

**THE USE OF SIMULATION TECHNIQUES
IN ADDRESSING AUTO WARRANTY PRICING
AND RESERVING ISSUES**

Simon J. Noonan

Title : The Use of Simulation Techniques in Addressing Auto Warranty Pricing and Reserving Issues

Abstract : Extended warranty contracts are generally quite difficult to evaluate because the factors affecting ultimate loss emergence tend to change quite considerably over time. The actuary is forced to extrapolate from historical data to take these changes into account whatever the methodology employed, and simulation techniques provide a powerful tool to model the changes in loss exposure in a way that is easy for the actuary and layman alike to grasp.

A. Policy Coverage

The coverage generally provides mechanical breakdown protection for new and used vehicles sold by automobile dealerships. Often, the dealership is legally the policyholder of the insurance company rather than the owner of the automobile, who instead purchases a service contract from the automobile dealer. The insurance policy reimburses the dealer for expenses incurred in fulfilling his obligations under the service contract. Despite the legal form of this arrangement, the insurance company is generally obligated to fulfill the terms of the service contract with the consumer should the automobile dealer fail to meet their contractual obligations, even if this is not specified in the service contract.

Coverage is nowadays generally limited to specified mechanical failures to eliminate coverage for parts which naturally wear out (e.g. shock absorbers) and to restrict in some fashion the automobile dealer's ability to make unnecessary and expensive repairs.

For new automobiles, the policy is essentially an umbrella coverage over the manufacturer's warranty, broadening the policy form with additional coverage such as the provision of a free rental car while repairs are being made, lengthening the time for which the coverage is valid and increasing the maximum mileage that may be driven before the auto owner must pay for repairs out of his own pocket.

B. Factors Affecting Consistency of Loss Emergence

More than with most lines of insurance, the factors affecting loss emergence tend to change considerably over time. The two most important changes are generally:

- The manufacturer's warranties have changed dramatically over time as auto manufacturers have sought to compete more or less heavily on the basis of quality. For example, one major manufacturer has offered the following coverage in recent years:

model year	basic coverage	powertrain only coverage
1986	12/12	36/36
1987	12/12	72/60 **
1988	12/12	72/60
1989	36/50	36/50
1990	36/50	36/50
1991	36/50	36/50

** meaning coverage is provided for 72 months or 60,000 miles, whichever expires sooner.

Clearly, changes of this magnitude have a considerable effect on loss emergence.

- ✦ In response to the above and other changes in the marketplace, insurers writing this line have adapted the coverage they offer to the changes in the underlying warranty. This has generally meant increasing both the duration of the policy and the mileage cap on the policy.

In many cases, companies that were offering 60 month/50,000 mile policies over 12 month/12,000 mile factory warranties find themselves offering 6 or 7 year contracts with 100,000 mile caps. The fact that the bulk of the exposure for this line occurs late in the policy term exacerbates the problem by requiring that the actuary develop loss projections from loss data that stems from policy forms that are several years old.

C. Methodology Employed

The loss data are aggregated by model year and losses are then divided according to which mileage band they fall into. Based on the number of contracts originally written, a pure premium is developed. Calculations based on hypothetical data are contained in the various Exhibits. As an example of the basic structure of the loss data, loss payments that have been made as of 4/30/91 (the evaluation date) for model year 1987 with mileage on the odometer of the vehicle of between 10,000 and 20,000 miles at the time of claim would total \$2,658,300 and the corresponding pure premium would be \$21.79 based on 122,000 contracts written for that model year (Exhibits 3, Parts A and B).

Each of the elements of this data matrix will tend to increase over time, until either all policies in the cohort have expired, or all automobiles have been driven a distance in excess of the upper mileage band. Basic questions of pricing or loss reserving therefore boil down into how to estimate the ultimate pure premium in each cell.

