The Analysis of "All-Prior" Data

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Abstract

Motivation. Some data sources, such as the NAIC Annual Statement – Schedule P as an example, contain a row of all-prior data within the triangle. While the CAS literature has a wealth of papers that have developed various methods for estimating tail factors, and the CAS Tail Factor Working Party recently published a report on tail factor methods, tail factors are not *directly* applicable to all-prior data.[1] Moreover, the author is not aware of any papers dealing directly with the analysis of all-prior data. Absent a defined methodology, it seems to be common practice for an analysis of data triangles that include an all-prior row to either exclude the all-prior data or to make the explicit assumption that the case reserves, or case plus IBNR reserves, for these claims are adequate. This may be reasonable in certain situations but given the potential materiality of this part of the reserve it would be a useful addition to the actuary's toolkit to develop some methods for analyzing the all-prior data or for testing the reasonability of assuming the case reserves, or case plus IBNR reserves, are adequate.

Method. The process followed in this paper is to both graphically and formulaically illustrate the data issues and analysis, then apply the concepts of a well-known method with three different data sets. While only a deterministic point estimate method is illustrated in this paper, the framework should be quite easily adaptable to other deterministic methods or stochastic models. The paper also illustrates the calculations for this method and examples in a companion Excel spreadsheet.

Conclusions. The methods used for any standard analysis can be adapted to accommodate all-prior data whenever it is present. Even in cases where the all-prior reserves prove adequate, the process of analyzing the all-prior data will help calibrate the tail factor used for all years by validating the selected tail factor using actual data.

Availability. The Excel spreadsheets created for this paper "All Prior Analysis.xlsm" and "Creating All Prior Data.xls" are available at http://www.casact.org/pubs/forum/14fforum/.

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1. INTRODUCTION

From our training in the art and science of actuarial practice, familiarity with basic data triangles and a wide variety of methods and models[2] for extrapolating that data to its ultimate value is a way of life for casualty actuaries. Recently, a significant portion of published CAS papers and research has been devoted to the analysis and quantification of the distribution of future payments[3] and tail factors[4] in order to greatly enhance the usefulness of a "standard" unpaid claim estimate analysis. However, the author is unaware of any research or papers related to the estimation of unpaid claims for the all-prior data found in some triangles.

[1] While it may be tempting to simply apply the tail factor to the all-prior data, we will see that this is not a sound practice.

[2] Keeping with the definitions of methods and models in [4], the primary feature that distinguishes a model from a method is that a model is used to calculate a "distribution of possible outcomes" whereas a method will only produce a single point estimate.

[3] See for example [4], which includes a large number of research papers in the Reference section.

[4] See for example [5].

Estimating future payments for unpaid claims is often referred to as "squaring" the triangle when there is no claim development beyond the end of the triangle. Development beyond the end of the triangle, or the calculation of tail factors, can be thought of as the analysis of what's beyond the end of or "to the right of" the square. Similarly, ratemaking and pricing can be thought of as the analysis of what comes after or "below" the triangle. The purpose of this paper is to introduce the analysis of what's before or "above" the triangle.

As we will see, the analyses "to the right of" and "above" the triangle are related, so this paper will build a bridge from the analysis and application of tail factors to the analysis of all-prior data. Once this bridge is built, it should be possible to adapt this framework to other deterministic methods and to stochastic models for estimating distributions of possible outcomes for the all-prior data.

1.1 Research Context

From a research perspective, this paper deals mainly with unpaid claim estimate analysis and presents a new method for a subset of the data in a typical analysis. Along the way, the paper will also review data organization related to unpaid claim estimates and then show its applicability for this new method. While not specifically addressed in this paper, other methods for calculating point estimates and models used for unpaid claim variability and the calculation of uncertainty and distributions could also be adapted to use the all-prior data in a similar fashion, although within the specific frameworks of those methods and models.

1.2 Objective

The two primary goals of this paper are to provide the practicing actuary with some new tools for the analysis of all-prior data and to develop the foundation for further research in this area.

1.3 Outline

In order to achieve these goals, Section 2 will start by reviewing and slightly expanding the notation used by recent CAS research Working Parties for describing unpaid claim estimation methods and models. Section 3 will then review the basic data structure of all-prior data and show, both graphically and formulaically, how the calculation of tail factors can be extended to include all-prior data. Section 4 will apply this basic methodology to the chain ladder method to illustrate that estimates of all-prior data are not only possible but a very useful extension of existing techniques. Finally, some possible areas for future research will be suggested in Section 5 and conclusions will be discussed in Section 6.

2. NOTATION

For the sake of uniform notation, we will use the notation from the CAS Working Party on Quantifying Variability in Reserve Estimates Summary Report [2] and expanded by the CAS Tail Factor Working Party [5], since it was intended to serve as a basis for further research. Many models visualize loss statistics as a two dimensional array. The row dimension is the period[5] by which the loss information is subtotaled, most commonly an accident period.[6] For each accident period, w, the (w,d) element of the array is the total of the loss information as of development age d.[7] For this discussion, we assume that the loss information available is an "upper triangular" subset of the two-dimensional array for rows $w = 1,2,\ldots,n$. For each row, w, the information is available for development ages 1 through $n - w + 1$. If we think of period n as not only the most recent accident period, but also the latest accounting period for which loss information is available, the triangle represents the loss information as of accounting dates 1 through n. The "diagonal" for which $w + d$ equals a constant, k, represents the loss information for each accident period w as of accounting period k.[8]

In general, the two-dimensional array will extend to columns $d = 1, 2,\ldots,n$.[9] For purposes of calculating tail factors, we are interested in understanding the development beyond the observed data for periods $d = n+1, n+2,\ldots,u$, where u is the ultimate time period for which any claim activity occurs – i.e., u is the period in which all claims are final and paid in full. As an aide to any reader not familiar with this notation, a graphical representation of each item is contained in Appendix F. [10]

The paper uses the following notation for certain important loss statistics:

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5 Most commonly the periods are annual (years), but as most methods can accommodate periods other than annual we will use the more generic term "period" to represent year, half-year, quarter, month, etc. unless noted otherwise.

6 Other exposure period types, such as policy period and report period, also utilize tail factor methods. For ease of description, we will use the generic term "accident" period to mean all types of exposure periods, unless otherwise noted.

7 Depending on the context, the (w,d) cell can represent the cumulative loss statistic as of development age d or the incremental amount occurring during the d^{th} development period.

8 For a more complete explanation of this two-dimensional view of the loss information see the *Foundations of Casualty Actuarial Science* [7], Chapter 5, particularly pages 210-226.

9 Some authors define $d = 0,1,\ldots,n-1$ which intuitively allows $k = w$ along the diagonals, but in this case the triangle size is $n \times n\text{-}1$ is not intuitive. With $d = 1, 2,\ldots,n$ defined as in this paper, the triangle size $n \times n$ is intuitive but then $k = w+1$ along the diagonals is not as intuitive. A way to think about this which helps tie everything together is to assume the w variables are the beginning of the accident periods and the d variables are at the end of the development periods. Thus, if we are using years then cell $c(n,1)$ represents accident year n evaluated at $12/31/n$, or essentially $1/1/n+1$.

10 Readers familiar with this notation could skip ahead to section 3.2. Even if you are not familiar with the notation, it is recommended to focus on the concepts in section 3.1 which should be familiar and not get bogged down in the notation. The Notation sheet in the "All Prior Analysis.xlsm" companion file should also be useful for gaining an understanding of the notation.

$c(w,d)$: cumulative loss from accident period w as of age d. Think "when" and "delay."

$q(w,d)$: incremental loss for accident period w during the development age from d - 1 to d. Note that $q(w,d) = c(w,d) - c(w,d-1)$.

$c(w,u) = U(w)$: total loss from accident period w when at the end of ultimate development u.

$R(w)$: future development after age $d = n - w + 1$ for accident period w, *i.e.*, = $U(w) - c(w, n - w + 1)$.

$D(k)$: future development after age $d = n - w + 1$ during calendar period k, *i.e.*, for all $q(w,d)$ where $w + d = k$ and $w + d > n + 1$.

$A(d)$: all-prior data by development age d.

$f(d) = 1 + v(d)$: factor applied to $c(w,d)$ to estimate $c(w,d+1)$ or more generally any factor relating to age d. This is commonly referred to as a link ratio. $v(d)$ is referred to as the 'development portion' of the link ratio, which is used to estimate $q(w,d+1)$. The other portion, the number one, is referred to as the 'unity portion' of the link ratio.

$F(d)$: ultimate development factor relating to development age d. The factor applied to $c(w,d)$ to estimate $c(w,u)$ or more generally any cumulative development factor relating to development age d. The capital indicates that the factor produces the ultimate loss level. As with link ratios, $V(d)$ denotes the 'development portion' of the loss development factor, the number one is the 'unity portion' of the loss development factor.

$T = T(n)$: ultimate tail factor at end of triangle data, which is applied to the estimated $c(w,n)$ to estimate $c(w,u)$.

$\hat{x}$ an estimate of any value or parameter x.

What are called factors here could also be summands, but if factors and summands are both used, some other notation for the additive terms would be needed. The notation does not distinguish paid *vs.* incurred, but if this is necessary, capitalized subscripts P and I could be used.

3. ALL-PRIOR ANALYSIS OVERVIEW

In order to analyze the all-prior data, we must start by understanding the make-up of this data and how it is related to the main triangle data as it is commonly understood. But before we delve into the all-prior data, we will start with a triangle array of cumulative data, illustrated in Table 3.1, and a typical method for estimating unpaid claims excluding any all-prior data.

Table 3.1 – Loss Triangle Data

w \ d	1	2	3	…	n-1	n
1	c(1,1)	c(1,2)	c(1,3)	…	c(1,n-1)	c(1,n)
2	c(2,1)	c(2,2)	c(2,3)	…	c(2,n-1)	
3	c(3,1)	c(3,2)	c(3,3)			
…	…	…				
n-1	c(n-1,1)	c(n-1,2)				
n	c(n,1)					

3.1 A Typical Unpaid Claim Estimate

As an example, a typical deterministic analysis of this data will start with an array of link ratios or development factors:

$$f(w,d) = \frac{c(w,d+1)}{c(w,d)}. \tag{3.1}$$

Then two key assumptions are made in order to make a projection of the known elements to their respective ultimate values. **First**, it is typically assumed that each accident period has the same development factor. Equivalently, for each $w = 1, 2, \ldots, n-d$:

$$f(w,d) = f(d).$$

Under this first assumption, one of the more popular estimators for the development factor is the weighted average:[11]

$$\hat{f}(d) = \frac{\sum_{w=1}^{n-d} c(w,d+1)}{\sum_{w=1}^{n-d} c(w,d)}. \tag{3.2}$$

Certainly there are other popular estimators in use, but they are beyond our scope at this stage and nothing is gained by exploring other estimators. Suffice it to say that many methods and their corresponding estimators are still consistent with our first assumption that each accident period has the same factor. There are, of course, methods that do not rely on this assumption that all accident periods use the same development factor,[12] but they are beyond the scope of this paper so that we can focus on a basic understanding of the analysis process.

Assuming there is no claim development beyond the end of the triangle, projections of the ultimate values, $\hat{c}(w,u)$ [or $\hat{c}(w,n)$ since $u = n$ in this case], for $w = 2, 3, \ldots, n$, are then computed using:

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[11] The popularity of this estimator may stem from it being unbiased as shown by Mack [8] and others.

[12] For example methods that trend the data can directly or indirectly result in different factors for each accident period.

$$\hat{c}(w,n) = c(w,d)\prod_{i=d}^{n-1} \hat{f}(i) \text{, for all } d = n - w + 1. \tag{3.3}$$

For completeness, carrying out the calculations for formula (3.3) sequentially for each $\hat{f}(i)$ is often done to estimate each future $\hat{c}(w,d)$, and then by subtraction each future $\hat{q}(w,d)$ is used to estimate cash flows (for paid data). Alternatively, ultimate development factors can be calculated as:

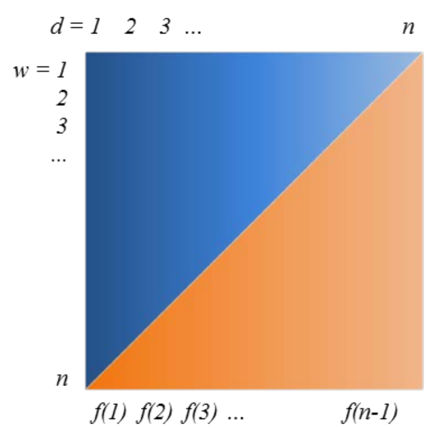
$$\hat{F}(d) = \prod_{i=d}^{n-1} \hat{f}(i) \text{, for each } d = 1, 2, \ldots, n-1. \tag{3.4}$$

And then formula (3.3) simplifies to:

$$\hat{c}(w,n) = c(w,d) \times \hat{F}(d) \text{, for all } d = n - w + 1. \tag{3.5}$$

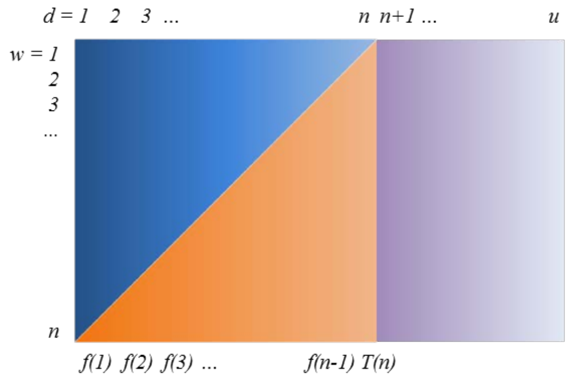
This part of the claim projection algorithm relies explicitly on the **second** assumption, namely that each accident period has a parameter representing its relative level. These level parameters are the current cumulative values for each accident period, or $c(w, n-w+1)$. Of course variations on this second assumption are also common, but the point is that every method has explicit assumptions that are an integral part of understanding the quality of that method. Graphically, our estimation model looks like Graph 3.1, where the blue triangle is the data we know and the orange triangle is estimated.

Graph 3.1 – Loss Estimation without a Tail



If the assumption of no claim development past the end of the triangle is true, then as we will see the analysis needs no further extensions as the all-prior data would similarly need no extrapolation beyond the end of the triangle. On the other hand, it is quite common to expect development beyond the end of the triangle, in which case a tail factor is generally used to extrapolate to the end of the expected development or the ultimate period, u. We can illustrate this graphically by expanding Graph 3.1 to include tail development, as shown in Graph 3.2, where the rectangle in purple is the tail extrapolation.

Graph 3.2 – Loss Estimation with a Tail



There are a variety of methods for estimating a tail factor, $T(n)$, but we will only use one of the common methods, namely, the exponential decay method.[13] The method utilizes link ratios, $f(d) = 1 + v(d)$, and assumes that the $v(d)$s decay at a constant rate, r, i.e., $v(d_{i+1}) = v(d_i) \times r$. The process consists of first fitting an exponential curve to the $v(d)$s, which can be accomplished by using a regression with the natural logarithms (natural log) of the $v(d)$s. Next, the decay constant r can be estimated as the inverse natural log of the slope of the fitted curve. The remaining development, from a given development age d, can be estimated as:

$$T(d) = \prod_{i=1}^{\infty}(1 + v(d) \times r^i), \text{ for } d \geq n. \qquad (3.6)$$

While formula (3.6) is infinite in theory, in practice the incremental factors in this formula, $\hat{f}(d) = 1 + v(d) \times r^i$, will get close enough to one[14] such that no new development is expected or the development is small enough to stop. Thus, one of the decision points for a typical tail factor selection is determining the ultimate number of periods or u. The goal of this analysis is to complete the "rectangle" and estimate the future cumulative values, as illustrated in Table 3.2.

Table 3.2 – Cumulative Loss Triangle Data with Estimated Ultimate Projections

		d							
		1	**2**	**3**	**…**	**n-1**	**n**	**…**	**u**
w	**1**	**c**(1,1)	**c**(1,2)	**c**(1,3)	…	**c**(1,n-1)	**c**(1,n)	…	*ĉ(1,u)*
	2	**c**(2,1)	**c**(2,2)	**c**(2,3)	…	**c**(2,n-1)	*ĉ(2,n)*	…	*ĉ(2,u)*
	3	**c**(3,1)	**c**(3,2)	**c**(3,3)	…	*ĉ(3,n-1)*	*ĉ(3,n)*	…	*ĉ(3,u)*
	…	…	…	…	…	…	…	…	…
	n-1	**c**(n-1,1)	**c**(n-1,2)	*ĉ(n-1,3)*	…	*ĉ(n-1,n-1)*	*ĉ(n-1,n)*	…	*ĉ(n-1,u)*
	n	**c**(n,1)	*ĉ(n,2)*	*ĉ(n,3)*	…	*ĉ(n,n-1)*	*ĉ(n,n)*	…	*ĉ(n,u)*

Of course for an analysis using cumulative data it is a simple step to subtract the last known value

13 For a more complete discussion of tail factor methods see [5]. The exponential decay method is shown in the "Tail Factors" sheet in the "All Prior Analysis.xlsm" file.

