Premium Deficiency Reserve Evaluation for Mortgage Insurers

David Kaye, FCAS, MAAA

Abstract

This paper will provide practical guidance for the actuary evaluating premium deficiency reserves for mortgage insurers. The paper includes a brief discussion of the premium deficiency accounting considerations for mortgage insurance, and introduces a practical deterministic approach for evaluating whether a premium deficiency reserve is necessary for mortgage insurers.

Keywords. Mortgage insurance; premium deficiency reserve; PDR.

1. INTRODUCTION

Beginning in 2007 and continuing for several years, the mortgage insurance (MI) industry experienced significant increases in losses, driven by the deterioration of several interrelated macroeconomic factors, principally, negative home price appreciation (HPA) and elevated unemployment levels. Given the prolonged period of elevated MI losses, mortgage insurers, their auditors and insurance regulators placed greater emphasis on the importance of evaluating whether a premium deficiency reserve (PDR) should be recorded on mortgage insurers' balance sheets.

The meaningful differences that exist between MI and short duration insurance products (e.g., a workers' compensation policy) require a specialized framework when evaluating MI PDR.

1.1 Objective

This objective of this paper is to provide the practicing actuary with:

- Sufficient background on the MI accounting requirements to understand the evaluation of premium deficiency reserve; and

- A simple deterministic framework for evaluating MI PDR.

This paper uses a simulated data set to familiarize the practicing actuary with the MI loss and premium process and provides a basic deterministic framework for analyzing PDR for MI companies. It should be noted, however, that there are various macroeconomic factors (principally, home price appreciation, unemployment and interest rates) that have a significant impact on MI claim and premium experience; these factors are not explicitly addressed in this paper but should be considered when evaluating PDR. These factors result in claim and premium processes that are

more complex than the simulated data utilized to demonstrate the methodology presented in this paper.

1.2 Outline

The remainder of the paper proceeds as follows: Section 2 provides MI background including a brief discussion regarding MI accounting framework with particular emphasis on PDR requirements and a description of common terminology used throughout this paper[1]; Section 3 provides a deterministic framework for evaluating MI PDR and discusses limitations and potential enhancements of the model presented herein.

2. BACKGROUND

2.1 Background on Mortgage Insurance

MI policies differ from typical short duration products familiar to most P&C actuaries in one key way: mortgage insurance policies have effectively unlimited terms and generate premium and losses for many years (in contrast to typical short duration contracts which generally remain in effect for one year or less). The implication for PDR for MI exposure is that the practitioner must project future premium and losses for loans originated on or before the evaluation date for many years into the future. Over the projection period, however, macroeconomic factors that influence claims and policy persistency can change significantly and result in significant deviation between historical performance and performance over the projection period.

Additional key features of MI policies include the following[2]:

- MI policies are issued at the time that the mortgage is issued and can either be paid by the borrower (most common) or lender (less common).

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[1] This paper presumes a level of familiarity with MI. For a more detailed primer on MI, see reference [1].
[2] Reprinted from [1].

- Premiums are paid on either a monthly (most common) or single up-front (less common) basis. The premium associated with monthly pay policies is typically paid as part of the monthly mortgage payment.

- The collected monthly premiums are generally recognized as income in the period in which they are collected (that is, the monthly premiums are written and earned at the same time) meaning that there is typically a very small (or no) unearned premium reserve associated with monthly paid MI policies. There is an unearned premium reserve associated with single up-front premium policies, which is amortized over the life of the MI contract as losses associated with the contract is expected to emerge; however monthly pay policies are more common than up front policies.

- MI coverage is typically expressed as a percentage of a loan's unpaid principal balance ("UPB"). These coverage percentages vary from loan to loan, but a typical average coverage percentage is around 25%.

- MI policies provide lenders coverage for a portion of the UPB stipulated in the contract (generally around 25%). In addition, the MI policy generally reimburses the coverage beneficiary for lost interest payments and certain foreclosure-related expenses.

- Unlike typical Property and Casualty insurance policies, which are generally in force for one year and have defined termination dates, MI policies often generate premiums and losses for a number of years and there is uncertainty with regard to how long each policy will remain in force. The MI policy holder may exit the insured population for a number of reasons, including defaulting on the mortgage (i.e., becoming a claim), refinancing the loan, or paying down the principal on the loan to the point that the loan no longer requires MI.

- MI losses are highly correlated with macroeconomic factors such as home price appreciation and unemployment. As was highly evident in 2007-2011, MI company results were adversely affected by a steep drop in home prices followed by rising levels of unemployment. Not surprisingly, the states with the sharpest decreases in home prices – CA, FL and NV – were significant drivers of adverse loss experience for the MI industry.

- As explained further below, MI loss reserves are recorded at the time when a borrower is "delinquent" in paying their mortgage. This results in an unusual accounting construct where the timing of premium earning and loss accrual are not matched. In other words, premium revenue from MI policies is recognized (i.e., earned) prior to the associated losses being recognized.
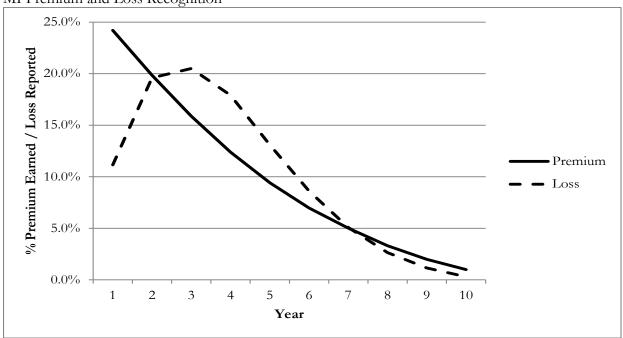
- For a cohort of monthly paid MI policies issued during a year, the premium revenue generated by the policies is the greatest during the first year and then decreases over the next ten years as policies exit the population. Those that exit the population through delinquency, thereby giving rise to the recording of MI loss reserves tend to rise through third or fourth year after loan origination. After peaking, incremental losses tend to decrease as policies continue to exit the population.

2.2 Accounting for Mortgage Insurance Losses

The accounting framework for MI results in a departure of one of the principal objectives of accounting: revenue and expense matching. For typical, single-year (or shorter duration) P&C insurance products, both revenue (premium) and expense (claim costs) are recognized uniformly through the period the policy is effective[3]. For the majority of MI policies written in the U.S., premium is earned on a monthly basis, while losses on MI policies are not recognized until the borrower stops paying the monthly mortgage payment and the lender or loan servicer notifies the MI company that the borrower is delinquent. The result of this accounting framework is that premiums are highest during the first year following the loan origination for a cohort of policies

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[3] This description is generally accurate, although there are exceptions such as property catastrophe cover where premium and loss might not be recognized uniformly through the policy period.

while losses generally rise through the first few years after origination, peak around the third or fourth year and then decline over time as loans from the cohort exit the population. Graph 1 demonstrates this relationship for a cohort of loans written during the same calendar year[4]:

Graph 1
MI Premium and Loss Recognition

While MI companies follow the general framework described above for general premium and loss accounting, Statement of Statutory Accounting Principle (SSAP) 58 provides statutory accounting guidance for PDR[5]. SSAP 58, paragraph 23 states:

> When the anticipated losses, loss adjustment expenses, commissions and other acquisition costs, and maintenance costs exceed the recorded unearned premium reserve, contingency reserve, and the estimated future renewal premium on existing policies, a premium deficiency reserve shall be recognized by recording an additional liability for the deficiency with a corresponding charge to operations. Commissions and other acquisition costs need not be considered in the premium deficiency analysis to the extent they have not been

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[4] In this paper, we refer to years in which loans are underwritten as "book year".

[5] GAAP accounting guidance does not specifically address MI, therefore, MI companies typically utilize statutory accounting guidance in preparing the GAAP accounting statements. A primary difference between statutory and GAAP accounting statements is the existence of a contingency reserve required by statutory accounting guidance, but not allowed under GAAP.

expensed. If an insurer utilizes anticipated investment income as a factor in the premium deficiency calculation, disclosure of such shall be made in the financial statements[6].

