Measurement of the Effect of Classification Factor Changes in Complex Rating Plans

David B. Schofield

Abstract

Insurance companies strive to distinguish themselves from their competitors. One way of doing so is to refine the rating plan so that it more precisely estimates the appropriate rate for each risk. This refinement process adds complexity to the rating plan and in turn makes the measurement of changes to the individual components of the rating plan (the classifications) more difficult. In this paper, several different types of rating plans are analyzed. The rating plans range from simple plans, with either multiplicative or additive classification factors, to more complex rating plans, with mixtures of each of these types of classification factors. Methods are developed for measuring the effect of changes to classification factors on the overall rate.

1. INTRODUCTION

When a rating plan is structured such that all classification factors are multiplicative, the effect of a change in an individual classification factor on the overall rate is easy to measure. When an additive classification factor like an expense fee is included in the rating plan, measuring the effect becomes more complex. The focus of the paper is on the accurate measurement of the percent change associated with a classification factor when the rating plan contains both additive and multiplicative classification factors. The reader will be presented two different, though mathematically equivalent, methods of presenting the percentage changes in average classification factors.

Some terms are used throughout the text. In the interest of clarity, those terms are defined here:

> **Base rate** – The dollar amount to which classification factors are applied to obtain the final rate.
>
> **Classification** – A type of characteristic of the policyholder, the insured property, or type or level of coverage (e.g. Increased Limits, Deductibles, Model Year, Amount of Insurance, Town Class, etc.) that affects the final rate through the Classification factor.
>
> **Classification level** – the specific value of a classification associated with a policy (e.g. a 100/300 Auto Bodily Injury Limit, a $500 deductible for Collision coverage).

Classification factor – a numeric quantity that adjusts the otherwise applicable rate to the level associated with the policy's particular classification level.

Rating plan – A mathematical model incorporating a base rate and classification factors in such a way as to produce an applicable rate.

This paper will begin with a review of the measurement of rating plan changes for four different rating models: a simple multiplicative model, a simple additive model, a simple combined additive and multiplicative model, and a more complex model with multiple additive and multiplicative classification factors. These models are represented algebraically as:

(1) $R = BM$

(2) $R = B + A$

(3) $R = BM + A$

(4) $R = B\mathbf{M} + \mathbf{A}$

Where

R = Average Rate
B = Average Base Rate
M = Average Multiplicative Classification Factor
A = Average Additive Classification Factor
$\mathbf{M}$ = The Product of All Average Multiplicative Classification Factors
$\mathbf{A}$ = The Sum of All Average Additive Classification Factors

Average classification factors are calculated using either exposures or premium that has been adjusted to remove the effect of the particular classification factor (See Appendix A for methods of calculating average classification factors).

In each of the four models, the goal will be to derive a set of multiplicative factors (f_i's), one for the base rate and each of the classifications, that, when applied to the present rate, R_0, give the proposed rate R_1, i.e.:

$$R_1 = R_0 \prod f_i \tag{1.1}$$

Equation 1.1 may also be represented as

$$R_1 = R_0\left(1 + \sum g_i\right) \tag{1.2}$$

Where the g_i's are the effects of the individual classification factors. The method for converting between the forms in Equations 1.1 and 1.2 will be shown in Section 3. Equations 1.1 and 1.2 represent the two different ways of looking at changes in classification factors. Each may be appropriate in different circumstances, as will be clear when we look at the heuristic examples associated with Model (1) and Model (2). In general, the Equation 1.1 seems more intuitive when the rating model contains all multiplicative classification factors, while Equation 1.2 seems more appropriate when there are predominantly additive classification factors. When the rating model is mixed (contains both additive and multiplicative classification factors), the actuary can choose the most appropriate method of representing the percent change.

2. MULTIPLICATIVE MODEL (Model 1)

Model (1) is the most basic of rating plans. It consists of a base rate and one multiplicative classification factor. Throughout this paper, the subscripts 0 and 1 represent current and proposed respectively. The current and proposed rating models are:

$$R_0 = B_0 M_0 \tag{2.1}$$

$$R_1 = B_1 M_1 \tag{2.2}$$

The goal is to find factors, f_B and f_M, representing the changes in average base rate and classification factor M, respectively, such that:

$$R_1 = R_0 f_B f_M \tag{2.3}$$

Rearranging Equations 2.1 – 2.3, we find:

$$f_B f_M = \frac{B_1 M_1}{B_0 M_0} \tag{2.4}$$

The following factors are selected:

$$f_B = \frac{B_1}{B_0} \tag{2.5}$$

$$f_M = \frac{M_1}{M_0} \tag{2.6}$$

This is the natural factorization of $f_B f_M$, the same factorization that most people would use without realizing assumptions are being made. Here, the factors are just (1 + change in classification factor). However, it should be kept in mind that it is not the only possible factorization. Consider, for instance,

$$f_B = \frac{2B_1}{B_0}; \quad f_M = \frac{M_1}{2M_0}$$

This is also a mathematically valid factorization of $f_B f_M$, though it makes little intuitive sense. This situation arises because we have one equation, Equation 2.3, with two unknowns, f_B and f_M. Additional assumptions are needed in order to restrict the possible factorizations of $f_B f_M$ to the one, intuitive, factorization we originally selected.

