Can Long Tailed Lines of Business Really Afford Higher Loss Ratios?

Jonathan P. Evans, FCAS, MAAA

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Abstract

Perhaps the most commonly accepted principle of modern property and liability insurance is that longer tailed lines of business are able to operate profitably at higher loss ratios, or almost equivalently higher combined ratios, than short tailed lines. A combined ratio of 120% might be devastating to an auto physical damage line of business but quite healthy for per occurrence excess liability reinsurance. However, this maxim may be eroding due to three real world forces:

1. The requirement that property and casualty insurers generally hold loss reserve liabilities at full undiscounted values.
2. The requirement that additional surplus capital be held to support risk in loss reserves on top of surplus held to support current writings.
3. The demands of investors, insurance executives, and modern capital markets that profits be high enough to support all invested capital at a cost per unit of capital judged to be commensurate with the perceived exposure to risk.

All of these factors may push necessary loss ratios for longer loss payment duration lines down to the levels necessary for short loss payment duration lines. In concrete terms, it may be that a per occurrence excess liability reinsurance line requires a combined ratio on the order of 95%, just like an auto physical damage line, to produce an equivalent return on invested capital. In this paper we review some modeling results for different sets of assumptions and examine this issue, but do not attempt to ultimately resolve it.

Note: Henceforth we shall use the terms "long duration" and "short duration" to refer to lines of business whose average times from policy inception to loss payments are long and short, respectively.

Caveat and Disclaimer: It is not the intent of this paper to strongly advocate the ultimate validity of a specific profitability model or specific values for model parameters such as surplus requirements. It does intend to show that within the range of different models and parameter assumptions, which may be appropriate according to contemporary actuarial practices and standards, there are frequent cases where longer duration lines require underwriting profit provisions equal to or greater than those for short duration lines.

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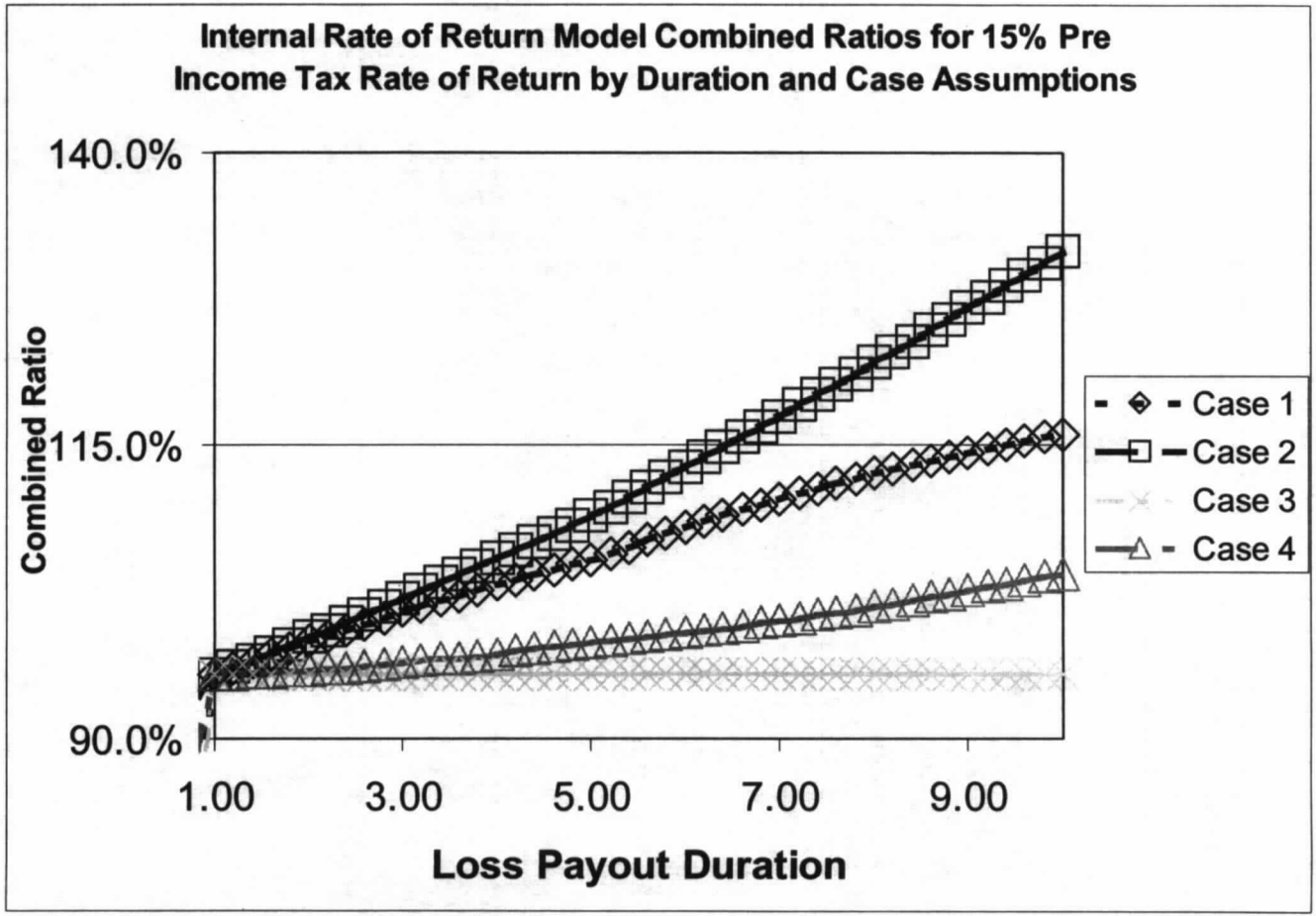
Some Results from an Internal Rate of Return Analysis