A key point to note is that a PDR is only required when the sum of projected future losses and expenses (outflows) exceeds the sum of projected future premiums (inflows), unearned premium reserves, and contingency reserves; nothing is recorded in the event that the opposite of a PDR (a "premium sufficiency") is estimated. In practice, when estimating PDR, MI companies project cash payments related to all loans in the portfolio as of the measurement date inclusive of cash payments related to loans that are delinquent (and therefore included in the recorded loss reserves) at the evaluation date (facilitating discounting of the projected cash flows). The recorded loss reserves are deducted from the projected cash flows to avoid double counting the outflows related to loans making up the recorded loss reserves at the financial statement date.

2.3 Terminology and Organization of Data

Before providing a framework for estimating MI PDR, it is important to introduce several additional terms as well as to lay out the key characteristics used to organize the data.

2.3.1 Terminology

Although the terminology below is not necessarily universal, it is used throughout the remainder of this paper.

- Book year (and book half year): The year (or half year) in which a cohort of MI policies is issued. For example, MI policies written during 2014 will be referred to as "book year 2014 policies". In Section 3, we organize the data by book half year with the format 20XX-1 representing loans originated during the first six months of 20XX and 20XX-2 representing loans originated during the second six months of 20XX[7].

- Risk in force ("RIF"): The exposure to loss faced by MI companies. The RIF is calculated by multiplying the MI's coverage percentage by the loan's UPB. In addition to the coverage percentage multiplied by the UPB, the MI company may also be required to pay lost interest

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[6] MI companies typically recognize investment income by discounting projected cash flows at an appropriate discount rate. The selection of an appropriate discount rate is beyond the scope of this paper; for illustrative purposes, a 2% discount rate has been used in the calculations shown in the Appendix.
[7] Although we use book half year in the Appendix, we note that the actuary could consider book year or book quarter.

and certain foreclosure expenses; for this reason, the ratio of claim payments to RIF may be greater than 100%.

- Policies in force ("PIF"): The total number of policies in force as of a particular evaluation date.

- Policy persistency (persistency): The portion of policies that remain in force from one period of time to the next.

- Submitted claim: A delinquent loan where the borrower has not made mortgage payments, the lending institution has foreclosed on the subject property, and a claim has been submitted to the MI company. Within this paper we will consider both claim payments (dollars) and claim counts.

- Outstanding delinquency: A loan reported to the MI company when the borrower has fallen two mortgage payments behind (note, there is some variance about when a loan is identified as delinquent in the MI industry, here we are assuming the MI company has set the definition as the borrower being behind two or more payments).

2.3.2 Data organization

For purposes of the method described in Section 3, the data will be organized by book half year with semi-annual evaluations. Further segmentation of the data in the Appendix is not addressed; however, in practice, the actuary should consider how to best segment the data for use in the model described in Section 3[8]. There are several items that an actuary might consider when determining appropriate segmentation for the data within the PDR analysis as described below:

- Unemployment: Varying levels of unemployment can have significant impacts on MI claim activity. States or regions with higher levels of unemployment are more likely to also experience elevated claim activity which might have a meaningful impact on the PDR analysis.

- Credit worthiness of borrowers: The credit worthiness of borrowers can be a significant predictor in determining borrower behavior. FICO score[9] or distinguishing between Prime and Subprime loans in developing estimates can result in better data stratification.

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[8] Actuarial literature contains a number of papers written about effective data segmentation. For example, see [2].
[9] FICO is a common credit scoring mechanism developed originally by the Fair Isaac Corporation. The FICO score is a numerical representation of the credit worthiness of a borrower.

- Home price appreciation or depreciation: Different states or regions might have different levels of home price appreciation that can impact both claim behavior and policy persistency.

During the 2008 housing market downturn, MI companies observed elevated MI claim submissions from states that had significant increases in home prices prior to the housing market downturn followed by significantly elevated unemployment levels resulting from the subsequent recession. For this reason, during the last market downturn, some MI companies chose to separately analyze California, Florida, and Nevada; these "sand states" were particularly hard hit by the combination of a significant housing market collapse and elevated unemployment and displayed similar, elevated claim characteristics.

Accounting principles require that the need for a PDR be evaluated at the level at which the MI company manages its insurance portfolio; the actuarial evaluation of PDR may or may not coincide with the level at which the MI company manages the insurance portfolio so the actuarial analysis may need to be aggregated in order to align with the Company's required PDR segmentation.

3. DETERMINISTIC FRAMEWORK FOR MI PDR

As described in the previous section we will estimate PDR by comparing:

a) the net present value of expected future losses and policy maintenance expenses for the MI company's business in force and

b) the net present value of expected future premiums, existing unearned premium reserves, unpaid claim reserves, DAC (if any), and contingency reserves (if any).

If, in our evaluation a) exceeds b), then the MI company should record a PDR.

We note that the focus of this paper is on the estimation of the present value of cash flows related to future premiums, policy maintenance expenses and losses. The financial statement items (unearned premium reserves, unpaid claim reserves and contingency reserves) are presumed to be known by the actuary at the financial statement date and their estimation is beyond the scope of this paper.

3.1 Estimating the Cash Flows for PDR

In developing the cash flows used to assess whether a PDR is necessary, we will separately estimate premiums (adding a provision for policy maintenance expenses) and losses and then

consider the financial statement items to determine whether a PDR is necessary. Sections 3.1.1 through 3.1.3 below provide a description of the PDR calculations and Section 3.1.4 provides a highly simplified sample PDR calculation. The Appendix includes a more realistic example based on simulated MI data (the sections below provide references to the calculations detailed in the Appendix).

3.1.1 Estimating future premiums

To estimate the future premiums, we will organize historical PIF data into a triangular data format familiar to P&C actuaries, with the rows representing book half year exposure periods and the columns representing semi-annual evaluation periods. The PIF data gathered to create the data triangle represents the remaining PIF at the end of each evaluation date, therefore, the PIF data decreases over time as the MI portfolio unwinds.

Estimating the future premiums as described in this paper is a three step process (the calculations are detailed in Appendix, Exhibit 3):

1. First, we evaluate the decline in PIF over time as the MI portfolio unwinds by estimating PIF persistency ("persistency"). The persistency is developed by calculating ratios of PIF at each evaluation period, i+1, divided by the PIF at the preceding evaluation period, i. The triangle of PIF is completed by selecting a persistency factor for each evaluation period and then applying the selected decay factor at each period to the PIF observed (or projected) at the end of the prior evaluation period. Performing these calculations allows the actuary to estimate the PIF for each exposure period at each future evaluation period.

   As an example, Appendix Exhibit 3, Table 1 indicates that at the end of December 2015, there are 6,923 policies in force for loans written during the second half (July 1 – December 31) of 2015 (the 2015-2 cohort). By using historical relationships of PIF decay (see Appendix Exhibit 3, Table 2), we estimate that 90.7% of policies will remain in force at the end of the next evaluation period. Therefore, we project 6,283 (=6,923 x 90.7%) policies in force at June 30, 2016 from the 2015-2 cohort. Proceeding in this manner for all projection periods, we can project PIF for each future period as shown in Appendix Exhibit 3, Table 4.

2. Based on the projected PIF, we calculate the average PIF for each future evaluation period as shown in Appendix Exhibit 3, Table 5. Note, that we have shifted the triangle

in Appendix Exhibit 3, Table 5 so that each column represents a future calendar half year; shifting the projections in this manner facilitates discounting of the projected premium cash flows (related to this point, below Appendix Exhibit 3, Table 5, discount factors are calculated assuming on average, premiums are collected in the middle of each projection period[10]).

3. Using the projected average PIF, we calculate the projected future premium by multiplying the average monthly premium for each book half year by the average PIF over each projection period and multiply the result by 6 (the premiums are monthly average premiums while each projection period represents 6 months of exposure). The calculations and results are shown in Appendix Exhibit 3, Table 6.

