Reserve Estimates Using Bootstrapped Statutory Loss Information

William C. Scheel

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by
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Abstract

The reserving methodology described in this paper produces minimum sufficiency levels for reserves that are risk adjusted both for uncertainty in claims payments and uncertainty in investments. The minimum sufficiency level is derived from measurements of correlation and other statistical properties of link ratios. These statistics are found using bootstrap methods. Because the approach relies on bootstrap methods, there is no explicit measurement of either process or parameter risk that ordinarily appears in dynamic financial analysis.

Introduction

The information in a property/casualty loss triangle is highly aggregated; individual claims information is lost during the summation processes both for accident and calendar periods. Ordinarily, bootstrap methods would be applied to raw claims information rather than to an aggregation such as the loss triangles found in Schedule P of the annual statement. However, published information about individual claims experience for companies is non-existent.

The paper describes how bootstrap methods can be applied to public loss information to produce range estimates for future losses.[2] This reserving methodology could use any of the popular reserving methods appearing in the literature. However, the focus of the paper is primarily on the use of bootstrapping to obtain adjustments both for uncertainty in claims payments and uncertainty in investments. The choice among the plethora of reserving methods was kept as simple as possible to illustrate more important principles. The chain ladder reserving method was used. The methods used in this paper are strictly mechanical; no actuarial judgment arises.

A correlation matrix for all lines of business evolves from the method of bootstrapping. Other statistics are derived during the same bootstrap process that produces estimates of correlation factors. The reserves that are estimated have adjustment for the correlation among lines of business, claims payments uncertainty and investment uncertainty. This

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[2] The data used in this paper were provided to authors participating in the 2001 DFA Seminar Call Papers contest held by the Casualty Actuarial Society. They include statutory Schedule P information for a hypothetical insurance company.

differs from conventional mono-line reserving approaches that often do not adjust for either source of uncertainty except by actuarial judgment.

The paper also introduces a new approach for reserve valuation that is tightly coupled to optimization methods applied to investment portfolios. It is difficult to separate where reserving leaves off and dynamic financial analysis (DFA) begins; in this regard they are inseparable.

Valuation Steps

There are six steps in the first phase of the reserving method:

1. Perform a bootstrap of link ratios for ultimate loss. Loss development factors neither were directly measured nor bootstrapped. Both the unfolding of ultimate loss and its relationship to paid loss were chosen as the bootstrap objects. The ultimate loss triangle contains potentially useful information not found in the paid loss triangle—it includes actuarial judgment.
2. Use bootstrapped ultimate link ratios to derive statistics including correlation coefficients, means and standard errors. Track the proportion of loss payments to ultimate loss as part of the bootstrap sampling of ultimate loss links.
3. Use the correlation matrices and statistics obtained in step 2 and simulate future ultimate development period links for each line of business using multinormal methods.
4. Apply the simulated ultimate link ratios to the latest loss triangle diagonal. The ultimate losses for the forecast period are obtained.
5. Perform a second-stage simulation using the probability distribution of paid-to-ultimate ratios (also derived as part of the bootstrap process in step 1).[3] The probability distribution for these ratios is a by-product of the ultimate link bootstrapping. The paid/ultimate ratios were tracked (and bootstrapped) during the bootstrap of the ultimate loss triangle. Each line of business has a probability distribution of these paid/ultimate ratios. It is used to simulate a payment proportion for the simulated ultimate losses. Forward period cash flows for each scenario in step 3 are obtained.
6. Use the cash flows determined in step 5 to calculate annuity-equivalent values for future loss cash flow. Do this at each forward calendar period. There is an annuity-equivalent valuation at each point in time that includes future estimated losses from that point in time onward. Repeat this step for each scenario. This produces a distribution of annuity-equivalent values or present values of future losses. These annuity distributions are discounted reserves. The discount rate is conservative. It could be zero.

[3] The probability distribution of the paid/ultimate loss ratio is a conditional one. The ratio was measured conditional on the bootstrapped ultimate link. Recall that the ultimate link ratios were bootstrapped. Each bootstrap sample involved resampling among accident periods. This was done independently for each development period link. The profile of resampled accident periods used for this ultimate link ratio bootstrapping was also used in connection with the payments triangle to calculate the ratio of paid to ultimate. There was a direct matching of accident periods for the paid and ultimate triangles in this process.

One might stop here. The distribution of the present value of future paid losses provides necessary information for a reserve range in which uncertainty in loss payment is recognized.[4] The distribution could be used to obtain ex ante estimates of reserves for future fiscal periods.[5]

But, a second phase that extends the measurement of uncertainty is useful, so we will not stop with just the uncertainty in claims payments. The distribution obtained in step 6 reflects only this source of uncertainty.[6] The second phase attempts to adjust the reserve levels for uncertainty in asset accumulations needed to back them. This secondary analysis seeks the sufficiency level for reserves.

Sufficiency Levels

Chance-constrained ranges can be set on the present value of future loss payments using the results of Step 6. Managerial judgment could be used to choose a percentile of this distribution. Because the percentile is a sample estimate, a conservative approach would pick the upper confidence level for the percentile. This choice is called the *minimum sufficiency target*. The present value of future payments (discounted reserves) is nominally sufficient to pay claims amounts within defined levels of confidence and sampling error. This result is a target, not the actual minimum sufficiency level because the target is risk adjusted *only* for uncertainty in claims payments. The target has a specified probability of sufficiency; but only to the extent of the amount of the liability for claims payment. The target is conditional on no risk in investment returns.[7]

The minimum sufficiency target for period t includes claims paid in period t and subsequent development periods, t+1, t+2, ... The target is a hurdle rate expressed as an end-of-period value.[8] Were assets at time t to equal the minimum sufficiency target, the

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[4] The distribution of the present value of future paid losses can be used to answer questions such as "What is the range in values within a 90 percent confidence band?" or "What is the loss level with a probability of no more than 0.05 of being exceeded (0.95 percentile)?" These and other chance-constrained questions concerning loss reserves can be answered using this distribution.

[5] The valuations for future periods do not include future business. There are many extensions of this reserving approach that can be done with DFA methods. One important extension is to include new business development. Others include separation among various sources of loss, such as allocated and unallocated loss adjustment, uncertainty in both frequency and severity of loss and the effects of reinsurance on loss transfer.