Consider the following results from a simple internal rate of return analysis (See Appendix I for modeling details):

Combined Ratios Necessary to Produce 15% Internal Rate of Return Before Income Taxes Under Different Assumptions

Case	Surplus Requirement	Loss Reserve Requirement	5.5 Years Undiscounted Loss Payout Duration	1.5 Years Undiscounted Loss Payout Duration
1	Released After Premium Earned	Undiscounted	106.2%	97.0%
2	Released After Premium Earned	Discounted	110.8%	97.1%
3	Held Until Loss Reserves Paid	Undiscounted	95.5%	95.5%
4	Held Until Loss Reserves Paid	Discounted	98.8%	95.6%

The traditional perspective is that Case 1 most accurately represents reality. Here we clearly see a higher combined ratio tolerance for the long duration line. However, Cases 3 or 4 may be closer to reality, for reasons which we will address subsequently. In Case 3 both lines must produce the same combined ratio to achieve their profitability objective.


Internal Rate of Return Model Combined Ratios for 15% Pre Income Tax Rate of Return by Duration and Case Assumptions
140.0%
115.0%
90.0%
Combined Ratio
1.00
3.00
5.00
7.00
9.00
Loss Payout Duration
Case 1
Case 2
Case 3
Case 4

Some Results from a Calendar Year Analysis

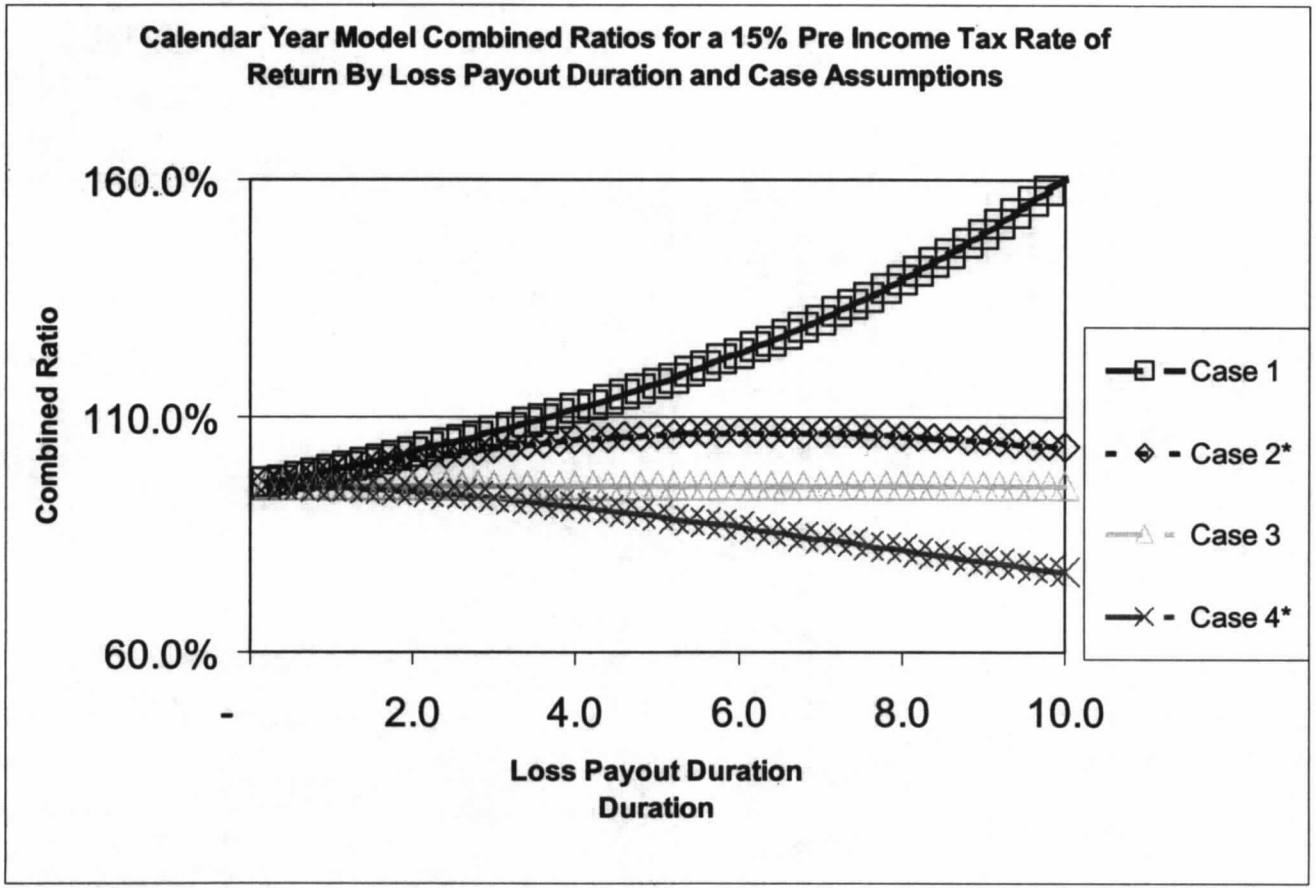
Now, we will alter our case assumptions slightly and consider results for an ongoing steady state calendar year analysis (See Appendix II for modeling details):

Combined Ratios Necessary to Produce 15% Calendar Year Return Before Income Taxes Under Different Assumptions

Case	5.5 Years Undiscounted Loss Payout Duration	1.5 Years Undiscounted Loss Payout Duration
1	120.0%	100.5%
2*	106.3%	99.7%
3	95.3%	95.3%
4*	87.8%	94.6%

Cases 1 and 3 embody basically the same surplus and reserve assumptions as in the previous internal rate of return analysis. Cases 2* and 4* are different from Cases 2 and 4 in the previous section. In Cases 2* and 4* loss reserves are actually held at undiscounted values in addition to surplus, but the loss reserve equity due to discount is included in the calculation of invested capital.