14 Under certain circumstances the regression can result in increasing factors with could become infinite, but when this happens the method is normally discarded as being unreasonable.

for each accident period from the estimated ultimate value to arrive at the estimated unpaid for each accident period w using formula (3.7).

$$\hat{R}_{(w)} = \hat{c}(w,u) - c(w,n-w+1) \qquad (3.7)$$

For our purposes, we will also take the additional step of converting the cumulative values to incremental values, as illustrated in Table 3.3.

Table 3.3 – Incremental Loss Triangle Data with Estimated Ultimate Projections

w \ d	1	2	3	…	n-1	n	…	u
1	**q**(1,1)	**q**(1,2)	**q**(1,3)	…	**q**(1,n-1)	**q**(1,n)	…	$\hat{q}(1,u)$
2	**q**(2,1)	**q**(2,2)	**q**(2,3)	…	**q**(2,n-1)	$\hat{q}(2,n)$	…	$\hat{q}(2,u)$
3	**q**(3,1)	**q**(3,2)	**q**(3,3)	…	$\hat{q}(3,n-1)$	$\hat{q}(3,n)$	…	$\hat{q}(3,u)$
…	…	…	…	…	…	…	…	…
n-1	**q**(n-1,1)	**q**(n-1,2)	$\hat{q}(n-1,3)$	…	$\hat{q}(n-1,n-1)$	$\hat{q}(n-1,n)$	…	$\hat{q}(n-1,u)$
n	**q**(n,1)	$\hat{q}(n,2)$	$\hat{q}(n,3)$	…	$\hat{q}(n,n-1)$	$\hat{q}(n,n)$	…	$\hat{q}(n,u)$

From the estimated incremental values we have an estimate of the unpaid claims for each accident period w using formula (3.8) to sum the estimated incremental values.

$$\hat{R}_{(w)} = \sum_{d=n-w+2}^{d=u} \hat{q}(w,d) \qquad (3.8)$$

Also, adding the estimates for each accident period, we can derive a formula for the total estimated unpaid as shown in formula (3.9).

$$\hat{R}_{(T)} = \sum_{w=1}^{w=n} \hat{R}_{(w)} = \sum_{w=1}^{w=n} \sum_{d=n-w+2}^{d=u} \hat{q}(w,d) \qquad (3.9)$$

Using the estimated incremental values we can also create an estimate of the future cash flows by calendar period k using formula (3.10) to sum the estimated incremental values along the diagonal instead of by row.

$$\hat{D}_{(k)} = \sum_{w=1}^{w=n} \hat{q}(w,k-w), \text{ for } n+2 \le k \le u+1$$
$$\hat{D}_{(k)} = \sum_{w=k-u}^{w=n} \hat{q}(w,k-w), \text{ for } u+2 \le k \le u+n \qquad (3.10)$$

For the formulas in (3.10), the first one is for complete diagonals (all rows) as k increases from $n+2$ to $u+1$, while in the second formula the diagonals are shrinking each period as k goes from $u+2$ to $u+n$.[15] Similarly, adding the estimates for each calendar period we can derive a formula for the total estimated unpaid as shown in formula (3.11).

$$\hat{R}_{(T)} = \sum_{k=n+2}^{k=n+u} \hat{D}_{(k)} = \sum_{k=n+2}^{k=u+1} \sum_{w=1}^{w=n} \hat{q}(w,k-w) + \sum_{k=u+2}^{k=n+u} \sum_{w=k-u}^{w=n} \hat{q}(w,k-w) \qquad (3.11)$$

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[15] Keep in mind that $k = w + d$ and the last row is contained in each diagonal sum, so the incremental values from $\hat{q}(n,2)$ to $\hat{q}(n,u)$ are part of the details in formulas (3.10) and (3.11).

3.2 The All-Prior Data

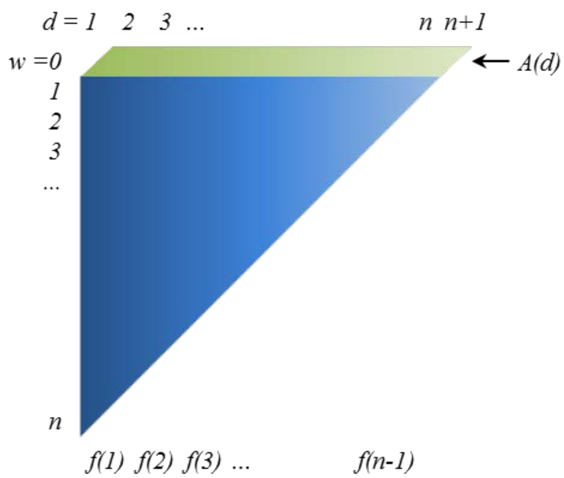
With this brief review complete, we can now expand the analysis by examining the all-prior data. First, the basic loss development triangle will include the extra row as shown in Table 3.4.

Table 3.4 – Loss Triangle Data with All-Prior Row

		d 1	2	3	…	n-1	n	n+1
w	0		**A**(2)	**A**(3)	…	**A**(n-1)	**A**(n)	**A**(n+1)
	1	**c**(1,1)	**c**(1,2)	**c**(1,3)	…	**c**(1,n-1)	**c**(1,n)	
	2	**c**(2,1)	**c**(2,2)	**c**(2,3)	…	**c**(2,n-1)		
	3	**c**(3,1)	**c**(3,2)	**c**(3,3)				
	…	…	…					
	n-1	**c**(n-1,1)	**c**(n-1,2)					
	n	**c**(n,1)						

Graphically the addition of all-prior data can be illustrated in Graph 3.3, with the all-prior data shown in green.

Graph 3.3 – Loss Triangle with All-Prior Data



The color and shape for the all-prior data is significant for three reasons. First, while the main triangle can be either cumulative or incremental values, the all-prior data could be either[16] but, more importantly, it is a combination of multiple periods and as such we need to introduce new notation, $A(d)$, for the cells in the all-prior row. Second, the addition of this extra row does not always include

[16] Technically, it is possible to use either incremental or cumulative data in the underlying data used to calculate the all-prior row. In addition, all development periods for $d = 1, 2, ..., u$ could be included or only the periods beyond the end of the triangle or $d = n+1, ..., u$. For purposes of this paper we will assume the underlying data is incremental and use all development periods.

any value in the first column(s)[17] so the overall shape is no longer strictly triangular. And third, because the data includes multiple periods at different stages of development we can't ***directly*** apply the factors from our typical analysis to extend it for the analysis of the all-prior row.

The all-prior data is included in accounting statements so that a triangle large enough to show all development can be truncated by collapsing the triangle down to a specific maximum size, while still including all of the relevant claim information for reconciliation with the balance sheet. Thus, the all-prior row is actually a summary of the claim activity for all claims that occurred prior to the first accident period ($w=1$) in the triangle as of the date of the financial statement.

As there can be different ways of compiling the all-prior data, the key to any analysis is to first understand exactly what is in the data or how it was created. As a common source of all-prior data is the NAIC Annual Statement Schedule P (for companies operating in the United States), we will use those rules here which result in each all-prior cell being the calendar period (i.e., diagonal) sum of all prior accident periods.[18] Rather than spending time and space here dissecting the NAIC rules [10], we direct the interested reader to the "Creating All Prior Data.xls" companion file, which uses one data set to walk through the rules for compiling Schedule P and then reconciles this with a more direct calculation. To illustrate this we can restate Table 3.4 as Table 3.5.

Table 3.5 – Loss Triangle Data with All-Prior Row Details

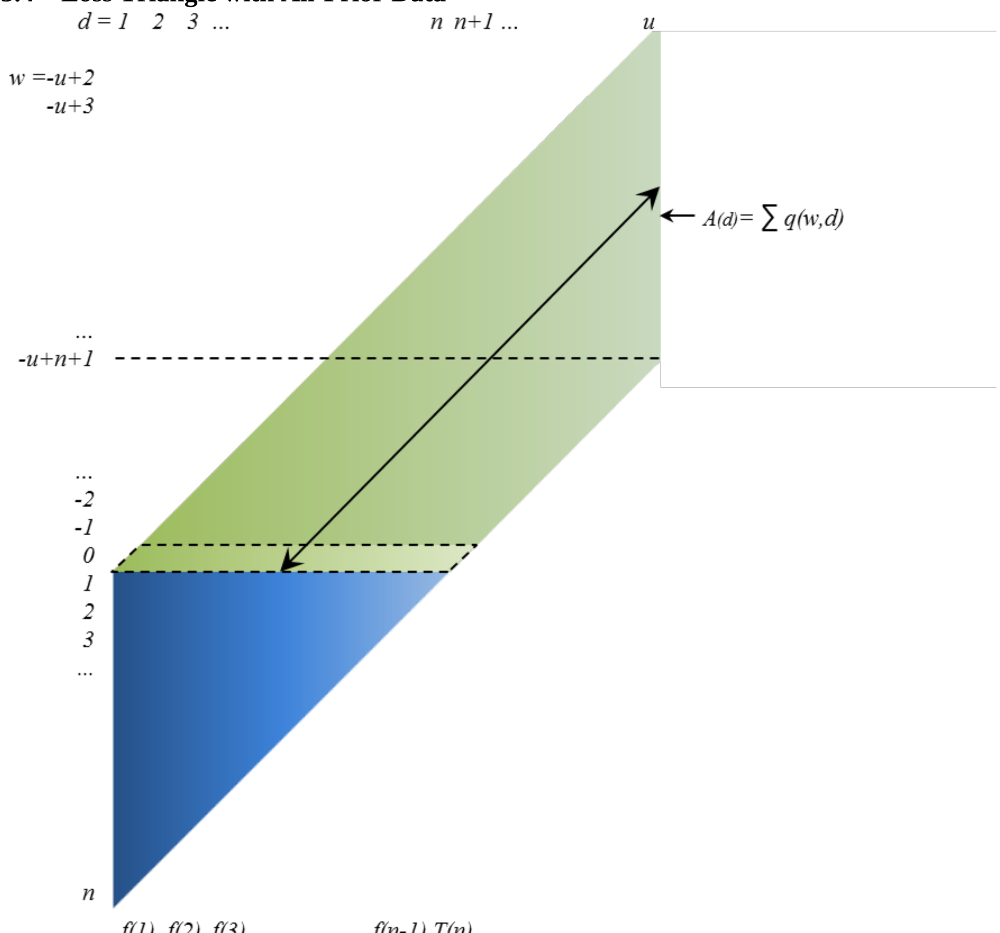
w \ d	1	2	3	…	n	n+1	n+2	n+3	…	u
-u+2										**q**(-u+2,u)
-u+3									…	**q**(-u+3,u)
…						…	…	…	…	
-2				…	**q**(-2,n)	**q**(-2,n+1)	**q**(-2,n+2)	**q**(-2,n+3)		
-1			**q**(-1,3)	…	**q**(-1,n)	**q**(-1,n+1)	**q**(-1,n+2)			
0		**q**(0,2)	**q**(0,3)	…	**q**(0,n)	**q**(0,n+1)				
1	**c**(1,1)	**c**(1,2)	**c**(1,3)	…	**c**(1,n)					
2	**c**(2,1)	**c**(2,2)	**c**(2,3)	…						
3	**c**(3,1)	**c**(3,2)	**c**(3,3)	…						
…	…	…								
n-1	**c**(n-1,1)	**c**(n-1,2)								
n	**c**(n,1)									

As we are assuming the all-prior data starts with $A(2)$, the first diagonal will include all incremental cells were $k = w + d = 2$, so the earliest accident period with data should be $-u+2$ and the earliest accident period with data in development period u should be $-u+n+1$. Graphically, we can illustrate this as shown in Graph 3.4.

[17] Of course none of the columns need to be missing or blank, but for purposes of this paper we will assume the first column $A(1)$ is blank and include data in columns $A(2)$ and later to be consistent with the NAIC Schedule P. In Schedule P the paid data for $A(2)$ is zero, but for incurred data it only contains reserves and no payments.

[18] Two useful references for understanding the all-prior data in the NAIC Schedule P are [6] and [10].

Graph 3.4 – Loss Triangle with All-Prior Data



Now we can more precisely define each cell in the all-prior row of data using formula (3.12), which is the diagonal sum of the claim activity in those periods.[19, 20]

$$A_{(k)} = \sum_{w=-u+k}^{w=0} q(w,k-w) \text{, for } k = 2,3,\ldots,n+1.^{21} \qquad (3.12)$$

It is not a coincidence that the diagonal sum of the all-prior row stretches out for the same number of periods, u, as we will expect for the tail factor. Indeed, if we can get the incremental data that was used to create the all-prior row then we can use this to calibrate the length of the tail factors.

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[19] Technically, $A_{(2)}$ could be the sum of all diagonals prior to $A_{(3)}$, thus the first cell in the graph would be a different color and Graphs 3.4 and 3.5 could be extended even further, but our focus will be on the incremental changes in the $A_{(k)}$, so we can ignore this technicality.

[20] Of course if the company did not start writing business that long ago, then claims for these older accident years would not exist at all and any estimates of the all-prior unpaid claims would need to be adjusted accordingly. For purposes of this paper we will assume business was written at least as early as is implied by the ultimate tail extrapolation.

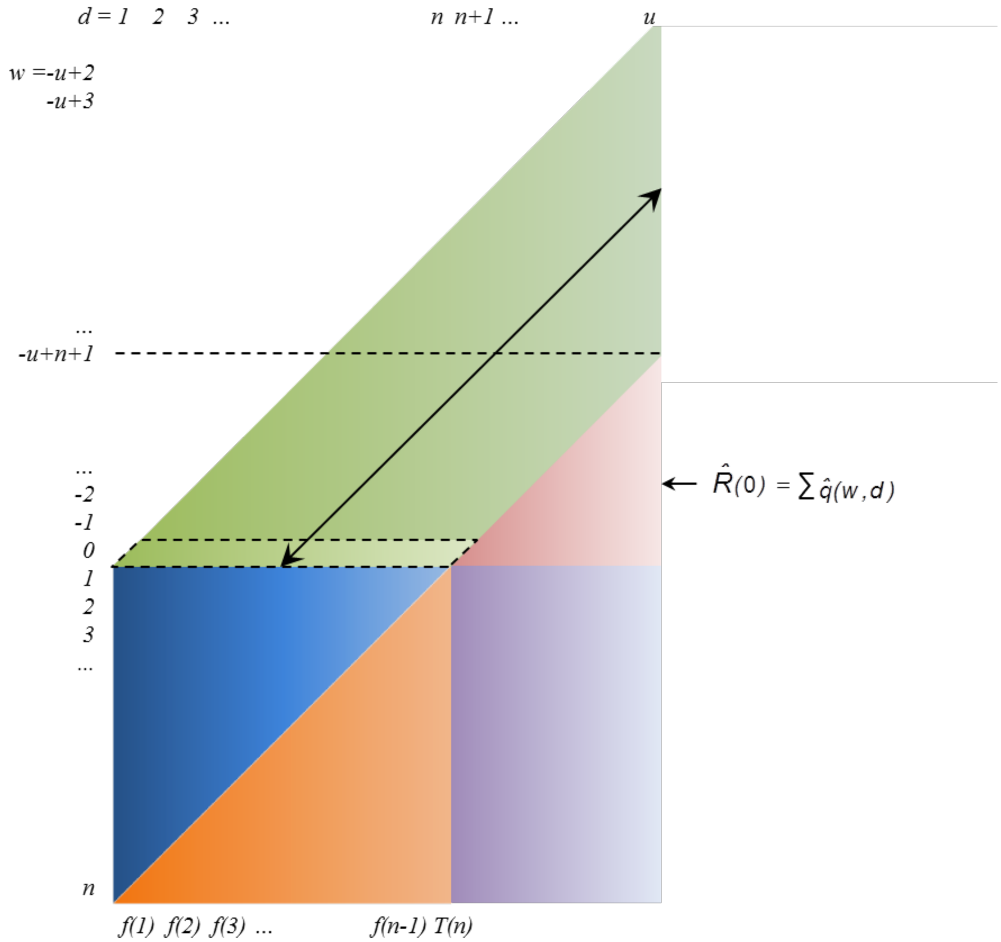
[21] In Graphs 3.3 and 3.4, we used d with our notation for the all-prior row, $A_{(d)}$, since it is used in those contexts consistent with development columns. In formula (3.12) and beyond we switch to using k in our notation for the all-prior row, $A_{(k)}$, since we are illustrating how this is a diagonal sum of the incremental values. For the all-prior row $d = k$, so they can be used interchangeably.

The last step in examining the all-prior row is to define the unpaid claims we need to estimate as the sum of the future all-prior diagonals. Graphically, we can combine Graph 3.2 with Graph 3.4 and illustrate the unpaid claim estimate we are working toward in red in Graph 3.5.

