Principles of the Chain-Ladder "Method" Selecting and Updating Claims Development Factors

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Abstract

There has been significant discussion recently regarding the roles of "models"[1] and "methods" in actuarial practice. I believe that much of this discussion is misguided as it is based on an imprecise and arbitrary distinction. I believe that "methods" are more appropriately considered to be a subclass of "models," rather than a wholly different class of estimation procedures. More specifically,as with "models", I believe that there **are** statistical assumptions underlying "methods."

If we accept this conclusion, then it becomes incumbent on actuaries to apply statistical theory when using methods. The most common method is the chain-ladder method. In this paper, as an example, I re-examine the process of selecting and updating claim[2] development factors under this new paradigm.

1. Methods versus Models

In the Fall 2005 CAS Forum, the CAS Working Party on Quantifying Variability in Reserve Estimates published *The Analysis and Estimation of Loss & ALAE Variability: A Summary Report*. This paper proposed the following definitions:

> *Method: A systematic procedure for estimating future payments for loss and allocated loss adjustment expense. Methods are algorithms or series of steps followed to determine an estimate; they do not involve the use of any statistical assumptions that could be used to validate reasonableness or to calculate standard error. Well known examples include the chain-ladder (development factors) method or the Bornhuetter-Ferguson method.* ***Within the context of*** **[the Working Party]** ***paper****, "methods" refer to algorithms for calculating future payment estimates, not methods for estimating model parameters. (emphasis added)*
>
> *Model: A mathematical or empirical representation of how losses and allocated loss adjustment expenses emerge and develop. The model accounts for known and inferred properties and is used to project future emergence and development. An example of a mathematical model is a formulaic representation that provides the best fit for the available historical data. Mathematical models may be parametric (see below) or non-parametric. Mathematical models are known as "closed form" representations, meaning that they are represented by mathematical formulas. An example of an empirical representation of how losses and allocated loss adjustment expenses emerge and develop is the frequency distribution produced by the set of all reserve values generated by a particular application of the chain ladder method. Empirical distributions are, by*

[1] The use of quotation marks is intended to indicate usages of the terms "models" and "methods" that the author believes to be incorrect.

[2] In this paper, we use the terms "claims" instead of "loss" in order to be consistent with Actuarial Standard of Practice (ASOP) No. 43, *Property/Casualty Unpaid Claim Estimates*.

> *construction, not in "closed form" as there is no underlying requirement that there be an underlying mathematical model.*[3]

It should be noted that these definitions were restricted to a specific context and that they were presented in a non-refereed paper. Despite this circumstance, the Actuarial Standards Board adopted these definitions in Actuarial Standard of Practice (ASOP) No. 43, *Property/Casualty Unpaid Claim Estimates*. ASOP No. 43 includes the following definitions:

> *2.5 Method—A systematic procedure for estimating the unpaid claims.*
>
> *2.6 Model—A mathematical or empirical representation of a specified phenomenon.*[4]

In addition, the ASOP document includes the following comment and response related to these definitions:

> *Section 2.5, Method and 2.6, Model*
>
> *Comment One commentator stated, "There are definite differences between 'methods' and 'models' that are much more substantial and fundamental than" what is in the proposed standard. The commentator suggested that more complete definitions be taken from the CAS Working Party paper on reserve variability.*
>
> *Response The definitions in the standard are abbreviated versions of what is in the referenced Working Party paper. The reviewers believe that further elaboration is unnecessary, although reference to various CAS publications has been added to appendix 1.* [5]

I believe that this was an unfortunate decision by the Actuarial Standards Board. These definitions appear to reinforce the notion that "methods" and "models" are actually different. The acceptance of these definitions within a binding document might also result in a *de facto* acceptance of these definitions without being subject to a refereed process.

"Methods" are defined as algorithms without statistical assumptions whereas "models" are defined as mathematical representations. The definition and cited examples imply that only an understanding of algebra and arithmetic are necessary to use "methods." In contrast, "models" appear to require more advanced statistical skills. These definitions are misguided. The definitions are also somewhat dangerous as a layperson would (rightly) question whether the training of an FCAS is required to use "methods."

For the "methods" crowd, this definition has the unfortunate result that they are not forced to statistically evaluate their estimation methodologies. After all, statistical tests cannot be performed in the absence of statistical assumptions. For the profession, this has a dangerous consequence as it devalues the skills required to perform actuarial calculations.