[6] It is not the intent of this paper to engage in a discussion of what uncertainties should properly be reflected in loss reserves. Suffice it to note that it still is a regulatory failure when an insurer set its reserves adequately in the sense that reserves for future claims payments were deemed to have a 95 percent chance of covering payments; but, unfortunately, the insurer's assets dwindled to insufficient levels. Policyholders or stockholders end up taking the fall anyway. When the original liability was established, it reflected uncertainty only in the magnitude of payments, not uncertainty in the ability to meet those payments. It is a moot issue both to the policyholder and to the stockholder whether insolvency occurred because the insurer cannot pay either an expected or unexpected loss payment.

[7] The minimum sufficiency target still has investment risk; so, it is a target. The target is not immunized because it involves discount assumptions. However, as a practical matter it might have been discounted at a riskless or near riskless rate and also be an immunized target sufficiency level.

[8] It is assumed that payments are end-of-period amounts for the purpose of this analysis.

liability would be covered at the indicated confidence level. Suppose that the distribution of required assets at time t were known. This target distribution could be discounted to get the distribution of beginning-of-period required assets. The discounted distribution is the premium distribution for a single premium deferred annuity. A confidence level associated with this asset distribution is referred to as the *minimum sufficiency confidence level.*

The minimum sufficiency level of assets funds future claims payments within specified levels of confidence. Both the minimum sufficiency level and targets are percentiles of probability distributions.[9]

Determination of Sufficiency Levels

The future claims are expressed as a present value using a conservative rate of discount. The minimum sufficiency target is an amount derived from this distribution of present values. Simulated link ratios lead to forecast-period cash flow estimation, and these cash flows are the source of the present value determinations.[10]

Determination of Link-Ratio Correlation Matrices

The period-to-period changes in estimated ultimate loss were bootstrapped in a special way so that a line-of-business correlation matrix could be obtained for each link ratio. A bootstrap sample of developed claims is drawn. This is done from the set of accident periods that can be used for the t^{th} development factor.

Table 1: Feasible Region for Bootstrap Sampling of a Link Ratio
the shaded area can be used for bootstrapping of the link for the 36-48 month development period. A bootstrap sample involves drawing with replacement from this region to create a pair of columns in which the rows are randomly sampled many times with replacement from the original set. The sampling scheme that unfolds for one line of business is used for the other lines too. For example if the rows in the region were numbered {1,2,3,..., 7}, a sampling scheme for the 36-48 month link could be {1,1,3,5,7,4,4} The corresponding column pairs from each line of business would be used and from them a link factor for the 36-48 development period for each line would be calculated. This technique of bootstrapping in a synchronous fashion from a multivariate sample space is reviewed in Scheel, et al [2000] and Laster [1998].

The derivation of the other development period links is done independently. For example, the bootstrap sample for the 72-84 month development period might use a sampling scheme of accident periods {4,2,2,1}. Table 1: Feasible Region for Bootstrap Sampling of a Link Ratio
illustrates this sampling scheme. But, other lines of business also would have this same replacement sampling for determining their 72-84 month link for this bootstrap sample.

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[9] Because the percentiles have sampling error, the sufficiency amounts are really confidence limits on the percentile. Respectively, they are the lower and upper confidence limits for the minimum sufficiency and target sufficiency percentiles.

[10] Statutory discounts might be at zero rates of interest.

Each development period link is an independent sample, and there is no scaling problem associated with exposure volumes in the various accident periods.

This sampling method is repeated many times to obtain many values for each calendar period link ratio within the sample space. The entire set of bootstrap samples can be used to derive statistics for the ratios. All of the link ratios for different calendar periods can be obtained by using the available accident periods for each transition link within the loss triangle.[11]

The bootstrap samples for different lines of business can be used to calculate all of the needed statistics for links. They also can be used to calculate line-of-business correlation coefficients for the links. Standard errors for these various statistics can be computed using bootstrap methods.

Correlations among lines of business are measured using the experimental sample space. In this case, the bootstrap samples being drawn in a synchronous fashion for all lines of business is that sample space. From a computational standpoint there is a great deal of housekeeping required, but the methods for obtaining a correlation matrix and estimates of the mean and standard deviation for a link are straightforward.[12]

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[11] Links for calendar periods 8 and 9 are not obtained from bootstrap sampling because of the sparse number of usable accident periods. Links for these periods are based on the actual loss triangle information and not bootstrap samples of it. The links for any forecast periods beyond 9 use actual $link_9$. The bootstrap sampling uses a decreasing number of accident years when calculating $link_1$, $link_2$, ..., $link_7$ for the transition in ultimate loss estimates.

[12] Calculations and simulations for this study were done using Microsoft Excel 2000. Multivariate normal simulations were performed with Excel 2000 and a DLL written with Compaq Visual Fortran Version 6.5. The multinormal simulation relies on a Cholesky factorization of the covariance matrix. See Rubinstein [1981] for a discussion of the multinormal simulation methods. Non-linear optimization was done with Frontline Systems Premium Solver Plus version 3.5, an add-in for Excel.

Table 1: Feasible Region for Bootstrap Sampling of a Link Ratio

	12	24	36	48	60	72	84	96	108	120
1990	92,906	123,086	121,828	121,312	120,960	120,786	120,667	120,986	120,907	120,685
1991	126,731	130,026	127,583	126,730	125,640	127,269	126,636	126,266	125,893	
1992	157,558	159,071	158,104	159,525	157,525	157,873	157,124	156,249		
1993	163,692	163,139	161,354	161,677	160,495	160,421	159,270			
1994	167,469	164,228	163,903	163,628	161,827	159,595				
1995	230,837	229,624	227,953	226,813	226,454					
1996	202,686	201,266	202,338	200,922						
1997	259,065	260,110	256,783							
1998	222,746	221,905								
1999	268,705									

Table 2: Example of Portions of a Bootstrap Sample in Shaded Regions

	12	24	36	48	60	72	84	96	108	120
1990	92,906	123,086	121,828	121,312	120,960	160,421	159,270	120,986	120,907	120,685
1991	126,731	130,026	121,828	121,312	125,640	127,269	126,636	126,266	125,893	
1992	157,558	159,071	158,104	159,525	157,525	127,269	126,636	156,249		
1993	163,692	163,139	163,903	163,628	160,495	120,786	120,667			
1994	167,469	164,228	202,338	200,922	161,827	159,595				
1995	230,837	229,624	161,354	161,677	226,454					
1996	202,686	201,266	161,354	161,677						
1997	259,065	260,110	256,783							
1998	222,746	221,905								
1999	268,705									

The 36-48 month link for the bootstrap sample in the shaded region of Table 2 is 0.99941. The 72-84 month link is .99547. Although these are members of the same bootstrap sample, the links for a development period are independent replacement sampling processes. The ratio of paid loss to ultimate loss for any development period also can be calculated for this same bootstrapped sample. It would use the same set of accident periods, but payment information for them is found in the payments triangle.

