Financial Pricing Models for Property-Casualty Insurance Products: Investment Yields

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by Sholom Feldblum and Neeza Thandi

The investment yield used in a financial pricing model greatly affects the indicated premium. The proper investment yield depends on the target return on capital. If the target return on capital compensates for both investment risk and insurance risk, the investment yield should be the pre-tax yield expected to be earned during the lifetime of the block of business; if the target return on capital compensates for insurance risk but not for investment risk, the investment yield should be a risk-free rate.

The Massachusetts models – that is, the underwriting beta model of Fairley and the risk adjustment discounted cash flow Myers/Cohn model – sought to disentangle investment risk from insurance risk, so that state could mandate uniform premium rates for all companies. The goal of separating investment risk from insurance risk has proved quixotic (Kozik [1994]); we use a target return on capital that reflects all risk of the insurance enterprise.

We speak of a benchmark investment yield; one may also speak of this as the expected yield adjusted for default risk, other investment risks, investment expenses, and federal income taxes. We review the major considerations for selecting a benchmark investment yield.

Expected Defaults vs Default Risk

The expected defaults from an investment portfolio are subtracted from the gross yield to determine the expected yield. We distinguish three types of adjustments for default risk.

- The expected defaults must be subtracted from the stated yield to get the expected yield.
- The risk that actual defaults will be higher than expected defaults is reflected in the target return on capital.
- The "worse case" default scenario is reflected in the risk-based capital requirements.

Illustration: The gross yield on a portfolio of fixed-income securities is 8% per annum. The expected annual defaults are 0.2%, and the expected return on defaulted bonds is 40¢ on the dollar. The expected annual cost of default is $0.2\% \times 60\% = 0.12\%$. The expected net yield on the bond portfolio is 7.88% per annum.[1]

INVESTMENT EXPENSES

Common practice is to subtract the insurance expenses from the yield of the security, and to use the net yield in the pricing model. This is in contrast to underwriting expenses, where the expenses are modeled explicitly. The rationale is that investment expenses are relatively uniform by type of security and do not vary with the insurance environment. If investment expenses are 0.20% for investment grade corporate bonds, the investment yield used in the pricing model is net of this investment expense.

We begin this discussion with the effects of federal income taxes on investment yields, since accurate consideration of the effective tax rates is important for policy pricing.

FEDERAL INCOME TAXES AND INVESTMENT YIELDS

The return on capital is the after-tax return; this is the pre-tax investment yield times the complement of the tax rate. Taxes may be applied to investment yields by two methods.

- The pricing actuary uses the gross investment yield and adjusts the tax rate on investment income by type of security.
- The pricing actuary uses the pre-tax equivalent yield and the full marginal tax rate (35%).

The latter approach avoids different tax rates in the pricing algorithms and makes the exhibits cleaner. The illustrations in this paper show conversion factors for several types of securities.

Federal income tax rates provide incentives for personal investors to hold stocks and for insurers (both life and property-casualty) to hold bonds.

- *Bonds:* The marginal tax rate on Treasury securities and corporate bonds is 35%. The marginal tax rate on municipal bonds is 5.25%, but the yield on municipal bonds is about 70% to 80% of the yield on comparable corporate bonds, and the after-tax yield on the two types of bonds is about equal, after adjusting for callability and liquidity.[2]
- *Stocks:* The marginal tax rate on dividends is 14.175% because of the dividends received deduction and the proration provision. The marginal tax rate on capital accumulation, assuming a ten year average holding period and a 12% average annual gain, is 25%.[3] Assuming a split of 15% dividends and 85% capital gains, the marginal tax rate on common stocks is 15% × 14.175% + 85% × 25% = 23.38%.

The marginal tax rates for high-tax bracket personal investors for bonds and stocks are:

- *Bonds:* The marginal tax rate on taxable bonds is about 32% to 39% before the tax amendments of 2003 and about 31% to 36% after the tax amendments. We use a 35% marginal tax rate here.[4] The marginal tax rate on municipal bonds is 0%.

- *Stocks:* The tax rate on shareholder dividends is 35%. Individual investors have a 20% tax rate on long-term capital gains; the marginal tax rate on capital accumulation, assuming a ten year average holding period and a 12% average annual gain, is 13.5%.[5] Assuming a split of 15% dividends and 85% capital gains, the marginal tax rate on common stocks is 15% x 35% + 85% x 13.5% = 16.73%.

Property-casualty insurers have a higher *relative* tax rate on stocks than individual investors have and a lower *relative* tax rate on bonds. Individual investors have a higher percentage of their investments in common stocks, and insurers have a higher proportion in bonds.[6]

Tax Rates

Property-casualty insurers have a slightly higher effective tax rate on investment income than other corporate investors have and most individual investors (if only federal income taxes are considered). The marginal tax rate on investment income for insurers is 35%, just as for other corporations, but insurers pay a higher tax rate on tax exempt investment income.

Personal investors have strong incentives – both tax and non-tax – to prefer common stocks to bonds; life insurers have equally strong incentives to prefer bonds to common stock. For property-casualty insurers, there is a tax incentive to hold bonds (especially municipal bonds, as explained below); the non-tax incentives are not compelling for either bonds or stocks.

PRORATION

For personal taxpayers and non-insurance company corporate taxpayers, municipal bond interest income is exempt from federal income taxes. Insurance companies do not receive the full exemption: the proration provision of the 1986 Tax Reform Act adds 15% of tax-exempt municipal bond income to regular taxable income for insurance company taxpayers.[7]

For an insurance company taxpayer in the regular tax environment, corporate bond income is taxed at a 35% rate. Municipal bond income is taxed at a 15% x 35% = 5.25% rate.

We can compare investment vehicles two ways. Given the pre-tax yield, we compare after-tax yields by taxpayer and asset class, or given the after-tax yield, we compare the *pre-tax equivalent yield* by taxpayer and asset class.

