Using Expected Loss Ratios in Reserving

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Abstract: The required loss reserve for a recent time period is estimated by using the recent loss experience plus two probability distributions. One distribution is of ultimate losses for the recent period, based on prior experience and rate adequacy changes. The other distribution is of the ratio of the estimator based on recent experience to the true ultimate loss.

Keywords: Loss reserving; Expected loss ratio.

1. Introduction

This paper prcsrnts a method of using expected loss ratios. together with prior and posterior distributions. in order to estimate loss reserves. This Bayesian method is especially useful for recent accident years and for lines of business with slow development. It incorporates. in a rigorous way. the degree of reliability of the expected loss ratio and of the loss development factors. Estimates of ultimate loss ratios for recent accident years can bc important factors in underwriting decisions.

A method of using expected loss ratios which is now well-known was presented by Bornhuetter and Ferguson (1972). The ultimate losses of an accident year are estimated by using the prior expectation of ultimate losses (expected losses) as well as the reported losses and the selected development factor to ultimate. The ultimate losses are estimated as

$$\text{reported losses} + (1 - z)(\text{expected lows}), \quad (1)$$

where z is the reciprocal of the development factor to ultimate.

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It is implicit in this method of estimation that the cxpecred development for an accident year in each future year is independent of the reported losses.

If 'developed losses' is defined as the product of the reported losses and the development factor to ultimate. then formula (1) can bc expressed as

$$z(\text{developed losses}) + (1 - z)(\text{expected losses}). \quad (2)$$

Bornhuetter–Ferguson and Bayesian estimates of loss reserves will be compared in an example later in this paper.

2. The model

In a Bayesian approach, the prior expectation of ultimate losses for an exposure period E may bc an estimate made several years after rhc beginning of E. If ultimate loss ratios are estimated for the same line of business for the insurer for previous periods, and industry-wide data as well as the insurer's changes in premium adequacy are taken into account, an estimate of the ultimate loss ratio for the period E can be made prior to considering the reported losses for E.

The following direct application of Bayes' the-"rem is basic to this discussion. Let $f(x)$ be the probability density function of the distribution of ultimate losses for exposure period E prior to considering the losses for E. Let $g(y|x)$ be the probability density function of the distribution of y, the developed losses defined previously, for E as of t months, given that the ultimate losses are x. Assume that this distribution has mean x. Let $h(x|y)$ be the probability density function of the distribution of the ultimate losses given that the developed losses are y. Then

$$h(x|y) = g(y|x)f(x) / \int_0^\infty g(y|x)f(x)\,dx. \quad (3)$$

In order to use the above proposition. it is necessary to estimate $g(y|x)$ and f(x). The mean of the distribution given by $h(x|y)$ will be the estimate of ultimate losses.

The variance of the distribution given by $g(y \mid x)$ can be estimated from a study of the historical variability of developed loss ratios at different stages of development. The variance of the distribution given by $f(x)$ can be estimated from the differences between prior expectations of ultimate losses for previous periods, based on the current method of predicting, and the latest developed losses for those periods. The estimated variances between the latest developed losses and the ultimate losses for those periods will also be considered. Historical data of the above types should be supplemented by judgement, experience, and related data.

If a method other than development factors is used for projecting the loss data to ultimate, Bayes' theorem can still be applied as above with $g(y \mid x)$ defined as necessary.

In order to apply Bayes' theorem to a set of accident years, a single development factor to ultimate for the period can be selected as follows. Estimate the ratios between the ultimate losses for each accident year by using the premium and the estimated relative rate adequacy for each year. Then use the reciprocal of the development factor for each year to estimate the ratio of the total ultimate losses for the period to the expected losses for the period at the stage of development. See Biihlmann's Cape Cod method [Schnieper (1991), Straub (1988)].

Biihlmann's (1967) formula for the least squares line estimate of the Bayesian estimates could be used to estimate the credibility of the actual developed losses. [This credibility approximation is exact Bayesian in certain useful cases. In the proof of formula (4), below, we use a special case of Jewell's result that credible means are exact Bayesian for exponential families. See Jewell (1974, 1975).] This method has the advantage of simplicity since 'it does not require the choice of particular distributions.

3. Lognormal distributions

Let $f(x)$, $g(y \mid x)$, and $h(x \mid y)$ be defined as for formula (3). For certain choices of $f(x)$ and $g(y \mid x)$, an explicit formula for the mean of $h(x \mid y)$ is known. An important example is the case in which $f(x)$ and $g(y \mid x)$ represent lognormal distributions. This is a reasonably good fit in many cases.