These results are even more stark. In Case 4*, where loss reserve equity is recognized as adding to invested capital and surplus is held to support loss reserves in addition to current writings, the allowed combined ratio is actually lower for the long duration line !


Calendar Year Model Combined Ratios for a 15% Pre Income Tax Rate of Return By Loss Payout Duration and Case Assumptions
160.0%
110.0%
60.0%
Combined Ratio
-
2.0
4.0
6.0
8.0
10.0
Loss Payout Duration
Duration
Case 1
Case 2*
Case 3
Case 4*

Loss Reserves

Higher loss ratios for longer duration lines are tolerated based on the justification that, from the calendar year perspective, the large reserves which build up to support long duration lines generate large amounts of investment income. Almost equivalently, from the internal rate of return perspective, one can say that much more investment income is earned between the time premium is collected and when losses are paid. This justification may be flawed. It ignores the additional cost of capital for large amounts of discount equity in the loss reserves. Loss reserves for long duration lines are generally held at undiscounted nominal values under both U.S. statutory accounting and GAAP.

It is generally true that return on equity and related profitability objectives set by insurance executives typically refer to return on GAAP equity, which excludes loss reserve discount equity, or a similar measure of return on invested capital. However, U.S. federal income tax accounting does consider equity in loss reserves. Such concerns are taken into account for valuations of books of loss reserves during acquisitions. They are also present in the minds of managers of long duration excess reinsurance companies.

Insurance companies must actually carry assets sufficient to cover nominal loss reserve liabilities in addition to their capital held as policyholder surplus and as deferred acquisition expense equity in their unearned premium reserves. In an economic sense the excess of nominal loss reserves over present value loss reserves is an additional contribution of invested capital by the insurer. The capital implicit in these nominal reserves demands much more profit be made to produce an overall rate of return consistent with the cost of invested capital.

Surplus Capital

Another consideration with regard to invested capital is the required level of statutory surplus held. Traditionally this required level has been set at a fixed ratio to yearly written or earned premium. This standard is regulated by the first NAIC IRIS test. Alone, it would imply that no surplus is needed for loss reserves. However, the recent addition of a Risk Based Capital (RBC) test requirement by the NAIC regulates that surplus also be required to support loss reserves. Although the RBC test does account for discount in its reserve risk component, this test is compared to an adjusted surplus where even tabular reserves are adjusted to nominal values. The RBC is therefore a requirement for assets in addition to undiscounted reserves and will generally be positive even in the case of a pure runoff portfolio. RBC generally results in a surplus requirement less than extending a leverage ratio to reserves in addition to premium. However, RBC will be higher, relative to annual premium levels, for a company writing long duration business. This requirement can add another large amount of invested capital, which must be supported at an appropriate cost per unit.

Even beyond the requirement imposed by RBC, credit rating agencies and financial analysts would be wary of large loss reserves unsupported by capital. Although the NAIC IRIS test 1 does not distinguish between different lines of business, many financial analysts do. It is not uncommon to see companies use different premium to surplus ratios, with long duration lines having lower leverage ratios, when doing internal allocations of surplus. For example, the overall premium to surplus ratio might be 1.00 with a 1.50 ratio for property lines and a 0.75 ratio for liability lines.

The argument may be made that the discount equity in the nominal loss reserves acts as a sufficient amount of capital at risk. However, if reserves are underestimated in any of a number of ways – neglect of IBNR, implicit discounting, etc. – this risk buffer may easily prove to be nonexistent. The discount buffer itself is highly sensitive to the effects of inflation and varying investment returns. Relying on this discount equity as the only risk buffer for loss reserves is often an unsuitable solution.

The Demands of Investors

Modern investment analysts and capital markets will recognize the total invested capital value of a company. If profits are not competitive with investments in the same broad category of risk, market forces will require divestiture or restructuring of operations.

It may be argued that recognition of a larger amount of capital leads to recognition of lower risk, and hence less pressure on profitability targets, due to a reduced cost per unit of capital. This argument is somewhat relevant when the comparison is a highly capitalized long duration line of business versus a minimal capitalization of the same long duration line. The same long duration line has the same underwriting obligations, and therefore the same volatility in its underwriting liabilities whether it is highly capitalized or minimally capitalized. More capital is likely to reduce the risk per unit of capital and hence the cost.

This same argument is usually not applicable if the relevant comparison is the larger capital invested in long duration insurance lines versus short duration lines. A long duration line, with its build up of volatile loss reserves or equivalently from the individual policy perspective the longer delay in reporting or payment of claims brings additional risk not present in a short duration line. The capital in both loss reserve discount and surplus supporting loss reserves may in fact be a reasonable requirement to cushion the extra risk at the same cost per unit of capital.