Assuming the mileage on the odometer of the vehicle is captured in the claims database of the insurance company at the time of each loss (without this, the loss data cannot, of course, be produced in the requisite form), it is possible to develop estimates of the distribution of the distance driven by a typical policyholder each year, and the correlation between successive years. Armed with this information, we can estimate the following quantities using simulation techniques:

- (A) the mean distance driven in each mileage band at the evaluation date while under the manufacturer's "basic" warranty.
- (B) the mean distance driven in each mileage band at the evaluation date while under the manufacturer's "powertrain-only" warranty.
- (C) the mean distance driven in each mileage band at the evaluation date while under the insurance company's warranty.
- (D) the mean distance driven in each mileage band at the expiration of all policies while under the manufacturer's "basic" warranty.

(E) the mean distance driven in each mileage band at the expiration of all policies while under the manufacturer's "powertrain-only" warranty.

(F) the mean distance driven in the mileage band at the expiration of all policies while under the insurance company's warranty.

Exhibit 2, Parts A to F shows the estimates of these quantities where the distance driven has a lognormal distribution with mean 10,000 miles and standard deviation 5,000 miles. Coverage was assumed to be the lesser of 5 years or 50,000 miles under the extended warranty contract, the lesser of 1 year or 10,000 miles under the manufacturer's basic coverage and the lesser of 2 years or 20,000 miles under the powertrain-only coverage provided by the manufacturer. The numbers contained therein were developed by performing 500 simulations for each data cell using add-in software in conjunction with a standard computer spreadsheet, a printout of which is shown in Exhibit 1. Information on the derivation of an appropriate distribution is contained in Appendix A.

As an example of the approach outlined above, one iteration of the simulation for the distance driven at policy expiration might generate the following data:

Year Driven	Mileage
1	6,000
2	12,000
3	8,000
4	20,000
5	3,000

Then the entries in the entries in Exhibit 2, Parts D, E and F would be:

Coverage	Distance Driven at Policy Expiration in Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
Basic	6,000				
Part D					
Powertrain	10,000	8,000			
Part E					
Insurance	10,000	10,000	10,000	10,000	9,000
Part F					

For example, the coverage for the powertrain-only warranty is the lesser of 2 years and 20,000 miles. For this example, the driver covered 18,000 miles at policy expiration, which implies that the full 10,000 miles were driven in the first mileage band, but only 8,000 miles in the second band from 10,000 to 20,000 miles.

We need to examine the question of what percentage of losses that are covered under the insurance company's policy form would also be covered by either the manufacturer's "basic" coverage or by the "powertrain-only" coverage assuming that all three coverages are in force at the time of a claim. Ranked in decreasing order of coverage, the three coverages would be the insurers coverage, the manufacturers basic coverage and the powertrain only coverage. Bearing this in mind, the results of such an analysis might hypothetically be as follows:

= > \$10 covered by
the insurer
alone

\$100 of losses
covered by
the insurer

= > \$30 covered by
both insurer
and basic coverage

= > \$60 covered by
insurer, basic
coverage and
powertrain

Stated another way, while both the basic warranty and the power-train coverage are in force, the insurer is responsible for 10% of the losses. Once the basic warranty expires, the insurer is responsible for 40% of the losses; the original 10% plus the 30% that were previously covered under the basic warranty. Finally, the insurer picks up 100% of the losses once both the manufacturer's warranties expire.

The hypothetical data above might be empirically derived from a study of the cause of actual losses.

Were the loss data available, we could, of course, analyze the losses separately according to which policy form they would be covered under, dividing our single claims matrix into 3 separate matrices. In the absence of such a division, we may estimate the effective exposure to loss at the evaluation date (g.) as:

$$\begin{aligned} (G) &= (C) - 90\% * (A) - 60\% * [(B) - (A)] \\ &= 10\% * (A) + 40\% * [(B) - (A)] + 100\% * [(C) - (B)] \end{aligned}$$

The effective exposure to loss at the expiration of all policies can similarly be calculated as:

$$(H) = (F) - 90\% * (D) - 60\% * [(E) - (D)]$$

These exposures are tabulated in Exhibit 2, Parts G and H respectively, with the ratio:

$$(I) = (H) / (G)$$

tabulated in Exhibit 2, Part I and a smoothed version of these factors-to-ultimate shown in Exhibit 2, Part J, where the factors of close to unity, caused by random errors in the simulation process, are rounded to 1.

