THE IMPLICATIONS OF MARKET RETURN PRICING STRATEGIES

UPON

PROFIT AND REQUIRED SURPLUS

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BIOGRAPHY:

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ABSTRACT:

As the length, amplitude and overall uncertainty of the underwriting cycle increases, firm profit and required surplus levels become less predictable. Actuarial pricing techniques commonly target expected returns which are impossible to achieve in the soft market. Market based pricing strategies which will maximize return over the entire cycle are not well understood. Extreme approaches have been taken in the past such as holding exposure levels constant or fixing price regardless of the long-term cost to profit or size of book. These strategies have not proved optimal. This paper attempts to determine the strategies which will accomplish various firm profitability goals for a model insurance economy subject to an underwriting cycle. These strategies are then examined with and without practical constraints on price growth, exposure growth, and surplus limitations to compare profits and required surplus levels. Selection criteria for ideal strategies are presented. Finally, ideal strategies are selected corresponding to different types of profitability goals.

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INTRODUCTION

In the last cycle, which hit bottom in 1984, many firms desperately held on to their books of business despite plummeting price levels, believing that investment income would overcome any amount of underwriting loss. The ultimate folly of this "cash-flow underwriting" has been well documented. During the current soft market, in apparent reaction to the disastrous strategy of holding exposures during the last down cycle, many firms are determined not to compromise on price, despite the unavoidable erosion in the size of their policy books. The pain of staff cutbacks, which inevitably accompanies this approach, is seen as the necessary antidote to the underwriting cycle. However, it is still not clear that firms adopting this "hold-price" strategy will fare any better than those who "held-exposures" during the last cycle. The bigger question is what sorts of strategies provide the ideal path through the underwriting cycle.

This paper focuses on the effect of various exposure strategies upon profit and required surplus. Strategies determined by adopting the standard actuarially determined or "expected return" price are contrasted with strategies which seek a long-term market return. Expected return approaches calculate the minimum price needed by a given firm to cover all fixed and variable costs and still provide an acceptable margin of return. The simplest forms assume all expense to be variable, effectively ignoring the impact of exposure levels on ultimate return. Even when exposure levels are considered, the expected return price may not be attainable in the market at the assumed exposure level. When the firm prices for its real economic market, considering the specific supply/demand structure and how it fluctuates over the course of the underwriting cycle, the ideal strategies selected may be entirely different from the naive strategies assumed with the expected return approach. Exposure and price levels may need to be adjusted significantly and at times and in directions different from expected return projections.

In this paper, we will define profit and required surplus and examine the difference between expected return and market return approaches on required surplus. Profitability goals will be defined based on various perspectives of different groups involved in the insurance economy. We will construct a simple monoline model of an insurance market and simulate the underwriting cycle by varying the demand curve over time.

Strategies which can be used to achieve the profitability goals in the model insurance market will be constructed. The resulting fluctuations in exposure levels, price, profit, and required surplus from following these strategies will be examined. Alterations to strategies in going from a small to a large firm or a low risk to high risk loss process will be explored. A few complications will be introduced to reflect practical concerns which may constrain strategies, for example the limitations of actual surplus levels, or the maximum variations in price and exposure levels which can be reasonably handled by firms or allowed by regulators in a given period of time.

REQUIRED SURPLUS

The problem of determining the amount of surplus required to safely back up a given amount of premium in a given line of business remains an open issue. Intuitively, surplus is required to absorb unforeseen operating losses. Operating losses could arise from underwriting experience which is worse than expected or investment performance which falls below pricing assumptions. Depending on one's risk adversity, required surplus for a set of policies is that amount which lowers the chance of bankruptcy or ruin (i.e. ruin, for a set of policies, occurs when losses and expenses exceeds the revenue derived from the set of policies) to an acceptably small percent.

Frequently, required surplus has been defined by focusing on the variability of losses and assuming that investment results will not contribute to the variability of operating results.[1] Put into a pricing perspective, required surplus is that amount which, when combined with projected operating profits, provides for an acceptably low probability of ruin based on the variability of the aggregate loss distribution. In equation form, required surplus (S_r) = [(the

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[1] See, for example, Finger, Robert J., "A Model for Calculating Minimum Surplus Requirements" 1979 CAS Discussion Paper Program, p. 123

aggregate loss with a cumulative probability equal to 1 - the ruin probability) - (projected loss) - (projected operating profit)]. Under this approach, exposure levels and projected return determine the ultimate required surplus. Assuming that projected return is positive, as exposures increase, required surplus will decrease as a percent of premium. In fact, if the projected return is a fixed percentage of premium, required surplus will eventually go to zero.

Using expected return approaches, required surplus and price are determined once exposure levels, probability of ruin, variability of aggregate losses and desired return are selected. However, if the calculated price is not obtainable in the marketplace, the required surplus calculated by the expected return method will not be sufficient! Required surplus, driven by the lower return allowed by the market, will have to increase. The attainable return has a huge influence on required surplus levels.

Clearly then, required surplus cannot be defined without considering the market price structure or demand curve facing the individual firm. Price, in combination with a demand curve, determines exposure levels (which determine the loss contribution to required surplus), projected return (the profit contribution to required surplus), and thus, required surplus.

As the demand curve shifts over time, long-term profitability goals will require exposure levels and required surplus to fluctuate, as well. And of course, different profitability goals will imply different exposure and required surplus levels over time.

THE EXPECTED RETURN PERSPECTIVE

Any number of return methods could be selected to illustrate the calculation of required surplus. We will present an example which illustrates the interdependence of ruin and return using an expected return on required surplus approach.[2]

Required surplus will be determined in the following way: Given a loss process, determine the aggregate loss distribution corresponding to a selected level of exposures. Pick an acceptable ruin probability. Find the amount of aggregate loss which has a cumulative probability equal to one minus the ruin probability. The difference between this aggregate loss and the expected loss is the loss contribution to the required surplus. Next determine an appropriate operating profit to provide the selected return. Required surplus equals the loss contribution less the operating profit.

For example, let:

1. Return on Required Surplus (RORS) = 20%
2. Expected loss cost (E{L}) = $80
3. Variable expense (v) = 30%. Fixed Expense (f) = $0
4. Frequency is binomial with p = 0.2
5. Severity equals $400
6. Probability of ruin (ε) = 1.0%
7. The number of exposures (q) = 3

Let L_ε = smallest aggregate loss, L, such that $P\{L > L_\varepsilon\} \le \varepsilon$

For the binomial distribution, this can be expressed as L_ε = (j)*400, j selected such that

$$\sum_{i=0}^{q-j-1} [\binom{q}{q-i}(.2)^{q-i}(.8)^{i}] \le \varepsilon < \sum_{i=0}^{q-j} [\binom{q}{q-i}(.2)^{q-i}(.8)^{i}]$$

when q=3, j=2 and L_ε=$800

TABLE A: BINOMIAL AGGREGATE LOSS DISTRIBUTION WHEN q=3

# of LOSSES	AGG LOSS	PROBABILITY	(1 - CUMUL PROB)
3	$1,200.	$(3\ 3)(.2)^3(.8)^0$ = 0.8%	0.8%
2	$800.	$(3\ 2)(.2)^2(.8)^1$ = 9.6%	10.4%
1	$400.	$(3\ 1)(.2)^1(.8)^2$ = 38.4%	48.8%
0	$0.	$(3\ 0)(.2)^0(.8)^3$ = 51.2%	100.0%

2 See, for example, Venter, Gary G., "Profit/Contingency Loadings and Surplus: Ruin and Return Implications" 1979 CAS Discussion Paper Program pp. 353-354

If premium is denoted by P_m, then

$P_m = (E\{L\}*q + RORS*L_r)/((1-v)*(1+RORS))$

This follows from the equations:

Operating profit $P_r = P_m*(1-v) - E\{L\}*q$, and

$RORS = P_r/(L_r - E\{L\}*q - P_r)$

When $q=3$,

$P_m = (80*3 + .2*800)/(.7*1.2) = 476.20$

$P_r = 476.20*.7 - 80*3 = 93.34$

Required Surplus, $S_r = 800 - 80*3 - 93.34 = 466.67$

$RORS = 93.34/466.67 = 20\%$

Prem to Required Surplus ratio, $P_m/S_r = 476.20/466.67 = 1.020$

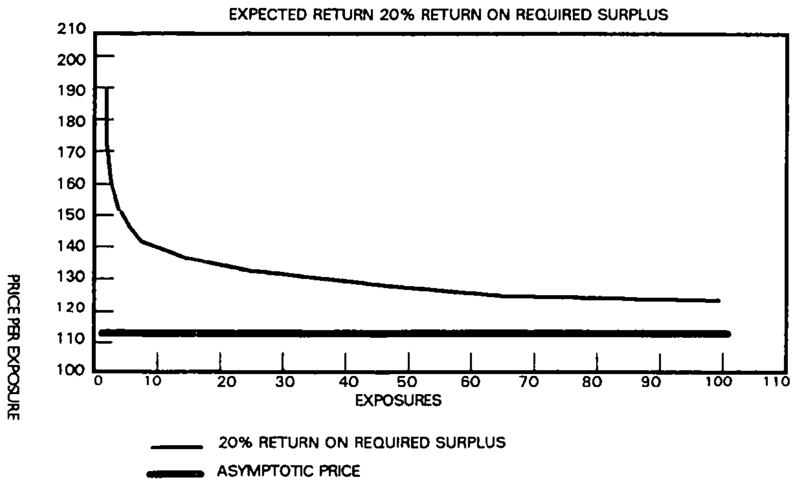
Price, $P = P_m/q = 158.73$

TABLE B: RELATIONSHIP OF EXPOSURE LEVEL TO KEY OPERATINGVALUES

# of EXPOSURES	PREMIUM	OPERATING PROFIT	REQUIRED SURPLUS	P/RS	PRICE
3	$476.20	$93.34	$466.67	1.02	$158.73
100	$12,380.95	$666.67	$3,333.33	3.714	$123.81
500	$59,142.86	$1,400.00	$7,000.00	8.449	$118.29
1,000.	$117,142.86	$2,000.00	$10,000.00	11.714	$117.14

Graph A depicts the relationship between exposure and price when price is calculated using the expected return on required surplus approach.

GRAPH A: EXPECTED RETURN CURVE



THE MARKET RETURN PERSPECTIVE

As we have seen, the expected return on required surplus method calculates a price based on the expected profit needed to return a satisfactory RORS. Since required surplus, as a percent of premium, decreases with increasing exposure levels, expected profit also declines and premium to required surplus ratios increase. Graph A depicts the price/exposure relationship for the expected RORS approach. Clearly this curve is asymptotic at the zero profit price ($80/0.7 = $114.29).

In fact, only a subset of the prices ("the feasible region") calculated by the expected RORS approach may be feasible when this curve is placed in the context of the marketplace. Using the same example as above, and assuming a market demand curve given by the equation: $p = -.5q + 155$, one can see on Graph B that for exposure levels below $q=5$ or above $q=50$, the expected RORS is not obtainable in the market. This implies that the market required surplus levels will be higher than the expected required surplus for any exposure levels outside the feasible region. (See Table C).

GRAPH B: EXPECTED RETURN WITH MARKET DEMAND CURVE

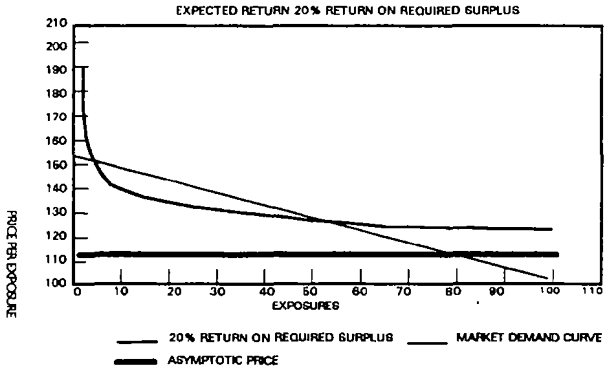


TABLE C: COMPARISON OF EXPECTED AND MARKET REQUIRED SURPLUS

	EXPECTED			MARKET			
EXPOSURES	RORS PRICE	REQ SURPLUS	P/RS	PRICE	REQ SURPLUS	P/RS	MKT - EXP REQ SURPLUS
1	$190.	$267.	0.714	$155.	$292.	0.529	$25.
5	$152.	$667.	1.143	$153.	$666.	1.144	($0.)
10	$143.	$1,000.	1.429	$150.	$950.	1.579	($50.)
15	$140.	$1,333.	1.571	$148.	$1,251.	1.768	($82.)
20	$133.	$1,333.	2.000	$145.	$1,170.	2.479	($163.)
25	$133.	$1,667.	2.000	$143.	$1,506.	2.365	($160.)
30	$130.	$1,667.	2.343	$140.	$1,460.	2.877	($207.)
35	$131.	$2,000.	2.286	$138.	$1,831.	2.628	($169.)
40	$129.	$2,000.	2.571	$135.	$1,820.	2.967	($180.)
45	$129.	$2,333.	2.490	$133.	$2,226.	2.678	($107.)
50	$128.	$2,333.	2.735	$130.	$2,250.	2.889	($83.)
55	$128.	$2,667.	2.643	$128.	$2,691.	2.606	$25.
60	$127.	$2,667.	2.857	$125.	$2,750.	2.727	$83.
65	$126.	$2,667.	3.071	$123.	$2,826.	2.817	$160.
70	$125.	$2,667.	3.286	$120.	$2,920.	2.877	$253.
75	$126.	$3,000.	3.143	$118.	$3,431.	2.568	$431.
80	$125.	$3,000.	3.333	$115.	$3,560.	2.584	$560.
85	$124.	$3,000.	3.524	$113.	$3,706.	2.58	$706.
90	$124.	$3,000.	3.714	$110.	$3,870.	2.558	$870.
95	$124.	$3,333.	3.543	$108.	$4,451.	2.294	$1,118.
100	$124.	$3,333.	3.714	$105.	$4,650.	2.258	$1,317.

Graph B illustrates the market determined feasible region for a specific return goal interacting with a specific market demand curve. Different return goals will have different return curves which will imply different feasible regions and different required surplus levels when faced with this specific demand curve. The demand curve is only instantaneously fixed in time. Over time, the market demand curve will shift up and down with the movement of the underwriting cycle.[3] Graphed with time as the third dimension, these demand curves will form a demand surface. The intersection of the demand surface at different points in time with the same return curve will generate different feasible regions, and thus different required surplus levels over time. The path that is taken through the underwriting cycle, i.e. the curve of exposure levels over time, I will call a "strategy." Clearly, a given return goal will determine a strategy through the intersection at each point in time with the demand surface, although the converse is not necessarily true. Each strategy will have its own set of required surplus levels. Some strategies will require more surplus and therefore will be riskier than others.

RETURN GOALS

I will define some return goals based on different perspectives (policyholder, stockholder, firm, and regulator) and environments (multi-line environment, heavily regulated environment, depleted actual surplus environment). These return goals will be placed in the context of a specific model of a market demand surface. The variety of strategies which result and their implications on profit and solvency will then be examined.

Policyholders seek the greatest security for their policy contract at the cheapest available price. This translates into determining the minimum price, that over the course of the underwriting cycle will never bring actual surplus below required surplus levels. If this strategy consistently depletes surplus over the course of a full underwriting cycle, eventually actual surplus will fall below required surplus levels. Therefore, in the long-term, this goal implies that actual surplus levels should not decrease over the course of a full underwriting cycle.

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3 See Feldblum, Sholom, "Underwriting Cycles and Business Strategies", 1990 CAS Forum, pp. 80-81

Stockholders seek the greatest return on invested capital at a given tolerable investment risk level. If actual surplus equals invested capital, the stockholder will seek to maximize the return on actual surplus. Maximizing the return on actual surplus while holding the risk level constant is equivalent to maximizing profit over all possible strategies where required surplus at the given risk level is less than actual surplus. Here, the probability of ruin is set to equate to the tolerable investment risk level. If stockholders sought to only invest the minimum capital required to maximize their return at a given risk level, the goal would be maximizing the return on required surplus. The "excess surplus" (difference between actual and required surplus) would be removed and reinvested elsewhere.

Regulators are concerned with the potentially opposing goals of maintaining the long-term solvency of insurance carriers and protecting policyholders from excessive rates. Capital will leave the insurance industry if investors perceive returns to be inadequate. The regulator must discern the target return which will be satisfactory to investors at a given risk level and then minimize price subject to this return. If more than one exposure level will provide the investors target return, the level that minimizes price will be preferred by regulators. If a fixed premium to surplus ratio is assumed in a given line of business, this translates into minimizing price subject to a target return on premium.

Return goals may vary by type of firm, as well. Stockholder owned companies will have the return goals discussed above. However, mutual companies may seek to maximize return on premium at a given price level so as to return the greatest percentage dividend. Or, if dividends are undistributed, they may minimize price while targeting whatever fixed addition to surplus is required to maintain the risk of insolvency at an acceptable level. For firms not purchasing reinsurance, surplus may need to be increased by a specified dollar amount to protect against catastrophe. Even non-profits may need to boost their surplus fund (especially in a growth phase) to maintain the security of their contracts.

Companies which operate in multiple lines may provide a discount for policies that package two or more different lines of business. A given return goal for all lines combined may not be achieved simply by applying the same return goal to each line individually. For example, if the return goal for all lines combined is maximizing profit, will this be accomplished by maximizing profit for each line individually? Not necessarily. By reducing profit in one line, the firm may attract a group of potential policyholders that need coverage in two or more lines and insist on package coverage. The profit earned on these extra policyholders may be greater than the profit foregone in the individual line. Essentially, there could be as many as 2^n demand curves facing an n-line firm. One for each line of business and potentially one for each combination of subsets of lines of business.