This paper will not detail the assumptions that are being made when factoring, but will instead use a common sense approach in factoring the more complex models.

Let's look at a numeric example. Suppose the current average base rate and average classification factor take on the following values, $B_0 = \$100$ and $M_0 = 1.0$. Also, suppose that each is being increased 10%, so that $B_1 = \$110$ and $M_1 = 1.1$. Using Equations 2.5 and 2.6, the multiplicative factors f_B and f_M are each 1.1. The overall change is 21.0%. The question is "What values of g_B and g_M should be selected to represent the percent change in base rate and classification factor M?" In our example, the base rate and the classification factor are increasing by the same percentage amount, so it makes sense to split the 21.0% evenly between those two components of the change and measure the change in each as 10.5%. So we have for Equations 1.1 and 1.2 respectively:

$$R_1 = R_0 f_B f_M = 100 \cdot (1.1)(1.1) = 121$$

$$R_1 = R_0 (1 + g_B + g_M) = 100 \cdot (1 + 0.105 + 0.105) = 121$$

This example is fairly simple. When the percent changes in the classification factors are different, determining appropriate values of g_B and g_M is more difficult. This subject will be discussed in further detail in Section 5, Multiple Additive and Multiplicative Model.

3. ADDITIVE MODEL (Model 2)

Model (2) is another simple rating plan. It consists of a base rate (B) and one additive classification factor (A). The current and proposed rating plans are:

$$R_0 = B_0 + A_0 \tag{3.1}$$

$$R_1 = B_1 + A_1 \tag{3.2}$$

Again, the goal is to find the factors representing the change in base rates and classification factor A, f_B and f_A, such that:

$$R_1 = R_0 f_B f_A \tag{3.3}$$

Rearranging Equations 3.1 – 3.3 we find:

$$f_B f_A = \frac{B_1 + A_1}{B_0 + A_0} \tag{3.4}$$

Factoring Equation 3.4 is a little more difficult than factoring Equation 2.4 in Model (1). Consider the example:

$$B_0 = 100; A_0 = 50$$

$$B_1 = 115; A_1 = 55$$

The overall change is 13.33%. There is a $15 change in the base rate and a five dollar change in the additive classification factor. How should $f_B f_A$ be factored? Two methods are readily apparent. These two methods are discussed in detail.

Additive Factoring Method I

Method I is the first of two methods that will be explored. While Method I makes some intuitive sense, it has several drawbacks that will be explored later. Method II is the author's preferred method of factoring and that method will be used primarily throughout the remainder of the paper. With Method I, we start with the percentage effect each of the dollar changes has on R_0. The percentage effects are:

$$\text{Change in base rate as a percent of present average rate} = \frac{B_1 - B_0}{R_0} = \frac{15}{150} = 10.0\%$$

$$\text{Change in classification factor as a percent of present average rate} = \frac{A_1 - A_0}{R_0} = \frac{5}{150} = 3.33\%$$

Multiplying the two factors together we get:

$$(1.1000)(1.0333) = 1.1366$$

Thus, these two factors overestimate the total change of 13.33%. The factors can be scaled to reach the 13.33% by way of the following:

$$1.1333 = 1.1366^{\alpha}$$

$$\alpha \cdot \ln 1.1366 = \ln 1.1333$$

$$\alpha = \frac{\ln 1.1333}{\ln 1.1366} = 0.9773$$

Once the scaling factor has been found, it can be applied to the individual percent changes:

$$f_B = (1.1000)^{0.9773} = 1.0976$$

$$f_A = (1.0333)^{0.9773} = 1.0325$$

The general form for calculating α is:

$$\alpha = \frac{\ln\left(\frac{R_0 + \Delta B + \Delta A}{R_0}\right)}{\ln\left(\frac{R_0 + \Delta B}{R_0}\right) + \ln\left(\frac{R_0 + \Delta A}{R_0}\right)} \tag{3.5}$$

Where

$$\Delta B = B_1 - B_0 ; \Delta A = A_1 - A_0$$

The final factors are:

$$f_B = \left(\frac{R_0 + \Delta B}{R_0} \right)^{\alpha} \tag{3.6}$$

$$f_A = \left(\frac{R_0 + \Delta A}{R_0} \right)^{\alpha} \tag{3.7}$$

Though Additive Method I certainly produces values in the range that we would expect, it has three drawbacks; the calculations are a little cumbersome, the use of the scaling factor is not intuitively appealing, and there is an interaction effect between classification factor changes. Additive Factoring Method II avoids these problems. The interaction effect will be made clear in Section 6, Interactions and Additive Factoring.