Completing the description for our all-prior estimate, we need to develop methods to solve for the future incremental cells for the all-prior data that will allow us to use formula (3.13) to estimate the total unpaid claims for the all-prior data.

$$\hat{R}_{(0)} = \sum_{k=n+2}^{k=u} \hat{A}_{(k)} = \sum_{k=n+2}^{k=u} \sum_{w=-u+k}^{w=0} \hat{q}(w, k-w) \qquad (3.13)$$

Graph 3.5 – Loss Estimation with All-Prior Data and a Tail



3.3 All-Prior Analysis

Even though we have more clearly delineated the problem, we can't just apply the tail factors we would use for the rest of the analysis because those factors are based on cumulative values and, even if we have the incremental details for the all-prior row, we can't calculate the appropriate cumulative values unless we have all of the claim data, not just the data used to calculate the all-prior row. In effect, to use a normal tail factor we would need the entire triangle for all periods – i.e., a $u \times u$ triangle[22] instead of an $n \times n$ triangle. If we had all of the data for the $u \times u$ triangle, then we could use formula (3.6) (or something similar) to successively apply a different factor $T(d)$ to each accident period for each $d > n$. Then again, if we have that data we would not need to calculate tail factors or use all-prior data.

[22] In keeping with the notation in Graph 3.5, the rows for the u x u triangle would run from $-u+n+1$ to n. Renumbering by adding $u-n$ to each row, the rows would then run from 1 to u.

Whenever we don't have complete cumulative data for every accident period that is part of the all-prior data, we will need to make some assumptions about the history prior to our data triangle in order to use our normal tail factors. For example, we could use the Bornhuetter-Ferguson [3] algorithm which uses an a priori estimate of the total losses and the loss development pattern to derive an estimate. With premium and/or exposure data prior to the data triangle, we can apply the Bornhuetter-Ferguson algorithm to estimate the cumulative values for the prior periods.

4. ALL-PRIOR METHODS

In order to illustrate the calculations for, and the usefulness of, the analysis of all-prior data within a typical deterministic analysis, three data sets were simulated, each with all of the historical data needed to estimate the all-prior unpaid claims.[23] While the data is simulated, it was done in a way to make it look real and tested using methods such as those suggested in Venter [12] and other sources to make sure it has realistic statistical properties. The three data sets approximate companies with three different case reserving philosophies, "medium" case reserves, "low" case reserves and "high" case reserves, respectively, as well as different exposures and development patterns. Within the body of the paper, we will only review and primarily discuss the "medium" scenario, but the analysis and results for the other two are contained in the Appendices.[24]

In addition to having simulated claim triangles for 10 years with an all-prior row, we are also assuming that we have 11 years of earned premium and expected loss ratios for the years in the all-prior row to approximate what you might find in practice (i.e., for the 11 years prior to the oldest year in the triangle). For the older periods where this information is unavailable (i.e., prior to those 11 years), we derive estimates for premium and expected loss ratios as you would need to do in practice. The paid data for the "medium" scenario is shown in Table 4.1.

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[23] The simulated data is for complete 30 x 30 rectangles, with different development, exposure growth, parameters, etc., but all of the simulated data is fully developed prior to 30 periods. This size was chosen to be consistent with the limits of flexibility set up in the companion Excel file. Each data set was then collapsed into 10 x 10 triangles, with an all-prior row, to illustrate the analysis. In addition, the prior 11 years of premiums and "ultimate" loss ratios are included to approximate the information you could obtain from the oldest accident years in the 11 Annual Statements prior to the current year.

[24] The complete details for all three scenarios are also included in the "All Prior Analysis.xlsm" file. The interested reader can select a different data set in cell "V1" on the Data sheet and recalculate the sheet to see the calculations for any of the scenarios.

Table 4.1 – "Medium" Paid Loss Triangle with All-Prior Data

	12	24	36	48	60	72	84	96	108	120	132
A-P		-	124,151	196,502	234,850	256,775	269,143	276,080	279,086	281,182	282,390
2004	74,998	189,335	252,351	284,850	301,895	311,600	317,040	319,748	321,762	322,784	
2005	92,015	216,237	283,370	316,672	335,600	346,804	352,535	356,275	357,748		
2006	90,909	191,270	262,856	289,054	310,018	319,763	325,725	328,463			
2007	100,503	215,220	271,927	315,048	333,808	343,553	348,988				
2008	94,647	225,979	295,390	330,250	349,553	359,694					
2009	99,464	204,539	271,740	308,343	329,792						
2010	83,463	200,265	274,434	309,186							
2011	76,140	184,681	255,177								
2012	112,865	243,840									
2013	100,689										

Extending the chain ladder method for a triangle of data that includes an all-prior row, the steps to our analysis can be summed up as follows:

1) Calculate the age-to-age factors excluding the all-prior row,
2) Extrapolate the age-to-age factors and select a tail factor,
3) Estimate the cumulative data for each prior accident period which is part of the all-prior row,
4) Estimate the incremental data for each prior accident period (from Step 3) and sum the diagonals to estimate the values in the all-prior row,
5) Use comparisons of the estimated all-prior row data to the actual all-prior row data to evaluate and calibrate the selected factors,
6) Re-select, re-estimate and re-calibrate (repeat Steps 2 through 5) as needed, and
7) Sum all future diagonals for each prior accident period to estimate the all-prior row reserves.

4.1 Calculate Age-to-Age Factors

The first step is to calculate the age-to-age factors or link ratios for the data triangle. Using formula (3.2), and excluding the all-prior (A-P) row, the weighted average age-to-age factors for this data are shown in Table 4.2. [25]

[25] Note that if you are trying to reproduce the calculated values in the Tables in this paper, the actual values are generally unrounded in Excel so you may encounter rounding differences.

Table 4.2 – "Medium" Paid Loss Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Tail
2004	2.525	1.333	1.129	1.060	1.032	1.017	1.009	1.006	1.003	
2005	2.350	1.310	1.118	1.060	1.033	1.017	1.011	1.004		
2006	2.104	1.374	1.100	1.073	1.031	1.019	1.008			
2007	2.141	1.263	1.159	1.060	1.029	1.016				
2008	2.388	1.307	1.118	1.058	1.029					
2009	2.056	1.329	1.135	1.070						
2010	2.399	1.370	1.127							
2011	2.426	1.382								
2012	2.160									
VWA	2.268	1.332	1.126	1.063	1.031	1.017	1.009	1.005	1.003	
5-Yr VWA	2.270	1.328	1.128	1.064	1.031	1.017	1.009	1.005	1.003	
3-Yr VWA	2.308	1.359	1.126	1.062	1.030	1.017	1.009	1.005	1.003	
TF Fitted	1.395	1.213	1.115	1.062	1.034	1.018	1.010	1.005	1.003	1.003
User	2.250									
Selected	2.250	1.332	1.126	1.063	1.034	1.018	1.010	1.005	1.003	1.0015
Ultimate	3.856	1.714	1.287	1.143	1.075	1.040	1.021	1.012	1.006	1.0033
% Paid	25.9%	58.4%	77.7%	87.5%	93.0%	96.2%	97.9%	98.9%	99.4%	99.7%
% Unpaid	74.1%	41.6%	22.3%	12.5%	7.0%	3.8%	2.1%	1.1%	0.6%	0.3%

In addition to the volume weighted average (VWA) factors from formula (3.2), other averages are shown in Table 4.2 to mimic a more typical process in practice where the actuary would compare different averages to select their age-to-age factors. A user entered row is also included and the selected factors by development period are outlined.

4.2 Select a Tail Factor

Using formula (3.6), we can also estimate a tail factor, including the incremental age-to-age factors that comprise the tail factor, which by itself is a factor to ultimate. The tail factor calculation for the paid data is illustrated in Table 4.3. Note that while the incremental factors that make up the tail factor could be ignored in an analysis without an all-prior row, they are a necessary part of this analysis since we need to estimate the incremental values that sum to the all-prior row data and we will need tail factors for $d > n$ in order to estimate the all-prior unpaid claims. Note also that age-to-age and tail factors can often be rounded to 3 decimal places in practice, but in order to calibrate the incremental tail factors with the ultimate development length of the data, u, more than 3 decimal places may be needed to help identify more precisely how many periods to include in the tail.

Table 4.3 – "Medium" Paid Tail Factor Calculation

		All Prior		
Tail Years: 12	Actual	282,390	Decay	0.540
Tail Factor: 1.0033	Estimated	303,022	Intercept	0.732
	Error %	7.3%		

Period	Factor	Dev	Log	Excl	Period	Log	Fitted	Selected	ATA	ATU
1	2.26832	1.26832	0.238	Y			1.395339		1.395339	2.155306
2	1.33162	0.33162	(1.104)	Y			1.213371		1.213371	1.544647
3	1.12622	0.12622	(2.070)		3	(2.070)	1.115159		1.115159	1.273022
4	1.06314	0.06314	(2.762)		4	(2.762)	1.062153		1.062153	1.141560
5	1.03099	0.03099	(3.474)		5	(3.474)	1.033545		1.033545	1.074760
6	1.01707	0.01707	(4.070)		6	(4.070)	1.018105		1.018105	1.039878
7	1.00923	0.00923	(4.685)		7	(4.685)	1.009771		1.009771	1.021386
8	1.00516	0.00516	(5.267)		8	(5.267)	1.005274		1.005274	1.011502
9	1.00318	0.00318	(5.752)		9	(5.752)	1.002846		1.002846	1.006195
10							1.001536		1.001536	1.003339
11							1.000829		1.000829	1.001800
12							1.000447		1.000447	1.000970
13							1.000242		1.000242	1.000523
14							1.000130		1.000130	1.000281
15							1.000070		1.000070	1.000151
16							1.000038		1.000038	1.000080
17							1.000020		1.000020	1.000042
18							1.000011		1.000011	1.000022
19							1.000006		1.000006	1.000011
20							1.000003		1.000003	1.000005
21							1.000002		1.000002	1.000002

4.3 Estimate Prior Cumulative Values

With the development factors and tail factor calculated it is a simple matter to "rectangle"[26] the triangle, so that will not be illustrated here.[27] Instead we will examine a process for estimating the incremental values that comprise the all-prior row of data shown in Table 4.1. To do this we can use the prior earned premiums, estimated ultimate loss ratios, estimated percent paid (from Table 4.2), and Bornhuetter-Ferguson methodology to estimate the cumulative paid for each prior year, as illustrated in Table 4.4.

For example, from the simulated data we know that the premium for 2003 is 468,659 and the estimated ultimate loss ratio is 71.6%.[28] Combining this with the estimated percent paid at 24 months from Table 4.2 of 54.8% we can estimate the cumulative losses for 2003 as 468,659 x .716 x .548 = 195,823. The estimated values for all years shown in Table 4.4, for development periods from 24 to 120 months were calculated using the same methodology. Using these estimated cumulative values at 120 months for each prior accident year, we can then use the incremental (age-to-age) tail factors from Table 4.3 to estimate the remaining cumulative values to ultimate.

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[26] Technically, it is more precise to say we are "rectangling" the triangle when we have a tail, but as a square is a type of rectangle, some may prefer to think of "squaring" in more general terms meaning turning the triangle into either a square or rectangle.

[27] While some calculations are skipped (or knowledge of the calculations is assumed) in the body of the paper, they are all contained in the companion Excel file "All Prior Analysis.xlsm" for easy reference.

[28] See the Data sheet in the "All Prior Analysis.xlsm" file.

Table 4.4 – "Medium" Paid All-Prior Projection (Cumulative)

	Premium	Loss Ratio	24	36	48	60	72	84	96	108	120	132
1984	*402,171*	*70.0%*	*164,287*	*218,768*	*246,380*	*261,937*	*270,724*	*275,625*	*278,319*	*279,786*	*280,583*	281,014
1985	*406,193*	*70.0%*	*165,930*	*220,956*	*248,844*	*264,557*	*273,431*	*278,382*	*281,102*	*282,584*	*283,389*	283,824
1986	*410,255*	*70.0%*	*167,589*	*223,165*	*251,332*	*267,202*	*276,165*	*281,165*	*283,913*	*285,410*	*286,222*	286,662
1987	*414,357*	*70.0%*	*169,265*	*225,397*	*253,846*	*269,874*	*278,927*	*283,977*	*286,752*	*288,264*	*289,085*	289,529
1988	*418,501*	*70.0%*	*170,958*	*227,651*	*256,384*	*272,573*	*281,716*	*286,817*	*289,619*	*291,147*	*291,975*	292,424
1989	*422,686*	*70.0%*	*172,667*	*229,927*	*258,948*	*275,299*	*284,534*	*289,685*	*292,516*	*294,058*	*294,895*	295,348
1990	*426,913*	*70.0%*	*174,394*	*232,226*	*261,537*	*278,052*	*287,379*	*292,582*	*295,441*	*296,999*	*297,844*	298,302
1991	*431,182*	*70.0%*	*176,138*	*234,549*	*264,153*	*280,832*	*290,253*	*295,508*	*298,395*	*299,969*	*300,823*	301,285
1992	*435,494*	*70.0%*	*177,899*	*236,894*	*266,794*	*283,640*	*293,155*	*298,463*	*301,379*	*302,968*	*303,831*	304,298
1993	439,848	69.1%	*177,368*	*236,187*	*265,998*	*282,794*	*292,280*	*297,572*	*300,479*	*302,064*	*302,924*	303,389
1994	472,929	64.9%	*179,117*	*238,515*	*268,620*	*285,581*	*295,161*	*300,505*	*303,441*	*305,041*	*305,910*	306,380
1995	412,911	75.1%	*180,964*	*240,975*	*271,390*	*288,526*	*298,205*	*303,604*	*306,570*	*308,187*	*309,064*	309,539
1996	460,127	68.0%	*182,592*	*243,143*	*273,831*	*291,122*	*300,888*	*306,335*	*309,328*	*310,960*	*311,845*	312,324
1997	471,803	67.0%	*184,472*	*245,646*	*276,651*	*294,120*	*303,986*	*309,490*	*312,514*	*314,162*	*315,056*	315,540
1998	443,804	71.9%	*186,215*	*247,968*	*279,265*	*296,899*	*306,858*	*312,414*	*315,467*	*317,130*	*318,033*	318,522
1999	448,454	71.9%	*188,166*	*250,565*	*282,191*	*300,009*	*310,073*	*315,687*	*318,772*	*320,453*	*321,365*	321,859
2000	439,491	74.1%	*190,048*	*253,071*	*285,013*	*303,010*	*313,174*	*318,844*	*321,960*	*323,658*	*324,579*	325,078
2001	499,204	65.9%	*191,981*	*255,646*	*287,912*	*306,092*	*316,360*	*322,088*	*325,235*	*326,950*	*327,881*	328,384
2002	447,766	74.2%	*193,888*	*258,184*	*290,772*	*309,132*	*319,502*	*325,286*	*328,465*	*330,197*	*331,137*	331,646
2003	468,659	71.6%	*195,823*	*260,762*	*293,675*	*312,218*	*322,691*	*328,534*	*331,744*	*333,493*	*334,443*	334,956

	Growth	Loss Ratio
Prior to 1993	*1.0%*	*70.0%*

Note that the cumulative projections in Table 4.4 extend 12 periods beyond 120 months to match the number of periods used for the tail factor selection in Table 4.3,[29] but we have included a total of 20 pre-2004 accident years since that's how many periods of all-prior data we will need to estimate the all-prior row in the next steps. Thus, in addition to the 11 years of prior earned premiums and estimated ultimate loss ratios we have, we need to make some additional assumptions for years prior to the those 11, namely a 1% growth rate and an expected loss ratio of 70% were assumed. Of course whether you have any premium and loss ratio data prior to the start of the triangle or not, the materiality of these assumptions can be stronger than the tail factor assumption when "calibrating" these assumptions by estimating the actual all-prior data.

4.4 Estimate Prior Incremental Values

After estimating the projected cumulative values, the projected incremental values are estimated by a simple subtraction, as illustrated in Table 4.5. With the incremental values, we can also sum along the diagonal using formula (3.11) to compare these estimated values with the actual incremental values from the data in Table 4.1.

[29] To keep Table 4.4 from becoming unreadable only projections to 132 months are shown, but all projections can be seen in the companion "All Prior Analysis.xlsm" file.

