigma\, r_i = 1$. As a consequence, in the model the error in estimating the outstanding claims reserve is of moderate size.

In reality, however, the ratios r_i often change in time, that is, they do in fact depend on u.

If we relaxed the assumption

$$r_{u,i} = r_i$$

and acknowledged that the insurer can not know how the ratios $r_{u,i}$ will change, we should get a model in which the errors in estimating the outstanding claims would be bigger. (Cf. Pentikäinen and Rantala, 1990).

3.3 INVESTMENT RETURN

The risks related to the yield and especially to the value of invested assets have an important effect on the insurers solvency and financial strength. Accordingly, evaluation of the asset risks is central in the study of solvency.

The purpose of the model is to describe the investment returns in such a way that, above all, the average level and the rate of fluctuation should correspond to the experienced ones. The objective of the model is largely to serve as a framework for simulation. The model aims neither at a deeper analysis nor at forecasting the future returns.

The investment returns are presented as a sum of the yield of investments and the change in value of assets. The change in value and the yield are treated separately, but by similar models.

When applying the model it is appropriate to divide the assets into homogeneous subcategories, such as bonds, shares and real estate. The very different returns of different investment categories can be described by adjusting the parameters of the model.

Inflation is used as the main external explanatory factor for investment returns. By calibrating the parameters of the model the degree of the inflation-linkage and the length of the time lags involved can be suitably adjusted. As in the Wilkie (1984) model, inflation is described as a first order autoregressive process.

For each investment category the change in value and the yield (separately) are presented as a product of three factors: the average value of the return, the effects of inflation and the inflation-independent factor. The last mentioned factor is presented by means of a second order autoregressive process. It is assumed to include the effects of all economic background factors other than inflation. Cf. the related model by Bonsdorff (1990), where the inflation-independent factor is modelled to have a drift to its mean value.

The following figures represent examples of investment returns generated by the model. They are not fitted to any data. The purpose of these examples is only to demonstrate possible outcomes of the model.