Additive Factoring Method II

Additive Method II takes a more direct approach. It uses exponential weighting to factor the overall percent change into f_B and f_A. Continuing with our previous example, we have a \$15 increase in the base rate and a five dollar increase in the additive classification factor. The Method II factorization is:

$$f_B f_A = 1.1333 = 1.1333^{\frac{15+5}{20}}$$

$$f_B f_A = (1.1333)^{\frac{15}{20}} (1.1333)^{\frac{5}{20}}$$

or

$$f_B = (1.1333)^{\frac{15}{20}} = 1.0984$$

$$f_A = (1.1333)^{\frac{5}{20}} = 1.0318$$

The general representation of this is:

$$f_B = \left(\frac{R_1}{R_0}\right)^{\frac{\Delta B}{\Delta B + \Delta A}} \tag{3.8}$$

$$f_A = \left(\frac{R_1}{R_0}\right)^{\frac{\Delta A}{\Delta B + \Delta A}} \tag{3.9}$$

when

$$\Delta B + \Delta A \neq 0$$

When $\Delta B + \Delta A = 0$, we get the following interesting result:

$$f_B = e^{\frac{\Delta B}{R_0}} \tag{3.10}$$

$$f_A = e^{\frac{\Delta A}{R_0}} \tag{3.11}$$

The derivation of this result is in Appendix B and can be generalized for use in the remainder of the models discussed in this paper.

Equations 3.8 and 3.9 are the heart of Method II. Conceptually, these can be written as

$$f_i = \left(\frac{\text{New Average Rate}}{\text{Old Average Rate}}\right)^{\frac{\text{Avg. Dollar Change in Rate Due to Change in Classification Factor i}}{\text{Average Dollar Change in Rate}}} \tag{3.12}$$

The Method II factorization is a much simpler calculation than Method I and gives results in the range we expect. No scaling factors are needed for this method. Again, another reason to prefer Method II over Method I will be discussed in Section 6, Interactions and

Additive Factoring. For the remainder of the paper, Additive Factoring Method II will be used.

We still need to find the solution in the form of Equation 1.2. Let:

$$1 + g_B + g_A = \frac{B_1 + A_1}{B_0 + A_0} \tag{3.13}$$

$$1 + g_B + g_A = \frac{R_0 + \Delta B + \Delta A}{R_0} = 1 + \frac{\Delta B}{R_0} + \frac{\Delta A}{R_0}$$

so:

$$g_B = \frac{\Delta B}{R_0} \tag{3.14}$$

$$g_A = \frac{\Delta A}{R_0} \tag{3.15}$$

Again, in our example, we have

$$B_0 = 100; A_0 = 50$$

$$B_1 = 115; A_1 = 55$$

So,

$$g_B = \frac{15}{150} = 0.1000$$

$$g_A = \frac{5}{150} = 0.0333$$

We can also use the results in Equation 3.13 to find a conversion method between those factors shown in Equation 1.1 and those of Equation 1.2. From Equation 3.8 we have:

$$f_B = \left(\frac{R_1}{R_0}\right)^{\frac{\Delta B}{\Delta R}} \tag{3.16}$$

Multiplying the exponent by a fancy form of one (R_0 / R_0) produces:

$$f_B = \left(\frac{R_1}{R_0}\right)^{\frac{\Delta B}{\Delta R}\cdot\frac{R_0}{R_0}} = \left(\frac{R_1}{R_0}\right)^{\frac{\Delta B}{R_0}\cdot\frac{R_0}{\Delta R}}$$

Substituting using Equation 3.14

$$f_B = \left(\frac{R_1}{R_0}\right)^{g_B\cdot\frac{R_0}{\Delta R}}$$

Solving for g_B gives:

$$g_B = \frac{\Delta R}{R_0}\cdot\frac{\ln f_B}{\ln\left(\frac{R_1}{R_0}\right)} \tag{3.17}$$

Under the Multiplicative model, calculating percentage changes using Equation 1.1 makes the most intuitive sense. In contrast, Equation 1.2 seems more appropriate under the Additive model. In a mixed model, the decision about the form of the percent change to use is less clear cut. Since there is a one to one correspondence between the two forms, it is up to the actuary to decide which method is most accurate in representing these changes.

