An Actuarial Model of Excess of Policy Limits Losses

Neil M. Bodoff, FCAS, MAAA

Abstract

Motivation. Excess of policy limits (XPL) losses is a phenomenon that presents challenges for the practicing actuary.

Method. This paper proposes using a classic actuarial framework of frequency and severity, modified to address the unique challenge of XPL.

Results. The result is an integrated model of XPL losses together with non-XPL losses.

Conclusions. A modification of the classic actuarial framework can provide a suitable basis for the modeling of XPL losses and for the pricing of the XPL loss component of reinsurance contracts.

Keywords. Excess of Policy Limits. XPL. ERM. Modeling.

1. INTRODUCTION

Excess of policy limits (XPL) losses is a phenomenon that presents challenges for the practicing actuary. For example, exposure rating, one of the standard actuarial methods for pricing reinsurance layers, seems to be completely unworkable for the challenge of pricing XPL losses; yet often, an exposure rating approach to reinsurance pricing is the only method that the practicing actuary has at his disposal.

In this paper, I propose an approach that incorporates XPL into the classic actuarial framework of frequency, severity, and limited expected value (LEV) of claims. In this way, XPL will simply be part of a broader landscape of claims behavior, and can draw upon and seamlessly integrate with standard actuarial tools for incorporating the price of XPL losses into the pricing of reinsurance contracts. In addition, using the classic actuarial framework allows one to incorporate XPL losses into stochastic economic capital models that are used for insurer enterprise risk management (ERM) purposes.

1.1 Research Context

The actuarial literature has very limited discussion of actuarial approaches to modeling of excess of policy limits losses. I have found only one paper by Braithwaite and Ware [1], which remains a crucially important paper.

1.2 Objective

In this paper, I propose a framework that builds upon the work of Braithwaite and Ware yet differs in some ways.

There are two main reasons for this difference in approach. The first reason relates to aligning resources with need. XPL is an important actuarial problem but by no means the paramount problem typically facing actuaries. As a result, I would like to propose a reasonable methodology that is more practicable than the one proposed in Braithwaite and Ware. Whereas Braithwaite and Ware's model required the actuary to build an additional, freestanding size-of-loss curve to describe XPL, this paper proposes a methodology that simply extends one's existing size-of-loss curve, greatly simplifying the implementation.

The second reason that the proposed approach differs from Braithwaite and Ware is the need to quantify XPL losses in the context of a broader insurance portfolio; one ought to model and price for XPL in conjunction with other non-XPL losses. Braithwaite and Ware, discussing clash reinsurance treaties, focuses entirely on XPL losses. Yet the practitioner actuary often desires to price for XPL losses in working layer reinsurance; only a small percentage of losses will be XPL whereas the majority of losses will be non-XPL. The task, then, is to price these reinsurance layers for the XPL losses in a framework that aligns with traditional actuarial pricing methods. Similarly, another situation that requires modeling of XPL losses together with non-XPL losses is enterprise risk management (ERM), in which one seeks to model all the insurance risk of the company. Modeling requires an integrated framework that covers XPL and non-XPL losses together, which will be facilitated by the proposed new approach.

2. ACTUARIAL MODEL OF SIZE OF LOSS DISTRIBUTION WITH EXTENSION TO XPL

We begin with the classic actuarial framework for evaluating loss costs in layers with a focus on limited expected value (LEV). Following Clark [2], we can write that

X = random variable for size of loss

$F_X(x)$ = probability that random variable X, the size of loss, is less than or equal to x

$f_X(x)$ = probability density function, first derivative of F(x)

E[X] = expected value or average unlimited loss

E[X;k] = expected value of loss capped at k

The expected value of loss capped at an amount k can be defined as follows:

$$LEV(X,k) = E[X;k] = \int_0^k xf(x)dx + \int_k^\infty kf(x)dx \qquad (2.1)$$

$$LEV(X,k) = E[X;k] = \int_0^k xf(x)dx + k[1-F(k)] \qquad (2.2)$$

2.1 Limited Expected Value (LEV)

Historically, actuaries needed to quantify the value of the average loss limited by the insurance policy; they adopted limited expected value (LEV) as the framework to calculate this value, under the assumption that a policy limit caps the insurance loss.

2.2 Incorporating XPL Losses

In light of our knowledge of XPL losses, we should revisit whether LEV is the ideal way to measure losses to an insurance policy. Let's describe the average loss accruing to an insurance policy as the Policy Limited Expected Value (PLEV). Until now, the implicit assumption has been that PLEV = LEV.