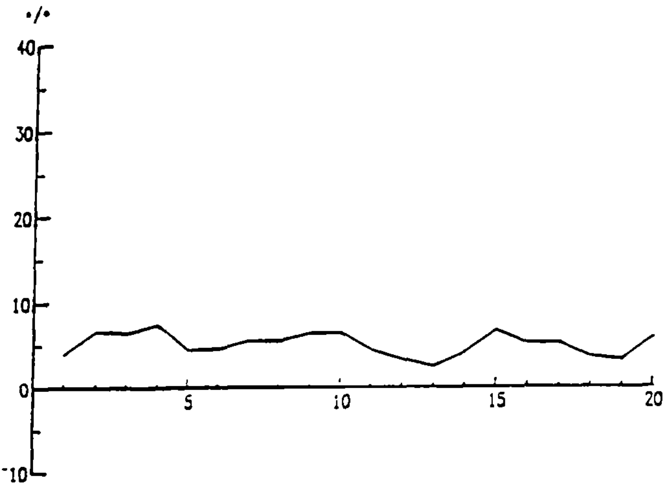

FIG.1. Simulation of yield of shares

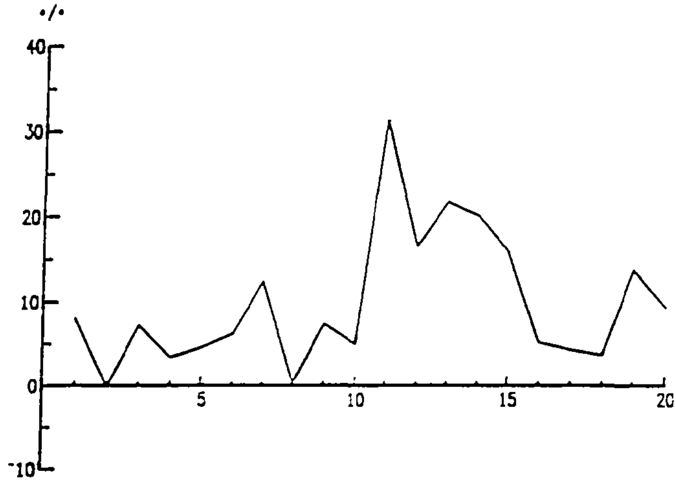

FIG.2. Simulation of change in value of real estate

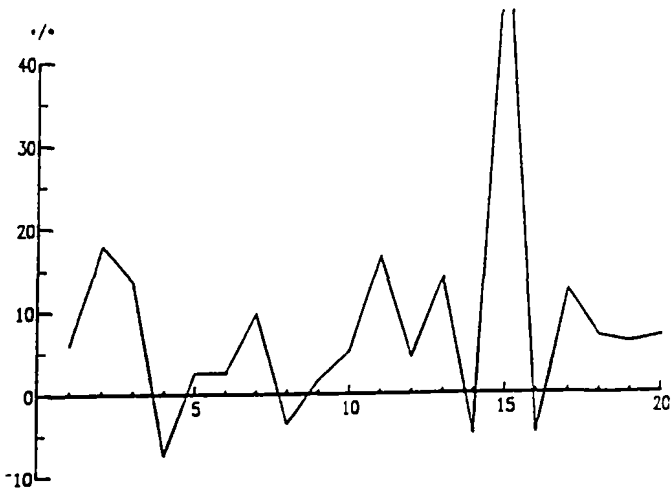

FIG.3. Simulation of change in value of shares

3.4. EXPENSES

A typical feature of expenses is their stability. However, it would be an oversimplification to assume that the expenses are constant.. For example, the changes in the rate of inflation will have a full effect on a company's expenses. Moreover, they provide a tool for the management to control the business. Let us also recall that we are using the concept expenses in a wide sense including also dividends. However, we do not consider this topic in greater detail here.

3.5 INFLATION

As mentioned, inflation affects with differing force and varying time lags all the central activities of the insurer. In general, the claims expenditure and operating costs react immediately to inflation, while the effect on premium income and investment income is delayed. In the case of investment income the inflation-linkage varies also with the type of investment. In fact, in case of certain investments inflation has only a minor effect.

As to inflation, it is necessary to distinguish between steady and more variable inflation. If inflation is steady the distortions caused by inflation probably remain slight, since most insurers have introduced index-linkage systems for premiums, deductibles and other provisions. But the consequences may be more serious if the rate increases suddenly. Then the index-linkage system may not react fast enough.

The sizes of individual claims are increased on one hand by normal inflation and on the other by special factors characteristic of particular lines of insurance. Variable inflation has an effect on the assessment of claims reserve as well. Ordinary valuation methods for the claims reserve generally take into account steady inflation correctly, but variable inflation may cause errors in the assessment.

The purpose of the inflation model is, in the first place, to produce simulation results which correspond reasonably well to empirical data. The model is neither aiming at explaining real inflation in a deeper sense nor at forecasting inflation. Due to need for different inflation rates for different lines of insurance, the model is constructed so that the general rate is simulated first and the special rates are computed using the general rate as a basis. The general rate of inflation is derived as a first order autoregressive process. By calibrating parameters, the average inflation rate and the degree of fluctuation can be adjusted.

The following figures give examples produced by the model.