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[3] CAS Working Party on Quantifying Variability in Reserve Estimates. The Analysis and Estimation of Loss & ALAE Variability: A Summary Report. Casualty Actuarial Society Forum (Fall 2005), 29-146. (Page 38)

[4] Actuarial Standards Board of American Academy of Actuaries, "Actuarial Standard of Practice No. 43, Property/Casualty Unpaid Claim Estimates (Doc. No. 106)," 2007. (Page 3)

[5] Ibid, Page 15

I believe that it is more appropriate to consider "methods" as a type or subclass of "models." Let us consider the plain-English definition of Model:

> *a simplified version of something complex used in analyzing and solving problems or making predictions*[6]

Cleary, the chain-ladder and Bornhuetter Ferguson methods, which are listed as examples of "methods," would also be considered models under this definition. Consider that the paid claims development method for estimating unpaid claim amounts may also be presented as:

$$\hat{U}_i = P_i \times (\prod_i \hat{f}_j) - 1$$

where: U = Unpaid Claims

P = Paid Claims

f_j = the estimated incremental claims development factor between j and $j + 1$

and i = the age of an accident period.

Under the definitions proposed by the Working Party and adopted by the Actuarial Standards Board, would this be considered a "method" or would it be considered a "model?" We should now see that the distinction is arbitrary.

I believe that it would have been more useful to focus on types or classes of models such as, but not limited to:

- arithmetic
- stochastic
- parametric
- deterministic
- empirical
- non-parametric

With this paradigm, we can better analyze **<u>deterministic models</u>** such as chain-ladder and Bornhuetter-Ferguson. An example of this analysis focused on the selection of the incremental claims development factors is presented in this paper. Other analyses, such as a review of the quality of the models themselves and correlations between development columns are beyond the scope of this paper – but they become possible under the new paradigm.

2. Review of the Properties of Statistical Estimators

We should now consider "selected incremental claims development factors" as estimators of the parameters of a model. We then consider the following properties of estimators in evaluating the quality of our claims development factors:

- Unbiasedness – An estimator ($\hat{\theta}$) is considered unbiased if its expected value is equal to the true value of the parameter (θ). That is:

  $$E[\hat{\theta}] = \theta$$

  A somewhat more relaxed constraint is that the estimator be asymptotically unbiased. That is:

[6] Encarta dictionary

$$\lim_{n \to \infty} E[\hat{\theta}] = \theta$$

- Efficiency – An estimator is considered efficient if its sampling distribution has a relatively small standard deviation.
- Consistency – An estimator is considered consistent if it is more likely to be close to its true value when the sample size is increased.
- Sufficiency – An estimator is considered sufficient if it uses all of the information in the sample.
- Robustness / Resistance – An estimator is considered resistant or robust if it is relatively unaffected by outliers.

3. Comparisons of Common Methods of Selecting Claims Development Factors

We now consider four common methods of selecting claims development factors: (i) all-year averages (weighted, or unweighted) (ii) averages of recent observations (iii) Ex hi/low averages and (iv) judgment. For purposes of this discussion, we should assume that there are no distorting influences on the data.

Table 1
Common Estimators of Claims Development Factors

	Estimator			
Property	All-Year Average	Average of Recent Observations	Ex-Hi/Low Averages	Judgment
Unbiasedness	Yes	Yes	Yes	Unknown
Efficiency	Unknown	Unknown	Unknown	Unknown
Consistency	Yes	Not Applicable (Fixed sample size)	Yes	Unknown
Sufficiency	Yes	No	No	Unknown
Robustness / Resistance	Unknown	Unknown	Yes	Probably

The conclusion that we should draw from this table is that, under current commonly used methods for estimating claims development factors, we understand very little about the quality of those factors. This situation is further exacerbated when we consider that the typical basis for selected claims development factors is "actuarial judgment" based on a review of various averages. This leads us to the unfortunate conclusion that we understand relatively little about the quality of the resulting estimates of ultimate claims.

4. Statistical Estimation Methods

We now consider two alternative statistical methods for estimating claims development: maximum likelihood and regression. We use the 12-24 month General Liability Excluding Mass Torts development experience published by the Reinsurance Association of America as our test data. The results of the estimation considering both of these methods and a comparison to the traditional techniques listed above are presented in Exhibit A.