- For corporate taxpayers other than insurance companies, the factor to adjust the municipal bond yield to the pre-tax equivalent yield is 1/(1 – 35%) = 153.85%.
- For insurance company taxpayers, the factor to adjust the municipal bond yield to the pre-tax equivalent yield is (1 – 5.25%)/(1 – 35%) = 145.77%.

Illustration: For corporate taxpayers other than insurance companies, a 5% municipal bond yield is equivalent to a 7.69% pre-tax equivalent yield. For insurance company taxpayers, a 5% municipal bond yield is equivalent to a 7.29% pre-tax equivalent yield.

Municipal bonds are exempt from federal income taxes and from state taxes of their domestic state. For example, a municipal bond issued by Massachusetts is exempt from federal income taxes and Massachusetts income taxes, but it is subject to state income taxes from other states. This would seem to provide a strong incentive for high tax brackets individual investors, whose combined federal plus state marginal tax rate may exceed 45% (before the 2003 tax amendments) to hold municipal bonds.

For several reasons, individuals do not invest heavily in municipal bonds. First, the lower tax rate and the tax deferral on long-term capital gains makes stocks a better investment for individuals than bonds, whether corporate bonds or municipal bonds. Second, stocks may be traded in units of several hundred dollars; bonds are traded in units of tens of thousands of dollars – beyond the reach of most individual investors. Third, bonds lack the pizzazz which stocks have. Large, unexpected gains in the bond markets are rare; large gains in the stock market occur frequently. Common stocks are a mix of investment and gambling; this enhances their appeal to individuals, though it somewhat dampens their appeal to corporations. Fourth, bonds are now packaged in *balanced funds* and sold to the average individual investor. But municipal bonds appeal only to investors in tax brackets of about 32% or higher. Since balanced funds are geared to the average investor who may have a marginal tax rate of 30%, they rarely include municipal bonds. Fifth, only investment houses with a large clientele can offer well diversified municipal bond funds.[8] The greater appeal of municipal bonds for property-casualty insurers than for personal investors affects investment strategy.

MUNICIPAL BONDS

Before the Tax Reform Act of 1986, property-casualty insurers had a tax rate of approximately 46% on corporate bonds and 0% on municipal bonds. If corporate bonds were yielding 10% per annum, municipal bonds of similar investment grade (and absent other differences) could attract property-casualty insurers with rates as low as 5.4% per annum.

Life insurers, annuity writers, and pension plans, with tax deferral or exemption on their investment income, had little use for municipal bonds. Bonds (whether corporate or municipal) are not commonly held by non-insurance companies, since they serve no business purpose; for insurance companies, bonds are used to back reserves.[9] As long as the yield on municipal bonds is higher than the after-tax yield on corporate bonds, demand for municipal bonds by property-casualty insurers and high tax bracket personal investors is high.[10] Individual investors in the highest tax brackets faced marginal tax rates as high as 50% before the Reagan tax reductions of the early 1980's.[11] Until the past two decades, it was difficult for individuals to invest in bonds, since even small trades involved thousands of dollars. In contrast, stock trades of a few hundred dollars are simple, though high commission rates

discourage them. The lower marginal tax rates on long-term capital gains and the trading difficulties led most individuals to invest in common stocks.

After 1986, the 35% tax rate faced by corporate taxpayers is again roughly the same as the personal tax rate faced by high income individuals. Property-casualty insurers lost 5.25% of the tax rate differential between corporate bonds and municipal bonds, rendering municipal bonds a slightly less efficient investment vehicle for them.[12]

Nevertheless, municipal bonds remain major components of insurance company investment portfolios.[13] Property-casualty insurers are the largest holders of municipal bonds, which constitute 46% of the aggregate industry portfolio. The current ratio of municipal bond yields to corporate bond yields is well above 68%, making municipal bonds a logical investment.[14]

Taxes and Invested Capital

If insurers held no capital – that is, if insurers held fair value reserves and no surplus, there were no underwriting (systematic) risk, and there were no costs of bankruptcy – the expected pre-tax income during the policy term would be zero, since the fair premium equals the present value of expected losses and expenses. In each year afterwards, the amortization of the interest discount in the reserves should offset the investment income, and pre-tax income would again be zero.[15]

But property-casualty insurers hold capital, either explicitly in surplus or implicitly in the gross unearned premium reserves and the full value loss reserves. The investment income on this capital is not offset by amortization of the interest discount in the loss reserves, and this investment income creates positive taxable income. Most investments are bonds and other fixed-income securities, common stock, and preferred stocks; state regulation prevents too much of the investment portfolio in other securities, such as venture capital. If the after-tax yield on municipal bonds for property-casualty insurance company taxpayers exceeds the after-tax yield on comparable corporate bonds, the insurer may hold part of its portfolio in municipal bonds. The maximum amount invested in municipal bonds is constrained by the alternative minimum income tax.[16]

For the insurance industry as a whole, the ratio of municipal bonds to capital is $237,079 million / $289,606 million = 81.86%. Some insurers do not hold municipal bonds, either because they prefer more aggressive investments, such as real estate, venture capital, high yield bonds, and common stocks, or because their have operating loss carryforwards that eliminate their expected taxable income. Other insurers hold enough municipal bonds that their alternative minimum taxable income is about 175% of their regular taxable income.

Table §.x: Bond Yields (June 2003)

	Volume	10 yr yield	20 yr yield

Treasury	$1.5 trillion	3.34%	4.00%
Corporate A-rated	$1.8 trillion	4.10%	5.70%
Municipal A-rated	$1.0 trillion	3.15%	4.40%

Table §.x: Composition of 2001 Bond Portfolio, Property-Casualty Insurers ($000,000)

	Percent of Total	*Statement Values*	*Interest Earned*
Treasuries	18.0%	$92,972	$5,606
Municipal Bonds	45.9%	$237,079	$9,559
Corporate Bonds	35.6%	$183,878	$16,696
Affiliates	0.5%	$2,583	$96
Total	100.0%	$516,511	$31,957

Source: Best's *Aggregates and Averages*, 2002.