The phenomenon of XPL losses shows us, however, that the policy limit written in the insurance policy contract is not always potent in capping losses. Thus the identity function, PLEV = LEV, is not fully accurate.

What could be a paradigm for how to think about the phenomenon of XPL losses? I propose that we begin to think of the effectiveness of the policy limit as being subject to a random variable.

Let's define a random variable Z, which follows a Bernoulli distribution. This random variable can have a value of 1, or "success", with probability p, and can have a value of 0, "failure", with probability 1-p. When Z=1 we have "success" and the policy limit caps the insurance loss; when Z=0 we have "failure" and the policy limit does not cap the insurance loss and we have an XPL situation.

Now we can say that the Policy Limited Expected Value is:

$$PLEV(X,k,Z)=\int_0^k xf(x)dx+P(Z=1)*\int_k^\infty kf(x\mid Z=1)dx+P(Z=0)*\int_k^\infty xf(x\mid Z=0)dx \tag{2.3}$$

Recalling that the probability that Z=1 is p and that Z=0 is 1-p, we write:

$$PLEV(X,k,Z)=\int_0^k xf(x)dx+p\int_k^\infty kf(x\mid Z=1)dx+(1-p)\int_k^\infty xf(x\mid Z=0)dx \tag{2.4}$$

If we let x = k + (x-k) in the final integral, we can rewrite equation (2.4) is as follows:

$$PLEV(X,k,Z)=\int_0^k xf(x)dx+k[1-F(k)]+(1-p)\int_k^\infty (x-k)\,f(x\mid Z=0)dx \tag{2.5}$$

One can say that on a fundamental level, equation (2.5) captures the approach crystallized in Braithwaite and Ware. The additional loss above and beyond the policy limit follows a different conditional probability density function than the initial size of loss distribution; as a result, the XPL loss component is a completely new entity that is grafted onto the non-XPL loss component.

3. A MORE PRACTICAL MODEL

How can we make this model more practical and easier to use? Let's revisit equation (2.4) and make some simplifying assumptions.

Let's assume that the probability density function above the policy limit is not conditional on whether or not an XPL scenario has been triggered. As explained in Braithwaite and Ware, the XPL situation arises when the policyholder is found liable for actual damage to a third party; the only question is whether or not the insurance company's conduct provides a basis for the courts to override the capping effect of the policy limit. Thus, this simplifying assumption should be reasonable for XPL (although perhaps not for extra-contractual obligations, ECO).

We can then substitute the unconditional f(x) into equation (2.4) by replacing the conditional f(x|Z=0) and f(x|Z=1) and rewrite equation (2.4) as follows:

$$PLEV(X,k,Z) = \int_0^k xf(x)dx + p\int_k^\infty kf(x)dx + (1-p)\int_k^\infty xf(x)dx \tag{3.1}$$

Thus we simply say that if random variable Z=1 we have a success and the policy limit caps the loss and if Z=0 we have a failure and the policy limit does not cap the loss. Unlike equation (2.5) and unlike the approach of Braithwaite and Ware, the XPL loss is not a completely new entity; rather, the XPL loss is simply an extension of the standard size-of-loss distribution that occurs when the policy limit's capping effect is ineffective. Such a framework would be much easier to work with when attempting to incorporate XPL losses.

3.1 Practical Applications: Insurance Risk Modeling

How can we apply the proposed paradigm of equation (3.1) in a practical way to achieve a tangible result? One possibility would be in a simulation environment.

3.1.1 Simulation Application #1: Collective Risk Model for Insurance Losses

Step #1: Define the size of loss distribution for an insurance policy or portfolio of policies on a gross of policy limit basis.

Step #2: Simulate individual losses and simulate the limit of the policy associated with each loss.

Step #3: For each loss, if the loss is greater than the policy limit, then simulate Z, a Bernoulli random variable. If Z=1, then cap the simulated loss at the policy limit. If Z=0, then do not cap the loss.

Notice that there is only one small new step here: rather than always capping the loss at the policy limit, let the capping be subject to the outcome of a random variable that reflects whether the policy limit will be effective at capping the loss or not.

3.1.2 Simulation Application #2: Catastrophe ("Cat") Modeling

The software vendors for cat modeling typically employ several steps in their calculations of the losses to an insurance portfolio for a given simulated cat event. After the software simulates a catastrophic ("cat") event, the software evaluates how the physical phenomenon affects the physical structures in its path. Then, in one of the final steps, the software overlays the insurance policy's contractual terms to achieve the financial loss to the company. Within this simulation environment, the final step could evolve away from the current deterministic view of the policy limit and towards a stochastic view of the policy limit. Moreover, one could consider correlating the individual probabilities that the policy limits fail; the correlation could depend upon geographical location and legal jurisdiction, among other factors. An approach to cat modeling simulations that treats policy limit capping of losses as a probable but not definite outcome would be more realistic and would show more severe risk metric output than current models.