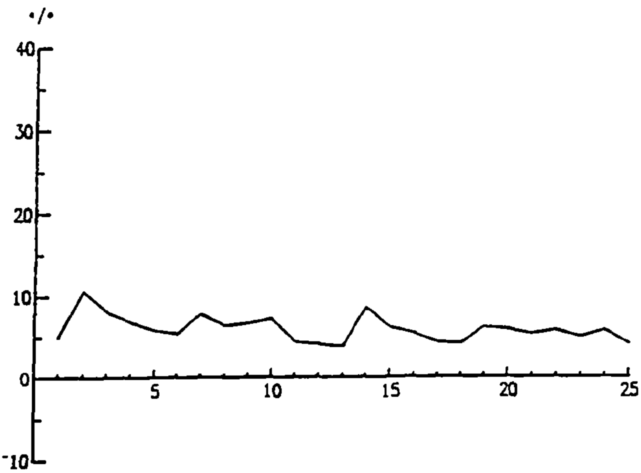

FIG.4. A moderately fluctuating inflation

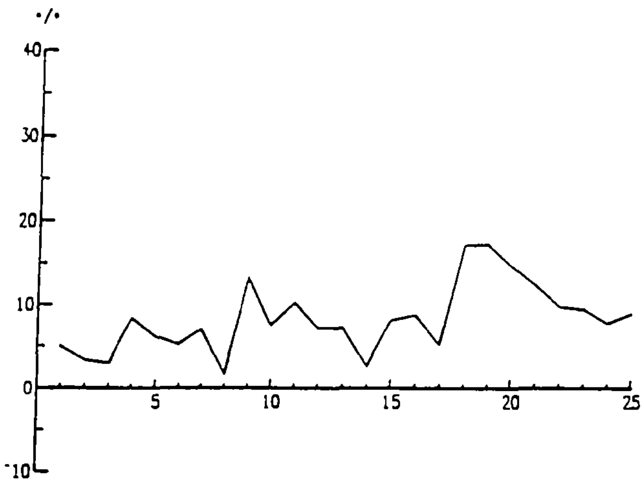

FIG.5. A strongly fluctuating inflation

3.6. BUSINESS CYCLES

Figure 6 shows the trading result of six largest Finnish non-life insurers (representing about 80 percent of the market share) as a ratio of premium income, and Figure 7 the corresponding solvency ratios. A predominant feature is the strong cyclical nature of the curves, the individual curves showing a similar pattern. The cycles are one of the most important factors affecting the financial strength of insurers, and even their solvency.

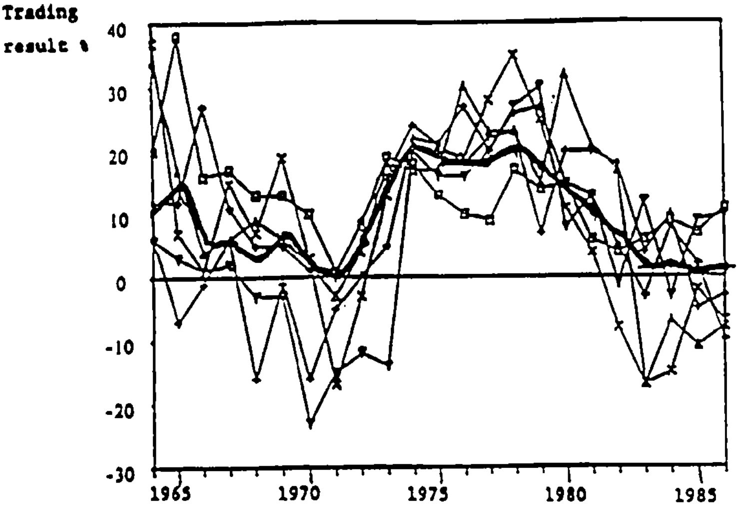

FIG.6. The trading result (underwriting result supplemented by the return on investments) of six largest Finnish non-life insurers, and the joint trading result for all insurers, as a percentage of the retained premium income