Note that Appendix Exhibit 3, Table 6 utilizes the discount factors calculated in Appendix Exhibit 3, Table 5 to determine the discounted projected future premium necessary for the PDR calculation. For illustrative purposes, the cash flows are discounted using a 2% discount rate assumption.

3.1.2 Estimating future claims and loss adjustment expenses

In order to estimate the future claim payments, we begin with a triangle of paid claim counts, which we will utilize to perform two standard actuarial methodologies – a traditional "chain ladder" development method (referred to in this paper and exhibits as Claim Development Method, or CDM and a Bornhuetter-Ferguson method, or BFM). The future claim estimates are outlined in Exhibit 2 of the Appendix.

The CDM should be recognizable by P&C actuaries. In preparing our estimates, we calculate a triangle of claim count development factors (Appendix Exhibit 2, Table 2), compute average development factors, select claim development factors and apply the calculated cumulative development factors to latest evaluation of the claim count triangle in order to develop an estimate of ultimate claim counts[11]. The claim count estimate represents the projected ultimate claims for each book half year.

The CDM results are shown in Appendix Exhibit 2, Table 3, Column 4. We utilize the CDM

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[10] Given that the policies are declining over the future periods, assuming the middle of each projection period results in slightly higher discount than what would be calculated using a more refined assumption (i.e., factoring in the declining portfolio). The mid-period assumption is utilized here for simplicity, although enhancements to the calculation could be made if the actuary chooses to do so.

[11] See [3] for a more detailed discussion of the CDM and BFM

results to provide guidance in developing expected loss estimates for use in the BFM by first dividing the CDM results by the number of loans originated in each half year and then selecting an expected claim count per loan count rate. The expected count per loan count and the claim development pattern underlying the CDM to develop a BFM as in Appendix Exhibit 2, Table 3, Column 7.

We select paid claims based on the results of the CDM and the BFM and then multiply the estimate by the expected average claims size. The expected average claim size, in turn, is calculated by dividing the claim dollars paid to date by the count of claims paid to date. In general, the claim severity tends to be more stable and predictable as claim payments are closely aligned with RIF and RIF tends to be stable over time, although the actuary should pay attention to observed or expected shifts in future claim severity relative to recent historical severity experience[12].

The estimate of ultimate claim costs is calculated by multiplying the ultimate claim count estimate by the projected average claim size; subtracting the claims paid as of the evaluation date results in the forecasted unpaid claims. The forecasted unpaid claims are discounted using the development factors underlying the CDM and a LAE factor is added to represent the total projected discounted loss and LAE cash flows.

3.1.3 PDR estimates

We use the discounted premium and loss cash flow projections to determine whether a PDR is necessary at the evaluation date. A PDR is recorded if the estimated net discounted loss and LAE) exceeds discounted premiums net of policy maintenance expenses plus financial statement items related to premiums and losses (unearned premium reserves, and recorded loss and LAE reserves)[13]. If the cash flows are negative and the absolute value of the cash flows is greater than the financial statement items related to premiums and losses, then we record a PDR (nothing is recorded if the conditions are not met). In the Appendix, Exhibit 1 displays the determination of whether a PDR is necessary at the evaluation date (in the example in the Appendix, no PDR is necessary).

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[12] For mortgage insurance, separate estimation of frequency and severity is often preferable since severity tends to be closely tied to RIF and is therefore generally more stable and easier to estimate than frequency.

[13] On a statutory basis, we also include contingency reserves.

3.1.4 A Simplified PDR Example

The Appendix to this paper contains a detailed sample calculation showing the PDR estimation framework described above, however a simplified sample is presented in this section to facilitate understanding of the process described herein.

The following data and assumptions are provided for this simple example (note, in this example, we are determining whether a PDR is necessary for a single cohort of policies written during a single calendar period, which is not consistent with actual practice where we would determine whether a PDR is necessary across a portfolio of MI policies):

A. During 2015, 1,100 MI policies are written. At year-end 2015 (the financial statement date), 1,000 of the 1,100 policies remain.

B. All of the policies are monthly pay premium and there is no unearned premium reserve related to single-pay policies.

C. We expect 250 policies to exit the population during each subsequent calendar year (i.e., 750 policies remain at year-end 2016, 500 policies remain at year-end 2017, etc.) until all policies exit the population by year-end 2019.

D. During 2015, the average monthly policy premium was $100 / policy / month.

E. The contingency reserve recorded at year-end 2015 for the loans in the cohort is $300,000.

F. At year-end 2015, there are 50 delinquent loans with average RIF on the loans of $40,000 and a recorded loss and LAE reserve of $400,000. The projected average payment date for the loss reserves is June 30, 2016. Further, we assume that $40,000 is a reasonable estimated severity for claims paid during subsequent calendar years.

G. Using historical claim data, we have projected 10 claims to be paid during 2017, 25 in 2018, 20 in 2019 and 5 in 2020. We assume the average payment date is June 30 for the paid claims.

H. Policy maintenance expenses are assumed to be 3% of the forecasted premiums and loss adjustment expenses are assumed to be 5% of the forecasted paid losses. Both items (policy maintenance expenses and loss adjustment expenses) are assumed to be expenses in the period in which the premiums and losses are paid.

I. The illustrative discount rate selected for the example is 1.5% / annum.

Table 1 outlines the methodology used to develop the projected and discounted premium flows over the projection period. Note that the notes referenced in Table 1 reference data provided in the assumptions and data list directly above.

		Note / formula	12/31/15	12/31/16	12/31/17	12/31/18	12/31/19	Total
Table 1: Projected and discounted premiums								
(1)	PIF	Given in (A) & (C)	1000	750	500	250	0	N/a
(2)	Average PIF	Average of CY values in (1)		875	625	375	125	N/a
(3)	Premium / policy / month	Given in (D)		100	100	100	100	N/a
(4)	Projected annual premium	(2)x(3)x12.0		1,050,000	750,000	450,000	150,000	2,400,000
(5)	Discount years			0.5	1.5	2.5	3.5	
(6)	Discount factor	1.0 /1.015 ^ (5)		0.993	0.978	0.963	0.949	
(7)	Discounted premium	(4)x(6)		1,042,213	733,436	433,558	142,384	2,351,590

Note that the premium presented in item D is presented as an average per policy per month, therefore in Table 1, item 4, the calculation is multiplied by 12 to represent a full year of premium for the average number of policies in force in each calendar year. The resulting discounted premium shown in the total column of line 7 of $2,351,590 is used in Table 3 below to determine if a PDR is needed at year-end 2015.

Table 2 outlines the methodology used to develop the projected and discounted loss cash flows over the projection period.

		Note / formula	6/30/16	6/30/17	6/30/18	6/30/19	6/30/20	Total
Table 2: Projected and discounted paid losses								
(1)	Paid claim counts	Given in (G)	N/a	10	25	20	5	60
(2)	Average size per claim	Given in (F)	40,000	40,000	40,000	40,000	40,000	N/a
(3)	Paid claim dollars	2016 from (F), other (1)x(2)	400,000	400,000	1,000,000	800,000	200,000	2,800,000
(4)	Discount years		0.5	1.5	2.5	3.5	4.5	
(5)	Discount factor	1.0 / 1.015 ^ (4)	0.993	0.978	0.963	0.949	0.935	
(6)	Discounted paid claim dollars	(4)x(6)	397,033	391,166	963,463	759,379	187,039	2,698,081

The resulting discounted paid claim dollars shown in the total column of line 6 of $2,698,081 is used in Table 3 below to determine if a PDR is needed at year-end 2015.

Table 3 uses the premium and paid claim amounts from Tables 1 and 2 to along with other amounts from the data and assumptions presented above to determine whether a PDR is necessary at year-end 2015.