The derivation of estimates of ultimate pure premiums is shown in Exhibit 3. The basic loss data is shown in Part A and paid pure premiums are calculated in Part B. Ultimate pure premiums for a policyholder who drives the maximum 10,000 mile distance in the cell with no underlying manufacturer's coverage are shown in Exhibit 3, Part C. It might be thought that this step could be bypassed; if the typical policyholder has driven 2,000 miles at the evaluation date and will ultimately drive 4,000 miles then an estimate for the ultimate pure premium is two times the paid pure premium. However, the ultimate pure premium for the 10,000 mile driver is a useful quantity to know when extrapolating experience from say a 5/50 policy to a 5/100 coverage in that the underlying exposure to loss from a driver who drives more than 100,000 miles can be estimated, before the typical distance driven in the higher mileage bands is considered.

Exhibit 3, Parts D, E and F, show a regression, performed to estimate ultimate pure premiums in each mileage band by averaging the data from all model years and then trending the average for the individual model years. The normalized pure premiums shown in Part F reflect the average pure premium, assumed to relate to model year 1987.5 trended forwards or backwards for the appropriate period of time using the trend rate derived in the regression. This step is desirable in computing estimates of ultimate loss using the Bornhuetter-Ferguson¹ method, as well as in determining a trend factor from historical data for ratemaking purposes. Obviously, the methodology can be refined to separate the overall claims trend into frequency and severity components, which is generally useful, since severity tends to be amenable to estimation, even for new components, leaving only frequency as the major unknown.

In Exhibit 3, Part G, we determine the ultimate pure premium for a typical driver, rather than for one who drives the full 10,000 miles in each exposure cell. With the hypothetical numbers shown, the pure premium for the typical driver declines at the higher mileage intervals even though the pure premium for the 10,000 mile driver rises with increasing mileage -- stemming from the reduction in distance driven by the typical driver in the higher mileage bands.

Exhibit 3, Parts H and I, show the derivation of estimates of ultimate pure premiums for the typical policyholder using two different approaches.

In Part H, the paid pure premiums as of the evaluation date are increased in the same proportion as the "ultimate effective exposure" bears to the "effective exposure at the evaluation date". In an analogous fashion to most forms of loss projection using triangular methods, the approach works best for those data cells where the factor to ultimate is not excessively large.

In Part I, ultimate losses are estimated using the Bornhuetter-Ferguson methodology adapted for current purposes :

$$\text{ultimate pure premium} = \text{paid pure premium} + \frac{(\text{ult. effective exposure} - \text{effective exposure at evaluation date}) * \text{normalized ult. pure premium for a 10,000 mile driver}}{10,000}$$

This approach works well for the more recent model years, where little in the way of ultimate loss emergence has taken place at the evaluation date, and where expected ultimate losses are taken from trended pure premium information from the older years.

D. Advantages of the Approach Used

There are several methods that make this approach quite useful in auto warranty work:

1. The financial effects of changes in several factors that have an impact on loss emergence can easily be modelled explicitly:

A. Changes in Manufacturer's Warranty

A change in the manufacturer's warranty in any given year can be dealt with in changing the parameters of the simulation. For example, if the basic warranty increases from 1 year/10,000 miles to 3 years/36,000 miles, then one needs merely to re-run Exhibit 2, Parts A and D.

B. Changes in the Insurance Company's Warranty

As with A., changes can be made in the simulation parameters to re-run Exhibit 2, Parts C and F. In particular, were we dealing with a company that was writing contracts with longer terms than those in the data, we could explicitly reflect this by calculating the increase in "effective exposure". In an instance where the insurer was covering high mileage bands never before covered, we could use the available data for an individual who drives the full 10,000 miles in the band to extrapolate into higher mileages.

C. *Changes in Driving Habits*

There is likely to be considerable adverse selection against the insurance company if a range of policies are offered, in that policies which offer high mileage caps tend to attract high-mileage drivers. Changes in the mix of coverage written will affect the distances driven by policyholders and these can be explicitly allowed for in the computations.