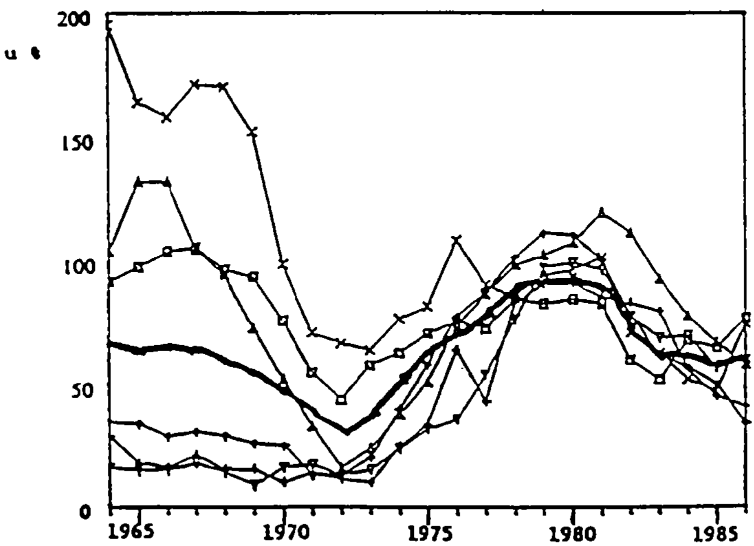

FIG.7. The solvency ratio (u = U/B) of six Finnish insurers (without hidden reserves)

The fluctuations can be the result of fluctuations in claims, investment income and premiums. Obviously, the setting of premiums is of central importance to explaining cyclic fluctuations. The aim of the insurer is, of course, to keep premiums in balance with claims, investment income and other variables. Usually, the actual claims outgo and investment income will differ from the amount anticipated when the premiums were set. A loss or profit will ensue. This leads to an adjustment of rates, contributing to a fluctuation of trading result. Note that even an application of the simple formula (4) may lead to a cyclic behavior of the solvency margin.

A major cause for cycles are market pressures. The capacity available in the market influences prices. Overcapacity has a tendency to decrease the premiums while an undercapitalized market will cause the premiums to go up. The capital available depends partly on the past profits or losses of the industry and partly on the anticipated profitability of the business, especially if there is free entry to the market. (Note that the six solvency ratios in Figure 7 are closer to each other in 1986 than in 1964. This may be caused by a strengthened price competition during the period.)

One way to model the effect of cycles on an insurer is to use a cyclical market pattern and set the premiums of the insurer partly dependent on the market premiums.

4. INTEGRATION OF THE VARIABLES INTO AN OVERALL MODEL

To arrive at a model that enables to handle problems related to the analysis of solvency the basic variables have to be integrated into an overall model. The problem of the interdependence of the basic variables such as claims, premiums, investment is very complex. Let us mention only two things.

1° All the main variables are correlated due to inflation.

2° Premiums are adjusted to the other variables and thuscorrelated with them.

In the model these are the main correlations. For instance investment income and claims are correlated only through inflation in the model.

When the model is used in studying general financial strength conditions, it is useful at first to specify a "basic case", in which certain specified values are fixed for the parameters of the model. Then sensitivity analysis can be carried out. By varying the size of the portfolio, its composition and other basic parameters, it is possible to study how the business reacts to various external and internal impulses.

Let us emphasize that when the model is intended for the use of a particular insurer, then the basic parameters, claim size distributions, etc. have to be chosen to correspond to the actual data.

In the following we give some simulation examples. In the basic case the insurer has 3 insurance lines with different claim size distribution, speed of claim settlement, etc. To take another example, the insurer has 4 different asset categories. We omit further details at this point. For details, the reader is referred to Pentikäinen et al. Ch 4, pp. 175-207, and especially to Appendix A, pp. 262-277 (See also p. 121). The basic case is described on p. 183 as "standard insurer".

The insurer is studied first as isolated and then as a part of the market. In the simulation model the market is dealt with as if it were just another, but much larger insurer. It is assumed that external variables, such as inflation are common both to the market and to the insurer.