		Note / formula	Statutory Basis	GAAP Basis
Table 3: PDR Calculation				
(1)	Discounted premium	Table 1, Line (7)	2,351,590	2,351,590
(2)	Policy maintenance costs	3% x (1), 3% given in (H)	70,548	70,548
(3)	Discounted paid claim dollars	Table 2, Line (6)	2,698,081	2,698,081
(4)	Loss adjustment expense	5% x (3), 5% given in (H)	134,904	134,904
(5)	**Net cash flows**	**(1)-(2)-(3)-(4)**	**(551,942)**	**(551,942)**
(6)	Contingency reserve	Given in (E)	300,000	-
(7)	Recorded loss & LAE reserve	Given in (F)	400,000	400,000
(8)	Unearned premium reserve	Given in (B)	-	-
(9)	**Total - Financial Statement Items**	**(6)+(7)+(8)**	**700,000**	**400,000**
(10)	Net Cash Flows Plus Financial Statement Items	(5)+(9)	148,058	(151,942)
(11)	**Premium deficiency reserve**	**ABS{Min(0,(10))}**	**-**	**151,942**

Table 3 presents the PDR calculation using U.S. GAAP and U.S. statutory accounting principles. Because contingency reserves are not permissible under U.S. GAAP, the calculations outlined in Table 3 indicate a PDR of $151,942 on a GAAP basis, but no PDR on a statutory basis.

3.2 Benefits and Limitations of the Methodology Described in Section 3.1

The triangular methods for estimating premium and claim cash flows should have an appeal for actuaries since the triangular arrangement of the data is familiar to all actuaries and the mechanics of the model are intuitive and straightforward. The methodology is also appealing because it is more straightforward to describe to a non-actuarial audience than methods that require an understanding of statistical concepts (e.g., regression). Statistical methods are often referred to by non-technical audiences as "black box" methods because the inputs and outputs of the model are easy to describe, but the actual model mechanics are difficult to describe; the deterministic model described in this paper does not have this limitation.

In addition to being straightforward to describe to a non-actuarial audience, the organization of the data as outlined above and in the Appendix allows the actuary and management to estimate and monitor loss ratios for book years through time. Monitoring current and historical book year loss ratios can give actuaries and management insight on changes in underwriting, claims experience or portfolio persistency that drives the profitability of the MI company's insurance portfolio.

The key limitations of the deterministic framework are:

- Using aggregate data does not allow the actuary to explicitly model the factors that are most correlated with persistency and claim behavior. For example, persistency is highly correlated with interest rates; if interest rates fluctuate significantly during the historical experience period, but are not expected to fluctuate over the projection period, then the historical experience may not be representative of future performance. Conversely, regression models can be developed that utilize interest rates as an explanatory variable, which allows the actuary to quickly develop alternative estimates assuming different future interest rate paths.
- Related to the first point, the deterministic model does not allow for explicit sensitivity testing of the results to changes in macroeconomic factors. For example, if the MI company is concerned about the effect of an increase in unemployment on the Company's results, the effect cannot be explicitly incorporated into the framework; such modeling may be required by Government Sponsored Entities (GSE's)'s to determine the MI company's capital requirements.

Statistical regression models have the distinct advantage over the method described in this paper in that they allow for direct modeling of premium and losses in different macro-economic environments. However, the deterministic methods utilized in this paper could be enhanced by

looking at the impact of historical macroeconomic "shock" events on MI claim and persistency and using those historical relationships to calculate "stressed" scenarios of future performance.

4. CONCLUSIONS

The actuary who prepares premium and loss forecasts for MI companies must understand the unique MI accounting framework, including the evaluation of whether a PDR is required. Although the accounting for MI differs from traditional P&C insurance products, deterministic triangular methods commonly used to develop estimates for P&C products can help actuaries project delinquent loan behavior. After the actuary has a strong grasp of MI data, the accounting model and persistency and claim behavior, more complex regression or generalized linear model procedures can be utilized to further enhance MI premium and loss forecasts.

Acknowledgment

Supplementary Material

The Appendix follows this paper is also available electronically on the CAS website accompanying this paper. The dataset provided within the Appendix was simulated using constraints generally consistent with the author's knowledge of MI premium and claim.

5. REFERENCES

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Abbreviations and notations

BFM, Bornhuetter-Ferguson method
CDM, claim development method
LAE, loss adjustment expenses
LTV, loan to value ratio
MI, mortgage insurance
PDR, premium deficiency reserve
PIF, policies in force
RIF, risk in force
UPB, unpaid principal balance

Biography of the Author

David Kaye is Director at PwC in Philadelphia, PA. He has a B.S. in Mathematics and a B.S. in Statistics from the Pennsylvania State University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. David participates on the CAS Committee on Professionalism Education.

Cash Flows

(1)	Discounted Premium Net of Policy Maintenance Flows		115,202,033
(2)	Discounted Loss & LAE Flows		92,714,955
(3)	Net Cash Flows	(1)-(2)	22,487,077

Financial Statement Items

(4)	Recorded Loss and LAE Reserves		42,153,568
(5)	Unearned Premium Reserve		7,515,352
(6)	Statutory Contingency Reserve		111,251,356
(7)	Total - Financial Statement Items	Sum (4) - (6)	160,920,276
(8)	Net Cash Flows Plus Financial Statement Items	(3)+(7)	183,407,353
(9)	Premium Deficiency Reserve	Abs{Min[0,(8)]}	-

Table 1: Cumulative Paid Claim Count Data

Book Half Year	1	2	3	4	5	6	7	8	9	10	11
2004-1	1	8	15	22	29	36	43	50	80	125	147
2004-2	1	8	15	22	29	36	43	50	78	120	141
2005-1	4	11	18	25	32	39	53	60	81	123	137
2005-2	4	11	18	25	32	39	54	61	98	120	135
2006-1	3	10	17	24	31	38	45	52	79	106	127
2006-2	3	10	17	24	31	38	45	59	80	101	122
2007-1	5	12	19	26	33	40	54	61	82	117	138
2007-2	5	12	19	26	33	40	55	70	100	145	175
2008-1	1	8	15	22	29	36	51	66	88	133	163
2008-2	1	8	15	22	29	36	43	50	86	108	130
2009-1	3	11	19	27	35	43	51	66	89	119	149
2009-2	3	10	17	24	31	38	52	59	95	138	152
2010-1	4	11	18	25	32	39	54	69	106	136	166
2010-2	5	12	19	26	33	40	47	61	95	116	130
2011-1	5	12	19	26	33	40	55	62	99	143	
2011-2	5	12	19	26	33	40	55	70	100		
2012-1	1	8	15	22	29	36	50	64			
2012-2	3	10	17	24	31	38	45				
2013-1	5	12	19	26	33	40					
2013-2	4	11	18	25	32						
2014-1	3	10	17	24							
2014-2	2	9	16								
2015-1	1	8									
2015-2	2										

12	13	14	15	16	17	18	19	20	21	22	23	24
162	177	184	191	198	205	212	219	224	227	228	229	229
162	176	190	197	204	211	218	225	230	233	234	235	
165	172	179	186	193	200	207	214	219	222	223		
157	164	171	178	185	192	199	206	211	214			
154	168	175	182	189	196	203	210	215				
143	157	171	178	185	192	199	206					
152	159	173	180	187	194	201						
197	204	219	226	233	240							
178	185	200	207	214								
145	152	167	174									
164	179	187										
181	195											
188												