Constraints may exist in a heavily regulated environment that are not present in an environment of open competition. Dramatic rates of growth, for example, may be looked upon unfavorably. A firm seeking to maximize profit may need to hold risks in the soft market by offering a price below the profit maximizing price in order to keep the growth rate at an acceptable level when the market hardens. Wide fluctuations in price may be as unacceptable as swings in growth. Firms may need to write more exposures in the hard market than indicated by a specific return strategy and less in the soft market to minimize price change.

When actual surplus falls below the surplus required to sustain a given return goal, will the return goal be most closely approached by utilizing the same strategy constrained to an allowable surplus level? Apart from maintaining a given return strategy, is there a minimum surplus level needed to survive in a given market environment?

SIMPLE MODEL OF MARKET PRICING

Assume that the individual firm faces:

1. Expected loss cost given by $E\{L\}$
2. Variable expenses denoted by v; fixed expense by f
3. Identical exposure units with binomial aggregate loss distribution, i.e. frequency equal to p, severity equal to $E\{L\}/p$
4. Linear market demand curve: Price $(P) = aq + b$, $a<0$, $b>0$

Define price (P), to be a function of exposure level and time (q and t). This "demand surface" will give the market price for any exposure level at any point in the underwriting cycle. The firm's strategy function (Q) will describe the choice of exposure levels written at each point in the underwriting cycle.

We can also define a premium surface (P_m), a profit surface (P_r), a required surplus surface (S_r), a market return surface (R_m), and an expected return function (R_e). The first four, like price, are functions of exposure and time, while the expected return (which doesn't give return at all, but rather the price needed to sustain a given return at a given exposure level) varies according to exposures alone. One method of determining strategy functions is to take the intersection of an expected return function extended across time (i.e. $R_e(q,t)=R_e(q)$) with the demand surface. Of course, if the expected return is not attainable at a given point in time, the function will not intersect the demand surface at that particular value of t.

For a given loss, expense, ruin probability, and demand surface, the premium, required surplus, and profit surfaces are determined. The market and expected return functions, however, depend on how the firm's return goal is measured. Every type of measurement implies different market and expected return functions. If the return goal is measured, for example, in units of return on required surplus, the R_m surface will give the return on required surplus for any value of q and t. The R_e curve would give the price needed to sustain the same selected expected return on required surplus at every value of q.

Symbolically,

$Q(t) = q$

$P(q,t) = aq + b$, a, b, and q are functions of t

$P_m(q,t) = q*P(q,t)$

$P_r(q,t) = P_m(q,t)*(1-v) - E\{L\}*q - f$

$S_r(q,t) = L_\varepsilon(q) - E\{L\}*q - P_r(q,t)$

$R_m(q,t) = P_r(q,t)$; profit units

$= P_r(q,t)/P_m(q,t)$; return on premium units

$= P_r(q,t)/S_r(q,t)$; return on req surplus units

$R_r(q) = (E\{L\}*q + f + k)/((1-v)*q)$; profit = k

$= (E\{L\}*q + f)/(q*(1-v-k))$; return on premium = k

$= (E\{L\}*q + (1+k)*f + k*L_r)/(q*(1-v)*(1+k))$; return on required surplus = k

Once a strategy function, Q(t), is selected, curves across the demand, premium, profit, required surplus and market return surfaces, as well as a single point (at each value of t) on the expected return curve, are determined. This occurs simply by replacing the variable q with Q(t), reducing each equation to a function of time, alone.

Line integrals over the paths implied by the selected strategy can provide useful information about key values for the firm over the course of the underwriting cycle. For example, the integral from 0 to t of Q(t) divided by t gives the average exposure level over the interval between 0 and t. The integral of $P_r(Q(t),t)$ with respect to time gives the expected gain in surplus arising from the strategy Q(t). Similarly, one can calculate the average profit, required surplus, premium to required surplus ratios, and market return resulting from the strategy Q(t) over a given period of time. Derivatives can also be helpful. The derivative of Q(t) with respect to time, for example, can provide a feel for the extent of growth.

In the following sections, we will consider five types of strategy functions. The first two are very simple forms derived by holding exposures and price, respectively, constant over time. The last three sets of strategy functions are derived from return functions for profit, return on premium, and return on required surplus. For each type of return measurement, two types of strategies will be shown corresponding to goals of maximizing return or targeting specific return levels. Of course, other strategies could be selected besides the those implied by these three return measures, for example, the strategy we mentioned earlier of minimizing price while holding surplus constant. Additional strategies will be considered later.

STRATEGY FUNCTIONS

Hold Exposures

This is the simplest of all strategy functions,

$Q(t) = k$; k, a fixed exposure level.

Hold Price

Price is held at a constant level over the course of the underwriting cycle.

$P = aQ(t) + b = k$, or

$Q(t) = (k - b)/a$, for k equal to a fixed price level.

Profit Return

In our simple model, profit which equals premium - loss - variable expense - fixed expense is a function of q and t.

At a particular point in time, t,

$P_r(q,t) = (a*q + b)*q*(1-v) - E\{L\}*q - f$

To find the exposure level, q, which will maximize $P_r(q,t)$ at t, set the partial derivative with respect to q, $\partial[P_r(q,t)]/\partial q$, equal to zero and solve for q.

$\partial[P_r(q,t)]/\partial q = 2*a*(1-v)*q + b*(1-v) - E\{L\}$

$Q(t) = (E\{L\} - b*(1-v))/(2*a*(1-v))$

Note that this strategy, like the Hold Exposure and Hold Price strategies is defined at every point in time.

Solving for a target profit level requires finding the roots of the equation $P_r(q,t) - k = 0$, for target value k.

$a*(1-v)*q^2 + (b*(1-v)-E\{L\})*q - (k+f) = 0$

$Q(t) = [E\{L\}-b*(1-v) \pm \sqrt{(E\{L\}-b*(1-v))^2 + 4*a(1-v)*(k+f)}]/2*a*(1-v)$

Under normal conditions, either both roots will be positive or no roots will exist. This is because the q^2 and constant coefficients of the quadratic equation are always negative, while the q term is ordinarily positive. To see this, consider the market conditions needed to achieve a target profit. Certainly the maximum possible market price, b, must be greater than the firm's shutdown price, E{L}/(1-v).[4]

$b > E\{L\}/(1-v) \Rightarrow b*(1-v) > E\{L\}$, since $(1-v) > 0$

This implies that two different strategies can be selected resulting in the same target profit. The choice of which of the two exposure levels is more appropriate depends on the firm's actual surplus and desired leverage. The larger exposure root will require more surplus to meet the ruin requirement, but generally provide a higher premium to required surplus ratio at a much lower price.

Note that all target-type strategies will be undefined when the maximum return allowed by the market is lower than the target return. For the purposes of having well-defined "target" strategies, we will assume the maximum strategy when maximum return is less than target return.

Return on Premium

When R_m equals return on premium,

$R_m(q,t) = P_r(q,t)/P_m(q,t) = ((a*q+b)*q*(1-v)-E\{L\}*q - f)/(a*q+b)*q$

To maximize return on premium, set $\partial[R_m(q,t)]/\partial q = 0$.

$\partial[R_m(q,t)]/\partial q = (a*E\{L\}*q^2 + 2*a*f*q + b*f)/(a*q+b)^2*q^2$

the partial derivative will equal zero at the roots of the numerator. Since the coefficient for q^2 is negative (since $a<0$) and the constant term is positive there is exactly one positive root.

$Q(t) = [-2*a*f + \sqrt{(2*a*f)^2 - 4*a*E\{L\}*b*f}]/2*a*E\{L\}$

Solving for a target return on premium requires finding the solutions to the equation $R_m(p,q) = k$.

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[4] The shutdown price actually declines with Q(t), approaching E{L}/(1-v) as $Q(t) \rightarrow \infty$. See Samuelson, Paul A. and Nordhaus, W.D., Economics, 13th edition (Manchester, MO: McGraw Hill Book Company, 1989), pp. 543-544.

$a*(k-(1-v))*q^2 + (b*(k-(1-v)) + E\{L\})*q + f = 0$

Again it can be argued that under normal market conditions this equation will have either two positive roots or none (if the maximum ROR is less than the target value). Thus, as with target profit, when the target strategy is defined, the strategy will have to be selected from among two different exposure options.

Return on Required Surplus

When R_m equals return on required surplus,

$R_m(q,t) = P_r(q,t)/S_r(q,t) = ((a*q+b)*q*(1-v)-E\{L\}*q-f)/(L_r - (a*q+b)*q*(1-v)+f)$

To maximize return on premium, set $\partial[R_m(q,t)]/\partial q = 0$.

To simplify the solution of this equation, approximate L_r by the equation

$(E\{L\}/p)*(p*q + Z_r*\sqrt{q*p*(1-p)})$; for p equal to the binomial probability, q equal to exposure level, and r equal to the ruin probability. Z_r denotes the number of standard deviations above the mean of the standard normal function corresponding to a cumulative probability of 1-r. The approximation is very good if p*q is greater than 5.

To further simplify the arithmetic, define a function G(q):

$G(q) = (E\{L\}/p)*Z_r*\sqrt{q*p*(1-p)}$

$R_m(q,t) = P_r(q)/(G(q)-P_r(q))$

$\partial[R_m(q,t)]/\partial q = (\partial[P_r]/\partial q*G-P_r*\partial[G]/\partial q)/(P_r+G)^2 = 0$

Clearly, the maximum will be reached when

$\partial[P_r]/\partial q*G-P_r*\partial[G]/\partial q = 0$, or

$3a(1-v)*q^2 + 2*(b(1-v)-E\{L\})*q + f = 0$

Again, since $a<0$ and $f>0$, there is exactly one positive root for each value of t. This root defines the strategy function Q(t).

Solving for a target return on required surplus requires finding the solutions to the equation $R_m(p,q) = k$.

$(1+k)*a*(1-v)*q^2 + (1+k)*(b*(1-v) - E\{L\})*q - k*E\{L\}/p*Z_r*\sqrt{qp*(1-p)} - (1+k)*f = 0$

This equation has no more than two positive roots which can be found by numerical techniques. Again, if there are defined solutions, a choice must be made on which exposure strategy will be taken.

EXAMPLE OF SIMPLIFIED MODEL

Assume, as in our earlier example:

1. Loss Cost: $E\{L\} = \$80$
2. Expenses: $v = 20\%$; $f = \$1,333$
3. Binomial probability: $p = .20$
4. Demand curve: $P = -.33q + 150 + 33.33*Sin(t*\pi/4)$

To simplify the following examples, we are assuming that only b varies with time, i.e. over the course of time the demand curve shifts up and down but does not change slope. The shifting demand curve is intended to simulate the underwriting cycle. In fact, for the insurance economy as a whole, it can be argued that the cycle is caused by firms raising or lowering rates and not by shifts in aggregate demand since demand is virtually fixed.[5] For the individual firm, however, demand is not fixed and fluctuations in competitors price levels are felt as shifts in the demand curve.

Hold Exposures

$Q(t) = 100$

$P_m(Q(t),t) = 100*(-.33*100 + (150+33.33*Sin(\pi*t/4)))$

$= 11,667 + 3,333*Sin(\pi*t/4)$

Avg. Price $= \int_0^8 [P_m(Q(t),t)\,dt] / \int_0^8 [Q(t)\,dt] = 116.67$

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5 See Feldblum, Sholom, "Underwriting Cycles and Business Strategies", 1990 CAS Forum, pp. 80-81

$P_r(Q(t),t) \quad = 100*P_m(Q(t),t)*.8 - 80*100 - 1,333$

$= 2,667*Sin(\pi*t/4)$

Surplus Gain $= \int_0^8 |P_r(Q(t),t)\, dt| = 0$

$S_r(Q(t),t) \quad = |80*100 + (80/.2)*2.326*\sqrt{100*.2*.8}| - 80*100 - P_r(Q(t),t)$

$= 3,722 - 2,667*Sin(\pi*t/4)$

Avg. Req Sur $= \int_0^8 |S_r(Q(t),t)\, dt| / \int_0^8 |dt| = 3,722$

Avg. Pm/Sr $= \int_0^8 |P_m(Q(t),t)\, dt| / \int_0^8 |S_r(Q(t),t)\, dt| = 3.13$

For this set of strategy functions characterized by constant exposures, there are only two values of k that set surplus gain equal to zero. Since surplus gain is the integral of profit over the underwriting cycle, and $P_r(t) = -.267*k^2 + 40*k - 1,333 + 26.67*k*Sin(\pi*t/4)$, it follows that the roots of $-.267*k^2 + 40*k - 1,333 = 0$ are the only constant exposure levels which will set surplus gain equal to zero. Of these two roots, k=100 and k=50, k=100 provides the lower average price level. Graphs and Tables 1.1 -1.5 depict these curves and their key values graphed over one complete underwriting cycle.

Hold Price

Assume Price, $a*Q(t) + b$, equals 116.67.

$Q(t) = (116.67 - (150 + 33.33*Sin(\pi*t/4)))/-.33$

$= 100*(1+Sin(\pi*t/4))$

Avg. Exposur $= \int_0^8 |Q(t)\, dt| / \int_0^8 |dt| = 100$

$P_m(Q(t),t) \quad = (Price)*Q(t) = 11,667*(1+Sin(\pi*t/4))$

Avg. Price $= \int_0^8 |P_m(Q(t),t)\, dt| / \int_0^8 |Q(t)\, dt| = 116.67$

$P_r(Q(t),t) = 100*P_m(Q(t),t)*.8 - 80*Q(t) - 1,333$

$= 1,333*\mathrm{Sin}(\pi*t/4)$

Surplus Gain $= \int_0^8 |P_r(Q(t),t)\,dt| = 0$

$Sr(Q(t),t) = [80*Q(t) + (80/.2)*2.326*\sqrt{Q(t)*.2*.8}] - 80*Q(t) - P_r(Q(t),t)$

$= 3,722*\sqrt{(1+\mathrm{Sin}(\pi*t/4))} - 1,333*\mathrm{Sin}(\pi*t/4)$

Avg. Req Sur $= \int_0^8 |S_r(Q(t),t)\,dt| / \int_0^8 |dt| = 3,351$

Avg. P_m/S_r $= \int_0^8 |P_m(Q(t),t)\,dt| / \int_0^8 |S_r(Q(t),t)\,dt| = 3.48$

Graphs and Tables 2.1 - 2.5 depict these strategies.

As with Hold Exposure strategies, for the set of strategies characterized by constant price levels, there are only two paths which hold surplus constant over the underwriting cycle, P=116.67 and P=148 (when Q(t) is not constrained to be greater than or equal to 0, P=150 is actually the higher price root, not P=148). Here we see that the minimum price level surplus strategy (MPLSS) for the Hold Price set of strategies has the same minimum price as the MPLSS strategy for Hold Exposures. While the minimum prices may be equal, required surplus is not. For the Hold Exposure MPLSS strategy, required surplus varies from 6,388 at the bottom of the cycle to 1,055 at the top of the cycle, averaging 3,722 throughout. Compare this to the Hold Price MPLSS strategy where required surplus varies from 3,920 at the top of the cycle to 1,333 at the bottom, averaging 3,351 throughout.

These are two different strategies that yield the same average profit to the firm at the same average price to the policyholder and yet demonstrate widely different risk levels. If surplus is more likely to be depleted at the bottom of the cycle (the point in the cycle where the greatest number of insolvencies occur[6]), clearly the Hold Price strategy is preferred. If investors want to take as much of their money elsewhere during the low return part of the cycle without increasing ,the risk to their remaining insurance investments, again the Hold Price strategy is preferred. On the other hand, the swing in exposure levels necessitated by the Hold Price strategy (200 to 0) raises difficult overhead expense and staffing issues not present in the hold exposure scenario.

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[6] See "Best's Insolvency Study: Property/Casualty Insurers 1969-1990," Best's Review, August 1991. p.16

Maximum Profit

$Q(t) = (80 - (150+33.33*Sin(\pi*t/4))*.8)/(2*-.33*.8)$

$= 75 + 50*Sin(\pi*t/4)$

Avg Exposure $= \int_0^8 [Q(t)\, dt] / \int_0^8 [dt] = 75$

$P_m(Q(t),t) = 9{,}375 + 7{,}500*Sin(\pi*t/4) + 833.33*Sin^2(\pi*t/4)$

Avg. Price $= \int_0^8 [P_m(Q(t),t)\, dt] / \int_0^8 [Q(t)\, dt] = 130.56$

$P_r(Q(t),t) = 666.67*Sin^2(\pi*t/4) + 2{,}000*Sin(\pi*t/4) + 166.67$

Surplus Gain $= \int_0^8 [P_r(Q(t),t)\, dt] = 4{,}000$

$S_r(Q(t),t) = [80*Q(t) + (80/.2)*2.326*\sqrt{Q(t)*.2*.8}] - Pr(Q(t),t)$

$= 3{,}72.16*\sqrt{75+50*Sin(\pi*t/4)} - P_r(Q(t),t)$

Avg. Req Sur $= \int_0^8 [S_r(Q(t),t)\, dt] / \int_0^8 [dt] = \$2{,}622$

Avg. P_m/S_r $= \int_0^8 [P_m(Q(t),t)\, dt] / \int_0^8 [S_r(Q(t),t)\, dt] = 3.73$

The equations for maximum return on premium, maximum return on required surplus as well as the target return goals are increasingly complex and are not shown here. Graphs and Tables 3.1 - 3.5 display the key information for the three maximum return goals.