The following figures present some simulation examples. The vertical axis represents the solvency margin proportioned to premium income. Each figure presents forty realizations of the development of the solvency of an insurer. The time span is thirty years. The solid horizontal line is the ruin barrier. (The ruin barrier is set at 22 percent of the unloaded premium income. Its value would be some 16-18 percent of the loaded income, corresponding thus the EC requirements.) The insolvency cases are marked by an asterisk. The realizations form a bundle which illustrates the development of the financial state of an insurer.

In the first examples the insurer is isolated, in other words, market effect is excluded. Figure 8 presents the solvency ratio in the basic case. In the following two diagrams some variables are taken as deterministic. In Figure 9 inflation and return on investment are kept deterministic but claims are stochastic. In Figure 10 claims are deterministic and inflation and investment are stochastic. It can be seen that in our example the uncertainties arising from claims and from investments are about the same size. Note that Figure 8 presents the situation where all variables are stochastic. Naturally, uncertainties are bigger in this case than in those cases where some variables were deterministic. In the basic case the time lag in premium rating is two years.

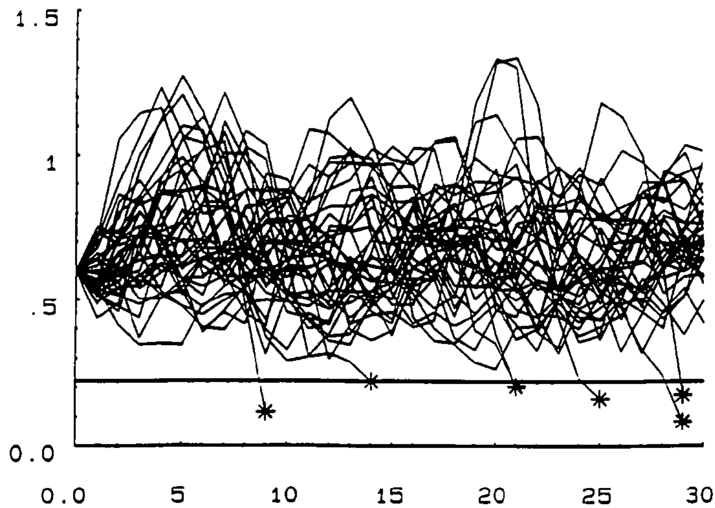

FIG.8. Basic case

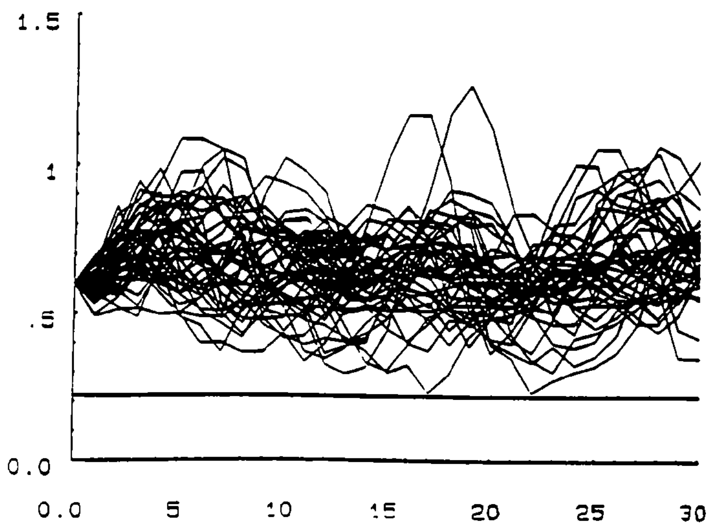

FIG.9. Deterministic inflation and return of investments

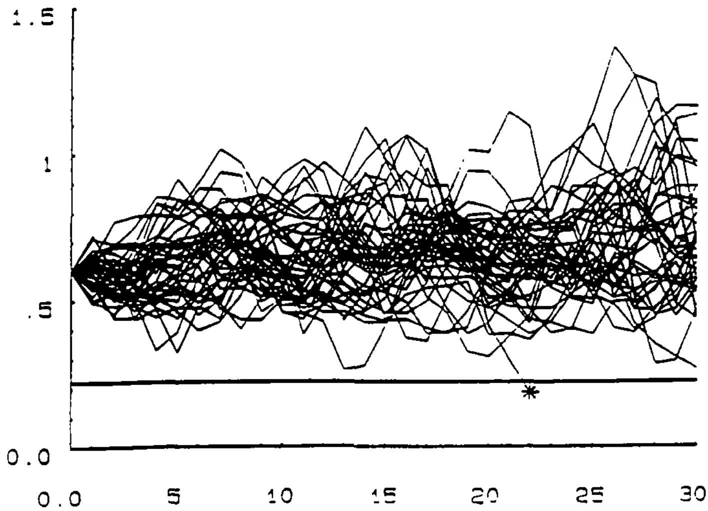

FIG.10. Deterministic claims

In the following simulation (see Figure 11) the effect of the time lag in premium rating is tested by extending the assumption from the standard 2 years to 3 years. As can be seen, the stability of the system is radically impaired, and the number of insolvencies remarkably increased. This demonstrates the importance of a quick control mechanism in premium rating.

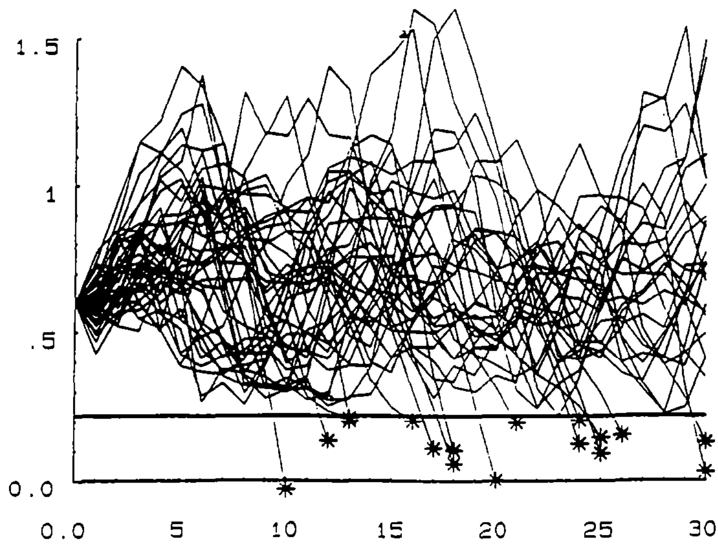

FIG.11. Time lag = 3

Finally, we present two diagrams demonstrating the market effect. In Figure 12 the lower diagram represents a situation where the insurer adopts the market premiums. In the upper diagram the insurer takes into account the market premiums but also controls the premiums by the solvency situation. As can be seen the number of insolvencies is smaller in the controlled case. (Note that different random number sequences were used in both diagrams).

In Figure 13 the solid line represents the solvency of the insurer and the dotted line that of the market. The diagram could be interpreted so that the better solvency of the market pushes the insurer under the ruin barrier.