The correlation matrix for one of the development period links is shown in Table 1: Statistics for $Link_1$. This table also includes general statistics for the paid-to-ultimate loss ratio. This ratio is developed during the bootstrap process along with the ultimate link factors. The distribution of the paid/ultimate ratio is used during a second-stage simulation to provide the transition from ultimate to paid loss. The second stage produces payment pattern variation; whereas, the first simulation stage works with the ultimate link ratios. During a second stage simulation a paid/ultimate ratio is determined and simulated cash flow is obtained for paid loss. This simulation methodology is discussed in detail later in the paper

Simulation Using Link Ratios

Links were simulated and applied to the most recent diagonal of the ultimate loss triangle to obtain forecast period ultimate losses. The means, standard deviations and correlation matrices used for the simulation are shown in Table 1: Statistics for $Link_1$. The links among lines of business were assumed to be multivariate normal with no serial correlation. Each simulation of a link was done independent of other link simulations; all were multivariate normal simulations.[13]

Each simulation had 3,000 trials so that a sample of 3,000 cash flows over a forecast period of 10 years was available for calculating the present value distributions used in subsequent analysis.

[13] See Rubinstein [1981] for a discussion of how the multivariate normal simulation is done. The algorithm used in this study is the IMSL fortran subroutine **DRNMVN**.

Table 1: Statistics for $Link_l$

	Line of Business Correlation Matrix									
	Home	PPA	CAL	WC	CMP	Spcl_Liab	OL_OCC	Reins_A	Reins_B	Reins_C
PPA	0.0089									
CAL	-0.0031	-0.0028								
WC	-0.0075	-0.0067	0.0016							
CMP	0.0119	0.0104	-0.0026	-0.0103						
Spcl_Liab	-0.0157	-0.0147	0.0108	0.0072	-0.0146					
OL_OCC	-0.0212	-0.0187	0.0049	0.0179	-0.0269	0.0244				
Reins_A	-0.0539	-0.0454	0.0172	0.0369	-0.0583	0.0684	0.1030			
Reins_B	-0.0047	-0.0132	0.0103	0.0059	-0.0043	-0.0670	0.0354	0.0052		
Reins_C	0 0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
Property_ShortTail	0.0026	0.0023	-0.0006	-0.0020	0.0031	-0.0027	-0.0050	-0.0146	-0.0042	0.0000

Line of Business	Ultimate Link1		Paid/Ultimate Ratio1	
	Expected Value	Standard Deviation	Expected Value	Standard Deviation
Home	1.0353	0.0795	0.8822	0.0159
PPA	0.9935	0.0713	0.5829	0.0431
CAL	0.9715	0.0475	0.4263	0.0370
WC	0.9585	0.0784	0.4345	0.0337
CMP	1.0177	0.1014	0.5408	0.0233
Spcl_Liab	1.1854	0.3476	0.7765	0.0877
OL_OCC	0.8596	0.1921	0.2174	0.0584
Reins_A	0.9823	0.3608	0.5491	0.1999
Reins_B	1.4092	0.5176	0.7865	0.0474
Reins_C	1.2188	0.0000	0.5461	0.2491
Property_ShortTail	1.0040	0.0270	0.9861	0.0075

Paid Loss Distributions

The variation in speed of payments is a source of uncertainty. Both this uncertainty and uncertainty in ultimate loss must be reflected in cash flow simulations during the forecast period. The distribution of the ratio of paid-to-ultimate also is by-product of the bootstrapping methods. Just as each bootstrap sample produces a link ratio for a development period, the same bootstrap sample develops the ratio of paid to ultimate. The ratio uses the same bootstrap sample accident periods as the ultimate link except that the same sampled accident periods are extracted from the paid loss triangle. The numerator of the paid/ultimate ratio is found in the bootstrapped accident periods of the paid triangle; the denominator is found in the ultimate triangle. The average of the ratios is used as that bootstrap sample's paid/ultimate ratio. The result of all bootstraps is the source of the conditional probability distribution of paid loss. The conditional operator here applies to paid loss given the ultimate loss linkage for the development period.

Payment Pattern Simulation

The ultimate-to-paid transition for cash flow determination occurs in a two-stage simulation. The first stage produces the ultimate link factors for all calendar period transitions. The second stage simulation produces a payment pattern in the form of paid/ultimate.

Each line of business has a set of bootstrap samples that represent a set of payment patterns in the form of paid/ultimate ratios. Once the change in ultimate loss estimates is determined from the first-stage simulation, a payment pattern is chosen during the second stage. In other words, the bootstrap samples of payment patterns are the source for a second stage simulation.

This second-stage simulation adjusts paid losses both for uncertainty in ultimate loss and for uncertainty in the speed of claims payments. The effect is simulation of a payment pattern associated with each ultimate loss level derived in the first-stage simulation. Finally, the cash flows for present value analysis can be assembled from the forecasted diagonals of the simulated paid loss triangle.

Discounting Simulated Paid Loss

Statistics for these present values are shown in Table 2: Statistics for Discounted Paid Losses. The 0.9 percentile of the distributions in this table are the minimum sufficiency targets used in subsequent optimizations. For example, the minimum sufficiency *target* at the end of period 1 would be $1,798,921. The minimum sufficiency target secularly declines, dropping to $484,940 by period 5 and $47,013 by period 10. As previously noted, all cash flows for losses were assumed to have occurred at the end of the period.

The distributions are risk adjusted only for uncertainty in the ultimate loss and variation in the speed of payments. Nevertheless, these results provide ranges for reserve estimation. Conventional reserve practice, both statutory and generally accepted accounting, is to use a point estimate of future paid losses as the basis for liability determination. The values shown in Table 2: Statistics for Discounted Paid Losses provide ranges and other chance-constrained values of what might be considered the conventional GAAP estimates.[14]

[14] A five-percent discount rate was used for present-value calculations of the paid loss cash flows.