Table 2: Paid Claim Count Development Factors

Book Half Year	2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9	11/10	12/11
2004-1	8.000	1.875	1.467	1.318	1.241	1.194	1.163	1.600	1.563	1.176	1.102
2004-2	8.000	1.875	1.467	1.318	1.241	1.194	1.163	1.560	1.538	1.175	1.149
2005-1	2.750	1.636	1.389	1.280	1.219	1.359	1.132	1.350	1.519	1.114	1.204
2005-2	2.750	1.636	1.389	1.280	1.219	1.385	1.130	1.607	1.224	1.125	1.163
2006-1	3.333	1.700	1.412	1.292	1.226	1.184	1.156	1.519	1.342	1.198	1.213
2006-2	3.333	1.700	1.412	1.292	1.226	1.184	1.311	1.356	1.263	1.208	1.172
2007-1	2.400	1.583	1.368	1.269	1.212	1.350	1.130	1.344	1.427	1.179	1.101
2007-2	2.400	1.583	1.368	1.269	1.212	1.375	1.273	1.429	1.450	1.207	1.126
2008-1	8.000	1.875	1.467	1.318	1.241	1.417	1.294	1.333	1.511	1.226	1.092
2008-2	8.000	1.875	1.467	1.318	1.241	1.194	1.163	1.720	1.256	1.204	1.115
2009-1	3.667	1.727	1.421	1.296	1.229	1.186	1.294	1.348	1.337	1.252	1.101
2009-2	3.333	1.700	1.412	1.292	1.226	1.368	1.135	1.610	1.453	1.101	1.191
2010-1	2.750	1.636	1.389	1.280	1.219	1.385	1.278	1.536	1.283	1.221	1.133
2010-2	2.400	1.583	1.368	1.269	1.212	1.175	1.298	1.557	1.221	1.121	
2011-1	2.400	1.583	1.368	1.269	1.212	1.375	1.127	1.597	1.444		
2011-2	2.400	1.583	1.368	1.269	1.212	1.375	1.273	1.429			
2012-1	8.000	1.875	1.467	1.318	1.241	1.389	1.280				
2012-2	3.333	1.700	1.412	1.292	1.226	1.184					
2013-1	2.400	1.583	1.368	1.269	1.212						
2013-2	2.750	1.636	1.389	1.280							
2014-1	3.333	1.700	1.412								
2014-2	4.500	1.778									
2015-1	8.000										
Sel. Claim Development Factor	**4.271**	**1.701**	**1.409**	**1.289**	**1.225**	**1.293**	**1.212**	**1.493**	**1.389**	**1.179**	**1.143**
Cumulative DF	100.801	23.601	13.874	9.850	7.639	6.237	4.824	3.981	2.666	1.920	1.628

13/12	14/13	15/14	16/15	17/16	18/17	19/18	20/19	21/20	22/21	23/22	24/23
1.093	1.040	1.038	1.037	1.035	1.034	1.033	1.023	1.013	1.004	1.004	1.000
1.086	1.080	1.037	1.036	1.034	1.033	1.032	1.022	1.013	1.004	1.004	
1.042	1.041	1.039	1.038	1.036	1.035	1.034	1.023	1.014	1.005		
1.045	1.043	1.041	1.039	1.038	1.036	1.035	1.024	1.014			
1.091	1.042	1.040	1.038	1.037	1.036	1.034	1.024				
1.098	1.089	1.041	1.039	1.038	1.036	1.035					
1.046	1.088	1.040	1.039	1.037	1.036						
1.036	1.074	1.032	1.031	1.030							
1.039	1.081	1.035	1.034								
1.048	1.099	1.042									
1.091	1.045										
1.077											

Factor to Ult

1.066	**1.065**	**1.039**	**1.037**	**1.036**	**1.035**	**1.034**	**1.023**	**1.014**	**1.004**	**1.004**	**1.002**	**1.002**
1.424	1.336	1.254	1.207	1.165	1.124	1.086	1.050	1.027	1.013	1.008	1.004	1.002

Table 3: Claim Estimates

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	Original Loan Population	Total Paid Claims	Development Factor	CDM Estimated Claims (2)x(3)	CDM Estimate / Loan Count (4)x(1)	Expected Claims / Loan Count	BFM Estimated Claims (A)	Selected Estimated Claims	Average Claim Size	Estimated Ultimate Claims (8)x(9)	Claims Paid A.o. 12/31/15	Forecasted Unpaid Claims (10)-(11)	Discounted Unpaid 2%	LAE Load 5%	Discounted Loss & LAE (13)x(14)
2004-1	7,483	229	1.002	229	3.1%	3.3%	229	229	38,952	8,937,848	8,920,008	17,840	17,752	1.05	18,640
2004-2	6,970	235	1.004	236	3.4%	3.3%	236	236	38,996	9,200,753	9,164,060	36,693	36,332	1.05	38,148
2005-1	6,923	223	1.008	225	3.2%	3.3%	225	225	40,050	9,005,735	8,931,150	74,585	73,690	1.05	77,374
2005-2	7,429	214	1.013	217	2.9%	3.3%	217	217	39,764	8,618,319	8,509,496	108,823	107,086	1.05	112,441
2006-1	6,845	215	1.027	221	3.2%	3.3%	221	221	38,862	8,577,167	8,355,330	221,837	218,499	1.05	229,424
2006-2	6,902	206	1.050	216	3.1%	3.3%	217	216	38,758	8,387,099	7,984,148	402,951	396,662	1.05	416,495
2007-1	7,095	201	1.086	218	3.1%	3.3%	220	218	39,146	8,546,174	7,868,346	677,828	666,119	1.05	699,425
2007-2	7,490	240	1.124	270	3.6%	3.3%	267	270	38,980	10,519,708	9,355,200	1,164,508	1,140,386	1.05	1,197,405
2008-1	7,498	214	1.165	249	3.3%	3.3%	249	249	39,647	9,881,804	8,484,458	1,397,346	1,362,634	1.05	1,430,765
2008-2	7,271	174	1.207	210	2.9%	3.3%	215	210	39,812	8,364,555	6,927,288	1,437,267	1,395,264	1.05	1,465,027
2009-1	7,500	195	1.254	245	3.3%	3.3%	245	245	39,564	9,674,521	7,714,980	1,959,541	1,893,593	1.05	1,988,272
2009-2	7,147	195	1.336	261	3.6%	3.3%	254	261	39,957	10,409,576	7,791,615	2,617,961	2,524,411	1.05	2,650,631
2010-1	7,405	188	1.424	268	3.6%	3.3%	261	268	39,218	10,501,088	7,372,984	3,128,104	3,006,236	1.05	3,156,548
2010-2	6,853	130	1.628	212	3.1%	3.3%	217	212	40,004	8,467,601	5,200,520	3,267,081	3,141,251	1.05	3,298,313
2011-1	7,359	143	1.920	275	3.7%	3.3%	259	275	38,868	10,670,000	5,558,124	5,111,876	4,909,406	1.05	5,154,876
2011-2	7,497	100	2.666	267	3.6%	3.3%	255	267	40,026	10,670,538	4,002,600	6,667,938	6,409,401	1.05	6,729,872
2012-1	6,869	64	3.981	255	3.7%	3.3%	234	255	39,138	9,972,839	2,504,832	7,468,007	7,161,251	1.05	7,519,314
2012-2	7,372	45	4.824	217	2.9%	3.3%	238	217	39,173	8,503,687	1,762,785	6,740,902	6,417,332	1.05	6,738,199
2013-1	7,244	40	6.237	249	3.4%	3.3%	241	241	39,931	9,616,498	1,597,240	8,019,258	7,582,637	1.05	7,961,769
2013-2	7,466	32	7.639	244	3.3%	3.3%	246	246	38,786	9,550,314	1,241,152	8,309,162	7,795,886	1.05	8,185,680
2014-1	7,274	24	9.850	236	3.2%	3.3%	240	240	40,049	9,602,935	961,176	8,641,759	8,046,733	1.05	8,449,069
2014-2	6,973	16	13.874	222	3.2%	3.3%	230	230	39,879	9,157,420	638,064	8,519,356	7,874,361	1.05	8,268,079
2015-1	7,027	8	23.601	189	2.7%	3.3%	230	230	38,906	8,955,273	311,248	8,644,025	7,932,295	1.05	8,328,910
2015-2	6,933	2	100.801	202	2.9%	3.3%	229	229	39,652	9,065,762	79,304	8,986,458	8,190,743	1.05	8,600,280
Total	172,825	3,333		5,632	3.3%		5,675	5,704		224,857,215	131,236,108	93,621,107	88,299,958		92,714,955
2004-1 - 2012-2	129,908	3,211		4,289	3.3%										

