# Measuring Rate Change

Neil M. Bodoff, FCAS, MAAA

**Abstract**

**Motivation.** Calculated rate changes can substantially affect loss ratio forecasts and thus are critical parameters for ratemaking. However, current methods are not well suited to a changing book of business.

**Method**. The analysis first explores the conceptual underpinnings of rate change and then applies the conclusions of this analysis to several practical problems.

**Results**. The proposed approach shows improved accuracy as compared to other methods, with particular significance for a nonstatic book of business.

**Conclusions**. I conclude that "rate change" measures the change in premium *relative to loss potential.* One can then apply this conceptual formulation in order to solve several problems that one confronts in practice: how to adjust for shifts in limits and deductibles, how to blend together changes in exposures when the portfolio uses several different exposure bases, and how to properly weight together granular measures of rate change (e.g., for each policy, subline, etc.) into an overall rate change for the entire portfolio.

**Availability**. Please contact the author at neil.bodoff@willis.com or neil_bodoff@yahoo.com

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## 1. INTRODUCTION

In theory, measuring rate change[1] ought to be straightforward: using the company's "manual," one can simply find the rates in effect during one time period and compare them to rates in effect during another period. Or, similarly, one can track over time the rate changes the company achieves through its periodic rate filings. In practice, however, measuring rate change is not this simple, for a variety of reasons. Some of these reasons are:

1. Some policies, such as "excess" policies (including "umbrella"), attach above an underlying policy. Rates for such policies often derive from the premium charged for the underlying policy, thus complicating the notion of a clearly defined rate for such business. Moreover, the factors used for excess policies often have a wide range of filed rates; the actual charged rate can vary quite significantly over time without any change to the rating plan.

2. More generally, the rating plans for commercial lines also incorporate a significant amount of underwriting judgment in the final rate that can be charged.[2] Therefore,

[1] In this paper, the terms "rate change" and "rate change factors" relate to the actual rate changes achieved by the company; they relate to the historical period and are descriptive. They do not refer to "indicated rate changes" or "required rate changes," which are both prospective and prescriptive.

[2] See Vaughn [5], pp. 498-502.

tracking the changes to the company's filed rates will provide an inaccurate picture of rate movements.

3. Even when dealing with rating plans that do not allow for judgmental rates, one can encounter other complications. For example, if one simply tracks over time the rate increases and decreases that a company files on any particular date, one may overlook the resulting shift in the company's mix of business.[3]

One approach to overcoming these problems is to discard the measuring of filed, manual rates and to focus instead on measuring changes in the premium the company actually charges. Under this approach, one matches each renewing policy to its corresponding expiring policy and measures the rate change for each policy.[4,5] Such an approach is often referred to as measuring "renewal rate change."

Measuring renewal rate change can introduce more granularity and precision to the measuring of rate change. Still, many questions persist, such as:

1. How do I account for changes to a policy's limit and deductible when measuring the renewal policy's rate change?
2. When I measure rate change for excess casualty policies, which cover auto liability and also general liability claims, how do I combine rate changes for these two sublines, which have different exposure bases? More generally, how do I combine any two sublines that have different exposure bases? Is it possible to obtain one overall number for "exposure change" when the sublines have different exposure bases?
3. When I measure rate changes for several different sublines or multiple individual policies, how do I weight them together to obtain one blended rate change factor for the overall portfolio?
4. When my firm implements rate increases and rate decreases for various classes of business, volume tends to grow in those classes that received rate decreases and volume

[3] See McCarthy [2], who notes this problem and provides an alternative solution.

[4] New policies, by definition, must be excluded and measured separately; measuring rate change for new policies is outside the scope of this paper.

[5] When premium rates are not unique for each individual policy but do vary by subline, then one need not measure the rate change of each policy but rather each subline. In such a situation, the only "new" business that would need to be excluded would be a new subline of business that did not exist in the prior rating plan. In contradistinction, new individual policies within existing sublines would not need to be excluded as "new" business but rather should be included as exposure growth within existing sublines.

tends to decline in those classes that received rate increases. Thus, rate changes tend to generate additional shifts in the mix of business in the firm's portfolio; how do I properly reflect this shift when calculating rate change for the total book of business?

## 2. THE THEORY AND PURPOSE OF RATE CHANGE FACTORS

In order to answer these detailed questions, we need to first examine the fundamental principles underlying the theory of rate change. How should one calculate a company's rate change factors? The answer to this question depends upon the answer to the following question: for what purpose will we use these rate change factors?