Table 2: Statistics for Discounted Paid Losses

All Lines Statistics Table	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
Mean	1,798,921	1,282,873	896,766	650,352	484,940	369,549	278,479	199,570	124,023	47,013
Standard Deviation	91,557	55,185	37,405	21,528	12,388	8,818	6,451	4,571	2,833	845
Median	1,798,343	1,281,362	895,937	649,768	484,449	369,120	278,192	199,447	123,920	46,980
5 percentile	1,649,169	1,195,019	837,174	615,604	465,394	355,541	268,231	192,349	119,508	45,660
10 percentile	1,682,185	1,214,548	850,383	623,497	469,507	358,660	270,411	193,797	120,432	45,956
25 percentile	1,737,040	1,244,796	870,057	634,645	476,207	363,456	274,039	196,427	122,083	46,435
75 percentile	1,860,955	1,319,754	920,838	664,331	492,981	375,434	282,777	202,531	125,786	47,557
90 percentile	***1,913,878***	***1,352,208***	***944,923***	***678,150***	***500,876***	***380,722***	***286,849***	***205,513***	***127,761***	***48,135***
95 percentile	1,948,887	1,374,690	960,082	686,303	506,422	384,138	289,258	207,396	128,899	48,491

Treatment of Incomplete Information

Some of the lines of business had incomplete Schedule P information. Some lines had either a few accident periods or accident periods with few or no losses. Only lines of business with at least fifty percent of completed ultimate and payments cells were used.[15]

In a few cases, the information provided was invalid—ultimate loss for some cells of the ultimate triangle did equal the sum of paid losses and reserves. Ad hoc methods were used in the cleansing of these few imbalanced cells. In general, the ultimate figures were taken to be valid and the paid loss was adjusted with reference to experience in near-by calendar periods. It is not likely that these adjustments had a material impact on the results.

Optimization

The distribution of present-valued claims payments was used to define target sufficiency levels. There is no risk adjustment in these levels for uncertainty in asset growth. We now turn to the interesting question of how such uncertainty might be recognized in the determination of reserves.

Reflecting Investment Uncertainty in Reserves

The distribution of present values for end-of-period valuations for future claims provides the means for assigning fair value to such claims given a conservative growth in investments backing them. The target sufficiency level constitutes a type of financial immunization. Because the target sufficiency is reckoned at a risk-free rate, the company could bank this level of assets and be assured of claims payment with the level of confidence used to determine the targets. Because there is little or no interest rate risk in the target sufficiency, the liability could be commuted; it is an actuarially fair value within defined confidence limits of the loss modeling mechanism.[16]

It remains for investment risk to be similarly bounded so that sufficient funds will exist at this target sufficiency level. The sought-for objective is an asset level at beginning-of-period that will grow to the required target with confidence. The main purpose of this study is not to eschew a particular asset modeling methodology. Any model can be used provided it can generate investment scenarios. This study uses prior work that derives

[15] The treatment of immature lines of business suffers from the problems plaguing any study using non-parametric methods. These approaches, including bootstrapping, rely on the availability of underlying data. Parametric procedures under these limitations also have a hard time determining appropriate choices for probability distributions or their parameters.

Another approach to handling this problem of unavailable or missing data would be to substitute "pure-play" data available from other companies or reference sources. These data would serve as proxies for the missing information and would have to be adjusted to the exposure volumes in existence for lines of business where such proxy data were deployed.

[16] The sufficiency target is the reinsurance pure premium for risk transfer at a level of confidence defined in the analysis. It includes risk margins for variation in loss payments, but no allowance for volatility in investment of those premiums from the discount rate of five-percent.

estimates of a covariance matrix for a mixture of assets [Scheel, et al, 2000]. The description of this database appears in Appendix A: Review of Data Sources. Other asset scenario models, which are based on time-dependent functions such as multi-factor models with mean reversion, could have been used. The approach described in this paper would remain unchanged even if another method of asset scenario generation had been deployed.

Investment returns were simulated using a bootstrapped estimation of the covariance matrix and expected values using monthly returns data for the 20-year period 1/1/1980-12/31/99. Multinormal simulation methods were used in the simulation; they were identical to the ones used for simulating calendar period links. Annualized rates of return were generated from the monthly data by assuming no serial correlation and compounding simulated monthly returns. Various statistics relating to this simulation appear in Table 3: Statistics for Simulated Asset Scenarios. This table shows investment performance for ten annual periods used in the study.

Table 3: Statistics for Simulated Asset Scenarios

Annualized Return	EAFEU	INTLUHD	S&P5	USTB	R_MID	HIYLD	CONV	LBCORP	LBGVT	LBMBS
Expected Value	0.1647	0.0958	0.1495	0.0659	0.1659	0.1018	0.1181	0.0940	0.0917	0.0964
Standard Deviation	0.1803	0.0814	0.1625	0.0084	0.1788	0.0904	0.1124	0.0859	0.0592	0.0587
0.25 Percentile	0.0375	0.0383	0.0330	0.0605	0.0392	0.0393	0.0406	0.0354	0.0509	0.0553
0.50 Percentile	0.1557	0.0958	0.1393	0.0658	0.1512	0.0968	0.1116	0.0909	0.0892	0.0953
0.75 Percentile	0.2764	0.1479	0.2553	0.0715	0.2828	0.1611	0.1932	0.1487	0.1317	0.1353

Correlation Matrix for Proxy Assets										
EAFEU	1.0000	0.4240	0.4124	-0.0297	0.3891	0.2689	0.3975	0.1823	0.1836	0.1150
INTLUHD	0.4240	1.0000	0.0075	0.0233	-0.0102	0.1407	-0.0253	0.2748	0.3301	0.2578
S&P5	0.4124	0.0075	1.0000	-0.0624	0.9435	0.4616	0.9313	0.3204	0.2374	0.2477
USTB	-0.0297	0.0233	-0.0624	1.0000	-0.0276	0.0130	-0.0918	0.1791	0.2622	0.2317
R_MID	0.3891	-0.0102	0.9435	-0.0276	1.0000	0.5063	0.9465	0.3156	0.2328	0.2549
HIYLD	0.2689	0.1407	0.4616	0.0130	0.5063	1.0000	0.5214	0.6655	0.5243	0.5248
CONV	0.3975	-0.0253	0.9313	-0.0918	0.9465	0.5214	1.0000	0.3314	0.2409	0.2500
LBCORP	0.1823	0.2748	0.3204	0.1791	0.3156	0.6655	0.3314	1.0000	0.9041	0.8112
LBGVT	0.1836	0.3301	0.2374	0.2622	0.2328	0.5243	0.2409	0.9041	1.0000	0.8424
LBMBS	0.1150	0.2578	0.2477	0.2317	0.2549	0.5248	0.2500	0.8112	0.8424	1.0000

Legend: EAFEU international equities; INTLHDG international fixed income; S&P5 large cap domestic equities; USTB cash; RMID mid-cap domestic equities, HIYLD high yield debt, CONV convertible securities, LBCORP corporate bonds, LBGOVT government bonds, LBMBS mortgage backed securities. Additional information about the proxy assets is in Appendix A: Review of Data Sources.