D. *Bivariate Approach of the Methodology*

Both the mileage limitation of the policy and the time limitation are taken into account. In some simpler methods, such as computing the "effective" mileage limitation of the policy, this is not the case. It is obviously not appropriate to reason: "if the average driver covers 20,000 miles per year, then there is no more exposure in a 6 year/100,000 mile policy than in a 5 year/100,000 mile policy because both have an effective mileage limit of 100,000 miles".

E. *Changes in Cancellation Rates*

The approach can readily be adapted to account specifically for cancellation rates. If one can track the percentage of policies in force at a given duration (time or mileage), then a change from say 80% to 70% can be expected to reduce loss emergence by a like amount.

F. *Timing of Auto Sales*

There are frequently considerable differences between model years in the timing of new car sales, primarily because of fluctuations in the strength of the economy. These can be explicitly allowed for in the analysis.

G. *Use of Up-To-Date Data*

Unlike most forms of actuarial study, no complicated adjustments are necessary for data recorded as of a date other than the anniversary of the model year. The most up-to-date data can be readily used.

E. Difficulties With the Method

There a number of practical problems that one is likely to face in employing the approach suggested:

A. *Discounting for Investment Income*

This is generally a relatively easy exercise in the normal course of actuarial events, but in this case becomes more difficult when the loss estimates are computed by mileage band rather than time interval. The approach we use is to use the simulation model to compute the expected value of:

$$\frac{\text{future miles driven}}{\text{in mileage band}} * \frac{\text{time to when the}}{\text{miles are driven}}$$

If we divide each of these quantities by the expected future mileage driven, we get an estimate of the average time to payment of unpaid losses and thus the discount can be computed.

B. *Settlement Lags*

While payment lags are generally modest, there are a few weeks elapsing between the time an incident gives rise to a claim and the time when that loss has been adjusted and coded into the insurance company's system. This needs to be allowed for when selecting an evaluation date for simulation purposes, and changes in administrative procedures or claims-handling practices cause similar problems to those encountered in other books of business.

References:

1. The Actuary and IBNR, R.L. Bornhuetter and R.E. Ferguson, PCAS LIX 1972

ANALYSIS OF INSURER AND MANUFACTURER COVERAGES

Exhibit 1

	Time Cap (years)	Mileage Cap (miles)
Insurance Policy Coverage	5	50,000
Manufacturer's Basic Coverage	1	10,000
Manufacturer's Powertrain-Only Coverage	2	20,000

DISTANCE DRIVEN

Mean Distance Driven :	10,000
Standard Deviation :	5,000
Distribution :	lognormal
Correlation Between Mileage :	0.5
Driven in Successive Years	

INCEPTION OF POLICY

Distribution :	uniform over one year
Inception	9/30 preceding model year

EVALUATION DATE

Date at which data are collected : 1991.33 i.e. 4/30/91

SIMULATION WORKTABLE

Band Investigated	Model Year	1989
	Low Miles	0
	High Miles	10,000

Projection of Current (C) exposure as of the evaluation date or Ultimate (U) exposure. C

Projection of Exposure Under Insurer's (I) Coverage, Manufacturer's Basic (B) Coverage or Manufacturer's Powertrain (P) Coverage. B

Random Time of Policy Inception 1989.25
Time Policy in Force at Projected Date 1.00 years

Year	Distance Driven	Portion of Year Applicable	Applicable Distance Driven
1	10,000	1.00	10,000
2	10,000	0.00	0
3	10,000	0.00	0
4	10,000	0.00	0
5	10,000	0.00	0

Total Distance Driven Capped by Coverage Limit 10,000

Distance Driven in Band Under Study 10,000

Variance -- Covariance Matrix

	1	2	3	4	5
1	1	0.5	0.5	0.5	0.5
2		1	0.5	0.5	0.5
3			1	0.5	0.5
4				1	0.5
5					1

ANALYSIS OF DISTANCE DRIVEN AT THE EVALUATION DATE

Exhibit 2

Distance Driven in Each Mileage Band
at 4/30/91 While Under the
Manufacturer's "Basic" Warranty

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	8,134	0	0	0	0
1986	8,134	0	0	0	0
1987	8,134	0	0	0	0
1988	8,132	0	0	0	0
1989	8,133	0	0	0	0
1990	7,645	0	0	0	0