Table 4.5 – "Medium" Paid All-Prior Projection (Incremental)

	12	24	36	48	60	72	84	96	108	120	132	144
1994												254
1995											475	257
1996										885	479	259
1997									1,648	894	484	262
1998								3,053	1,664	903	489	264
1999							5,614	3,085	1,681	912	494	267
2000						10,164	5,670	3,116	1,698	921	499	270
2001					18,180	10,268	5,728	3,147	1,715	931	504	272
2002				32,587	18,360	10,370	5,785	3,179	1,732	940	509	275
2003			64,939	32,913	18,543	10,473	5,842	3,210	1,750	949	514	278
Totals:	(144+)	(36-132)	36	48	60	72	84	96	108	120	132	144
Estimated	1,309	303,022	138,094	73,886	41,383	23,068	12,720	6,947	3,774	2,044	1,106	*598*
Actual		282,390	124,151	72,351	38,348	21,925	12,368	6,937	3,006	2,096	1,208	
Differences		20,632	13,943	1,535	3,035	1,143	352	10	768	(52)	(102)	
Cumulative Percent Difference			7.3%	4.2%	6.0%	4.5%	3.8%	4.7%	9.7%	-4.6%	-8.4%	
Weights			0.25	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	
Weighted Average			0.4%									

4.5 Compare to Actual & Calibrate

Comparing the estimates to the actual all-prior data we can see in Table 4.5 that the differences are not too far off.[30] The totals for both the actual and estimated all-prior row are also included in Table 4.3, which shows the estimates are 7.3% higher than the actual values. While the cumulative percentage difference of 7.3% is useful for gauging all of the assumptions for the all-prior row, it tends to be heavily influenced by the early development periods and is, thus, not usually responsive to changes in the tail factor assumptions. To calibrate the tail factor assumptions, it is much better to focus on the cumulative percent differences close to the end of the triangle, or use a weighted average of all cumulative differences with much more weight given to later development periods, which shows a difference of 0.4%, as illustrated in Table 4.5.

The process of using the all-prior estimates to help "calibrate" the tail factor assumptions (i.e., what are reasonable for $v(d)$ and u) can be quite useful in practice. For example, if we had used only 3 decimal places in the tail factors in Table 4.3, and thus only 2 years appear to be needed in the tail,[31] the weighted average of the cumulative percentage differences changes to -14.9% instead of +0.4%. Of course either $v(d)$ or u, or both, can be adjusted to see whether changing the tail factor assumption improves the fit of the estimated all-prior data to the actual data, thus validating the tail factor

[30] Again for readability values beyond 144 months of development are excluded from Table 4.5 so the diagonal values will not sum to the values in the Incremental row without referencing all of the values in the companion Excel file.

[31] Since all fitted factors beyond the 11th period in Table 4.3 would round to 1.000.

assumption with actual data in the all-prior row.[32]

To illustrate a more complete validation process, Table 4.6 summarizes key results when changing the number of years in the tail estimation from 1 to 14 years. Of course the actual validation process in practice can include other assumptions and methods for calculating the tail, but in the end judgment is required for making the final selections.

Table 4.6 – "Medium" Paid Tail Calibration Summary

		All-Prior Projection					Change in IBNR	
Tail Years	(*u*) Ultimate	Total Difference	Cumulative Percent	Weighted Percent	IBNR	Total IBNR	All-Prior	Total
1	11	16,039	5.7%	-28.1%	(1,323)	176,381		
2	12	18,173	6.4%	-14.9%	(1,045)	179,629	278	3,248
3	13	19,311	6.8%	-7.8%	(746)	181,532	299	1,903
4	14	19,920	7.1%	-4.0%	(506)	182,639	241	1,107
5	15	20,245	7.2%	-2.0%	(334)	183,279	172	640
6	16	20,419	7.2%	-0.9%	(218)	183,647	116	368
7	17	20,512	7.3%	-0.4%	(143)	183,857	75	211
8	18	20,562	7.3%	0.0%	(97)	183,978	47	120
9	19	20,588	7.3%	0.1%	(68)	184,046	29	68
10	20	20,602	7.3%	0.2%	(51)	184,085	17	39
11	21	20,619	7.3%	0.3%	(31)	184,116	20	31
12	22	20,632	7.3%	0.4%	(14)	184,139	17	23
13	23	20,642	7.3%	0.4%	(2)	184,155	13	16
14	24	20,648	7.3%	0.5%	7	184,166	9	11

4.6 Estimate All-Prior Reserves

Finally, summing all of the diagonals below the diagonal line in Table 4.5, using formula (3.13), allows us to derive an independent estimate of the unpaid claims for all-prior years, as shown in Table 4.5.[33] Using this estimate of all-prior unpaid claims, we can complete the typical summary of our chain ladder estimates, as illustrated in Table 4.7.[34]

[32] While calibrating and validating could be used somewhat interchangeably, I think it is more useful to think of them as different yet related processes. In this case, calibration is the process of adjusting the parameters used to estimate a tail factor and validation is the process of checking the tail factor against the actual data in the all-prior row.

[33] As Table 4.5 is truncated beyond 144 months for readability, the interested reader can refer to the Excel file for the details beyond 144 months of development which sum to derive the all-prior row estimate.

[34] Note that the columns in Table 4.8 are a continuation of Table 4.7, so the column (7) referenced in Table 4.7 can be found in Table 4.8.

Table 4.7 – "Medium" Paid Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Paid Data
***All-Prior Estimate in Separate Exhibit**

	(1) Paid to Date	(2) Paid CDF	(3) (1) x (2) Ultimate	(4) (3) - (1) Estimated Unpaid	(5) (7) - (1) Case Reserve	(6) (4) - (5) Estimated IBNR
A-P*	282,390	*1.0046*	*283,699*	*1,309*	1,323	(14)
2004	322,784	1.0033	323,862	1,078	1,132	(54)
2005	357,748	1.0062	359,964	2,216	2,030	186
2006	328,463	1.0115	332,241	3,778	3,473	305
2007	348,988	1.0214	356,451	7,463	6,054	1,409
2008	359,694	1.0399	374,038	14,344	11,865	2,479
2009	329,792	1.0748	354,447	24,655	19,049	5,607
2010	309,186	1.1426	353,283	44,097	34,772	9,326
2011	255,177	1.2868	328,373	73,196	61,512	11,684
2012	243,840	1.7136	417,840	174,000	118,332	55,669
2013	100,689	3.8556	388,215	287,525	189,983	97,542
				633,661	449,522	184,139

The all-prior (A-P) row in Table 4.7 is highlighted to signify that it was not calculated the same as the remaining rows. For the all-prior row, the estimated unpaid amount is the sum of the future diagonals from Table 4.5, the ultimate is (1) plus (4) and the Paid CDF is (3) divided by (1), which is only included for comparison purposes with the other CDFs in column (2). Note that simply using the tail factor for the all-prior row (1.0033 instead of 1.0046) would have misestimated the all-prior unpaid claims, perhaps significantly in some cases.

The analysis in Tables 4.1 to 4.7 used paid data. Analogous work using incurred data is included in Appendix A as Tables A.1 to A.7, respectively. For ease of comparison, the summary of results for the incurred data (Table A.7) is repeated here as Table 4.8.

Table 4.8 – "Medium" Incurred Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Incurred Data
***All-Prior Estimate in Separate Exhibit**

	(7) Incurred to Date	(8) Incurred CDF	(9) (7) x (8) Ultimate	(10) (11) + (12) Estimated Unpaid	(11) (7) - (1) Case Reserve	(12) (9) - (7) Estimated IBNR
A-P*	283,713	*1.0001*	*283,735*	1,344	1,323	*21*
2004	323,915	1.0001	323,948	1,164	1,132	33
2005	359,778	1.0002	359,866	2,118	2,030	88
2006	331,936	1.0006	332,131	3,668	3,473	195
2007	355,042	1.0014	355,543	6,555	6,054	501
2008	371,559	1.0039	373,025	13,331	11,865	1,466
2009	348,841	1.0093	352,096	22,304	19,049	3,255
2010	343,957	1.0226	351,733	42,548	34,772	7,776
2011	316,689	1.0525	333,326	78,149	61,512	16,637
2012	362,172	1.1214	406,131	162,291	118,332	43,959
2013	290,672	1.2840	373,216	272,527	189,983	82,544
				605,997	449,522	156,475

Comparing the results in Tables 4.7 and 4.8, it seems fair to conclude that the case reserves for the all-prior years are adequate and that an IBNR reserve near zero for these years would be reasonable.[35]

Appendices B and C include analyses for the "low" case reserve simulated data for paid and incurred data, respectively. For ease of comparison, Tables B.7 and C.7 are repeated here as Tables 4.9 and 4.10, respectively.

Table 4.9 – "Low" Paid Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Paid Data
***All-Prior Estimate in Separate Exhibit**

	(1) Paid to Date	(2) Paid CDF	(3) (1) x (2) Ultimate	(4) (3) - (1) Estimated Unpaid	(5) (7) - (1) Case Reserve	(6) (4) - (5) Estimated IBNR
A-P*	546,393	*1.0122*	*553,046*	*6,653*	6,075	578
2004	386,452	1.0114	390,872	4,420	3,476	944
2005	434,642	1.0185	442,661	8,020	5,946	2,074
2006	407,012	1.0306	419,475	12,463	7,684	4,779
2007	457,165	1.0518	480,866	23,701	16,130	7,571
2008	398,617	1.0892	434,190	35,574	23,671	11,903
2009	431,152	1.1550	497,975	66,823	33,566	33,257
2010	400,155	1.2794	511,940	111,786	63,349	48,437
2011	304,450	1.5237	463,877	159,427	94,442	64,985
2012	231,388	2.2836	528,388	297,000	159,371	137,629
2013	105,488	5.0838	536,281	430,793	206,653	224,140
				1,156,658	620,362	536,296

Table 4.10 – "Low" Incurred Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Incurred Data
***All-Prior Estimate in Separate Exhibit**

	(7) Incurred to Date	(8) Incurred CDF	(9) (7) x (8) Ultimate	(10) (11) + (12) Estimated Unpaid	(11) (7) - (1) Case Reserve	(12) (9) - (7) Estimated IBNR
A-P*	552,468	*1.0019*	*553,494*	7,101	6,075	*1,026*
2004	389,928	1.0025	390,883	4,432	3,476	955
2005	440,588	1.0045	442,586	7,944	5,946	1,998
2006	414,696	1.0084	418,178	11,166	7,684	3,482
2007	473,295	1.0164	481,067	23,902	16,130	7,772
2008	422,287	1.0298	434,869	36,252	23,671	12,581
2009	464,718	1.0551	490,328	59,176	33,566	25,610
2010	463,503	1.1028	511,172	111,017	63,349	47,669
2011	398,892	1.1871	473,531	169,080	94,442	74,639
2012	390,758	1.3800	539,250	307,862	159,371	148,491
2013	312,141	1.7137	534,926	429,438	206,653	222,785
				1,167,370	620,362	547,007

Comparing the results in Tables 4.9 and 4.10, we have evidence that the case reserves for the all-prior years are inadequate, so we have the ability to compare our estimates to any held IBNR to see if it is sufficient.

[35] Some tables in the Appendices have also been reduced for readability, so the reader is directed to the companion Excel file for all of the details.

Appendices D and E include the analysis for the "high" case reserve simulated data for paid and incurred data, respectively.[36] For ease of comparison, Tables D.7 and E.7 are repeated here as Tables 4.11 and 4.12, respectively.

Table 4.11 – "High" Paid Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Paid Data
***All-Prior Estimate in Separate Exhibit**

	(1)	(2)	(3)	(4)	(5)	(6)
			(1) x (2)	(3) - (1)	(7) - (1)	(4) - (5)
	Paid to Date	Paid CDF	Ultimate	Estimated Unpaid	Case Reserve	Estimated IBNR
A-P*	2,028,756	*1.0040*	*2,036,779*	*8,024*	13,009	(4,985)
2004	962,203	1.0093	971,173	8,969	11,874	(2,904)
2005	898,591	1.0184	915,098	16,508	21,878	(5,370)
2006	907,581	1.0363	940,536	32,955	42,994	(10,040)
2007	977,881	1.0722	1,048,462	70,581	83,430	(12,849)
2008	1,040,208	1.1459	1,191,977	151,769	140,745	11,025
2009	914,456	1.2918	1,181,321	266,865	257,107	9,758
2010	732,524	1.7372	1,272,516	539,993	528,128	11,865
2011	496,043	2.6041	1,291,769	795,726	696,830	98,896
2012	271,729	5.2619	1,429,810	1,158,081	933,516	224,565
2013	99,365	14.9591	1,486,405	1,387,040	1,129,608	257,432
				4,436,510	3,859,117	577,393

Table 4.12 – "High" Incurred Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Incurred Data
***All-Prior Estimate in Separate Exhibit**

	(7)	(8)	(9)	(10)	(11)	(12)
			(7) x (8)	(11) + (12)	(7) - (1)	(9) - (7)
	Incurred to Date	Incurred CDF	Ultimate	Estimated Unpaid	Case Reserve	Estimated IBNR
A-P*	2,041,764	*0.9996*	*2,040,912*	12,156	13,009	*(853)*
2004	974,077	0.9989	973,045	10,841	11,874	(1,032)
2005	920,468	0.9981	918,726	20,135	21,878	(1,742)
2006	950,576	0.9972	947,946	40,364	42,994	(2,630)
2007	1,061,310	0.9942	1,055,132	77,251	83,430	(6,179)
2008	1,180,953	0.9933	1,173,030	132,822	140,745	(7,923)
2009	1,171,563	0.9942	1,164,732	250,275	257,107	(6,832)
2010	1,260,651	1.0042	1,265,965	533,442	528,128	5,314
2011	1,192,873	1.0589	1,263,124	767,081	696,830	70,252
2012	1,205,245	1.1466	1,381,967	1,110,238	933,516	176,722
2013	1,228,972	1.2667	1,556,760	1,457,395	1,129,608	327,787
				4,412,001	3,859,117	552,884

Comparing the results in Tables 4.11 and 4.12, we have evidence that the case reserves for the all-prior years are more than adequate, and again we have the ability to assess any held IBNR.

[36] Note that the exponential decay method (3.6) of estimating tail factors is not well suited to fitting development factors less than 1.000. Thus, the selected tail factor in Table E.3 needed to be estimated using a different method.

5. FUTURE RESEARCH

As this is the first paper outlining a process for estimating unpaid claims for all-prior data, there is much that can be done to expand this in various ways. Only a few suggestions for such future research are offered here.

- The historical estimation process could also incorporate assumptions from other estimation methods such as Berquist and Sherman [3].
- Closed-form estimates for the standard deviation as in Mack [8] or alternative assumptions for age-to-age factors as in Murphy [9] may be adaptable to all-prior data.
- The Over-Dispersed Poisson (ODP) Bootstrap models such as those discussed in Shapland and Leong [11] could incorporate the all-prior data analysis to simulate a distribution for the all-prior claims.
- The incremental log models in Barnett and Zehnwirth [1] or Zehnwirth [13] can be extended backwards to simulate a distribution for the all-prior claims.

6. CONCLUSIONS

Whenever data being used to estimate unpaid claims includes an all-prior row and a tail factor is needed, the starting point to analyzing the all-prior data is understanding the data (i.e., how was it created and what is included). Once the data is understood, the methods introduced in this paper can be used to analyze the all-prior row. Regardless of whether the unpaid claims in the all-prior row are a significant portion of the total unpaid claims or not, the value of the methodology in helping to calibrate the tail factor should not be underestimated. Indeed, the process of calibrating the tail factor and validating it by comparing estimates of the all-prior data to the actual all-prior data may reveal that the tail factor is different than otherwise expected, which will have an impact on estimates for all accident periods.

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Supplementary Material

A more complete review of the notation, data and examples used in this paper are contained in the companion Excel file "All Prior Analysis.xlsm". An example of how all-prior data is compiled for the NAIC Schedule P is contained in the "Creating All Prior Data.xls" file.

REFERENCES

[1] Barnett, Glenn and Ben Zehnwirth. 2000. Best Estimates for Reserves. PCAS LXXXVII: 245-321.
[2] Berquist, James R. and Richard E. Sherman. 1977. Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach. PCAS LXIV: 123-84.
[3] Bornhuetter, Ronald and Ronald Ferguson. 1972. The Actuary and IBNR. PCAS LIX: 181-95.
[4] CAS Working Party on Quantifying Variability in Reserve Estimates. 2005. The Analysis and Estimation of Loss & ALAE Variability: A Summary Report. *CAS Forum* (Fall): 29-146.
[5] CAS Tail Factor Working Party. 2013. The Estimation of Loss Development Tail Factors: A Summary Report. *CAS Forum* (Fall): 1-111.
[6] Feldblum, Sholom. 1991. Completing and Using Schedule P. *CAS Forum* (Fall): 1-34.
[7] *Foundations of Casualty Actuarial Science*, 4th ed. 2001. Arlington, Va.: Casualty Actuarial Society.
[8] Mack, Thomas. 1993. Distribution-Free Calculation of the Standard Error of Chain Ladder Reserve Estimates. *ASTIN Bulletin* 23, no. 2: 213-25.
[9] Murphy, Daniel. 1994. Unbiased Loss Development Factors. PCAS LXXXI: 154-222.
[10] NAIC Annual Statement Instructions for Property/Casualty Companies. 2011. National Association of Insurance Commissioners.
[11] Shapland, Mark R. and Jessica Leong. 2010. Bootstrap Modeling: Beyond the Basics. *CAS Forum* (Fall-1): 1-66.
[12] Venter, Gary G. 1998. Testing the Assumptions of Age-to-Age Factors. PCAS LXXXV: 807-47.
[13] Zehnwirth, Ben. 1994. Probabilistic Development Factor Models with Applications to Loss Reserve Variability, Prediction Intervals and Risk Based Capital. *CAS Forum* (Spring-2): 447-605.