Table 3 illustrates the statistical properties of the annualized asset scenarios for just one of the annual periods in the analysis. However, because each annual period's asset scenarios were independently calculated from the same multinormal distribution of returns, the statistical properties for other periods were approximately the same. A small sample of some of the investment scenarios appears in Appendix B: Example of Asset Scenarios for an Annual Period.

Optimization Methods

Non-linear optimization was used. The optimizer posits trial solution set of weights for the investments. All of the simulated investment scenarios were weighted with this investment profile, and a portfolio return was calculated for each scenario. The result is a distribution of portfolio returns for a period. The portfolio return for each scenario is a discount rate that can be used to determine beginning-of-period sufficiency requirements.

The minimum sufficient asset level (beginning-of-period) can be calculated using the portfolio discount rate applied to the (end-of-period) target sufficiency level. When this is done for each investment scenario, a distribution of minimum sufficiency levels is obtained. That distribution then is used to choose a chance-constrained minimum sufficient level. It is a reserve that is risk-adjusted both for uncertainty in claim payments and in investment return.

The minimum sufficiency level (beginning-of-period) is returned to the optimizer as the objective value. The non-linear optimizer continues to posit different investment weights until a minimum for this objective value is found. Such an optimized minimum is the risk-adjusted reserve being sought.

Optimization Constraints

The optimizer was given a standard set of feasibility constraints for investments; all component asset weights were constrained to lie between 0 and 1 and the weights must add to 1. No short sales were allowed.

Optimization Objective Function Calculation

The optimization objective function was the present value of the target sufficiency level. It was minimized by the optimizer. The objective value was calculated for each trial solution of the optimizer using a separate instance of Excel.[17]

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[17] The computational method used to derive the objective function values involved use of a separate instance of Excel as a COM object for the Excel instance running Solver. Although these are programming issues, they are important to the study and warrant some explanation. When the optimizer supplies the workbook with a trial solution for the portfolio weights, it recalculates the workbook. This recalculation is supposed to produce a value for the objective function cell.

The goal cell contained a cell function, a call to an Excel macro that must be within the same workbook as Solver. This macro has restrictions on what it can do with the workbook cells while it is executed during a recalculation of the workbook. The macro only can read sections of the workbook, it cannot modify the contents of any cell during its execution. It only can return a value to the cell from which the macro was called. Although this limits what might be done while Solver executes, COM objects running in separate processes provide exceptional flexibility that ordinarily would be missing were just the solver workbook to

Reserves

The minimum sufficiency reserve levels for each period are shown in Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios. The reserve level is "Minimum Beginning of Period" values. It is the amount, which with confidence .9, will grow to the "Required End-of-Period" value—the target level for sufficiency. The weights for components of the optimized proxy portfolio also appear in the table. Statistics for the end-of-period portfolio values are shown.

For example, an asset level of $1,591,549 at the beginning of period 1 is the nominal amount needed to provide for payments of claims in this period and fund the present value of all future claims in periods 2, 3,.... The target level declines as the magnitude of future claims payments dwindles over time. For example, by period 5, the target sufficiency has declined to $296,149, and by period 9 it drops to $37,711.

There is a .1 probability of assets not growing to the target sufficiency level.[18] Further, that target level has a .2 probability of being inadequate for claims payment because it

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be used. The calculation of the objective value given the weights, for example, is complex. However, it can be easily done in its own instance of Excel. This instance is being controlled by the macro of the workbook running solver.

The objective cell macro uses, as an argument, the reference to the cell range containing the weights being suggested by the optimizer for the current trial solution. The fact that an argument was used in the macro call is extremely important...it assures that the macro function will not be executed until after the optimizer has written the trial solution weights to the referenced cell area Because the macro can read cells within the Solver workbook, the macro can copy the weights into the separate instance of Excel. Previously, that instance was also provided the sufficiency target, rates of return for simulated asset scenarios and other information about confidence levels. The separate instance is recalculated and the results are available to the macro for return to the objective function cell.

The separate instance if Excel has all of the information it needs to perform its own calculation. This calculation is driven by the Solver macro after it has done the necessary setup in the separate instance. The investment returns for all scenarios contained in the separate workbook are weighted by the trial solution set of weights. The recalculation of this instance develops the distribution of present values for the sufficiency target. Finally, the upper confidence limit for the percentile of that distribution is calculated. The percentile is binomially distributed. With adequate numbers of investment scenarios, the upper confidence limits for the percentile can be calculated using normal approximation methods. The 2,500 investment scenarios used in this calculation were more than adequate for a normal approximation.

To summarize, the Solver goal cell is recalculated along with other cells in the workbook during a trial solution. The weights are passed through the macro to a separate instance of Excel. The separate instance is recalculated by the macro to produce the answer that is returned to the goal cell. Solver does not know that a separate computational environment was used to derive the complex goal calculation.

Of course, this calculation is repeated many times as the optimizer tests trial combinations of the weights. Furthermore, it is done for each period in the analysis. This trick of using a separate instance of Excel and COM techniques is useful for deriving complex calculations associated with optimizer objective function calculations. It is essential when these calculations require multiple workbook recalculation or involve their own macros that may be difficult to otherwise order correctly within the cell recalculation hierarchy used by Excel during a workbook recalculation.

[18] The probability of inadequacy is actually more conservative because the stated percentile of the distribution was adjusted for sampling error in measuring that stated percentile. So, the minimum

was based on the sufficiency payment target set at a confidence level of .8. Higher confidence levels could have been used both for the payments and investments.