4. SIMPLE ADDITIVE AND MULTIPLICATIVE MODEL (Model 3)

Model (3) is a simple combination of Model (1) and Model (2). Under Model (3), the current and proposed rating plans are:

$$R_0 = B_0 M_0 + A_0 \tag{4.1}$$

$$R_1 = B_1 M_1 + A_1 \tag{4.2}$$

The objective is to find a set of factors that accurately represent the multiplicative effect of changes to B, M and A. We want to find f_B, f_M and f_A that satisfy:

$$R_1 = R_0 f_B f_M f_A \tag{4.3}$$

Consider the following numeric example. Table 1 contains the information about the current and proposed rates needed to determine the individual effects of each classification factor change.

Table 1

Variable	Current	Proposed	Δ	%Δ
R	$200	$240	$40	20.00%
B	$100	$110	$10	10.00%
M	1.65	1.80	0.15	9.10%
A	$35	$42	$7	20.00%

Finding the factors is a two-step process. First, determine the effects of the two additive components (BM) and A, i.e., we find values for $f_B f_M$ and f_A. Next, the multiplicative sub-components of the (BM) component are partitioned into f_B and f_M. There is a total dollar change of $40. The change in A accounts for seven dollars of the total change; the changes in B and M account for the remaining $33. Measuring these changes individually results in the following percent changes by classification factor:

$$\text{Additive}: \frac{\Delta A}{R_0} = \frac{7}{200} = 0.035 \quad \text{or } 3.5\%$$

$$\text{Multiplicative}: \frac{\Delta(\text{BM})}{\text{R}_0} = \frac{33}{200} = 0.165 \quad \text{or } 16.5\%$$

$$\begin{aligned}\Delta(BM) &= B_1 M_1 - B_0 M_0 = (B_0 + \Delta B)(M_0 + \Delta M) - B_0 M_0 \\ &= B_0 \Delta M + \Delta B M_0 + \Delta B \Delta M \\ &= 100 \cdot 0.15 + 10 \cdot 1.65 + 10 \cdot 0.15 = 33\end{aligned}$$

The total change in R, of 20.0%, is factored into additive and multiplicative effects by using Method II factorization:

$$\frac{R_1}{R_0} = 1.2^1 = 1.2^{\frac{\Delta A + \Delta(BM)}{\Delta R}} = 1.2^{\frac{7+33}{40}}$$

We let

$$f_A = 1.2^{\frac{7}{40}} = 1.0324$$

$$f_B f_M = 1.2^{\frac{33}{40}} = 1.1623$$

If there were no additive classification factor in our model, $f_B f_M$ would be the same as in Model (1), the product of the two percentage changes associated with the base rate change and the multiplicative classification factor change:

$$\frac{B_1 M_1}{B_0 M_0} = 1.2$$

But we know the overall effect of the base rate and multiplicative classification factor change is 1.1623. So we factor this multiplicative part of our model just as was done in Model (1), then scale the factors to produce the overall effect of 1.1623. To determine the individual factors f_B and f_M while retaining the relative effect of the underlying classification factors, let:

$$f_B f_M = \left(\frac{B_1}{B_0} \cdot \frac{M_1}{M_0}\right)^{\alpha} \tag{4.4}$$

$$f_B f_M = \left(\frac{B_1}{B_0}\right)^{\alpha} \left(\frac{M_1}{M_0}\right)^{\alpha} \tag{4.5}$$

Factoring results in:

$$f_B = \left(\frac{B_1}{B_0}\right)^{\alpha} \tag{4.6}$$

$$f_M = \left(\frac{M_1}{M_0}\right)^{\alpha} \tag{4.7}$$

Solving Equation 4.4 for α :

$$\alpha = \frac{\ln(f_B f_M)}{\ln\left(\frac{B_1 M_1}{B_0 M_0}\right)} \tag{4.8}$$

$$\alpha = \frac{\ln(1.1623)}{\ln(1.2)}$$

$$\alpha = 0.8249$$

Plugging this result back into Equations 4.6 and 4.7 yields:

$$f_B = \left(\frac{B_1}{B_0}\right)^{\alpha} = \left(\frac{110}{100}\right)^{0.8249} = 1.0818$$

$$f_M = \left(\frac{M_1}{M_0}\right)^{\alpha} = \left(\frac{1.80}{1.65}\right)^{0.8249} = 1.0744$$

The effects of the changes in the base rate and classification factors on the overall rate are shown in Table 2.

Table 2

	Equation 1.1	Equation 1.2
Base Rate (B):	8.18%	8.63%
Classification Factor (M):	7.44%	7.87%
Classification Factor (A):	3.24%	3.50%

Model (3) has presented all the tools necessary to measure the effect of even very complex rating plans. The basic idea is to group whatever rating plan model you may have into a series of additive components (we know how to measure these from Model (2)), and then calculate the multiplicative effects within each of the additive components. Model (4) is a slightly more complex than Model (3), and shows how factors can be determined when there are multiple additive and multiplicative classification factors.