There is another point about cost of capital, aside from the arguments of changing risk and cost per unit of capital which might accompany changes in requirements for capital, or just changes in the recognition of total invested capital. It is probably unrealistic to expect markets, analysts, and possibly even executives to quickly adjust their targeted rates of return for such subtleties. That is to say any of these parties is very likely to fix on a standard such as: "Insurance operations should return 15% on investment." They are likely to apply the same standard of 15% to a larger amount of recognized capital, at least over the short term, for a specific company or a specific line of business.

It is difficult to dismiss the possibility that more absolute dollars of profit must be made to support a much larger capital base at roughly the same cost per unit of capital, for a long duration line.

The Risk in Large Loss Reserves

At this point it is warranted, based on the above discussions of surplus and required rates of return, to briefly consider in more detail the issue of risk in loss reserves. There is frequently a confusion that mature loss reserves for older accident or policy years are always less volatile than losses for current writings or reserves for more recent years. In some cases, where there is no possibility of pure IBNR and most claim cases have been closed, this may be true. It is often not true if risk is measured by an appropriate relative measurement such as the coefficient of variation of loss reserves. The confusion arises because older, mature accident or policy years are usually less volatile relative to their ultimate total losses. However, most of these ultimate losses have already been paid and are not being held as loss reserve liabilities. Relative to their loss reserves, older, mature accident or policy years may easily be as volatile as recent years' or next year's writings.

Consider the following hypothetical example (See Appendix IV for details):

	Coefficients of Variation		
Years After Policy Inception	Total Accident/Policy Year Losses	Incremental Losses	Calendar Year Loss Reserves
1	*15.7%*	7.4%	17.5%
2	*15.1%*	9.0%	18.9%
3	*14.4%*	11.0%	20.6%
4	*13.5%*	13.5%	22.5%
5	*12.4%*	16.6%	24.7%
6	*11.0%*	20.3%	27.4%
7	*9.3%*	24.8%	30.9%
8	*7.2%*	30.4%	36.0%
9	*4.6%*	37.3%	45.6%
10	*0.0%*	45.6%	NA
Correlated Totals		16.2%	17.2%

In the example above the total calendar year reserves of a company have a coefficient of variation, at 17.2%, which is higher than for the ultimate of a single accident/policy year's losses at inception, which is 16.2%. What declines over time is the coefficient of variation for the ultimate total losses for a given accident/policy year.

Inadequate Profits Versus Operating Losses, a Possible Mitigating Factor

A possible mitigating factor for the dangers of running long duration lines at high loss ratios may be found by examining what happens when loss ratios are high. Consider the internal rate of return results when we revisit Case 3 with a 120% combined ratio (See Appendix V for modeling details):

Internal Rates of Return Before Income Taxes Corresponding to 120% Combined Ratios for Case 3.

Case	Surplus Requirement	Loss Reserve Requirement	5.5 Years Undiscounted Loss Payout Duration	1.5 Years Undiscounted Loss Payout Duration
3	Held Until Loss Reserves Paid	Undiscounted	5.17%	-8.43%

Similarly, here are the calendar year results when we revisit Case 3 with a 120% combined ratio (See Appendix V for modeling details):

Calendar Year Returns Before Income Taxes Implied by a 120% Combined Ratio for Case 3 Assumptions

Case	5.5 Years Undiscounted Loss Payout Duration	1.5 Years Undiscounted Loss Payout Duration
3	7.08%	-3.70%

Previously, we had shown that both the calendar year and internal rate of return models indicated a combined ratio of slightly over 95% was needed for a pre-tax return of 15% in Case 3. When we change the combined ratio to 120% we see a consistent difference in both models between the different loss payout durations. The long duration line still produces a gross profit, although lower than our 15% target. The short duration line actually produces an operating loss.

A partial explanation of the insurance industry's general tolerance of higher loss or combined ratios for long duration lines may be that the consequence is only an inadequate rate of return, rather than actual dollar losses as would be the consequence for a short duration line.

Implications for Actuarial Practice

Actuarial practitioners doing profitability analyses, with emphasis on loss reserve payout durations, should take special care with the following considerations:

- What exactly is total invested capital ? What asset components such as unearned premium reserve equity, statutory surplus, loss reserve discount equity, etc. should be included in invested capital ?
- For what periods of time after policy inception must invested capital remain committed, and to what specific lines/exposures is invested capital allocated ? When exactly must capital be contributed and when exactly can it be released from corporate assets ?
- What is an appropriate rate of rate of return on invested capital ? Does this rate apply to all the components of invested capital or just a fraction of total invested capital ? Does this rate differ for different components ?

These questions are not new. There has been much discussion of these considerations by actuaries doing profitability analyses. However, as we have shown, differences in how these considerations are addressed by modeling assumptions may dramatically and qualitatively alter results for long duration lines of business. Specifically, it may change the relative performance benchmarks of long duration versus short duration lines of business

Conclusion

In this paper we have raised the question of whether long duration lines of business can run higher loss ratios than short duration lines and be equally profitable. We have shown that this principle is dependent on assumptions about invested capital and its associated cost per unit. Some common assumptions about these two considerations, which lead to higher loss ratio tolerances for long duration lines, may not be valid in the real world. These assumptions may be inconsistent with regulatory requirements, demands of investors, or perhaps even financial economic theory. The acceptance of higher loss ratios for long duration lines may be partially explained by the property that such cases tend to produce lower rates of return but not actual dollar operating losses. It is beyond the scope of this paper to propose a definite solution or take a specific stance on this issue. It is clear that in the insurance industry there is a great deal of confusion and disagreement about which assumptions should be used for profitability modeling. There are sets of assumptions, which are not entirely outlandish, implying that long duration lines of business should produce loss ratios equal to, or even below, loss ratios for short duration lines of business.