In theory, rate change factors can be used for several different purposes. For example, one potential use of rate change factors is to enable management to better run the company. Under this approach, rate change factors indicate how the company is performing: they tell management where performance is improving and where it is slipping, thus allowing for better steering of the business and better implementation of strategy. If in fact this is the purpose of the rate change factors, then consider the dynamic situation in which policies currently issued by the company have higher deductibles than policies issued in the past. As the deductibles increase, the stable volume of losses in the deductible layer disappears and the company covers policies that have more variability, lower premium volume, and (because of fixed costs) higher expense ratios. Therefore, if the goal of the company is to understand the true nature of its performance, then traditional rate change factors, which ignore shifts in required risk load and shifts in expense ratios, will fall short of the desired goal. Rather, the company must implement an approach whereby each policy in the portfolio, accounting for risk load and fixed expenses, is priced to a target premium; then, the company can evaluate how the actual premium compares to the target premium and how this ratio of "actual to target" changes over time.

Table 1

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 = (3+4+5) / (1-6) | 8 = 3 / 7 | 9 | 10 = 3 / 9 | 11 = 9 / 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Limit | Deductible | Expected Loss | Target Risk Load | Fixed Expenses | Variable Expenses | Target Premium | ELR to Target Premium | Actual Premium | ELR to Actual Premium | Actual / Target |
| Expiring | 2,000,000 | 1,000 | 7,601 | 1,383 | 1,000 | 15% | 11,746 | 65% | 12,500 | 61% | 1.064 |
| Renewing | 2,000,000 | 100,000 | 3,045 | 1,133 | 1,000 | 15% | 6,091 | 50% | 5,900 | 52% | 0.969 |

"Rate Adequacy Change" (Change in Ratio of Actual Premium to Target Premium) -9.0%

Table 1 shows an example in which the company's expected loss ratio (ELR) improves. By measuring the change in the ratio of Actual to Target, however, one can determine that rate adequacy has actually deteriorated. In a dynamic environment with changing policy provisions, only such an approach can give complete information to management about the company's "rate adequacy change."

Given that most rate change factors do not typically account for all the aspects of shifts in target risk load and shifts in expense ratios, the question persists: what good are rate change factors, for what purpose can we use them, and how does this affect how we ought to calculate them?

Traditional rate change factors therefore appear to be much more relevant to a second purpose: formulating a loss ratio projection for a book of business. Such a projection is often helpful for operational needs, such as estimating initial loss reserves, or for transactional purposes, such as effecting reinsurance treaties. In order to forecast the projected loss ratio, the actuary often begins by looking at historical experience data; in order to make the data relevant to the projected period, the losses and premium are adjusted to current level.

Therefore, in order to understand the role of rate change factors, we must investigate the nature of the traditional loss ratio projection and articulate its assumptions.

## 3. PROJECTING LOSS RATIO USING ADJUSTED HISTORICAL DATA

What is the nature of the loss ratio projection framework? Losses (in aggregate for any given historical year) are simply adjusted to current cost level; they are typically not adjusted in any way to incorporate changes in mix of business or changes in policy provisions such as deductibles and limits.[6] Premium is adjusted to what it "would be" had the historical policies been written today (or, more precisely, during the projected period).[7] Just as with losses, there seem to be no adjustments for shifts in the mix of business or in policy features. Thus traditional methods appear to be relevant only for the limited situations of a static book of business or one that changes only glacially.

How can traditional loss ratio projection be appropriate, then, for the many books of business that sustain significant changes in policies, classes of business, exposures, limits, and deductibles?

One response to this challenge is simply to concede: using historical data to project the future only makes sense when the portfolio is reasonably static, but not when it undergoes significant changes. This conclusion appears especially relevant to the "extended exposures" method for adjusting premium to current level. After all, the extended exposures approach takes historical policies and simply re-rates the policies at today's rates;[8] but if the types of policies in the portfolio have changed, the mix of business has shifted, and the limits and deductibles are different, what is the relevance of re-rating the policies of the historical portfolio?

Nevertheless, I believe that one can defend the use of historical data and adjusting for rate change by advancing the following reasoning. The goal of analyzing adjusted historical data is not to measure the **amount** of losses and premium that would occur from the historical portfolio, adjusted to today's dollars; rather, the goal is to measure premium and losses with respect to each other, i.e., the **interrelationship** of premiums to losses, and to measure what this relationship from the historical period would be in today's environment. Thus, even when the insurer's portfolio of policies undergoes significant change, when traditional adjustments to historical data do not accurately measure the projected amounts of losses and premium, the loss ratio projection can still be quite relevant; its relevance is rooted in its focus on measuring the relationship between premium and losses. This understanding of the purpose of using adjusted historical premium and losses, in turn, has ramifications for our understanding of what rate change factors should do and how we should calculate them, as we shall see in the following section.