5. MULTIPLE ADDITIVE AND MULTIPLICATIVE MODEL (Model 4)

Model (4) expands on all that has been learned in the first three models. It is very similar to Model (3), but contains both multiple additive and multiplicative classification factors. In its simplest form, the model can be written as:

$$R = B\mathbf{M} + \mathbf{A}$$

M represents the product of all the multiplicative classification factors (other than the base rate) and **A** is the sum of the additive classification factors. So the present and proposed model can be represented as:

$$R_0 = \left(B_0 \prod M_{j0}\right) + \sum A_{k0} \tag{5.1}$$

$$R_1 = \left(B_1 \prod M_{j1}\right) + \sum A_{k1} \tag{5.2}$$

We want to find f_B, f_M and f_A, a set of factors that satisfy:

$$R_1 = R_0 \prod f_i \tag{5.3}$$

Consider the following numeric example. Table 3 contains the information about the current and proposed rates needed to determine the individual effects of each classification factor change.

Table 3

Variable	Current	Proposed	Δ	%Δ

R	$233.25	$279.80	$46.55	19.96%
B	$100.00	$110.00	$10.00	10.00%
M1	1.65	1.80	0.15	9.09%
M2	1.05	1.10	0.05	4.76%
A1	$35.00	$42.00	$7.00	20.00%
A2	$10.00	$8.00	($2.00)	-20.00%
A3	$15.00	$12.00	($3.00)	-20.00%

Table 3 is similar to the example shown in Section 4, Table 1. In Table 3, multiplicative classification factor M2 has been added as well as additive classification factors, A2 and A3. The base rate and other classification factors remain as they did in Table 1. Finding the factors is again a two-step process. First partition all the additive components and determine each of those effects. Then calculate the effects of the multiplicative sub-components of each (if any). The model for this rating plan is:

$$R = B \cdot M1 \cdot M2 + A1 + A2 + A3$$

We calculate $f_B f_{M1} f_{M2}$, f_{A1}, f_{A2} and f_{A3} using Additive Factoring Method II. First note:

$$\Delta A1 = 7$$
$$\Delta A2 = -2$$
$$\Delta A3 = -3$$
$$\Delta(B \cdot M1 \cdot M2) = \Delta R - (\Delta A1 + \Delta A2 + \Delta A3) = 46.55 - (7 - 2 - 3) = 44.55$$

So

$$f_B f_{M1} f_{M2} = (1.1996)^{\frac{44.55}{46.55}} = 1.1903$$
$$f_{A1} = (1.1996)^{\frac{7}{46.55}} = 1.0277$$
$$f_{A2} = (1.1996)^{\frac{-2}{46.55}} = 0.9922$$
$$f_{A3} = (1.1996)^{\frac{-3}{46.55}} = 0.9883$$

The scaling factor for $f_B f_{M1} f_{M2}$ is:

$$\alpha = \frac{\ln(f_B f_{M1} f_{M2})}{\ln\left(\frac{B_1 M1_1 M2_1}{B_0 M1_0 M2_0}\right)}$$

$$\alpha = \frac{\ln(1.1903)}{\ln(1.2571)}$$

$$\alpha = 0.7614$$

Giving us:

$$f_B = \left(\frac{B_1}{B_0}\right)^{\alpha} = \left(\frac{110}{100}\right)^{0.7614} = 1.0753$$

$$f_{M1} = \left(\frac{M1_1}{M1_0}\right)^{\alpha} = \left(\frac{1.80}{1.65}\right)^{0.7614} = 1.0685$$

$$f_{M2} = \left(\frac{M2_1}{M2_0}\right)^{\alpha} = \left(\frac{1.10}{1.05}\right)^{0.7614} = 1.0361$$

The effect of the changes in the base rate and classification factors on the overall rate is show in Table 4.

Table 4

	Equation 1.1	Equation 1.2
Base Rate (B):	7.53%	7.96%
Classification Factor (M1):	6.85%	7.27%
Classification Factor (M2):	3.61%	3.89%
Classification Factor (A1):	2.77%	3.00%
Classification Factor (A2):	-0.78%	-0.86%
Classification Factor (A3):	-1.17%	-1.29%

A comparison of these results with those of the example in Section 4, shows that the addition of classifications can have an impact on the measurement of the effect of the changes in the original classification factors.

6. INTERACTIONS AND ADDITIVE FACTORING

The major motivation for using Additive Factoring Method II, over Method I, is how each treats interaction effects between additive portions of the rating plan. For instance, suppose we have the following rating model:

$$R = B + A1 + A2 \tag{6.1}$$

Now consider the numeric example shown in Table 5. It contains data about the current rates but has different proposed rates for classifications A1 and A2.