Appendix I

An internal Rate of Return Model

Here are some details of this specific IRR model:

- Time is measured in discrete years and each transaction is at year beginning or equivalently last year end.
- Premium is collected at year 0.
- Losses are reported and paid at the same time.
- All underwriting expenses are a fixed 30% of premium.
- All underwriting expenses are paid at year 0 and correspond to an investment of capital for equity in the unearned premium reserve for the time between year 0 and year 1.
- Initial surplus is an investment of capital equal to 50% of premium or equivalently 50% of the initial unearned premium reserve.
- Depending on the case assumptions surplus in subsequent years is either 0 or 50% of loss reserves.
- Depending on the case assumptions loss reserves are either held at discounted or undiscounted values.
- Invested assets correspond to the total of loss reserves, unearned premium reserves, and surplus.
- Investment income is a fixed 5% of the prior year's invested assets.
- The underwriting profit provision, or equivalently the loss ratio or combined ratio, is chosen to produce a 15% internal rate of return before income taxes.
- Income taxes are not explicitly modeled, but they could be reasonably modeled as a factor adjustment to the internal rate of return. (i.e. If income tax is 30% we are solving for a 10.5% after tax rate of return.)
- Although we have fixed premium and solved for loss ratio, the same underwriting profit provisions result if loss cost is fixed and we solve for premium.
- Although we have modeled all underwriting expenses as a variable, that is a fixed percentage of premium, we could have modeled fixed dollar expenses as a deduction to the loss cost resulting in an adjustment to the resulting loss ratio. The combined ratio would be unaffected.

Case 1 - 5.5 Year Loss Payment Duration

Surplus	Released
Loss Reserves	Nominal
UW provision	-6.2%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	76.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	-	-	-	-	-	-	-	-	-	-
Invested Capital	800	-	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	686	610	534	457	381	305	229	152	76	-
Total Invested Assets	1,500	686	610	534	457	381	305	229	152	76	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	76	76	76	76	76	76	76	76	76	76
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	34	30	27	23	19	15	11	8	4
Release of Earnings	(800)	813	34	30	27	23	19	15	11	8	4
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	707	26	20	15	11	8	6	4	2	1
Discounted Loss Reserve		542	493	441	387	330	270	208	142	73	-

Case 1 - 1.5 Year Loss Payment Duration

Surplus	Released										
Loss Reserves	*Nominal*										
UW provision	3.0%										
	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	67.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	-	-	-	-	-	-	-	-	-	-
Invested Capital	800	-	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	335	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	335	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	335	335	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	17	-	-	-	-	-	-	-	-
Release of Earnings	(800)	905	17	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	787	13	-	-	-	-	-	-	-	-
Discounted Loss Reserve		319	-	-	-	-	-	-	-	-	-

Case 2 - 5.5 Year Loss Payment Duration

Surplus	Released
Loss Reserves	Discounted
UW provision	-10.8%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	80.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	-	-	-	-	-	-	-	-	-	-
Invested Capital	800	-	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	727	646	566	485	404	323	242	162	81	(0)
Total Invested Assets	1,500	574	522	467	410	350	286	220	150	77	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	81	81	81	81	81	81	81	81	81	81
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	29	26	23	21	17	14	11	8	4
Release of Earnings	(800)	920	0	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	800	-	-	-	-	-	-	-	-	-
Discounted Loss Reserve		574	522	467	410	350	286	220	150	77	-

Case 2 - 1.5 Year Loss Payment Duration

Surplus	Released
Loss Reserves	Discounted
UW provision	2.9%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	67.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	-	-	-	-	-	-	-	-	-	-
Invested Capital	800	-	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	335	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	320	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	335	335	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	16	-	-	-	-	-	-	-	-
Release of Earnings	(800)	920	0	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	800	-	-	-	-	-	-	-	-	-
Discounted Loss Reserve		320	-	-	-	-	-	-	-	-	-

Case 3 - 5.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Nominal
UW provision	4.5%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	65.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	295	262	229	197	164	131	98	66	33	-
Invested Capital	800	295	262	229	197	164	131	98	66	33	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	590	524	459	393	328	262	197	131	66	-
Total Invested Assets	1,500	884	786	688	590	491	393	295	197	98	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	66	66	66	66	66	66	66	66	66	66
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	44	39	34	29	25	20	15	10	5
Release of Earnings	(800)	625	77	72	67	62	57	52	47	43	38
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	544	58	47	38	31	25	20	16	12	9
Discounted Loss Reserve		466	423	379	332	284	232	178	122	62	-

Case 3 - 1.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Nominal
UW provision	4.5%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	65.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	164	-	-	-	-	-	-	-	-	-
Invested Capital	800	164	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	328	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	491	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	328	328	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	25	-	-	-	-	-	-	-	-
Release of Earnings	(800)	756	188	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	658	142	-	-	-	-	-	-	-	-
Discounted Loss Reserve		312	-	-	-	-	-	-	-	-	-