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[6] Patrik [4] recommends that trending reflect all changes "that might affect the loss potential"; however, this step is difficult to implement and is often not done in practice.
[7] McClenahan [3], p. 88, describes the on-level premium as the premium "that would have resulted for the experience period had the current rates been in effect for the entire period." Thus we see that on-level premium is defined as historical premium adjusted solely for changes in rate level; apparently, no adjustments are made for changes in the portfolio's composition.
[8] See McClenahan [3], p. 94.

## 4. MEASURING RATE CHANGE FOR USE IN LOSS RATIO PROJECTION

Before proceeding to the derivation of the relevant formulas, let us articulate several observations, caveats, and limitations of scope.

1. Nothing in this paper intends to relate to the question of converting rate changes from a policy year, written premium basis to an accident year, earned premium basis; nor does this paper have any connection to rate level calculations based upon geometric techniques that rely on parallelograms and rectangles. These issues are addressed extensively elsewhere in the actuarial literature and are outside the scope of this paper.[9] Therefore, one should interpret all references to premium as references to policy year, written premium.
2. As noted in Section 2, how one ought to calculate rate change factors depends upon their intended purpose. Our discussion in this section presupposes that one will use the rate change factors in the context of projecting a loss ratio. However, if one were to use these factors for a different purpose, then the procedure of calculating the rate change factors may very well need to be different.
3. This paper does not intend to address the issue of inflation-sensitive exposure bases. Therefore, the reader should interpret the exposure base information as having already been converted from a nominal basis to a real (i.e., inflation-adjusted) basis.
4. When using historical data to project a loss ratio, actuaries often use multiple years of data; for simplicity, we will discuss the case of using data of one historical year (period t). In addition, we will simplify by discussing the procedure of adjusting this data one year forward (to period t+1).

### 4.1 Algebraic Representation

Let:

- Premium(observation(t), portfolio(t), rates(t)) = premium for historical period t, reflecting the portfolio in force and rates in effect during period t
- Loss(observation(t), portfolio(t), cost(t)) = losses for historical period t, reflecting the

[9] See McClenahan [3].

portfolio in force and claim cost inflation level in force during period t

- LP(portfolio(t)) = loss potential for the portfolio for historical period t; reflects the portfolio's propensity for loss given its limits, deductibles, and exposure base units, but does not reflect claim cost inflation[10]
- LP(portfolio(t+1)) = loss potential for portfolio for projected period t+1; reflects the portfolio's propensity for loss given its limits, deductibles, and exposure base units, but does not reflect claim cost inflation
- LP(portfolio(t+1))/ LP(portfolio(t)) = "shift in loss potential" = multiplier that adjusts the loss potential for the portfolio at time t to the loss potential for the portfolio at time t+1
- Trend(t, t+1) = claim cost inflation level during period t+1 / claim cost inflation level during period t = cost(t+1) / cost(t)

Let's assume that there are changes in the book of business relating to exposures, limits, and deductibles.

We want to take observed premium and losses from historical period t and to adjust them to the basis of period t+1, so we must calculate:

$$\begin{aligned}
&\text{Fully Adjusted Losses}(t \rightarrow t+1) = \\
&\text{Loss}(\text{observation}(t), \text{portfolio}(t+1), \text{cost}(t+1)) = \\
&\text{Loss}(\text{observation}(t), \text{portfolio}(t), \text{cost}(t)) * \frac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))} * \text{Trend}(t, t+1)
\end{aligned} \tag{4.1}$$

And

[10] Loss potential is essentially the expected loss cost. However, "loss cost" is usually measured in dollar units and thus tends to emphasize a particular numerical dollar value. In contrast, "loss potential" emphasizes the underlying real exposure to loss (and, as a result, changes to dollars of loss cost arising from inflation will not here be classified as a change in loss potential).

$$\text{Fully Adjusted Premium}(t \rightarrow t+1) = \text{Premium}(\text{observation}(t), \text{portfolio}(t+1), \text{rates}(t+1)) \tag{4.2a}$$

Multiplying both the numerator and denominator by equal quantities, we derive:

$$\text{Fully Adjusted Premium}(t \rightarrow t+1) = \text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \frac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))} * \frac{\text{Premium}(\text{observation}(t), \text{portfolio}(t+1), \text{rates}(t+1))}{\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \frac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))}} \tag{4.2b}$$