Graphs and Tables 4.1 - 4.5 show the key values for the target return goals. The actual target levels have been selected for these examples to provide zero surplus gain at the higher exposure root (i.e, the MPLSS for each type of target return). As I mentioned earlier, the target strategies that result are actually a combination of target and maximum strategies. When the maximum return allowed by the market falls below the target level during the down part of the cycle (i.e. the instantaneous demand curve lies below the target $R_e(q)$ curve), the target strategy seeks the highest return level which has an $R_e(q)$ curve tangent to the demand curve. This, of course, is the exposure level dictated by the maximum return strategy.

The following table summarizes the key indicators for the 5 sets of strategies considered above:

TABLE D: DIMENSION VALUES FOR EXAMPLE STRATEGIES

	HOLD EXPOS	HOLD EXPOS	HOLD PRICE	HOLD PRICE	MAXIMUM			ZERO GAIN TARGET		
DECISION DIMENSIONS	@100	@75	@117	@130	PROFIT	ROR	RORS	PROFIT	ROR	RORS
RANGE IN EXPOSURES	0	0	200	159	100	19	50	186	141	156
AVERAGE # OF EXPOS	100	75	100	67	75	72	74	102	111	109
RANGE IN PROFITS	$2,666.	$4,000.	$2,668.	$3,865.	$4,000.	$3,828.	$4,000.	$2,000.	$2,747.	$2,417.
SURPLUS GAIN	$0.	$1,333.	$0.	$2,357.	$4,000.	$2,222.	$3,333.	$0.	$0.	$0.
RANGE IN PRICE	$67.	$66.	$0.	$0.	$33.	$60.	$50.	$17.	$30.	$26.
AVERAGE PRICE	$117.	$124.	$117.	$130.	$130.	$127.	$129.	$116.	$115.	$115.
RANGE IN REQ SUR	$5,333.	$4,000.	$2,597.	$1,421.	$1,929.	$3,407.	$2,910.	$1,546.	$1,089.	$1,247.
AVERAGE REQ SUR	$3,722.	$3,056.	$3,351.	$2,115.	$2,622.	$2,861.	$2,760.	$3,523.	$3,795.	$3,735.
AVG PREM/RS RATIO	3.13	3.07	3.48	4.12	3.73	3.2	3.47	3.35	3.34	3.35

A few observations on these strategies:

- When the premium to required surplus ratio is high, it indicates that relatively less surplus is required to safely back up a given amount of premium. This happens naturally as exposure levels increase due to two processes: (1) the loss portion of required surplus increases in proportion to the square root of exposures while premium at a given price level increases directly with exposures and, (2) for the strategies considered above, exposures increase during the up part of the cycle when price is rising and thus the profit contribution to required surplus is declining. When comparing two strategies with very different average P_m/S_r ratios, generally the strategy with the higher value is less risky (e.g. compare average and variation in S_r for maximum profit and maximum ROR). The same is true in comparing loss processes. A loss process with a lower coefficient of variation generally has higher P_m/S_r ratios for similar strategies. Graph 5 illustrates this principle.
- It is often felt that strategies which require larger changes in exposure levels are inherently riskier.[7] If exposures levels rise and decline in phase with the underwriting cycle;

[7] Companies experiencing unusual premium growth accounted for 81% of all insolvencies. Ibid. pp.19-20.

increasing when price is high and decreasing when price is low, this variation may minimize risk. Take the maximum profit Hold Exposure strategy. This strategy returns one-third the surplus gain and requires a much wider range and higher average surplus than the maximum profit strategy. Clearly it has a higher risk. Obviously, strategies which cause variations in growth out of phase with the underwriting cycle may indeed increase risk (e.g. when companies enter the market during the down-turn and then attempt to buy exposures[8]).

- Seemingly important parameters can change without altering strategy curves for a given goal. For example, when the coefficient of variation of the loss process is increased, the premium to required surplus ratio declines but the strategy curve remains the same. The strategy curve is dictated by the return goal, the firm's expected expense and loss structure, and the market demand, not the variability of the loss process. As another example, suppose the size of the firm is increased. Again, the premium to required surplus ratio will go up[9] but the strategy curve will not change (see tables and graphs 6.1 - 6.5). This presumes that the fixed expense structure increases in proportion to the size of the firm (i.e. the firms expected expense structure is unaltered). If economies of scale are assumed, the firms expected expense structure is altered and the strategy curves will begin to change shape (interestingly, assuming 50% economies of scale on fixed expense, both the maximum profit and maximum RORS strategies improve dramatically relative to the maximum ROR approach. See graphs and tables 7.1 - 7.5).
- Referencing back to the introduction, our model shows that neither the "hold-exposure" strategy dominant in the last cycle nor the "hold-price" strategy which many firms have adopted this time around could be considered ideal from a maximizing return perspective. The maximum profit obtainable under a "hold-exposures" strategy, for example, falls well short of the maximum profit strategy ($1,333 vs. $4,000) at a higher average risk level. The maximum profit obtainable under a "hold-price" strategy fares better, providing close to two-thirds the maximum profit at a slightly lower risk level.

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8 Ibid.

9 Consistent with Best's finding that failure frequency decreased with size of company. Ibid.

- It is desirable to be able to approximate the P_m/S_r for a given firm, utilizing a given strategy in a line with a given coefficient of variation at a given point in the underwriting cycle. This sort of information would allow regulators and firms alike to monitor the safety of a firm's course of action. Essentially, if we symbolize the premium to required surplus ratio sought by: $P_m/S_r(F_w,S_x,C_y,t_z)$; F,S,C,t=firm size, strategy, coefficient of variation, and time respectively, then we are seeking: $P_m/S_r(F_w,S_x,C_y,t_z)=f(f_1(F),f_2(S),f_3(C),f_4(t))$. Determining the form of $f,...f_4$ is left as an open-ended problem. Ideally, $f,...f_4$ could be replaced with constants or simple functions of t without sacrificing much accuracy. Unfortunately this does not seem to be possible with any of the strategy sets considered in this paper.
- The 1991 Best's Insolvency study[10] identified five firm characteristics significantly related to insolvencies: size, ownership, personal versus commercial, age and growth. While P_m/S_r ratios clearly increase for larger firms and firms writing lines with lower coefficients of variation (which would simplistically differentiate between personal and commercial lines), increased risk of insolvency due to age, growth or ownership doesn't fall out so cleanly from this analysis. I've already mentioned that growth in phase with the cycle lowers risk. None of the strategies here illustrate growth out of phase with the cycle. Of the strategies considered here, the fixed exposure strategies, ironically enough, give the clearest idea of how required surplus would respond if growth were to occur out of phase. Not only would the range in required surplus widen but the maximum required surplus would soar during the soft part of the market. The maximum strategies give no insight into the higher failure rate of stock versus mutual firms. The maximum profit strategy (which I've ascribed to stock companies) is actually less risky than maximum ROR which I've ascribed to mutuals. Assumptions could certainly be made about stock firm strategies that would imply greater risk, e.g. inclination to resist falling revenues during the soft market or greater willingness to write risky lines of business, but these would be conjecture only.

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[10] Ibid.

Expected Return Pricing

Again, I would like to provide a contrast between expected value pricing, devoid of any consideration of market forces, and the market return perspectives discussed above. Graph 8.1 shows $R_c(q)$ curves (set at the target levels shown on Tables 4.1 - 4.5) for the three return perspectives compared to the demand curve at the top of the cycle. At this point in the cycle, the target price, where $R_c(q)$ intersects the demand curve, is attainable. Here, the market demand forces interact with expected return pricing only to the extent that exposure choices and thus price levels are limited.

Eventually the demand curve drops below the $R_c(q)$ curve and expected return pricing is impossible. Graphs and Tables 8.2 - 8.5 illustrate the profit and required surplus levels that would result if the firm could get the $R_c(q)$ price at each value of Q(t) for the MPLSS target strategies. Of course, during the down part of the market this return is fictional, providing a distorted picture of overstated profits and understated required surplus levels. Risk level is misperceived to be much smaller than it actually is. During the up part of the cycle when the exposure level Q(t) falls in the feasible region of the $R_c(t)$ curve, expected profit and required surplus values are attainable and thus equal to the target return values.

IDEAL STRATEGIES

Given that for any goal there is a set of strategies which achieve the goal (the set may have only one member, e.g. maximum profit), it is natural to seek some other characteristics besides the ostensible goal to rank these qualifying strategies and select one best or "ideal" strategy. I will call these ranking characteristics, secondary dimensions. I will call the characteristics used to define the original goal, primary dimensions. The following is a brief list of dimensions which could be used to define goals and rank qualifying strategies for the ten strategies shown above. Of course, every firm may have a different list of key dimensions.

- Range in Exposure levels: a wide range adds instability to firm staffing levels, overhead expenses, and loyalty of potential long-term policyholders. In a strict regulatory environment, dramatic swings in exposure levels could be a cause for detailed examination. Smallest variation scored as a "1".
- Average number of Exposures: the higher the exposure level the lower the price (due to the market demand dynamic). From a policyholder's perspective, highest average is scored as a "1".
- Range in Profits: investors look for steady returns (if firm is a stock firm) or policyholders look for steady dividends (if firm is a mutual firm). Smallest variation scored as a "1".
- Surplus Gain: from the perspective of firm solvency and return to long-term stockholders, the higher the better. From the perspective of policyholders, lower is better if it ensures lower prices, although not so low as to threaten insolvency. Highest scored as a "1".
- Range in Price: the narrower the range, the greater the stability for long-term policyholders and the firm's image in the marketplace. In a strict regulatory environment, it may be difficult to change price frequently or charge a wide range of price. Smallest variation scored as a "1".
- Average Price: from the standpoint of firm solvency and return to long-term stockholders, the higher the price the better (assuming efficient expense structures). To policyholders, the lowest price that doesn't threaten solvency is preferred. Lowest scored as a "1".
- Range in Required Surplus: a narrow range is more stable from a firm planning perspective and capital attraction standpoint (if a stock firm). Assume hypothetically that the entire surplus is supported by one investor who seeks to invest only up to the required amount and then reinvest elsewhere. To obtain the highest investment returns elsewhere (the long-term rates), he needs to know the precise surplus required for the insurance investment. Thus a narrow range is preferable. In the context of many investors, a narrow range is more conducive to efficient capital allocation. Smallest variation scored as a "1".
- Average Required Surplus: the lower the average, the more likely that actual surplus will be sufficient. The lower the required surplus for a given strategy, the more accessible the

strategy will be to similar sized companies subject to the same demand curve (i.e. the more competitive the market). Lowest scored as a "1".

TABLE E: DIMENSION RANKING FOR EXAMPLE STRATEGIES

	HOLD EXPOS	HOLD EXPOS	HOLD PRICE	HOLD PRICE	MAXIMUM			ZERO GAIN TARGET		
DECISION DIMENSIONS	@100	@75	@117	@130	PROFIT	ROR	RORS	PROFIT	ROR	RORS
RANGE IN EXPOSURES	1	1	10	8	5	3	4	9	6	7
AVERAGE # OF	4	6	4	10	6	9	8	3	1	2
RANGE IN PROFITS	3	8	3	7	8	6	8	1	5	2
SURPLUS GAIN	6	5	6	3	1	4	2	6	6	6
RANGE IN PRICE	10	9	1	1	6	8	7	3	5	4
AVERAGE PRICE	3	6	3	9	9	7	8	3	1	1
RANGE IN REQ SUR	10	9	6	3	5	8	7	4	1	2
AVERAGE REQ SUR	8	5	6	1	2	4	3	7	10	9
AVG PREM/RS RATIO	9	10	3	1	2	8	4	5	5	5

Naturally, these strategies have been designed to achieve different goals and thus are not directly comparable. For example, it isn't fair to compare a target strategy that has been designed to maintain constant surplus with a strategy designed to maximize profit along the surplus gain dimension. That's precisely the point. There are a host of potential goals, each goal directed towards maximizing the result along one or more dimensions. Once the primary dimensions (e.g. minimum price with no surplus gain) have been selected, one seeks to find the set of all strategies equivalent along these primary dimensions. These "qualifying" strategies can then be contrasted along selected secondary dimensions to choose the "ideal" strategy.

For example, policyholders may seek a strategy which minimizes price with no appreciable increase to the risk of insolvency. If this is interpreted as MPLSS, and the set of strategies the policyholder can pick from is limited to that shown above, two of the above strategies would qualify. Thus two strategies exist in our primary dimension set. Looking along the secondary dimensions, target ROR provides a tighter exposure range and higher average exposure level. Target RORS has a tighter profit range, and lower average required surplus. A stock firm policyholder may put greater weight on lower required surplus levels and tighter profit ranges feeling these characteristics will lower the risk of stockholder capital outflow, and therefore insolvency. Mutual policyholders may put greater weight on maximizing dividends in the up part of the cycle without dislocating policyholders and thus choose the target ROR approach. Ultimately, the choice of secondary dimensions depends on one's perspective.

TABLE F: IDEAL STRATEGIES FOR VARIOUS PERSPECTIVES

PERSPECTIVE	GOAL	PRIMARY DIMENSIONS	SECONDARY DIMENSIONS	IDEAL STRATEGY
STOCKHOLDER	MAXIMIZE PROFIT	SURPLUS GAIN	ONLY ONE PRIMARY STRATEGY	MAX PROFIT
MINIMUM INVESTOR	MAXIMIZE RORS	RORS	ONLY ONE PRIMARY STRATEGY	MAX RORS
MUTUAL FIRM	MAXIMIZE ROR	ROR	ONLY ONE PRIMARY STRATEGY	MAX ROR
MUTUAL POLICYHOLDER	MAX ROR FOR MINIM PRICE W/ZERO SUR GAIN	AVG PRICE SURPLUS GAIN AVG ROR	RANGE IN ROR EXPOSURE RANGE PRICE RANGE	TARGET ROR = 5.2%
STOCK POLICYHOLDER	MINIMIZE PRICE W/ZERO SUR GAIN	AVG PRICE SURPLUS GAIN	AVG REQ SURPLUS RANGE REQ SURPLUS RANGE PROFITS	TARGET RORS = 5.0%
REGULATOR	MINIMIZE PRICE W/CON-STRAINTS	AVG PRICE SURPLUS GAIN PRICE RANGE EXPOSURE RANGE TARGET RETURN RANGE	AVG REQ SURPLUS RANGE REQ SURPLUS	TARGET ROR = 5.2%

CONSTRAINED STRATEGIES

In a more realistic setting, strategies can be subject to any number of external constraints. As we mentioned earlier, in a regulated environment, a target return strategy may be acceptable only if exposure and price vary within a certain range around their average values. A strategy that implies too much variation in growth may be infeasible from a practical operating standpoint, as well. The hiring and layoff of staff required over the course of the cycle, may be too extreme for the firm to handle practically or even desire. Apart from regulatory restrictions on price variation, firm systems may not be equipped to process large and frequent changes to price structure. Ruin requirements may also constrain certain strategies. If required surplus exceeds actual surplus, the risk level may be too great for the firm, investor or policyholder to bear.

Graphs and Tables 9.1 - 9.5 examine the maximum profit return goal when constraints are placed on the extent of allowable variation in exposure levels. The unconstrained maximum profit strategy, Q(t), shows a 133% variation around its average, i.e. $(Q_{MAX} - Q_{MIN})/Q_{AVG} = 133\%$. By limiting exposures on both the high and low ends of the cycle and maintaining the maximum strategy in between, we solve for the maximum profit strategies with growth constraints. Constraining growth to 50% and 25% lowers the surplus gain by 20% and 40%, respectively. When compared to the unconstrained strategy, the average premium to required surplus ratio drops (indicating increased risk, which is not surprising). However, it is interesting to note that required surplus increases precisely during the low part of the cycle, when borderline companies are most vulnerable to insolvency.

Graphs and Tables 10.1 - 10.5 consider the effect on the maximum profit strategy when price is constrained within certain limits of average price. Price varies by 26% for the unconstrained maximum profit strategy. In general, to reduce price variation without changing average price, exposure levels must be increased during the hard market (thereby driving price down when the price is high) and decreased during the soft market (thereby driving price up when the price is low). Using a multiplicative factor adjusted for location in the cycle, we reshaped the maximum profit strategy to suit 20% and 12.5% variations in price. As with the growth constraints, profit and premium to required surplus ratios dropped off significantly.

Graphs and Tables 11.1 - 11.5 consider the maximum profit goal when required surplus exceeds actual surplus at the midpoint of the cycle. Surprisingly, actual surplus can fall significantly below required surplus with very little drop-off in surplus gain. However, at a critical value below required surplus, a point is reached where the ruin requirement and the market dynamic causes exposures and actual surplus to spiral to zero. When actual surplus is above this critical value, the disturbance to the unconstrained strategy is minimal. In our example, when actual surplus = 2,344 at time equal to zero (the maximum profit strategy requires surplus of 3,056 at t=0), exposures will spiral to zero before t=1. However, if actual surplus = 2,345, just one dollar more, then actual surplus will grow so rapidly that exposure levels will rejoin the maximum profit strategy curve before t=1. Surplus gain will decrease by less than 15%. This phenomena depends on the location in the cycle. If actual surplus is less than required surplus at the top of the cycle, the difference will expand as the cycle heads down. Further work needs to be done to show how the critical value as a percent of required surplus varies over the course of the cycle.

CONCLUSION

Actuarial or "expected return" pricing, by ignoring the price demand forces at work in the market, squanders the opportunity to determine strategies most likely to achieve the firm's goals in the real economic world. In reality, insurance firms are faced with an ever changing economic environment the terrain of which must be considered and understood before attempting to find the optimal course to a particular location. Extreme goals such as holding exposures or fixing price regardless of the cost to profit or size of book will not optimize a firm's profitability.

In this paper, I have attempted to set the terrain for an extremely simplified version of an insurance economy, define target locations or goals which could apply to different groups involved in the insurance economy, and then determine paths or strategies most likely to get one across the landscape to the target location. The goals, like the model itself, have been simplified to better illustrate the basic process. Further work needs to be done to extend the model to consider empirical size of loss curves, parameter variance, investment income, taxes, non-linear demand curves, and multi-line environments.