Biography of the Author

Mark R. Shapland is a Senior Consulting Actuary in Milliman's Dubai office. He is responsible for various reserving, pricing and risk modeling projects for a wide variety of clients. He has a B.S. degree in Integrated Studies (Actuarial Science) from the University of Nebraska-Lincoln. He is a Fellow of the Casualty Actuarial Society, a Fellow of the Society of Actuaries and a Member of the American Academy of Actuaries. He has previously served the CAS as a chair of the CAS Committee on Reserves, a chair of the Dynamic Risk Modeling Committee, a co-chair of the CAS Loss Simulation Model Working Party and a co-chair of the CAS Tail Factor Working Party. He is a co-creator and co-presenter for the CAS Reserve Variability Limited Attendance Seminar and a frequent speaker on reserve variability at actuarial meetings in the United States and many other countries.

Appendix A – Incurred Analysis for "Medium" Case Reserve Data

Table A.1 – "Medium" Incurred Loss Triangle with All-Prior Data

	12	24	36	48	60	72	84	96	108	120	132
A-P		226,614	253,212	272,185	278,519	281,496	283,003	283,520	283,663	283,741	283,713
2004	250,529	286,453	307,218	317,077	321,489	322,467	323,628	323,685	323,858	323,915	
2005	277,084	325,918	342,040	353,268	356,648	358,593	359,498	359,761	359,778		
2006	271,418	298,981	316,852	323,994	328,877	330,662	331,705	331,936			
2007	284,989	320,743	335,916	347,257	352,265	354,693	355,042				
2008	297,906	334,537	353,299	365,298	369,420	371,559					
2009	277,237	307,715	333,225	343,673	348,841						
2010	270,103	313,682	337,891	343,957							
2011	255,515	292,838	316,689								
2012	323,902	362,172									
2013	290,672										

Table A.2 – "Medium" Incurred Loss Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Tail
2004	1.143	1.072	1.032	1.014	1.003	1.004	1.000	1.001	1.000	
2005	1.176	1.049	1.033	1.010	1.005	1.003	1.001	1.000		
2006	1.102	1.060	1.023	1.015	1.005	1.003	1.001			
2007	1.125	1.047	1.034	1.014	1.007	1.001				
2008	1.123	1.056	1.034	1.011	1.006					
2009	1.110	1.083	1.031	1.015						
2010	1.161	1.077	1.018							
2011	1.146	1.081								
2012	1.118									
VWA	1.133	1.065	1.029	1.013	1.005	1.003	1.001	1.000	1.000	
5-Yr VWA	1.131	1.068	1.028	1.013	1.005	1.003	1.001	1.000	1.000	
3-Yr VWA	1.140	1.080	1.028	1.014	1.006	1.002	1.001	1.000	1.000	
TF Fitted	1.157	1.066	1.027	1.011	1.005	1.002	1.001	1.000	1.000	1.000
User	1.145									
Selected	1.145	1.065	1.029	1.013	1.005	1.003	1.001	1.000	1.000	1.0001
Ultimate	1.284	1.121	1.053	1.023	1.009	1.004	1.001	1.001	1.000	1.0001
% Reported	0.779	0.892	0.950	0.978	0.991	0.996	0.999	0.999	1.000	1.000
% Unrptd	0.221	0.108	0.050	0.022	0.009	0.004	0.001	0.001	0.000	0.000

Table A.3 – "Medium" Incurred Tail Factor Calculation

Tail Years:	5	Actual	57,099	Decay	0.417
Tail Factor:	1.0001	Estimated	63,910	Intercept	0.377
		Error %	11.9%		

Period	Factor	Dev	Log	Excl	Period	Log	Fitted	Selected	ATA	Ultimate
1	1.13328	0.13328	(2.015)		1	(2.015)	1.157232		1.157232	1.291574
2	1.06541	0.06541	(2.727)		2	(2.727)	1.065522		1.065522	1.116089
3	1.02927	0.02927	(3.531)		3	(3.531)	1.027304		1.027304	1.047457
4	1.01315	0.01315	(4.331)		4	(4.331)	1.011378		1.011378	1.019618
5	1.00537	0.00537	(5.228)		5	(5.228)	1.004741		1.004741	1.008147
6	1.00253	0.00253	(5.979)		6	(5.979)	1.001976		1.001976	1.003390
7	1.00054	0.00054	(7.521)		7	(7.521)	1.000823		1.000823	1.001411
8	1.00028	0.00028	(8.184)		8	(8.184)	1.000343		1.000343	1.000587
9	1.00018	0.00018	(8.646)		9	(8.646)	1.000143		1.000143	1.000244
10							1.000060		1.000060	1.000101
11							1.000025		1.000025	1.000041
12							1.000010		1.000010	1.000016
13							1.000004		1.000004	1.000006
14							1.000002		1.000002	1.000002

Table A.4 – "Medium" Incurred All-Prior Projection (Cumulative)

	Premium	Loss Ratio	24	36	48	60	72	84	96	108	120	132
1991	*431,182*	*70.0%*	*269,158*	*286,762*	*295,154*	*299,037*	*300,641*	*301,402*	*301,650*	*301,754*	*301,797*	301,815
1992	*435,494*	*70.0%*	*271,849*	*289,630*	*298,106*	*302,027*	*303,648*	*304,416*	*304,667*	*304,771*	*304,815*	304,833
1993	*439,848*	*69.1%*	*271,038*	*288,765*	*297,216*	*301,125*	*302,741*	*303,507*	*303,757*	*303,861*	*303,905*	303,923
1994	*472,929*	*64.9%*	*273,709*	*291,611*	*300,146*	*304,094*	*305,725*	*306,499*	*306,751*	*306,856*	*306,900*	306,919
1995	*412,911*	*75.1%*	*276,532*	*294,619*	*303,241*	*307,230*	*308,878*	*309,660*	*309,915*	*310,021*	*310,065*	310,084
1996	*460,127*	*68.0%*	*279,020*	*297,269*	*305,969*	*309,993*	*311,657*	*312,445*	*312,703*	*312,810*	*312,855*	312,873
1997	*471,803*	*67.0%*	*281,893*	*300,330*	*309,120*	*313,186*	*314,866*	*315,663*	*315,923*	*316,031*	*316,076*	316,095
1998	*443,804*	*71.9%*	*284,557*	*303,168*	*312,040*	*316,145*	*317,841*	*318,645*	*318,908*	*319,017*	*319,063*	319,082
1999	*448,454*	*71.9%*	*287,538*	*306,344*	*315,310*	*319,457*	*321,171*	*321,984*	*322,249*	*322,360*	*322,406*	322,425
2000	*439,491*	*74.1%*	*290,414*	*309,408*	*318,463*	*322,652*	*324,383*	*325,204*	*325,472*	*325,583*	*325,630*	325,649
2001	*499,204*	*65.9%*	*293,368*	*312,556*	*321,703*	*325,934*	*327,683*	*328,512*	*328,783*	*328,895*	*328,942*	328,962
2002	*447,766*	*74.2%*	*296,281*	*315,660*	*324,897*	*329,171*	*330,937*	*331,775*	*332,048*	*332,162*	*332,209*	332,229
2003	*468,659*	*71.6%*	*299,239*	*318,811*	*328,141*	*332,457*	*334,241*	*335,087*	*335,363*	*335,478*	*335,526*	335,546

Table A.5 – "Medium" Incurred All-Prior Projection (Incremental)

	12	24	36	48	60	72	84	96	108	120	132	144
1994												8
1995											18	8
1996										44	18	8
1997									106	44	18	8
1998								257	107	45	19	8
1999							797	260	108	45	19	8
2000						1,696	804	262	109	46	19	8
2001					4,147	1,714	813	265	111	46	19	8
2002				9,055	4,189	1,731	821	268	112	47	19	8
2003			19,188	9,147	4,232	1,749	829	270	113	47	20	8
Totals:	**(144+)**	**(36-132)**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**
Estimated	21	63,910	35,321	16,297	7,222	3,021	1,285	460	192	80	33	*14*
Actual		57,099	26,597	18,973	6,334	2,976	1,507	517	143	77	(28)	
Differences		6,811	8,724	(2,677)	888	44	(222)	(57)	49	2	61	
Cumulative Percent Difference			11.9%	-6.3%	6.6%	-2.4%	-7.6%	7.6%	57.7%	126.1%	219.1%	
Weights			0.25	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	
Weighted Average			90.0%									

Table A.6 – "Medium" Incurred Tail Calibration Summary

		All-Prior Projection					Change in IBNR	
Tail	***(u)***	**Total**	**Cumulative**	**Weighted**		**Total**		
Years	**Ultimate**	**Difference**	**Percent**	**Percent**	**IBNR**	**IBNR**	**All-Prior**	**Total**
1	11	6,698	11.7%	62.1%	-	156,306		
2	12	6,766	11.9%	79.0%	8	156,403	8	97
3	13	6,795	11.9%	86.0%	15	156,447	7	44
4	14	6,806	11.9%	88.8%	19	156,466	4	20
5	15	6,811	11.9%	90.0%	21	156,475	2	9
6	16	6,813	11.9%	90.5%	23	156,479	1	4
7	17	6,814	11.9%	90.7%	23	156,480	1	2
8	18	6,814	11.9%	90.8%	24	156,481	0	1
9	19	6,815	11.9%	90.8%	24	156,482	0	0
10	20	6,815	11.9%	90.9%	24	156,482	0	0

Table A.7 – "Medium" Incurred Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Incurred Data
***All-Prior Estimate in Separate Exhibit**

	(7)	(8)	(9)	(10)	(11)	(12)
			(7) x (8)	(11) + (12)	(7) - (1)	(9) - (7)
	Incurred to Date	Incurred CDF	Ultimate	Estimated Unpaid	Case Reserve	Estimated IBNR
A-P*	283,713	*1.0001*	*283,735*	1,344	1,323	*21*
2004	323,915	1.0001	323,948	1,164	1,132	33
2005	359,778	1.0002	359,866	2,118	2,030	88
2006	331,936	1.0006	332,131	3,668	3,473	195
2007	355,042	1.0014	355,543	6,555	6,054	501
2008	371,559	1.0039	373,025	13,331	11,865	1,466
2009	348,841	1.0093	352,096	22,304	19,049	3,255
2010	343,957	1.0226	351,733	42,548	34,772	7,776
2011	316,689	1.0525	333,326	78,149	61,512	16,637
2012	362,172	1.1214	406,131	162,291	118,332	43,959
2013	290,672	1.2840	373,216	272,527	189,983	82,544
				605,997	449,522	156,475

Appendix B – Paid Analysis for "Low" Case Reserve Data

Table B.1 – "Low" Paid Loss Triangle with All-Prior Data

	12	24	36	48	60	72	84	96	108	120	132
A-P		-	224,096	349,441	428,145	476,471	506,620	525,072	535,370	541,985	546,393
2004	59,477	172,635	254,266	309,215	335,168	355,021	372,113	378,908	383,860	386,452	
2005	95,293	190,721	287,897	338,580	382,595	407,187	421,132	429,650	434,642		
2006	73,884	165,497	266,958	318,469	366,483	387,022	397,578	407,012			
2007	81,811	222,270	329,320	389,660	419,385	442,175	457,165				
2008	119,772	205,222	277,631	333,442	373,116	398,617					
2009	111,735	225,388	329,885	394,175	431,152						
2010	89,494	212,010	339,510	400,155							
2011	73,009	200,877	304,450								
2012	115,736	231,388									
2013	105,488										

Table B.2 – "Low" Paid Loss Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Tail
2004	2.903	1.473	1.216	1.084	1.059	1.048	1.018	1.013	1.007	
2005	2.001	1.510	1.176	1.130	1.064	1.034	1.020	1.012		
2006	2.240	1.613	1.193	1.151	1.056	1.027	1.024			
2007	2.717	1.482	1.183	1.076	1.054	1.034				
2008	1.713	1.353	1.201	1.119	1.068					
2009	2.017	1.464	1.195	1.094						
2010	2.369	1.601	1.179							
2011	2.751	1.516								
2012	1.999									
VWA	2.226	1.499	1.191	1.108	1.060	1.036	1.021	1.012	1.007	
5-Yr VWA	2.109	1.483	1.190	1.112	1.060	1.036	1.021	1.012	1.007	
3-Yr VWA	2.316	1.526	1.191	1.095	1.059	1.032	1.021	1.012	1.007	
TF Fitted	1.539	1.313	1.182	1.105	1.061	1.035	1.021	1.012	1.007	1.011
User										
Selected	2.226	1.499	1.191	1.108	1.060	1.036	1.021	1.012	1.007	1.0044
Ultimate	5.084	2.284	1.524	1.279	1.155	1.089	1.052	1.031	1.018	1.0114
% Paid	19.7%	43.8%	65.6%	78.2%	86.6%	91.8%	95.1%	97.0%	98.2%	98.9%
% Unpaid	80.3%	56.2%	34.4%	21.8%	13.4%	8.2%	4.9%	3.0%	1.8%	1.1%

Table B.3 – "Low" Paid Tail Factor Calculation

Paid Tail Factor Analysis

All Prior

Tail Years:	15	Actual	546,393	Decay	0.580
Tail Factor:	1.0114	Estimated	548,874	Intercept	0.930
		Error %	0.5%		

Period	Factor	Dev	Log	Excl	Period	Log	Fitted	Selected	ATA	ATU
1	2.22626	1.22626	0.204	Y			1.539467		1.539467	3.051586
2	1.49874	0.49874	(0.696)	Y			1.313031		1.313031	1.982236
3	1.19095	0.19095	(1.656)	Y			1.181640		1.181640	1.509664
4	1.10768	0.10768	(2.229)	Y			1.105398		1.105398	1.277601
5	1.06036	0.06036	(2.807)		5	(2.807)	1.061159		1.061159	1.155783
6	1.03556	0.03556	(3.337)		6	(3.337)	1.035488		1.035488	1.089171
7	1.02078	0.02078	(3.874)		7	(3.874)	1.020592		1.020592	1.051843
8	1.01230	0.01230	(4.398)		8	(4.398)	1.011949		1.011949	1.030621
9	1.00675	0.00675	(4.998)		9	(4.998)	1.006933		1.006933	1.018451
10							1.004023	1.004440	1.004440	1.011439
11							1.002335	1.002640	1.002640	1.006968
12							1.001355	1.001940	1.001940	1.004316
13							1.000786	1.000940	1.000940	1.002372
14							1.000456	1.000640	1.000640	1.001430
15							1.000265	1.000340	1.000340	1.000790
16							1.000154	1.000240	1.000240	1.000450
17							1.000089		1.000089	1.000210
18							1.000052		1.000052	1.000120
19							1.000030		1.000030	1.000069
20							1.000017		1.000017	1.000039
21							1.000010		1.000010	1.000021
22							1.000006		1.000006	1.000011
23							1.000003		1.000003	1.000005
24							1.000002		1.000002	1.000002

Table B.4 – "Low" Paid All-Prior Projection (Cumulative)

	Premium	Loss Ratio	24	36	48	60	72	84	96	108	120	132
1994	408,252	74.4%	*133,011*	*199,349*	*237,416*	*262,981*	*278,854*	*288,769*	*294,715*	*298,237*	*300,305*	301,638
1995	421,696	74.2%	*137,022*	*205,361*	*244,575*	*270,911*	*287,263*	*297,476*	*303,602*	*307,230*	*309,360*	310,734
1996	426,540	75.5%	*141,024*	*211,359*	*251,718*	*278,824*	*295,653*	*306,165*	*312,470*	*316,203*	*318,396*	319,809
1997	435,782	76.2%	*145,416*	*217,941*	*259,557*	*287,507*	*304,860*	*315,699*	*322,200*	*326,050*	*328,311*	329,768
1998	445,319	76.8%	*149,768*	*224,463*	*267,326*	*296,112*	*313,984*	*325,148*	*331,843*	*335,809*	*338,137*	339,638
1999	479,330	73.5%	*154,280*	*231,225*	*275,379*	*305,032*	*323,443*	*334,943*	*341,840*	*345,925*	*348,323*	349,870
2000	482,332	75.2%	*158,837*	*238,055*	*283,513*	*314,042*	*332,996*	*344,836*	*351,937*	*356,142*	*358,612*	360,204
2001	508,950	73.4%	*163,591*	*245,180*	*291,998*	*323,440*	*342,963*	*355,157*	*362,470*	*366,801*	*369,345*	370,984
2002	499,443	77.0%	*168,409*	*252,401*	*300,598*	*332,966*	*353,063*	*365,617*	*373,146*	*377,604*	*380,222*	381,910
2003	552,073	71.8%	*173,584*	*260,156*	*309,834*	*343,198*	*363,912*	*376,851*	*384,612*	*389,207*	*391,906*	393,646