In June 2003, the average yield on A-rated 20 year municipal bonds was 4.4%, which is 77% of the average yield on comparable grade corporate bonds (5.7%). For non-insurance company taxpayers, the pre-tax equivalent 20 year municipal bond yield is 4.4% / (1 – 35%) = 6.77%; for insurance company taxpayers, the pre-tax equivalent 20 year municipal bond yield is 4.4% x (1 – 5.25%) / (1 – 35%) = 6.41%.

Municipal bonds generally yield about 70% to 80% of the pre-tax yield on corporate bonds of comparable quality; from a pure tax analysis, they should yield 65% to 68.60%. Several other items affect the relative yields on corporate vs municipal securities:

- *Callability:* Nearly all municipal bonds are callable; few corporate bonds are now callable. When bond yields declined in the 1980's, many corporate bonds were called; investors began demanding higher call premiums, making the call option expensive. When interest rates continued to fall, corporate issuers saw little need to include call provisions. Municipal bonds continued to use call provisions, and their yields reflect this option.[17]
- *Liquidity:* Municipal bonds are less liquid than corporate bonds and much less liquid than Treasuries, perhaps lowering their market values and raising their yields.[18]
- *Tax legislation:* In 1986, the proration provision reduced the tax advantage of municipal bond for insurance company taxpayers (though bonds acquired before August 8, 1986, were grand-fathered). Investors may believe that the interest income on corporate bonds may receive some tax reduction from the increasingly Republican Congress, reducing the relative tax advantage of municipal bonds.

COMMON STOCK DIVIDENDS

Stockholder dividends received by corporate taxpayers are partially exempt from federal income tax, to avoid triple taxation of a single income flow.

- *Double* taxation of common stock dividends is the imposition of both corporate income taxes and personal income taxes on the same income. Dividends are paid from after-tax corporate earnings, but they are taxed again when they are received by the equityholders.
- *Triple* taxation of dividends received by one company from another is the imposition of two layers of corporate income tax and one layer of personal income tax on the same income.

Illustration: Company A earns $10 million, which it pays to its shareholders. Company B, which owns 1% of company A, pays its earnings to its shareholders, who have an average personal tax rate of 32%. Company A pays federal income taxes on its $10 million of earnings at a 35% rate; the remaining $6.5 million is paid to its owners, of which Company B receives $65,000. Were there no dividends received deduction, company B would pay federal income taxes on the $65,000 at the regular tax rate of 35%, or $6,500 × 35% = $2,275. The remaining money, $4,225, is paid to the owners of company B, who pay personal income taxes at a 32% rate, or $4,225 × 32% = $1,352. The net income received is $4,225 – $1,352 = $2,873. The total effective tax rate is 1 – $2,873/$10,000 = 71.27%.

Since such high marginal tax rates degrade economic efficiency, the Congress enacted the dividends received deduction, which exempts 70% of common stock dividends received from taxation. For non-insurance company corporate taxpayers, the effective tax rate on dividends from unaffiliated entities is 30% × 35% = 10.50%.[19] For insurance company taxpayers, the proration provision adds 15% of the tax-exempt income to regular taxable income; the tax rate on dividends from non-affiliated entities is *(30% × 35%) + (70% × 15% × 35%) = 14.175%*.

The factor to adjust the dividend yield to the tax equivalent yield is (1 – 14.175%)/(1 – 35%) = 132.04%. A 2% dividend yield is equivalent to a 2.64% pre-tax equivalent yield. The effective tax rate for the three layers of tax (two corporate and one personal) is 1 – (1 – 35%) × (1 – 14.175%) × (1 – 32%) = 62.07%.[20]

CAPITAL GAINS

Capital gains received by corporate taxpayers are subject to a 35% tax rate when they are realized; taxes are not paid on unrealized capital gains. The value of the tax deferral of the unrealized capital gains is inversely related to the stock turnover rates.[21] If the stocks are traded frequently, there is little value to the tax deferral. If the stocks are intended to be held indefinitely, the value of the tax deferral is large.[22]

The expected capital accumulation is the expected stock yield minus the expected dividend. For high income personal taxpayers, the tax rate on long-term capital gains of 20% is lower than the tax rate on regular income. Many common stocks now pay little dividends, and the average capital accumulation is about 1 to 1½ percentage points below the stock yield.[23]

Illustration – High Turnover: A stock portfolio worth $1,000 has an expected capital accumulation yield of 10% per annum and a turnover rate of 25% a year. To simplify, we

assume that each stock is held for four years and then sold to realize the capital gains.[24] At the end of four years, the stock are worth $\$1{,}000 \times 1.100^4 = \$1{,}464.10$. The pre-tax income is \$464.10, and the after-tax income is $\$464.10 \times (1 - 35\%) = \$301.67$. The annual after-tax yield necessary to achieve this income is $(1 + \$301.67 / \$1000)^{¼} - 1) = 6.81\%$ per annum. The pre-tax equivalent yield is 6.81% / (1–35%) = 10.48% per annum.

Illustration – Low Turnover: With a turnover rate of 5% per annum, the stocks are held for 20 years (on average). At the end of twenty years, they are worth $\$1{,}000 \times 1.100^{20} = \$6{,}727.50$. The pre-tax income is \$5,727.50, and the after-tax income is $\$5{,}727.50 \times (1 - 35\%) = \$3{,}722.88$. The annual after-tax yield necessary to achieve this income is $(1 + \$3{,}722.88 / \$1000)^{0.05} - 1 = 8.07\%$ per annum. The pre-tax equivalent yield is 8.07% / (1–35%) = 12.42%.

The implications of the differential tax rates on common stocks are:

- Individual investors should prefer growth stocks with high capital accumulation rather than income paying stocks with high dividends.
- Non-insurance corporate taxpayers should prefer income paying stocks to growth stocks.
- Property-casualty insurers are between individual and corporate taxpayers.