Target strategies were constructed and used to highlight the way in which an expected return goal is altered by the market environment and the shortfall in return that naturally results. Even when an expected return goal utilizes an optimal exposure strategy the difference in perception of profitability and risk can be hazardous.

A simple decision process was defined to help select the best or "Ideal" strategy when there was more than one path available to take the firm to the goal in question. The effect upon Ideal strategies of altering the model parameters to reflect riskier loss processes and larger firms was considered. Finally, a few complications were introduced into the model to illustrate constraints that are likely to exist in the real world such as surplus, growth and pricing limitations. The effect of these constraints on the Ideal strategy for a given goal was considered.

CURVE VALUES FOR HOLD EXPOSURES

GRAPH 1.1: EXPOSURES, Q(t)

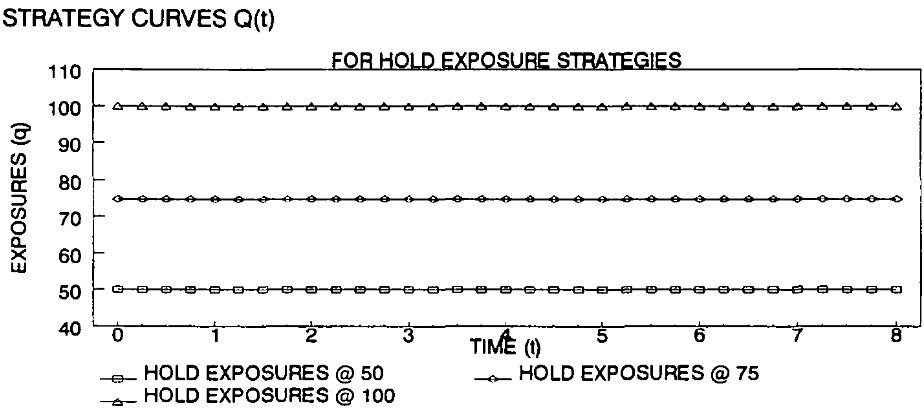


TABLE 1.1: EXPOSURES, Q(t)

	TIME =					AVG # EXPOSURE
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	
EXPOS = 50	50	50	50	50	50	50
EXPOS = 75	75	75	75	75	75	75
EXPOS = 100	100	100	100	100	100	101

GRAPH 1.2: PROFIT, Pr(Q(t),t)

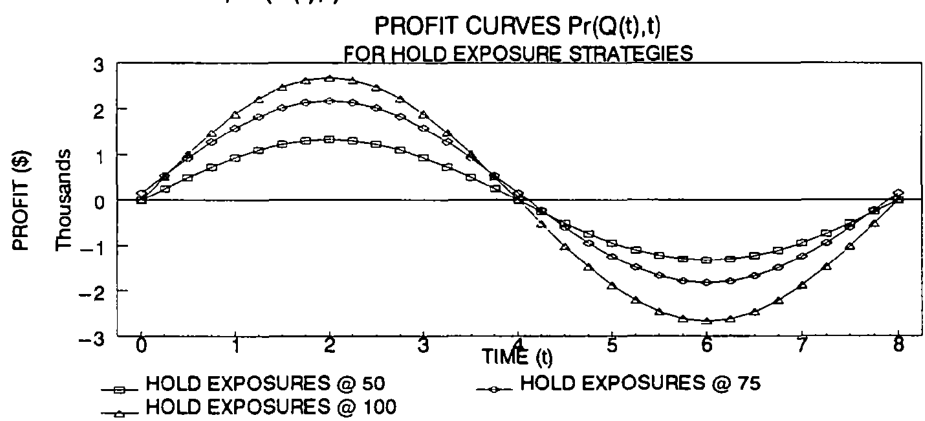


TABLE 1.2: PROFIT, Pr(Q(t),t)

	TIME =					SURPLUS GAIN
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	
EXPOS = 50	$0	$943	$1,333	($943)	($1,333)	$0
EXPOS = 75	$167	$1,581	$2,167	($1,248)	($1,833)	$1,342
EXPOS = 100	$0	$1,886	$2,667	($1,886)	($2,667)	$0

CURVE VALUES FOR HOLD EXPOSURES

GRAPH 1.3: PRICE, Pr(Q(t),t)/Q(t)

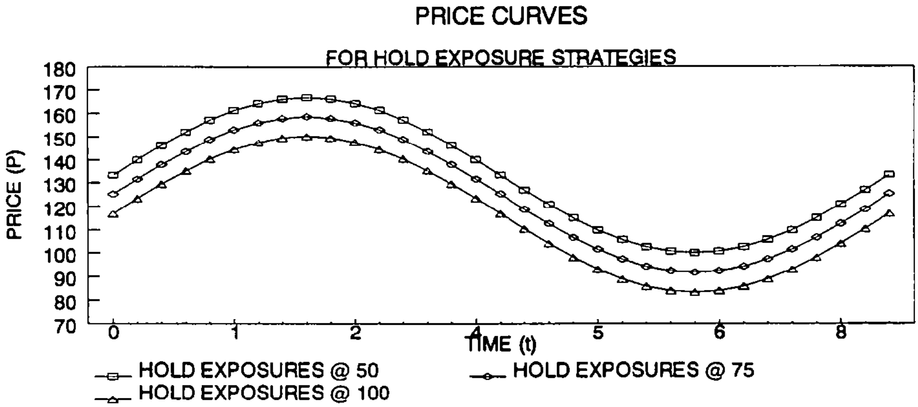


TABLE 1.3: PRICE, Pr(Q(t),t)/Q(t)

	TIME =					AVG PRICE
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	
EXPOS = 50	$133	$157	$167	$110	$100	$133
EXPOS = 75	$125	$149	$158	$101	$92	$124
EXPOS = 100	$117	$140	$150	$93	$83	$116

GRAPH 1.4: REQUIRED SURPLUS, Sr(Q(t),t)

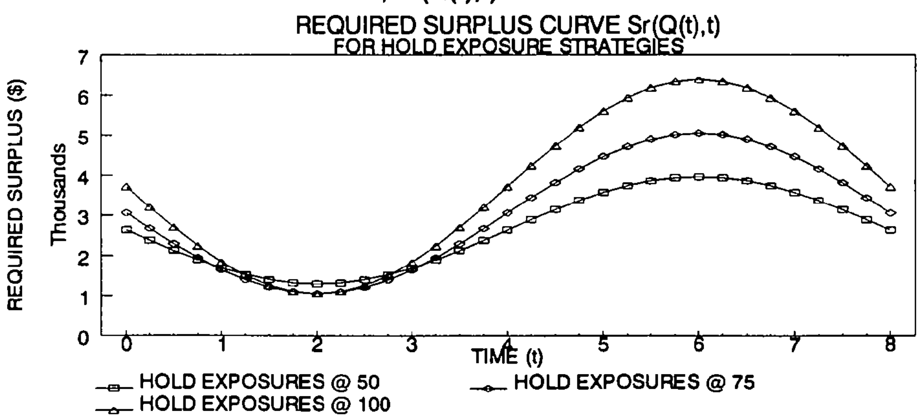


TABLE 1.4: REQUIRED SURPLUS, Sr(Q(t),t)

	TIME =					AVG REQ SUR
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	
EXPOS = 50	$2,632	$1,689	$1,298	$3,574	$3,965	$2,632
EXPOS = 75	$3,056	$1,642	$1,056	$4,471	$5,056	$3,056
EXPOS = 100	$3,722	$1,836	$1,055	$5,607	$6,388	$3,722

CURVE VALUES FOR HOLD EXPOSURES

GRAPH 1.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

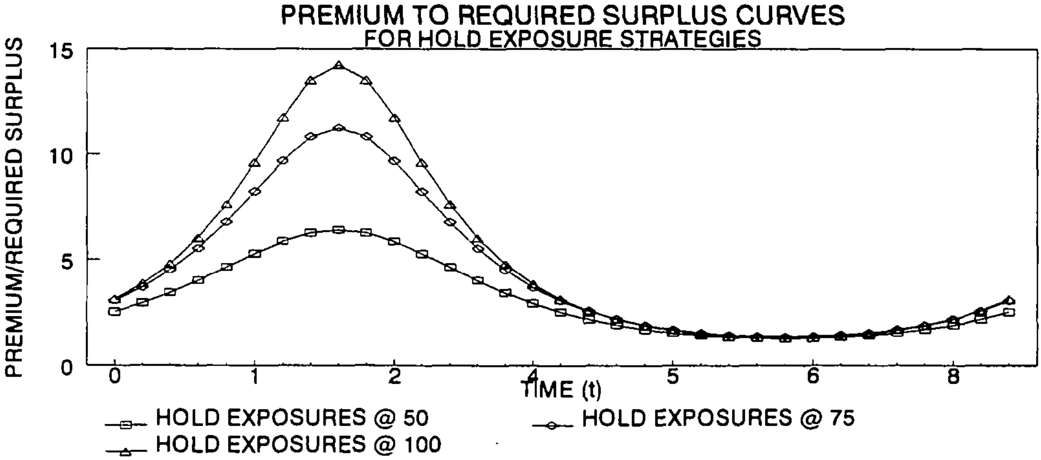


TABLE 1.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	PREM/RS
EXPOS = 50	2.53	4.65	6.42	1.54	1.26	2.53
EXPOS = 75	3.07	6.79	11.24	1.70	1.36	3.07
EXPOS = 100	3.13	7.64	14.22	1.66	1.30	3.13

CURVE VALUES FOR HOLD PRICE

GRAPH 2.1: EXPOSURES, Q(t)

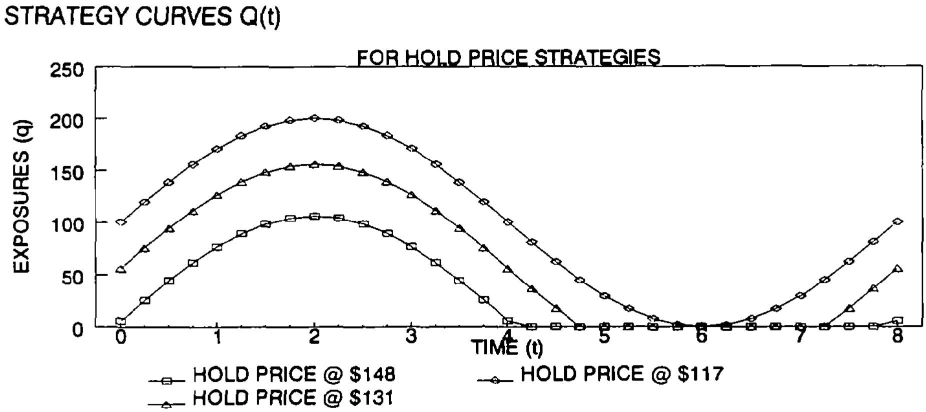


TABLE 2.1: EXPOSURES, Q(t)

	TIME =					AVG #
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	EXPOSURE
PRICE = 116.67	100	171	200	29	0	101
PRICE = 131	57	128	157	0	0	66
PRICE = 148	6	77	106	0	0	35

GRAPH 2.2: PROFIT, Pr(Q(t),t)

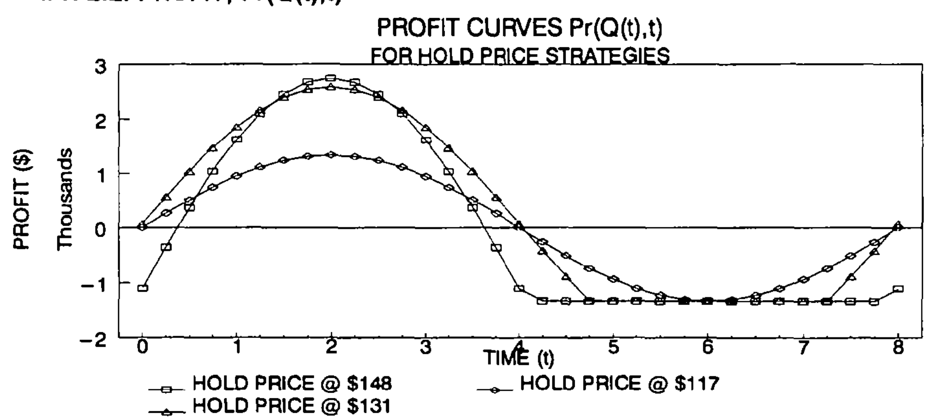


TABLE 2.2: PROFIT, Pr(Q(t),t)

	TIME =					SURPLUS
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	GAIN
PRICE = 116.67	($0)	$943	$1,333	($943)	($1,333)	($0)
PRICE = 131	$80	$1,834	$2,560	($1,333)	($1,333)	$2,364
PRICE = 148	($1,102)	$1,613	$2,737	($1,333)	($1,333)	($0)

CURVE VALUES FOR HOLD PRICE

GRAPH 2.3: PRICE, Pr(Q(t),t)/Q(t)

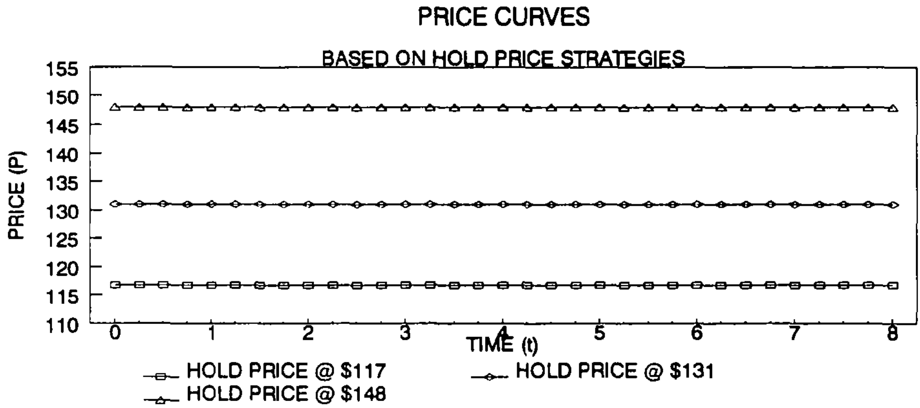


TABLE 2.3: PRICE, Pr(Q(t),t)/Q(t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	PRICE
PRICE = 116.67	$117	$117	$117	$117	$117	$116
PRICE = 131	$131	$131	$131	ERR	ERR	$130
PRICE = 148	$148	$148	$148	ERR	ERR	$148

GRAPH 2.4: REQUIRED SURPLUS, Sr(Q(t),t)

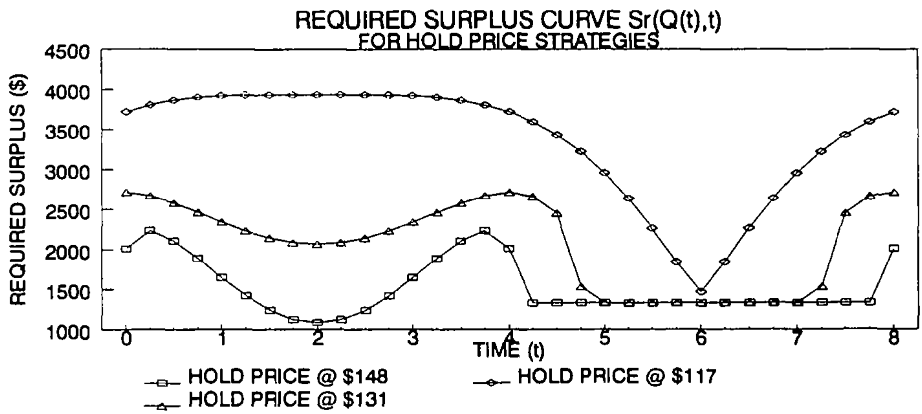


TABLE 2.4: REQUIRED SURPLUS, Sr(Q(t),t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	REQ SUR
PRICE = 116.67	$3,722	$3,920	$3,930	$2,957	$1,333	$3,351
PRICE = 131	$2,729	$2,372	$2,103	$1,333	$1,333	$2,085
PRICE = 148	$2,016	$1,647	$1,095	$1,333	$1,333	$1,507

CURVE VALUES FOR HOLD PRICE

GRAPH 2.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

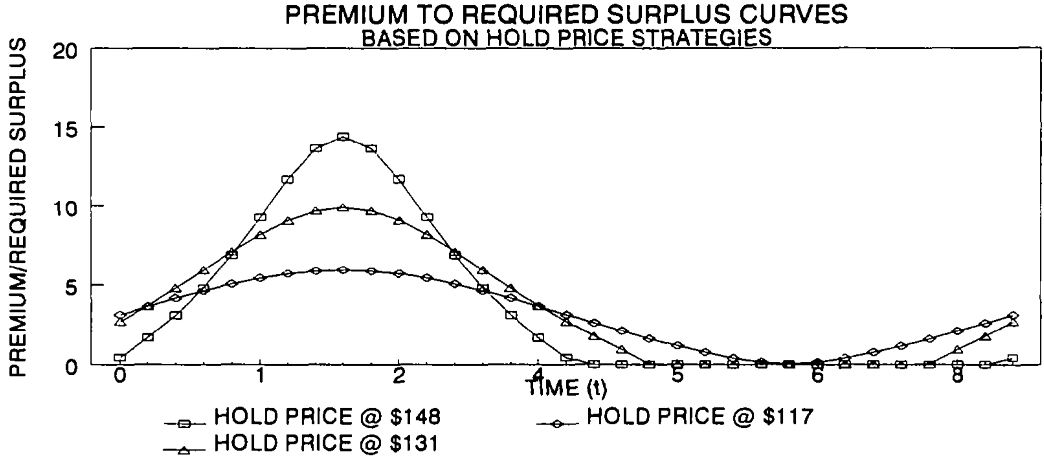


TABLE 2.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	PREM/RS
PRICE = 116.67	3.13	5.08	5.94	1.16	0.00	3.48
PRICE = 131	2.74	7.05	9.78	0.00	0.00	4.12
PRICE = 148	0.44	6.89	14.33	0.00	0.00	3.43