Then dividing losses by premium, we derive:

$$\text{Fully Adjusted Loss Ratio}(t \rightarrow t+1) = \frac{\text{Fully Adjusted Losses}(t \rightarrow t+1)}{\text{Fully Adjusted Premium}(t \rightarrow t+1)} \tag{4.3a}$$

As stated above, and as implied by Equation (4.1), in theory the losses should be adjusted to reflect all changes in loss potential, whether from changes in exposures, mix of business, limits, deductibles, etc. Nevertheless, if we focus on the interrelationship of losses and premium, we note that the shift in loss potential [i.e., LP(portfolio(t+1)) / LP(portfolio(t))] appears both in Equation (4.1) for Fully Adjusted Losses and in Equation (4.2b) for Fully Adjusted Premium. Dividing Equation (4.1) by Equation (4.2b) and canceling the factor for shift in loss potential, we derive:

$$\text{Fully Adjusted Loss Ratio}(t \to t+1) = \frac{\text{Fully Adjusted Losses}(t \to t+1)}{\text{Fully Adjusted Premium}(t \to t+1)}$$

$$= \text{Adjusted Loss Ratio}(t \to t+1) = \frac{\text{Adjusted Losses}(t \to t+1)}{\text{Adjusted Premium}(t \to t+1)} \tag{4.3b}$$

Such that:

$$\text{Adjusted Losses}(t \to t+1) = \text{Loss}(\text{observation}(t), \text{portfolio}(t), \text{cost}(t)) * \text{Trend}(t, t+1) \tag{4.4}$$

And:

$$\text{Adjusted Premium}(t \to t+1) =$$

$$\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) *$$

$$\frac{\text{Premium}(\text{observation}(t), \text{portfolio}(t+1), \text{rates}(t+1))}{\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \frac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))}} \tag{4.5}$$

Note that Equation (4.4) for adjusted losses is similar to Equation (4.1) for fully adjusted losses; however, it no longer has any factor for changes in loss potential from exposures, limits, and deductibles. Therefore, the practice of not adjusting losses for these shifts in loss potential is sustainable, but only if one simultaneously defines adjusted premium in a corresponding fashion, per Equation (4.5).

Now, let us define the Rate Change Factor as the multiplier which converts historical premium to adjusted premium.

Therefore:

$$\text{Adjusted Premium}(t \to t+1) = \text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \text{Rate Change Factor}(t \to t+1) \quad (4.6a)$$

And:

$$\text{Rate Change Factor}(t \to t+1) = \frac{\text{Adjusted Premium}(t \to t+1)}{\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t))} \quad (4.6b)$$

Then combining Equations (4.5) and (4.6a), we derive:

$$\text{Rate Change Factor}(t \to t+1) = \frac{\text{Premium}(\text{observation}(t), \text{portfolio}(t+1), \text{rates}(t+1))}{\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \dfrac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))}} \quad (4.7a)$$

The premium observed during any period reflects the portfolio and rates in effect at the time; however, in contradistinction to losses, premium is not a stochastic process and is not subject to random observation.[11] Therefore, we can drop the reference to "observation(t)" from Equation 4.7a and write:

$$\text{Rate Change Factor}(t \to t+1) = \frac{\text{Premium}(\text{portfolio}(t+1), \text{rates}(t+1))}{\text{Premium}(\text{portfolio}(t), \text{rates}(t)) * \dfrac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))}} \quad (4.7b)$$

Or, equivalently,

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[11] One exception to this general rule occurs if a policy's premium is "loss sensitive": then the observed premium is a function of the observed losses. Policies with loss sensitive premium are outside the scope of this analysis.

$$\text{Rate Change Factor}(t \rightarrow t+1) = \frac{\text{Premium}(t+1)}{\text{Premium}(t) * \text{Shift in Loss Potential}} \qquad (4.8)$$

Equation (4.8) demonstrates that one must calculate the rate change factor using the ratio of 2 quantities:

1) Actual premium in period (t+1)
2) Actual premium in period (t) "restated" for all shifts in loss potential, reflecting changes in exposures, limits, deductibles, etc.

To summarize, we have demonstrated three points:

1) To obtain an Adjusted Loss Ratio, the losses in the numerator do not need to be adjusted for changes in loss potential, thus somewhat exonerating current practice.
2) The Rate Change Factor is defined by Equation (4.8), which shows that when measuring rate change, one must first restate premium from the prior period for changes in loss potential.
3) Per Equation (4.6a), Adjusted Premium for use in loss ratio projection equals actual historical premium multiplied by the Rate Change Factor.