Part A

Distance Driven in Each Mileage Band
at 4/30/91 While Under the
Manufacturer's "Powertrain-Only" Warranty

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	9,876	6,530	0	0	0
1986	9,867	6,701	0	0	0
1987	9,884	6,606	0	0	0
1988	9,857	6,645	0	0	0
1989	9,860	6,348	0	0	0
1990	8,171	2,259	0	0	0

Part B

Distance Driven in Each Mileage Band
at 4/30/91 While Under the
Insurance Company Warranty

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	10,000	9,998	9,469	7,590	5,246
1986	10,000	9,987	9,280	7,217	5,056
1987	10,000	9,896	8,431	5,706	3,369
1988	9,999	9,228	6,040	3,153	1,415
1989	9,843	6,918	2,896	942	281
1990	8,172	2,092	278	88	3

Part C

ANALYSIS OF DISTANCE DRIVEN AT POLICY EXPIRATION

Exhibit 2 (cont)

Distance Driven in Each Mileage Band
at Policy Expiration While Under the
Manufacturer's "Basic" Warranty

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	8,134	0	0	0	0
1986	8,134	0	0	0	0
1987	8,134	0	0	0	0
1988	8,134	0	0	0	0
1989	8,134	0	0	0	0
1990	8,134	0	0	0	0

Part D

Distance Driven in Each Mileage Band
at Policy Expiration While Under the
Manufacturer's "Powertrain-Only" Warranty

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	9,861	6,640	0	0	0
1986	9,861	6,640	0	0	0
1987	9,861	6,640	0	0	0
1988	9,861	6,640	0	0	0
1989	9,861	6,640	0	0	0
1990	9,861	6,640	0	0	0

Part E

Distance Driven in Each Mileage Band
at Policy Expiration While Under the
Insurance Company Warranty

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	10,000	9,996	9,405	7,439	5,194
1986	10,000	9,996	9,405	7,439	5,194
1987	10,000	9,996	9,405	7,439	5,194
1988	10,000	9,996	9,405	7,439	5,194
1989	10,000	9,996	9,405	7,439	5,194
1990	10,000	9,996	9,405	7,439	5,194

Part F

ANALYSIS OF EFFECTIVE EXPOSURE TO LOSS

Exhibit 2 (cont)

Effective Exposure to Loss
at 4/30/91 in Miles

(g.) = (c.) - 90% * (a.) - 60% * [(b.) - (a.)]

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	1,634	6,080	9,469	7,590	5,246
1986	1,640	5,966	9,280	7,217	5,056
1987	1,629	5,932	8,431	5,706	3,369
1988	1,645	5,241	8,040	3,153	1,415
1989	1,487	3,109	2,896	942	281
1990	976	737	278	88	3

Part G

Effective Exposure to Loss
at Policy Expiration

(h.) = (f.) - 90% * (d.) - 60% * [(e.) - (d.)]

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	1,643	6,012	9,405	7,439	5,194
1986	1,643	6,012	9,405	7,439	5,194
1987	1,643	6,012	9,405	7,439	5,194
1988	1,643	6,012	9,405	7,439	5,194
1989	1,643	6,012	9,405	7,439	5,194
1990	1,643	6,012	9,405	7,439	5,194

Part H

DEVELOPMENT FACTORS TO ULTIMATE

Exhibit 2 (cont)

Unsmoothed Factor to Ultimate
(i.) = (h.) / (g.)

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	1.006	0.989	0.993	0.980	0.990
1986	1.002	1.008	1.013	1.031	1.027
1987	1.008	1.013	1.116	1.304	1.542
1988	0.999	1.147	1.557	2.359	3.671
1989	1.105	1.934	3.248	7.897	18.484
1990	1.684	8.162	33.831	84.534	1731.333

Part I

Note : in the table above, the factors have not been smoothed
to remove random fluctuations in the simulation.

