Valuing Stochastic Cash Flows: A Thought Experiment

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How should we value stochastic cash flows? Without a common framework our discussions generate more heat than light. The following thought experiment seeks to filter out the extraneous and to focus on the essential.

Imagine that you wish to value stochastic cash flow C that has n possible outcomes. The i^{th} outcome is the receipt of x_i dollars t_i years from now, and the probability of the outcome is p_i. Hence, $\sum_{i=1}^{n} p_i = 1$. The value of C, $V[C]$, is a dollar value, and you would be willing to pay for C as much as, but not more than, $V[C]$. Really, it matters not whether your beliefs about n, x, t, and p are the "right" ones; indeed, others might disagree with you. What matters is that you are willing to act on your beliefs.

Take for granted that you can value deterministic cash flows, i.e., that you can value the guaranteed receipt of x dollars t years from now. Let the function $v(x, t)$ denote that value. For stochastic cash flow C, with $n > 1$ and all $p_i > 0$,[1] your value should lie within the range of the deterministic values; in symbols:

$$\min_i\{v(x_i, t_i)\} \le V[C] \le \max_i\{v(x_i, t_i)\}$$

How could anyone dispute this inequality? One who purchases a stochastic cash flow for its minimum possible value can do no worse than that, and might do better. And one who purchases it for its maximum can do no better, and might do worse.

[1] Outcomes of zero probability are impossible outcomes, and do not affect the range of value.

As a corollary, if all the outcomes have the same deterministic value, the value of the stochastic cash flow must equal that value. For example, suppose that you value at 100 dollars both the receipt of 104 dollars one year from now and the receipt of 109 dollars two years from now. The stochastic cash flow is to flip a coin and to receive 104 dollars in one year if the coin lands heads, and to receive 109 dollars in two years if it lands tails. You should value the stochastic cash flow at 100 dollars. If the coin were flipped immediately after you paid the 100 dollars and you observed the outcome, you would think, "Fine. In effect, I bought this deterministic cash flow at its value." If your $v(x,t)$ function were compatible with the U.S. Treasury market, you could even cash in and regain your 100 dollars. But does it matter whether you will know the outcome immediately afterward? After all, no matter what will happen and no matter when you will learn of it, you will certainly receive something that right now you value at 100 dollars. That your $v(x,t)$ function may change (indeed, it almost certainly will change) makes no difference to the present.

True, one cannot hedge this stochastic cash flow so as at any moment to cash it in for 100 dollars (regardless of risk-free short-term interest). Some argue from this that the value of the flow should be less than 100 dollars. But the argument proves nothing because it proves too much. For it would undermine the $v(x,t)$ function itself. Just as little can one hedge the receipt of 104 dollars in one year so as at will to recover one's cost. So the hedging argument is no more germane to stochastic cash flows than it is to deterministic ones.

The implications of the stochastic-value inequality, $\min_i\{v(x_i,t_i)\} \le V[C] \le \max_i\{v(x_i,t_i)\}$, are revolutionary. Most likely, $V[C]$ should be not just *bounded* by the deterministic values, but even *determined* by them. In other words, the only property of outcome (x_i, t_i) that affects stochastic value is $v(x_i, t_i)$.[2] Therefore, the value of a stochastic cash flow should be invariant to any change in its outcomes that preserves their deterministic values. Furthermore, since nothing honors a minimum and a maximum better than a weighted average, it seems that stochastic value should be of the form:

$$V[C] = \sum_{i=1}^{n} q_i \cdot v(x_i, t_i)$$

Thus, to value a stochastic cash flow you should weight the deterministic values of its outcomes according to artificial, or risk-adjusted, probabilities. How to do this the author has shown in a full-length paper.[3] Suffice to say, a revolution is mounting against the ruling method of discounting expected cash flows at some risk-adjusted rate of return.[4] One should gauge the ruling method's right to his or her intellectual allegiance according to its eagerness and ability to respond to simple thought experiments like this one.

[2] Of course, the probabilities must not change. However, they are extrinsic to the outcomes.

[3] "The Valuation of Stochastic Cash Flows," CAS Forum (Reinsurance Discussion Papers, Spring 2003), 1-68.

[4] For detailed arguments against risk-adjusted discounting and its consort, capital allocation, see the author's paper "A Critique of Risk-Adjusted Discounting," *2001 ASTIN Colloquium*, www.casact.org/coneduc/reinsure/astin/2000/halliwell1.doc.