Release of Capital in Reserves

The assets, before payment of claims, were ***expected*** to grow from \$1,591,549 to \$1,770,939 given the optimized portfolio weights shown in the table. This expected value for asset growth is ***higher*** than the target minimum sufficiency level of \$1,683,785. The chance-constrained level imposes a higher standard than expected value; there only can be a .1 probability of the growth being inadequate, which was the confidence level chosen for objective function valuation. The built-in margins in the reserve are a source of expected capital release as the reserves are released.

A higher volume of initial assets is required to assure the confidence levels sought.[19] As a result, the higher initial reserve level that must be maintained is ***expected*** to grow to be more than is required. Of course, it may not grow at the expected rate, but the optimization sets a higher confidence level so that the reserve will be sufficient both with respect to claims and investment experience (asset growth confidence is 0.9). There is an expectation of a favorable release of the contingency margins, both claims and investment, in this reserving method.

A local optimum is shown for the minimum reserve in the first row of Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios. Other weights of assets produce different local optima. In general, the other local optima are similar. The variations are discussed in detail in the section "Variation among Local Minima".

Investment Portfolio Rebalancing

Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios shows the changes that would occur were portfolio rebalancing to track the changes in the optimal portfolios each period. It will be noted at a later point in the paper that the result shown in the table is a local optimum. Other solutions of the non-linear optimizer produce different optimum values. A high volatility in portfolio composition could be found among optimizer solutions to what amounts to the same problem. Many different portfolios could lead to approximately the same optimized solution; the local optima, although clustered, were constructed from rather different portfolio compositions.

This makes generalizations about changes in portfolio composition over time very difficult to make. Even the rank order of asset weights was highly volatile. In general, higher risk investments give way to lower risk ones such as U.S. Treasury bills and debt.

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sufficiency level was an empirical observation somewhat lower than the observed .1 percentile of the distribution. A similar conservative adjustment was made in the choice of the empirically determined target sufficiency level. Both adjustments to the stated percentiles were done because of sampling error in measuring them.

[19] There are two confidence levels: (1) that the investments will grow to a target sufficiency level, and (2) that the target sufficiency level will be adequate to fund future claims assuming only a risk-free level of return thereafter.

The Sharpe ratio, which is measure of risk-adjusted return, also reflects a rebalancing scheme that tends to move from higher risk to lower risk.

An intuitive explanation for this rebalancing scheme is that the portfolio cannot be placed at risk during later periods when losses have less chance of being recouped. Higher risk in the early periods, however, may be an acceptable tradeoff both because there is a longer period to recover early investment losses and because the higher expected returns assist in meeting the higher demands for cash flow at early stages of claims development.

Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios

	Period 1	*Period 2*	*Period 3*	*Period 4*	*Period 5*	*Period 6*	*Period 7*	*Period 8*	*Period 9*	*Period 10*
Min Sufficiency Level	1,591,549	1,064,347	680,513	445,919	296,149	198,432	129,609	78,977	37,711	0
Required EOP target assets	1,683,785	1,128,672	722,708	471,934	313,611	210,181	137,047	83,439	39,879	0
EAFEU	.067	.073	.146	.208	.009	.144	.011	.063	.015	
INTLUHD	.189	.039	.126	.198	.166	.134	.154	.226	.036	
S&P5	.163	.023	.0	.147	.063	.0	.084	.082	.039	
USTB	.067	.465	.063	.167	.341	.141	.199	.058	.559	
R_MID	.050	.0	.097	.083	.034	.040	.023	.119	.023	
HIYLD	.122	.148	.002	.004	.0	.012	.179	.098	.159	
CONV	.036	.055	.131	.033	.067	.140	.061	.041	.003	
LBCORP	.032	.068	.023	.032	.046	.033	.025	.105	.0	
LBGVT	.123	.011	.325	.116	.096	.205	.107	.046	.148	
LBMBS	.151	.117	.088	.011	.179	.152	.157	.164	.017	
Expected Return	1,770,939	1,162,538	759,151	499,672	323,787	219,901	142,558	87,867	40,883	
Standard Deviation	105,186	40,051	44,333	33,730	11,656	11,316	6,513	5,284	1,176	
0.1 Percentile	1,635,357	1,112,691	704,420	457,192	308,825	205,665	134,405	81,043	39,420	
0.2 Percentile	1,683,802	1,128,682	722,719	471,996	313,612	210,183	137,059	83,447	39,880	
0.25 Percentile	1,699,140	1,135,369	729,442	478,066	315,443	211,864	138,069	84,296	40,067	
0.5 Percentile	1,767,097	1,160,296	756,702	498,949	323,608	219,678	142,282	87,783	40,827	
0.75 Percentile	1,842,846	1,189,563	787,478	520,928	331,522	227,456	146,862	91,338	41,673	
0.8 Percentile	1,859,232	1,197,558	795,862	526,399	333,689	229,322	148,179	92,295	41,892	
0.9 Percentile	1,908,091	1,215,841	816,402	542,054	338,831	234,158	151,193	94,726	42,410	
Sharpe ratio	.712	.697	.759	.727	.688	.740	.682	.697	.580	

Effect of Using Line-of-Business Correlated Links

The bootstrap method used in this study produced a correlation matrix for each link factor. The correlations were small and very often insignificant. This can be seen in the correlation matrix for one of the links shown in Table 1: Statistics for $Link_1$. However, the measured correlations were more often positive than negative. For example, Spcl_Liab and OL_OCC have positive correlation with Reins_A exceeding 0.07. In general, small positive correlations were found for many other lines and for other development periods. It is reasonable to characterize the ultimate links for lines of business in this study as being generally uncorrelated, but occasionally having isolated pockets of positively correlated loss among certain lines.

The experimental results were recast so that line-of-business independence was assumed. The same expected values and variances were used, but the multinormal link ratios were simulated with a zero correlations among the lines. The minimum sufficiency levels were calculated using the same investment scenarios. So, the only difference in treatment was the removal of the generally slight positive correlation. The results appear in Table 5: Effects of Removal of By-Line Correlation. This table should be compared with Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios.