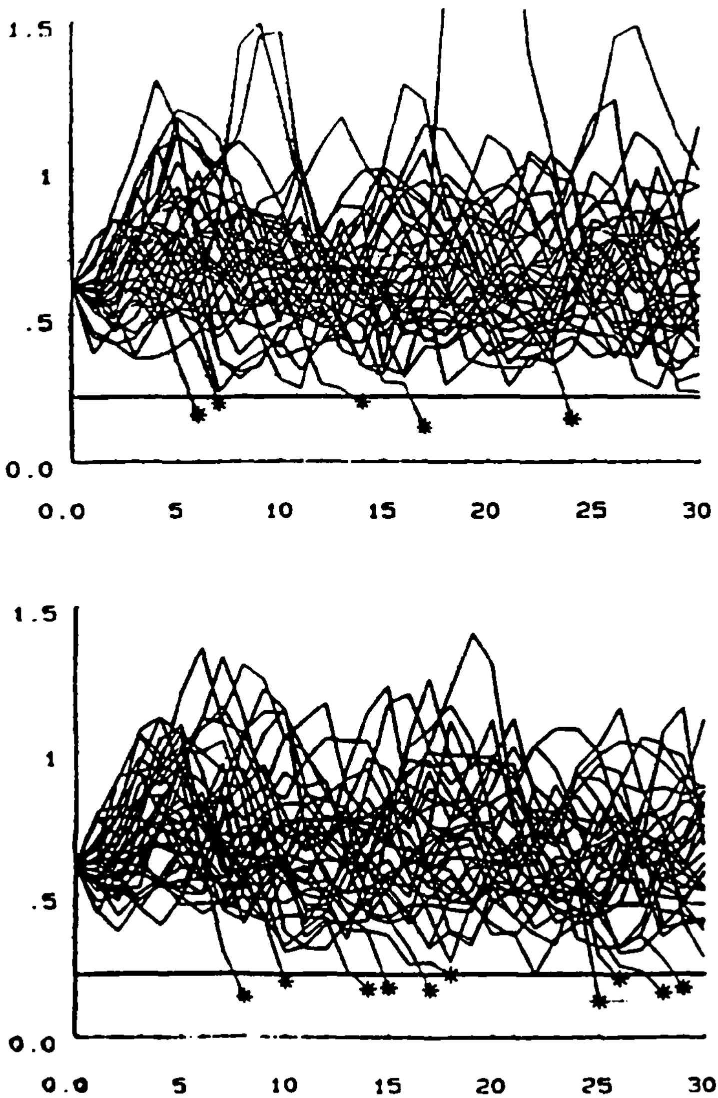

FIG.12.

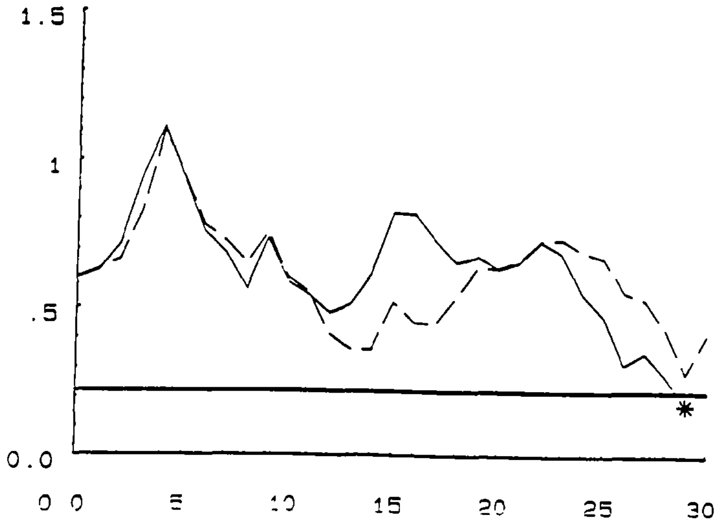

FIG.13.

5. PUBLIC SOLVENCY CONTROL

The main purpose of the public solvency control is to safeguard the interests of the policyholders from the consequences of insolvencies. Supervision carries out regular tests, normally annually, in order to make sure that the financial position of the insurer is healthy enough. In this presentation we concentrate on the problem how much the assets have to exceed the liabilities in order that the risk of insolvency during a test period would be reasonably small. Before going into this question we consider briefly two different ways to define insolvency, the going-concern and run-off approaches.

In the going-concern alternative a company is insolvent if the best estimate of assets does not exceed the best estimate of liabilities by a safety margin. In the run-off alternative it is required, in addition, that if the writing of new business would be stopped at the test time point, the assets would be sufficient for paying the outstanding claims. The possible errors in valuations of assets and liabilities have to be taken into account, as well as the matching of assets and liabilities, including a possible adverse development of the asset values during the run-off of the business.