CURVE VALUES FOR MAXIMUM RETURN

GRAPH 3.1: EXPOSURES, Q(t)

STRATEGY CURVES Q(t)

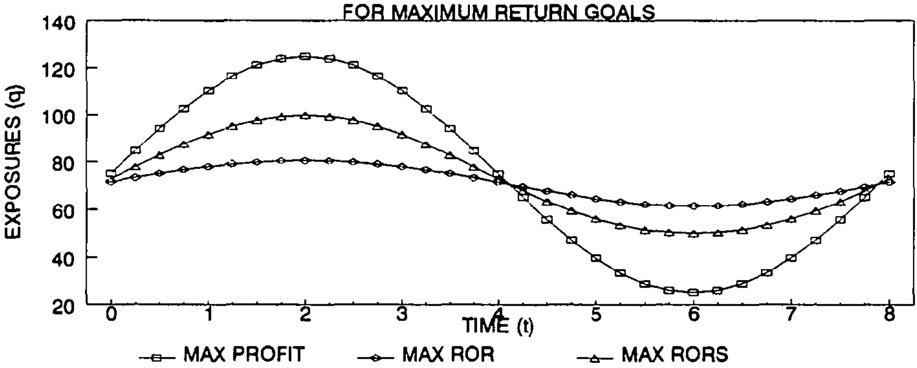


TABLE 3.1: EXPOSURES, Q(t)

	TIME =					AVG # EXPOSURE
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	
MAX PROFIT	75	110	125	40	25	75
MAX ROR	72	78	81	65	62	72
MAX RORS	73	92	100	56	50	74

GRAPH 3.2: PROFIT, Pr(Q(t),t)

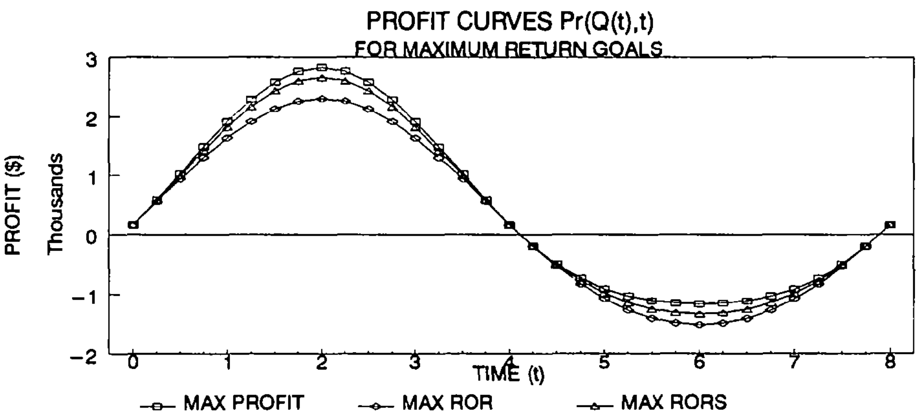


TABLE 3.2: PROFIT, Pr(Q(t),t)

	TIME =					SURPLUS GAIN
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	
MAX PROFIT	$167	$1,914	$2,833	($914)	($1,167)	$4,008
MAX ROR	$163	$1,635	$2,306	($1,080)	($1,522)	$2,228
MAX RORS	$165	$1,822	$2,667	($987)	($1,333)	$3,343

CURVE VALUES FOR MAXIMUM RETURN

GRAPH 3.3: PRICE, Pr(Q(t),t)/Q(t)

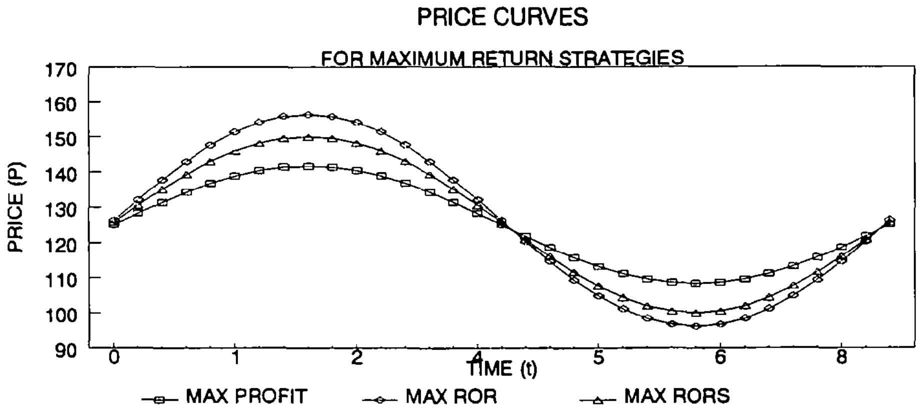


TABLE 3.3: PRICE, Pr(Q(t),t)/Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG PRICE
MAX PROFIT	$125	$137	$142	$113	$108	$130
MAX ROR	$126	$148	$156	$105	$96	$127
MAX RORS	$126	$143	$150	$108	$100	$129

GRAPH 3.4: REQUIRED SURPLUS, Sr(Q(t),t)

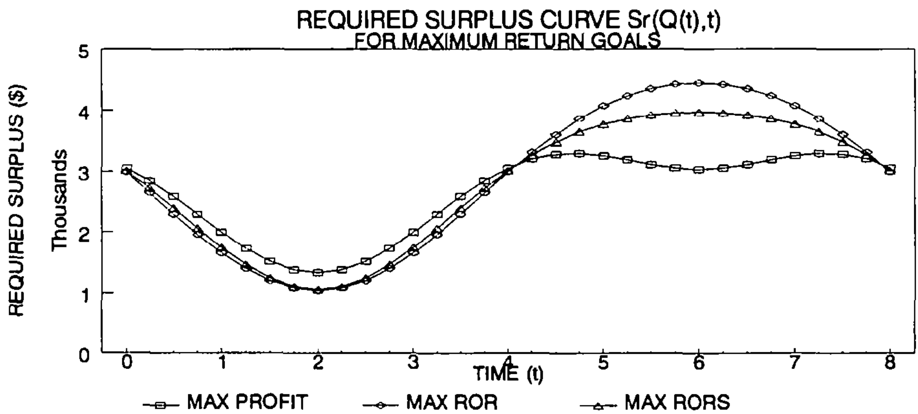


TABLE 3.4: REQUIRED SURPLUS, Sr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG REQ SUR
MAX PROFIT	$3,056	$1,995	$1,328	$3,257	$3,027	$2,622
MAX ROR	$2,984	$1,652	$1,034	$4,070	$4,441	$2,861
MAX RORS	$3,011	$1,743	$1,055	$3,775	$3,965	$2,760

CURVE VALUES FOR MAXIMUM RETURN

GRAPH 3.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

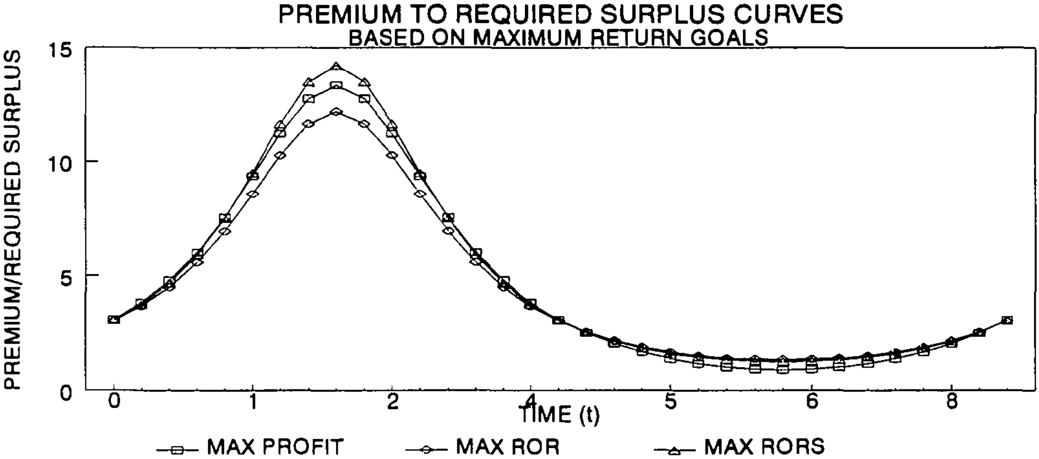


TABLE 3.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG PREM/RS
MAX PROFIT	3.07	7.57	13.34	1.38	0.89	3.73
MAX ROR	3.02	6.97	12.19	1.66	1.33	3.20
MAX RORS	3.04	7.53	14.22	1.60	1.26	3.47

CURVE VALUES FOR TARGET RETURN

TARGETS SET FOR ZERO SURPLUS GAIN

GRAPH 4.1: EXPOSURES, Q(t)

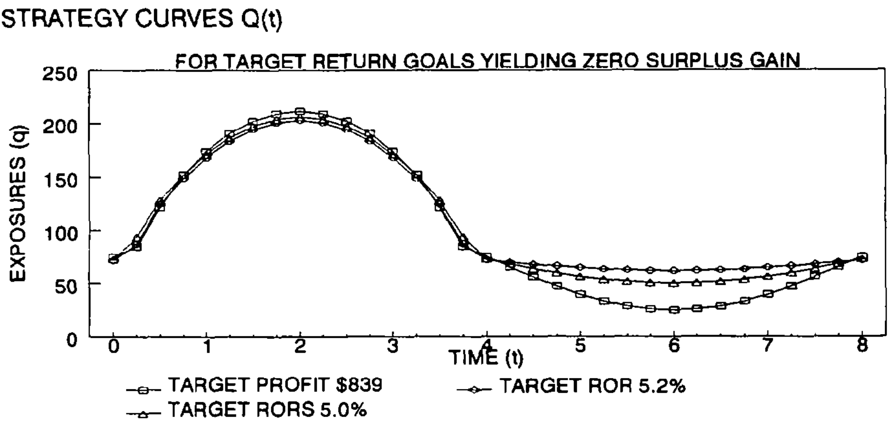


TABLE 4.1: EXPOSURES, Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG # EXPOSURE
PROFIT = 839	75	174	211	40	25	102
ROR = 5.2%	72	168	203	65	62	111
RORS = 5.0%	73	171	206	56	50	109

GRAPH 4.2: PROFIT, Pr(Q(t),t)

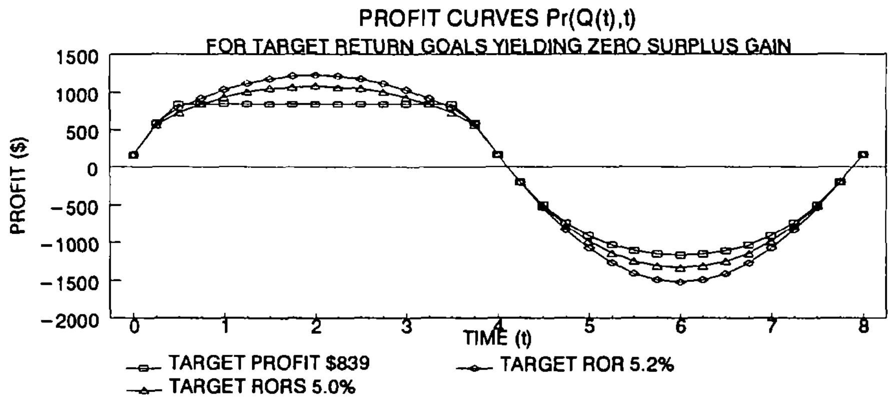


TABLE 4.2: PROFIT, Pr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	SURPLUS GAIN
PROFIT = 839	$167	$839	$839	($914)	($1,167)	($0)
ROR = 5.2%	$163	$1,031	$1,225	($1,080)	($1,522)	$0
RORS = 5.0%	$165	$933	$1,084	($987)	($1,333)	($0)

CURVE VALUES FOR TARGET RETURN

TARGETS SET FOR ZERO SURPLUS GAIN

GRAPH 4.3: PRICE, Pr(Q(t),t)/Q(t)

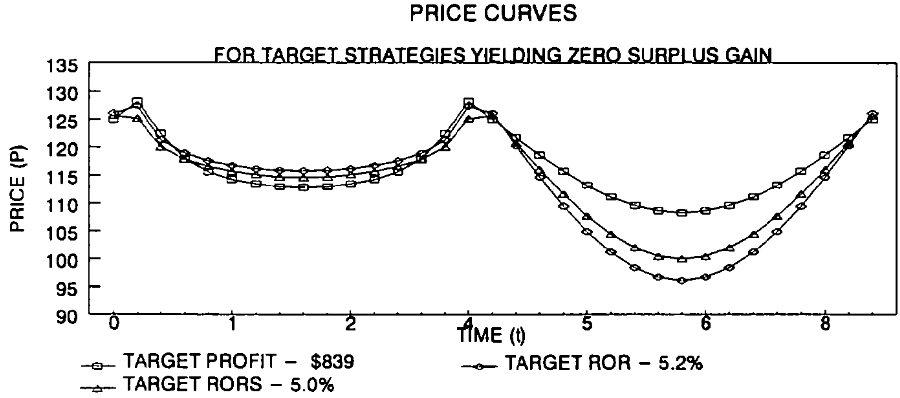


TABLE 4.3: PRICE, Pr(Q(t),t)/Q(t)

	TIME =					AVG PRICE
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	
PROFIT = 839	$125	$116	$113	$113	$108	116
ROR = 5.2%	$126	$118	$116	$105	$96	115
RORS = 5.0%	$126	$117	$115	$108	$100	115

GRAPH 4.4: REQUIRED SURPLUS, Sr(Q(t),t)

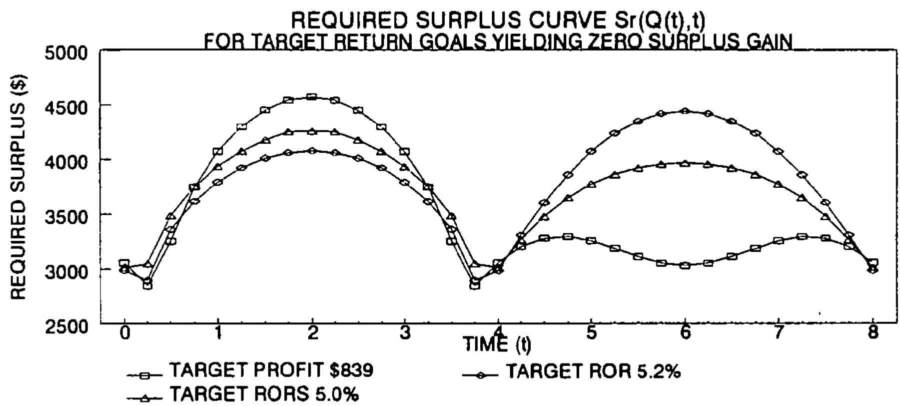


TABLE 4.4: REQUIRED SURPLUS, Sr(Q(t),t)

	TIME =					AVG REQ SUR
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	
PROFIT = 839	$3,056	$4,068	$4,573	$3,257	$3,027	$3,523
ROR = 5.2%	$2,984	$3,792	$4,073	$4,070	$4,441	$3,795
RORS = 5.0%	$3,011	$3,933	$4,258	$3,775	$3,965	$3,735

CURVE VALUES FOR TARGET RETURN

TARGETS SET FOR ZERO SURPLUS GAIN

GRAPH 4.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

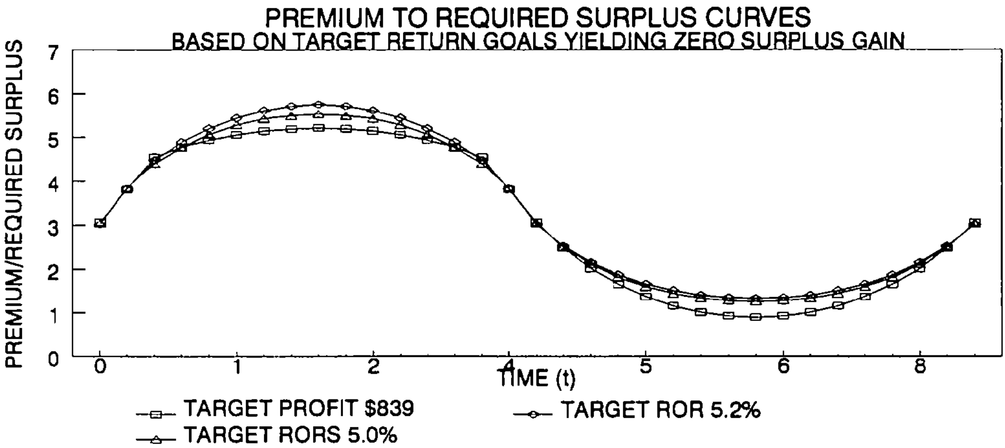
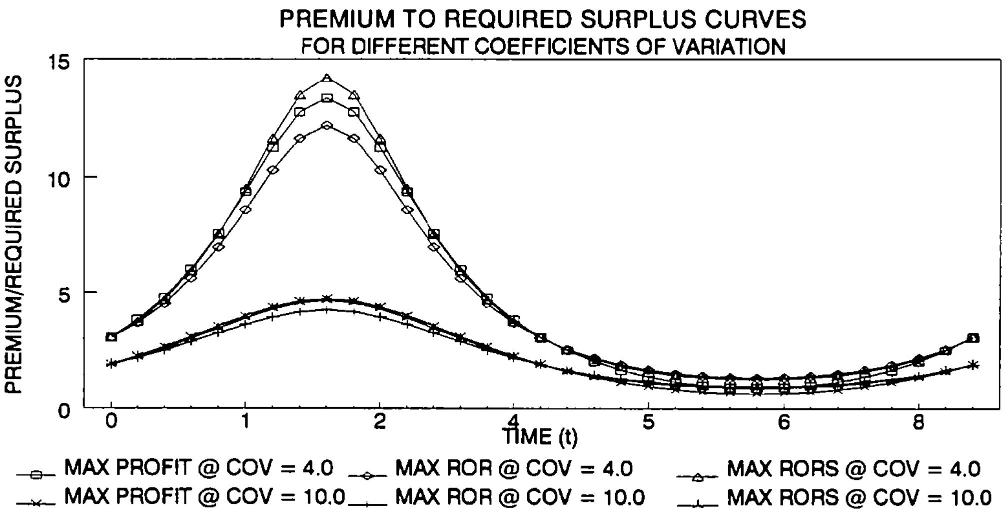