Table 5

	Variable	Current	Proposed	Δ	%Δ
Scenario	R	$200.00	$230.00	$30.00	15.00%
1	B	100.00	110.00	10.00	10.00%
	A1	50.00	60.00	10.00	20.00%
	A2	50.00	60.00	10.00	20.00%
Scenario	R	$200.00	$230.00	$30.00	15.00%
2	B	100.00	110.00	10.00	10.00%
	A1	50.00	50.00	0.00	0.00%
	A2	50.00	70.00	20.00	40.00%

Under each scenario, the overall rate change and base rate change are 15.0% and 10.0% respectively. The total dollar change for classification factors A1 and A2 is constant over the two scenarios (i.e. $\Delta A1 + \Delta A2 = 20$). We have the following changes under each scenario:

Scenario 1: $\Delta R = \Delta B + \Delta A1 + \Delta A2 = 10 + 10 + 10 = 30$

Scenario 2: $\Delta R = \Delta B + \Delta A1 + \Delta A2 = 10 + 0 + 20 = 30$

We would expect that the measured effect of the change in the base rate would be the same under both scenarios. Under Additive Factoring Method I we have for the base rate change:

$$f_B = \left(\frac{R_0 + \Delta B}{R_0}\right)^\alpha = \left(\frac{200+10}{200}\right)^\alpha = 1.05^\alpha$$

$$\alpha = \frac{\ln\left(\frac{R_0 + \Delta B + \Delta A1 + \Delta A2}{R_0}\right)}{\ln\left(\frac{R_0 + \Delta B}{R_0}\right) + \ln\left(\frac{R_0 + \Delta A1}{R_0}\right) + \ln\left(\frac{R_0 + \Delta A2}{R_0}\right)}$$

Under Scenario 1 we have:

$$\alpha = \frac{\ln\left(\frac{230}{200}\right)}{\ln\left(\frac{210}{200}\right) + \ln\left(\frac{210}{200}\right) + \ln\left(\frac{210}{200}\right)} = \frac{0.1398}{0.0488 + 0.0488 + 0.0488} = 0.9549$$

$$f_B = 1.05^{0.9549} = 1.0477 \quad \text{or} \quad 4.77\%$$

Under Scenario 2:

$$\alpha = \frac{\ln\left(\frac{230}{200}\right)}{\ln\left(\frac{210}{200}\right) + \ln\left(\frac{200}{200}\right) + \ln\left(\frac{220}{200}\right)} = \frac{0.1398}{0.0488 + 0.0000 + 0.0953} = 0.9702$$

$$f_B = 1.05^{0.9702} = 1.0485 \quad \text{or} \quad 4.85\%$$

This difference between the results shown under these scenarios is one reason for preferring Additive Factoring Method II. Under Method II, each scenario produces:

$$f_B = \left(\frac{R_1}{R_0}\right)^{\frac{\Delta B}{\Delta R}} = \left(\frac{230}{200}\right)^{\frac{10}{30}} = 1.0477 \quad \text{or} \quad 4.77\%$$

This is, of course, the same result we obtained under Scenario 1 above (because all the dollar changes in classification factors were the same under that scenario). The value of f_B will remain constant for all $\Delta A1$ and $\Delta A2$, provided $\Delta A1 + \Delta A2 = 20$. It is this invariance property that makes Additive Factoring Method II favorable to Method I.

7. MULTIPLE EFFECTS OF A SINGLE CLASSIFICATION

Suppose, a rating plan can be represented by the following model:

$$R = B \cdot M1 + B \cdot M2 + A1 \tag{7.1}$$

Here the base rate has effects in two of the components. We still want one value for f_B. How can this be accomplished? The answer is to simply calculate the effect of the base rate change in each of the components and multiply them together. Consider the heuristic example shown in Table 6.

Table 6

Variable	Current	Proposed	Δ	%Δ
R	$250.00	$313.00	$63.00	25.20%
B	$100.00	$110.00	$10.00	10.00%
M1	1.00	1.10	0.10	10.00%
M2	1.00	1.20	0.20	20.00%
A1	$50.00	$60.00	$10.00	20.00%

First find the factors for the components, $f_B f_{M1}$, $f_B f_{M2}$ and f_{A1}.

