The Pricing of Commutations

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Pricing commutations is complex; one must consider cash flows, federal income taxes, deferred tax assets, capital requirements, the cost of holding capital, the required return on invested capital, and implied equity flows. Vincent F. Conner and Richard A. Olsen outline a pricing method in their 1991 *Proceedings* paper, "Commutation Pricing in the Post Tax-Reform Era," and Lee Steeneck has adapted their method for his CAS Exam 6 study note on commutations.[1] Conner and Olsen consider expected cash flows and taxes. This paper expands upon their method by considering the other pricing items listed above.

Some of Conner and Olsen's conclusions are counter-intuitive. The authors say on page 96:

> *In certain instances, the commutation price developed under this methodology can be negative. . . . In cases of reinsurance of long-tailed lines, such as workers' compensation, . . . , negative commutation values can be expected frequently.*[2]

Presumably, a negative commutation price means that the primary insurer pays the reinsurer for the privilege of assuming back the loss liability. Before the commutation, the reinsurer has the reserve liability and the obligation to pay the claimant. By paying cash, the primary company gets to re-assume the liability and the obligation to pay the claimant.

A result this strange gives one pause. The Conner and Olsen paper deals with a complex issue – the handling of federal income taxes. Other items that must be considered are:

1. Conner and Olsen use after-tax interest rates. The better method is to use pre-tax interest rates for pre-tax revenues and expenditures and to explicitly model the federal income tax cash flows. This is particularly true for modeling the deferred tax assets stemming from IRS loss reserve discounting.
2. Conner and Olsen do not consider the statutory accounting requirements for long-tailed casualty full value loss reserves.
3. Conner and Olsen do not consider capital requirements for holding loss reserves, such as risk-based capital requirements and rating agency capital requirements.
4. Conner and Olsen do not consider the cost of holding capital, consisting of the double taxation of investment income on capital and surplus funds as well as any difference between the cost of equity capital and the company's investment yield.

Omitting the last three items understates the commutation price. This discussion provides a more complete analysis of commutation pricing in a post-RBC and post-codification era. It provides guidance for reinsurance actuaries pricing commutations, and it should help actuarial candidates understand the financial theory behind these transactions.

INVESTMENT YIELDS AND FEDERAL INCOME TAXES

Casualty actuaries are often berated for ignoring the federal income tax implications in their pricing analyses. Some actuaries, stung by this criticism, use after-tax interest rates, on the presumption that the lower discount rate accounts for the federal income tax liabilities.

The use of after-tax interest rate is as likely to obscure as to illuminate. It is not always a good proxy for the explicit modeling of taxes. Rather

a. Pre-tax cash flows should be discounted at pre-tax interest rates, and
b. The federal income tax cash flows should be modeled explicitly.

These principles apply to discounted cash flow pricing models [Myers and Cohn: 1987; Butsic and Lerwick, 1992]; Mahler [1985; 1998], internal rate of return models [Kahley and Halliwell, 1992], Robbin 1992, algorithm 7], and loss reserve discounting models [Butsic: 1988]. Butsic says:

> *We have demonstrated that the appropriate interest rate for reserve discounting under income taxation should be the same as that without taxes . . .* (page 177).
>
> *The discounting interest rate must be a pretax value* (page 183).

Similarly, Actuarial Standard of Practice No. 20, "Discounting of Property and Casualty Loss and Loss Adjustment Expense Reserves," paragraph 5.4.4, says:

> *Effect of Income Taxes – The actuary normally should use an interest rate or rates consistent with investment returns that are available before the payment of income taxes.*[3]

If used correctly, after-tax interest rates are not necessarily incorrect – but it is very hard to use them correctly. It is particularly hard to account for deferred tax assets and liabilities when using after-tax interest rates. Throughout this discussion, we use pre-tax interest rates and we explicitly model the federal income tax cash flows.

Conner and Olsen use an after-tax investment yield, and they back out the federal income tax credit stemming from loss reserve discounting. They note that this procedure is equivalent to using pre-tax discount rates if the IRS loss reserve discount factors are the same as the actuarial discount factors (page 92):

If the payment pattern and nominal interest rate used to determine the present value of the losses are identical to the factors used to develop the tax-basis discounted reserves, then the commutation price will equal the present value of the losses using the nominal interest rate.[4]

The apparent equivalence leaves out the effects of statutory accounting, capital requirements, and the cost of holding capital.

IDEALIZED ILLUSTRATION

If there were no surplus requirements and the insurer held fair value reserves, it could charge premiums equal to the present value of losses and expenses. The statutory requirement for full value loss reserves and the risk-based capital requirements necessitate equityholder funded capital to support the insurance business.

If there were no corporate income taxes, these regulatory requirements might have little effect on pricing. The equityholder provided capital is invested, and the equityholders receive the investment income.

Illustration: Suppose that the owners (equityholders) of an insurance company provide $100 million of capital. In a world without taxes or insurance regulation, the insurance company invests this money in securities chosen by the equityholders and passes along the investment yield to them.

Federal income taxes change the analysis. The multiple layers of taxation give the IRS about 60% to 70% of the underwriting profit margin for long-tailed commercial lines of business.

This paper analyzes the cash flows and taxes related to claim commutations. We use a series of simple illustrations, so that the mathematics does not obscure the intuition. We then re-examine the illustrations in the Conner and Olsen paper and in the Steeneck study note.

STATUTORY ACCOUNTING

We begin with the implication of the statutory requirements for full value loss reserves.

SINGLE YEAR ILLUSTRATION

A loss is commuted on January 1, 20X2. The loss will be paid for $105,000 on December 31, 20X2. The risk-free interest rate is 5% per annum, and the federal income tax rate is 35%. No risk adjustments or risk loads are used.

One might surmise that the appropriate commutation price is $100,000, or the present value of the loss payment. We examine the effects of federal income taxes to confirm Butsic's remarks on discount rates cited above.

We assume first that the insurer holds fair value reserves and that it does not need surplus to support the reserves. On January 1, 20X2, the insurer receives $100,000 in cash and sets up a loss reserve of $100,000. The two accounting entries offset each other, and there is no effect on income.

During the year, the $100,000 in cash is invested at the 5% per annum risk-free interest rate, yielding investment income of $5,000. On December 31, 20X2, the loss is paid for $105,000 and the loss reserve is taken down to zero. The additional incurred losses between January 1 and December 31 equals the paid loss plus the change in reserves, or $105,000 + ($0 – $100,000) = $5,000. The incurred loss offsets the investment income, and the taxable income during the year is zero.

We have not yet considered statutory accounting, capital requirements, and the cost of holding capital. This illustration puts all the cash flows into a single calendar year so that we can avoid the IRS loss reserve discount factors and their effect on the commutation price. The tax cash flows are crucial to commutation pricing, but we want to first examine the effects of statutory accounting and capital requirements without the complexities of the IRS discount factors.

NO CAPITAL REQUIREMENTS AND NO ADDITIONAL COST OF CAPITAL

We assume that the primary insurer must hold full value (undiscounted) loss reserves, but that it does not need to hold surplus to support the loss reserve. We initially assume that its equityholders are satisfied with a risk-free rate of return; we relax this assumption below. We examine the implications of a $100,000 commutation price.

On January 1, 20X2, the primary company receives $100,000 and it records a $105,000 statutory reserve to its books. The insurer is missing $5,000 of assets to back the statutory reserves. The shareholders of the insurance company contribute $5,000 on that date.

Investment income: The $105,000 is invested in risk-free securities yielding 5% per annum. During 20X2, the company earns 5% × $105,000 = $5,250.00 of interest income.

Incurred loss: The loss reserve is zero on December 31, 20X1 (before the commutation) and zero on December 31, 20X2, after the loss is paid.

- The statutory incurred loss in 20X2 is the paid loss plus the change in reserves, or $105,000 + ($0 – $0) = $105,000.
- The tax-basis incurred loss in 20X2 is the paid loss plus the change in discounted reserves, which is also 105,000 + ($0 – $0) = $105,000.

The components of the tax liability are as follows:

- The tax-basis underwriting income in 20X2 is the premium minus the paid loss, or $100,000 – $105,000 = –$5,000.
- The investment income during 20X2 is $105,000 × 5% = $5,250.
- The net income to the company in 20X2 is $5,250 – $5,000 = $250.
- The federal income tax on the net income is 35% × $250 = $87.50.

The net after-tax gain to the company in 20X2 is $250 – $87.50 = $162.50. At the end of the year, the equityholders receive back their $5,000 investment plus the net income of $162.50. This is a return of $162.50 / $5,000 = 3.25%.

If there is no risk in the insurance operations, the shareholders expect a risk-free return of 5% per annum, not a return of 3.25%. The shareholders could get a 5% return by investing directly in risk-free securities instead of investing indirectly through the insurance company. The opportunity cost of the capital to the shareholders is 5% per annum, not 3.25% per annum.

For the primary insurance company to attract capital, it must provide a 5% rate of return to its shareholders. This means that the commutation price must be higher than $100,000. We set the commutation price to be $100,000 + *z*, and we solve for *z*. There are several changes to the example's cash flows and accounting entries.

A. The net underwriting income to the company during 20X2 is the earned premium minus the incurred losses, or $100,000 + *z* – $105,000 = *z* – $5,000.
B. The investment income during 20X2 is $5,250, since the company holds $105,000 of assets to back the $105,000 of reserves. Of this $105,000, $100,000+z is paid by the reinsurer and $5,000–z is contributed by the equityholders of the primary insurer.
C. The cash received from the reinsurer is a pre-tax cash flow, and it is taxed in 20X2. The funds received from the equityholders are an after-tax cash flow, and they are not taxed.[5] The federal income taxes on underwriting income in 20X2 equals 35% × ($100,000 + *z* – $105,000) = 35% × (*z* – $5,000) = 0.35z – $1,750.
D. The $105,000 of assets at the beginning of the year grow to $110,250 by December 31, 20X2; the investment income is $5,250. The company pays $105,000 to the claimant and it pays taxes on the investment income of 35% × $5,250 = $1,837.50 to the U.S. Treasury. The total tax is $1,837.50 + 0.35z – $1,750 = 0.35z + $87.50. The rest of the money is returned to the equityholders: this is $110,250 – $105,000 – $87.50 – 0.35*z* = $5,162.50 – 0.35*z*.
E. The company's shareholders expect a 5% annual return. This means that

$$(\$5{,}162.50 - 0.35z) / (\$5{,}000 - z) = 105\%$$
$$\$5{,}162.50 - 0.35z = \$5{,}250 - 1.05z$$
$$z = \$87.50 / 0.70 = \$125.00$$

The proper commutation price is not $100,000 but $100,125, or an increase of 0.125%. The difference is small, since the loss is paid one year after the commutation. As the subsequent illustrations show, the costs are substantial when we add the effects of statutory regulation and reserve requirements to long-tailed casualty lines of business.

The difference in the commutation price is in the right direction. The primary insurance company faces additional costs of double taxation on the cash contributed by equityholders.

INTUITION

We show the intuition for this result. The equityholders contribute $5,000 at the beginning of the year. The investment income is $250, and the tax on the investment income is 35% × $250 = $87.50. The reinsurer pays this tax by a profit margin in the commutation price. If the reinsurer did not pay this tax, the primary company would have no interest in the commutation. The commutation price is taxed as underwriting income, so the reinsurer must pay $87.50/(1–35%) = $134.62. The present value of this amount at policy inception is $134.62 / 1.05 = $128.21.

This is slightly too high, since the more that the reinsurer pays, the less must be contributed by the equityholders of the primary company. The money is not given to the equityholders until the end of the year. During the year, the money supports the full value loss reserves.

Since the reinsurer pays $100,125, the equityholder contribution is $4,875, not $5,000. The investment income is $4,875 × 5% = $243.75. The tax on the investment income is $243.75 × 35% = $85.31. To fund the double taxation on the investment income on the equityholder supplied funds, the margin in the commutation price must be $85.31 / (1–35%) = $131.25. This is the margin that would be needed at the end of the year. The margin needed at the beginning of the year, when the commutation is effected, is $131.25 / 1.05 = $125.00.

In multi-period illustrations, the money is paid to equityholders incrementally over the years. An algebraic formula for the commutation price becomes increasingly complex as the number of periods increases; spreadsheet pricing techniques are easier.

We examined the $100,125 commutation price indirectly by looking at the margin needed to fund the double taxation on the investment income on the equityholder provided capital. As a complementary perspective, we examine the direct cash flows for the commutation price.

The reinsurer pays $100,125, and the equityholders contribute $4,875.00. The underwriting income is $100,125 – $105,000.00 = –$4,875. The IRS takes –$4,875 × 35% = –$1,706.25. The underwriting income is negative, so the tax liability is negative. This is a tax refund, which is an offset against the federal income taxes on investment income.

The investment income earned during the year is 5% x $105,000 = $5,250.00. The IRS takes 35% x $5,250 = $1,837.50. The total tax liability is $1,837.50 – $1,706.25 = $131.25.[6]

At the end of the year, the equityholders receive back their initial contribution plus the commutation price plus the investment income minus the loss payment minus the income taxes, or

$$\$4,875 + \$100,125 + \$5,250 - \$105,000 - \$131.25 = \$5,118.75.$$

The return on the invested capital is $5,118.75 / $4,875 – 1 = 5.00%.

Timing of the Equity Flows

Most of the cash flows and accounting flows for insurance transactions occur continuously. This is true for loss incurral and settlement, expenses, investment earnings, and premium earnings. Even if the cash flow for a particular event is discrete, such as the settlement of a loss, the expected cash flows are continuous.

An ideal pricing model would have continuous functions. Some life actuarial models use forces of mortality and interest. In practice, it is easier to work with discrete valuation dates, particularly for spreadsheet based applications.

The valuation dates and valuation periods reflect a compromise between accuracy and expediency. This discussion uses annual valuation periods and year-end valuation dates. More accurate results may be obtained with a quarterly pricing model.

When using discrete periods, some actuaries use present values of year end figures. For instance, Atkinson and Dallas [2000], argue that capital requirements are examined only at year end. At the beginning of the year, the company needs the present value of the year-end capital requirements, where the present value is calculated at the after-tax investment yield. Similarly, one might argue that the full value loss reserves need be held only at year end. At the beginning of the year, the company must hold the present value of the loss reserves.

This argument is not applicable to our illustration, for two reasons.

- Full value loss reserves are required at all times, not just at the end of the year. Insurers make quarterly disclosure of loss reserves to state regulators in Schedule X.
- We chose the January 1 commutation date simply to avoid the complexities of IRS loss reserve discounting. We change the illustration below to a commutation date of December 31, 20X1. On a December 31 commutation date, the insurer must surely hold full value loss reserves and risk-based capital requirements.

Reinsurance Regulation

If the reinsurer is subject to the same statutory accounting, capital requirements, and tax provisions as the primary insurer, the accounting for the reinsurer is the mirror image of the accounting for the primary company.[7] Just as the primary company incurs the cost of double taxation on equityholder supplied capital, the reinsurer saves the cost of double taxation on funds which its equityholders would otherwise have had to contribute. The additional premium for the commutation transfers the funds saved by the reinsurer to the primary company which now incurs the costs.

If the reinsurer is not domiciled in the U.S., and particularly if it is domiciled in a jurisdiction with less stringent reserve regulations and capital requirements, it may not be subject to the same costs as a U.S. domiciled primary insurance company. If the reinsurer is domiciled in the Bermudas, it does not save the costs of double taxation by a commutation. The reinsurer may not be willing to pay more than $100,000 for the commutation.

A commutation is feasible only if the benefits from the commutation outweigh any additional costs to the companies. These additional costs stem from statutory accounting, capital requirements, and federal income taxes, which may be incurred only by the primary company (or primarily by the primary company).