TABLE 4.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	PREM/RS
PROFIT = 839	3.07	4.94	5.22	1.38	0.89	3.35
ROR = 5.2%	3.02	5.21	5.76	1.66	1.33	3.34
RORS = 5.0%	3.04	5.07	5.55	1.60	1.26	3.35

GRAPH 5: PREM/RS FOR DIFFERENT COEFFICIENTS OF VARIATION



CURVE VALUES FOR MAXIMUM RETURNS

WITH a = −0.033 AND f = 13,333

GRAPH 6.1: EXPOSURES, Q(t)

STRATEGY CURVES Q(t)

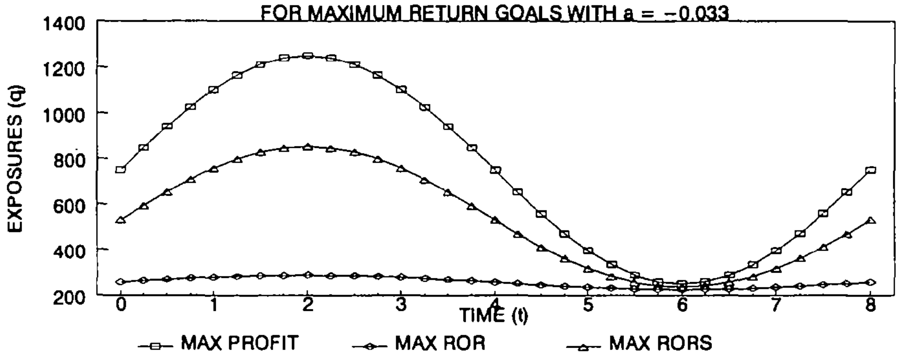


TABLE 6.1: EXPOSURES, Q(t)

	TIME =						AVG # EXPOSURE
RETURN GOAL	0.0	1.0	2.0	5.0	6.0		
MAX PROFIT	750	1104	1250	396	250		755
MAX ROR	715	780	805	646	615		717
MAX RORS	729	917	1000	561	500		744

GRAPH 6.2: PROFIT, Pr(Q(t),t)

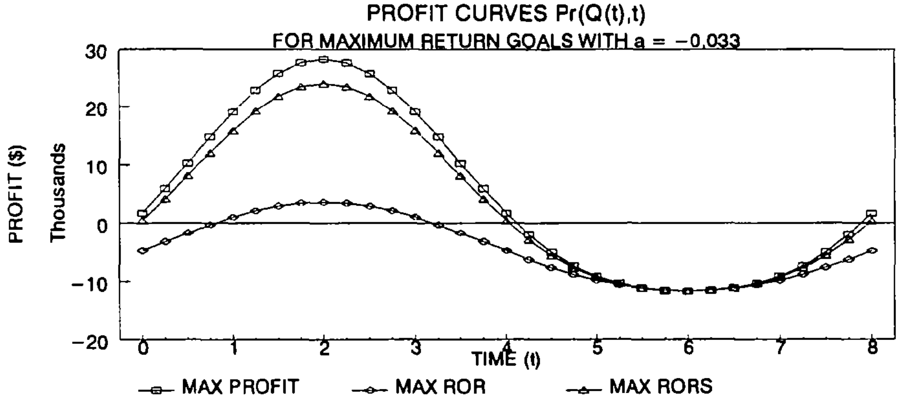


TABLE 6.2: PROFIT, Pr(Q(t),t)

	TIME =						SURPLUS GAIN
RETURN GOAL	0.0	1.0	2.0	5.0	6.0		
MAX PROFIT	$1,667	$19,142	$28,333	($9,142)	($11,667)		$40,083
MAX ROR	$1,634	$16,345	$23,056	($10,799)	($15,221)		$22,280
MAX RORS	$1,655	$18,218	$26,667	($9,866)	($13,333)		$33,428

CURVE VALUES FOR MAXIMUM RETURNS

WITH a = −0.033 AND f = 13,333

GRAPH 6.3: PRICE, Pr(Q(t),t)/Q(t)

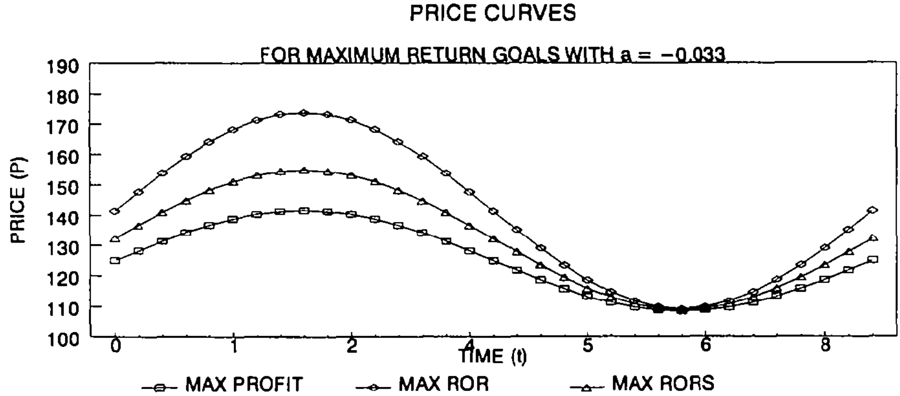


TABLE 6.3: PRICE, Pr(Q(t),t)/Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG PRICE
MAX PROFIT	$125	$137	$142	$113	$108	130
MAX ROR	$126	$148	$156	$105	$96	127
MAX RORS	$126	$143	$150	$108	$100	129

GRAPH 6.4: REQUIRED SURPLUS, Sr(Q(t),t)

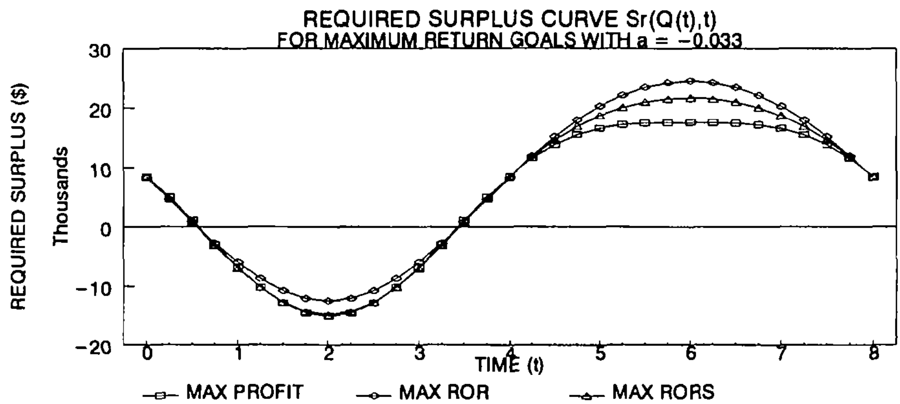


TABLE 6.4: REQUIRED SURPLUS, Sr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG REQ SUR
MAX PROFIT	$8,525	($6,779)	($15,175)	$16,552	$17,551	$4,872
MAX ROR	$8,319	($5,954)	($12,496)	$20,256	$24,450	$7,149
MAX RORS	$8,392	($6,946)	($14,898)	$18,683	$21,655	$5,877

CURVE VALUES FOR MAXIMUM RETURNS

WITH a = −0.033 AND f = 13,333

GRAPH 6.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

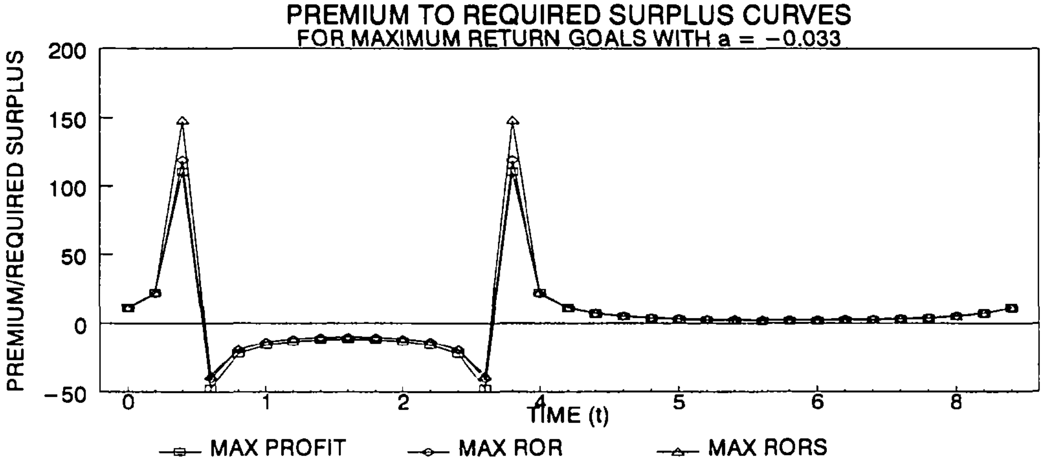


TABLE 6.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	PREM/RS
MAX PROFIT	11.00	−22.27	−11.67	2.71	1.54	20.10
MAX ROR	10.85	−19.33	−10.08	3.34	2.42	12.78
MAX RORS	10.92	−18.89	−10.07	3.24	2.31	16.30

CURVE VALUES FOR MAXIMUM RETURNS

WITH a = −0.033,f = 6,667 TO MODEL LARGE FIRM W/ 50% FIXED EXPENSE SAVINGS

GRAPH 7.1: EXPOSURES, Q(t)

STRATEGY CURVES Q(t)

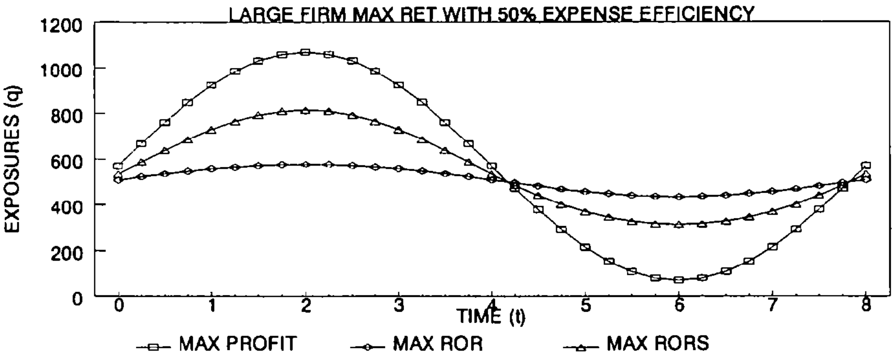


TABLE 7.1: EXPOSURES, Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG # EXPOSURE
MAX PROFIT	570	924	1070	216	70	574
MAX ROR	510	558	576	458	435	511
MAX RORS	536	730	816	370	313	553

GRAPH 7.2: PROFIT, Pr(Q(t),t)

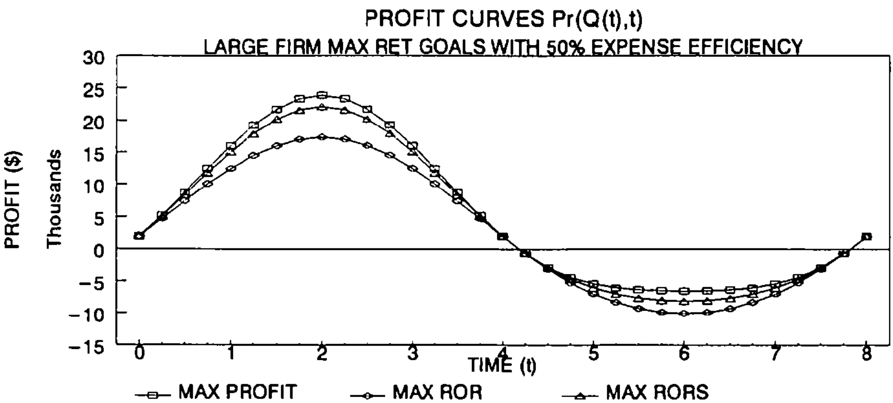


TABLE 7.2: PROFIT, Pr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	SURPLUS GAIN
MAX PROFIT	$1,997	$16,079	$23,864	($5,417)	($6,536)	$42,745
MAX ROR	$1,901	$12,509	$17,368	($6,973)	($10,088)	$22,248
MAX RORS	$1,966	$15,078	$22,137	($6,044)	($8,110)	$36,126

CURVE VALUES FOR MAXIMUM RETURNS

WITH a = −0.033,f = 6,667 TO MODEL LARGE FIRM W/ 50% FIXED EXPENSE SAVINGS

GRAPH 7.3: PRICE, Pr(Q(t),t)/Q(t)

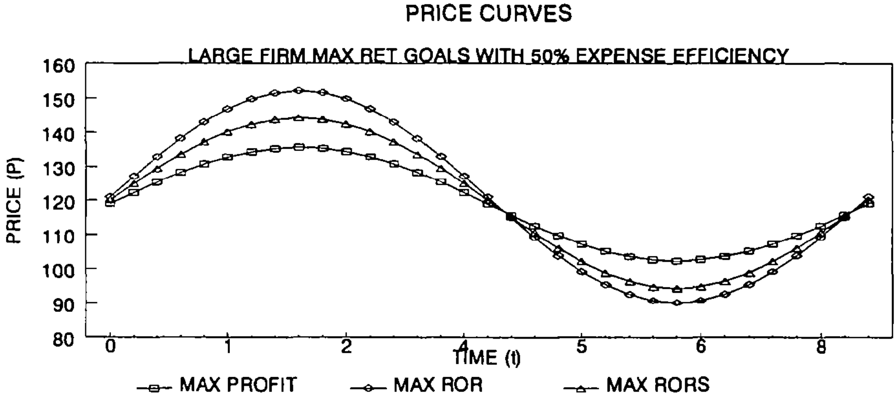


TABLE 7.3: PRICE, Pr(Q(t),t)/Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG PRICE
MAX PROFIT	$119	$131	$136	$107	$102	126
MAX ROR	$121	$143	$152	$99	$90	122
MAX RORS	$120	$137	$144	$102	$94	125

GRAPH 7.4: REQUIRED SURPLUS, Sr(Q(t),t)

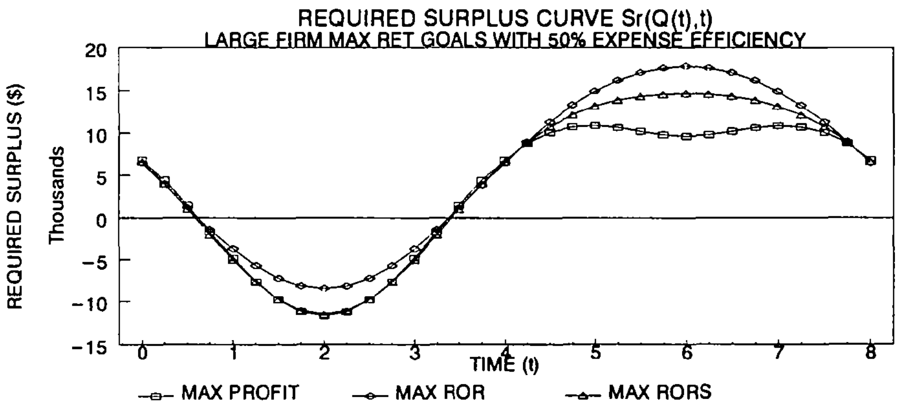


TABLE 7.4: REQUIRED SURPLUS, Sr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG REQ SUR
MAX PROFIT	$6,888	($4,769)	($11,690)	$10,893	$9,650	$3,000
MAX ROR	$6,503	($3,720)	($8,432)	$14,937	$17,849	$5,607
MAX RORS	$6,647	($5,024)	($11,509)	$13,199	$14,694	$4,104

CURVE VALUES FOR MAXIMUM RETURNS

WITH a = −0.033,f = 6,667 TO MODEL LARGE FIRM W/ 50% FIXED EXPENSE SAVINGS

GRAPH 7.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

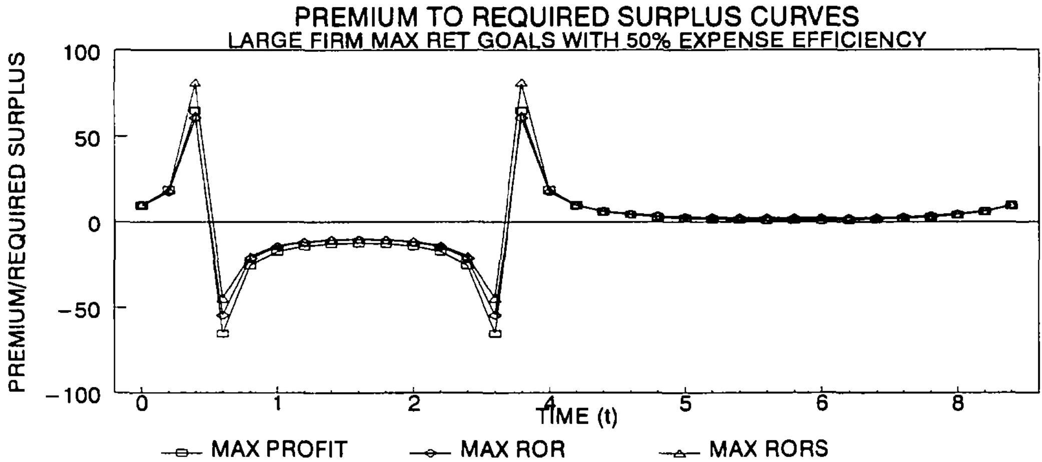


TABLE 7.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	PREM/RS
MAX PROFIT	9.85	−25.33	−12.42	2.13	0.74	24.00
MAX ROR	9.49	−21.43	−10.40	3.04	2.20	11.16
MAX RORS	9.68	−19.94	−10.21	2.86	2.01	16.80