Suppose that the prior probability distribution of logs of ultimate losses has mean μ and variance ν^2. Suppose that for all x, the distribution, given ultimate losses x, of logs of actual developed losses has variance σ^2. Note that if x is the mean of a lognormal distribution and m and s^2 are the mean and variance of the distribution of the logs, then $\log x = m + s^2/2$. Therefore, for all x the distribution of logs of actual developed losses has mean $\log x - \sigma^2/2$. Then the mean of the distribution given by $h(x \mid y)$ (and thus the estimate of ultimate losses) is

$$\exp(\mu_1 + \nu_1^2/2), \tag{4}$$

where

$$\mu_1 = (1-z)\mu + z(\log y + \sigma^2/2), \tag{5}$$

$$\nu_1^2 = \sigma^2 z, \tag{6}$$

$$z = \nu^2/(\sigma^2 + \nu^2). \tag{7}$$

The derivation is given in the appendix.

Example. Assume that, based on historical experience as described previously, the prior distribution for an insurer's overall ultimate loss ratio for 1987–91 for medical malpractice has a mean of 0.90 (i.e. 90%) and a variance of 0.16. Suppose the selected development factor to ultimate for 1987-91 reported losses as of 12/31/91 is 2.065 and the probability distribution for the ratio of the developed losses to the ultimate losses has a variance of 0.075.

If both of the above distributions are lognormal, then μ, ν^2 and σ^2 in equations (5) and (6) can be found by solving the following equations for the mean and variance of lognormal distributions:

$$0.90 = \exp(\mu + \nu^2/2), \tag{8}$$

$$0.16 = \exp(2\mu + \nu^2)(\exp(\nu^2) - 1), \tag{9}$$

$$1.00 = \exp(m + \sigma^2/2), \tag{10}$$

$$0.075 = \exp(2m + \sigma^2)(\exp(\sigma^2) - 1). \tag{11}$$

By squaring both sides of equation (8) and then dividing by the corresponding sides of equation (9), we get

$$(0.90)^2/0.16 = 1/(\exp(\nu^2) - 1). \tag{12}$$

Table 1
Comparison of methods of estimation.

Actual developed loss ratio	Bayesian estimate of ultimate loss ratio	Bornhuetter–Ferguson estimate of ultimate loss ratio
20%	32%	56%
40%	52%	66%
80%	85%	85%
160%	139%	124%
320%	229%	201%

Solving for ν^2 and μ is then immediate. The same method can be used for σ^2 and m. The solutions are 0.180, -0.195, and 0.072, respectively, for ν^2, μ, and σ^2, so formula 4 becomes $\exp(-0.004 + 0.714 \log y)$. So, if y = 20%. for example, the estimated ultimate loss ratio is 32%. Table 1 compares three methods of estimation.

Appendix: Derivation of formula (4)

Tbe following lemma will be used.

Lemma. *Suppose that an element is chosen at random from a normal distribution for which the value of the mean θ is unknown ($-\infty < \theta < \infty$) and the value of the variance σ^2 is known ($\sigma^2 > 0$). Suppose also that the prior distribution of θ is a normal distribution with given values of the mean μ and the variance ν^2. Then the posterior distribution of θ, given that the element chosen equals x_1, is a normal distribution for which the mean μ_1 and the variance ν_1^2 are as follows:*

$$\mu_1 = (\sigma^2\mu + \nu^2 x_1)/(\sigma^2 + \nu^2), \tag{A.1}$$

$$\nu_1^2 = (\sigma^2\nu^2)/(\sigma^2 + \nu^2). \tag{A.2}$$

See DeGroot (1986) for the-proof of the above.

Proof of formula (4). The mean and variance of the distribution, given ultimate losses x, of $\sigma^2/2$ + log(developed losses). are $\log x$ and σ^2, respectively. The prior distribution of log(ultimate losses) has mean μ and variance ν^2. Therefore, the posterior distribution of log(ultimate losses), given $\sigma^2/2$ + log(developed losses) = x_1, has mean μ_1 and variance ν_1^2 given in the Lemma, where $x_1 = \sigma^2/2$ + log(developed losses). Therefore, the distribution of ultimate losses has mean $\exp(\mu_1 + \nu_1^2/2)$.

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