Notes

(A) (1)x(6)x[1.0-1.0/(3)]+(2)

Table 1: Policies in Force (PIF) Data

Book Half Year	Evaluation										
	1	2	3	4	5	6	7	8	9	10	11
2004-1	7,474	6,951	6,256	5,568	4,956	4,213	3,539	2,831	2,406	2,045	1,575
2004-2	6,965	6,269	5,956	5,360	4,878	4,293	3,692	3,212	2,730	2,321	1,810
2005-1	6,916	6,432	5,724	5,094	4,636	3,941	3,507	3,016	2,413	1,955	1,564
2005-2	7,420	6,678	6,077	5,530	5,088	4,477	3,805	3,158	2,716	2,173	1,717
2006-1	6,839	6,497	5,847	5,262	4,683	4,262	3,708	3,078	2,524	2,070	1,697
2006-2	6,892	6,065	5,459	4,804	4,468	3,842	3,419	2,838	2,242	1,794	1,417
2007-1	7,089	6,238	5,552	5,052	4,547	3,910	3,363	2,690	2,233	1,742	1,446
2007-2	7,485	6,961	6,335	5,575	5,073	4,515	3,838	3,186	2,676	2,221	1,843
2008-1	7,492	6,818	6,477	6,088	5,297	4,555	3,963	3,369	2,864	2,263	1,788
2008-2	7,266	6,467	5,691	5,008	4,357	3,834	3,336	2,669	2,189	1,751	1,383
2009-1	7,490	7,041	6,689	5,953	5,417	4,659	3,820	3,094	2,599	2,183	1,768
2009-2	7,141	6,427	5,977	5,618	4,944	4,499	3,869	3,134	2,539	2,057	1,584
2010-1	7,399	6,585	5,861	5,392	4,637	4,034	3,469	2,845	2,333	1,936	1,588
2010-2	6,847	6,368	6,050	5,385	4,631	3,890	3,190	2,743	2,359	1,982	1,645
2011-1	7,350	6,542	5,888	5,417	5,038	4,282	3,597	3,021	2,507	2,006	
2011-2	7,488	6,814	6,405	6,021	5,359	4,823	4,244	3,692	3,175		
2012-1	6,864	6,384	5,873	5,521	4,969	4,224	3,675	3,050			
2012-2	7,365	6,629	6,298	5,857	5,388	4,849	4,267				
2013-1	7,239	6,732	6,261	5,510	5,069	4,309					
2013-2	7,457	6,860	6,174	5,433	4,672						
2014-1	7,269	6,469	5,887	5,475							
2014-2	6,968	6,132	5,825								
2015-1	7,021	6,600									
2015-2	6,923										

Table 1 (cont'd): Policies in Force (PIF) Data

12	13	14	15	16	17	18	19	20	21	22	23	24
1,197	886	611	403	258	157	88	53	28	14	7	3	1
1,412	1,031	722	484	329	197	118	65	34	16	8	4	
1,189	856	608	413	285	182	111	59	32	15	7		
1,322	1,018	743	505	313	194	109	61	35	19			
1,290	942	659	468	290	174	110	58	31				
1,091	851	604	417	271	173	97	51					
1,070	781	547	356	235	143	82						
1,456	1,092	819	549	346	225							
1,413	1,102	815	554	377								
1,106	852	622	429									
1,326	1,008	685										
1,236	902											
1,255												

Table 2: Incremental PIF Decay
          Evaulation

	1	2	3	4	5	6	7	8	9	10	11
2004-1	100.0%	93.0%	90.0%	89.0%	89.0%	85.0%	84.0%	80.0%	85.0%	85.0%	77.0%
2004-2	100.0%	90.0%	95.0%	90.0%	91.0%	88.0%	86.0%	87.0%	85.0%	85.0%	78.0%
2005-1	100.0%	93.0%	89.0%	89.0%	91.0%	85.0%	89.0%	86.0%	80.0%	81.0%	80.0%
2005-2	100.0%	90.0%	91.0%	91.0%	92.0%	88.0%	85.0%	83.0%	86.0%	80.0%	79.0%
2006-1	100.0%	95.0%	90.0%	90.0%	89.0%	91.0%	87.0%	83.0%	82.0%	82.0%	82.0%
2006-2	100.0%	88.0%	90.0%	88.0%	93.0%	86.0%	89.0%	83.0%	79.0%	80.0%	79.0%
2007-1	100.0%	88.0%	89.0%	91.0%	90.0%	86.0%	86.0%	80.0%	83.0%	78.0%	83.0%
2007-2	100.0%	93.0%	91.0%	88.0%	91.0%	89.0%	85.0%	83.0%	84.0%	83.0%	83.0%
2008-1	100.0%	91.0%	95.0%	94.0%	87.0%	86.0%	87.0%	85.0%	85.0%	79.0%	79.0%
2008-2	100.0%	89.0%	88.0%	88.0%	87.0%	88.0%	87.0%	80.0%	82.0%	80.0%	79.0%
2009-1	100.0%	94.0%	95.0%	89.0%	91.0%	86.0%	82.0%	81.0%	84.0%	84.0%	81.0%
2009-2	100.0%	90.0%	93.0%	94.0%	88.0%	91.0%	86.0%	81.0%	81.0%	81.0%	77.0%
2010-1	100.0%	89.0%	89.0%	92.0%	86.0%	87.0%	86.0%	82.0%	82.0%	83.0%	82.0%
2010-2	100.0%	93.0%	95.0%	89.0%	86.0%	84.0%	82.0%	86.0%	86.0%	84.0%	83.0%
2011-1	100.0%	89.0%	90.0%	92.0%	93.0%	85.0%	84.0%	84.0%	83.0%	80.0%	
2011-2	100.0%	91.0%	94.0%	94.0%	89.0%	90.0%	88.0%	87.0%	86.0%		
2012-1	100.0%	93.0%	92.0%	94.0%	90.0%	85.0%	87.0%	83.0%			
2012-2	100.0%	90.0%	95.0%	93.0%	92.0%	90.0%	88.0%				
2013-1	100.0%	93.0%	93.0%	88.0%	92.0%	85.0%					
2013-2	100.0%	92.0%	90.0%	88.0%	86.0%						
2014-1	100.0%	89.0%	91.0%	93.0%							
2014-2	100.0%	88.0%	95.0%								
2015-1	100.0%	94.0%									
2015-2	100.0%										
Selection	**100.0%**	**90.7%**	**92.3%**	**90.5%**	**90.0%**	**87.5%**	**86.7%**	**85.0%**	**84.2%**	**82.0%**	**80.8%**

Table 2 (cont'd): Incremental PIF Decay

12	13	14	15	16	17	18	19	20	21	22	23	24
76.0%	74.0%	69.0%	66.0%	64.0%	60.9%	56.1%	60.2%	52.8%	50.0%	50.0%	42.9%	33.3%
78.0%	73.0%	70.0%	67.0%	68.0%	59.9%	59.9%	55.1%	52.3%	47.1%	50.0%	50.0%	
76.0%	72.0%	71.0%	67.9%	69.0%	63.9%	61.0%	53.2%	54.2%	46.9%	46.7%		
77.0%	77.0%	73.0%	68.0%	62.0%	62.0%	56.2%	56.0%	57.4%	54.3%			
76.0%	73.0%	70.0%	71.0%	62.0%	60.0%	63.2%	52.7%	53.4%				
77.0%	78.0%	71.0%	69.0%	65.0%	63.8%	56.1%	52.6%					
74.0%	73.0%	70.0%	65.1%	66.0%	60.9%	57.3%						
79.0%	75.0%	75.0%	67.0%	63.0%	65.0%							
79.0%	78.0%	74.0%	68.0%	68.1%								
80.0%	77.0%	73.0%	69.0%									
75.0%	76.0%	68.0%										
78.0%	73.0%											
79.0%												
78.0%	**76.0%**	**72.5%**	**67.3%**	**65.5%**	**62.4%**	**58.2%**	**53.6%**	**54.3%**	**49.6%**	**48.9%**	**46.4%**	**33.3%**