I do not intend to imply that these are the only available statistical tools that may be used to estimate claims development factors. They are presented here as two possible examples. A discussion of the advantages and disadvantages between maximum likelihood, regression and alternative parameter estimation methodologies is beyond the scope of this paper.

Furthermore, I recognize that these estimation methods do not always have all of the desired properties listed in the prior section. For example, the maximum likelihood estimator is not always unbiased. However, what is important is that we realize where these methods fall short as compared to the (almost complete) lack of knowledge associated with traditional estimators.

Most importantly, the knowledge that we have about these estimators will allow us to update the development factors only when appropriate. That is, I believe that, too often, unpaid claim estimates are impacted by differences in judgments applied year-to-year or quarter-to-quarter. This (understandably) reduces the confidence that stakeholders have in actuarial work product.

4.1. Maximum Likelihood Estimators

The advantage of maximum likelihood estimators (MLE) is that they are: (i) asymptotically unbiased (ii) asymptotically efficient, (iii) consistent and, (iv) for large samples, the MLE is normally distributed. The principal difficulty with maximum likelihood estimation is that the procedure requires the assumption of a model form. However, this does provide a benefit in that we would then expect the MLE to be robust / resistant.

There are three steps to develop the MLE for claims development factors. First, we must determine the appropriate distribution form for the claims development factors. Then, using this distribution, we must formulate the maximum likelihood function. Finally we must determine the parameters that maximize the likelihood function.

In the attached example we assume the following distributional form:

$$\text{(claims development factor} - 1) \sim \text{LogNormal}\ (\mu, \sigma)$$

The likelihood functions and log-likelihood functions may then be, respectively, written as:

$$L = \prod f(x; \mu, \sigma)$$

$$\ln L = \sum \ln f(x; \mu, \sigma),:$$

We then can use numerical methods to solve for the parameters that maximize the likelihood or equivalently maximize the log-likelihood[7].

4.2. Regression (Least Squares Estimator)

We can also use regression techniques to estimate the claims development. For convenience, we will refer to the resulting estimator as the "regression estimator" (RE). Under the assumptions of chain ladder method that claims at a given age are proportional to the claims at the prior age, the RE will be unbiased. Heuristically, we would also expect it to be asymptotically efficient, consistent, sufficient and robust.

REs are developed by solving for the *X*-coefficient of the following regression equation:

$$Y = mX + \varepsilon$$

This is the equation for regression through the origin (intercept=0). *Y* and *X* are the claims at 24 and 12 months, respectively. The X coefficient, $m = Y / X$, represents the estimate of the claims development factor.

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[7] In this particular example it is well known that the MLE for the μ and σ parameters of the lognormal distribution are the mean and standard deviations of the logarithms of the data.

5. Updating Claims Development Factors

A significant benefit of defining a model in terms of statistical estimators is that it provides valuable guidance in updating the model. That is, it removes the arbitrariness associated with updates to development factors determined using traditional methods.

For example, assume that with the RAA-GL data presented in the example, we had observed a development factor of X in the next period. The question then becomes: should we revise our estimator of the claims development factor? Too often, that question is answered "yes" without thought. In fact, "yes" may be the only possible answer if our claims development factor is based on a "traditional approach." The answer should be: "Only if our new observation results in an updated estimator that is statistically significantly different from the prior estimator." We can use hypothesis testing to determine whether a change in the claims development factor is warranted.

5.1. Maximum Likelihood Estimators

In the example presented, we use the Likelihood Ratio test to determine whether a development factor estimator developed using maximum likelihood should be updated. That is, we test the null hypothesis that there should be no change to the estimator. The alternative hypothesis is that, the estimator should be updated.

The Likelihood Ratio test statistic is calculated as 2 times the difference in the log-likelihood values. This test statistic has a chi-square distribution with degrees of freedom equal to the number of parameters. The log-likelihoods are calculated including the new data.

Furthermore, it should be noted that there is no restriction on the data used in the calculation of the test statistic. That is, even if the initial parameters are calculated using all available observations, we are free to test for whether an update is required using, for example, only the most recent five observations. Stated differently, the decision as to the data used in the estimation process is independent of the hypothesis test.

Exhibits B1 and B2 present examples where the new observation does not support and does support, respectively, a change to the claims development factor estimator.