This should not be a surprise. Reinsurance with off-shore companies can be financially beneficial, even if there is no significant transfer of risk, as long as the reinsurance contract passes the SFAS 113 tests. Companies benefit from finite reinsurance transactions with off-shore reinsurers. Undoing the reinsurance with a commutation can be costly.

COST OF HOLDING CAPITAL

The illustration above assumes that shareholders are satisfied with a risk-free rate of return on their invested capital. Many actuaries assume that shareholders require a higher rate of return for capital supplied to insurance enterprises.[8]

For our illustrations in this discussion, we assume that investors in an insurance enterprise require a rate of return equal to 100 basis points less than the average return for publicly traded stock companies. In a CAPM perspective, this assumes that insurers investing in risk-free securities have a market beta of about 85% to 90%.

We assume the risk-free interest rate on Treasury bills is 5% per annum and the market risk premium is 8% per annum.[9] The return on capital demanded by insurance company stockholders is 5% + 8% – 1% = 12%. We solve for z using the following equation:

$$(\$5{,}162.50 - 0.35z) / (\$5{,}000 - z) = 112\%$$
$$\$5{,}162.50 - 0.35z = \$5{,}600.00 - 1.12z$$

$$z = \$437.50/\ 0.77 = \$568.18$$

Double Taxation and the Cost of Holding Capital

If the equityholders are satisfied with a risk-free return, the double taxation of the equityholder provided capital is the only cost. If the equityholders demand a higher equity-type return, the additional premium is greater.

The tax on equityholder provided capital appears in two places.

- the tax on investment income on the equityholder provided capital
- the tax on underwriting income stemming from the commutation transaction.

Illustration: Suppose an insurer invests in Treasury bills yielding 5% per annum. The equityholders could obtain a 5% yield by investing in Treasury bills on their own. If they give their funds to the insurance company, the insurance company pays corporate taxes before remitting dividends to the equityholders. The yield received by the equityholders is (1 – 35%) × 5% = 3.25%. The difference of 5% – 3.25% = 1.75% must be paid by the policyholders.[10]

Were the policyholders to pay this amount directly to the equityholders, this would be the cost of double taxation. But this is not the actual cash flow. The policyholders pay this cost to the insurance company as part of the policy premium, which then remits these funds to the equityholders. This round-about flow of funds induces an additional layer of taxation on the underwriting profits. The cost to the policyholders is 1.75% / (1 – 35%) = 2.69% of the capital.

If the return on capital demanded by equityholders is 12% instead of 5%, the cost of holding capital is the required rate of return minus the after-tax investment yield, or 12% – (1 – 35%) × 5% = 8.75%.[11] Since the policyholders pay this though the policy premium, they subject the funds to an additional layer of taxation. The additional premium paid by the policyholders is 8.75% / (1 – 35%) = 13.46% of the capital provided by equityholders.

This cost in incurred at the end of the year. The policy premium is paid at the beginning of the year. The present value of this additional premium is 13.46% / 1.050 = 12.82%.[12]

Capital Requirements

State regulation imposes capital requirements that depend on the insurance operations. An increase in loss reserves causes an increase in the NAIC risk-based capital requirements.

The capital requirements depend on the line of business and the covariance adjustment, along with the capital philosophy of the insurer. Some insurers are satisfied with carrying capital only slightly in excess of the risk-based capital requirements; others set their target capital

levels at a multiple of the risk-based capital requirements. Most well-rated property-casualty companies hold capital about twice their RBC requirements.

The marginal capital requirements from the commutation stem from the difference between the reserving risk charge (R_4) and the credit risk charge (R_3), which includes the charge for reinsurance recoverables.

The marginal effect of a change in an RBC charge on overall capital requirements is proportional to the size of that RBC charge in relation to the other RBC charges; see Feldblum [1996: RBC, pages 362-365]. For the average company, the reserving risk R_4 charge is about 10 times the size of the credit risk R_3 charge. Even for companies with large reinsurance recoverables, the remaining credit risk charge after half of the original amount has been transferred to the reserving risk charge is generally less than 10% of the final reserving risk charge (see the following paragraph). For the computations in this paper, we assume that the credit charge has 10% of the marginal effect as the reserving risk charge.

The 10% credit risk charge for reinsurance recoverables is split into two parts. Half the charge, or 5% of reinsurance recoverables, is transferred to the reserving risk category in the covariance adjustment. The other half remains in the credit risk category; it is equivalent to a ½ x 10% x 10% = 0.5% reserving risk charge. The combined charge is equivalent to a 5.5% reserving risk charge.

The reserving risk charge varies by line of business. We consider two scenarios: a commutation in a general liability line and a workers' compensation commutation.

The risk-based capital reserving risk charges for general liability, products liability, and medical malpractice are shown in the table below. The average reserving risk charge for these lines, weighted by industry premium volume, is 25.9% of held reserves.

Line of Business	Other Liability	Products Liability	Medical Malpractice
RBC Charge	52.0%	53.2%	56.5%
Interest Discount	83.2%	83.2%	80.8%
Reserving Risk Charge	26.46%	27.46%	26.45%
Unpaid Losses	$63,821 million	$10,423 million	$20,999 million
Claims-Made Percent			57.563%
Charge after C/M Offset			23.407%

The average reserving risk charge is

($63,821 x 26.464% + $10,423 x 27.462% + $20,999 x 23.407%) / $95,242 = 25.90%.

The workers' compensation reserving risk RBC percentage is 27.3% and the investment income offset is 87.2%. The reserving risk charge is [(1 + 27.3%) × 87.2%] – 1 = 11.0%.

The marginal effect of the reserving risk charge is reduced by the loss concentration factor and the covariance adjustment, reflecting the company's diversification.

- A monoline insurer gets no reduction from the loss concentration factor. A multi-line insurer with reserves split evenly among five different lines of business has a 24% reduction from the loss concentration factor.[13]
- The effect of the covariance adjustment depends primarily on the size of the written premium risk charge. It would be greatest for a property predominating insurer which writes a limited amount of workers' compensation or commercial liability business.

For most companies, the marginal effect of the reserving risk charge on overall capital requirements ranges from 60% to 80%, depending on their mix of business and the size of their other risk charges. A greater marginal effect increases the equityholder contribution and the commutation price.[14]

Few companies hold surplus just equal to their risk-based capital requirements. Most companies with Best's rating of A– or higher have risk-based capital ratios of 175% to 250%. For the illustrations here, we assume the company targets a risk-based capital ratio of 200%.

PRICING WITH FULL CAPITAL CONTRIBUTION

The average marginal risk-based capital charge from a commutation in general liability, products liability, or medical malpractice is (25.9% – 5.5%) × 60% × 200% = 24.48%. We round this to 25% to simplify the computations. The shareholders contribute 25% × $105,000 = $26,250 on January 1, 20X2, besides the $5,000 needed to fund the full value statutory loss reserves. The total contribution is $31,250.

If the commutation price equals the present value of the future loss payments, with no profit for the equityholders of the ceding company, the equityholders receive back their contribution at the end of the year plus the after-tax investment income, or

$$\$31{,}250 + 5\% \times \$31{,}250 \times 65\% = \$32{,}265.625.$$

This is a $32,265.625 / $31,250 – 1 = 3.25% return, which is inadequate.

For an adequate commutation price, the reinsurer pays $100,000+z. The company needs $131,250 of assets: $105,000 to back the statutory loss reserves and $26,250 as surplus capital. The equityholders provide $31,250 – z. We examine each cash flow:

A. The underwriting income in 20X2 is \$100,000 + z − \$105,000 = z − \$5,000. The tax on underwriting income is 35% × (z − \$5,000) = 35% × z − \$1,750.
B. The investment income is 5% × \$131,250 = \$6,562.50. The tax on investment income is \$6,562.50 × 35% = \$2,296.875.
C. On December 31, \$105,000 is paid to the claimant. The total tax paid to the U.S. Treasury is \$2,296.875 + 0.35z − \$1,750 = 0.35z + \$546.875. The rest of the money is returned to the equityholders; this \$131,250 − \$105,000 + \$6,562.50 − \$546.875 − 0.35z = \$32,265.625 − 0.35z.
D. The company's equityholders expect a 5% annual return. This means that

$$(\$32{,}265.625 - 0.35z) / (\$31{,}250 - z) = 105\%$$
$$\$32{,}265.625 - 0.35z = \$32{,}812.50 - 1.05z$$
$$z = \$546.875 / 0.70 = \$781.25$$

The RBC requirements raise the commutation price because they increase the tax liability of the primary company. The tax liability is proportional to the equityholders contribution, so the additional premium z is also roughly proportional to the equityholders contribution.

Illustration: If there are no surplus requirements, the equityholder contribution is \$4,875 and the margin in the commutation price is \$125. With a 25% reserving risk charge, the equityholder contribution is \$31,250 − \$781.25 = \$30,468.75 and the margin in the commutation price is \$781.25. As expected, \$4,875 : \$125 :: \$30,468.75 : \$781.25.

If equityholders require an equity-type return (vs. a risk-free return), the additional premium is

$$(\$32{,}265.625 - 0.35z) / (\$31{,}250 - z) = 112\%$$
$$\$32{,}265.625 - 0.35z = \$35{,}000 - 1.12z$$
$$z = \$2{,}734.375 / 0.77 = \$3{,}551.14$$

Workers' Compensation Commutations

Workers' compensation has a reserving risk charge of 11%. The additional charge from the commutation is 11% − 5.5% = 5.5%. The marginal capital requirements are 5.5% × 60% × 200% = 6.60%.

The equityholders contribute 6.6% × \$105,000 = \$6,930 on January 1, 20X2, to support the additional loss reserves, besides the capital embedded in the full value loss reserves.

- The after-tax investment income earned on the \$5,000 embedded in the full value loss reserves is 5% × \$5,000 × (1 − 35%) = \$162.50.
- The after-tax investment income earned on the \$6,930 capital requirement is 5% × \$6,930 × (1 − 35%) = \$225.225.

The offsetting of the investment income on the fair value (discounted) reserves against the incurred loss simplifies the calculations.

- The investment income on the fair value loss reserves is $100,000 x 5% = $5,000. The tax liability on this investment income is $5,000 x 35% = $1,750.
- The underwriting income during the year is $100,000 + z – $105,000 = z – $5,000. The tax liability on the underwriting income is (z – $5,000) x 35% = 0.35z – $1,750.
- The net pre-tax income from these two pieces is $5,000 + z – $5,000 = z. The net after-tax income from these two pieces is 0.65z.

The equityholder provided capital on January 1 is $5,000 + $6,930 – z. The amount returned to the equityholders on December 31 is their capital contribution plus the after-tax net income of $162.50 + $225.225 + 0.65z.

If the equityholders require only a risk-free return, the equation is:

$$(\$5{,}000 + \$162.50 + \$6{,}930 + \$225.225 - z + 0.65z) / (\$5{,}000 + \$6{,}930 - z) = 105\%$$
$$(\$5{,}162.50 + \$7{,}155.225 - 0.35z) / (\$5{,}000 + \$6{,}930 - z) = 105\%$$
$$\$12{,}317.725 - 0.35z = \$12{,}526.50 - 1.05z$$
$$z = \$208.775 / 0.70 = \$298.25$$

If the equityholders require an "equity" return (versus a risk-free return), the equation is

$$(\$5{,}162.50 + \$7{,}155.225 - 0.35z) / (\$5{,}000 + \$6{,}930 - z) = 112\%$$
$$\$12{,}317.725 - 3.5z = \$13{,}361.60 - 1.12z$$
$$z = \$1{,}043.875 / 0.77 = \$1{,}355.68$$

For workers' compensation pension claims with tabular discounts, the commutation price is lower, since less capital is needed to back the discounted reserve. We discuss this issue further below.

The statutory line of business to which a claim is coded affects the commutation price. The commutation price has two components: (i) the present value of future losses and expenses, and (ii) a margin to cover the cost of holding capital.

- The present value of future losses and expenses, which forms the bulk of the commutation price, does not depend on the statutory line of business.[15]
- The margin to cover the cost of holding capital depends on the risk-based capital requirements for the line of business.

The margin for a commutation in medical malpractice or products liability is more than twice the margin for a workers' compensation commutation.[16]

TAX DISCOUNT FACTORS

The most complex part of the commutation calculations involves the IRS loss reserve discount factors. We continue the illustration with one modification: the premium for the commutation is paid on December 31 of the previous year.

> *A loss reserve is commuted on December 31, 20X2. The loss will be paid for $105,000 on December 31, 20X3. The risk-free interest rate is 5% per annum, and the federal income tax rate is 35%. No risk adjustments or risk loads are used. The IRS loss reserve discount factor is 1/1.05 = 95.238095%, which is the discount based on the payment pattern for this risk and the risk-free interest rate.*[17]

NO CAPITAL REQUIREMENTS AND NO ADDITIONAL COST OF CAPITAL

We assume first that the primary insurer does not need surplus to support the loss reserve and that its equityholders are satisfied with a risk-free rate of return. We examine the financial implications of a $100,000 commutation price.

When the commutation is done on December 31, 20X2, the primary company receives $100,000 and records a $105,000 statutory reserve on its books. The tax-basis reserve is 95.238095% × $105,000 = $100,000. The $100,000 of cash exactly offsets the $100,000 of increased tax-basis incurred losses, so taxable income is not affected by the commutation.

The December 31, 20X2, the statutory loss reserve is $105,000, so the equityholders of the insurance company contribute $5,000 on that date. The cash is invested in risk-free securities yielding 5% per annum. During 20X3, the company earns $5,250 of interest income.

The incurred loss offset to tax-basis underwriting income in 20X3 is the paid loss plus the change in tax-basis reserves, or $105,000 + ($0 – $100,000) = $5,000. This is often referred to as the unwinding of the interest discount.[18] In practice, it is an offset to other underwriting (taxable) income. There is no other underwriting income in this illustration, so the total tax-basis underwriting income is –$5,000.

The taxable income in 20X3 is $5,250 – $5,000 = $250. The federal income tax is 35% × $250 = $87.50.

The after-tax income in 20X3 is $250 – $87.50 = $162.50. This is the reward to the shareholders for their contribution of $5,000 at the beginning of the year.

If there is no risk in the insurance operations, the shareholders expect a risk-free rate of return of 5% per annum, not a return of $162.50 / $5,000 = 3.25% per annum. The shareholders could get a 5% per annum return by investing directly in risk-free securities. This is the same as in the previous illustration.

For the primary insurance company to provide a 5% rate of return to its shareholders, the commutation price must be higher than $100,000. Let the commutation price be $100,000+*z*.

A. The taxable income on December 31, 20X2 is $0 + *z*, and the tax is 35% × *z*.

B. The company holds assets of $105,000 to fund the reserve. It has $100,000 + (1 – 35%) × *z* from the commutation transaction, or $100,000 + 65% × *z*. Shareholders contribute $5,000 – 65% × *z* at the beginning of the year to fund the $105,000 statutory reserve.

C. The company's shareholders expect a 5% annual return. The amount returned to the shareholders on December 31, 20X3 is $5,162.50, so $5,162.50 / ($5,000 – 65% × *z*) = 105%. This means that

$$\$5,162.50 / (\$5,000 - 0.65z) = 105\%$$
$$\$5,162.50 = \$5,250 - 0.6825z$$
$$z = \$87.50 / 0.6825 = \$128.21$$

The proper commutation price is not $100,000.00 (the present value of the future loss payments) or $100,125.00 (the commutation price for an effective date of January 1) but $100,128.21. This is an increase of 0.128% over the discounted loss payments. The commutation price rises by $128.21 – $125 = $3.21 when the premium payment changes from January 1, 20X3, to December 31, 20X2. Since the premium payment is moved to the preceding calendar year, the tax on the premium payment is moved up one year, and the primary insurance company loses investment income on the tax payment. The present values of the pre-tax cash flows have not changed, but the after-tax cash flows are different.