A deferral of the tax liability increases the effective pre-tax equivalent yield for a given stated yield. The difference between the effective yield and the stated yield varies with the length of the tax deferral, or the holding period of the securities.

We write E = f(Y, T, L), where E = pre-tax equivalent yield, Y = pre-tax stated yield, T = tax rate, L = length of the deferral period. The partial derivative of E with respect to each of the input variables is positive: $\partial E/\partial Y > 0$, $\partial E/\partial T > 0$, and $\partial E/\partial L > 0$.

- *Stated yield (Y):* The higher the stated yield, the higher is the pre-tax equivalent yield.
- *Tax rate (T):* The higher the tax rate, the greater is the gain from tax deferral.[25]
- *Length of deferral:* The longer the deferral, the greater is the gain from deferral.

Illustration: An insurer can invest \$100 million in bonds yielding 10% per annum or in common stocks that pay no dividends but that will increase in value by 10% per annum. To avoid issues of investment risk, we assume that both investments are risk-free.

One Year Holding Period: During the first year, the bond interest is \$10 million. The federal income tax liability is \$3.5 million, and the increase in statutory surplus is \$6.5 million. The after-tax investment yield is 6.5% per annum.

During the first year, the change in the stock value is +\$10 million. The deferred tax liability is \$3.5 million, so the change in the statutory balance sheet is a \$6.5 million increase in surplus. For a one-year holding period, the after-tax investment yield is 6.5% for the stocks.

Although the change in surplus is the same for the two investments in the first year, the cash flows are not the same. The stock scenario has an extra $10 million in marketable securities, partly offset by a $3.5 million non-cash deferred tax liability.[26]

For a multi-year scenario, the yields on the bonds and the common stocks are not identical. For the two year scenario, we assume the interest income is reinvested in the same bonds.

Two Year Holding Period: The bond portfolio pays interest of $10 million the first year; $3.5 million is paid to the U.S. Treasury; and $6.5 million is reinvested in the bond portfolio. During the second year, the bond portfolio pays interest of $106.5 million × 10% = $10.65 million; $10.65 million × 35% = $3.7275 million is paid to the U.S. Treasury; and $10.65 million × 65% = $6.9225 million is reinvested in the bond portfolio. The totals bonds after two years is $106.5 million + $6.9225 = $113.4225. This is a 6.5% annual yield, since $1.065^2 = 1.134225$. The pre-tax equivalent yield is 6.5% / (1 – 35%) = 10% per annum.

The common stock portfolio appreciates to $110 million the first year. Nothing is paid to the U.S. Treasury, and the company sets up a $3.5 million deferred tax liability on its balance sheet. During the second year, the common stock portfolio appreciates to $121 million. Nothing is yet paid to the U.S. Treasury, and the company increases the deferred tax liability to $21 million × 35% = $7.35 million. The net common stock asset is $121 million – $7.35 million = $113.65 million. This is a 6.607% annual yield, since $1.06607^2 = 1.1365$. The pre-tax equivalent yield is 6.607% / (1 – 35%) = 10.16%.

The effect of the tax deferral on the effective investment yield is small for a short holding period and greater for longer holding periods. Let y be the pre-tax yield, r be the tax rate, and h be the holding period in years. The pre-tax return after h years is $(1+y)^h - 1$; the after-tax return after h year is $((1+y)^h - 1) \times (1 - r)$; the equivalent after-tax yield is $(((1+y)^h - 1) \times (1 - r) + 1)^{1/h} - 1$; and the pre-tax equivalent yield is $((((1+y)^h - 1) \times (1 - r) + 1)^{1/h} - 1)/(1-r)$. For example, the pre-tax equivalent yield of a 30 year 8% annual bond to fund a pension liability with taxes deferred until its maturity is $((((1.08)^{40} - 1) \times (1 - 0.35) + 1)^{0.025} - 1)/(1-0.35) = 10.63\%$.

GROWTH VS INCOME STOCKS

We compare the marginal tax rates on growth stocks vs income stocks for personal investors, insurers, and non-insurance companies. Growth stocks, such as high-tech firms, provide little or no dividends but high capital accumulation. Income stocks, such as municipal utilities, pay regular dividends. For the illustration below, we assume the growth stock pays no dividends but has an average growth rate of 12% per annum, and the income stock pays an 8% dividend yield and grows at 4% per annum. We consider two turnover rates: 10% per annum and 20% per annum. We assume that investors are in a high tax bracket with a 35% marginal tax rate; this is reasonably accurate and it simplifies the comparison with corporate investors.[27]

Personal investors pay the full 35% tax rate on dividends and a 20% rate on realized capital gains. The 8% pre-tax dividend yield is an 8% x 65% = 5.20% after-tax yield. The after-tax yield from capital accumulation of 12% or 4% in a 5 year or 10 year horizon is

- 12% with 5 year holding period: $(1.12^5 - 1) \times 0.80 + 1)^{0.2} - 1 = 9.991\%$
- 12% with 10 year holding period: $(1.12^{10} - 1) \times 0.80 + 1)^{0.1} - 1 = 10.380\%$
- 4% with 5 year holding period: $(1.04^5 - 1) \times 0.80 + 1)^{0.2} - 1 = 3.248\%$
- 4% with 10 year holding period: $(1.04^{10} - 1) \times 0.80 + 1)^{0.1} - 1 = 3.305\%$

The corporate investor has a 35% marginal tax rate on capital gains; the after-tax yields are

- 12% with 5 year holding period: $(1.12^5 - 1) \times 0.65 + 1)^{0.2} - 1 = 8.382\%$
- 12% with 10 year holding period: $(1.12^{10} - 1) \times 0.65 + 1)^{0.1} - 1 = 9.007\%$
- 4% with 5 year holding period: $(1.04^5 - 1) \times 0.65 + 1)^{0.2} - 1 = 2.670\%$
- 4% with 10 year holding period: $(1.04^{10} - 1) \times 0.65 + 1)^{0.1} - 1 = 2.754\%$

The marginal tax rate on stockholder dividends is 10.50% for non-insurance companies and 14.175% for insurance companies; the 8% pre-tax yield is an 8% x 89.5% = 7.160% for non-insurance companies and 8% x 85.825% = 6.866% for insurance companies.