Table 3: Actual and Projected Loan Persistency Factors

Evaulation	1	2	3	4	5	6	7	8	9	10	11
2004-1	100.0%	93.0%	90.0%	89.0%	89.0%	85.0%	84.0%	80.0%	85.0%	85.0%	77.0%
2004-2	100.0%	90.0%	95.0%	90.0%	91.0%	88.0%	86.0%	87.0%	85.0%	85.0%	78.0%
2005-1	100.0%	93.0%	89.0%	89.0%	91.0%	85.0%	89.0%	86.0%	80.0%	81.0%	80.0%
2005-2	100.0%	90.0%	91.0%	91.0%	92.0%	88.0%	85.0%	83.0%	86.0%	80.0%	79.0%
2006-1	100.0%	95.0%	90.0%	90.0%	89.0%	91.0%	87.0%	83.0%	82.0%	82.0%	82.0%
2006-2	100.0%	88.0%	90.0%	88.0%	93.0%	86.0%	89.0%	83.0%	79.0%	80.0%	79.0%
2007-1	100.0%	88.0%	89.0%	91.0%	90.0%	86.0%	86.0%	80.0%	83.0%	78.0%	83.0%
2007-2	100.0%	93.0%	91.0%	88.0%	91.0%	89.0%	85.0%	83.0%	84.0%	83.0%	83.0%
2008-1	100.0%	91.0%	95.0%	94.0%	87.0%	86.0%	87.0%	85.0%	85.0%	79.0%	79.0%
2008-2	100.0%	89.0%	88.0%	88.0%	87.0%	88.0%	87.0%	80.0%	82.0%	80.0%	79.0%
2009-1	100.0%	94.0%	95.0%	89.0%	91.0%	86.0%	82.0%	81.0%	84.0%	84.0%	81.0%
2009-2	100.0%	90.0%	93.0%	94.0%	88.0%	91.0%	86.0%	81.0%	81.0%	81.0%	77.0%
2010-1	100.0%	89.0%	89.0%	92.0%	86.0%	87.0%	86.0%	82.0%	82.0%	83.0%	82.0%
2010-2	100.0%	93.0%	95.0%	89.0%	86.0%	84.0%	82.0%	86.0%	86.0%	84.0%	83.0%
2011-1	100.0%	89.0%	90.0%	92.0%	93.0%	85.0%	84.0%	84.0%	83.0%	80.0%	80.8%
2011-2	100.0%	91.0%	94.0%	94.0%	89.0%	90.0%	88.0%	87.0%	86.0%	82.0%	80.8%
2012-1	100.0%	93.0%	92.0%	94.0%	90.0%	85.0%	87.0%	83.0%	84.2%	82.0%	80.8%
2012-2	100.0%	90.0%	95.0%	93.0%	92.0%	90.0%	88.0%	85.0%	84.2%	82.0%	80.8%
2013-1	100.0%	93.0%	93.0%	88.0%	92.0%	85.0%	86.7%	85.0%	84.2%	82.0%	80.8%
2013-2	100.0%	92.0%	90.0%	88.0%	86.0%	87.5%	86.7%	85.0%	84.2%	82.0%	80.8%
2014-1	100.0%	89.0%	91.0%	93.0%	90.0%	87.5%	86.7%	85.0%	84.2%	82.0%	80.8%
2014-2	100.0%	88.0%	95.0%	90.5%	90.0%	87.5%	86.7%	85.0%	84.2%	82.0%	80.8%
2015-1	100.0%	94.0%	92.3%	90.5%	90.0%	87.5%	86.7%	85.0%	84.2%	82.0%	80.8%
2015-2	100.0%	90.7%	92.3%	90.5%	90.0%	87.5%	86.7%	85.0%	84.2%	82.0%	80.8%

Table 3 (cont'd): Actual and Projected Loan Persistency Factors

12	13	14	15	16	17	18	19	20	21	22	23	24
76.0%	74.0%	69.0%	66.0%	64.0%	60.9%	56.1%	60.2%	52.8%	50.0%	50.0%	42.9%	33.3%
78.0%	73.0%	70.0%	67.0%	68.0%	59.9%	59.9%	55.1%	52.3%	47.1%	50.0%	50.0%	33.3%
76.0%	72.0%	71.0%	67.9%	69.0%	63.9%	61.0%	53.2%	54.2%	46.9%	46.7%	46.4%	33.3%
77.0%	77.0%	73.0%	68.0%	62.0%	62.0%	56.2%	56.0%	57.4%	54.3%	48.9%	46.4%	33.3%
76.0%	73.0%	70.0%	71.0%	62.0%	60.0%	63.2%	52.7%	53.4%	49.6%	48.9%	46.4%	33.3%
77.0%	78.0%	71.0%	69.0%	65.0%	63.8%	56.1%	52.6%	54.3%	49.6%	48.9%	46.4%	33.3%
74.0%	73.0%	70.0%	65.1%	66.0%	60.9%	57.3%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
79.0%	75.0%	75.0%	67.0%	63.0%	65.0%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
79.0%	78.0%	74.0%	68.0%	68.1%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
80.0%	77.0%	73.0%	69.0%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
75.0%	76.0%	68.0%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	73.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
79.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%
78.0%	76.0%	72.5%	67.3%	65.5%	62.4%	58.2%	53.6%	54.3%	49.6%	48.9%	46.4%	33.3%

Table 4: Actual and Projected PIF
Evaulation

Book Half Year	1	2	3	4	5	6	7	8	9	10	11
2004-1	7,474	6,951	6,256	5,568	4,956	4,213	3,539	2,831	2,406	2,045	1,575
2004-2	6,965	6,269	5,956	5,360	4,878	4,293	3,692	3,212	2,730	2,321	1,810
2005-1	6,916	6,432	5,724	5,094	4,636	3,941	3,507	3,016	2,413	1,955	1,564
2005-2	7,420	6,678	6,077	5,530	5,088	4,477	3,805	3,158	2,716	2,173	1,717
2006-1	6,839	6,497	5,847	5,262	4,683	4,262	3,708	3,078	2,524	2,070	1,697
2006-2	6,892	6,065	5,459	4,804	4,468	3,842	3,419	2,838	2,242	1,794	1,417
2007-1	7,089	6,238	5,552	5,052	4,547	3,910	3,363	2,690	2,233	1,742	1,446
2007-2	7,485	6,961	6,335	5,575	5,073	4,515	3,838	3,186	2,676	2,221	1,843
2008-1	7,492	6,818	6,477	6,088	5,297	4,555	3,963	3,369	2,864	2,263	1,788
2008-2	7,266	6,467	5,691	5,008	4,357	3,834	3,336	2,669	2,189	1,751	1,383
2009-1	7,490	7,041	6,689	5,953	5,417	4,659	3,820	3,094	2,599	2,183	1,768
2009-2	7,141	6,427	5,977	5,618	4,944	4,499	3,869	3,134	2,539	2,057	1,584
2010-1	7,399	6,585	5,861	5,392	4,637	4,034	3,469	2,845	2,333	1,936	1,588
2010-2	6,847	6,368	6,050	5,385	4,631	3,890	3,190	2,743	2,359	1,982	1,645
2011-1	7,350	6,542	5,888	5,417	5,038	4,282	3,597	3,021	2,507	2,006	1,620
2011-2	7,488	6,814	6,405	6,021	5,359	4,823	4,244	3,692	3,175	2,604	2,103
2012-1	6,864	6,384	5,873	5,521	4,969	4,224	3,675	3,050	2,570	2,107	1,702
2012-2	7,365	6,629	6,298	5,857	5,388	4,849	4,267	3,627	3,055	2,506	2,023
2013-1	7,239	6,732	6,261	5,510	5,069	4,309	3,738	3,177	2,676	2,195	1,773
2013-2	7,457	6,860	6,174	5,433	4,672	4,088	3,546	3,014	2,539	2,082	1,682
2014-1	7,269	6,469	5,887	5,475	4,927	4,311	3,740	3,179	2,678	2,196	1,774
2014-2	6,968	6,132	5,825	5,272	4,744	4,151	3,601	3,061	2,579	2,115	1,708
2015-1	7,021	6,600	6,089	5,510	4,959	4,339	3,764	3,199	2,695	2,210	1,785
2015-2	6,923	6,283	5,796	5,245	4,720	4,130	3,583	3,045	2,566	2,104	1,699