Case 4 - 5.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Discounted
UW provision	1.2%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	68.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	310	275	241	206	172	138	103	69	34	-
Invested Capital	800	310	275	241	206	172	138	103	69	34	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	619	551	482	413	344	275	206	138	69	-
Total Invested Assets	1,500	799	720	639	556	470	382	291	197	100	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	69	69	69	69	69	69	69	69	69	69
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	40	36	32	28	24	19	15	10	5
Release of Earnings	(800)	707	50	48	46	45	43	41	40	38	36
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	615	38	32	27	22	19	16	13	11	9
Discounted Loss Reserve		489	445	398	349	298	244	187	128	66	-

Case 4 - 1.5 Year Loss Payment Duration

Surplus	Held										
Loss Reserves	Disounted										
UW provision	4.4%										
Time Period	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%										
Loss Ratio	65.6%										
Surplus	500	164	-	-	-	-	-	-	-	-	-
Invested Capital	800	164	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	328	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	477	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	328	328	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	24	-	-	-	-	-	-	-	-
Release of Earnings	(800)	770	172	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	670	130	-	-	-	-	-	-	-	-
Discounted Loss Reserve		313	-	-	-	-	-	-	-	-	-

Appendix II

A Calendar Year Rate of Return Analysis

Here are some details of this specific calendar year rate of return model:

- Loss reserves are held at nominal undiscounted values.
- All underwriting expenses are a fixed 30% of premium.
- All underwriting expenses are paid up front and correspond to an investment of capital for equity in the unearned premium reserve.
- Depending on the case assumptions surplus is 50% of either premium, or premium and loss reserves.
- The unearned premium reserve is equal to 50% of the premium.
- Loss reserves are equal to the product of the premium, loss ratio, and duration. (See Appendix III.)
- The discount factor for computing loss reserve discount equity is based on a uniform payout pattern lasting for twice the payout duration. (See Appendix III.)
- Depending on the case assumptions loss reserve discount equity is or is not included in invested capital.
- Invested assets correspond to the total of loss reserves, unearned premium reserves, and surplus.
- Investment Income is a fixed 5% of Invested Assets.
- The underwriting profit provision, or equivalently the loss ratio or combined ratio, is chosen to produce a 15% calendar year rate of return before income taxes.
- Income taxes are not explicitly modeled, but they could be reasonably modeled as a factor adjustment to the internal rate of return. (i.e. If income tax is 30% we are solving for a 10.5% after tax rate of return.)
- Although we have fixed premium and solved for loss ratio, the same underwriting profit provisions result if loss cost is fixed and we solve for premium.
- Although we have modeled all underwriting expenses as a variable, that is a fixed percentage of premium, we could have modeled fixed dollar expenses as a deduction to the loss cost resulting in an adjustment to the resulting loss ratio. The combined ratio would be unaffected.

Case 1 Premium leverage based on premium, with no loss reserve discount equity included in invested assets.
Case 2* Premium leverage based on premium, with loss reserve discount equity included in invested assets.
Case 3 Premium leverage based on premium and loss reserves, with no loss reserve discount equity included in invested assets.
Case 4* Premium leverage based on premium and loss reserves, with loss reserve discount equity included in invested assets.

Calendar Year Model for 5.5 Year Loss Payout Duration

Case	Target ROR	Premium	Expense Ratio	Investment Rate of Return	Duration	Leverage	Combined Ratio
1	15%	1,000	30%	5.0%	5.50	2.00	120.0%
2*	15%	1,000	30%	5.0%	5.50	2.00	106.3%
3	15%	1,000	30%	5.0%	5.50	2.00	95.3%
4*	15%	1,000	30%	5.0%	5.50	2.00	87.8%

Case	UW Margin	Loss Ratio	UEPR	UEPR Equity	Loss Reserves Held	Loss Reserve Discount Equity	Statutory Surplus
1	-20.00%	90%	500	150	4,950	-	500
2*	-6.34%	76%	500	150	4,199	660	500
3	4.75%	65%	500	150	3,589	-	2,294
4*	12.24%	58%	500	150	3,177	500	2,088

Case	Invested Assets	Invested Capital	UW income	Investment Income	Earnings	ROR
1	5,950	650	(200)	298	97	15.0%
2*	5,199	1,310	(63)	260	197	15.0%
3	6,383	2,444	48	319	367	15.0%
4*	5,765	2,738	122	288	411	15.0%

Calendar Year Model for 1.5 Year Loss Payout Duration

Case	Target ROR	Premium	Expense Ratio	Investment Rate of Return	Duration	Leverage	Combined Ratio
1	15%	1,000	30%	5.0%	1.50	2.00	100.5%
2*	15%	1,000	30%	5.0%	1.50	2.00	99.7%
3	15%	1,000	30%	5.0%	1.50	2.00	95.3%
4*	15%	1,000	30%	5.0%	1.50	2.00	94.6%

Case	UW Margin	Loss Ratio	UEPR	UEPR Equity	Loss Reserves Held	Loss Reserve Discount Equity	Statutory Surplus
1	-0.54%	71%	500	150	1,058	-	500
2*	0.26%	70%	500	150	1,046	49	500
3	4.75%	65%	500	150	979	-	989
4*	5.43%	65%	500	150	968	46	984

Case	Invested Assets	Invested Capital	UW income	Investment Income	Earnings	ROR
1	2,058	650	(5)	103	98	15.0%
2*	2,046	699	3	102	105	15.0%
3	2,468	1,139	48	123	171	15.0%
4*	2,453	1,180	54	123	177	15.0%

Appendix III

Loss Reserves Held at a Point in Time and Discount Factor

We will calculate the average ratio of outstanding loss reserves to the rate of losses currently being incurred. The motivation behind this is to show that without growth or decline in written exposures the product of the premium, loss ratio, and duration is a reasonable estimate of loss reserves.