	Five year holding period			Ten year holding period		
	Personal	Insurer	Corporate	Personal	Insurer	Corporate
Growth	9.99%	8.38%	8.38%	10.38%	9.01%	9.01%
Income	8.45%	9.54%	9.83%	8.51%	9.62%	9.91%
Difference	1.54%	-1.15%	-1.45%	1.88%	-0.61%	-0.91%

A personal investor should prefer the growth stock, whose after-tax yield is 150 to 190 basis points higher. Corporate investors should prefer income stocks, whose after-tax yield is 60 to 115 basis points higher for insurers and 90 to 145 basis points higher for other companies.

ACCOUNTING ISSUES

Several accounting issues affect the investment yield used in a pricing model. We discuss below (i) asset allocation, (ii) the investment generation method, (iii) books yields vs new money yields, and (iv) the valuation of securities.

ASSET ALLOCATION

Some life insurance companies allocate assets to product line by means of segregated accounts. Property-casualty insurers do not allocate specific assets with segregated accounts, though they may use a *nominal* allocation of assets.

- *Actual asset allocation:* Specific assets are purchased to support specific product lines.
- *Nominal asset allocation:* The asset returns are ascribed to specific lines for pricing.

Illustration: Some insurers allocate fixed-income securities to back loss reserves and equities to back capital and surplus funds. The fixed-income securities may be further allocated by line of business so that the durations of the bonds match the payment pattern of the liabilities.

If assets are allocated by line of business, the pricing actuary may use a weighted average benchmark investment yield for each line. Alternatively, the pricing model may apply the appropriate investment yield to each class of investments.

Illustration: An insurer uses intermediate term fixed-income securities with an average yield of 8% per annum to back its workers' compensation loss reserves and common stocks with an average pre-tax equivalent yield of 12% per annum to back surplus funds. The reserves to surplus ratio for workers' compensation is three to one.

- The pricing actuary may use a weighted average investment yield of $\frac{1}{4} \times (3 \times 8\% + 1 \times 12\%) = 9\%$ applied to all the investable assets in the pricing model.
- The pricing model may apply the 8% investment yield to assets backing loss reserves and the 12% investment yield to assets backing surplus.

INVESTMENT GENERATION APPROACHES

Permanent life insurance products have premium inflows over multiple years. Life insurance actuaries often use investment year techniques to model the interest earned in each calendar year. Policyholder funds received in year X receive the year X investment yield; policyholder funds received in year X+1 receive the year X+1 investment yield; and so forth.

The investment generation method is useful for performance measurement, which tracks the actual investment yield. For prospective pricing, the current investment yield is generally the best estimate off future investment yields, and the investment generation method adds little.

For pricing a single policy, we assume that all premium is received at policy inception and earns the same investment yield. Only for performance measurement of a cohort of policies would the investment generation method be appropriate.

BOOK YIELDS AND NEW MONEY YIELDS

The investment yield is the market yield (the *new money rate*). Portfolio yields, book yields, and amortized yields are accounting constructs that have no place in a financial pricing model.

Illustration: A 10% coupon bond has five years remaining to maturity, a $1,000 par value, a $900 market value, and a $950 amortized value. Assume that the expected market value one year hence is $920, and the amortized value one year hence is $960.

- The coupon yield is $100 / $1,000 = 10%.
- The current yield is $100 / $950 = 10.53%.
- The amortized yield is ($100 + $10)/$950 = 11.58%.
- The market yield is ($100 + $20) / $900 = 13.33%.

Some analysts use average investment yields instead of new money investment yields. The IRS uses a 60 month rolling average of federal mid-term rates to set the discount rate for tax basis loss reserves. Similarly, Harwayne [1979], page 380, uses a five year average investment yield for determining the investment income on policyholder supplied funds.[28]

An average yield may be preferable if investment yields fluctuate widely; a new money rate is preferable if investment yields are more stable. Bond yields are relatively stable, and new money rates are proper. Stock returns fluctuate greatly. For stock returns we use an expected return based on a return factor model, such as the CAPM; we do not use the most recent actual return or even an average return over the past several years.

VALUATION OF SECURITIES

The implied equity flows depend on statutory accounting principles. For determining statutory capital requirements, assets are valued according to statutory rules.

For newly purchased assets, the statutory accounting value equals the market value, whether or not the assets are bought in the primary market or the secondary market, and whether or not the assets are purchased at par. The statutory amortization of fixed-income securities are not relevant to prospective policy pricing.[29]

The statutory valuation of bonds at amortized value and of stocks at market value affects the investment portfolio mix. Common stock values fluctuate with market movements, and the fluctuations directly affect statutory surplus. The market values of long term bond fluctuate with interest rate movements, but these fluctuations are not shown in statutory financial statements. Insurers who seek stable assets values on their balance sheets have a preference for bonds.

States may impose limits by asset class. Some states limit the investments in derivative securities or restrict the statement values of common stock investments to a percentage of policyholders' surplus. These restrictions do not affect the pricing actuary's work.

INVESTMENT STRATEGY

Insurers differ in their investment strategies. One insurer might invest in Treasury bonds and investment grade corporate bonds with an average investment yield of 7% per annum. A second insurer might invest in lower grade corporate bonds plus common stocks, real estate, and venture capital, for an average investment yield of 12% per annum.

Different investment yields give different indicated premiums, if all other parameters were kept the same. Higher investment yield generate a lower policy premium. But this is valid only if the other pricing assumptions are not changed. The target return on capital depends on both underwriting risk and investment risk. A change in the investment strategy means a change in the mix of securities by asset class, which is offset by a change in the target return on capital. A riskier investment strategy necessitates a higher target return on capital to induce investors to fund the insurance operations. If the pricing parameters are chosen correctly, the added investment return is just offset by the higher target return on capital.