$$\Delta(B \cdot M1) = B_1 M1_1 - B_0 M1_0 = 121 - 100 = 21$$
$$\Delta(B \cdot M2) = B_1 M2_1 - B_0 M2_0 = 132 - 100 = 32$$

$$\Delta A1 = 60 - 50 = 10$$

Using Additive Factoring Method II, we get:

$$f_B f_{M1} = \left(\frac{313}{250}\right)^{\frac{21}{63}} = 1.0778.$$

$$f_B f_{M2} = \left(\frac{313}{250}\right)^{\frac{32}{63}} = 1.1209$$

$$f_{A1} = \left(\frac{313}{250}\right)^{\frac{10}{63}} = 1.0363$$

Factoring $f_B f_{M1}$:

$$f_B f_{M1} = \left(\frac{B_1}{B_0}\cdot\frac{M1_1}{M1_0}\right)^{\alpha} = \left(\frac{110}{100}\cdot\frac{1.1}{1.0}\right)^{\alpha} = 1.21^{\alpha}$$

$$\alpha = \frac{\ln(f_B f_{M1})}{\ln\left(\frac{B_1 M1_1}{B_0 M1_0}\right)} = \frac{\ln(1.0778)}{\ln(1.21)} = 0.3930$$

$$f_B = \left(\frac{110}{100}\right)^{0.3930} = 1.0382$$

$$f_{M1} = \left(\frac{1.1}{1.0}\right)^{0.3930} = 1.0382$$

Next, factor $f_B f_{M2}$:

$$f_B f_{M2} = \left(\frac{B_1}{B_0}\cdot\frac{M2_1}{M2_0}\right)^{\alpha} = \left(\frac{110}{100}\cdot\frac{1.2}{1.0}\right)^{\alpha} = 1.32^{\alpha}$$

$$\alpha = \frac{\ln(f_B f_{M2})}{\ln\left(\frac{B_1 M2_1}{B_0 M2_0}\right)} = \frac{\ln(1.1209)}{\ln(1.32)} = 0.4111$$

$$f_B = \left(\frac{110}{100}\right)^{0.4111} = 1.0400$$

$$f_{M2} = \left(\frac{1.2}{1.0}\right)^{0.4111} = 1.0778$$

To calculate the total effect of f_B:

$$f_B = (1.0382)(1.0400) = 1.0797$$

The effects of each of the classification factor changes are shown in Table 7.

Table 7

	Equation 1.1	Equation 1.2
Base Rate	7.97%	8.60%
Classification Factor (M1):	3.82%	4.20%
Classification Factor (M2):	7.78%	8.40%
Classification Factor (A1):	3.63%	4.00%

8. PROCESS SUMMARY

The methodology has now been developed to measure the effect of changes in classification factors (CF's) where the form of the rating plan can be represented as:

$$R = \sum_{i=1}^{n} \prod_{j=1}^{m_i} CF_{ij} \qquad CF_{ij} \in (\mathrm{B}, \mathrm{M}_1, \ldots, \mathrm{M}_p, \mathrm{A}_1, \ldots, \mathrm{A}_q) \tag{8.1}$$

Where there are p multiplicative and q additive classifications; n represents the number of additive components; m_i is the number of multiplicative sub-components within additive component i. Note that each of the four original models can be represented using Equation 8.1. In order to calculate the effects of the classification factor changes, the following steps should be followed:

1) Determine the mathematical representation of the rating plan and put it in the form of Equation 8.1.

2) Calculate the average classification factor change (Appendix A).

3) Determine the effects of each of the additive components of the rating plan (Section 3, Additive Factoring Method II).

4) Determine the effects for each set of multiplicative sub-components (Section 4).

5) Combine the effects of classifications that are represented in more than one additive component (Section 7).

6) Transform the result into either Equation 1.1 or Equation 1.2 format if necessary.

The result of this process is the accurate partition of the total rate change into the effects of the changes in each of the classification factors.

9. CONCLUSION

Measuring the effect of classification factor changes can be difficult in anything other than a simple rating plan. This paper has shown methods for accurately measuring the effects of individual classification factor changes in the context of more complex rating plans. The rating models included combinations of both additive and multiplicative classification factors. It was also shown that the percent change for the classification factors within a rating plan could be viewed from a factor point of view (Equation 1.1) or as an additive percent (Equation 1.2). The techniques described in this paper can of course be used in models other than those shown.

Appendix A

Methods for Calculating the Average Classification Factor

An average classification factor is the weighted average of the factors for the individual levels of the classification. There are two different weighting schemes used in calculating this weighted average, exposures and adjusted premium at current rate level. Using exposures as weights assumes that the classification's factors are uncorrelated with the factors of the other classifications. If the actuary does not wish to make this assumption, then adjusted premium should be used. The adjustment removes the effect of the particular classification on the premium. If unadjusted premiums are used as weights, the effect of the classification factor is, in effect, doubled. The actuary must also decide which of either written or earned weights is appropriate; this will generally depend on the type of data being used for the analysis, Accident Year, Calendar Year or Policy Year data (of course, this is all conditioned on the actuary having the appropriate data available). Table A.1 provides the data for the first example of the calculation of an average classification (Limit) factor using each of exposures and adjusted premium.