The additional $3.21 premium stems from two items.

1. Since the $125 premium margin is moved up one year, the insurer pays taxes of 35% × $125 = $43.75 one year earlier. It loses the investment income of 5% × $43.75 = $2.1875. This investment income would be received at the end of the year. The present value of this investment income at the beginning of the year is $2.1875 / 1.05 = $2.0833.

2. The lost investment income must be compensated by additional premium, or the equityholders of the primary company would not agree to the commutation. If the reinsurance company were to pay the $2.0833 directly to the equityholders of the primary company, this would be the total charge. But the reinsurance company pays the money indirectly through the commutation price. The premium received through the commutation price is taxed at the 35% corporate tax rate. To pay $2.0833 to the equityholders fo the primary company, the additional premium must be

$$\$2.0833 / (1 - 35\%) = \$3.21.$$

The accounting entries clarify the implied equity flows. The reinsurer transfers $105,000 of loss reserves with a present value of $100,000 to the primary company, paying $100,128.21 in cash. The tax-basis discounted reserves are also $100,000.

The primary company receives $100,128.21 in cash and it increases its tax-basis reserves by $100,000.00, for an increase in taxable income of $128.21. The tax rate is 35%, so its tax liability is 35% x $128.21 = $44.87.

The primary company is left with $100,128.21 – $44.87 = $100,083.34 from the reinsurer. It must hold assets of $105,000 to back the statutory liabilities of $105,000. The primary company's shareholders contribute $105,000 – $100,128.21 + $44.87 = $4,916.66 to complete the funding of the statutory liabilities.

During the following year, the assets of $105,000 earn $5,250 of interest. The tax-basis reserves increase from $100,000 to $105,000, for a tax-basis incurred loss of $5,000. The net income is $250, and the federal income tax on this income is $250 x 35% = $87.50. The loss is paid for $105,000, and the remaining $5,250 – $87.50 = $5,162.50 is returned to the primary company's shareholders.

The return to the shareholders is $5,162.50 / $4,916.66 – 1 = 5.00%.[19]

COST OF HOLDING CAPITAL

If shareholders require an equity return instead of a risk-free return, the additional premium is

$$\$5,162.50 / (\$5,000 - 0.65z) = 112\%$$
$$\$5,162.50 = \$5,600 - 0.728z$$
$$z = \$437.50 / 0.728 = \$600.96.$$

As before, we have assumed a 8% market risk premium and an implicit beta of 87.5% for the property-casualty insurance industry, for a cost of capital of 5% x 87.5% x 8% = 12%.

CAPITAL REQUIREMENTS

We use the same assumptions for capital requirements as in the first illustration. The illustration below uses the 25% marginal effect of the reserving risk charge for the commercial liability lines of business (general liability, products liability, medical malpractice). A workers' compensation commutation would use the same formulas with 6.6% substituted for 25%.

The shareholders contribute 25% x $105,000 = $26,250 on December 31, 20X2, to support the additional loss reserves. On December 31, 20X3, the shareholders receive back this amount plus the after-tax investment income, or

$$\$26{,}250 + 5\% \times \$26{,}250 \times (1 - 35\%) = \$27{,}103.125.$$

If the shareholders require only a risk-free rate of return, the appropriate equation is:

$$(\$5{,}162.50 + \$27{,}103.125) / (\$5{,}000 + \$26{,}250 - 0.65z) = 105\%$$
$$\$32{,}265.625 = \$32{,}812.50 - 0.6825z$$
$$z = \$546.875 / 0.6825 = \$801.28$$

If the shareholders require an equity return, the appropriate equation is:

$$(\$5{,}162.50 + \$27{,}103.125) / (\$5{,}000 + \$26{,}250 - 0.65z) = 112\%$$
$$\$32{,}265.625 = \$35{,}000.00 - 0.728z$$
$$z = \$2{,}734.375 / 0.728 = \$3{,}756.01$$

The pricing method is straight-forward. The numerical result depends on the marginal effect of the reserving risk charge, ranging from 6% to 25%, and on the company's target return on capital, ranging from a 0% margin over the risk-free rate to a 7% margin over the risk-free rate. The price would also depend on any discounts in the statutory loss reserve, such as tabular discounts on workers' compensation claims.

DEFERRED TAX ASSETS AND LIABILITIES

The previous analysis is still incomplete, since we have not yet considered deferred tax assets and liabilities.

The deferred tax asset stemming from IRS loss reserve discounting has a material effect on the implied equity flows and the commutation price. The discussion here proceeds along the following path:

a. Accounting theory: current tax liability vs accrued taxes
b. Quantifying the deferred tax asset from IRS loss reserve discounting
c. One year illustration using the gross deferred tax asset
d. Statutory admissibility rules for deferred tax assets reversing over more than 12 months
e. Multi-year illustration using the admitted portion of the deferred tax asset

Current Taxes vs Accrued Taxes

The deferred tax asset stemming from IRS loss reserve discounting is unique to property-casualty insurance. To clarify the intuition, we use an example of deferred tax assets and liabilities stemming from realized capital gains and losses.

Illustration: An insurance company purchases 20,000 shares of common stock for $50 apiece on December 31, 20X2. By December 31, 20X3, each share of stock appreciates to $60. The company sells 10,000 shares at this price, and keeps the other 10,000 stocks.

- The realized capital gain is 10,000 x ($60 – $50) = $100,000. The tax liability on the realized capital gain is 35% x $100,000 = $35,000.
- The unrealized capital gain is 10,000 x ($60 – $50) = $100,000. The tax liability on the unrealized capital gain is zero, since capital gains are not taxed until they are realized.

On its balance sheet, the company shows common stocks at their market values. At December 31, 20X3, the remaining 10,000 shares are valued at $60 apiece. The income implied by the statutory balance sheet is the number of shares times the difference between the current price and the purchase price, or 10,000 x ($60 – $50) = $100,000.

If the company sells these shares, it must pay tax of $35,000 to the U.S. Treasury. Its after-tax income would be $65,000.

- The current tax liability is the tax on the realized gains, or $100,000 x 35% = $35,000.
- The accrued tax liability is the tax on the realized capital gains plus the deferred tax liability on the unrealized capital gains, for a total of $70,000.

Only realized capital gains flow through the statutory income statement. The unrealized capital gains and losses are direct charges and credits to surplus. For common stock gains and losses, there is no difference between statutory and taxable income.

Whether or not a balance sheet change is reflected in the statutory income statement is not relevant to deferred tax assets and liabilities. We compare taxable income to the *income implied by the statutory balance sheet*, not to actual statutory income; see SFAS 109.[20]

GAAP vs Statutory Accounting

Deferred tax assets and liabilities stem from timing differences in the realization of income. The unrealized capital gains or losses are recognized on the statutory balance sheet, but they are not recognized for taxable income until they are realized. When they are realized, the timing difference reverses: there is a gain or loss in future taxable income that is not reflected in the balance sheet changes of that accounting period.

- GAAP financial statements use the accrued tax basis of accounting. All deferred tax assets and liabilities that are expected to be realized in the future must be recognized on the balance sheet in the current accounting period.[21]
- Until 2001, statutory financial statements did not recognize deferred tax assets or liabilities. After codification of statutory accounting, the deferred tax liabilities and a

portion of the deferred tax assets are recognized on the statutory balance sheet; see SSAP No. 10, "Income Taxes."

Deferred tax assets and liabilities do not affect the cash flows of the company. However, they affect admitted assets and the implied equity flows, and thereby affect the commutation price.[22]

DEFERRED TAX ASSETS AND LOSS RESERVE DISCOUNTING

We illustrate the computation of the deferred tax asset stemming from loss reserve discounting and then examine the application to the commutation price.[23]

Illustration: On January 1, 20X2, a company writes a policy for a $12,000 premium and pays $2,000 in expenses. On December 31, 20X2, the company records a case reserve of $10,000, and it expects the loss to be paid on December 31, 20X3.

The risk-free interest rate is 5% per annum. For simplicity, we assume that the 60 month moving average of federal mid-term rates is the same as the current risk-free interest rate of 5% per annum, and that the IRS loss payment pattern matches the actual loss payment pattern for the block of business. The IRS loss reserve discount factor is 1/1.05 = 95.238095%.[24]

The statutory underwriting income is earned premium minus expenses minus incurred loss. In 20X2, this is $12,000 – $2,000 – $10,000 = $0. The *accrued* tax liability is the tax rate times the booked income, or $0 × 35% = $0.[25]

The tax basis incurred loss in 20X2 is 95.238095% × $10,000 = $9,523.81. The tax basis underwriting income in 20X2 is $12,000 – $2,000 – $9,523.81 = $476.19. The *current* tax liability is the actual amount owed to the IRS, or $476.19 × 35% = $166.67.

The timing difference between taxable income and statutory income is $166.67. The current tax liability is greater than the accrued tax liability, meaning that the company pays more tax than it would pay were the tax computed on the basis of its statutory balance sheet.

The timing difference reverses in 20X3. In 20X3, the statutory income is zero. The tax basis incurred loss is the paid loss plus the change in the discounted reserves, or

$$\$10{,}000 + (\$0 - \$9{,}523.81) = \$476.19.$$

The tax liability in 20X3 is –35% × $476.19 = –$166.67, or a tax refund of $166.67. In 20X2, the company holds a deferred tax asset of $166.67 on its statutory balance sheet.

DEFERRED TAX ASSETS AND COMMUTATION PRICING

One may conceive of the deferred tax asset as an "IOU" that is secured by the expectation of receiving a $166.67 tax refund in 20X3. It is not an investable asset, and it earns no investment income. It is an admitted asset, since it is expected to reverse within the next 12 months. We deal with the statutory admissibility constraints further below.

As an admitted asset, the deferred tax asset can back the loss reserves and it can back the risk-based capital needed to support the loss reserves. The deferred tax asset of $166.67 reduces the equityholder provided capital by $166.67.

We re-work the commutation illustration that we began earlier.

> *A loss reserve is commuted on December 31, 20X2. The loss will be paid for $105,000 on December 31, 20X3. The risk-free interest rate is 5% per annum, and the federal income tax rate is 35%. No risk adjustments or risk loads are used. The IRS loss reserve discount factor is 95.238095%, which is equal to the actual discount based on the true payment pattern for this risk and the current risk-free interest rate.*[26]

The changes from the previous calculation are as follows. On December 31, 20X2, the company holds a deferred tax asset of

$$35\% \times (\$105{,}000 - \$105{,}000 \times 95.238095\%) = \$1{,}750$$

The equityholder contribution on December 31, 20X2, is $1,750 lower. The deferred tax asset is not investable. The investment income in 20X3 is reduced by $1,750 × 5% = $87.50. The tax on investment income in 20X3 is reduced by $87.50 × 35% = $30.625.

The commutation is now priced in the same manner as before. We show the indicated commutation prices for four scenarios: a workers' compensation commutation with a 6.6% capital requirement vs. a commercial liability commutation with a 25% capital requirement and a 5% risk-free return to equityholders vs. a 12% "equity" return to equityholders.

The step-by-step documentation below explains the calculations for the commercial liability commutation with a 12% equity return. The computations for other scenarios are similar.

COMMERCIAL LIABILITY COMMUTATION WITH RISK-FREE RETURN: YEAR 20X2

- The commutation price is $100,000 + z.
- The undiscounted reserve is $105,000.
- The IRS loss reserve discount factor is 1/1.05 = 95.238095%.
- The tax basis loss reserve $105,000 × 1/1.05 = $100,000.
- The tax basis underwriting income in 20X2 is $100,000 + z – $100,000 = z.

- The tax on underwriting income in 20X2 is 35% × z.
- The deferred tax asset stemming from loss reserve discounting is 35% × ($105,000 – $100,000) = $1,750. The deferred tax asset does not depend on "z."
- The investment income in 20X2 is zero, and the tax on investment income in 20X2 is zero.
- The total tax paid in 20X2 is 35% × z.
- The capital requirement is 25% of the undiscounted reserves, or 25% × $105,000 = $26,250.00.
- The assets needed on December 31, 20X2, are $105,000 (reserves) + $26,250 (surplus) = $131,250.
- The cash received from the reinsurer is $100,000 + Z.
- The cash paid to the IRS is 35% × z.
- The net cash available is $100,000 + z – 35% × z = $100,000 + 65% × z.
- The non-cash asset (the DTA) is $1,750.
- The capital contribution needed from the equityholders is $131,250 – ($100,000 + 65% × z) – $1,750 = $29,500 – 65% × z.

YEAR 20X3

- The cash assets (investable assets) at the beginning of the year are $131,250 – $1,750 = $129,500.
- The investment yield is 5% per annum.
- The investment income in 20X3 is 5% × $129,500 = $6,475.00.
- The tax on investment income is 35% × $6,475 = $2,266.25.
- The tax basis incurred loss in 20X3 is the paid loss plus the change in reserves, or $105,000 + ($0 – $100,000) = $5,000.
- The tax basis underwriting income in 20X3 equals –$5,000.
- The tax liability on underwriting income in 20X3 is –$5,000 × 35% = –$1,750.
- The net taxes paid in 20X3 are $2,266.25 – $1,750 = $516.25.
- The cash available at the end of the year before payment of the loss is $129,500 + 6,475 – $516.25 = $135,458.75.
- The loss is paid for $105,000.
- The cash remaining after payment of the loss is $30,458.75.
- The required return on capital is 12% per annum.
- We solve for "z" as

$$\$30{,}458.75 / (\$29{,}500 - 65\% \times z) = 1.12$$
$$65\% \times z = \$2{,}304.6875$$
$$z = \$3{,}545.67$$

The addition to the commutation price depends on the capital requirements and the cost of equity capital. We show two choices for each of these dimensions:

- *Capital requirements:* 25% of reserves for commercial liability vs 6.6% of reserves for workers' compensation
- *Cost of equity capital:* 5% risk-free return vs 12% equity return

The value of $3,545.67 as the addition to the commutation price is the high end of the range. The four possibilities from the two dimensions listed above are shown in the matrix below, where the vertical axis represents the capital requirements and the horizontal axis represents the cost of equity capital.

Capital Requirement	*5% cost of equity capital*	*12% cost of equity capital*
Commercial Liability: 25%	$756.41	$3,545.67
Workers' Compensation: 6.6%	$261.03	$1,223.56

We summarize the results of the pricing analysis as follows.

- The commutation price is higher than the present value of the commuted reserves, where the present value is taken at the pre-tax interest rate.
- If there were no capital requirements, the commutation price would be the present value of the commuted reserves, except for variances between the IRS loss reserve discount factor and the actuarially determined discount factor for the commuted reserves. Over the long-term, the IRS loss reserve discount factors are not materially biased, and the variance should be small for the commutation of an entire block of business.
- When individual claims are commuted, the actual loss payment pattern is generally slower than the pattern assumed by the IRS for the entire line of business. The tax basis reserves are higher than fair value reserves (providing a benefit to the company holding the reserves), and the commutation price is lower.
- The required return on capital has a large effect on the indicated commutation price. Changing the required return on capital from a 5% risk-free rate to a 12% equity return causes about a five fold increase in the addition to the commutation price.
- The capital requirements also have a considerable effect on the commutation price. This capital requirement is the additional requirement from the commutation, or the difference in capital requirements between the reserving risk charge and the reinsurance charge. Changing the capital requirement from 6.6% of reserves to 25% of reserves causes about a three fold increase in the commutation price.