Let f(t) be the probability density for the time between when a certain amount of exposure is earned and the time when the corresponding losses are paid. Let F(t) be the corresponding cumulative distribution for f(t). Let D be the undiscounted duration or average time to loss payment, which we shall refer to as the "duration". Hence the following integral relations hold:

$$\int_0^\infty f(t)dt = 1$$

$$D = \int_0^\infty t \cdot f(t)dt$$

$$F(s) = \int_0^s f(t)dt$$

We define v(t) to the rate at which exposure (measured in incurred losses) is earned at time t. Consequently we can calculate the average outstanding loss reserves R at time 0 based on previously earned exposure :

$$R = \int_0^\infty \upsilon(-t) \cdot [1 - F(t)]dt$$

Next we let v(t) follow a constant exponential rate of growth and solve the integral using integration by parts:

$$\upsilon(t) = e^{\alpha t}$$

$$R = \int_0^{\infty} e^{-\alpha t} \cdot [1 - F(t)]dt$$

$$= \left[\frac{e^{-\alpha t}}{-\alpha} [1 - F(t)] \right]_0^{\infty} - \int_0^{\infty} \frac{e^{-\alpha t}}{-\alpha} f(t)dt$$

$$= \frac{1}{\alpha} - \frac{M_T(-\alpha)}{\alpha}$$

$$= \frac{1 - M_T(-\alpha)}{\alpha}$$

where $M_T(\ )$ is the moment generating function of the density f(t). Finally, we can use L'Hospital's Rule to evaluate this expression for the steady state case, where growth is zero:

$$R = \left. \frac{M_T'(-\alpha)}{1} \right|_{\alpha=0} = E[T] = D$$

Since we defined our exposure to be 1 unit of loss per time period at time 0, the duration is a reasonable estimate of the ratio of outstanding loss reserves to the rate of losses incurred at a point in time, when there has been 0 growth for a long time prior.
Now we will address the issue of the average discount factor for loss reserves. If we denote the discount factor for dollars paid at time s, by the symbol a(s), the following expression holds.

$$PV(Loss\,\mathrm{Re}serves) =$$

$$\int_0^\infty v(-t)dt \int_0^\infty f(t+s)a(s)ds$$

We will set the discount factor to correspond to continually compounded interest, the loss payout density to be uniform between 0 and 2D, and the exposure to be uniformly earned at a rate of 1 :

$$a(s) = e^{-\beta s}$$

$$f(t) = \frac{1}{2D} \qquad t \in [0,2D]$$

$$f(t) = 0 \qquad t \notin [0,2D]$$

$$v(t) = 1$$

Now we can evaluate the present value of loss reserves:

$$PV(Loss\ \mathrm{Re}serves) = \int_0^{2D} dt \int_0^{2D-t} \frac{e^{-\beta s}}{2D} ds$$

$$= \int_0^{2D} \frac{1-e^{-\beta(2D-t)}}{2\beta D} dt$$

$$= \frac{1}{\beta} + \frac{e^{-2\beta D} - 1}{2\beta^2 D}$$

We can divide this by the nominal amount of reserves, which we have previously shown to be D, to get an overall discount factor:

$$\frac{PV(Loss\ \mathrm{Re}serves)}{Loss\ \mathrm{Re}serves} = \frac{\frac{1}{\beta} + \frac{e^{-2\beta D} - 1}{2\beta^2 D}}{D}$$

$$= \frac{1}{\beta D} + \frac{e^{-2\beta D} - 1}{2\beta^2 D^2}$$

Appendix IV

Hypothetical Demonstration of Volatility of Loss Reserves

- This demonstration uses a flat dollar amount reporting/payment pattern over 10 years after policy inception.
- Time is discrete and losses are reported and paid at the same time
- The number of claim counts reported/paid for a given policy year in a given calendar year after policy inception is Poisson distributed.
- The severity of claims is uniformly distributed between 0 and twice the average severity.
- As the policy year matures the expected number of claims reported/paid in a given calendar year decreases and their severity increases.
- The incremental dollar amounts of losses for different calendar periods after policy inception for the same policy year have a correlation coefficient of 50%. This is used to determine the total variance for the losses of a policy year.
- Similarly, the total dollar amounts of loss reserves for different accident/policy years have a 50% correlation. This is used to determine the total variance for the loss reserves of a calendar year, assuming no growth or decline in written exposure volume.