Smoothed Factor to Ultimate

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	1.000	1.000	1.000	1.000	1.000
1986	1.000	1.008	1.013	1.031	1.027
1987	1.000	1.013	1.116	1.304	1.542
1988	1.000	1.147	1.557	2.359	3.671
1989	1.105	1.934	3.248	7.897	18.484
1990	1.684	8.162	33.831	84.534	1731.333

Part J

ANALYSIS OF PURE PREMIUM FOR A 10,000 MILE DRIVER

Exhibit 3

Paid Losses at 4/30/91 in \$000's

Model Year	Mileage Band					Number of Contracts Originally Written
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	
1985	507.1	2,027.4	4,013.9	3,756.7	2,893.1	102,000
1986	592.2	2,337.5	4,578.6	4,196.9	3,184.1	112,000
1987	666.7	2,658.3	4,848.3	3,903.6	2,519.1	122,000
1988	779.4	2,769.6	4,021.1	2,427.2	1,213.5	132,000
1989	818.5	1,838.3	2,157.0	842.5	277.4	142,000
1990	603.7	489.5	228.3	91.8	3.4	152,000

Part A

Paid Pure Premiums at 4/30/91 in \$'s

Model Year	Mileage Band					Total
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	
1985	\$4.97	\$19.88	\$39.35	\$36.83	\$28.36	\$129.39
1986	\$5.29	\$20.87	\$40.88	\$37.47	\$28.43	\$132.94
1987	\$5.47	\$21.79	\$39.74	\$32.00	\$20.65	\$119.64
1988	\$5.90	\$20.98	\$30.46	\$18.39	\$9.19	\$84.93
1989	\$5.76	\$12.95	\$15.19	\$5.93	\$1.95	\$41.79
1990	\$3.97	\$3.22	\$1.50	\$0.60	\$0.02	\$9.32

Part B

Ultimate Pure Premiums for a 10,000 mile driver

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	\$30.42	\$32.69	\$41.56	\$48.53	\$54.07
1986	\$32.25	\$34.98	\$44.05	\$51.92	\$56.23
1987	\$33.54	\$36.73	\$47.14	\$56.08	\$61.29
1988	\$35.89	\$40.03	\$50.43	\$58.32	\$64.97
1989	\$38.76	\$41.64	\$52.45	\$62.98	\$69.52
1990	\$40.70	\$43.72	\$54.03	\$68.65	\$75.08
Average	\$35.26	\$38.30	\$48.28	\$57.75	\$63.52

Part C

Ultimate Pure Premium for 10,000 mile driver =
 [Paid Pure Premium at 4/30/91]
 * 10,000 / [Effective Exposure at 4/30/91]

Ultimate Pure Premiums / Average for Mileage band for the 10,000 Mile Driver

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	0.863	0.854	0.861	0.840	0.851
1986	0.915	0.913	0.912	0.899	0.885
1987	0.951	0.959	0.976	0.971	0.965
1988	1.018	1.045	1.045	1.010	1.023
1989	1.099	1.087	1.086	1.091	1.094
1990	1.154	1.142	1.119	1.189	1.182

Part D

REGRESSION TO DETERMINE PURE PREMIUM TREND FACTOR

Exhibit 3 (cont)

Year	Ultimate /Average (U/A)	year - 1987.5	ln(U/A)
1985	0.863	-2.5	-0.147
1985	0.854	-2.5	-0.158
1985	0.861	-2.5	-0.150
1985	0.840	-2.5	-0.174
1985	0.851	-2.5	-0.161
1986	0.915	-1.5	-0.089
1986	0.913	-1.5	-0.091
1986	0.912	-1.5	-0.092
1986	0.899	-1.5	-0.106
1986	0.885	-1.5	-0.122
1987	0.951	-0.5	-0.050
1987	0.959	-0.5	-0.042
1987	0.976	-0.5	-0.024
1987	0.971	-0.5	-0.029
1987	0.965	-0.5	-0.036
1988	1.018	0.5	0.018
1988	1.045	0.5	0.044
1988	1.045	0.5	0.044
1988	1.010	0.5	0.010
1988	1.023	0.5	0.022
1989	1.099	1.5	0.095
1989	1.087	1.5	0.084
1989	1.086	1.5	0.083
1989	1.091	1.5	0.087
1989	1.094	1.5	0.090
1990	1.154	2.5	0.143
1990	1.142	2.5	0.132
1990	1.119	2.5	0.113
1990	1.189	2.5	0.173
1990	1.182	2.5	0.167