Statutory Recognition of Deferred Tax Assets

All deferred tax liabilities are recognized on the statutory balance sheet. For most deferred tax assets, the admitted statutory portion equals the entire asset, and statutory accounting is the same as GAAP. In certain instances, only a portion of the deferred tax assets are recognized on the statutory balance sheet. This applies particularly to the deferred tax asset stemming from IRS loss reserve discounting for medium- and long-tailed lines of business.

SSAP No. 10, "Income Taxes," paragraph 10, says:

Gross DTAs shall be admitted in an amount equal to the sum of:

a *Federal income taxes paid in prior years that can be recovered through loss carrybacks for existing temporary differences that reverse by the end of the subsequent calendar year;*
b *The lesser of:*

   i *The amount of gross DTAs, after the application of paragraph 10 a., expected to be realized within one year of the balance sheet date; or*
   ii *Ten percent of statutory capital and surplus as required to be shown on the statutory balance sheet of the reporting entity for its most recently filed statement with the domiciliary state commissioner adjusted to exclude any net DTAs, EDP equipment and operating system software and any net positive goodwill; and*

c. *The amount of gross DTAs, after application of paragraphs 10 a. and 10 b., that can be offset against existing gross DTLs.*

A gross deferred tax asset is admissible if it will reverse within one year, as required by paragraph (a) and by paragraph (b.i).

The limitation of 10% of surplus in paragraph (b.ii) may affect companies with long-tailed lines of business, pre-paid annual policies, and high premium to surplus ratios.[27]

Illustration: An insurer with $100 million of surplus writes annual policies with effective dates spread evenly through the year. Its premium to surplus ratio is 2 to 1, and its reserves to surplus ratio is 4 to 1. Its average loss reserve discount factor is about 80%, and 30% of its deferred tax asset from loss reserve discount will reverse within 12 months. We work out its gross deferred tax asset and the portion admitted on the statutory balance sheet.

Annual premium is twice surplus, or $200 million. The unearned premium reserve is $200 million x 50% = $100 million, and the deferred tax asset stemming from revenue offset is $100 million x 20% x 35% = $7 million.

The reserves to surplus ratio is 4 to 1, so the held loss reserves are $400 million. The average loss reserve discount factor is 80%, so the discounted reserves are $400 million x 80% = $320 million. The gross deferred tax asset from IRS loss reserve discounting is ($400 million – $320 million) x 35% = $28 million. The portion admitted on the statutory balance sheet is $28 million x 30% = $8.4 million.

The total statutory deferred tax asset is $7 million + $8.4 million = $15.4 million. This is limited to 10% of statutory surplus, or $10 million, so an additional $5.4 million is not admitted.

Various changes in the scenario would mitigate the "10% of surplus" restriction.

- *Premium to surplus ratio:* The U.S. property-casualty insurance industry has a premium to surplus ratio of about 1 to 1. With this premium to surplus ratio, the 10% of surplus restriction has no effect in the illustration above.
- *Policy term:* The unearned premium reserve for six month policies and the deferred tax asset from revenue offset would be only half the size of those for annual policies.
- *Lines of business:* Property lines do not have material deferred tax assets from loss reserve discounting.
- *Effective dates:* Policies effective on January 1 have much lower unearned premium reserves and deferred tax assets at the end of the year.

The offsetting against existing gross deferred tax liabilities mentioned in paragraph (c) is relevant for companies with large unrealized capital gains from common stock holdings. The actuary should take this provision into account when quantifying the admitted portion of the deferred tax asset.

Common stock that has suffered an unrealized capital loss may be sold within the next 12 months to realize the tax benefits. A literal reading of the SSAP would permit the recognition of the deferred tax asset only if the company expects to realize the capital loss during the coming calendar year. In practice, most auditors do not require an explicit expectation to realize the loss in order to admit the deferred tax asset.

LOSS RESERVE DISCOUNTING

Statutory incurred losses are the paid losses plus the change in the undiscounted loss reserves. The tax basis incurred losses are the paid losses plus the change in the *discounted* loss reserves. The difference between statutory and tax basis incurred losses is a timing difference. The change in the deferred tax asset is 35% of this difference.

Illustration: A policy is issued on January 1, 20XX, for a premium of $1000 and expenses of $200. Losses of $800 are incurred in 20XX, of which half are paid in 20XX and half are paid in 20XX+1. The IRS loss reserve discount factor at the 12 month valuation is 90%. For simplicity, we assume that the companies earns no investment income.

- The statutory incurred losses in 20XX are $400 of paid losses plus $400 of loss reserve change = $800. Statutory income is $1000 – $200 – $800 = $0. The accrued taxes are 35% × $0 = $0.
- The taxable incurred losses in 20XX are $400 of paid losses plus $400 × 90% = $360 of change in tax basis (discounted) loss reserves = $760. Taxable income is $1000 – $200 – $760 = $40. The tax liability is 35% × $40 = $14

The difference between the income implied by the statutory balance sheet and taxable income is $0 – $14 = –$14. The gross deferred tax asset is $14.

The portion of the deferred tax asset that reverses within 12 months is admitted on the statutory balance sheet. We examine the statutory income and taxable income for 20XX+1.

- The statutory incurred losses in 20XX+1 are $400 of paid losses plus –$400 of loss reserve change = $0. There is no premium or expense in 20XX+1, so statutory income is $0. The accrued taxes are 35% × $0 = $0.
- The taxable incurred losses in 20XX+1 are $400 of paid losses plus –$360 of change in discounted loss reserves = $40. There is no premium or expense in 20XX+1, so taxable income is $0 – $40 = –$40. The tax liability is 35% × (–$40) = –$14.

The full difference between statutory and taxable income reverses in 20XX+1, so the full deferred tax asset of $14 is admitted on the statutory balance sheet.

TWELVE MONTH REVERSAL

We compute the admitted portion of the deferred tax asset from loss reserve discounting separately by line of business and accident year.

Illustration: For accident year 20XX in a given line of business, the loss reserve discount factors are Z_1 at December 31, 20YY, and Z_2 at December 31, 20YY+1. Let "R" be the held loss reserves at December 31, 20YY. Let "P" be the percentage of accident year 20XX reserves that will be paid during calendar year 20XX.

- At December 31, 20YY, the difference between statutory and taxable income for accident year 20XX is $R \times (1 - Z_1)$. The gross deferred tax asset is $35\% \times R \times (1 - Z_1)$.
- At December 31, 20YY+1, the difference between statutory and taxable income for accident year 20XX is $R \times (1 - P) \times (1 - Z_2)$. The gross deferred tax asset is $35\% \times R \times (1 - P) \times (1 - Z_2)$.
- The admitted portion of the deferred tax asset on the statutory balance sheet at December 31, 20YY is $35\% \times R \times [(1 - Z_1) - (1 - P) \times (1 - Z_2)]$.

The value of *P* depends on the company's estimated loss payment pattern, not the IRS loss payment pattern. The pattern should be based on actuarially justified discount factors.

ILLUSTRATION

Suppose accident year 20X5 incurred losses are $180,000, of which $15,000 is paid in 20X5. The paid loss development factors are 8.000 from 12 months to ultimate and 5.000 from 24 months to ultimate. The IRS loss reserve discount factors for accident year 20X5 are 77.8022% at 12 months and 78.7611% at 24 months. We determine the statutory and GAAP deferred tax assets on December 31, 20X5.

The accident year 20X5 loss reserves for statutory and GAAP balance sheets on December 31, 20X5 are $180,000 – $15,000 = $165,000. The discounted tax basis loss reserves are

$$\$165{,}000 \times 77.8022\% = \$128{,}373.63.$$

The difference between the GAAP loss reserves and the tax basis loss reserves is

$$\$165{,}000.00 - \$128{,}373.63 = \$36{,}626.37.$$

The addition to taxable income stemming from loss reserve discounting is $36,626.27 x 35% = $12,819.23. This is the deferred tax asset on the GAAP balance sheet.

The admitted portion of the deferred tax asset on the statutory balance sheet depends on the portion of the loss reserve that remains unpaid in one year's time. This is an actuarial estimate; it is not the IRS provision used in the loss reserve discounting calculation. We use the paid loss development factors to project the losses remaining unpaid at 24 months.

- At 12 months of development, 1/8.000 = 12.5% of incurred losses have been paid and 1 – 1/8.000 = 87.5% of incurred losses are still unpaid.
- At 24 months of development, 1/5.000 = 20.0% of incurred losses have been paid and 1 – 1/5.000 = 80.0% of incurred losses are still unpaid.

We expect 80.0% / 87.5% = 91.428571% of the December 31, 20X5, accident year 20X5 loss reserves to remain unpaid at December 31, 20X6. This amount is $165,000 × 91.4285714% = $150,857.14. The expected IRS discounted reserves at December 31, 20X6 equal this amount times the IRS loss reserve discount factor for accident year 20X5 at 24 months of development, or 78.7611%:

$150,857.14 × 78.7611% = $118,816.75.

IMPLICIT DISCOUNTING

Some companies implicitly discount reserves for long-tailed lines of business. Implicit discounting means that the company consciously holds less than full value loss reserves (for capital management purposes), not that the company mis-estimates the reserve indication.

One might be tempted to think that the amount of the implicit reserve discount should be taken into consideration when calculating the deferred tax asset. This is not correct. The deferred tax asset must be calculated as if the company held full value loss reserves.

Illustration: An insurer expects to pay a loss for $100,000 in three years. The IRS loss reserve discount factor for this line of business and accident year is 80% for the current valuation date and 85% for the valuation date 12 months hence.

- The (gross) deferred tax asset on the GAAP financial statements is 35% × $100,000 × (1 – 80%) = $7,000.
- The (net admitted) deferred tax asset on the statutory financial statements is 35% × $100,000 × (85% – 80%) = $1,750.

If the insurer implicitly discounts reserves at 5% per annum, its held reserves are $100,000 / 1.05^3 = $86,383.76, and its tax basis reserves are 80% × $100,000 / 1.05^3 = $69,107.01. Its expected held reserves one year hence are $100,000 / 1.05^2 = $90,702.95, and its expected tax basis reserves at that time are 85% × $100,000 / 1.05^2 = $77,097.51.

One might think that the gross (GAAP) and net admitted (statutory) deferred tax assets should be computed as follows:

- Gross (GAAP): 35% × ($100,000 – $69,107.01) = $10,812.55.
- Net admitted (statutory): 35% × ($77,097.51 – $69,107.01) = $2,796.68.

This is not correct. If the company shows a reserve of $86,383.76 on its statutory financial statements, it must treat that reserve at though it were a full value loss reserve for calculating the deferred tax asset. The appropriate calculations are as follows:

- The (gross) deferred tax asset on the GAAP financial statements is 35% × $86,383.76 × (1 – 80%) = $6,046.86.
- The (net admitted) deferred tax asset on the statutory financial statements is 35% × $86,383.76 × (85% – 80%) = $1,511.72.

DEFERRED TAX ASSETS: MULTIPLE PERIODS

Actual commutations have cash flows extending for many future years. To price the commutations, we determine the projected deferred tax assets at each future valuation date.

The deferred tax assets depend on the IRS loss reserve discount factors. These discount factors may be either industry factors or company specific factors. The industry factors are based on industry-wide loss payment patterns by line of business. They are promulgated by the Secretary of the Treasury each year. The company specific factors are based on the company's own loss payment patterns by line of business.

The loss reserve discount factors are determined for a specific accident year. Once the factors are determined, they are frozen (or "vintaged" in tax parlance).

> *Illustration:* By mid-December 20XX–1, all the required information is available to determine accident year 20XX loss reserve discount factors.[28] Between 11 and 16 loss reserve discount factors are computed, applicable to valuation dates of 12/31/20XX, 12/31/20XX+1, . . . , 12/31/20XX+15. These factors are applicable to accident year 20XX losses only, and they are not changed after the initial determination.

The calculation of the *gross* deferred tax assets stemming from loss reserve discounting is based solely on the loss reserve discount factors.[29] Just as the loss reserve discount factors are determined and frozen at the beginning of the accident year, the gross deferred tax asset as a percentage of the loss reserves at each valuation date is determined at the inception of the accident year.

In early 20XX, the company determines the loss reserve discount factors for accident year 20XX at each future valuation date. The deferred tax asset factor for each valuation date (as a percentage of the held reserves) is 35% of loss reserve discount at that valuation date.

Illustration: If the loss reserve discount factor for accident year 20XX at valuation date December 31, 20XX+2, is 75%, the (gross) deferred tax asset factor is 35% × (1 – 75%) =

8.75%. That is, the deferred tax asset for accident year 20XX on the December 31, 20XX+2, GAAP balance sheet is 8.75% of the held loss reserves.

The admitted portion of the deferred tax asset for the statutory balance sheet depends on the actuary's estimate of the loss liquidation pattern at the valuation date. These are statutory accounting numbers, not tax accounting numbers, and there is no "vintaging."

For pricing commutations, we must estimate the deferred tax asset as a percentage of the full value loss reserves at each future valuation date. Projecting loss liquidation patterns is a staple of casualty loss reserving, and the estimation of future deferred tax assets presents no unusual complications. We explain the estimation process by means of an illustration.

Illustration: An insurer commutes a workers' compensation permanent total disability claim with the following characteristics:

- Date of loss occurrence: July 1, 2000.
- Date of commutation: December 31, 2005.
- Annual indemnity benefits: $40,000 with no cost of living adjustments
- Annual medical benefits: None
- Life expectancy: 20 years
- Full value loss reserve: $800,000

The IRS loss reserve discount factors for accident year 2000 are shown below:

Tax Year	Valuation Date	IRS Loss Reserve Discount Factor	Tax Year	Valuation Date	IRS Loss Reserve Discount Factor
AY + 0	2000	0.819398	AY + 5	2005	0.675118
AY + 1	2001	0.807648	AY + 6	2006	0.661927
AY + 2	2002	0.802667	AY + 7	2007	0.670194
AY + 3	2003	0.761583	AY + 8	2008	0.703333
AY + 4	2004	0.710909	AY + 9	2009	0.739426

We examine three versions of this illustration:

- The company holds full value loss reserves with no tabular discount.
- The company holds reserves net of a tabular discount, and the tabular discount for all workers' compensation business in accident year 2000 is less than the IRS loss reserve discount for the company's workers' compensation business in this accident year. This would be true for the most recent five to ten accident years.
- The company holds reserves net of a tabular discount, and the tabular discount for all workers' compensation business in accident year 2000 is greater than the IRS loss

reserve discount for the company's workers' compensation business in this accident year. This is generally true only for old accident years, such as the Schedule P prior years row.

To ease the exposition, we initially assume that the probability of death in calendar year 2006 is insignificant. Once the intuition is clear, we relax this constraint.

DEFERRED TAX ASSET AT DECEMBER 31, 2005

The full value loss reserve at 12/31/2005 is $800,000. The IRS loss reserve discount factor for accident year 2000 at 72 months is 0.675118. The tax basis reserve is $800,000 × 0.675118 = $540,094.

If we assume the probability of death in 2006 is 0%, the full value loss reserve at 12/31/2006 is $800,000 – $40,000 = $760,000.[30] The IRS loss reserve discount factor for accident year 2000 at 84 months is 0.661927. The tax basis reserve is $760,000 × 0.661927 = $503,065.

The loss reserve discount is $800,000 – $540,094 = $259,906 at 12/31/2005 and $760,000 – $503,065 = $256,935 at 12/31/2006. The change in the discount over the 12 months following 12/31/2005 is $259,906 – $256,935 = $2,159. The deferred tax asset admitted on the statutory balance sheet on 12/31/2005 is $2,159 × 35% = $756.