CURVE VALUES FOR EXPECTED RETURN

BASED ON ZERO SURPLUS GAIN TARGET STRATEGIES

GRAPH 8.1: Re(q)

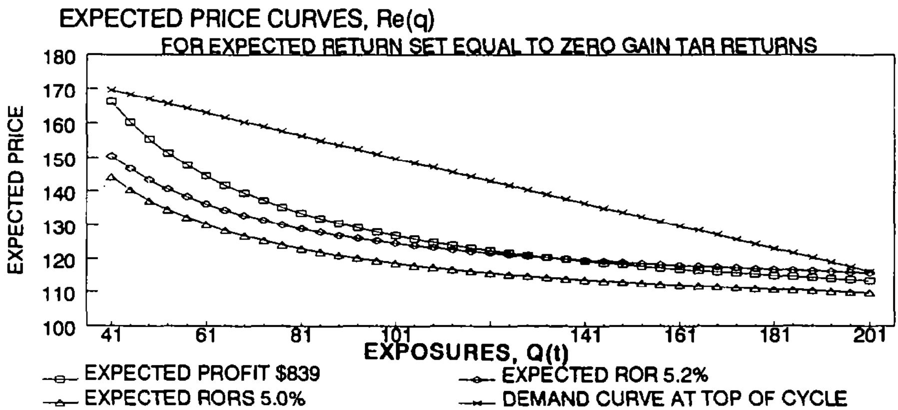


TABLE 8.1: EXPOSURES, Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG # EXPOSURE
EXP PRO = 839	75	174	211	40	25	102
EXP ROR=5.2%	72	168	203	65	62	111
EXP RORS=5.0%	73	171	206	56	50	109

TABLE 8.2: PROFIT, Pr(Q(t),t)

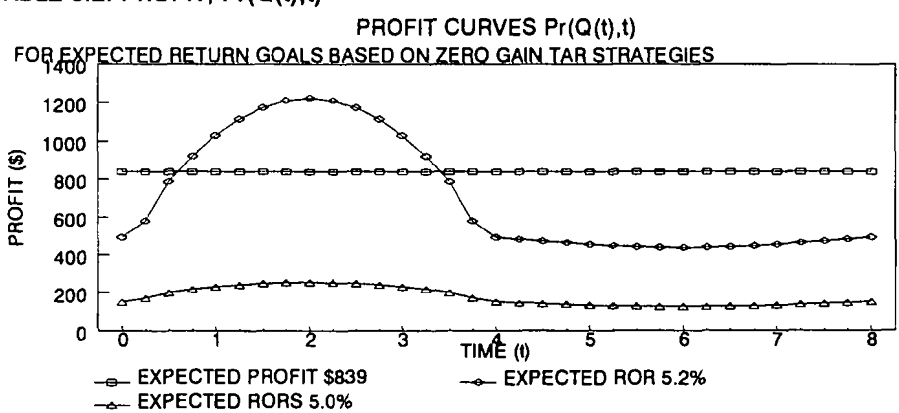


TABLE 8.2: PROFIT, Pr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	SURPLUS GAIN
EXP PRO = 839	$839	$839	$839	$839	$839	$6,753
EXP ROR=5.2%	$492	$1,031	$1,225	$454	$437	$5,692
EXP RORS=5.0%	$151	$232	$254	$133	$125	$1,430

CURVE VALUES FOR EXPECTED RETURN

BASED ON ZERO SURPLUS GAIN TARGET STRATEGIES

GRAPH 8.3: PRICE, Pr(Q(t),t)/Q(t)

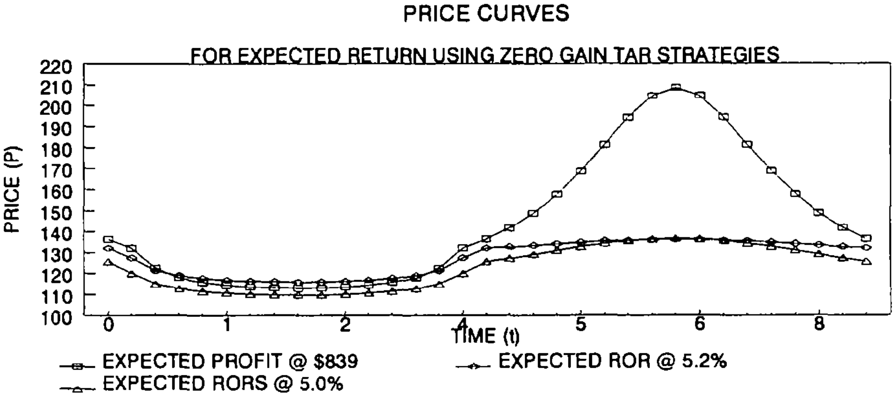


TABLE 8.3: PRICE, Pr(Q(t),t)/Q(t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	PRICE
EXP PRO = 839	$136	$116	$113	$168	$209	116
EXP ROR=5.2%	$132	$118	$116	$135	$136	115
EXP RORS=5.0%	$125	$111	$110	$133	$136	115

GRAPH 8.4: REQUIRED SURPLUS, Sr(Q(t),t)

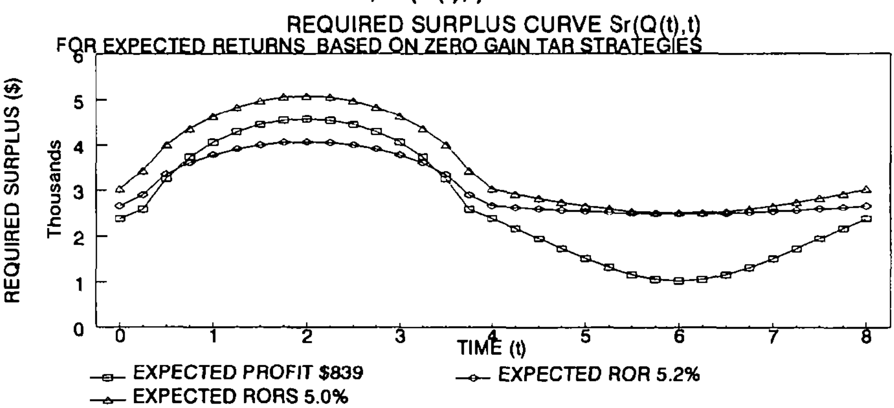


TABLE 8.4: REQUIRED SURPLUS, Sr(Q(t),t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	REQ SUR
EXP PRO = 839	$2,384	$4,068	$4,573	$1,504	$1,022	$2,683
EXP ROR=5.2%	$2,655	$3,792	$4,073	$2,537	$2,482	$3,085
EXP RORS=5.0%	$3,026	$4,635	$5,087	$2,655	$2,506	$3,556

CURVE VALUES FOR EXPECTED RETURN

BASED ON ZERO SURPLUS GAIN TARGET STRATEGIES

GRAPH 8.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

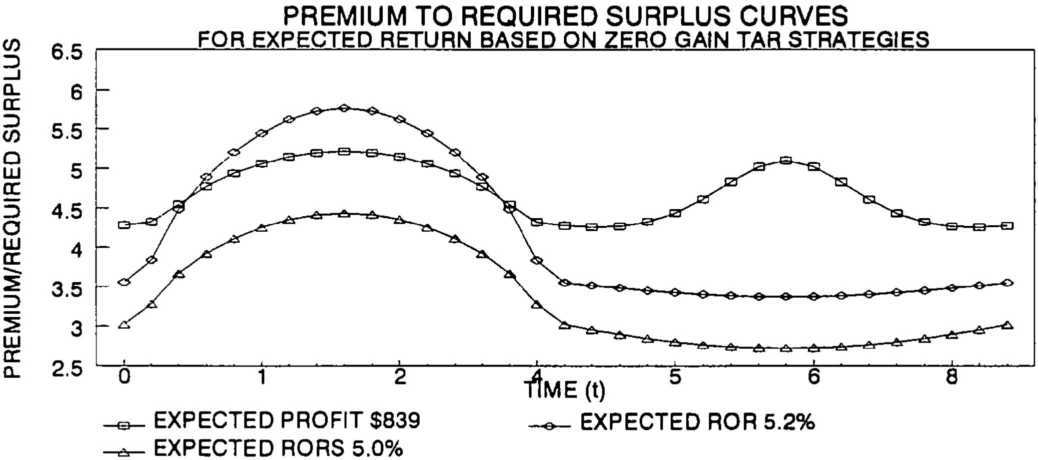


TABLE 8.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	PREM/RS
EXP PRO = 839	4.28	4.94	5.22	4.44	5.10	4.39
EXP ROR=5.2%	3.55	5.21	5.76	3.43	3.37	4.11
EXP RORS=5.0%	3.02	4.11	4.44	2.80	2.72	3.52

CURVE VALUES FOR MAXIMUM PROFIT

WITH GROWTH CONSTRAINTS

GRAPH 9.1: EXPOSURES, Q(t)

STRATEGY CURVES Q(t)

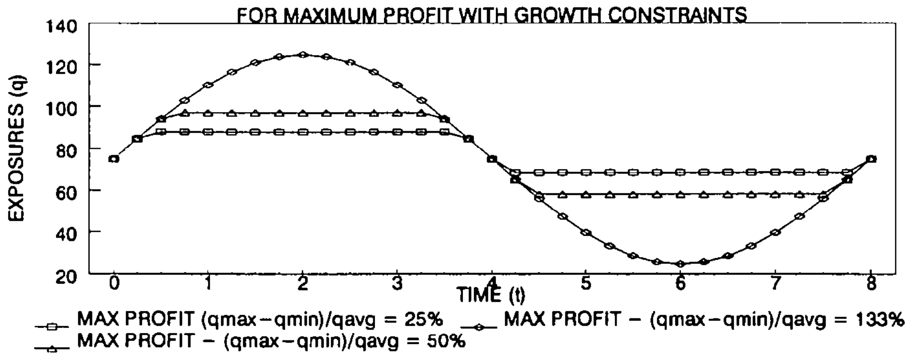


TABLE 9.1: EXPOSURES, Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG # EXPOSURE
MAX PROFIT	75	110	125	40	25	75
50% GROWTH	75	97	97	58	58	77
25% GROWTH	75	88	88	68	68	78

GRAPH 9.2: PROFIT, Pr(Q(t),t)

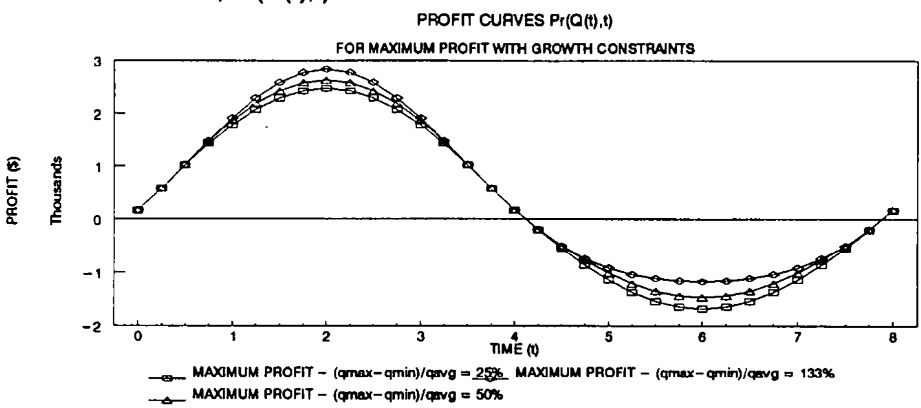


TABLE 9.2: PROFIT, Pr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	SURPLUS GAIN
MAX PROFIT	$167	$1,914	$2,833	($914)	($1,167)	$4,008
50% GROWTH	$167	$1,867	$2,624	($1,007)	($1,461)	$3,228
25% GROWTH	$167	$1,781	$2,468	($1,135)	($1,670)	$2,455

CURVE VALUES FOR MAXIMUM PROFIT

WITH GROWTH CONSTRAINTS

GRAPH 9.3: PRICE, Pr(Q(t),t)/Q(t)

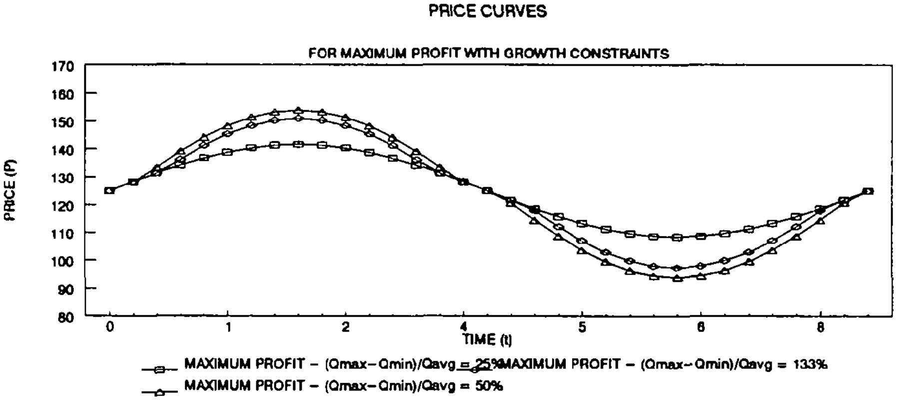


TABLE 9.3: PRICE, Pr(Q(t),t)/Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG PRICE
MAX PROFIT	$125	$137	$142	$113	$108	130
50% GROWTH	$125	$141	$151	$107	$97	127
25% GROWTH	$125	$144	$154	$104	$94	126

GRAPH 9.4: REQUIRED SURPLUS, Sr(Q(t),t)

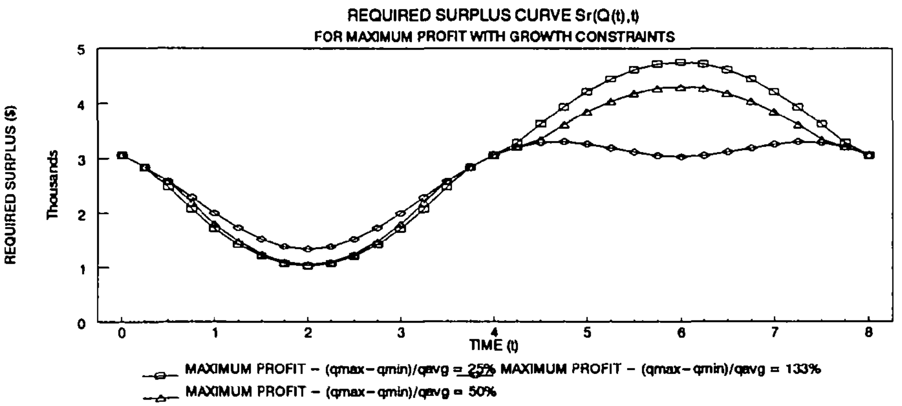


TABLE 9.4: REQUIRED SURPLUS, Sr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG REQ SUR
MAX PROFIT	$3,056	$1,995	$1,328	$3,257	$3,027	$2,622
50% GROWTH	$3,056	$1,799	$1,041	$3,847	$4,302	$2,841
25% GROWTH	$3,056	$1,710	$1,023	$4,214	$4,749	$2,971

CURVE VALUES FOR MAXIMUM PROFIT

WITH GROWTH CONSTRAINTS

GRAPH 9.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

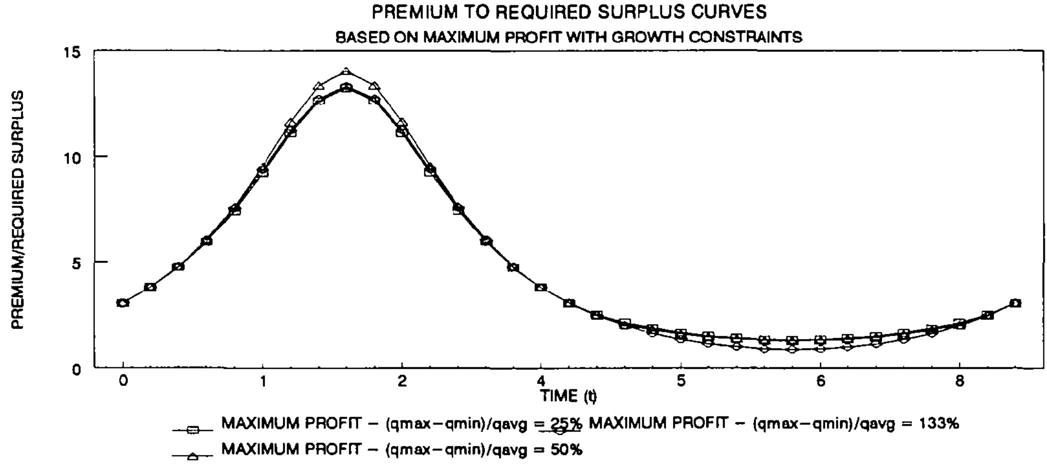


TABLE 9.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

	TIME =					AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0	PREM/RS
MAX PROFIT	3.07	7.57	13.34	1.38	0.89	3.73
50% GROWTH	3.07	7.62	14.07	1.62	1.32	3.47
25% GROWTH	3.07	7.42	13.25	1.68	1.35	3.31