Table 5: Effects of Removal of By-Line Correlation

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
Min Sufficiency Level	1,586,858	1,067,462	682,914	444,120	295,781	198,493	129,681	78,428	37,512	0
Required EOP target assets	1,680,031	1,129,561	723,563	471,676	313,792	210,037	137,006	83,388	39,792	0
EAFEU	.027	.226	.251	.111	.082	.172	.092	.060	.054	
INTLUHD	.139	.124	.071	.018	.112	.142	.146	.036	.118	
S&P5	.012	.156	.025	.059	.050	.069	.038	.003	.041	
USTB	.002	.007	.057	.402	.452	.004	.117	.738	.371	
R_MID	.142	.087	.097	.0	.090	.014	.011	.020	.034	
HIYLD	.004	.073	.103	.030	.004	.093	.091	.005	.072	
CONV	.072	.049	.011	.089	.008	.157	.132	.006	.003	
LBCORP	.023	.120	.250	.011	.010	.047	.220	.0	.002	
LBGVT	.001	.124	.099	.087	.006	.148	.104	.079	.212	
LBMBS	.579	.034	.035	.193	.186	.154	.050	.052	.093	
Expected Return	1,761,302	1,206,609	768,268	487,332	324,459	221,731	143,432	84,694	41,001	
Standard Deviation	97,050	93,454	54,729	19,170	12,377	14,050	7,681	1,602	1,453	
0.1 Percentile	1,637,518	1,091,963	700,940	462,841	308,686	204,174	133,796	82,668	39,139	
0.2 Percentile	1,680,084	1,129,562	723,598	471,678	313,795	210,037	137,009	83,389	39,792	
0.25 Percentile	1,693,157	1,141,484	731,828	474,425	315,561	211,698	138,070	83,591	40,029	
0.5 Percentile	1,759,289	1,202,290	765,699	486,883	324,270	221,319	143,077	84,669	40,934	
0.75 Percentile	1,826,979	1,268,604	803,468	499,658	332,806	230,934	148,617	85,766	41,966	
0.8 Percentile	1,843,977	1,285,234	812,933	502,672	334,830	233,406	150,128	86,021	42,200	
0.9 Percentile	1,889,191	1,329,506	837,534	511,794	340,621	239,733	153,711	86,799	42,873	
Sharpe ratio	.724	.735	.735	.735	.734	.721	.682	.683	.697	

The positive correlation seen in this study generally increased the minimum sufficiency levels. Chance-constrained reserves must be higher in the presence of correlation among lines of business. The company should have a higher level of capital attribution for the collection of correlated lines of business than what would be needed were they to be independent. But, the effect for the company in this study was small. For example, the beginning (Period 1) minimum sufficiency level dropped from $1,591,549 to $1,586,858. Comparison of Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios and Table 5: Effects of Removal of By-Line Correlation discloses that there generally are higher minimum sufficiency levels when correlation in the ultimate links is considered. However, the effect is modest and not always consistent. For example, periods 2,3 and 7 have modestly higher minimum sufficiency levels when independence was assumed among the lines of business.

Caveats Regarding this Study

Variation among Local Minima

The optimization problem requires non-linear methods. Many combinations of asset weightings are likely to yield the same objective value, and the optimizer will produce varying optima for the same problem. These local optima arise when the optimizer randomly seeds different paths to a solution.

A separate test of the optimization procedures was done to better understand the nature of the local minima. There was variation in the answers produced by the optimizer when all aspects of the problem were held constant except one: the optimizer was seeded in different ways at the start of the optimization by giving it different starting values for the portfolio allocation. Optimum values found by the optimizer are dependent on many empirical properties of the data being optimized—generalizations are difficult.

Variation among local optima of minimum sufficiency levels was studied for two of the periods. The results are summarized in Table 6: Variation in Local Optima.

Table 6: Variation in Local Optima

Values of Optimization	Variation Observed
Objective function (minimum sufficiency level)	Based on 266 replications of the optimization using different seeding for portfolio allocations, the local optima had modest variation.[20]

[20] The period means are higher than reported in other Tables because this test of local optima was based on different confidence levels. But, the overall results are insensitive to the choice of sufficiency probabilities; the variance in local optima is small.

Values of Optimization	Variation Observed
	1 1,805,772 51,733 3 832,612 22,321 From an operational standpoint, the standard deviations are small; sufficiency estimates are substantially unchanged regardless of the local optimum chosen by the optimizer.
Asset allocation weights	There was high volatility in portfolio allocation among the local optima.[21] Even rank shifts among asset category weights were large. In the following table, statistics for rank order of asset appearance in Period 1 results are given. Observe that the mean ranks are very close and standard deviations of the ranks are high. This indicates a high volatility in the ranking of any given asset category among the local optima. Period 1 Asset Category Ranks EAFEU 5.60 2.28 INTLUHD 5.37 2.37 S&P5 5.50

[21] The fact that many different portfolio allocations result in similar objective values was indirectly observed in Scheel, et al [2000]. In that study, efficient frontiers often were found not to be particularly efficient in a forecast sense. Off-frontier points in that study had portfolio weights, which produced results comparable to those of on-frontier points having different weights. A similar insensitivity to portfolio allocation was found in this study, because many different portfolio allocations resulted in local optima that differed by insubstantial amounts.

Values of Optimization	Variation Observed	
		2.69
	USTB	
		4.91
		2.63
	R_MID	
		5.45
		2.80
	HIYLD	
		5.23
		3.08
	CONV	
		5.39
		3.22
	LBCORP	
		5.63
		3.27
	LBGVT	
		6.64
		3.23
.	LBMBS	
		5.28
		2.67

The results shown in Table 6: Variation in Local Optima show that at least for this study, the variation in the minimum sufficiency levels is small; the local optima appear to be clustered. However, there were numerous portfolio profiles with some rank order stability but still considerable differences in weight magnitudes. Although the optimization methods seem to be reasonably robust from an operational standpoint, this may not generally be true for other empirical datasets.

Other Assumptions and Limitations

Because this study was focused on methodology and not on precision of the actual valuations of reserves, some shortcuts were made. The following limitations may be important if greater precision is desired:

1. No adjustments for uncertainty respecting inflation were made. In fact, no business scenario generation, other than investment returns, was done in this analysis.