DEFERRED TAX ASSET AT DECEMBER 31, 2007

The deferred tax asset in the illustration above is less than 0.1% of the full value loss reserve. The small size stems from the decline in the loss reserve discount factor from AY+5 to AY+6 and the slow payment pattern for this workers' compensation pension case.

If the commutation is effected on 12/31/2007 instead of 12/31/2005, the deferred tax asset is greater. We redo the computation with the new valuation date and without changing the remaining life expectancy of 20 years.

The full value loss reserve at 12/31/2007 is 20 × $40,000 = $800,000. The IRS loss reserve discount factor for accident year 2000 at 96 months is 0.670194. The tax basis reserve is $800,000 × 0.670194 = $536,155.

If we assume the probability of death in 2008 is 0%, the full value loss reserve at 12/31/2008 is $800,000 – $40,000 = $760,000. The IRS loss reserve discount factor for accident year 2000 at 108 months is 0.703333. The tax basis reserve is $760,000 × 0.703333 = $534,533.

The loss reserve discount is $800,000 – $536,155 = $263,845 at 12/31/2007 and $760,000 – $534,533 = $225,467 at 12/31/2008. The change in the discount over the 12 months following 12/31/2005 is $263,845 – $225,467 = $38,378. The deferred tax asset that is admitted on the statutory balance sheet on 12/31/2005 is $38,378 × 35% = $13,432.

The deferred tax asset is $13,432 / $756 = 17.8 times larger in 2007 than in 2005. The difference does not stem from any change in the full value loss reserve (which is $800,000 in both cases) or the tax basis loss reserve (which is about $535,000 to $540,000 in both cases). The difference stems from the change in the relative size of the current year's loss reserve discount factor and the next year's loss reserve discount factor.

Probability of Death

When the probability of death is considered, the deferred tax asset is computed with a mortality table. The explanation below shows the intuition; it is not an exact calculation. To keep the intuition clear, we assume that deaths occur only at year end.

We resume the illustration described above, with a current valuation date of December 31, 2005. We assume that the probability of the injured worker's death in 2006 is 1%.

On December 31, 2005, the injured worker's life expectancy is 20 years. There are two scenarios for the coming year.

- The worker dies on December 31, 2006, and his life expectancy on December 31, 2005 is 1 year.
- The worker does not die in 2006, and his life expectancy on December 31, 2005 is 1 year plus his life expectancy on December 31, 2006.

Let "Z" represent the conditional life expectancy on December 31, 2006, given that he is alive on that date. We determine "Z" as

$$20 \text{ years} = 1\% \times 1 \text{ year} + 99\% \times (1 \text{ year} + Z \text{ years})$$
$$19 \text{ years} = 99\% \times Z \text{ years}$$
$$Z = 19/0.99 = 19.191919 \text{ years}$$

The life expectancy for the this worker on December 31, 2006, based on the uncertainty present on December 31, 2006, is 1% x 0 years + 99% x 19.191919 years = 19.000 years.

This result is true for all scenarios. The probability of death during the coming year does not affect the expected life expectancy at the next valuation date. The probability of death during the coming year does not affect the deferred tax asset at the current valuation date.[31]

Tabular Discount

Workers' compensation claim may have tabular discounts, which affect the held reserve. The tabular discount reduces equityholder capital embedded in the reserves, thereby reducing the commutation price. It also reduces the reserving risk charge, further reducing the commutation price.

If the tabular discount is explicitly shown in the Annual Statement, the tabular discount does not affect the tax basis loss reserves, unless the held reserves are lower than the IRS discounted reserves.[32] Because the tabular discount changes the loss reserves on the statutory balance sheet but not the tax basis reserves, the deferred tax asset changes.

We explain the computations by means of the illustration begun above. Assume the discount rate for the tabular discount is 5% per annum. The reciprocal of the discount rate is 1/1.05 = 0.952381. The discounted reserve at December 31, 2005 is approximately

$$\$40{,}000 \times (1 - 0.952381^{20})/(1-0.952381) = \$523{,}413.04.^{33}$$

In this case, the statutory reserves are lower than the tax basis reserves of $540,094. There are two possible scenarios.

Scenario A: The permanent total disability cases are not the only workers' compensation loss reserves in this accident year. Other workers' compensation loss reserves are being held at full value. For all claims combined, the statutory held reserves are greater than the tax basis loss reserves. In this scenario, we use the tax basis reserve of $540,094 to determine the deferred tax asset or liability. We do not limit the IRS loss reserve discount, since the limit is offset by the full value loss reserves on other claims.

The *gross* deferred tax asset or liability is a deferred tax *liability* of ($540,094 – $523,413) × 35% = $5,838. This deferred tax liability would appear on the GAAP balance sheet. For the statutory balance sheet, the amount of the deferred tax asset or liability depends on the amount that reverses over the coming 12 months. In the illustration used here, this becomes a deferred tax asset, since the IRS loss reserve discount factors decline from 72 months to 84 months, whereas the pension discount factors increase from 72 months to 84 months. We show the appropriate calculation:

The IRS loss reserve discount factors are 67.5118% at 72 months and 66.1927% at 84 months. The tabular discount factors at these two dates are

- *72 months:* $(1 - 0.952381^{20})/[20 \times (1–0.952381)] = 65.4266\%$.
- *84 months:* $(1 - 0.952381^{19})/[19 \times (1–0.952381)] = 66.7873\%$.[34]

The expected change in the tax basis loss reserves over the coming 12 months is

$$\$800{,}000 \times 67.5118\% - \$760{,}000 \times 66.1927\% = \$37{,}030.$$

The tax basis incurred loss is $40,000 – $37,030 = $2,970.

The expected change in the statutory loss reserves over the coming 12 months is

$$\$800{,}000 \times 65.4266\% - \$760{,}000 \times 66.7873\% = \$15{,}829.$$

The statutory incurred loss is \$40,000 – \$15,829 = \$24,171.

The statutory deferred tax asset is 35% × (\$24,171 – \$2,970) = \$7,420.

Scenario B: The permanent total disability cases are the dominant portion of the workers' compensation loss reserves in this accident year, as is true for the Schedule P prior years row. For all claims combined, the statutory held reserves are lower than the tax basis loss reserves, so the tax basis reserve is set equal to the held reserve. There is no deferred tax asset on either the statutory or the GAAP balance sheet.

THE CONNOR AND OLSEN ILLUSTRATION

We redo the illustration in the Connor and Olsen paper, using the techniques outlined here. Because the Connor and Olsen technique does not consider risk-based capital requirements and the cost of holding capital, Connor and Olsen significantly overstate the proper commutation price. The inclusion of deferred tax assets in post-codification statutory accounting slightly reduces the proper commutation price, but it does not materially offset the bias in the Connor and Olsen technique.

Illustration: A company commutes a block of loss reserves with expected payments of \$20,000 each year starting one year from now. The IRS loss reserve discount factors for this line of business and accident year are shown in the table below for the current valuation date and each of the five subsequent valuation dates. The investment yield is 8.5% per annum, and the federal income tax rate is 35%. (Connor and Olsen use a 34% tax rate, since their paper appeared in the late 1980's, when the rate was 34%, not 35%.) The figures are taken from Table 1 on page 86 of the Connor and Olsen paper.

Calendar Year	*Paid Loss*	*Year End Reserve*	*IRS Discount Factor*
1990	\$0	\$100,000	0.79812
1991	\$20,000	\$80,000	0.77935
1992	\$20,000	\$60,000	0.75561
1993	\$20,000	\$40,000	0.73577
1994	\$20,000	\$20,000	0.70271
1995	\$20,000	\$0	0.68950

PRICING ASSUMPTIONS

We add two items to price the commutation: the cost of equity capital and the capital requirements. We assume that equityholders demand a return 400 basis points above the investment yield, or 12.5% for this illustration.[35] For capital requirements, we choose low figures, to avoid any perception that the results are dependent on over-stated assumptions.[36] Specifically, we assume that the company holds 10% of written premium plus 15% of held reserves as its surplus.[37]

The commutation price depends somewhat on the effective date of the transaction. For simplicity, we assume that taxpayers remit the taxes at end of the year, so an effective date later in the year slightly raises the commutation price.[38] We use a December 31 date for the commutation, to match the Connor and Olsen illustration. Exhibits using a January 1 date for the commutation are included in the appendix.

The commutation price is the price which provides a return to equityholders commensurate with the cost of equity capital. If the primary company and the reinsurer are both subject to the same insurance regulation, and if both companies have the same cost of equity capital, then the savings gained by the reinsurance company are transferred to the primary company. The magnitude of the investment yield, the cost of equity capital, and the capital requirements do not affect this relationship.

Illustration: The primary company has a 12.5% cost of equity capital, and the pre-tax investment yield is 8.5% per annum. By accepting the commuted claims, the primary company must allocate supporting capital, and it needs additional profits to defray the cost of this capital. Conversely, the reinsurance company frees up an equal amount of capital, and it needs correspondingly less profit to cover its cost of capital.[39]

The commutation price is most easily determined by an iterative procedure.[40] We do not show the derivation of the commutation price by algebraic methods. Rather, we show that the indicated commutation price provides a 12.5% return on capital. Since a lower price would provide a lower return on capital and a higher price would provide a higher return on capital, the solution is unique.

The indicated commutation price is $87,962; in contrast, Connor and Olsen determine a commutation price of $79,437. The change in the tax rate from 34% to 35% has a minor effect on the commutation price. The major cause of the higher commutation price is the additional capital requirements stemming from holding additional reserves and the gap between the cost of equity capital and the company's investment yield.

The exhibits below show the sensitive of the commutation price to the pricing assumptions. The alternative illustration for the Connor and Olsen example uses capital requirements equal

to 25% of premium and 20% of reserves. This gives about a one to one premium to surplus ratio.[41]

SURPLUS REQUIREMENTS

Surplus requirements have a material effect on the commutation price. Table 1 shows the commutation price for the illustration in the Connor and Olsen paper under varying surplus requirements (expressed as leverage ratios), ranging from 0% of held reserves and of written premium to 25% of held reserves and of written premium.

The reserving risk charge has a greater effect than the written premium risk charge, since the held reserves stay on the company's books for several years, whereas the written premium risk charge is in effect for a single year. The commutation price for the surplus requirements in the illustration above (10% written premium risk charge and 15% reserving risk charge) is shown in the boxed cell.

- As the written premium risk charge ranges from 0% to 25%, the commutation price ranges from $87,123 to $89,251, for a difference of $2,128.
- As the reserving risk charge ranges from 0% to 25%, the commutation price ranges from $84,219 to $90,457, for a difference of $6,238.

Table 1: Sensitivity Analysis – Surplus Requirements

Reserving Risk	*Written Premium Risk*					
	0%	5%	10%	15%	20%	25%
0%	83,416	83,816	84,219	84,627	85,038	85,454
5%	84,652	85,057	85,467	85,880	86,298	86,720
10%	85,887	86,299	86,715	87,134	87,558	87,986
15%	87,123	87,541	**87,962**	88,388	88,817	89,251
20%	88,359	88,782	89,210	89,641	90,077	90,517
25%	89,594	90,042	90,457	90,894	91,337	91,783

Table 2 shows the figures as percentages of the commutation price in the boxed cell. For example, the $89,641 commutation price with a 15% written premium risk charge and a 20% reserving risk charge is 1.91% higher than the $87,962 commutation price with a 10% written premium risk charge and a 15% reserving risk charge.

Table 2: Sensitivity Analysis – Surplus Requirements (Percentages)

Reserving Risk	Written Premium Risk					
	0%	5%	10%	15%	20%	25%
0%	-5.17%	-4.71%	-4.26%	-3.79%	-3.32%	-2.85%
5%	-3.76%	-3.30%	-2.84%	-2.37%	-1.89%	-1.41%
10%	-2.36%	-1.89%	-1.42%	-0.94%	-0.46%	0.03%
15%	-0.95%	-0.48%	**0.00%**	0.48%	0.97%	1.47%
20%	0.45%	0.93%	1.42%	1.91%	2.40%	2.90%
25%	1.86%	2.34%	2.84%	3.33%	3.84%	4.34%

TARGET RETURN ON CAPITAL AND BENCHMARK INVESTMENT YIELD

Table 3 below shows the sensitivity of the commutation price to the benchmark investment yield and the target return on capital; Table 4 shows the corresponding percentage changes. As the benchmark investment yield increases, the present value of the reserves decreases. Since the reserves in this illustration have a duration of about three years, a 100 basis point rise in the benchmark investment yield leads to about a 3% decline in the commutation price. The 3% change can be seen between adjacent columns along any row of the exhibit.

As the investment yield increases, the target return on capital increases as well. The higher target return on capital consumes about 45% of the reduction in the commutation price.

Illustration: A increase in the benchmark investment yield from 8.5% to 11.5% leads to a 9.20% reduction in the commutation price. If the 300 basis point increase in the investment yield is coupled with a 300 basis point increase in the target return on capital, the net reduction in the commutation price is only 5.14%, which is about 55% of 9.20%.

Table 3: Sensitivity Analysis – Yields and Returns

Return on Capital	Investment Yield					
	6.5%	7.5%	8.5%	9.5%	10.5%	11.5%
9.5%	90,177	87,319	84,467	81,619	78,777	75,940
11.0%	91,819	89,035	86,257	83,484	80,716	77,953
12.5%	93,381	90,669	**87,962**	85,260	82,562	79,870
14.0%	94,780	92,226	89,587	86,952	84,322	81,697
15.5%	96,291	93,711	89,137	88,566	86,000	83,439
17.0%	97,647	95,129	92,616	90,108	87,603	85,103

Table 4: Sensitivity Analysis – Yields and Returns (Percentages)

Return on Capital	*Investment Yield* 6.5%	7.5%	8.5%	9.5%	10.5%	11.5%
9.5%	2.52%	-0.73%	-3.97%	-7.21%	-10.44%	-13.67%
11.0%	4.38%	1.22%	-1.94%	-5.09%	-8.24%	-11.38%
12.5%	6.16%	3.08%	**0.00%**	-3.07%	-6.14%	-9.20%
14.0%	7.85%	4.85%	1.85%	-1.15%	-4.14%	-7.12%
15.5%	9.47%	6.52%	3.61%	0.69%	-2.23%	-5.14%
17.0%	11.01%	8.15%	5.29%	2.44%	-0.41%	-3.25%

STEENECK'S ILLUSTRATION

Lee Steeneck's CAS Exam 6 study note is the source from which most new casualty actuaries learn how to price commutations. Steeneck uses the Connor and Olsen method to price the commutation illustration in his study note. We reprice his illustration, using the method described in this discussion.

Illustration: A block of $1,000,000 in nominal reserves has a three year liquidation pattern, with the payment pattern shown below. The IRS loss reserve discount factors for this accident year at the current and two subsequent valuation dates are shown with the expected loss payments. The tax rate is 35%. The risk-free interest rate is 5% per annum.

Calendar Year	*Paid Loss*	*Year End Reserve*	*IRS Discount Factor*
1998	$0	$1,000,000	0.730
1999	$500,000	$500,000	0.723
2000	$300,000	$200,000	0.741
2001	$200,000	$0	——

To price the commutation, we must add surplus assumptions and the cost of equity capital. We use the same surplus assumptions as for the Connor and Olsen illustration: 10% of written premium plus 15% of held reserves. For the cost of equity capital, we choose 12% per annum. This gives a 700 basis point spread between the risk-free interest rate and the cost of equity capital, which is about right for the insurance industry.

The discounted reserves are

$$\$500{,}000 / 1.05 + \$300{,}000 / 1.05^2 + \$300{,}000 / 1.05^3 = \$921{,}066.84.$$

Steeneck arrives at a commutation price of $921,770. This is expected. If no account is taken of capital requirements or the cost of holding capital, the commutation price is about equal to the present value of the reserves, using a pre-tax discount rate.