CURVE VALUES FOR MAXIMUM PROFIT

WITH PRICE CONSTRAINTS

GRAPH 10.1: EXPOSURES, Q(t)

STRATEGY CURVES Q(t)

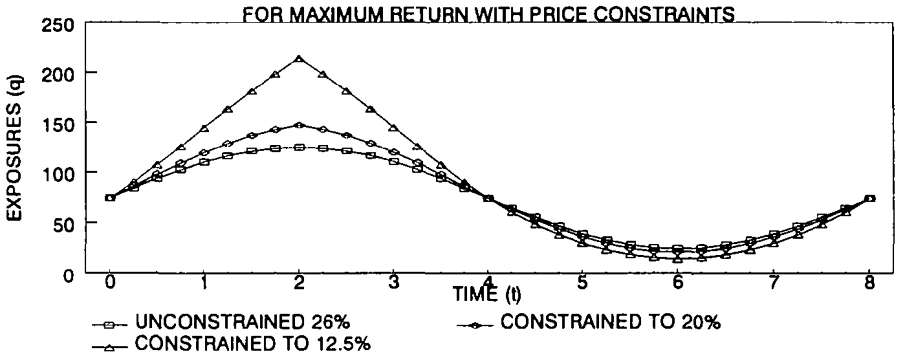


TABLE 10.1: EXPOSURES, Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG # EXPOSURE
UNCONSTD	75	110	125	40	25	75
20% CONSRT	75	120	147	37	21	79
12.5% CONSRT	75	145	215	30	15	90

GRAPH 10.2: PROFIT, Pr(Q(t),t)

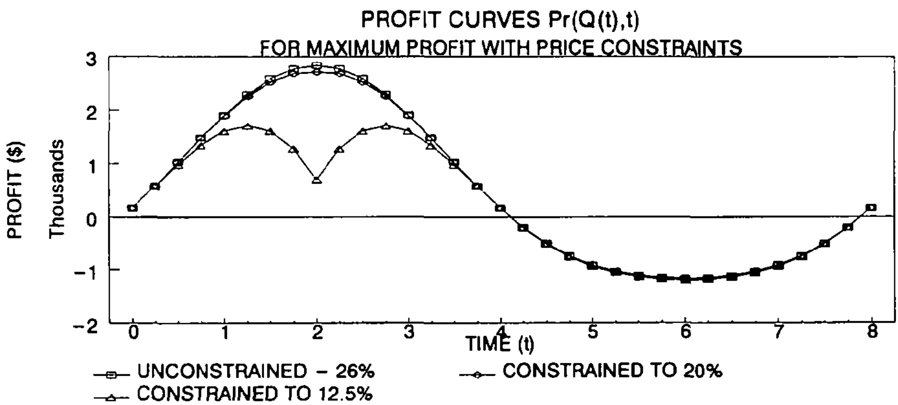


TABLE 10.2: PROFIT, Pr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	SURPLUS GAIN
UNCONSTD	$167	$1,914	$2,833	($914)	($1,167)	$4,008
20% CONSRT	$167	$1,891	$2,702	($917)	($1,170)	$3,846
12.5% CONSRT	$167	$1,602	$697	($938)	($1,196)	$1,641

CURVE VALUES FOR MAXIMUM PROFIT

WITH PRICE CONSTRAINTS

GRAPH 10.3: PRICE, Pr(Q(t),t)/Q(t)

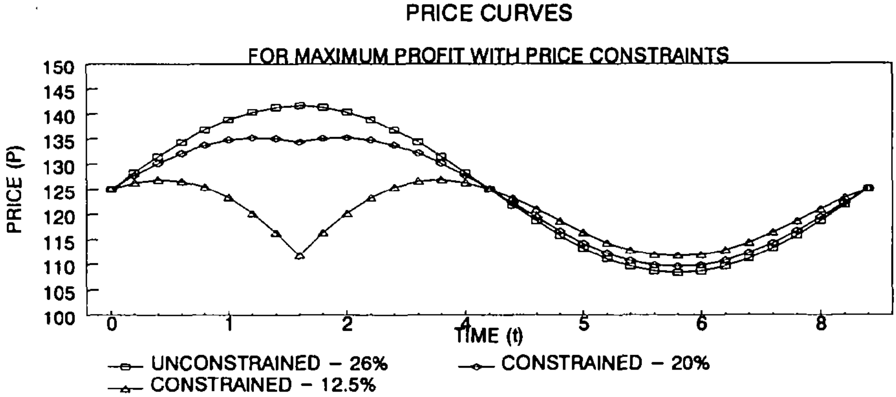


TABLE 10.3: PRICE, Pr(Q(t),t)/Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG PRICE
UNCONSTD	$125	$137	$142	$113	$108	130
20% CONSRT	$125	$134	$134	$114	$110	128
12.5% CONSRT	$125	$125	$112	$116	$112	121

GRAPH 10.4: REQUIRED SURPLUS, Sr(Q(t),t)

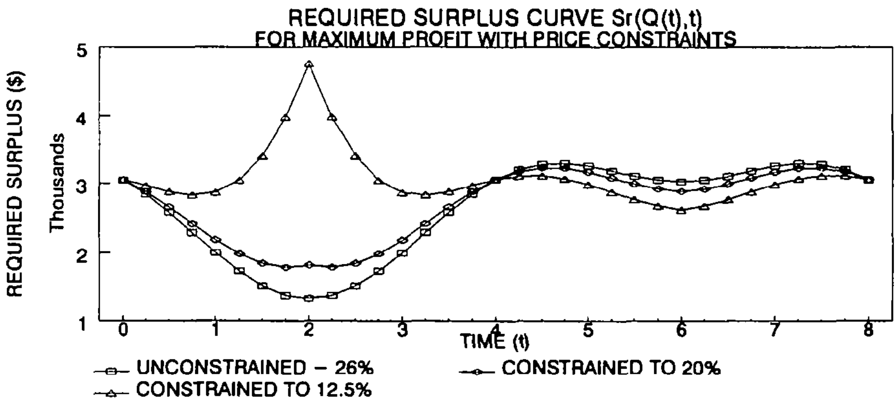


TABLE 10.4: REQUIRED SURPLUS, Sr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG REQ SUR
UNCONSTD	$3,056	$1,995	$1,328	$3,257	$3,027	$2,622
20% CONSRT	$3,056	$2,182	$1,812	$3,166	$2,885	$2,683
12.5% CONSRT	$3,056	$2,873	$4,754	$2,985	$2,616	$3,080

CURVE VALUES FOR MAXIMUM PROFIT

WITH PRICE CONSTRAINTS

GRAPH 10.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

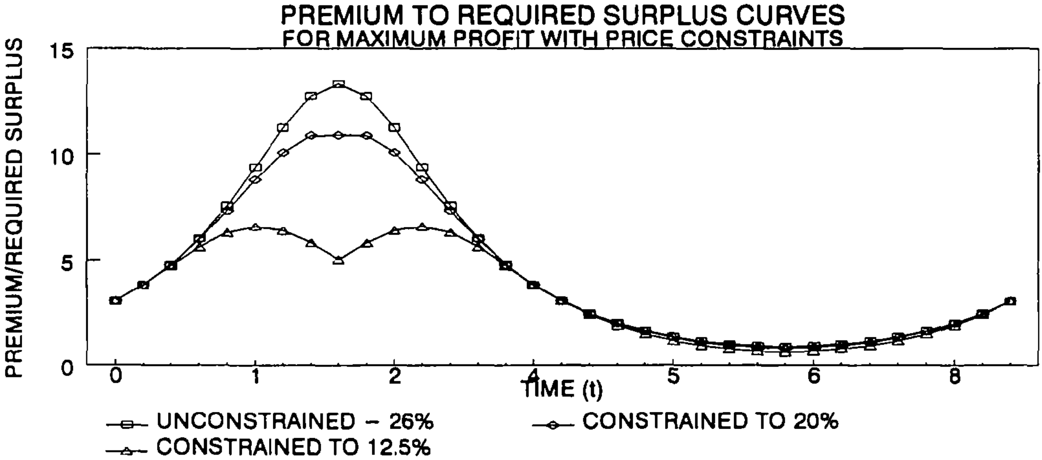


TABLE 10.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

	TIME =						AVG
RETURN GOAL	0.0	1.0	2.0	5.0	6.0		PREM/RS
UNCONSTD	3.07	7.57	13.34	1.38	0.89		3.73
20% CONSRT	3.07	7.34	10.90	1.32	0.81		3.78
12.5% CONSRT	3.07	6.31	5.05	1.18	0.62		3.55

CURVE VALUES FOR MAXIMUM PROFIT

WITH SURPLUS CONSTRAINTS

GRAPH 11.1: EXPOSURES, Q(t)

STRATEGY CURVES Q(t)

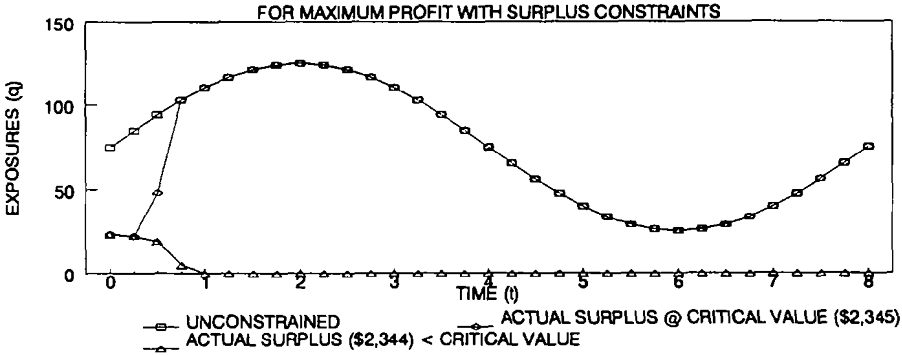


TABLE 11.1: EXPOSURES, Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG # EXPOSURE
MAX PROFIT	75	110	125	40	25	75
Sa @ CRITICAL	23	110	125	40	25	71
Sa < CRITICAL	23	0	0	0	0	2

GRAPH 11.2: PROFIT, Pr(Q(t),t)

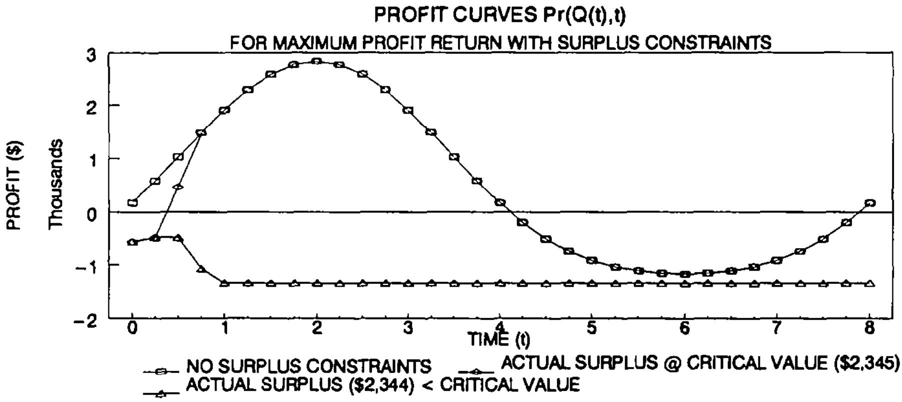


TABLE 11.2: PROFIT, Pr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	SURPLUS GAIN
MAX PROFIT	$167	$1,914	$2,833	($914)	($1,167)	$4,008
Sa @ CRITICAL	($554)	$1,914	$2,833	($914)	($1,167)	$3,461
Sa < CRITICAL	($554)	($1,333)	($1,333)	($1,333)	($1,333)	($10,116)

CURVE VALUES FOR MAXIMUM PROFIT

WITH SURPLUS CONSTRAINTS

GRAPH 11.3: PRICE, Pr(Q(t),t)/Q(t)

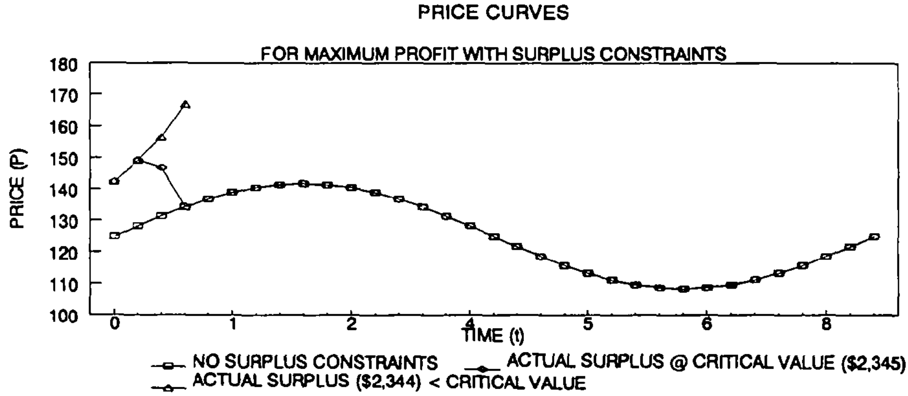


TABLE 11.3: PRICE, Pr(Q(t),t)/Q(t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG PRICE
MAX PROFIT	$125	$137	$142	$113	$108	130
Sa @ CRITICAL	$142	$137	$142	$113	$108	130
Sa < CRITICAL	$142	ERR	ERR	ERR	ERR	152

GRAPH 11.4: REQUIRED SURPLUS, Sr(Q(t),t)

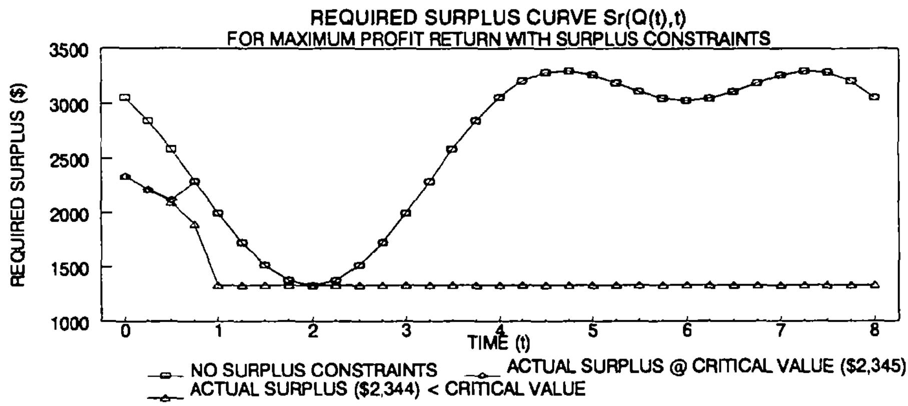


TABLE 11.4: REQUIRED SURPLUS, Sr(Q(t),t)

RETURN GOAL	TIME = 0.0	1.0	2.0	5.0	6.0	AVG REQ SUR
MAX PROFIT	$3,056	$1,995	$1,328	$3,257	$3,027	$2,622
Sa @ CRITICAL	$2,339	$1,997	$1,330	$3,259	$3,029	$2,575
Sa < CRITICAL	$2,339	$1,333	$1,333	$1,333	$1,333	$1,425

CURVE VALUES FOR MAXIMUM PROFIT

WITH SURPLUS CONSTRAINTS

GRAPH 11.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

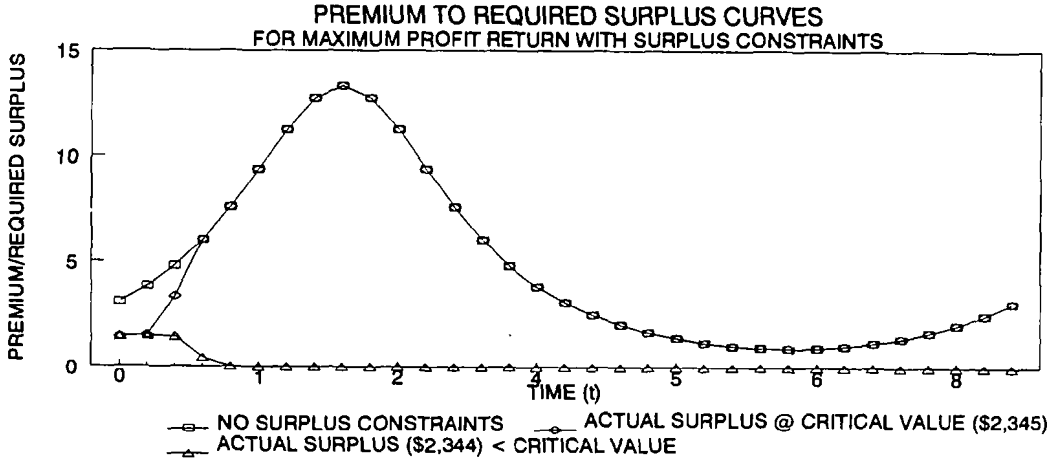


TABLE 11.5: PREM/RS, Pm(Q(t),t)/Sr(Q(t),t)

	TIME =						AVG PREM/RS
RETURN GOAL	0.0	1.0	2.0	5.0	6.0		
MAX PROFIT	3.07	7.57	13.34	1.38	0.89		3.73
Sa @ CRITICAL	1.40	7.56	13.33	1.38	0.90		3.60
Sa < CRITICAL	1.40	0.00	0.00	0.00	0.00		0.20