The target return on capital compensates for both investment risk and underwriting risk; we have no method of disentangling the two types of risk. We have no objective measure of insurance risk, and we don't even have an accepted measure of investment risk, so we do not quantify the relationship between the target return on capital and these two types of risk.[30]

Illustration: An insurer investing in Treasury bonds and investment grade corporate bonds with an average investment yield of 7% per annum might have a target return on capital of 10% per annum, whereas an insurer investing in lower grade corporate bonds plus common stocks, real estate, and venture capital, for an average investment yield of 12% per annum, might have a 15% target return on capital.[31]

The fair value of the insurance liabilities does not depend on the assets held by the insurer.[32] Similarly, the price of the policy generating the liabilities does not depend on the assets held.

If two investment strategies with different expected returns are equally appropriate for the insurance company, the higher target return on capital should just offset the higher investment yield. If the investment strategy is not sound, the indicated premium rate may change.

Illustration: A workers' compensation carrier invests only in Treasury bills and cash equivalents. If the insurer has no sound rationale for its conservative investment portfolio, the return on capital demanded by its equityholders will not decline sufficiently to offset the lower investment yield, and the indicated premium rate will rise.

COMPANY VERSUS INDUSTRY YIELDS

The benchmark investment yield may be determined as a weighted average of the new money investment yields on either the company's targeted investment portfolio or the industry's

aggregate investment portfolio. A company pricing its own business might use a company specific benchmark investment yield. An industry aggregate investment yield would be appropriate for a regulatory pricing model or for a rating bureau's pricing model.[33]

Some actuaries use risk-free interest rates for insurance pricing models, citing the Fairley formula and the Myers/Cohn discounted cash flow model, which assume that the insurer invests solely in Treasury securities; see Woll [1987] for an early example.[34] But these two models use a cost of capital equal to the risk-free interest rate adjusted for underwriting risk only, not for investment risk. This procedure was designed to meet the specifications set by Dr James Stone, the 1976 Massachusetts Commissioner of Insurance. This procedure is (perhaps) appropriate for state-made rates in a non-competitive environment such as Massachusetts; it is not appropriate for company pricing models in other states.[35]

In a return on capital pricing model, both investment risk and underwriting risk are reflected in the capital requirements and the cost of equity capital. The replacement of actual company expected yields with risk-free yields or risk adjusted yields is a duplicate offset and overstates the premium rate indications. If one wishes to use a risk-free investment yield, one must divide the risk of insurance operations into underwriting risk and investment risk.

ILLUSTRATION: BENCHMARK INVESTMENT YIELD

A company's investment portfolio consists of the following asset classes, with their expected yields and weights. The table shows three entries for each asset class:

- The weights in the target investment portfolio.
- The actual pre-tax yield.
- The pre-tax equivalent yield.

Asset Class	*Percentage of Portfolio*	*Market Yield*	*Pre-Tax Equivalent*
Fixed-Income Securities			
Five year Treasury notes	6%	5.0%	5.00%
Mortgage backed securities and CMO's	22%	6.3%	6.30%
Corporate and foreign bonds	32%	6.4%	6.40%
Municipal bonds	4%	4.32%	6.30%
Equities			
Common stocks – Dividend yield	23%	2%	2.64%
– Capital accumulation		10%	11.07%
Venture capital – Capital accumulation	5%	15%	18.21%
Real estate – Rental Income	8%	6%	6.00%
– Capital Accumulation		5%	5.55%
Total (weighted average yields)	100%	8.30%	8.97%

The relatively high percentage of equities illustrates the difference between the risk-free rate and the company's investment yield. The expected new money pre-tax yields are shown in the column captioned *actual yields*. The equivalent pre-tax yields would provide the same after-tax income were these investments fully taxed at a 35% marginal rate.

Fixed-Income Securities: The company holds 64% of its investment portfolio in fixed-income securities; the industry average is between 75% and 80%.[36]

Treasuries: The company holds 6% of its investment portfolio in Treasury securities with maturities averaging about five years. Property-casualty insurers need liquid investments for catastrophe loss payments; routine claim payments are funded by premium inflows.

Mortgage-Backed Securities and Collateralized Mortgage Obligations are favored by some property-casualty insurers; they have little default risk, since the collateral is backed by federal agencies. Their yield depends on the prepayment risk: During the 1980's, the prepayment risk was high, and these securities had high yields. The low interest rates in the late 1990's and early 2000's reduces the prepayment risk.

Bonds: We have grouped corporate and foreign bonds together. In practice, we would use separate categories for investment grade corporate bonds, high yield corporate bonds, sovereign deby, and short term money market securities, since their yields and risks differ. The tax treatment is the same for all, so we do not subdivide them in this illustration.

Municipal bonds coupon income is exempt from federal income taxes. For non-insurance company taxpayers, the pre-tax equivalent yield is slightly above the yield on comparable corporate bonds; for insurance company taxpayers, the pre-tax equivalent yield is slightly lower.[37] The pre-tax equivalent yield here is 4.32% × (1 – 5.25%) / (1 – 35%) = 6.30%.

Equities: The company holds 36% of its investment portfolio in common stocks, venture capital, and investment real estate; the industry average is between 20% and 25%. The company's percentage partly reflects the rapid capital appreciation during the 1990's. State investment regulations sometimes prevent insurers from a high percentage of equities.

Common Stocks: The 12% yield is 2% dividends and 10% capital accumulation; the average turnover rate is 12.5% per annum. The pre-tax equivalent dividend yield is 2.0% × (1 – 14.175%) / (1 – 35%) = 2.64%. The pre-tax capital accumulation after eight years is 1.10^8 = 2.144, the tax is 1.144 × 35% = 0.400, and the after-tax accumulation factor is 2.144 – 0.400 = 1.743. The annual after-tax yield is $1.743^{(1/8)}$ = 1.072, and the pre-tax equivalent yield is (1.072 – 1) / (1 – 35%) = 0.1107, or 11.07%.