Table A.1

Limit of Liability	Limit Factor	Exposures	Premium	Adjusted Premium
1,000	1.0	1,000	100,000	100,000
2,000	1.4	800	123,200	88,000
5,000	1.6	500	112,000	70,000
10,000	1.8	200	72,000	40,000
Total		2,500	407,200	298,000

For each Limit of Liability we calculate (assuming all multiplicative classification factors):

$$\text{Adjusted Premium} = \frac{\text{Premium}}{\text{Limit Factor}}$$

Using exposures as weights yields an average classification factor (in this case, Limit factor) of:

$$\begin{aligned}\text{Average Limit Factor} &= \frac{\sum_{\text{all limits}} (\text{Limit Factor}) \cdot (\text{Exposures})}{\sum_{\text{all limits}} \text{Exposures}} \\ &= \frac{3280}{2500} \\ &= 1.312\end{aligned}$$

When the limit factor is correlated with other classification factors, adjusted premiums are used as weights, yielding:

$$\begin{aligned}\text{Average Limit Factor} &= \frac{\sum_{\text{all limits}} (\text{Limit Factor}) \cdot (\text{Adjusted Premium})}{\sum_{\text{all limits}} \text{Adjusted Premium}} \\ &= \frac{\sum_{\text{all limits}} \text{Premium}}{\sum_{\text{all limits}} \text{Adjusted Premium}} \\ &= \frac{407{,}200}{298{,}000} \\ &= 1.366\end{aligned}$$

Care should be taken when choosing which premium to use and the adjustments to be made. As in the above example, when the rating plan has all multiplicative classification factors, the premium used can be total premium; the adjusted premium has the effect of the classification removed. Suppose that the premium in the example shown in Table A.1 includes a fixed expense fee of $20 per exposure. Table A.2 gives the data to show the effect of this difference in rating plans on the premium to be used as a weighting variable.

Table A.2

Limit of Liability	Limit Factor	Exposures	Expense Fees	Premium with Exp. Fees	Premium without Exp. Fees	Adjusted Premium
1,000	1.0	1,000	20,000	100,000	80,000	80,000
2,000	1.4	800	16,000	123,200	107,200	76,571
5,000	1.6	500	10,000	112,000	102,000	63,750
10,000	1.8	200	4,000	72,000	68,000	37,778
Total		2,500	50,000	407,200	357,200	258,099

Here the Adjusted Premium is calculated as:

$$\text{Adjusted Premium} = \frac{\text{Premium without Expense Fees}}{\text{Limit Factor}}$$

The average classification factor (Limit Factor in this case) is calculated as:

$$\begin{aligned}\text{Averge Limit Factor} &= \frac{\sum_{\text{all limits}} (\text{Limit Factor}) \cdot (\text{Adjusted Premium})}{\sum_{\text{all limits}} \text{Adjusted Premium}} \\ &= \frac{\sum_{\text{all limits}} \text{Premium without Expense Fees}}{\sum_{\text{all limits}} \text{Adjusted Premium}} \\ &= \frac{357{,}200}{258{,}099} \\ &= 1.384\end{aligned}$$

When using premium as a weight, it is important to use only that portion of the premium to which the classification factor is being applied; then adjust that premium to remove the effects of the classification factor. The ratio of the applicable premium to adjusted premium is the average classification factor.

Appendix B

Derivation of Percent Changes for Additive Components When $\Delta R = 0$

A revenue neutral rate change ($\Delta R = 0$) must be handled differently than a non-zero change when using Additive Factoring Method II. The general form of Additive Factoring Method II, given additive classification factor A and $\Delta R = R_1 - R_0$, is:

$$f_A = \left(\frac{R_1}{R_0} \right)^{\frac{\Delta A}{\Delta R}}$$

This can be rewritten as:

$$f_A = \left(1 + \frac{\Delta R}{R_0} \right)^{\frac{\Delta A}{\Delta R} \cdot \frac{R_0}{R_0}}$$

$$= \left(1 + \frac{\Delta R}{R_0} \right)^{\frac{R_0}{\Delta R} \cdot \frac{\Delta A}{R_0}}$$

Taking the limit as $\Delta R \to 0$:

$$f_A = \lim_{\Delta R \to 0} \left(1 + \frac{\Delta R}{R_0} \right)^{\frac{R_0}{\Delta R} \cdot \frac{\Delta A}{R_0}}$$

$$= \left(\lim_{\Delta R \to 0} \left(1 + \frac{\Delta R}{R_0} \right)^{\frac{R_0}{\Delta R}} \right)^{\frac{\Delta A}{R_0}}$$

$$= e^{\frac{\Delta A}{R_0}} \quad \therefore$$