Table B.5 – "Low" Paid All-Prior Projection (Incremental)

	12	24	36	48	60	72	84	96	108	120	132	144
1994												796
1995											1,374	820
1996										2,192	1,414	844
1997									3,850	2,261	1,458	871
1998								6,696	3,965	2,328	1,501	897
1999							11,500	6,897	4,085	2,398	1,547	924
2000						18,955	11,840	7,101	4,205	2,469	1,592	951
2001					31,443	19,522	12,194	7,313	4,331	2,543	1,640	979
2002				48,197	32,369	20,097	12,553	7,529	4,459	2,618	1,688	1,008
2003			86,573	49,678	33,363	20,715	12,939	7,760	4,596	2,699	1,740	1,039
Totals:	**(144+)**	**(36-132)**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**
Estimated	6,653	548,874	212,814	130,042	82,786	50,912	31,107	18,716	11,286	6,892	4,319	*2,657*
Actual		546,393	224,096	125,345	78,704	48,327	30,149	18,451	10,298	6,615	4,409	
Differences		2,480	(11,282)	4,697	4,082	2,585	958	264	987	277	(89)	
Cumulative Percent Difference			0.5%	4.3%	4.6%	4.2%	3.4%	3.6%	5.5%	1.7%	-2.0%	
Weights			0.25	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	
Weighted Average			2.2%									

Table B.6 – "Low" Paid Tail Calibration Summary

		All-Prior Projection					Change in IBNR	
Tail Years	***(u)* Ultimate**	**Total Difference**	**Cumulative Percent**	**Weighted Percent**	**IBNR**	**Total IBNR**	**All-Prior**	**Total**
1	11	(14,527)	-2.7%	-33.9%	(6,075)	497,077		
2	12	(7,802)	-1.4%	-19.7%	(5,031)	510,460	1,044	13,383
3	13	(3,044)	-0.6%	-9.6%	(3,521)	521,061	1,510	10,602
4	14	(822)	-0.2%	-4.9%	(2,439)	526,557	1,082	5,496
5	15	635	0.1%	-1.7%	(1,471)	530,533	968	3,976
6	16	1,380	0.3%	-0.1%	(837)	532,766	634	2,233
7	17	1,887	0.3%	1.0%	(308)	534,424	529	1,658
8	18	2,068	0.4%	1.4%	(82)	535,070	226	645
9	19	2,170	0.4%	1.6%	66	535,461	148	391
10	20	2,226	0.4%	1.7%	161	535,697	95	236
11	21	2,300	0.4%	1.9%	277	535,895	116	198
12	22	2,366	0.4%	2.0%	383	536,048	105	153
13	23	2,417	0.4%	2.1%	467	536,161	85	113
14	24	2,455	0.4%	2.2%	532	536,241	64	80
15	25	2,480	0.5%	2.2%	578	536,296	46	56
16	26	2,498	0.5%	2.2%	610	536,334	32	38
17	27	2,509	0.5%	2.3%	632	536,359	22	25
18	28	2,516	0.5%	2.3%	647	536,376	15	17
19	29	2,521	0.5%	2.3%	657	536,387	10	11
20	30	2,524	0.5%	2.3%	664	536,394	7	7

Table B.7 – "Low" Paid Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Paid Data
***All-Prior Estimate in Separate Exhibit**

	(1)	(2)	(3)	(4)	(5)	(6)
			(1) x (2)	(3) - (1)	(7) - (1)	(4) - (5)
	Paid to Date	**Paid CDF**	**Ultimate**	**Estimated Unpaid**	**Case Reserve**	**Estimated IBNR**
A-P*	546,393	*1.0122*	*553,046*	*6,653*	6,075	578
2004	386,452	1.0114	390,872	4,420	3,476	944
2005	434,642	1.0185	442,661	8,020	5,946	2,074
2006	407,012	1.0306	419,475	12,463	7,684	4,779
2007	457,165	1.0518	480,866	23,701	16,130	7,571
2008	398,617	1.0892	434,190	35,574	23,671	11,903
2009	431,152	1.1550	497,975	66,823	33,566	33,257
2010	400,155	1.2794	511,940	111,786	63,349	48,437
2011	304,450	1.5237	463,877	159,427	94,442	64,985
2012	231,388	2.2836	528,388	297,000	159,371	137,629
2013	105,488	5.0838	536,281	430,793	206,653	224,140
				1,156,658	620,362	536,296

Appendix C – Incurred Analysis for "Low" Case Reserve Data

Table C.1 – "Low" Incurred Loss Triangle with All-Prior Data

	12	24	36	48	60	72	84	96	108	120	132
A-P		313,964	419,793	474,098	509,975	528,993	540,336	546,327	549,438	551,508	552,468
2004	229,846	286,253	326,645	356,188	367,977	378,068	384,592	387,096	389,206	389,928	
2005	272,625	317,769	373,881	395,845	419,735	430,657	435,569	439,389	440,588		
2006	239,240	296,287	343,883	375,203	398,283	406,375	411,221	414,696			
2007	273,614	361,153	416,886	443,360	456,786	467,440	473,295				
2008	280,215	326,745	365,787	389,743	411,549	422,287					
2009	299,423	361,656	410,220	449,671	464,718						
2010	301,843	364,457	432,227	463,503							
2011	263,437	341,716	398,892								
2012	318,040	390,758									
2013	312,141										

Table C.2 – "Low" Incurred Loss Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Tail
2004	1.245	1.141	1.090	1.033	1.027	1.017	1.007	1.005	1.002	
2005	1.166	1.177	1.059	1.060	1.026	1.011	1.009	1.003		
2006	1.238	1.161	1.091	1.062	1.020	1.012	1.008			
2007	1.320	1.154	1.064	1.030	1.023	1.013				
2008	1.166	1.119	1.065	1.056	1.026					
2009	1.208	1.134	1.096	1.033						
2010	1.207	1.186	1.072							
2011	1.297	1.167								
2012	1.229									
VWA	1.229	1.155	1.076	1.045	1.025	1.013	1.008	1.004	1.002	
5-Yr VWA	1.220	1.153	1.077	1.047	1.025	1.013	1.008	1.004	1.002	
3-Yr VWA	1.242	1.162	1.078	1.039	1.023	1.012	1.008	1.004	1.002	
TF Fitted	1.283	1.153	1.083	1.045	1.024	1.013	1.007	1.004	1.002	1.002
User										
Selected	1.242	1.162	1.076	1.045	1.025	1.013	1.008	1.004	1.002	1.0011
Ultimate	1.714	1.380	1.187	1.103	1.055	1.030	1.016	1.008	1.005	1.0025
% Reported	0.584	0.725	0.842	0.907	0.948	0.971	0.984	0.992	0.995	0.998
% Unrptd	0.416	0.275	0.158	0.093	0.052	0.029	0.016	0.008	0.005	0.002

Table C.3 – "Low" Incurred Tail Factor Calculation

Incurred Tail Factor Analysis

			All Prior		
Tail Years:	10	Actual	238,504	Decay	0.541
Tail Factor:	1.0025	Estimated	230,023	Intercept	0.522
		Error %	-3.6%		

Period	Factor	Dev	Log	Excl	Period	Log	Fitted	Selected	ATA	Ultimate
1	1.22940	0.22940	(1.472)	Y			1.282709		1.282709	1.763247
2	1.15526	0.15526	(1.863)		2	(1.863)	1.152996		1.152996	1.374628
3	1.07641	0.07641	(2.572)		3	(2.572)	1.082798		1.082798	1.192222
4	1.04524	0.04524	(3.096)		4	(3.096)	1.044809		1.044809	1.101057
5	1.02458	0.02458	(3.706)		5	(3.706)	1.024250		1.024250	1.053836
6	1.01316	0.01316	(4.331)		6	(4.331)	1.013123		1.013123	1.028886
7	1.00796	0.00796	(4.834)		7	(4.834)	1.007102		1.007102	1.015558
8	1.00400	0.00400	(5.521)		8	(5.521)	1.003844		1.003844	1.008396
9	1.00185	0.00185	(6.290)		9	(6.290)	1.002080		1.002080	1.004535
10							1.001126		1.001126	1.002450
11							1.000609		1.000609	1.001323
12							1.000330		1.000330	1.000713
13							1.000178		1.000178	1.000384
14							1.000097		1.000097	1.000205
15							1.000052		1.000052	1.000109
16							1.000028		1.000028	1.000056
17							1.000015		1.000015	1.000028
18							1.000008		1.000008	1.000013
19							1.000004		1.000004	1.000004

Table C.4 – "Low" Incurred All-Prior Projection (Cumulative)

	Premium	Loss Ratio	24	36	48	60	72	84	96	108	120	132
1994	*408,252*	*74.4%*	*220,100*	*255,864*	*275,415*	*287,875*	*294,952*	*298,833*	*301,211*	*302,368*	*302,997*	303,338
1995	*421,696*	*74.2%*	*226,737*	*263,579*	*283,720*	*296,556*	*303,846*	*307,844*	*310,293*	*311,486*	*312,134*	312,485
1996	*426,540*	*75.5%*	*233,359*	*271,278*	*292,006*	*305,218*	*312,721*	*316,835*	*319,356*	*320,584*	*321,250*	321,612
1997	*435,782*	*76.2%*	*240,626*	*279,725*	*301,100*	*314,722*	*322,459*	*326,702*	*329,301*	*330,567*	*331,255*	331,627
1998	*445,319*	*76.8%*	*247,828*	*288,097*	*310,112*	*324,142*	*332,110*	*336,480*	*339,157*	*340,461*	*341,169*	341,553
1999	*479,330*	*73.5%*	*255,294*	*296,776*	*319,454*	*333,907*	*342,115*	*346,616*	*349,374*	*350,717*	*351,446*	351,842
2000	*482,332*	*75.2%*	*262,834*	*305,542*	*328,889*	*343,769*	*352,220*	*356,854*	*359,694*	*361,076*	*361,827*	362,234
2001	*508,950*	*73.4%*	*270,701*	*314,687*	*338,733*	*354,058*	*362,761*	*367,534*	*370,459*	*371,883*	*372,656*	373,076
2002	*499,443*	*77.0%*	*278,673*	*323,955*	*348,709*	*364,485*	*373,445*	*378,359*	*381,369*	*382,835*	*383,632*	384,063
2003	*552,073*	*71.8%*	*287,236*	*333,909*	*359,424*	*375,685*	*384,920*	*389,985*	*393,088*	*394,599*	*395,420*	395,865

Table C.5 – "Low" Incurred All-Prior Projection (Incremental)

	12	24	36	48	60	72	84	96	108	120	132	144
1994												174
1995											331	179
1996										629	341	185
1997									1,193	648	351	190
1998								2,521	1,227	667	362	196
1999							4,243	2,600	1,266	688	373	202
2000						7,968	4,369	2,678	1,304	708	384	208
2001					14,453	8,208	4,501	2,758	1,343	730	396	214
2002				23,347	14,880	8,451	4,634	2,840	1,382	751	407	221
2003			43,986	24,046	15,325	8,704	4,773	2,925	1,424	774	419	227
Totals:	**(144+)**	**(36-132)**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**
Estimated	1,026	230,023	99,036	56,696	33,627	18,849	10,448	5,845	3,008	1,631	883	*478*
Actual		238,504	105,829	54,304	35,877	19,019	11,343	5,991	3,110	2,071	960	
Differences		(8,481)	(6,793)	2,392	(2,251)	(170)	(894)	(146)	(102)	(439)	(77)	
Cumulative Percent Difference			-3.6%	-1.3%	-5.2%	-4.3%	-7.1%	-6.3%	-10.1%	-17.0%	-8.0%	
Weights			0.25	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	
Weighted Average			-9.4%									

Table C.6 – "Low" Incurred Tail Calibration Summary

		All-Prior Projection					Change in IBNR	
Tail Years	***(u)* Ultimate**	**Total Difference**	**Cumulative Percent**	**Weighted Percent**	**IBNR**	**Total IBNR**	**All-Prior**	**Total**
1	11	(11,900)	-5.0%	-34.8%	-	539,749		
2	12	(10,267)	-4.3%	-22.7%	227	542,847	227	3,097
3	13	(9,411)	-3.9%	-16.4%	470	544,643	243	1,797
4	14	(8,963)	-3.8%	-13.0%	664	545,678	194	1,035
5	15	(8,729)	-3.7%	-11.3%	802	546,272	138	593
6	16	(8,606)	-3.6%	-10.3%	894	546,610	92	339
7	17	(8,541)	-3.6%	-9.9%	953	546,803	59	192
8	18	(8,508)	-3.6%	-9.6%	990	546,911	37	109
9	19	(8,490)	-3.6%	-9.5%	1,012	546,973	22	61
10	20	(8,481)	-3.6%	-9.4%	1,026	547,007	13	35
11	21	(8,470)	-3.6%	-9.3%	1,041	547,034	15	27
12	22	(8,462)	-3.5%	-9.3%	1,054	547,053	13	19
13	23	(8,456)	-3.5%	-9.2%	1,063	547,066	10	13
14	24	(8,452)	-3.5%	-9.2%	1,070	547,075	7	9
15	25	(8,449)	-3.5%	-9.2%	1,075	547,081	5	6
16	26	(8,447)	-3.5%	-9.2%	1,078	547,084	3	4
17	27	(8,446)	-3.5%	-9.2%	1,080	547,086	2	2
18	28	(8,434)	-3.5%	-9.2%	1,081	547,088	1	1
19	29	(8,421)	-3.5%	-9.2%	1,082	547,088	1	1
20	30	(8,407)	-3.5%	-9.2%	1,082	547,089	0	1

Table C.7 – "Low" Incurred Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Incurred Data
***All-Prior Estimate in Separate Exhibit**

	(7)	(8)	(9)	(10)	(11)	(12)
			(7) x (8)	(11) + (12)	(7) - (1)	(9) - (7)
	Incurred to Date	**Incurred CDF**	**Ultimate**	**Estimated Unpaid**	**Case Reserve**	**Estimated IBNR**
A-P*	552,468	*1.0019*	*553,494*	7,101	6,075	*1,026*
2004	389,928	1.0025	390,883	4,432	3,476	955
2005	440,588	1.0045	442,586	7,944	5,946	1,998
2006	414,696	1.0084	418,178	11,166	7,684	3,482
2007	473,295	1.0164	481,067	23,902	16,130	7,772
2008	422,287	1.0298	434,869	36,252	23,671	12,581
2009	464,718	1.0551	490,328	59,176	33,566	25,610
2010	463,503	1.1028	511,172	111,017	63,349	47,669
2011	398,892	1.1871	473,531	169,080	94,442	74,639
2012	390,758	1.3800	539,250	307,862	159,371	148,491
2013	312,141	1.7137	534,926	429,438	206,653	222,785
				1,167,370	620,362	547,007

Appendix D – Paid Analysis for "High" Case Reserve Data

Table D.1 – "High" Paid Loss Triangle with All-Prior Data

	12	24	36	48	60	72	84	96	108	120	132
A-P		-	694,326	1,233,322	1,605,148	1,798,756	1,911,906	1,969,504	2,002,311	2,019,120	2,028,756
2004	79,078	195,201	376,363	563,604	760,099	854,132	909,879	940,170	953,400	962,203	
2005	55,011	166,607	338,389	508,834	706,763	803,987	853,722	883,714	898,591		
2006	62,645	195,873	369,571	541,058	719,526	811,071	874,968	907,581			
2007	75,825	190,645	413,211	587,344	815,442	914,584	977,881				
2008	81,654	244,999	466,821	694,938	922,414	1,040,208					
2009	81,003	235,834	436,030	702,479	914,456						
2010	100,835	239,091	488,580	732,524							
2011	74,250	228,057	496,043								
2012	91,294	271,729									
2013	99,365										

Table D.2 – "High" Paid Loss Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Tail
2004	2.468	1.928	1.497	1.349	1.124	1.065	1.033	1.014	1.009	
2005	3.029	2.031	1.504	1.389	1.138	1.062	1.035	1.017		
2006	3.127	1.887	1.464	1.330	1.127	1.079	1.037			
2007	2.514	2.167	1.421	1.388	1.122	1.069				
2008	3.000	1.905	1.489	1.327	1.128					
2009	2.911	1.849	1.611	1.302						
2010	2.371	2.043	1.499							
2011	3.071	2.175								
2012	2.976									
VWA	2.805	1.996	1.499	1.345	1.127	1.069	1.035	1.015	1.009	
5-Yr VWA	2.843	2.021	1.499	1.344	1.127	1.069	1.035	1.015	1.009	
3-Yr VWA	2.774	2.021	1.531	1.336	1.126	1.070	1.035	1.015	1.009	
TF Fitted	2.991	2.013	1.516	1.262	1.134	1.068	1.035	1.018	1.009	1.009
User										
Selected	2.843	2.021	1.499	1.345	1.127	1.069	1.035	1.018	1.009	1.0046
Ultimate	14.959	5.262	2.604	1.737	1.292	1.146	1.072	1.036	1.018	1.0093
% Paid	6.7%	19.0%	38.4%	57.6%	77.4%	87.3%	93.3%	96.5%	98.2%	99.1%
% Unpaid	93.3%	81.0%	61.6%	42.4%	22.6%	12.7%	6.7%	3.5%	1.8%	0.9%