5.2. Regression (Least Squares Estimator)

Similarly, we can use hypothesis testing to determine whether a development factor estimator developed using regression should be updated. We perform this test by calculating the predicted Y values using the following relationship:

$$\hat{Y} = mX$$

We then fit the following regression line to the predicted-Y values:

$$Y - \hat{Y} = aY + \varepsilon$$

We can then test for significance of the regression coefficient. If the regression coefficient is significant, we then reject the null hypothesis. Exhibits C1 and C2 present examples where the new observation does not support and does support, respectively, a change to the claims development factor estimator.

6. Acknowledgments

The author wishes to thank Katy Siu and Bernard Chan for their reviews of this paper. Any errors that remain herein are the responsibility of the author. As with many research topics, the concepts presented herein are a "work-in-progress." The author would welcome your comments. Please consult the CAS member directory for contact information.

RAA
General Liability Excluding Mass Torts
Selecting Claims Development Factors
Reported Incurred Claims

(1)	(2)	(3)	(4)	(5)	(6)	(7)
			(3) / (2)	ln [(4) - 1]		ln [(6)]
				Statistics for Maximum Likelihood		
Accident Year	at 12 mos. (*A*)	at 24 mos. (*B*)	Observed (*X*)	Y = ln(*X* - 1)	f(y; μ, σ)	Log-Likelihood
1989	49,997	139,166	2.7835	0.578570442	0.620243308	-0.477643445
1990	70,104	201,662	2.8766	0.629467965	0.801247311	-0.221585627
1991	79,614	208,748	2.6220	0.483660668	0.307094784	-1.180598835
1992	56,265	190,867	3.3923	0.872249604	0.850514412	-0.161913922
1993	68,133	199,866	2.9335	0.65931547	0.895211433	-0.110695351
1994	68,530	241,658	3.5263	0.9267596	0.661954402	-0.412558604
1995	69,055	253,640	3.6730	0.983206773	0.461090693	-0.774160524
1996	102,320	295,607	2.8890	0.636070974	0.823202378	-0.194553205
1997	115,360	330,745	2.8671	0.624369451	0.783933695	-0.243430835
1998	138,160	468,526	3.3912	0.871788697	0.851967055	-0.160207421
1999	151,311	565,163	3.7351	1.006171101	0.386360609	-0.950984125
2000	178,943	562,916	3.1458	0.763504918	1.050062421	0.048849611
2001	187,203	671,424	3.5866	0.950347825	0.576352116	-0.551036493
2002	183,601	692,642	3.7725	1.019763639	0.34516186	-1.063741814
2003	149,925	494,121	3.2958	0.831076094	0.963885674	-0.036782587

	Estimator Value
Traditional Estimators	
All-Year Weighted Average	3.3064
Five Year Weighted Average	3.5092
X-Hi/Low Average	3.2381
Statistical Estimators	
Maximum Likelihood	
Model Form *assumes LDF-1 is lognormally distributed*	
μ	0.7891
σ	0.175177205
LDF	3.2014
Log-Likelihood	-6.491043178
Regression through the origin	
Rgression Model	*B* = *m***A* + ε
Coefficient (*m*)	3.3743
Standard Error of Coefficient *SE* (*m*)	0.0896

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(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
			(3) / (2)	ln [(4) - 1]			ln [(6)]	ln [(7)]
					Statistics for Maximum Likelihood			
Accident Year	at 12 mos. (*A*)	at 24 mos. (*B*)	Observed (*X*)	Y = ln(*X* - 1)	f(y; μ_0, σ_0)	f(y; μ_a, σ_a)	Log-Likelihood	Log-Likelihood
					H_0	H_a	H_0	H_a
1989	49,997	139,166	2.7835	0.578570442	0.609738605	0.558088576	-0.49472493	-0.583237591
1990	70,104	201,662	2.8766	0.629467965	0.805137803	0.747959337	-0.216741832	-0.290406665
1991	79,614	208,748	2.6220	0.483660668	0.285169761	0.255255323	-1.254670623	-1.365490968
1992	56,265	190,867	3.3923	0.872249604	0.873308617	0.895304603	-0.135466272	-0.11059128
1993	68,133	199,866	2.9335	0.65931547	0.908646836	0.852297506	-0.095798779	-0.159819627
1994	68,530	241,658	3.5263	0.9267596	0.6702392	0.706931176	-0.400120615	-0.346821965
1995	69,055	253,640	3.6730	0.983206773	0.456795588	0.497404281	-0.78351928	-0.698352141
1996	102,320	295,607	2.8890	0.636070974	0.829189261	0.771901938	-0.18730685	-0.25889776
1997	115,360	330,745	2.8671	0.624369451	0.786225562	0.729238497	-0.240511553	-0.315754444
1998	138,160	468,526	3.3912	0.871788697	0.874878068	0.896706774	-0.133670753	-0.109026367
1999	151,311	565,163	3.7351	1.006171101	0.378578156	0.417908939	-0.971332738	-0.87249172
2000	178,943	562,916	3.1458	0.763504918	1.086090261	1.059350837	0.082584332	0.057656303
2001	187,203	671,424	3.5866	0.950347825	0.57879619	0.618480075	-0.546804867	-0.480490303
2002	183,601	692,642	3.7725	1.019763639	0.335822123	0.373795744	-1.091173654	-0.98404577
2003	149,925	494,121	3.2958	0.831076094	0.995671493	1.000599101	-0.004337902	0.000598921
2004	**New Obseravtion**		**3.7000**	0.993251773	0.421722988	0.461945273	-0.863406607	-0.772308852
Statistical Estimators					0.000650999	0.000682683	-7.337002925	-7.289480228