An important consequence of these results relates to <u>when</u> one can accurately measure the true rate change from period t ("the expiring period") to period t+1 ("the renewing period"). Formula (4.8) makes clear that one must take the premium from the expiring period and restate it based upon the shift in loss potential in the renewing period; however, the shift in loss potential cannot be known until the end of the renewing period. Therefore, when one implements rate changes to various segments of the portfolio at the beginning of a period, one can only estimate the rate change; the true rate change cannot be precisely calculated until the end of the period.

## 5. APPLICATIONS

We will now apply the conclusions of the discussion above to solve the problems raised at the beginning of this paper.

## 5.1 Weighting Together Multiple Rate Changes

This section will discuss how to measure the rate change for an entire portfolio in light of the rate changes of the portfolio's individual components.

Exhibit 1A: Change in Exposures

Expiring Period

| | Premium | Exposures | Premium per Exposure |
|---|---|---|---|
| Red Trucks | 12,000,000 | 600 | 20,000 |
| Green Trucks | 4,000,000 | 400 | 10,000 |
| Total | 16,000,000 | 1,000 | 16,000 |

Renewing Period

| | Premium | Exposures | Premium per Exposure |
|---|---|---|---|
| Red Trucks | 8,640,000 | 360 | 24,000 |
| Green Trucks | 4,480,000 | 560 | 8,000 |
| Total | 13,120,000 | 920 | 14,261 |

Exhibit 1B: Traditional Rate Change Calculations

Method 1: Average Rate per Exposure Unit

| [1] | [2] | [3] | [4] = [3] / [2] -1 |
|---|---|---|---|
| | Expiring Premium Per Exposure | Renewing Premium Per Exposure | Change |
| Red Trucks | 20,000 | 24,000 | 20.00% |
| Green Trucks | 10,000 | 8,000 | -20.00% |
| Total | 16,000 | 14,261 | -10.87% |

Methods 2 and 3: Weighted Average of Rate Changes

| [1] | [2] | [3] | [4] |
|---|---|---|---|
| | Change | Expiring Premium Weight | Renewing Premium Weight |
| Red Trucks | 20.00% | 75.00% | 65.85% |
| Green Trucks | -20.00% | 25.00% | 34.15% |
| Weighted Average | | 10.00% | 6.34% |

In this example, we show three traditional methods of measuring rate change:

1) Calculate the weighted average premium per exposure; measure this quantity for the renewal portfolio relative to the expiring portfolio for the rate change.[12]

2) Measure the rate change of each class or policy in the portfolio; blend these rate changes together using a weighted average; use expiring premium as the weights.[13]

3) Measure the rate change of each class or policy in the portfolio; blend these rate changes together using a weighted average; use renewing premium as the weights.[14]

Note that all of the traditional methods produce different answers; all of them measure the rate change approximately, but not one of them measures the rate change precisely.

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[12] See Jones [1], pp. 9 – 10, who focuses on average premium per exposure as a measure of rate change.
[13] See http://www.casact.org/education/reinsure/2008/handouts/schober.ppt. On slide 33, discussing commercial property, Schober suggests one "re-rate to expiring," which refers to taking renewal policies and re-rating them on the basis of the expiring coverage. The wording appears to imply that one should use expiring premium as the weighting basis.
[14] Vaughn [5], p. 503.

The exhibit below shows the proposed approach.

Exhibit 1C: Proposed Approach to Calculating Rate Change

| [1] | [2] | [3] | [4] = [3] * [2] | [5] | [6] = [5]/[4] - 1 |
|---|---|---|---|---|---|
| | Expiring Premium | Renewing Exposures / Expiring Exposures | Expiring Premium Restated For Change in Exposure | Renewing Premiums | Rate Change |
| Red Trucks | 12,000,000 | 0.60 | 7,200,000 | 8,640,000 | 20.00% |
| Green Trucks | 4,000,000 | 1.40 | 5,600,000 | 4,480,000 | -20.00% |
| Total | 16,000,000 | | 12,800,000 | 13,120,000 | 2.50% |

Exhibit 1D: Comparison Exhibit

| Method | Description | Calculated Rate Change |
|---|---|---|
| 1 | Ratio of Average Rate per Exposure Unit | -10.87% |
| 2 | Expiring Premium Weighted Average of Rate Changes | 10.00% |
| 3 | Renewing Premium Weighted Average of Rate Changes | 6.34% |
| Proposed | Restate Expiring Premium for Change in Loss Potential | 2.50% |

The proposed approach builds upon the prior conceptual discussion and Equation (4.8); thus, expiring premium must be "restated" for all shifts in loss potential before measuring rate change.[15] In Exhibit 1D, we see that the proposed approach can generate significantly different rate change factors than other methods.