Hypothetical Demonstration of Loss Reserve Volatility

Correlation Coefficient of Incremental Losses for an Accident/Policy Year	50%
Correlation of Between Loss Reserves for Different Accident/Policy Years	50%

Incremental Policy/Accident Year Losses

Years After Policy Inception	Mean Incremental Losses on Policy	Poisson Frequency of Claims	Mean Claim Severity	Claim Severity Variance	Variance of Incremental Losses	Standard Deviation of Incremental Losses	Coefficient of Variation of Incremental Losses
1	1,000,000	200.0	5,000	2,083,333	5,416,666,667	73,598	7.4%
2	1,000,000	133.3	7,500	4,687,500	8,125,000,000	90,139	9.0%
3	1,000,000	88.9	11,250	10,546,875	12,187,500,000	110,397	11.0%
4	1,000,000	59.3	16,875	23,730,469	18,281,250,000	135,208	13.5%
5	1,000,000	39.5	25,313	53,393,555	27,421,875,000	165,596	16.6%
6	1,000,000	26.3	37,969	120,135,498	41,132,812,500	202,812	20.3%
7	1,000,000	17.6	56,953	270,304,871	61,699,218,750	248,393	24.8%
8	1,000,000	11.7	85,430	608,185,959	92,548,828,125	304,218	30.4%
9	1,000,000	7.8	128,145	1,368,418,407	138,823,242,188	372,590	37.3%
10	1,000,000	5.2	192,217	3,078,941,417	208,234,863,281	456,328	45.6%
Total	10,000,000	589.6	16,961	4,373,583,686	2,638,178,325,928	1,624,247	16.2%

Policy/Accident Year Reserves

Years After Policy Inception	Mean Loss Reserves	Variance of Loss Reserves	Standard Deviation of Loss Reserves	Coefficient of Variation of Loss Reserves
1	9,000,000	2,479,259,701,390	1,574,567	17.5%
2	8,000,000	2,291,258,965,740	1,513,691	18.9%
3	7,000,000	2,070,957,081,014	1,439,082	20.6%
4	6,000,000	1,816,070,055,659	1,347,616	22.5%
5	5,000,000	1,526,288,345,750	1,235,430	24.7%
6	4,000,000	1,204,964,483,487	1,097,709	27.4%
7	3,000,000	861,801,932,280	928,333	30.9%
8	2,000,000	517,081,159,368	719,084	36.0%
9	1,000,000	208,234,863,281	456,328	45.6%
10	-	-	-	NA
Calendar Year Totals	45,000,000	59,654,966,719,124	7,723,663	17.2%

Ultimate Losses

Years After Policy Inception	*Coefficient of Variation of Ultimate Accident/Policy Year Losses*
1	*15.7%*
2	*15.1%*
3	*14.4%*
4	*13.5%*
5	*12.4%*
6	*11.0%*
7	*9.3%*
8	*7.2%*
9	*4.6%*
10	*0.0%*

Appendix V

Calendar Year Model Case 3 at 120% Combined Ratio for 5.5 Year Loss Payout Duration

Case	Target ROR	Premium	Expense Ratio	Investment Rate of Return	Duration	Leverage	Combined Ratio
3	7.08%	1,000	30%	5.0%	5.50	2.00	120.0%

Case	UW Margin	Loss Ratio	UEPR	UEPR Equity	Loss Reserves Held	Loss Reserve Discount Equity	Statutory Surplus
3	-20.00%	90%	500	150	4,950	-	2,975

Case	Invested Assets	Invested Capital	UW income	Investment Income	Earnings	ROR
3	8,425	3,125	(200)	421	221	7.1%

Calendar Year Model Case 3 at 120% Combined Ratio for 1.5 Year Loss Payout Duration

Case	Target ROR	Premium	Expense Ratio	Investment Rate of Return	Duration	Leverage	Combined Ratio
3	-3.70%	1,000	30%	5.0%	1.50	2.00	120.0%

Case	UW Margin	Loss Ratio	UEPR	UEPR Equity	Loss Reserves Held	Loss Reserve Discount Equity	Statutory Surplus
3	-20.03%	90%	500	150	1,350	-	1,175

Case	Invested Assets	Invested Capital	UW income	Investment Income	Earnings	ROR
3	3,026	1,325	(200)	151	(49)	-3.7%

120% Combined Ratio for Case 3 - 5.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Nominal
UW provision	-20.0%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	90.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	405	360	315	270	225	180	135	90	45	(0)
Invested Capital	800	405	360	315	270	225	180	135	90	45	(0)
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	810	720	630	540	450	360	270	180	90	(0)
Total Invested Assets	1,500	1,215	1,080	945	810	675	540	405	270	135	(0)
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	90	90	90	90	90	90	90	90	90	90
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	61	54	47	41	34	27	20	14	7
Release of Earnings	(800)	270	106	99	92	86	79	72	65	59	52
IRR	5.2%										
Discount Factor	1.000	0.951	0.904	0.860	0.817	0.777	0.739	0.703	0.668	0.635	0.604
Cash Flow of Earnings and Capital	(800)	257	96	85	75	66	58	51	44	37	31
Discounted Loss Reserve		640	582	521	457	390	319	245	167	86	-

120% Combined Ratio for Case 3 - 1.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Nominal
UW provision	-20.0%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	90.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	225	-	-	-	-	-	-	-	-	-
Invested Capital	800	225	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	450	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	675	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	450	450	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	34	-	-	-	-	-	-	-	-
Release of Earnings	(800)	450	259	0	0	0	0	0	0	0	0
IRR	-8.4%										
Discount Factor	1.000	1.092	1.193	1.302	1.422	1.553	1.696	1.852	2.023	2.209	2.412
Cash Flow of Earnings and Capital	(800)	491	309	-	-	-	-	-	-	-	-
Discounted Loss Reserve		429	-	-	-	-	-	-	-	-	-

Bibliographic Note

There have been many papers published in the Proceedings of the Casualty Actuarial Society and Casualty Actuarial Society Forum which address underwriting profit provision models. By no means are the points discussed in this paper entirely original. However, the author does not endorse any particular pricing model presented in this paper or elsewhere. The reader interested in further information should consult the research sections of the Casualty Actuarial Society website: www.casact.org