Table D.3 – "High" Paid Tail Factor Calculation

Paid Tail Factor Analysis

			All Prior		
Tail Years:	13	Actual	2,028,756	Decay	0.509
Tail Factor:	1.0093	Estimated	1,885,275	Intercept	3.912
		Error %	-7.1%		

Period	Factor	Dev	Log	Excl	Period	Log	Fitted	Selected	ATA	ATU
1	2.80509	1.80509	0.591		1	0.591	2.990863		2.990863	14.957482
2	1.99552	0.99552	(0.004)		2	(0.004)	2.013295		2.013295	5.001059
3	1.49908	0.49908	(0.695)		3	(0.695)	1.515739		1.515739	2.484017
4	1.34473	0.34473	(1.065)		4	(1.065)	1.262497		1.262497	1.638816
5	1.12735	0.12735	(2.061)		5	(2.061)	1.133604		1.133604	1.298075
6	1.06876	0.06876	(2.677)		6	(2.677)	1.068001		1.068001	1.145086
7	1.03521	0.03521	(3.347)		7	(3.347)	1.034611		1.034611	1.072178
8	1.01541	0.01541	(4.173)		8	(4.173)	1.017616		1.017616	1.036310
9	1.00923	0.00923	(4.685)		9	(4.685)	1.008966		1.008966	1.018371
10							1.004563		1.004563	1.009321
11							1.002323		1.002323	1.004736
12							1.001182		1.001182	1.002408
13							1.000602		1.000602	1.001224
14							1.000306		1.000306	1.000622
15							1.000156		1.000156	1.000316
16							1.000079		1.000079	1.000160
17							1.000040		1.000040	1.000081
18							1.000021		1.000021	1.000040
19							1.000010		1.000010	1.000020
20							1.000005		1.000005	1.000009
21							1.000003		1.000003	1.000004
22							1.000001		1.000001	1.000001

Table D.4 – "High" Paid All-Prior Projection (Cumulative)

	Premium	Loss Ratio	24	36	48	60	72	84	96	108	120	132
1994	669,311	83.4%	*106,085*	*214,352*	*321,331*	*432,104*	*487,131*	*520,628*	*538,647*	*548,135*	*553,050*	555,574
1995	715,259	82.0%	*111,464*	*225,223*	*337,626*	*454,017*	*511,834*	*547,029*	*565,962*	*575,932*	*581,096*	583,748
1996	758,317	81.2%	*117,021*	*236,451*	*354,458*	*476,652*	*537,352*	*574,302*	*594,178*	*604,645*	*610,067*	612,851
1997	811,833	79.6%	*122,811*	*248,150*	*371,996*	*500,235*	*563,938*	*602,716*	*623,576*	*634,561*	*640,251*	643,172
1998	853,244	79.5%	*128,914*	*260,480*	*390,480*	*525,092*	*591,960*	*632,665*	*654,562*	*666,092*	*672,064*	675,131
1999	890,376	80.0%	*135,370*	*273,526*	*410,036*	*551,390*	*621,607*	*664,350*	*687,344*	*699,452*	*705,723*	708,943
2000	986,176	75.9%	*142,251*	*287,429*	*430,878*	*579,417*	*653,203*	*698,119*	*722,281*	*735,005*	*741,595*	744,979
2001	984,188	79.8%	*149,259*	*301,589*	*452,105*	*607,961*	*685,383*	*732,511*	*757,864*	*771,214*	*778,129*	781,680
2002	984,698	83.8%	*156,821*	*316,870*	*475,013*	*638,766*	*720,110*	*769,627*	*796,264*	*810,291*	*817,556*	821,287
2003	1,041,477	83.2%	*164,676*	*332,742*	*498,806*	*670,761*	*756,180*	*808,177*	*836,148*	*850,878*	*858,507*	862,424

	Growth	Loss Ratio
Prior to 1993	*5.0%*	*80.0%*

Table D.5 – "High" Paid All-Prior Projection (Incremental)

	12	24	36	48	60	72	84	96	108	120	132	144
1994												1,290
1995											2,652	1,356
1996										5,421	2,784	1,423
1997									10,985	5,689	2,922	1,494
1998								21,897	11,531	5,972	3,067	1,568
1999							42,743	22,994	12,108	6,271	3,221	1,647
2000						73,787	44,916	24,162	12,724	6,590	3,384	1,730
2001					155,856	77,422	47,128	25,353	13,350	6,915	3,551	1,816
2002				158,142	163,753	81,344	49,516	26,637	14,027	7,265	3,731	1,908
2003			168,066	166,064	171,956	85,419	51,997	27,971	14,729	7,629	3,918	2,003
Totals:	**(144+)**	**(36-132)**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**
Estimated	8,024	1,885,275	642,062	497,792	348,365	185,260	104,849	55,504	28,914	14,896	7,632	*3,901*
Actual		2,028,756	694,326	538,996	371,826	193,608	113,149	57,599	32,807	16,809	9,635	
Differences		(143,480)	(52,263)	(41,204)	(23,461)	(8,349)	(8,300)	(2,094)	(3,892)	(1,913)	(2,003)	
Cumulative Percent Difference			-7.1%	-6.8%	-6.3%	-6.3%	-7.9%	-8.5%	-13.2%	-14.8%	-20.8%	
Weights			0.25	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	
Weighted Average			-13.3%									

Table D.6 – "High" Paid Tail Calibration Summary

		All-Prior Projection					Change in IBNR	
Tail	**(*u*)**	**Total**	**Cumulative**	**Weighted**		**Total**		
Years	**Ultimate**	**Difference**	**Percent**	**Percent**	**IBNR**	**IBNR**	**All-Prior**	**Total**
1	11	(162,400)	-8.0%	-34.1%	(13,009)	514,079		
2	12	(152,459)	-7.5%	-23.4%	(11,001)	543,201	2,008	29,122
3	13	(147,751)	-7.3%	-18.2%	(9,006)	559,028	1,995	15,828
4	14	(145,524)	-7.2%	-15.7%	(7,519)	567,564	1,487	8,536
5	15	(144,473)	-7.1%	-14.5%	(6,533)	572,140	986	4,576
6	16	(143,977)	-7.1%	-13.9%	(5,920)	574,580	613	2,440
7	17	(143,744)	-7.1%	-13.6%	(5,554)	575,876	366	1,296
8	18	(143,634)	-7.1%	-13.5%	(5,342)	576,562	212	686
9	19	(143,583)	-7.1%	-13.4%	(5,222)	576,923	121	362
10	20	(143,559)	-7.1%	-13.4%	(5,154)	577,114	68	190
11	21	(143,526)	-7.1%	-13.3%	(5,082)	577,248	72	134
12	22	(143,499)	-7.1%	-13.3%	(5,025)	577,337	57	89
13	23	(143,480)	-7.1%	-13.3%	(4,985)	577,393	40	56
14	24	(143,468)	-7.1%	-13.3%	(4,959)	577,428	26	35
15	25	(143,461)	-7.1%	-13.3%	(4,942)	577,449	17	21
16	26	(143,457)	-7.1%	-13.3%	(4,932)	577,461	10	12
17	27	(143,454)	-7.1%	-13.3%	(4,926)	577,468	6	7
18	28	(143,453)	-7.1%	-13.3%	(4,922)	577,472	4	4
19	29	(143,452)	-7.1%	-13.3%	(4,920)	577,475	2	2
20	30	(143,452)	-7.1%	-13.3%	(4,919)	577,476	1	1

Table D.7 – "High" Paid Chain Ladder Summary, with All-Prior

Estimate of Total Unpaid Claims Using Paid Data

***All-Prior Estimate in Separate Exhibit**

	(1)	(2)	(3)	(4)	(5)	(6)
			(1) x (2)	(3) - (1)	(7) - (1)	(4) - (5)
	Paid to Date	**Paid CDF**	**Ultimate**	**Estimated Unpaid**	**Case Reserve**	**Estimated IBNR**
A-P*	2,028,756	*1.0040*	*2,036,779*	*8,024*	13,009	(4,985)
2004	962,203	1.0093	971,173	8,969	11,874	(2,904)
2005	898,591	1.0184	915,098	16,508	21,878	(5,370)
2006	907,581	1.0363	940,536	32,955	42,994	(10,040)
2007	977,881	1.0722	1,048,462	70,581	83,430	(12,849)
2008	1,040,208	1.1459	1,191,977	151,769	140,745	11,025
2009	914,456	1.2918	1,181,321	266,865	257,107	9,758
2010	732,524	1.7372	1,272,516	539,993	528,128	11,865
2011	496,043	2.6041	1,291,769	795,726	696,830	98,896
2012	271,729	5.2619	1,429,810	1,158,081	933,516	224,565
2013	99,365	14.9591	1,486,405	1,387,040	1,129,608	257,432
				4,436,510	3,859,117	577,393

Appendix E – Incurred Analysis for "High" Case Reserve Data

Table E.1 – "High" Incurred Loss Triangle with All-Prior Data

	12	24	36	48	60	72	84	96	108	120	132
A-P		1,874,645	1,989,030	2,049,323	2,067,607	2,056,452	2,052,137	2,046,479	2,044,469	2,042,713	2,041,764
2004	770,485	871,259	892,079	959,581	981,362	979,974	979,594	975,287	974,890	974,077	
2005	755,139	837,212	871,723	909,541	920,876	927,887	924,599	921,732	920,468		
2006	778,857	837,074	908,267	945,531	951,361	950,469	952,152	950,576			
2007	835,631	969,389	991,007	1,048,260	1,058,442	1,062,825	1,061,310				
2008	980,023	1,039,677	1,099,087	1,178,784	1,185,561	1,180,953					
2009	958,889	1,052,715	1,105,673	1,164,752	1,171,563						
2010	1,007,229	1,087,877	1,213,688	1,260,651							
2011	974,991	1,102,902	1,192,873								
2012	1,091,849	1,205,245									
2013	1,228,972										

Table E.2 – "High" Incurred Loss Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Tail
2004	1.131	1.024	1.076	1.023	0.999	1.000	0.996	1.000	0.999	
2005	1.109	1.041	1.043	1.012	1.008	0.996	0.997	0.999		
2006	1.075	1.085	1.041	1.006	0.999	1.002	0.998			
2007	1.160	1.022	1.058	1.010	1.004	0.999				
2008	1.061	1.057	1.073	1.006	0.996					
2009	1.098	1.050	1.053	1.006						
2010	1.080	1.116	1.039							
2011	1.131	1.082								
2012	1.104									
VWA	1.104	1.061	1.054	1.010	1.001	0.999	0.997	0.999	0.999	
5-Yr VWA	1.095	1.067	1.053	1.008	1.001	0.999	0.997	0.999	0.999	
3-Yr VWA	1.105	1.083	1.054	1.007	1.000	0.999	0.997	0.999	0.999	
TF Fitted	1.192	1.062	1.020	1.006	1.002	1.001	1.000	1.000	1.000	0.999
User Selected	1.105	1.083	1.054	1.010	1.001	0.999	0.997	0.999	0.999	0.9995
Ultimate	1.267	1.147	1.059	1.004	0.994	0.993	0.994	0.997	0.998	0.9989
% Reported	0.789	0.872	0.944	0.996	1.006	1.007	1.006	1.003	1.002	1.001
% Unrptd	0.211	0.128	0.056	0.004	(0.006)	(0.007)	(0.006)	(0.003)	(0.002)	(0.001)

Table E.3 – "High" Incurred Tail Factor Calculation

Incurred Tail Factor Analysis

All Prior

Tail Years:	8	Actual	167,119	Decay	0.322
Tail Factor:	0.9989	Estimated	130,156	Intercept	0.597
		Error %	-22.1%		

Period	Factor	Dev	Log	Excl	Period	Log	Fitted	Selected	ATA	Ultimate
1	1.10429	0.10429	(2.261)		1	(2.261)	1.191966		1.191966	1.301527
2	1.06108	0.06108	(2.796)		2	(2.796)	1.061760		1.061760	1.091916
3	1.05445	0.05445	(2.911)		3	(2.911)	1.019870		1.019870	1.028402
4	1.01011	0.01011	(4.595)		4	(4.595)	1.006393		1.006393	1.008366
5	1.00088	0.00088	(7.031)		5	(7.031)	1.002057		1.002057	1.001961
6	0.99911	(0.00089)	7.022	Y			1.000662		1.000662	0.999905
7	0.99694	(0.00306)	5.788	Y			1.000213		1.000213	0.999243
8	0.99912	(0.00088)	7.041	Y			1.000068		1.000068	0.999031
9	0.99917	(0.00083)	7.089	Y			1.000022		1.000022	0.998962
10							1.000007	0.999460	0.999460	0.998940
11							1.000002	0.999780	0.999780	0.999480
12							1.000001	0.999870	0.999870	0.999700
13							1.000000	0.999910	0.999910	0.999830
14							1.000000	0.999960	0.999960	0.999920
15							1.000000	0.999980	0.999980	0.999960
16							1.000000	0.999990	0.999990	0.999980
17							1.000000	0.999990	0.999990	0.999990

Table E.4 – "High" Incurred All-Prior Projection (Cumulative)

	Premium	Loss Ratio	24	36	48	60	72	84	96	108	120	132
1994	*669,311*	*83.4%*	*486,823*	*527,159*	*555,862*	*561,479*	*561,975*	*561,474*	*559,754*	*559,264*	*558,797*	558,496
1995	*715,259*	*82.0%*	*511,511*	*553,892*	*584,051*	*589,953*	*590,474*	*589,947*	*588,140*	*587,625*	*587,135*	586,817
1996	*758,317*	*81.2%*	*537,012*	*581,507*	*613,169*	*619,365*	*619,912*	*619,359*	*617,462*	*616,921*	*616,406*	616,073
1997	*811,833*	*79.6%*	*563,582*	*610,278*	*643,506*	*650,009*	*650,583*	*650,003*	*648,012*	*647,444*	*646,904*	646,555
1998	*853,244*	*79.5%*	*591,586*	*640,602*	*675,482*	*682,308*	*682,911*	*682,301*	*680,211*	*679,615*	*679,049*	678,682
1999	*890,376*	*80.0%*	*621,214*	*672,685*	*709,311*	*716,479*	*717,112*	*716,472*	*714,277*	*713,652*	*713,057*	712,672
2000	*986,176*	*75.9%*	*652,790*	*706,878*	*745,366*	*752,898*	*753,563*	*752,891*	*750,584*	*749,927*	*749,302*	748,897
2001	*984,188*	*79.8%*	*684,950*	*741,702*	*782,086*	*789,989*	*790,687*	*789,981*	*787,561*	*786,872*	*786,215*	785,791
2002	*984,698*	*83.8%*	*719,655*	*779,283*	*821,713*	*830,017*	*830,750*	*830,009*	*827,466*	*826,742*	*826,052*	825,606
2003	*1,041,477*	*83.2%*	*755,702*	*818,316*	*862,872*	*871,592*	*872,362*	*871,583*	*868,913*	*868,152*	*867,428*	866,960

Table E.5 – "High" Incurred All-Prior Projection (Incremental)

	12	24	36	48	60	72	84	96	108	120	132	144
1994												(111)
1995											(288)	(117)
1996										(467)	(302)	(123)
1997									(515)	(490)	(317)	(129)
1998								(1,897)	(541)	(515)	(333)	(136)
1999							(581)	(1,991)	(567)	(540)	(349)	(142)
2000						603	(609)	(2,090)	(596)	(567)	(367)	(149)
2001					7,168	633	(640)	(2,195)	(625)	(595)	(385)	(157)
2002				38,488	7,532	665	(672)	(2,307)	(657)	(626)	(405)	(165)
2003			56,752	40,384	7,903	698	(706)	(2,420)	(690)	(656)	(425)	(173)
Totals:	**(144+)**	**(36-132)**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**
Estimated	(853)	130,156	99,014	44,355	4,165	(3,926)	(4,857)	(4,357)	(2,034)	(1,411)	(793)	*(386)*
Actual		167,119	114,384	60,293	18,285	(11,156)	(4,315)	(5,658)	(2,010)	(1,756)	(949)	
Differences		(36,963)	(15,370)	(15,938)	(14,120)	7,229	(542)	1,300	(24)	344	156	
Cumulative Percent Difference			-22.1%	-40.9%	-74.8%	32.8%	8.4%	17.1%	10.1%	18.5%	16.5%	
Weights			0.25	0.50	1.00	2.00	3.00	4.00	5.00	6.00	7.00	
Weighted Average			11.7%									