[15] For the total portfolio, the premium must be restated for the shift in the total loss potential, which in turn depends upon the expected loss ratios of the various components of the portfolio. Here, however, we do not use any explicit assumptions about the components' loss ratios. Thus, implicitly, we presume that the expiring expected loss ratios for all the components are equal. Given that one has chosen to combine the various components into one portfolio for measuring loss ratio, the assumption of equal loss ratios by component is usually reasonable. However, if one were to combine different segments of business with clearly different expected loss ratios, one would need to explicitly reflect the different loss ratios by component when measuring the "shift in loss potential" for the total portfolio.

## 5.2 Dealing with a Portfolio of Heterogeneous Exposure Bases

The proposed framework for measuring rate change also allows us to solve the problem of how to deal with a portfolio with multiple, dissimilar exposure bases.

The exhibits below demonstrate the proposed approach.

Exhibit 2A: Dissimilar Exposure Bases

Expiring

| | Premium | Exposure Base | Exposures | Premium per Exposure |
|---|---|---|---|---|
| Jane's Contracting | 12,000,000 | sales (000s) | 600 | 20,000 |
| Jill's Stores | 4,000,000 | square feet (000s) | 400 | 10,000 |
| Total | 16,000,000 | undefined | undefined | undefined |

Renewing

| | Premium | Exposure Base | Exposures | Premium per Exposure |
|---|---|---|---|---|
| Jane's Contracting | 8,640,000 | sales (000s) | 360 | 24,000 |
| Jill's Stores | 4,480,000 | square feet (000s) | 560 | 8,000 |
| Total | 13,120,000 | undefined | undefined | undefined |

Exhibit 2B: Measuring "Change in Premium from Change in Exposure Base Units"

Proposed Approach to Measuring Rate Change

| [1] | [2] | [3] | [4] | [5] = [4] * [2] | [6] | [7] = [6] / [5] - 1 |
|---|---|---|---|---|---|---|
| | Expiring Premium | Exposure Base | Renewing Exposures / Expiring Exposures | Expiring Premium Restated For Change in Exposure | Renewing Premiums | Rate Change |
| Jane's Contracting | 12,000,000 | sales (000s) | 0.600 | 7,200,000 | 8,640,000 | 20.00% |
| Jill's Stores | 4,000,000 | square feet (000s) | 1.400 | 5,600,000 | 4,480,000 | -20.00% |
| Total | 16,000,000 | loss potential | 0.800 | 12,800,000 | 13,120,000 | 2.50% |

Measuring Exposure Change for Total Book

| [1] | [2] | [3] | [4] = [3] / [2] | [5] = [3] / [2] -1 |
|---|---|---|---|---|
| | Expiring Premium | Expiring Premium Restated For Change in Exposure | Ratio | Change in Premium from Changes in Exposure Base Units |
| Total | 16,000,000 | 12,800,000 | 0.800 | -20.00% |

Initially, the disparate exposure bases of the classes of business prevent us from measuring the exposure base change for the total book. However, by restating the expiring premium for shifts in exposure bases, we create a new way to measure total exposure base change; we simply measure the total change in premium arising from changes in exposure bases. Thus, the proposed procedure of restating expiring premium for shifts in loss potential provides a framework for measuring the total exposure base change for a portfolio that has multiple, incongruous exposure bases.

## 5.3 Measuring Rate Change When Limits and Deductibles Change

The proposed framework for measuring rate change also allows us to solve the problem of how to measure rate change when values of the limit and deductible of a renewing policy change from their values under an expiring policy, as demonstrated in the exhibits below:

## Exhibit 3A: Change in Deductibles

Expiring

| | Premium | Square Feet (000s) | Limit | Deductible | ILF Index = ILF(Limit) - ILF(Deductible) | Premium per Exposure |
|---|---|---|---|---|---|---|
| Joe's Stores | 13,500,000 | 900 | 1,000,000 | - | 1.00 | 15,000 |
| Bill's Stores | 9,000,000 | 900 | 1,000,000 | 250,000 | 0.50 | 10,000 |
| Total | 22,500,000 | 1,800 | | | | 12,500 |