3.2 Reinsurance Pricing

A second practical application of the proposed paradigm of equation (3.1) could be reinsurance pricing.

Recall that traditional exposure rating is viewed as not producing loss cost indications that encompass XPL. After all, XPL losses by definition exceed the policy limit and thus exceed the exposure; how could exposure rating possibly incorporate XPL within its framework?

Let's revisit equation (3.1):

$$PLEV(X,k,Z) = \int_0^k xf(x)dx + p\int_k^\infty kf(x)dx + (1-p)\int_k^\infty xf(x)dx \tag{3.1}$$

If we multiply the first term on the right side of equation (3.1) by 1 and let 1 = p + 1 – p and rearrange terms, we can rewrite equation (3.1) as follows:

$$PLEV(X,k,Z) = p[\int_0^k xf(x)dx + \int_k^\infty kf(x)dx] + (1-p)[\int_0^k xf(x)dx + \int_k^\infty xf(x)dx] \tag{3.2}$$

This is also the same as the following:

$$PLEV(X,k,Z) = p[\int_0^k xf(x)dx + \int_k^\infty kf(x)dx] + (1-p)[\int_0^\infty xf(x)dx] \tag{3.3}$$

And:

$$PLEV(X,k,Z) = p(LEV(X,k)) + (1-p)E[X] \tag{3.4}$$

Equations (3.3) and (3.4) demonstrate that in the presence of XPL losses, we have a loss severity that has probability p of being limited by the policy limit and probability (1-p) of not being limited by the policy limit.

We can use this framework to calculate expected layer loss for excess-of-loss reinsurance exposure rating.

Following Clark, for each policy we want to calculate the exposure factor, i.e. the percentage of the policy's total loss that is covered by the reinsurance layer.

$$\textit{Exposure Factor} = \frac{\textit{layer loss}}{\textit{total loss}} \tag{3.5}$$

Now let's calculate the layer loss.

Layer loss = Loss limited at the top of the reinsurance layer – loss limited at the bottom of the reinsurance layer (3.6)

Here, we have a probability p that the policy limit will cap the loss and a 1-p probability that the policy limit will not cap the loss. While these probabilities apply to the primary policy, we assume that they do not apply at all to the reinsurance limit and attachment point.

Thus, when estimating the loss limited by the top of the reinsurance layer, we have a probability p that the loss will be capped by the lesser of the policy limit and the top of the reinsurance layer; we also have a probability 1-p that the loss will be capped solely by the top of reinsurance layer, with no application of the policy limit.

Loss limited at top of reinsurance layer = p * LEV (X, min(policy limit, reinsurance exit point)) + (1-p) * LEV (X, reinsurance exit point) (3.7)

Note: Reinsurance exit point = reinsurance attachment point + reinsurance limit

Similarly, when estimating the loss limited by the bottom of the reinsurance layer, we have a probability p that the loss will be capped by the lesser of the policy limit and the bottom of the reinsurance layer; we also have a probability 1-p that the loss will be capped solely by the bottom of reinsurance layer.

Loss limited at bottom of reinsurance layer = p * LEV (X, min(policy limit, reinsurance attachment point)) + (1-p) * LEV (X, reinsurance attachment point) (3.8)

Thus:

Layer loss = p * LEV (X, min(policy limit, reinsurance exit point)) + (1-p) * LEV (X, reinsurance exit point) – {p * LEV (X, min(policy limit, reinsurance attachment point)) + (1-p) * LEV (X, reinsurance attachment point)} (3.9)

Thus:

Layer loss = p * traditional exposure rating layer LEV subject to primary policy limit + (1-p) * layer LEV not subject to primary policy limit (3.10)

Having calculated the layer loss, which is the numerator of the exposure factor, we now need to calculate the denominator, the policy's total loss.

Recall that the exposure factor produces layer loss by multiplying the policy's total loss; total loss is usually calibrated based on policy premium multiplied by an Expected Loss Ratio (ELR). Therefore, whether or not the ELR was calculated to include a provision for XPL losses will affect how one ought to calculate the denominator of the exposure factor.

For our discussion, let's proceed under the assumption that the ELR does not include a provision for XPL loss. As a result, when calculating the "total loss" for the denominator of the exposure factor, we will calculate it based only on non-XPL losses.