For the revised pricing, we assume capital requirements equal to 10% of written premium and 15% of held reserves, along with a 12% target return on capital. The revised commutation price is $983,671, as shown in the exhibit at the end of this paper.[42]

Multi-Year Illustration

Connor and Olsen show a multi-year illustration as their final example. We re-price the same multi-year illustration using the methods in this discussion. The illustration assumes an 8% pre-tax investment yield. We use a 35% tax rate, a 12.5% cost of equity capital, and surplus requirements equal to 10% of written premium and 15% of held reserves.

The indicated commutation price of $20,717,770 is 16.70% higher than the price derived by Connor and Olsen of $17,753,000; see the pricing worksheets appended to this paper.

THE COMMUTATIONS MARKETPLACE

The commutations transacted in the reinsurance marketplace do not always take account of the cost of holding capital. For primary insurance contracts, differences between the indicated premium and the premium charged stem from market pressures or unusual attributes of the insured. In contrast, commutations are generally priced by actuaries or with actuarial advice. One might expect the transaction prices to closely reflect the indicated prices.

We categorize these reasons for these discrepancies into five groups: (i) financial knowledge, (ii) cost allocation, (iii) reinsurance advice, (iv) foreign reinsurers, and (v) accounting practice.

FINANCIAL KNOWLEDGE

The tax implications of claim commutations and the effects on capital requirements are not simple. Many claims department personnel handling commutation negotiations and actuaries aiding them are unaware of the multiple taxation effects and the other costs of holding capital.

Some actuaries estimate commutation prices from the discounted values of the future loss payments. Pricing "rules of thumb" – such as mark-ups over the cost of goods sold – are used in all industries; insurance is no exception.

One might wonder: "Wouldn't subsequent results show the gain or loss from the commutation? Wouldn't the claims personnel and actuaries learn from the subsequent profit measurement?"

Tax effects are rarely allocated to the particular operations that caused them. This is surely true for the double taxation effects on investment income from capital requirements, whether this capital is embedded in statutory reserves or it remains in policyholders' surplus.

To this day, many actuaries fail to incorporate double taxation costs and the costs of holding capital in their pricing analyses. These topics are not covered on the casualty actuarial syllabus, and many practicing actuaries find this subject perplexing.[43]

Business personnel learn the subjects by which they are measured, and they may avoid spending time on matters that do not affect their performance review. A claims examiner not measured by the return on capital could hardly be expected to have any interest in this subject.

COST ALLOCATION

Cost allocation issues are not simple. The double taxation on equityholders' capital is a real cash flow; it is money paid by the company to the U.S. Treasury. But this money is paid regardless of how the capital is used. Taxes are paid whether the capital is embedded in statutory reserves, forms part of risk-based capital requirements, or sits idly as excess capital.

Some analysts might say that the cost of holding capital is a sunk cost, not a marginal cost, and it should not be considered in pricing. Others say that the cost of holding capital is indeed a marginal cost, since if the capital were not tied up in the commutation transaction, it could be returned to shareholders, thereby avoiding the double taxation (and other costs of holding capital). In economic parlance, this is an opportunity cost, since if the capital were not tied up in the commutation transaction, it could be used to support other endeavors; see Feldblum and Thandi [2003A]. In practice, not all actuaries fully consider the opportunity costs of capital.

REINSURANCE ADVICE

It is sometimes heard in reinsurance circles that the commutation price may be lower than the discounted value of the future claim payments. This is the impression one gets from a superficial reading of the Conner and Olsen paper. In truth, Conner and Olsen say that the commutation price is less than the discounted value at the *after-tax* discount rate. It is approximately equal to the discounted value at the *pre-tax* discount rate, unless

- the IRS loss discount rate (the 60 month moving average of federal mid-term rates) differs material from the actuarial loss discount rate or
- the IRS loss payment pattern differs materially from the actuarial loss payment pattern.

Nonetheless, the impression from the paper is that the actuarial price may be well below the fully discounted value. Many practicing actuaries and claims personnel do not understand the reasoning of the authors. They sense that the result is not correct, but they can not specify exactly what is wrong.

We do not imply that reinsurance actuaries consciously distort their analyses to lower the indicated premiums their companies pay. Rather, all persons' judgments are affected by self-interest or the interests of companies and clients. A reinsurance actuary has no incentive to consider the effects of capital requirements and double taxation. It is human nature to omit these items from research papers and educational notes.[44]

FOREIGN REINSURERS

This discussion implicitly assumes that the reinsurer is subject to the same accounting, tax, and regulatory constraints as the ceding company, as is true for U.S. domiciled reinsurers. Alien reinsurers, particularly those domiciled in the Bermudas or the Cayman Islands, may face more lenient accounting, tax, and surplus requirements.[45]

If the reinsurer is not subject to U.S. accounting and tax constraints, the costs of holding capital for the primary company may be greater than the costs to the reinsurer. The disparity may be so great that it deters the commutation.

The Conner and Olsen method implicitly takes the viewpoint of a lightly regulated reinsurer. In contrast, this discussion is written from the viewpoint of a U.S. regulated primary company. The Conner and Olsen ambivalence point is lower for a lightly regulated reinsurer than for a U.S. domiciled primary insurer.

ACCOUNTING PRACTICE

This implicitly discussion assumes that companies hold full-value loss reserves, with the exception of tabular discounts on workers' compensation pension cases. Some insurers include the medical portions of these claims in the tabular discounts or implicitly discount the medical portions. Other long-term claims, such as medical malpractice claims, products liability claims, and general liability claims, may also be implicitly discounted.

We distinguish three types of discounting and examine their effects on capital requirements and federal income taxes.

	Capital Requirements	*Double Discounting*
Explicit Discounting – Non-tabular	no change	no effect
Explicit Discounting – Tabular	decreases	no effect
Implicit Discounting	decreases	increases present value of taxes

- Explicit non-tabular discounting does not affect NAIC risk-based capital requirements or the IRS loss reserve discounting procedure.
  - Explicit non-tabular discounts are removed from surplus and added to reserves when computing RBC requirements and adjusted surplus.
  - Explicit non-tabular discounts are added to reserves before computing the IRS discounted reserves.

- Explicit tabular discounting decreases the NAIC risk-based capital requirements, and it does not affect IRS loss reserve discounting.
  - Explicit tabular discounts are not removed from surplus and they are not added to reserves when computing RBC requirements and adjusted surplus.
  - Explicit non-tabular discounts are added to reserves before computing the IRS discounted reserves.

- Implicit discounting decreases the NAIC risk-based capital requirements, and it decreases the tax basis reserves, thereby raising the present value of the tax liability. Since the discount is not disclosed in the Annual Statement, it is not removed from surplus or added to reserves.

DISCOUNTING AND DOUBLE TAXATION

In the simplified one year illustration with which this paper begins, the commutation is effected on January 1, and the claim is settled by December 31. If the primary company reports discounted reserves on its statutory financial statements (with no disclosure), it holds assets equal to the discounted reserves. If the discounting is at the pre-tax investment yield, and if no additional capital is needed (that is, if the company needs no surplus), the shareholders need not contribute any capital when the commutation is effected. In this case, the commutation price is the discounted value of the future loss payments.

If the commutation involves cash flows in more than one tax year and the discounting is implicit, tax payments are speeded up, and the primary company suffers a tax loss from the commutation.

DISCOUNTING AND CAPITAL REQUIREMENTS

Explicit non-tabular discounts are removed from surplus and added to reserves before calculating the risk-based capital reserving risk charge. Neither the capital requirements nor the risk-based capital ratio are affected by explicit non-tabular discounts. Implicit discounts reduce the RBC capital requirements. Rating agencies which discern the discounting may lower the company's rating, so implicit discounting is not a panacea for capital constraints.

Explicit tabular discounts have the best of both worlds. They reduce capital requirements by more than the amount of the discount, are they are added to reserves for IRS loss reserve

discounting. Explicit discounts are carefully prescribed by the NAIC accounting rules: They may be used only on the indemnity portion of workers' compensation permanent total disability and fatality cases and on long term disability health claims. They may not be applied to medical benefits or loss adjustment expenses; see Feldblum [2002: SchP].

Conclusion: Insurance Pricing, Capital Requirements, and Federal Income Taxes

In the past, casualty actuaries priced commutations and other products without considering capital requirements and federal income taxes. Common rationales were the following.

- Before the Tax Reform Act of 1986, most property-casualty insurance companies paid little federal income tax. When the volume of business was growing, whether because of increases in exposures or inflation, the underwriting losses occasioned by increasing amounts of pre-paid acquisition costs and full value loss reserves offset most of the taxable investment income. Three provisions of the 1986 Tax Reform Act substantially raised the tax liabilities for property-casualty insurance companies:

  - The revenue offset provision defers the tax deduction for pre-paid acquisition costs and spreads it over the policy term.
  - The loss reserve discounting provision allows an offset to taxable income only for the change in discounted reserves, not for the change in full value loss reserves.
  - The proration provision reduces the benefit of tax exempt investment income for insurance companies, effectively eliminating this investment vehicle for them.

  Before 1986, property-casualty insurance companies were in an enviable tax position. After 1986, they are in a "tax-plus" position, with an effective tax rate above that of most other industries.[46] Taxes are now a critical part of accurate pricing.

- Taxes are complex, and casualty actuaries lack the expertise to deal with them. This statement may be true, but the lesson is inverted. Since taxes are complex, actuaries must be sure to properly account for them in their pricing procedures. Just as ignorance of the law is no excuse for trespass, complexity of the law is no excuse for disregard.

- Before the advent of risk-based capital requirements in 1994, it was difficult to measure capital requirements for property-casualty insurance products. Actuaries developed theoretical models showing the amount of capital that *ought* to be held by an insurance company. These models may have been wonderful research, but they did not address the issue of the capital *required* to be held by insurance companies. After 1994, with the NAIC risk-based capital requirements and the similar rating agency capital requirements, the analysis of required capital can be included in actuarial ratemaking.[47]

This discussion deals with commutations, which are the reverse of retroactive reinsurance. The pricing of commutations is equally applicable to reinsurance pricing, whether prospective or retroactive and whether finite or standard.

[1] Vincent P. Connor and Richard A. Olsen, "Commutation Pricing in the Post Tax-Reform Era," *Proceedings of the Casualty Actuarial Society*, Volume 78 (1991), pages 81-109; Lee Steeneck, "Commutation of Claims," CAS Exam 6 study note, 1998.

[2] The full quotation is as follows: "In certain instances, the commutation price developed under this methodology can be negative. This can occur when there is a great mismatch between the payment profit / interest rate used to develop tax-basis discounted reserves and the payment profit / interest rate used to calculate the present value of the losses. Specifically, the tax-basis discounted reserves are substantially higher than the present value of the losses. This leads to the tax on the underwriting gain/loss becoming greater than the cost of not commuting. In cases of reinsurance of long-tailed lines, such as workers' compensation, where the overall industry average reinsurance payment profile is quite short relative to the actual payment profile, negative commutation values can be expected frequently. In these situations, commutations are not favored."

[3] The Standard of Practice adds: "The actuary may consider adjusting this rate if the amount of discount for tax purposes differs significantly from the amount of discount determined in accordance with this standard."

[4] Conner and Olsen use the term "nominal interest rate" to mean the pre-tax interest rate. See also page 94: "If the IRS payment profiles and interest rates equal the factors used to determine the present value of the losses, then the commutation price will equal the present value of the losses using the nominal interest rate."

[5] See also Atkinson and Dallas [2000], who have the same perspective.

[6] One might wonder: "There is $5,250 of investment income in this illustration. Were there no commutation, the IRS would receive 35% of the investment income, or 35% x $5,250 = $1,837.50. If the commutation is effected, the IRS receives only $131.25. Where did the rest of the tax liability go?" Answer: The reinsurer's cash payment is about $5,000 less than its reserve takedown, giving it an underwriting gain for this amount. It pays federal income taxes on this underwriting gain; the IRS has not lost any money.

The IRS loss reserve discounting procedure modifies the underwriting gain or loss of the reinsurer and the primary company by offsetting amounts. The total tax liability is not changed, unless one of the parties is not subject to U.S. federal income taxation.

[7] There are some differences, such as the loss reserve discount factors used by ceding and assuming companies for the same transaction; these are generally not material.

[8] Some analysts argue that a risk-free rate is appropriate if there is no systematic risk to the underwriting operations. More precisely, they argue that shareholders ought to be satisfied with a risk-free rate of return if the insurer invests only in Treasury securities and if the market value effects of the loss cash flows are not correlated with the cash flows of the overall securities markets.

This perspective, commonly associated with the pricing models of Fairley, Kahane, Hill, Myers, and Cohn, has been used in Massachusetts private passenger automobile and workers' compensation ratemaking. It has not been used in competitive insurance markets, nor has it found acceptance in the actuarial community. For

discussion of this "underwriting beta" perspective, see Kozik [1994].

[9] The exact magnitude of the market risk premium is unclear; figures between 7% and 9% are commonly used. Some analysts claim that the market risk premium has narrowed in recent years, resulting in the high P/E multiples common in the late 1990's. Other analysts believe that the high P/E multiples stem from the heady bull markets of the 1990's and will gradually subside to their longer term averages. The stock market decline in 2000-2002 supports the latter view.

[10] Cf. Myers and Cohn [1987], who first drew attention to this topic.

[11] See Atkinson and Dallas [2000], chapters 8 and 11, for a life insurance pricing example.

[12] It may seem surprising that cost of holding capital is greater than the required return on capital. Half of this stems from the market risk premium of 7% per annum. The other half stems from the two layers of additional taxes: one on the underwriting income and one on the investment income. For each dollar of policy premium in excess of discounted losses and expenses in the casualty lines of business, the IRS takes between 60¢ and 80¢.

[13] The loss concentration factor equals 70% + 30% x (reserves in largest line)/(total reserves). If the total reserves are split evenly among five lines of business, the loss concentration factor is 70% + 30% x 1/5 = 76%, giving a 24% reduction in the reserving risk charge.

[14] In practice, companies do not always book full value loss reserves for long duration claims, which form the bulk of many commutations. Company booking practices vary with the stage of the underwriting cycle and the reserving philosophy of the company. The average implicit discount ranges from zero percent for the most conservative companies to about 20% for less conservative companies.

Implicit reserve discounts reduce capital requirements. For exact pricing, one should use the marginal effect of the reserving risk charge along with the implicit discount in the company's reserves. For the illustrations here, we assume full value loss reserves and we use the "60%" low end of the marginal effects.

[15] The present value of the taxes on underwriting income does depend on the line of business, since the IRS loss reserve discount factors differ by line.

[16] For the illustrations here, the relative sizes of the margins is $781.25 / $298.25 = 2.619 if the equityholders demand a risk-free rate and $3,551.14 / $1,355.68 = 2.619 if the equityholders demand an equity return. The relative sizes of the margin does *not* depend on the target return on capital.

[17] Overall, the IRS loss reserve discount factors are not materially biased over the long run (though they be biased in the short run or for a specific block of business). Since we are using a simplified illustration, we use corresponding discount factors.

[18] See, for instance, Conner and Olsen (page 86): "Thus, the present value of the tax benefit on the unwinding of the discount is calculated to be . . . ." and *passim* through much of the paper.

[19] After all the mathematics, it pays to examine the actual rate of return by writing out the cash flows from the commutation. If the actual rate of return is lower than expected, the pricing calculations may not be correct.

[20] In most scenarios, we add the direct charge or credit to surplus to the reported statutory income when making the comparison with taxable income. If future tax rate changes are anticipated, we use the expected tax rate when the timing difference are expected to reverse.