Renewing

| | Premium | Square Feet (000s) | Limit | Deductible | ILF Index = ILF(Limit) - ILF(Deductible) | Premium per Exposure |
|---|---|---|---|---|---|---|
| Joe's Stores | 8,977,500 | 800 | 1,000,000 | 250,000 | 0.50 | 11,222 |
| Bill's Stores | 14,400,000 | 1,000 | 1,000,000 | - | 1.00 | 14,400 |
| Total | 23,377,500 | 1,800 | | | | 12,988 |

## Exhibit 3B: Traditional Rate Change Calculations

| Class | Change | Expiring Premium Weight | Renewing Premium Weight |
|---|---|---|---|
| Joe's Stores | 49.6% | 60.0% | 38.4% |
| Bill's Stores | -28.0% | 40.0% | 61.6% |
| Weighted Average | | 18.6% | 1.8% |

Exhibit 3C: Proposed Approach to Calculating Rate Change

| | [1] | [2] | [3] = [1] * [2] | [4] | [5] = [3] * [4] | [6] | [7] = [6] / [5] | [8] = [6] / [5] - 1 |
|---|---|---|---|---|---|---|---|---|
| | Expiring Premium | Renewing Exposures / Expiring Exposures | Expiring Premium Restated For Change in Exposure | Renewing ILF Index / Expiring ILF Index | Expiring Premium Restated For Change in Exposure **and** Change in Limits & Deductibles | Renewing Premium | Rate Change Factor | Rate Change |
| Joe's Stores | 13,500,000 | 0.889 | 12,000,000 | 0.50 | 6,000,000 | 8,977,500 | 1.496 | 49.6% |
| Bill's Stores | 9,000,000 | 1.111 | 10,000,000 | 2.00 | 20,000,000 | 14,400,000 | 0.720 | -28.0% |
| Total | 22,500,000 | | 22,000,000 | | 26,000,000 | 23,377,500 | 0.899 | -10.1% |

| | |
|---|---|
| Change in Premium from Change in Exposure ( = [3] total / [1] total - 1 ) | -2.2% |
| Change in Premium from Change in Limits, Deductibles ( = [5] total / [3] total - 1 ) | 18.2% |
| Change in Premium from Rate Change ( = [6] total / [5] total - 1 ) | -10.1% |

Exhibit 3D: Comparison Exhibit

| Method | Description | Calculated Rate Change |
|---|---|---|
| 1 | Expiring Premium Weighted Average of Rate Changes | 18.6% |
| 2 | Renewing Premium Weighted Average of Rate Changes | 1.8% |
| Proposed Approach | Adjust Expiring Premium for Change in Loss Potential | -10.1% |

Again, we see the importance of measuring rate change only after restating expiring premium for changes in loss potential.

### 5.3.1 Clarifying Which ILFs to Use

In the numerical example above (Exhibits 3A through 3D), we use ILFs (increased limits factors) to measure the change in loss potential from changing limits and deductibles. However, there is more than one type of ILF. "Loss ILFs" measure the relationship of loss costs at different limits and deductibles; they derive from measures of Limited Expected Value (LEV, aka LAS or Limited Average Severity). "Premium ILFs," however, measure the relationship of the premium the company charges for different limits and deductibles; they incorporate LEVs, risk load, and expenses. So when measuring rate change and restating premium for changes to limits and deductibles, which ILFs should one use?

Equation (4.8) demonstrates that when measuring rate change one must restate expiring premium for changes in loss potential. Therefore, when measuring rate change, it is more precise to restate expiring premium via Loss ILFs than via Premium ILFs; after one has used Loss ILFs to restate the expiring premium, one can then calculate the rate change factor as the ratio of renewing premium to restated expiring premium.

### 5.3.2 Tracking All Sources of Change

Exhibit 3C highlights another benefit of the proposed approach: the ability to completely track all changes to premium. Other methods for measuring rate change do not necessarily provide the framework to fully track the changes in premium and to connect the expiring premium to the renewing premium in a comprehensive way; nor do they identify the catalysts that are driving the changes in premium.