Denominator of Exposure Factor = Same as traditional exposure rating = Policy total loss excluding XPL = LEV(X, policy limit)	(3.11)

Then, combining equations (3.9) and (3.11), we derive:

Exposure Factor = [p * LEV (X, min(policy limit, reinsurance exit point)) + (1-p) * LEV (X, reinsurance exit point) – {p * LEV (X, min(policy limit, reinsurance attachment point)) + (1-p) * LEV (X, reinsurance attachment point)}] / LEV(X, policy limit)	(3.12)

Or, more simply, combining equations (3.10) and (3.11), we derive:

Exposure Factor = [p * traditional exposure rating layer LEV subject to primary policy limit + (1-p) * layer LEV not subject to primary policy limit] / traditional exposure rating ground up LEV capped at policy limit	(3.13)

3.2.1 Reinsurance Pricing: Numerical Example

Now let's do a numerical example of the proposed algorithm. The goal is to generate layer loss costs via exposure rating that include a loss provision for XPL losses.

First, let's stipulate some hypothetical numerical values for our policy limits distribution:

Exhibit 1

1	2	3
Policy Limit	% of premium	ELR%
50,000	1.0%	65.0%
100,000	1.0%	65.0%
500,000	2.0%	65.0%
1,000,000	80.0%	65.0%
2,000,000	10.0%	65.0%
3,000,000	1.0%	65.0%
4,000,000	1.0%	65.0%
5,000,000	3.0%	65.0%
10,000,000	1.0%	65.0%

We also need values for our size-of-loss severity curve:

Exhibit 2

Item #	Description	Value
1	Curve	Pareto
2	Theta	50,000
3	Alpha	1.50

Finally, we need to input parameter values for probability p that a policy limit will successfully cap losses and 1-p that the policy limit will not cap losses; the values may vary for each policy. Here we select a simple parameter structure in which all the policies in our limits table have the same value for p.

Exhibit 3

	p	1-p
All Policy Limits < $25M	99%	1.00%
Policy Limit = $25M	100%	0.00%

We now apply the proposed methodology to the numerical values to produce the following output in Exhibit 4.

Exhibit 4

1	2	3	4	5	6
			Layer Losses as % of total ground up losses	Layer Losses as % of total ground up losses	Implied Loading for XPL
Layer	Limit	Attachment	Traditional Exposure Rating	Proposed Method Including XPL	Proposed / Traditional - 1
1	500,000	-	88.420%	88.440%	0.023%
2	500,000	500,000	10.067%	10.074%	0.072%
3	1,000,000	1,000,000	1.150%	1.219%	5.989%
4	3,000,000	2,000,000	0.333%	0.403%	21.057%
5	5,000,000	5,000,000	0.031%	0.068%	119.369%
6	15,000,000	10,000,000	0.000%	0.033%	#N/A
Total			100.000%	100.237%	0.237%

Column 6 of Exhibit 4 shows the "loading factor" for each layer loss attributable to XPL. What is notable about this output is that choosing one simple value for p creates layer loading factors for XPL that are different for the various layers. Also, these loading factors for XPL would be different for other portfolios with different policy limits distributions, even with no change in the underlying value of the p parameters.[1]

[1] A copy of the Microsoft Excel workbook with the supporting calculations is available from the author upon request.

4. CONCLUSIONS

In this paper, I propose an actuarial paradigm for describing excess of policy limits (XPL) losses. The central idea is that one can envision a random variable governing the application of the policy limit; most of the time the policy limit is enforced as it is written in the insurance contract, whereas other times the policy limit is superseded. This paradigm is quite parsimonious; therein lies its attractiveness. At the same time, this simple framework can generate nuanced, differentiated, useful, and non-obvious output information for practicing actuaries. One practical application would be to incorporate XPL losses into actuarial exposure rating estimates for casualty excess-of-loss reinsurance layers; the output values vary based on the attachment point and limit of the reinsurance layer being priced as well as the granular policy limits usage of the particular insurance portfolio under review. A second practical application would be to incorporate XPL losses in a simulation environment such as commercial software for estimating losses arising from natural catastrophes; envisioning policy limits as being random variables can affect the cat modeling and thus the critical risk metrics of an insurer's portfolio.

5. REFERENCES

[1] Braithwaite, Paul, and Bryan C. Ware, "Pricing Extra-Contractual Obligations and Excess of Policy Limits Exposures in Clash Reinsurance Treaties", *CAS Forum*, Spring 1997.

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Biography of the Author

Neil Bodoff is Executive Vice President at Willis Re Inc. He is a Fellow of the Casualty Actuarial Society.

Contact the author at neil.bodoff@willis.com and neil_bodoff@yahoo.com