[21] If the reversal of the deferred tax asset or liability is uncertain, GAAP financial statements may use a "valuation allowance" to eliminate or reduce the deferred tax asset or liability. If the reversal is expected to occur several years in the future, some GAAP accountants use the present value of the deferred tax asset or liability (though this is not common GAAP practice). See the Financial Accounting Standards Board, Discussion Memorandum, an analysis of issues related to *Present Value-Based Measurements in Accounting* (December 1990) and White, Gerald I., Ashwinpaul C. Sondhi, and Dov Fried, *The Analysis and Use of Financial Statements*, 2nd edition (Wiley 1998).

[22] The financial community uses the term "free cash flow" instead of implied equity flow. Atkinson and Dallas [2000], chapter 11, use the term "distributable earnings" instead of implied equity flows.

The Actuarial Standards Board, "Actuarial Standard of Practice No. 19: Actuarial Appraisals" (October 1991), page 4, has the same view of distributable earnings: "5.2.1 Distributable Earnings – For insurance companies, statutory earnings form the basis for determining distributable earnings, since the availability of dividends to owners is constrained by the amount of accumulated earnings and minimum capital and surplus requirements, both of which must be determined on a statutory accounting basis. Distributable earnings consist of statutory earnings, adjusted as appropriate to allow for the retention of a portion thereof or the release of a portion of prior accumulated earnings therein, in recognition of minimum capital and surplus levels necessary to support existing business. . . . Economic value generally is determined as the present value of future cash flows. Statutory accounting determines the earnings available to the owner. Hence, while future earnings calculated according to generally accepted accounting principles (GAAP) will often be of interest to the user of an actuarial appraisal, as may other patterns of earnings, the discounted present-value calculations contemplated within the definition of actuarial appraisal in this standard should be developed in consideration of statutory earnings, rather than some other basis. . . . The actuary's report should include a discussion of factors, such as capital needs (whether perceived by the actuary or imposed by an external entity such as a regulator), that may cause the earnings available for shareholder or policyholder distribution to be different from statutory earnings."

[23] For a summary of U.S. tax law pertaining to property-casualty insurance companies, along with the post-codification statutory accounting rules pertaining to federal income taxes, see Appendix A of Feldblum and Thandi [2002], "Modeling the Equity Flows."

[24] For the multi-period illustrations below, we use actual IRS loss reserve discount factors.

[25] The booked income, or the book income, is the income shown on the company's accounting books. Booked income may be either statutory income or GAAP income. For computing deferred tax assets and liabilities, the booked income is not the income in the earnings statement (or income statement). Rather, it is the income implied by the balance sheet entries at the beginning of the year and the end of the year. In other words, the booked income used for computing deferred tax assets and liabilities is the income on the earnings statement plus direct credits to surplus minus direct charges to surplus. Credits and charges to surplus stemming from changes in the deferred tax assets and liabilities are not included in this computation.

[26] IRS loss reserve discount factors are computed to six decimal places. We show eight significant digits in this illustration to avoid rounding errors.

[27] This is one of the few occasions when the implied equity flows depend not just on the book of business being priced but on all operations of the company.

[28] The IRS promulgation of the industry factors is generally delayed until the summer months, if not later, though the formula for the factors is specified in the Internal Revenue Code.

[29] By gross deferred tax assets, we mean gross of statutory admissibility tests. The gross deferred tax assets are shown on GAAP financial statements.

[30] As noted earlier, using the actual probability of death does not change the result.

[31] The probabilities of death in each year do affect the tabular discount on the reserve. However, once the tabular discount has been determined, the probabilities of death do not affect the admitted portion of the deferred tax asset on the statutory balance sheet.

[32] The tabular discounts are disclosed in the notes to the financial statements; only non-tabular discounts are disclosed in Schedule P, Part 1.

[33] A more exact calculation would use actuarial present values, which include the mortality pattern.

[34] For simplicity, we estimate the discounted reserve using an annuity certain with a term equal to the claimant's life expectancy; the slight inaccuracy is not material.

[35] We assume a cost of equity capital for property-casualty insurers about 700 basis points above the Treasury bill rate. The average investment yield for property-casualty insurers is about 300 basis points above the Treasury bill rate. The 400 basis point spread is the difference between these two figures.

The commutation price depends on whether one uses the company's investment yield for discounting or the risk-free interest rate for discounting. Connor and Olsen uses the investment yield for discounting, so one must use the full market risk premium (times the equity beta for property-casualty insurance companies) to determine the equity return for investors.

[36] The exhibits at the end of this paper show the results for higher surplus assumptions as well.

[37] The premium to surplus ratio here is about 2 to 1, which accords with the (revised) Kenney rule of thumb. A lower premium to surplus ratio increases the commutation price.

[38] The assumption that taxes are paid at the end of the year is not actually correct, since corporate taxpayers pre-pay their taxes over the course of the year.

[39] The commutation actually causes a slight release of capital equal to the credit charge for reinsurance recoverables.

[40] This might have been a hindrance years ago; now it is no longer a concern. Most computer spreadsheets use iterative methods for all computations of the sort done here.

[41] Since the loss reserves are shown at undiscounted values on the statutory balance sheet, whereas much of the capital requirements in this illustration are needed in future years, we use the ratio of premium to discounted capital.

The overall industry premium to surplus ratio has been about one to one during the 1990's and the early years of the twenty-first century. However, much of the equity supports pricing risk, not reserving risk. A commutation does not have pricing risk, and a higher premium to surplus ratio is appropriate.

[42] Steeneck uses a risk-free rate instead of the company's investment yield. This does not materially change the capital requirements, but it does change the target return on capital expected by investors.

[43] This paper was stimulated by a fellow actuary's request to examine a proposed commutation price. The actuary knew the price was unreasonable, but he was stymied by the tax rationale provided by the reinsurer.

[44] The early 20th century social theorist Max Weber highlighted the difficulties inherent in objective analysis of social behavior; see especially Weber [1975: The Interpretation of Social Reality].

[45] Conner and Olsen mention that even if both the primary company and the reinsurer are domiciled in the U.S., differences in line of business coding for non-proportional reinsurance affects the IRS loss reserve discount factors and therefore the ambivalence point. The differences in the tax liabilities and capital requirements for U.S. vs off-shore reinsurers are even greater.

[46] Other industries pay no tax on tax-exempt investment income, and they enjoy various tax benefits for accelerated depreciation, investment tax credits, and similar items. The property-casualty insurance industry has a 5.25% effective tax rate on tax exempt investment income, and it has no tax benefits applicable to its operations.

[47] Cf Daykin, Pentikäinen, and Pesonen [1994] and Philbrick [2001].

Feldblum Example
Commuted on January 1 of Year 1

Price 105,959 0

20% of WP
25% of Reserves

Inv Yield 5.0%

Tax 35%

1

105,000

Time	WP	Paid Loss	Nominal Reserve	Surplus	Held Asset	Non-income producing	Income producing	Inv Income	IRS Disc factors	Taxable UW income
0	105,959	0	105,000	47,442	152,442	0	152,442	0	1.000	
1	0	105,000	0	0	0	0	0	7,622	1.000	959

ROE 12.0%

NPV 0
IRR 12.00%

COMPANY FLOWS

Time	Tax paid UW	Taxable Inv Income	Tax paid Inv income	Total tax paid	DTA Disc Reserve	UW	Inv Inc	Asset	Tax	DTA	Equity Flow
0		0	0	0		105,959	0	152,442	0	0	-46,483
1	336	7,622	2,668	3,003	0	-105,000	7,622	-152,442	-3,003	0	52,060

Feldblum Example
Commuted on December 31 of Year 0

Price 106,096 0

20% of WP
25% of Reserves

Inv Yield 5.0%

Tax 35%

1

105,000

Time	WP	Paid Loss	Nominal Reserve	Surplus	Held Asset	Non-income producing	Income producing	Inv Income	IRS Disc factors	Taxable UW income
0	106,096	0	105,000	47,469	152,469	1,750	150,719	0	0.952	6,096
1	0	105,000	0	0	0	0	0	7,536	1.000	-5,000

ROE 12.0%

NPV 0
IRR 12.00%

COMPANY FLOWS

Time	Tax paid UW	Taxable Inv income	Tax paid Inv income	Total tax paid	DTA Disc Reserve	UW	Inv Inc	Asset	Tax	DTA	Equity Flow
0	2,134	0	0	2,134	1,750	106,096	0	152,469	-2,134	1,750	-46,757
1	-1,750	7,536	2,638	888	0	-105,000	7,536	-152,469	-888	-1,750	52,368

Connor&Olsen Table 2 Example
Commuted on January 1 of Year 1

Price 89,978 0

79,854

20% of WP
25% of Reserves

1

Inv Yield 8.5%

Tax 35%

100,000

Time	WP	Paid Loss	Nominal Reserve	Surplus	Held Asset	Non-income producing	Income producing	Inv Income	IRS Disc factors	Taxable UW income
0	89,978	0	100,000	42,996	142,996	0	142,996	0	0.788	
1	0	20,000	80,000	20,000	100,000	1,949	98,051	12,155	0.819	4,466
2	0	20,000	60,000	15,000	75,000	1,520	73,480	8,334	0.851	-5,569
3	0	20,000	40,000	10,000	50,000	1,054	48,946	6,246	0.886	-4,342
4	0	20,000	20,000	5,000	25,000	548	24,452	4,160	0.922	-3,011
5	0	20,000	0	0	0	0	0	2,078	0.922	-1,567

ROE 12.5%

NPV 0
IRR 12.50%

COMPANY FLOWS

Time	Tax paid UW	Taxable Inv income	Tax paid Inv income	Total tax paid	DTA Disc Reserve	UW	Inv Inc	Asset	Tax	DTA	Equity Flow
0		0	0	0		89,978	0	142,996	0	0	-53,017
1	1,563	12,155	4,254	5,817	1,949	-20,000	12,155	-42,996	-5,817	1,949	31,282
2	-1,949	8,334	2,917	968	1,520	-20,000	8,334	-25,000	-968	-429	11,937
3	-1,520	6,246	2,186	666	1,054	-20,000	6,246	-25,000	-666	-466	10,114
4	-1,054	4,160	1,456	402	548	-20,000	4,160	-25,000	-402	-505	8,253
5	-548	2,078	727	179	0	-20,000	2,078	-25,000	-179	-548	6,351

Connor&Olsen Table 2 Example
Commuted on December 31 of Year 0

Price	91,846	0			
	79,854		20% of WP 25% of Reserves	Inv Yield 8.0%	Tax 35%
		100,000	1		

Time	WP	Paid Loss	Nominal Reserve	Surplus	Held Asset	Non-Income producing	Income producing	Inv Income	IRS Disc factors	Taxable UW income
0	91,846	0	100,000	43,369	143,369	2,236	141,133	0	0.799	11,992
1	0	20,000	80,000	20,000	100,000	1,855	98,145	11,291	0.828	-6,388
2	0	20,000	60,000	15,000	75,000	1,443	73,557	7,852	0.859	-5,299
3	0	20,000	40,000	10,000	50,000	999	49,001	5,885	0.892	-4,123
4	0	20,000	20,000	5,000	25,000	519	24,481	3,920	0.926	-2,853
5	0	20,000	0	0	0	0	0	1,959	0.926	-1,481

ROE	12.5%
NPV	0
IRR	12.50%

COMPANY FLOWS

Time	Tax paid UW	Taxable Inv income	Tax paid Inv income	Total tax paid	DTA Disc Reserve	UW	Inv Inc	Asset	Tax	DTA	Equity Flow
0	4,197	0	0	4,197	2,236	91,846	0	143,369	-4,197	2,236	-53,485
1	-2,236	11,291	3,952	1,716	1,855	-20,000	11,291	-43,369	-1,716	-381	32,563
2	-1,855	7,852	2,748	893	1,443	-20,000	7,852	-25,000	-893	-412	11,547
3	-1,443	5,885	2,060	616	999	-20,000	5,885	-25,000	-616	-445	9,824
4	-999	3,920	1,372	373	519	-20,000	3,920	-25,000	-373	-480	8,067
5	-519	1,959	685	167	0	-20,000	1,959	-25,000	-167	-519	6,273

Steeneck Example
Commuted on January 1 of Year 1

Price 974,956

921,770

10% of WP
15% of Reserves

Inv Yield 5.0%

Tax 35%

1,000,000

Time	WP	Paid Loss	Nominal Reserve	Surplus	Held Asset	Non-income producing	Income producing	Inv Income	IRS Disc factors	Taxable UW income
0	974,956	0	1,000,000	247,496	1,247,496	0	1,247,496	0	0.730	
1	0	500,000	500,000	75,000	575,000	30,393	544,607	62,375	0.723	113,690
2	0	300,000	200,000	30,000	230,000	18,164	211,836	27,230	0.741	-86,837
3	0	200,000	0	0	0	0	0	10,592	1.000	-51,897

ROE 12.0%

NPV 0
IRR 12.00%

COMPANY FLOWS

Time	Tax paid UW	Taxable Inv income	Tax paid Inv income	Total tax paid	DTA Disc Reserve	UW	Inv Inc	Asset	Tax	DTA	Equity Flow
0		0	0	0		974,956	0	1,247,496	0	0	-272,540
1	39,791	62,375	21,831	61,623	30,393	-500,000	62,375	-672,496	-61,623	30,393	203,641
2	-30,393	27,230	9,531	-20,862	18,164	-300,000	27,230	-345,000	20,862	-12,229	80,864
3	-18,164	10,592	3,707	-14,457	0	-200,000	10,592	-230,000	14,457	-18,164	36,885

Steeneck Example
Commuted on December 31 of Year 0

Price 983,671

921,770

10% of WP
15% of Reserves

Inv Yield 5.0%

Tax 35%

1,000,000

Time	WP	Paid Loss	Nominal Reserve	Surplus	Held Asset	Non-income producing	Income producing	Inv Income	IRS Disc factors	Taxable UW Income
0	983,671	0	1,000,000	248,367	1,248,367	45,945	1,202,422	0	0.730	253,676
1	0	500,000	500,000	75,000	575,000	30,393	544,607	60,121	0.723	-131,271
2	0	300,000	200,000	30,000	230,000	18,164	211,836	27,230	0.741	-86,837
3	0	200,000	0	0	0	0	0	10,592	1.000	-51,897

ROE 12.0%

NPV 0

IRR 12.00%

COMPANY FLOWS

Time	Tax paid UW	Taxable Inv Income	Tax paid Inv Income	Total tax paid	DTA Disc Reserve	UW	Inv Inc	Asset	Tax	DTA	Equity Flow
0	88,786	0	0	88,786	45,945	983,671	0	1,248,367	-88,786	45,945	-307,538
1	-45,945	60,121	21,042	-24,902	30,393	-500,000	60,121	-673,367	24,902	-15,552	242,839
2	-30,393	27,230	9,531	-20,862	18,164	-300,000	27,230	-345,000	20,862	-12,229	80,864
3	-18,164	10,592	3,707	-14,457	0	-200,000	10,592	-230,000	14,457	-18,164	36,885