The pre-tax yield on common stocks is assumed here to be a 700 basis point premium over five year Treasury notes. The pre-tax equivalent yield is higher because of the dividends received deduction and the deferral of taxes on unrealized capital gains.

Venture Capital: The venture capital yield is all capital accumulation. Venture capital funds are invested for an average of twelve years per project before the gain (or loss) is realized.

The pre-tax yield after twelve years is 1.15^{12} = 5.350. The tax is 4.350 × 35% = 1.523, and the after-tax accumulation factor is 5.350 – 1.523 = 3.828. The annual after-tax yield is $3.828^{(1/12)}$ = 1.118. The pre-tax equivalent yield is (1.118 – 1) / (1 – 35%) = 0.1821, or 18.21%.

Investment Real Estate: The real estate is properties held as investments, not Home Office real estate.[38] The real estate is held for an average of 15 years before being sold.

The pre-tax yield after fifteen years is 1.05^{15} = 2.079. The tax is 1.079 × 35% = 0.378, and the after-tax accumulation factor is 2.079 – 0.378 = 1.701. The annual after-tax yield is $1.701^{(1/15)}$ = 1.036. The pre-tax equivalent yield is (1.036 – 1) / (1 – 35%) = 0.0555, or 5.55%.

We do not model the term structure of interest rates or the shape of the credit spread curve, and we do not assume any investments in derivative securities. These are important issues for investment strategy, but they are outside the scope of this paper.

The yields are net of investment expenses, and the yields on corporate bonds and municipal bonds are net of expected default rates. For instance, if the expected default rate on the corporate bonds in the portfolio is 0.5% per annum and the expected recovery upon default is 40¢ on the dollar, the gross yield is multiplied by *1 – 0.5% × (1 – 40%) = 99.7%.*

Expected default rates by investment grade and by type of fixed-income security are available from commercial investment houses.[39] The analysis of default rates and investment expenses would normally be done by the investment department, not by the pricing actuary.[40]

The weighted average pre-tax equivalent yield in this illustration is 8.973% per annum. This insurer has an aggressive investment portfolio, with a high percentage of its assets in equities (common stock, venture capital, and real estate). A more conservative investment strategy, with most assets in fixed-income securities, would show a pre-tax equivalent yield of about 6% to 8% per annum.

[1] This does not mean that the bond portfolio is equivalent to a risk-free bond portfolio yielding 7.88% per annum. The actual bond portfolio has significant variance in its returns, which reduces the worth of these securities. The certainty equivalent of the bond portfolio is less than 7.88% per annum, assuming that investors are compensated for assuming default risk.

[2] If the yield on municipal bonds is (1 – 35%) / (1 – 5.25%) = 68.60% of the yield on corporate bonds, the after tax yield is the same.

[3] In ten years, one dollar accumulates to \$3.105848 at a 12% annual rate. The after-tax gain upon realization is 35% × (\$3.105848 – \$1) = \$1.368801. The after-tax investment yield needed to achieve this gain is $(\$1.368801 + \$1)^{0.1} - 1 = 9.0066\%$. This is a 25% tax rate on the pre-tax investment yield of 12%.

[4] A more accurate (but complicated) analysis would include state income taxes. Municipal bonds are generally exempt from their domestic state's income tax, though the tax rules and the tax rates vary by state.

[5] In ten years, one dollar accumulates to \$3.105848 at a 12% annual rate. The after-tax gain upon realization is 80% × (\$3.105848 – \$1) = \$1.684679. The after-tax investment yield needed to achieve this gain is $(\$1.684679 + \$1)^{0.1} - 1 = 10.3797\%$. This is a 13.5% tax rate on the pre-tax investment yield of 12%.

[6] To be exact, we should include state income taxes and investment expenses in the analysis; see below.

[7] More precisely, 15% of tax-exempt income is deducted from the loss reserves offset to taxable income. Municipal bonds bought before August 8, 1986, are *grandfathered* and exempt from the proration provision.

[8] Vanguard has a municipal bond fund geared to high tax bracket investors; not all mutual fund families have municipal bond funds, though they generally have balanced funds.

[9] Holding large cash reserves does not make sense for most corporations, since these funds incur double taxation; shareholders are better off receiving the funds as dividends or share repurchases. Companies in cyclical industries, such as auto manufacturing and airlines, sometimes hold large cash reserves.

[10] The exposition here is simplified; we explain why property-casualty insurers provided the greatest demand for municipal bonds; we do not mean to say that municipal bonds were held only by property-casualty insurers.

[11] In 1981, prior to Reagan's tax cuts, the top tax rate on investment (and most other) income was 70% (on taxable income over \$215,400 for married filing joint); the bottom rate was 14%. In 1982, after, or as a result of, Reagan's tax cuts, the top tax rate was 50% on income in excess of \$169,020; the bottom rate was 11%.

[12] Proration applies to life insurers as well, though since their bond holdings generally back policy reserves, the tax deferral makes municipal bonds inefficient investment vehicles. For example, suppose corporate bonds yield 10% per annum and equivalent grade municipal bonds yield 6.5% per annum. With no tax deferral and a tax rate of 35%, the two investments provide the same after-tax return. With a ten year tax deferral, the municipal bond still yields 6.5% after-tax. The corporate bonds yield $(1 + (1.1^{10} - 1) \times 65\%)^{0.1} - 1 = 7.37\%$ after-tax. The advantage of the tax deferral increases as the deferment lengthens.

[13] The incidence of the proration provision is difficult to judge. The Congress passed the proration provision to exact greater tax revenue from property-casualty insurers, who are the major purchasers of municipal bonds. Who actually pays the tax is unclear. If municipal bond yields do not change, insurers pay the tax. If municipal bond yields increase to offset the loss to insurers, the states pay the tax as a higher cost of debt capital.

[14] The 68.6% uses the proration provision for property-casualty insurers: $68.6\% = 65\%/(1 - 15\% \times 35\%)$.