Maximum Likelihood

Model Form *assumes LDF-1 is lognormally distributed*

	Hyothesis Testing		
	H_0	H_a	
μ	0.7891	0.8018	
σ	0.169237259	0.171153501	
LDF	3.2014	3.2297	
Log-Likelihood	-7.337002925	-7.289480228	
Change in Log_likelihood	0.047522697		
Likelihhod Ratio Test Statistic	0.095045395		
Critical Value	5.991464547	Chi-Square (2 d.f.)	
	Accept H_0		

RAA
General Liability Excluding Mass Torts
Selecting Claims Development Factors
Reported Incurred Claims

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
			(3) / (2)	ln [(4) - 1]			ln [(6)]	ln [(7)]
					Statistics for Maximum Likelihood			
Accident Year	at 12 mos. (*A*)	at 24 mos. (*B*)	Observed (*X*)	Y = ln(*X* - 1)	f(y; μ_0, σ_0)	f(y; μ_a, σ_a)	Log-Likelihood	Log-Likelihood
					H_0	H_a	H_0	H_a
1989	49,997	139,166	2.7835	0.578570442	0.609738605	0.526960015	-0.49472493	-0.640630606
1990	70,104	201,662	2.8766	0.629467965	0.805137803	0.611993102	-0.216741832	-0.491034267
1991	79,614	208,748	2.6220	0.483660668	0.285169761	0.35433519	-1.254670623	-1.037511951
1992	56,265	190,867	3.3923	0.872249604	0.873308617	0.680420218	-0.135466272	-0.385044704
1993	68,133	199,866	2.9335	0.65931547	0.908646836	0.654525535	-0.095798779	-0.42384468
1994	68,530	241,658	3.5263	0.9267596	0.6702392	0.606929497	-0.400120615	-0.499342644
1995	69,055	253,640	3.6730	0.983206773	0.456795588	0.511158897	-0.78351928	-0.671074784
1996	102,320	295,607	2.8890	0.636070974	0.829189261	0.621970701	-0.18730685	-0.474862293
1997	115,360	330,745	2.8671	0.624369451	0.786225562	0.604091376	-0.240511553	-0.504029808
1998	138,160	468,526	3.3912	0.871788697	0.874878068	0.680931052	-0.133670753	-0.384294223
1999	151,311	565,163	3.7351	1.006171101	0.378578156	0.469315257	-0.971332738	-0.756480547
2000	178,943	562,916	3.1458	0.763504918	1.086090261	0.734687447	0.082584332	-0.308310113
2001	187,203	671,424	3.5866	0.950347825	0.57879619	0.568643622	-0.546804867	-0.564501365
2002	183,601	692,642	3.7725	1.019763639	0.335822123	0.444295045	-1.091173654	-0.811266421
2003	149,925	494,121	3.2958	0.831076094	0.995671493	0.717284422	-0.004337902	-0.332282833
2004	**New Obseravtion**		**5.6000**	1.526056303	3.90759E-05	0.006127827	-10.15000459	-5.094915038
Statistical Estimators					6.03201E-08	1.54664E-06	-16.62360091	-13.37942628