Table E.6 – "High" Incurred Tail Calibration Summary

		All-Prior Projection					Change in IBNR	
Tail	**(*u*)**	**Total**	**Cumulative**	**Weighted**		**Total**		
Years	**Ultimate**	**Difference**	**Percent**	**Percent**	**IBNR**	**IBNR**	**All-Prior**	**Total**
1	11	(34,285)	-20.5%	35.3%	-	559,823		
2	12	(35,485)	-21.2%	24.7%	(173)	557,075	(173)	(2,748)
3	13	(36,159)	-21.6%	18.8%	(372)	555,354	(199)	(1,721)
4	14	(36,603)	-21.9%	14.9%	(574)	554,098	(202)	(1,255)
5	15	(36,791)	-22.0%	13.2%	(691)	553,513	(117)	(585)
6	16	(36,880)	-22.1%	12.4%	(763)	553,208	(71)	(305)
7	17	(36,923)	-22.1%	12.0%	(805)	553,049	(42)	(159)
8	18	(36,963)	-22.1%	11.7%	(853)	552,884	(48)	(165)
9	19	(36,963)	-22.1%	11.7%	(853)	552,884	0	0
10	20	(36,963)	-22.1%	11.7%	(853)	552,884	0	0
11	21	(36,963)	-22.1%	11.7%	(853)	552,884	0	0
12	22	(36,963)	-22.1%	11.7%	(853)	552,884	0	0
13	23	(36,963)	-22.1%	11.7%	(853)	552,884	0	0

Table E.7 – "High" Incurred Chain Ladder Summary, with All-Prior

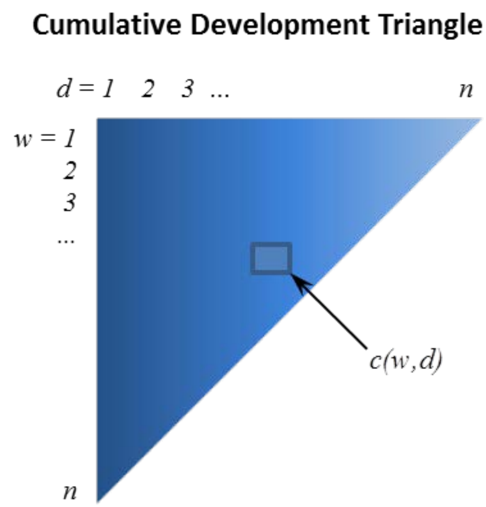
Estimate of Total Unpaid Claims Using Incurred Data
***All-Prior Estimate in Separate Exhibit**

	(7)	(8)	(9)	(10)	(11)	(12)
			(7) x (8)	(11) + (12)	(7) - (1)	(9) - (7)
	Incurred to Date	**Incurred CDF**	**Ultimate**	**Estimated Unpaid**	**Case Reserve**	**Estimated IBNR**
A-P*	2,041,764	*0.9996*	*2,040,912*	12,156	13,009	*(853)*
2004	974,077	0.9989	973,045	10,841	11,874	(1,032)
2005	920,468	0.9981	918,726	20,135	21,878	(1,742)
2006	950,576	0.9972	947,946	40,364	42,994	(2,630)
2007	1,061,310	0.9942	1,055,132	77,251	83,430	(6,179)
2008	1,180,953	0.9933	1,173,030	132,822	140,745	(7,923)
2009	1,171,563	0.9942	1,164,732	250,275	257,107	(6,832)
2010	1,260,651	1.0042	1,265,965	533,442	528,128	5,314
2011	1,192,873	1.0589	1,263,124	767,081	696,830	70,252
2012	1,205,245	1.1466	1,381,967	1,110,238	933,516	176,722
2013	1,228,972	1.2667	1,556,760	1,457,395	1,129,608	327,787
				4,412,001	3,859,117	552,884

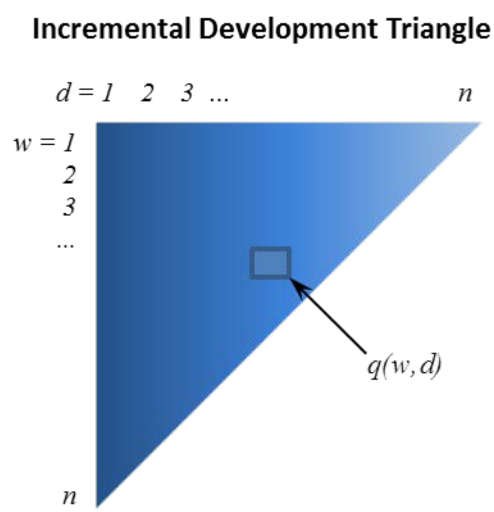
Appendix F – Graphical Representation of Notation

The paper uses the following notation for certain important loss statistics which is also represented graphically:

$c(w,d)$: cumulative loss from accident period w as of age d. Think "when" and "delay."

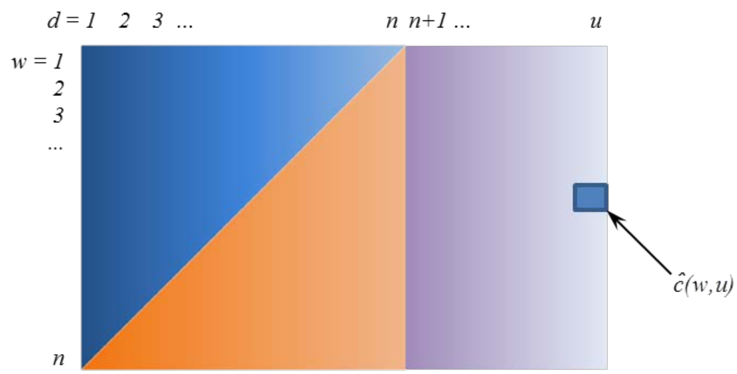


$q(w,d)$: incremental loss for accident period w during the development age from d - 1 to d. Note that $q(w,d) = c(w,d) - c(w,d-1)$.



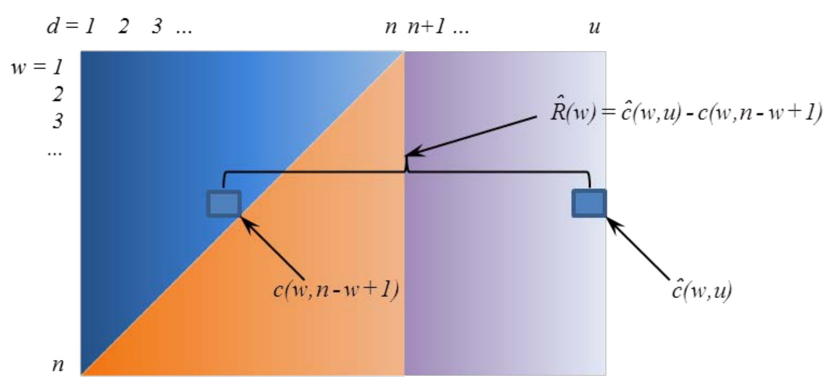
$c(w,u) = U(w)$: total loss from accident period w when at the end of ultimate development u.

Cumulative Development Triangle, estimated to Ultimate

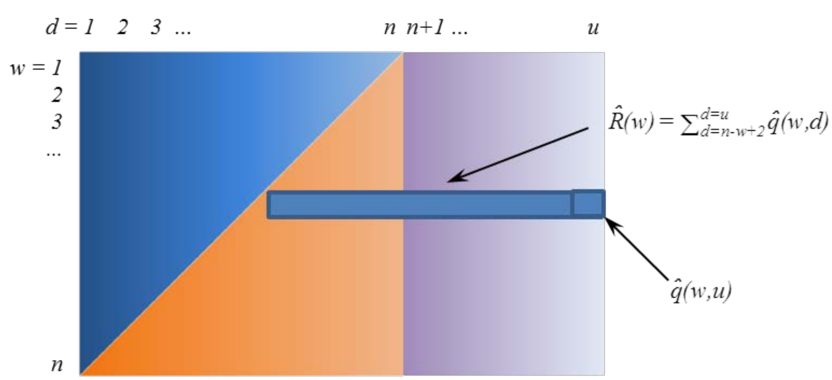


$R(w)$: future development after age $d = n - w + 1$ for accident period w, *i.e.*, $= U(w) - c(w, n - w + 1)$.

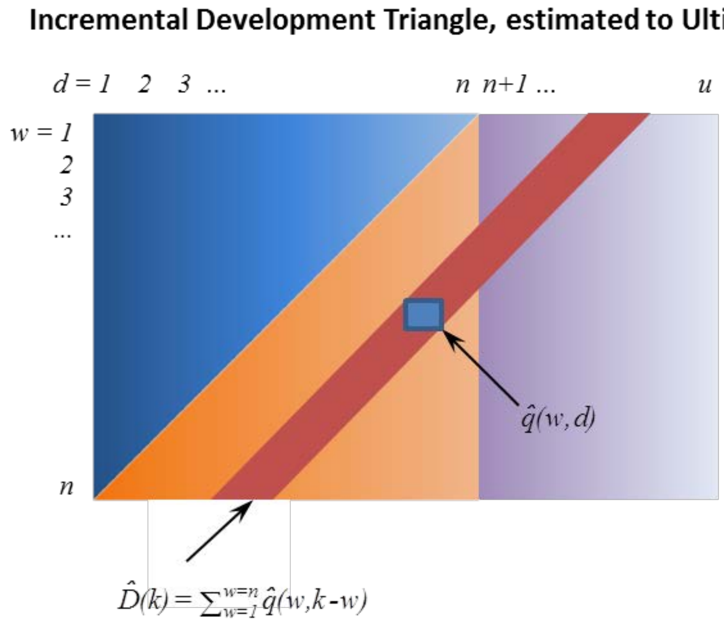
Cumulative Development Triangle, estimated to Ultimate



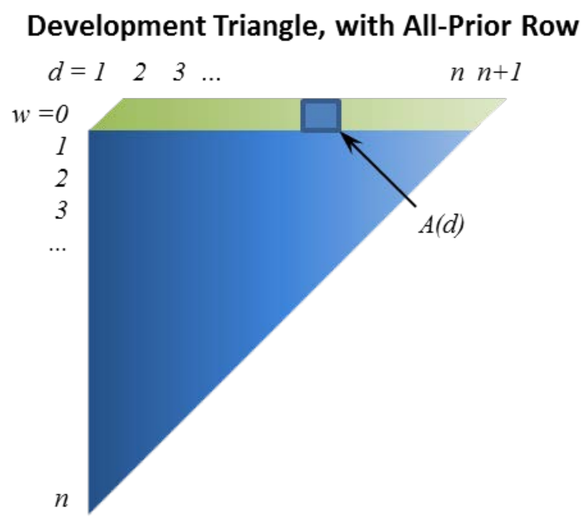
Incremental Development Triangle, estimated to Ultimate



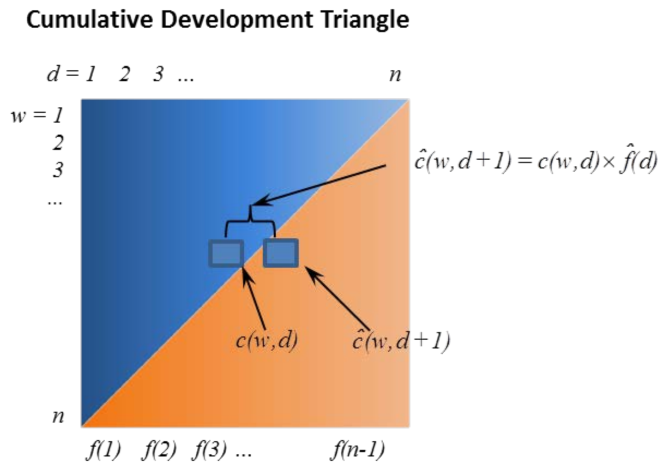
$D(k)$: future development after age $d = n - w + 1$ during calendar period k, *i.e.*, for all $q(w,d)$ where $w + d = k$ and $w + d > n + 1$.



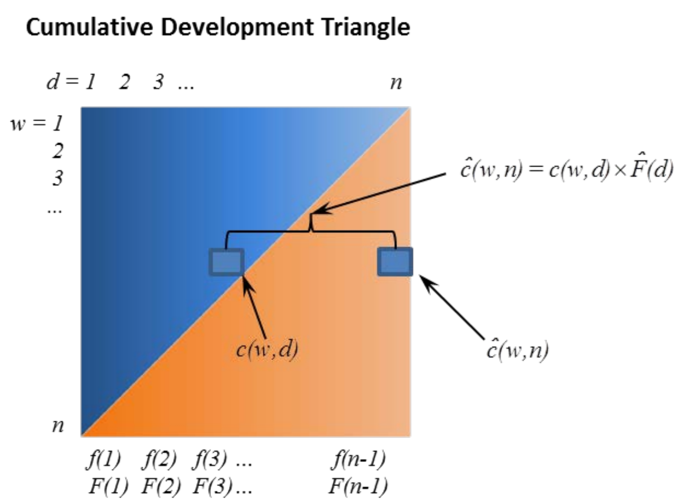
$A(d)$: all-prior data by development age d .

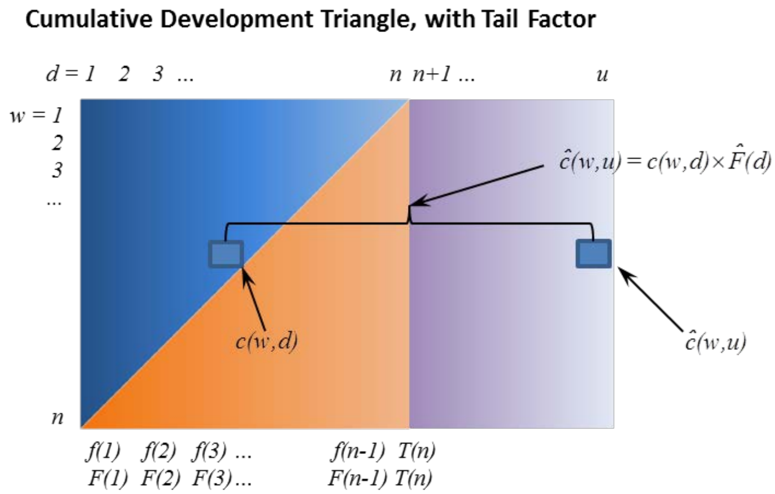


$f(d) = 1 + v(d)$: factor applied to $c(w,d)$ to estimate $c(w,d+1)$ or more generally any factor relating to age d . This is commonly referred to as a link ratio. $v(d)$ is referred to as the 'development portion' of the link ratio, which is used to estimate $q(w,d+1)$. The other portion, the number one, is referred to as the 'unity portion' of the link ratio.

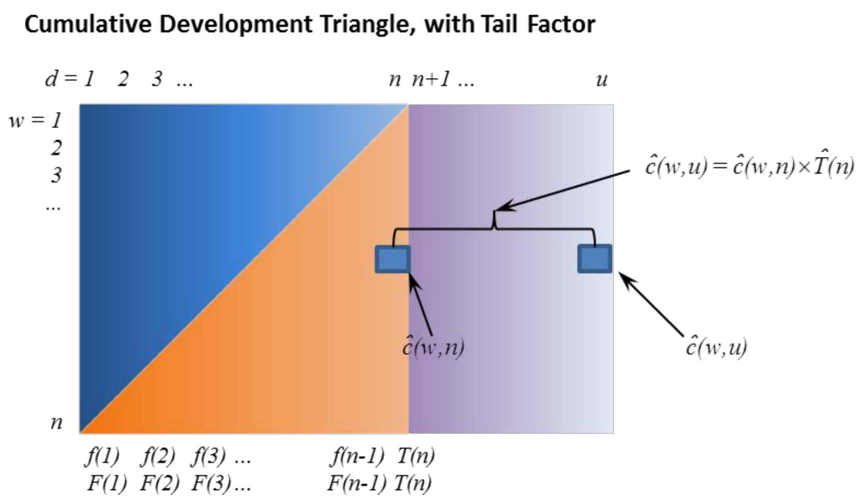


$F(d)$: ultimate development factor relating to development age d. The factor applied to $c(w,d)$ to estimate $c(w,u)$ or more generally any cumulative development factor relating to development age d. The capital indicates that the factor produces the ultimate loss level. As with link ratios, $V(d)$ denotes the 'development portion' of the loss development factor, the number one is the 'unity portion' of the loss development factor.





$T = T(n)$: ultimate tail factor at end of triangle data, which is applied to the estimated $c(w,n)$ to estimate $c(w,u)$.



$\hat{x}$ an estimate of any value or parameter x.