[15] In practice, differences between tax basis and present value of liabilities creates minor taxable income.

[16] We are not implying that or capital funds should be invested in municipal bonds. Some insurers notionally allocate fixed-income securities to policyholder reserves and equities to capital and surplus funds. We are saying only that the maximum investment in municipal bonds depends on the capital held by the company; the more invested capital, the more taxable income, and more funds may be invested in municipal bonds.

[17] In theory, we could price the cost of the options embedded in municipal bonds, if we can make reasonable assumptions about interest rate volatility. In practice, the current interest rate environment in the United States of extremely low interest rates during a period of relative prosperity is unique. The other period of such low interest rates in the 1930's was a period of economic recession. Similarly, the low rates now in Japan reflect the recession in Japan.

[18] There is no accepted model for pricing liquidity; it is hard to know if it has much effect on yields. Most buyers of municipal bonds intend to hold them to maturity and do not need extra liquidity.

[19] A non-affiliated entity is an entity that is less than 20% owned by the taxpayer.

[20] Since the funds used to pay the tax on investment income come from policyholders, they are subject to tax on underwriting income. The effective tax rate is $1 - (1 - 35\%) \times (1 - 35\%) \times (1 - 14.175\%) \times (1 - 32\%) = 75.34\%$. For the commercial casualty lines of business, the effective tax rate on the profit margin in the policyholder premium is between 70% and 80% when both personal and corporate income taxes are included. This underscores the need to be cognizant of tax effects when pricing insurance products.

[21] Jeffrey [1995] examines the relation between the stock turnover rate and the effective tax rate.

[22] The turnover rate in the stock portfolio is the reciprocal of the average time that the stocks are held. If the annual turnover rate in the stock portfolio is 1/n, the average stock is held for n years before being sold.

[23] To avoid the higher tax on dividends, corporations can use share repurchases. Alternatively, investors can produce virtual dividends by selling a fixed percentage of their shares each quarter. Either alternative produces a higher after-tax yield.

[24] A more accurate analysis would consider the distribution of stock holding periods. A 25% common stock turnover rate might be modeled as 25% of stocks have 1 year holding period; $(1 - 25\%) \times 25\% = 18.75\%$ have a 2 year holding period; $(1 - 25\%)^2 \times 25\% = 14.06\%$ have a 3 year holding period; and so forth.

[25] The higher the tax rate, the lower is the after-tax yield, but the higher is the pre-tax equivalent yield. One might suppose that the pre-tax equivalent yield must reach a maximum at some point, since when the tax rate is 100%, the investor gets no income, so the pre-tax equivalent yield must be zero. That is not correct; the pre-tax equivalent yield is not defined when the tax rate is 100%.

[26] The $10 million marketable security in the stock scenario is an investable asset, though it remains invested in the same security (the stock).

[27] The disparity between the growth stock and the income stock in this illustration is deliberately made stark, to show the effects of tax rates on investment strategy.

[28] "Five year average investment yields . . . is a reasonable reflection of the need for stability in considering that payment amounts are subject to substantial fluctuation and extend over long and fluctuating durations . . . A five year average investment return provides some stability . . ." Harwayne, Frank, "Restatement of the Consideration of Investment Income in Workers' Compensation Insurance Ratemaking," (Casualty Actuarial Society 1979 Discussion Paper Program), page 380.

[29] For retrospective analyses of profitability, statutory valuation rules are relevant. For the variance stemming from interest rate changes, the statutory valuation rules affect the economic value added. If bonds are held at amortized cost, the portfolio yield should be used instead of the market yield in each year.

[30] Some theorists have tried to quantify the systematic risk of underwriting using a CAPM-based approach (termed the Massachusetts method of calculating underwriting betas); see Fairley [1987], Kahana [1987], and Hill [1987]. This approach has not generally been successful; see Kozik [1994].

[31] We distinguish a change in investment strategy from a change in the investment yields. The effect of changes in asset class *returns* – investment *strategy* – are discussed below.

[32] Butsic uses this reasoning to argue that the loss reserve discount rate should not depend on the assets carried by the company.

[33] Some actuaries contend that the pricing assumptions should depend on industry figures; see Bault [19xx].

[34] See Fairley, William, "Investment Income and Profit Margins in Property-Liability Insurance: Theory and Empirical Results," *The Bell Journal of Economics* 10 (Spring 1979) pages 192-210, reprinted in J. David Cummins and Scott E. Harrington (eds.), *Fair Rate of Return in Property-Liability Insurance* (Boston: Kluwer*Nijhoff Publishing, 1987), pages 1-26, and Myers, Stewart and Richard Cohn, "A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulation," in J. David Cummins and Scott E. Harrington (eds.), *Fair Rate of Return in Property-Liability Insurance* (Boston: Kluwer*Nijhoff Publishing, 1987), pages 55-78.

[35] See Feldblum [discussion of D'Arcy and Dyer paper] for a full discussion of this issue.

[36] Life insurance companies hold larger percentages in fixed-income securities because of the need to perform asset adequacy analyses for many products; property-casualty companies do not have this restriction.

[37] We do not model any exemptions from state income taxes, since the statutes differ by jurisdiction.

[38] Real estate is more commonly held by life insurance companies than by property-casualty insurance companies, and it has been somewhat in disfavor after the early 1980's downturn in the real estate market. More recently, some property-casualty insurance companies are seeking higher yielding alternative investments, such as venture capital and real estate.

[39] Periodic studies are published by financial analysts, such as those by David Altman.

[40] A common assumption is that efficient markets lead to the same present value of investment yields from all fixed-income securities. If used too broadly, this assumption is not suitable for a pricing model. The yield on Treasury bills and notes is lower than the yields on investment grade corporate securities, even after adjusting for expected default rates. The demand for Treasury securities is particularly high because of their low risk (for overseas investors and certain institutional investors) and their use to fulfil margin requirements for derivative securities. Most insurance companies prefer higher yield corporate and mortgage backed securities, since the additional risk can be managed with sound investment strategy.