Connor&Olsen Exhibit Example
Commuted on June 30 of Year 1

Price
20,717,770 0

17,753,000

10% of WP
15% of Reserves

1

24,501,000

		3,498,000	6,999,000	6,001,000	8,003,000						
Time	WP	Acc Year 85 Paid Losses	Acc Year 86 Paid Losses	Acc Year 87 Paid Losses	Acc Year 88 Paid Losses	Acc Year 85 Nominal Reserve	Acc Year 86 Nominal Reserve	Acc Year 87 Nominal Reserve	Acc Year 88 Nominal Reserve	Surplus	Held Asset
0	20,717,770	-	-	-	-	3,498,000	6,999,000	6,001,000	8,003,000	5,746,927	30,247,927
1	0	330,000	556,000	449,000	736,000	3,168,000	6,443,000	5,552,000	7,267,000	3,364,500	25,794,500
2	0	594,000	1,111,000	816,000	1,011,000	2,574,000	5,332,000	4,736,000	6,256,000	2,834,700	21,732,700
3	0	528,000	1,000,000	816,000	920,000	2,046,000	4,332,000	3,920,000	5,336,000	2,345,100	17,979,100
4	0	396,000	889,000	735,000	920,000	1,650,000	3,443,000	3,185,000	4,416,000	1,904,100	14,598,100
5	0	330,000	667,000	653,000	828,000	1,320,000	2,776,000	2,532,000	3,588,000	1,532,400	11,748,400
6	0	264,000	556,000	490,000	736,000	1,056,000	2,220,000	2,042,000	2,852,000	1,225,500	9,395,500
7	0	198,000	444,000	408,000	552,000	858,000	1,776,000	1,634,000	2,300,000	985,200	7,553,200
8	0	198,000	333,000	327,000	460,000	660,000	1,443,000	1,307,000	1,840,000	787,500	6,037,500
9	0	198,000	333,000	245,000	368,000	462,000	1,110,000	1,062,000	1,472,000	615,900	4,721,900
10	0	132,000	333,000	245,000	276,000	330,000	777,000	817,000	1,196,000	468,000	3,588,000
11	0	132,000	222,000	245,000	276,000	198,000	555,000	572,000	920,000	336,750	2,581,750
12	0	66,000	222,000	163,000	276,000	132,000	333,000	409,000	644,000	227,700	1,745,700
13	0	66,000	111,000	163,000	184,000	66,000	222,000	246,000	460,000	149,100	1,143,100
14	0	66,000	111,000	82,000	184,000	0	111,000	164,000	276,000	82,650	633,650
15	0	-	111,000	82,000	92,000	0	0	82,000	184,000	39,900	305,900
16	0	-	-	82,000	92,000	0	0	0	92,000	13,800	105,800
17	0	-	-	-	92,000	0	0	0	0	0	0

Inv Yield
8.0%

Tax
35%

Time	Non-income producing	Income producing	Inv income	Acc Year 85 IRS Disc factors	Acc Year 86 IRS Disc factors	Acc Year 87 IRS Disc factors	Acc Year 88 IRS Disc factors	Total Disc. Reserve	Taxable UW Income	Tax paid UW	Taxable Inv Income	Tax paid Inv income
0	0	30,247,927	0	0.729	0.759	0.777	0.787	18,818,030		0	0	0
1	200,296	25,594,204	1,209,917	0.717	0.729	0.759	0.764	16,728,634	1,918,136	671,348	1,209,917	423,471
2	233,253	21,499,447	2,047,536	0.714	0.717	0.729	0.745	13,768,908	-572,274	-200,296	2,047,536	716,638
3	296,662	17,682,438	1,719,956	0.716	0.714	0.717	0.713	11,171,346	-666,437	-233,253	1,719,956	601,985
4	285,154	14,312,946	1,414,595	0.747	0.716	0.714	0.704	9,078,951	-847,605	-296,662	1,414,595	495,108
5	291,970	11,456,430	1,145,036	0.780	0.747	0.716	0.697	7,415,675	-814,724	-285,154	1,145,036	400,763
6	218,400	9,177,100	916,514	0.818	0.780	0.747	0.731	6,203,874	-834,199	-291,970	916,514	320,780
7	168,931	7,384,269	734,168	0.860	0.818	0.780	0.766	5,225,874	-624,000	-218,400	734,168	256,959
8	130,396	5,907,104	590,742	0.909	0.860	0.818	0.805	4,390,533	-482,659	-168,931	590,742	206,760
9	89,958	4,631,942	472,568	0.966	0.909	0.860	0.850	3,619,093	-372,560	-130,396	472,568	165,399
10	52,790	3,535,210	370,555	0.966	0.966	0.909	0.902	2,890,117	-257,024	-89,958	370,555	129,694
11	8,941	2,572,809	282,817	0.966	0.966	0.966	0.963	2,165,945	-150,828	-52,790	282,817	98,986
12	6,431	1,739,269	205,825	0.966	0.966	0.966	0.963	1,464,489	-25,544	-8,941	205,825	72,039
13	5,462	1,137,638	139,142	0.966	0.966	0.966	0.963	958,863	-18,373	-6,431	139,142	48,700
14	3,490	630,160	91,011	0.966	0.966	0.966	0.963	531,469	-15,606	-5,462	91,011	31,854
15	2,163	303,737	50,413	0.966	0.966	0.966	0.963	256,441	-9,973	-3,490	50,413	17,644
16	1,182	104,618	24,299	0.966	0.966	0.966	0.963	88,621	-6,180	-2,163	24,299	8,505
17	0	0	8,369	0.966	0.966	0.966	0.963	0	-3,379	-1,182	8,369	2,929

											ROE	12.5%
											NPV	0
							COMPANY FLOWS					
Time	Total tax paid	Acc Year 85 DTA Reserve Disc	Year 86 DTA Reserve Disc	Year 87 DTA Reserve Disc	Year 88 DTA Reserve Disc	Total DTA	UW	Inv Inc	Asset	Tax	DTA	Equity Flow
0	0						20,717,770	0	30,247,927	0	0	-9,530,157
1	1,094,819	55,965	83,805	19,079	41,447	200,296	-2,071,000	1,209,917	-4,453,427	-1,094,819	200,296	2,697,821
2	516,342	54,871	94,219	61,537	22,626	233,253	-3,532,000	2,047,536	-4,061,800	-516,342	32,957	2,093,952
3	368,731	56,837	92,385	69,250	78,190	296,662	-3,264,000	1,719,956	-3,753,600	-368,731	63,409	1,904,233
4	198,447	44,732	95,700	67,862	76,859	285,154	-2,940,000	1,414,595	-3,381,000	-198,447	-11,508	1,645,640
5	115,609	34,129	75,320	70,333	112,189	291,970	-2,478,000	1,145,036	-2,849,700	-115,609	6,816	1,407,943
6	28,810	25,344	57,399	55,328	80,329	218,400	-2,046,000	916,514	-2,352,900	-28,810	-73,570	1,121,034
7	38,559	20,959	42,625	42,260	63,086	168,931	-1,602,000	734,168	-1,842,300	-38,559	-49,470	886,440
8	37,829	15,609	35,250	31,366	48,172	130,396	-1,318,000	590,742	-1,515,700	-37,829	-38,534	712,078
9	35,003	1,578	26,251	25,940	36,189	89,958	-1,144,000	472,568	-1,315,600	-35,003	-40,438	568,728
10	39,736	1,578	2,655	19,320	29,236	52,790	-986,000	370,555	-1,133,900	-39,736	-37,169	441,551
11	46,196	789	2,655	1,949	3,547	8,941	-875,000	282,817	-1,006,250	-46,196	-43,849	324,021
12	63,098	789	1,327	1,949	2,365	6,431	-727,000	205,825	-836,050	-63,098	-2,510	249,267
13	42,269	789	1,327	981	2,365	5,462	-524,000	139,142	-602,600	-42,269	-969	174,504
14	26,392	0	1,327	981	1,182	3,490	-443,000	91,011	-509,450	-26,392	-1,972	129,098
15	14,154	0	0	981	1,182	2,163	-285,000	50,413	-327,750	-14,154	-1,327	77,681
16	6,342	0	0	0	1,182	1,182	-174,000	24,299	-200,100	-6,342	-981	43,077
17	1,747	0	0	0	0	0	-92,000	8,369	-105,800	-1,747	-1,182	19,240

Connor&Olsen Exhibit Example
Commuted on December 31 of Year 0

Price 17,476,775 0

17,753,000

20% of WP
25% of Reserves

0

24,501,000

		3,498,000	6,999,000	6,001,000	8,003,000						
Time	WP	Acc Year 85 Paid Losses	Acc Year 86 Paid Losses	Acc Year 87 Paid Losses	Acc Year 88 Paid Losses	Acc Year 85 Nominal Reserve	Acc Year 86 Nominal Reserve	Acc Year 87 Nominal Reserve	Acc Year 88 Nominal Reserve	Surplus	Held Asset
0	17,476,775	-	-	-	-	3,498,000	6,999,000	6,001,000	8,003,000	0	24,501,000
1	0	330,000	556,000	449,000	736,000	3,168,000	6,443,000	5,552,000	7,267,000	0	22,430,000
2	0	594,000	1,111,000	816,000	1,011,000	2,574,000	5,332,000	4,736,000	6,256,000	0	18,898,000
3	0	528,000	1,000,000	816,000	920,000	2,046,000	4,332,000	3,920,000	5,336,000	0	15,634,000
4	0	396,000	889,000	735,000	920,000	1,650,000	3,443,000	3,185,000	4,416,000	0	12,694,000
5	0	330,000	667,000	653,000	828,000	1,320,000	2,776,000	2,532,000	3,588,000	0	10,216,000
6	0	264,000	556,000	490,000	736,000	1,056,000	2,220,000	2,042,000	2,852,000	0	8,170,000
7	0	198,000	444,000	408,000	552,000	858,000	1,776,000	1,634,000	2,300,000	0	6,568,000
8	0	198,000	333,000	327,000	460,000	660,000	1,443,000	1,307,000	1,840,000	0	5,250,000
9	0	198,000	333,000	245,000	368,000	462,000	1,110,000	1,062,000	1,472,000	0	4,106,000
10	0	132,000	333,000	245,000	276,000	330,000	777,000	817,000	1,196,000	0	3,120,000
11	0	132,000	222,000	245,000	276,000	198,000	555,000	572,000	920,000	0	2,245,000
12	0	66,000	222,000	163,000	276,000	132,000	333,000	409,000	644,000	0	1,518,000
13	0	66,000	111,000	163,000	184,000	66,000	222,000	246,000	460,000	0	994,000
14	0	66,000	111,000	82,000	184,000	0	111,000	164,000	276,000	0	551,000
15	0	-	111,000	82,000	92,000	0	0	82,000	184,000	0	266,000
16	0	-	-	82,000	92,000	0	0	0	92,000	0	92,000
17	0	-	-	-	92,000	0	0	0	0	0	0

Inv Yield 8.0%

Tax 34%

Non-income producing	Income producing	Inv income	Acc Year 85 IRS Disc factors	Acc Year 86 IRS Disc factors	Acc Year 87 IRS Disc factors	Acc Year 88 IRS Disc factors	Total Disc. Reserve	Taxable UW income	Tax paid UW	Taxable Inv income	Tax paid Inv income	Total tax paid
-6,255	24,507,255	0	0.729	0.759	0.777	0.787	18,818,030	-1,341,255	-456,027	0	0	-456,027
194,573	22,235,427	1,960,580	0.717	0.729	0.759	0.764	16,728,634	18,396	6,255	1,960,580	666,597	672,852
226,589	18,671,411	1,778,834	0.714	0.717	0.729	0.745	13,768,908	-572,274	-194,573	1,778,834	604,804	410,230
288,186	15,345,814	1,493,713	0.716	0.714	0.717	0.713	11,171,346	-666,437	-226,589	1,493,713	507,862	281,274
277,006	12,416,994	1,227,665	0.747	0.716	0.714	0.704	9,078,951	-847,605	-288,186	1,227,665	417,406	129,220
283,628	9,932,372	993,359	0.780	0.747	0.716	0.697	7,415,675	-814,724	-277,006	993,359	337,742	60,736
212,160	7,957,840	794,590	0.818	0.780	0.747	0.731	6,203,874	-834,199	-283,628	794,590	270,161	-13,467
164,104	6,403,896	636,627	0.860	0.818	0.780	0.766	5,225,874	-624,000	-212,160	636,627	216,453	4,293
126,671	5,123,329	512,312	0.909	0.860	0.818	0.805	4,390,533	-482,659	-164,104	512,312	174,186	10,082
87,388	4,018,612	409,866	0.966	0.909	0.860	0.850	3,619,093	-372,560	-126,671	409,866	139,355	12,684
51,281	3,068,719	321,489	0.966	0.966	0.909	0.902	2,890,117	-257,024	-87,388	321,489	109,306	21,918
8,685	2,236,315	245,497	0.966	0.966	0.966	0.963	2,165,945	-150,828	-51,281	245,497	83,469	32,188
6,247	1,511,753	178,905	0.966	0.966	0.966	0.963	1,464,489	-25,544	-8,685	178,905	60,828	52,143
5,306	988,694	120,940	0.966	0.966	0.966	0.963	958,863	-18,373	-6,247	120,940	41,120	34,873
3,391	547,609	79,096	0.966	0.966	0.966	0.963	531,469	-15,606	-5,306	79,096	26,892	21,586
2,101	263,899	43,809	0.966	0.966	0.966	0.963	256,441	-9,973	-3,391	43,809	14,895	11,504
1,149	90,851	21,112	0.966	0.966	0.966	0.963	88,621	-6,180	-2,101	21,112	7,178	5,077
0	0	7,268	0.966	0.966	0.966	0.963	0	-3,379	-1,149	7,268	2,471	1,322

ROE 8.0%

NPV 0
IRR 8.00%

Acc Year 85 DTA Reserve Disc	Year 86 DTA Reserve Disc	Year 87 DTA Reserve Disc	Year 88 DTA Reserve Disc	Total DTA	COMPANY FLOWS: UW	Inv Inc	Asset	Tax	DTA	Equity Flow
17,899	-20,268	-321	-3,565	-6,255	17,476,775	0	24,501,000	456,027	-6,255	-6,574,453
54,366	81,411	18,534	40,263	194,573	-2,071,000	1,960,580	-2,071,000	-672,852	200,828	1,488,556
53,303	91,527	59,779	21,980	226,589	-3,532,000	1,778,834	-3,532,000	-410,230	32,016	1,400,619
55,213	89,745	67,271	75,956	288,186	-3,264,000	1,493,713	-3,264,000	-281,274	61,597	1,274,036
43,454	92,966	65,924	74,663	277,006	-2,940,000	1,227,665	-2,940,000	-129,220	-11,179	1,087,265
33,154	73,168	68,323	108,983	283,628	-2,478,000	993,359	-2,478,000	-60,736	6,621	939,245
24,620	55,759	53,748	78,033	212,160	-2,046,000	794,590	-2,046,000	13,467	-71,468	736,589
20,361	41,407	41,053	61,284	164,104	-1,602,000	636,627	-1,602,000	-4,293	-48,056	584,278
15,163	34,243	30,469	46,796	126,671	-1,318,000	512,312	-1,318,000	-10,082	-37,433	464,796
1,533	25,501	25,199	35,155	87,388	-1,144,000	409,866	-1,144,000	-12,684	-39,282	357,900
1,533	2,579	18,768	28,401	51,281	-986,000	321,489	-986,000	-21,918	-36,107	263,464
767	2,579	1,893	3,446	8,685	-875,000	245,497	-875,000	-32,188	-42,596	170,713
767	1,289	1,893	2,297	6,247	-727,000	178,905	-727,000	-52,143	-2,438	124,324
767	1,289	953	2,297	5,306	-524,000	120,940	-524,000	-34,873	-941	85,127
0	1,289	953	1,149	3,391	-443,000	79,096	-443,000	-21,586	-1,915	55,594
0	0	953	1,149	2,101	-285,000	43,809	-285,000	-11,504	-1,289	31,015
0	0	0	1,149	1,149	-174,000	21,112	-174,000	-5,077	-953	15,083
0	0	0	0	0	-92,000	7,268	-92,000	-1,322	-1,149	4,797