In contrast, the proposed approach allows one (as in Exhibit 3C) to measure all changes of premium, properly weighting together the changes of each policy or segment of the portfolio. In addition, applying all sources of change to the expiring premium will actually balance to the renewing premium. In other words, one can begin with expiring premium and then calculate:

*Expiring premium * (1+change in premium from exposure change) * (1+change in premium from change in limits & deductibles) * … * (1+ rate change) = Renewing premium [excluding new business]*

## 5.4 Change in Share

Sometimes a company writes a portion of a policy; for example, one company might take only a 50% "share" or "participation" in a given excess policy. The following exhibit describes such a situation:

Exhibit 4A: Change in Share

Expiring

| | Premium @100% share | Square Feet (000s) | Limit | Deductible | ILF Index = ILF(Limit) - ILF(Deductible) | Premium per Exposure | Company Share | Premium @Company share |
|---|---|---|---|---|---|---|---|---|
| Joe's Stores | 13,500,000 | 900 | 1,000,000 | - | 1.00 | 15,000 | 50% | 6,750,000 |
| Bill's Stores | 9,000,000 | 900 | 1,000,000 | 250,000 | 0.50 | 10,000 | 50% | 4,500,000 |
| Total | 22,500,000 | 1,800 | | | | 12,500 | | 11,250,000 |

Renewing

| | Premium @100% share | Square Feet (000s) | Limit | Deductible | ILF Index = ILF(Limit) - ILF(Deductible) | Premium per Exposure | Company Share | Premium @Company share |
|---|---|---|---|---|---|---|---|---|
| Joe's Stores | 8,977,500 | 800 | 1,000,000 | 250,000 | 0.50 | 11,222 | 25% | 2,244,375 |
| Bill's Stores | 14,400,000 | 1,000 | 1,000,000 | - | 1.00 | 14,400 | 75% | 10,800,000 |
| Total | 23,377,500 | 1,800 | | | | 12,988 | | 13,044,375 |

In Exhibit 4A, the values are the same as in Exhibit 3A, but with one important change: the company's share declines for the policy that receives a rate increase, whereas the company's share increases for the policy that receives a rate decrease. The following exhibit demonstrates the proposed approach of measuring rate change in such a situation:

Exhibit 4B: Proposed Approach to Calculating Rate Change

| | [1] | [2] | [3] | [4] | [5] = [3] * [4] | [6] | [7] = [6] / [5] | [8] = [7]-1 |
|---|---|---|---|---|---|---|---|---|
| | Expiring Premium @Company Share | Expiring Premium Restated For Change in Exposure | Expiring Premium Restated For Change in Exposure **and** Change in Limits & Deductibles | Renewing Share / Expiring Share | Expiring Premium Restated For Change in Exposure **and** Change in Limits & Deductibles **and** Change in Share | Renewing Premium | Rate Change Factor | Rate Change |
| Joe's Stores | 6,750,000 | 6,000,000 | 3,000,000 | 0.50 | 1,500,000 | 2,244,375 | 1.496 | 49.6% |
| Bill's Stores | 4,500,000 | 5,000,000 | 10,000,000 | 1.50 | 15,000,000 | 10,800,000 | 0.720 | -28.0% |
| Total | 11,250,000 | 11,000,000 | 13,000,000 | | 16,500,000 | 13,044,375 | 0.791 | -20.9% |

| | |
|---|---|
| Change in Premium from Change in Exposure ( = [2] total / [1] total - 1 ) | -2.2% |
| Change in Premium from Change in Limits, Deductibles ( = [3] total / [2] total - 1 ) | 18.2% |
| Change in Premium from Change in Company Share ( = [5] total / [3] total - 1 ) | 26.9% |
| Change in Premium from Rate Change ( = [6] total / [5] total - 1 ) | -20.9% |

Note that the rate change for each individual policy is unaffected by the change in company share; thus, each policy's rate change in Exhibit 4B is exactly equal to the value calculated in Exhibit 3C. However, there is now a significant difference in the rate change for the overall portfolio. Thus accurately measuring rate change for the portfolio requires that one use information about each policy's share; conversely, measuring rate change by first "grossing up" each policy's share to a common 100% basis can potentially lead to an imprecise rate change calculation for the portfolio.

## 6. SUMMARY

Quantitative analysis that projects an expected loss ratio often makes use of historical experience data and rate change factors. The appropriate application of such an analysis and the accurate calculation of rate change factors require a clear understanding of the conceptual foundations that undergird these methods. Having explored these foundational concepts, we conclude that the key goal of analyzing historical data is to forecast the interrelationship of losses and premiums for the projected book of business. Thus, when calculating rate change factors, one must first restate expiring premium for changes in all sources of loss potential, including changes in exposure base units, limits and deductibles, company share, etc. As a result, one can take the theory of measuring rate change factors and apply it towards solving problems in practice.

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### Biography of the Author

**Neil Bodoff** is senior vice president at Willis Re; he currently focuses on client solutions, with emphasis on property and casualty insurance and reinsurance. Neil is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. Please contact the author at neil.bodoff@willis.com or neil_bodoff@yahoo.com.