Table 4 (cont'd): Actual and Projected PIF

12	13	14	15	16	17	18	19	20	21	22	23	24
1,197	886	611	403	258	157	88	53	28	14	7	3	1
1,412	1,031	722	484	329	197	118	65	34	16	8	4	1
1,189	856	608	413	285	182	111	59	32	15	7	3	1
1,322	1,018	743	505	313	194	109	61	35	19	9	4	1
1,290	942	659	468	290	174	110	58	31	15	8	3	1
1,091	851	604	417	271	173	97	51	28	14	7	3	1
1,070	781	547	356	235	143	82	44	24	12	6	3	1
1,456	1,092	819	549	346	225	131	70	38	19	9	4	1
1,413	1,102	815	554	377	235	137	73	40	20	10	4	1
1,106	852	622	429	281	175	102	55	30	15	7	3	1
1,326	1,008	685	461	302	188	110	59	32	16	8	4	1
1,236	902	654	440	288	180	105	56	30	15	7	3	1
1,255	954	691	465	305	190	111	59	32	16	8	4	1
1,283	975	707	476	312	194	113	61	33	16	8	4	1
1,264	960	696	468	307	192	111	60	32	16	8	4	1
1,640	1,247	904	608	398	249	145	78	42	21	10	5	2
1,327	1,009	731	492	322	201	117	63	34	17	8	4	1
1,578	1,200	869	585	383	239	139	75	41	20	10	5	2
1,383	1,051	762	512	336	210	122	65	36	18	9	4	1
1,312	997	723	486	318	199	116	62	34	17	8	4	1
1,383	1,052	762	513	336	210	122	65	36	18	9	4	1
1,332	1,012	734	494	323	202	118	63	34	17	8	4	1
1,392	1,058	767	516	338	211	123	66	36	18	9	4	1
1,325	1,007	730	491	322	201	117	63	34	17	8	4	1

Table 5: Average Projected PIF
Evauation

Book Half Year	1	2	3	4	5	6	7	8	9	10	11
2004-1	1										
2004-2	3	1									
2005-1	5	2	1								
2005-2	14	7	3	1							
2006-1	23	11	5	2	1						
2006-2	39	21	10	5	2	1					
2007-1	63	34	18	9	4	2	1				
2007-2	178	101	54	29	14	7	3	1			
2008-1	306	186	105	57	30	15	7	3	1		
2008-2	355	228	139	78	42	22	11	5	2	1	
2009-1	573	381	245	149	84	45	24	12	6	2	1
2009-2	778	547	364	234	142	80	43	23	11	5	2
2010-1	1,104	823	578	385	247	150	85	46	24	12	6
2010-2	1,464	1,129	841	591	394	253	154	87	47	25	12
2011-1	1,813	1,442	1,112	828	582	388	249	152	86	46	24
2011-2	2,889	2,353	1,871	1,443	1,075	756	503	323	197	111	60
2012-1	2,810	2,338	1,904	1,515	1,168	870	612	407	262	159	90
2012-2	3,947	3,341	2,780	2,264	1,801	1,389	1,035	727	484	311	189
2013-1	4,024	3,458	2,927	2,436	1,984	1,578	1,217	906	637	424	273
2013-2	4,380	3,817	3,280	2,777	2,311	1,882	1,497	1,154	860	604	402
2014-1	5,201	4,619	4,026	3,459	2,928	2,437	1,985	1,579	1,218	907	637
2014-2	5,548	5,008	4,448	3,876	3,331	2,820	2,347	1,911	1,520	1,172	873
2015-1	6,344	5,799	5,235	4,649	4,052	3,482	2,947	2,453	1,998	1,589	1,225
2015-2	6,603	6,039	5,520	4,983	4,425	3,857	3,314	2,805	2,335	1,902	1,512
	0.25	0.75	1.25	1.75	2.25	2.75	3.25	3.75	4.25	4.75	5.25
Discount Factor	0.995	0.985	0.976	0.966	0.956	0.947	0.938	0.928	0.919	0.910	0.901

Table 5 (cont'd): Average Projected PIF

12	13	14	15	16	17	18	19	20	21	22	23	24
1												
2	1											
6	2	1										
12	6	2	1									
32	16	7	3	2								
48	26	13	6	3	1							
107	58	30	15	7	3	2						
166	94	50	27	13	6	3	1					
259	157	89	48	25	12	6	3	1				
424	273	166	94	50	27	13	6	3	1			
614	409	263	160	90	49	26	13	6	3	1		
913	641	427	275	167	94	51	27	13	6	3	1	
1,166	869	611	406	261	159	90	48	25	13	6	3	1
5.75	6.25	6.75	7.25	7.75	8.25	8.75	9.25	9.75	10.25	10.75	11.25	11.75
0.892	0.884	0.875	0.866	0.858	0.849	0.841	0.833	0.824	0.816	0.808	0.800	0.792

Table 6: Premium Estimate

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
					Discounted	Policy	Discounted
Average Monthly	Premium	Projected Future	Premium	Total	Projected Future	Maintenance	Projected Prem.
Policy Premium	Scaling Factor	Premium	A.o. 12/31/15	Premium	Premium	Expense Ratio	Net of Expenses
				(3)+(4)			(6)x[1.0-(7)]
75	6	450	23,446,397	23,446,847	448	5%	425
75	6	1,807	23,538,827	23,540,634	1,792	5%	1,703
76	6	3,795	23,607,461	23,611,256	3,757	5%	3,569
76	6	11,492	23,694,210	23,705,701	11,360	5%	10,792
76	6	19,897	23,756,921	23,776,818	19,648	5%	18,665
76	6	35,928	23,859,775	23,895,703	35,440	5%	33,668
77	6	60,088	23,931,198	23,991,285	59,220	5%	56,259
77	6	178,270	23,860,998	24,039,268	175,543	5%	166,766
77	6	328,722	23,782,663	24,111,386	323,347	5%	307,180
78	6	411,422	23,820,521	24,231,943	404,192	5%	383,982
78	6	709,735	23,570,671	24,280,407	696,350	5%	661,533
78	6	1,045,062	23,356,747	24,401,809	1,023,843	5%	972,651
78	6	1,627,253	22,847,761	24,475,014	1,591,486	5%	1,511,912
79	6	2,363,358	22,234,031	24,597,389	2,307,077	5%	2,191,723
79	6	3,195,937	21,499,842	24,695,779	3,113,575	5%	2,957,896
79	6	5,539,850	19,254,711	24,794,562	5,385,751	5%	5,116,464
80	6	5,849,567	19,068,968	24,918,535	5,674,436	5%	5,390,715
80	6	8,869,340	16,123,950	24,993,290	8,584,457	5%	8,155,234
80	6	9,719,246	15,324,030	25,043,277	9,385,388	5%	8,916,119
80	6	11,348,862	13,744,501	25,093,363	10,933,164	5%	10,386,506
81	6	14,531,726	10,662,011	25,193,737	13,965,083	5%	13,266,829
81	6	16,708,141	8,535,983	25,244,124	16,015,431	5%	15,214,659
81	6	20,599,330	4,720,526	25,319,857	19,692,463	5%	18,707,840
81	6	22,931,142	2,515,314	25,446,456	21,862,046	5%	20,768,944
		126,090,420	**460,758,018**	**586,848,438**	**121,265,297**		**115,202,033**