Part E

Regression Model

$(U/A) = (1+t)^{(year - 1987.5)}$ where t is the annual trend factor
 $\ln(U/A) = \ln(1+t) * (year - 1987.5)$

Regression Output:

Constant	0	selected under model
Std Err of Y Est	0.01467163	
R Squared	0.98131894	
No. of Observations	30	
Degrees of Freedom	29	
X Coefficient(s)	0.06130442 = ln(1+t) => t=	6.3%
Std Err of Coef.	0.00156846	

NORMALISED ULTIMATE PURE PREMIUMS

Exhibit 3 (cont)

Normalised Ultimate Pure Premiums
for the 10,000 Mile Driver

Model Year	Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	\$30.25	\$32.86	\$41.42	\$49.54	\$54.50
1986	\$32.16	\$34.93	\$44.04	\$52.67	\$57.94
1987	\$34.20	\$37.14	\$46.82	\$56.00	\$61.61
1988	\$36.36	\$39.49	\$49.78	\$59.54	\$65.50
1989	\$38.66	\$41.99	\$52.93	\$63.31	\$69.64
1990	\$41.10	\$44.64	\$56.27	\$67.31	\$74.05
1987.5	\$35.26	\$38.30	\$48.28	\$57.75	\$63.52

Part F

Normalised Ultimate Pure Premiums
for the Typical Driver

Model Year	Mileage Band					Total
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	
1985	\$4.97	\$19.75	\$38.95	\$36.85	\$28.31	\$128.84
1986	\$5.28	\$21.00	\$41.42	\$39.18	\$30.10	\$136.98
1987	\$5.62	\$22.33	\$44.03	\$41.66	\$32.00	\$145.64
1988	\$5.97	\$23.74	\$46.82	\$44.29	\$34.02	\$154.85
1989	\$6.35	\$25.24	\$49.78	\$47.09	\$36.17	\$164.64
1990	\$6.75	\$26.84	\$52.92	\$50.07	\$38.46	\$175.05

Part G

Ultimate Pure Premium for Typical Driver =
Pure Premium for the 10,000 Mile Driver
* Effective Exposure at Policy Expiration / 10,000

PROJECTED ULTIMATE PURE PREMIUM

Exhibit 3 (cont)

Projected Ultimate Pure Premium Using "Factor to Ultimate" Methodology

Model Year	Mileage Band					Total
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	
1985	\$4.97	\$19.88	\$39.35	\$36.83	\$28.36	\$129.39
1986	\$5.29	\$21.03	\$41.43	\$38.62	\$29.21	\$135.58
1987	\$5.47	\$22.08	\$44.33	\$41.71	\$31.83	\$145.43
1988	\$5.90	\$24.07	\$47.43	\$43.38	\$33.74	\$154.53
1989	\$6.37	\$25.03	\$49.33	\$46.85	\$36.11	\$163.69
1990	\$6.69	\$26.28	\$50.81	\$51.07	\$38.99	\$173.85

Part H

Ultimate Pure Premium for Typical Driver =
Paid Pure Premium at 4/30/91

* Smoothed Factor to Ultimate from Exhibit 2j

Projected Ultimate Pure Premium Using
Bornhuetter-Ferguson Methodology

Model Year	Mileage Band					Total
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	
1985	\$5.00	\$19.65	\$39.09	\$36.08	\$28.08	\$127.90
1986	\$5.30	\$21.03	\$41.43	\$38.64	\$29.23	\$135.63
1987	\$5.51	\$22.08	\$44.30	\$41.70	\$31.89	\$145.49
1988	\$5.90	\$24.03	\$47.21	\$43.91	\$33.95	\$154.99
1989	\$6.37	\$25.13	\$49.64	\$47.06	\$36.17	\$164.37
1990	\$6.71	\$26.77	\$52.86	\$50.08	\$38.46	\$174.89

Part J

Ultimate Pure Premium =

Paid Pure Premium

+ (Ultimate Effective Exposure - Effective Exposure at 4/30/91)

* Ultimate Pure Premium for the 10,000 Mile Driver / 10,000