In what follows we mainly consider testing of solvency on the going-concern basis. To formalize the problem, we ask how big the solvency margin U has to be in order that the probability that it will fall below the zero level during a test period T is smaller than a given probability ε. We denote the minimal margin satisfying this requirement by U_{req}. The length of T is usually one year, normally 2 years at the most. The probability ε has to be chosen to be reasonably small, a possible choice might be $\varepsilon = 0.01$. It is often practical to scale the solvency margin by expressing it as a percentage of premium income.

The choice of T influences the amount of U_{req}. If T = 18 months is applied instead of T = 12 months, a margin about 20-30 percent bigger is needed, cf. Pentikäinen and Rantala (1982), Vol. I, p. 4.2-28 and Beard, Pentikäinen and Pesonen (1984), p. 280. Correspondingly, the choice of the probability ε influences the level of U_{req}. Comparing to case $\varepsilon = 0.01$, a 20-30 percent smaller margin would be sufficient in case $\varepsilon = 0.05$, and a 20-40 percent bigger margin would be required in case $\varepsilon = 0.001$ (cf. Pentikäinen and Rantala 1982, Vol.I, p. 4.2-26 and p. 4.2-44).

It is evident that given some ε and T there does not exist any simple formula for the required solvency margin. Numerous features of the business affect the required margin, e.g. volume and quality of the insurance portfolio, especially the mix of insurance lines and possible catastrophe risks, possible errors in underwriting reserves in long-tail business, reinsurance cover, the phase of marketing cycle and the risks involved in the invested assets.

Because of the complexity of the problem, the determining of the required solvency margin has to be súpplemented by an analysis of a qualified expert (cf. Daykin et al., 1987). Obviously, the expert analysis cannot be made without well-defined quantitative standards. These standards can, at least partly, be determined by the kind of model as described here. A comprehensive model can also be used in building an early warning system.

The quantitative analyses suggest relatively high required margins: On the going-concern basis, a margin from about 30 percent even to 100 percent of premium income in the riskiest cases (e.g. when a considerable part of the portfolio consists of credit insurance). Adopting the run-off basis increases the required margin at least some 20-30 percent of premium income, cf. Daykin et al. (1984), (1987,) and Pentikäinen and Rantala (1986).

6. FINANCIAL STRENGTH

The role of the management is to maintain financial strength, in the long term, on a suitable level. This managerial view differs considerably from that needed in short term solvency considerations. First of all, a longer time span is necessary. Further, the suitable level of the financial strength differs, in general, from that which would be necessary in order to barely avoid insolvency. In addition, financial strength is only one of the goals of the management.

When considering the different goals of the management we limit ourselves to what is called general management process or strategic planning. A strategic plan may be based e.g. on high financial strength, expansion of business and a policy of distributing dividends or bonuses. These aspects, being partly contradictionary, are in an intimate interaction with each other.

The advantages of a high financial strength are evident. A high solvency margin allows freedom in strategic planning, e.g. giving possibilities to expand the business by marketing and price competition. Further financial strength is an advantage when dealing with distribution of dividends, acquisition of new capital and maintaining net retention.

The model described here is a useful tool in strategic planning as it gives quantified information to support decision making. The model can be used for scenarios by studying the effects of sudden changes and outcomes, such as inflation peaks, catastrophes, adverse marketing cycles etc. Scenarios can help to avoid over-risky strategies and also to reveal an adverse development early enough. Scenario technique applies to determining a minimal desired solvency margin, as well.

Scenarios can also be used to determine a biggest desirable solvency margin. A surplus exceeding this limit should be used to other purposes than to strengthen the solvency margin, e.g. to dividends, bonuses, marketing and price competition. Of course, shareholders, policyholders, supervisors and taxation authorities, even if having differing views, are interested in whether the solvency margin is growing unreasonably big.

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