<u>Maximum Likelihood</u>

Model Form *assumes LDF-1 is lognormally distributed*

	Hyothesis Testing	
	H_0	H_a
μ	0.7891	0.8351
σ	0.169237259	0.242228661
LDF	3.2014	3.3052
Log-Likelihood	-16.62360091	-13.37942628
Change in Log_likelihood	3.244174636	
Likelihhod Ratio Test Statistic	6.488349272	
Critical Value	5.991464547	Chi-Square (2 d.f.)
	Reject H_0	

RAA
General Liability Excluding Mass Torts
Selecting Claims Development Factors
Reported Incurred Claims

(1)	(2)	(3)	(4)	(5)	(6)	(7)
			(3) / (2)	(3) / (1)	(5) * (2)	(6) - (3)
Accident Year	at 12 mos. (*A*)	at 24 mos. (*B*)	Observed (*X*)	Estiamtor H_0	Predicted *B*	Residuals
1989	49,997	139,166	2.7835	3.3743	168,703	29,537
1990	70,104	201,662	2.8766	3.3743	236,550	34,888
1991	79,614	208,748	2.6220	3.3743	268,639	59,891
1992	56,265	190,867	3.3923	3.3743	189,853	-1,014
1993	68,133	199,866	2.9335	3.3743	229,899	30,033
1994	68,530	241,658	3.5263	3.3743	231,239	-10,419
1995	69,055	253,640	3.6730	3.3743	233,010	-20,630
1996	102,320	295,607	2.8890	3.3743	345,255	49,648
1997	115,360	330,745	2.8671	3.3743	389,256	58,511
1998	138,160	468,526	3.3912	3.3743	466,189	-2,337
1999	151,311	565,163	3.7351	3.3743	510,564	-54,599
2000	178,943	562,916	3.1458	3.3743	603,802	40,886
2001	187,203	671,424	3.5866	3.3743	631,673	-39,751
2002	183,601	692,642	3.7725	3.3743	619,519	-73,123
2003	149,925	494,121	3.2958	3.3743	505,887	11,766
2004	175,000	647,500	3.7000	3.3743	590,497	-57,003

Statistical Estimators

<u>Regression through the origin</u>

	Hyothesis Testing H_0	H_a
LDF	3.3743	3.4141
Test Statistic	-0.020898502	
Standard Error	0.024561098	
t Statistic	-0.850878164	
d.f.	15	
Critical Value at 5%	2.131449536	
	Accept H_0	

RAA
General Liability Excluding Mass Torts
Selecting Claims Development Factors
Reported Incurred Claims

(1)	(2)	(3)	(4)	(5)	(6)	(7)
			(3) / (2)	(3) / (1)	(5) * (2)	(6) - (3)
Accident Year	at 12 mos. (*A*)	at 24 mos. (*B*)	Observed (*X*)	Estiamtor H_0	Predicted *B*	Residuals
1989	49,997	139,166	2.7835	3.3743	168,703	29,537
1990	70,104	201,662	2.8766	3.3743	236,550	34,888
1991	79,614	208,748	2.6220	3.3743	268,639	59,891
1992	56,265	190,867	3.3923	3.3743	189,853	-1,014
1993	68,133	199,866	2.9335	3.3743	229,899	30,033
1994	68,530	241,658	3.5263	3.3743	231,239	-10,419
1995	69,055	253,640	3.6730	3.3743	233,010	-20,630
1996	102,320	295,607	2.8890	3.3743	345,255	49,648
1997	115,360	330,745	2.8671	3.3743	389,256	58,511
1998	138,160	468,526	3.3912	3.3743	466,189	-2,337
1999	151,311	565,163	3.7351	3.3743	510,564	-54,599
2000	178,943	562,916	3.1458	3.3743	603,802	40,886
2001	187,203	671,424	3.5866	3.3743	631,673	-39,751
2002	183,601	692,642	3.7725	3.3743	619,519	-73,123
2003	149,925	494,121	3.2958	3.3743	505,887	11,766
2004	175,000	980,000	5.6000	3.3743	590,497	-389,503

Statistical Estimators

Regression through the origin

	Hyothesis Testing H_0	H_a
LDF	3.3743	3.6462
Test Statistic	-0.116444321	
Standard Error	0.049658948	
t Statistic	2.34488095	
d.f.	15	
Critical Value at 5%	2.131449536	
	Reject H0	