2. The possibility of future inflation differing from expected inflation was not considered. There was no common economic tie binding future loss projections and future asset valuations.
3. Tax frictions were not analyzed.
4. The finesse of actuarial reserving involves many considerations such as actuarial judgment, the appropriate reserving model to be used with aggregate loss triangle information, and many other loss reserve details.
5. The reserving method deployed was a simple average chain ladder approach; some actuaries may think it naive, but other reserving methods could be applied using the approach laid out here.
6. Claim frequency was not studied.
7. The implicit assumption is that the proxy asset portfolio used in this study is a reasonable representation of assets used to back reserves. Other asset proxies and approaches to investment scenario generation might yield materially different results.
8. The optimizer constraint set did not limit the extent of portfolio allocations for specific asset classes such as equities or international securities within the proxy portfolio. In general, the mix of assets in the optimal portfolios was under 20 percent equities. Allocations to mortgaged backed securities got as high as 18 percent. These and some allocations to convertible and high yield bonds may not be consistent with statutory limitations on such asset classes. These asset class-specific constraints were deemed to be beyond the scope of this study; however, appropriate changes could be made in the constraints set to limit the asset allocation weight for a class of investments.

Appendix A: Review of Data Sources

Liabilities

The source of financial information used for this paper is the data provided to participants in the CAS Call Paper program for the 2001 DFA Seminar. The data consisted primarily of Schedule P information.

Proxy Assets

This paper uses monthly time series of asset class total returns. A selection of broad asset classes typical of P&C insurance company asset portfolios was chosen for examination. The time series all begin January 1, 1970. However, certain asset classes (e.g., mortgage backed securities) do not have a history that extends back this far. For these classes the time series were backfilled to the January 1, 1970 start date by an investment consultant. The backfill process was based on a consideration of the market conditions of the time (e.g. interest rates, fixed income spreads, inflation expectations) and how the particular sector would have performed given those market conditions.

Table 6 Asset Components

Class	Code	Source	Start Date
International Equities	EAFEU	MSCI EAFE Index	1/1970
International Fixed Income	INTLHDG	JP Morgan Non-US Traded Index	1/1970
Large Cap Domestic Equities	S&P5	S&P 500 Index	1/1970
Cash	USTB	90 Day US Treasury Bill	1/1970
Mid Cap Domestic Equities	RMID	S&P Mid Cap 400 Index	1/1982
Mid Cap Domestic Equities	RMID	S&P Mid Cap 400 Index	1/1982
High Yield	HIYLD	CSFB High Yield Bond Index	1/1986

Class	Code	Source	Start Date
International Equities	EAFEU	MSCI EAFE Index	1/1970
Convertible Securities	CONV	CSFB Convertible Index	1/1982
Corporate Bonds	LBCORP	Lehman Brothers Corporate Bond Index	1/1973
Government Bonds	LBGOVT	Lehman Brothers Government Bond Index	1/1973
Mortgage Backed Securities	LBMBS	Lehman Brothers Mortgage Backed Securities Index	1/1986

Appendix B: Example of Asset Scenarios for an Annual Period

Specimen Annualized Returns										
Scenario	EAFEU	INTLHDG	S&P5	USTB	R_MID	HIYLD	CONV	LBCORP	LBGVT	LBMBS
1	-0.0027	0.0623	-0.0158	0.0802	0.0022	-0.0290	0.0297	0.0129	0.0757	0.0647
2	0.1580	-0.1165	0.1401	0.0621	0.1595	0.1326	0.1681	0.0600	0.0645	0.0513
3	0.3420	0.0894	0.2213	0.0707	0.1533	0.1743	0.1572	0.1284	0.1234	0.1386
4	-0.1184	0.0985	-0.1215	0.0656	-0.0601	0.0159	-0.0607	0.0657	0.0823	0.0778
5	0.2593	0.0544	0.2091	0.0625	0.2001	0.2195	0.1990	0.1681	0.1452	0.1144
6	0.0529	0.1815	-0.0815	0.0640	-0.1208	-0.0084	-0.0478	-0.0194	0.0714	0.0716
7	0.0352	0.0257	0.4263	0.0561	0.3197	0.1684	0.2330	0.0785	0.0928	0.1219
8	-0.0263	-0.0213	0.1301	0.0654	0.1293	0.2067	0.1132	0.1714	0.1567	0.1576
9	0.0622	0.0899	-0.0052	0.0630	-0.0161	0.0749	0.0622	0.0689	0.0691	0.0900
10	-0.1113	0.0308	-0.0285	0.0687	-0.0737	-0.0644	-0.0015	-0.0181	0.0659	0.0232
11	0.4261	0.0867	0.3720	0.0594	0.3460	0.1973	0.2650	0.1624	0.1031	0.1247
12	0.3469	0.0412	0.2036	0.0610	0.2292	0.2230	0.1441	0.1319	0.0908	0.1537
13	0.1085	0.0008	0.0505	0.0662	0.1505	-0.0178	0.0839	-0.0024	0.0036	0.0287
14	0.1889	0.1957	0.0605	0.0611	-0.0622	0.1085	-0.0074	0.2271	0.2147	0.1889
15	0.7624	0.1837	0.4632	0.0765	0.6217	0.2408	0.3528	0.2786	0.2308	0.1944
16	0.4232	0.1025	0.0522	0.0760	-0.0179	-0.0002	0.0297	0.1637	0.1714	0.1520
17	0.2868	0.1125	0.1013	0.0718	0.0766	0.1287	0.1499	0.0728	0.0873	0.1445
18	0.2166	0.0479	0.3057	0.0728	0.2180	0.0654	0.1420	0.0621	0.1379	0.0866
19	-0.0120	0.0196	0.2636	0.0705	0.2442	0.0827	0.1536	0.0235	0.0456	0.0105
20	0.2230	0.0416	0.0891	0.0515	0.1085	0.2218	0.1033	0.1687	0.1087	0.0765
21	-0.0172	-0.0164	-0.0093	0.0747	0.0470	0.0555	0.0214	-0.0214	0.0483	0.0812
22	-0.0505	0.1184	0.0768	0.0765	0.0528	0.0757	0.0090	0.1442	0.1424	0.1624
23	0.0845	-0.0059	0.0561	0.0692	0.0730	-0.0704	0.0608	0.0045	0.0513	0.0376
24	0.2747	0.1734	0.2725	0.0582	0.2774	0.0052	0.2158	0.0623	0.1011	0.0947
25	0.1503	0.1671	-0.0037	0.0650	0.0446	0.0852	0.0275	0.1147	0.1007	0.0244

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