An Algebraic Reserving Method for Paid Loss Data

by Alfred O. Weller

AN ALGEBRAIC RESERVING METHOD

FOR PAID LOSS DATA

by Alfred O. Weller

Sooner or later a casualty actuary is confronted by the question, "Given a history of paid loss amounts by calendar year, what should reserves be?" Often, it is not possible to accurately gather and analyze additional data within the time constraints for the reserving decision. The algebraic reserving method presented in this paper offers one approach to rapidly addressing this problem. The paper consists of four sections - General Considerations, Formulas, Examples, and Conclusion.

General Considerations

In general, reserve estimates will prove more accurate to the extent that they reflect information from a variety of sources and several actuarial methods. In any reserving situation, available data and information is limited by practical constraints (e.g., design of systems) and time constraints (e.g., financial reporting deadlines). In addition, the question of whether the benefits of better actuarial estimates are worth the costs of gathering better information is implicit in any reserving situation.

The situation to which the algebraic reserving method applies is one in which available information is paid losses by calendar year and there is some basis on which the actuary can assess the annual change in the level of incurred losses by accident year.

For example, a history of earned premiums might be used to create an index of loss levels by accident year. Or, the assumption regarding loss levels by accident year might even be weaker. For example, losses for similar business might have increased an average of 10% per year for the period for which paid losses are available.

The information on loss levels need not be detailed to afford an algebraic solution to the reserving problem. However, in general the more accurate the assumed relative loss levels, the more accurate the estimated reserves will be.

In addition to requiring an assumption regarding relative loss levels by accident year, the method assumes that there is a stable development pattern across all accident years. Thus, as the number of calendar years increases or the numbers of claims whose payments comprise calendar year paid amounts decreases, the possibility of fluctuations in actual payment patterns becomes more important in evaluating the results of the algebraic method.

The information on paid losses should cover all calendar years from the inception of the program. Otherwise, the method cannot estimate reserves without ad hoc adjustments. For example, if data started with the third year of the program, the method would estimate the portion of accident year losses paid through 36 months maturity instead of 12 months. Since the most recent accident year would be at 12 months maturity, the estimate through 36 months would have to be allocated to the maturities 0-12 months, 12-24 months, and 24-36 months using other techniques in order to derive reserve estimates for all accident years.

Tail factors are beyond the scope of the algebraic method. For n calendar year periods, the algebraic method derives development through maturity n years and leaves the tail factor to further analysis. Unless a parameterized payment pattern is assumed and the structure of the equations changed, the tail factor will require separate actuarial analysis.

Finally, the method is called the "algebraic method" because it is based on the algebraic solution of n linear equations in n unknowns. Thus, for any set of assumed relative loss levels, there is a unique solution for unpaid (unreported) losses that will be paid (reported) on or before accident years attain maturity n years. Reserve estimates based on successful mathematical solutions of the equations may differ from reasonable actuarial estimates. The algebraic method can provide useful input into actuarial decisions on appropriate reserves, but should not be used as an algorithm without professional scrutiny.

Formulas

The following equations define the "algebraic method."

$$I_j = \text{Incurred amount for accident year } j. \tag{1}$$

$$\begin{aligned} n &= \text{Number of calendar years for which data is available} \\ &= \text{Number of accident years affecting data} \end{aligned} \tag{2}$$

$$f_i = \text{Fraction of accident year loss paid during year } i \text{ after start of accident year.} \quad (3)$$

Because the algebraic method estimates development through maturity n and because the sum of the fractions of losses at maturity n paid in each calendar year must total unity (i.e., 100%),

$$f_n = 1 - \sum_{i=1}^{n-1} f_i \quad (4)$$

Calendar year payments can now be expressed in terms of accident year components.

$$\begin{aligned} P_j &= \text{Amount paid during calendar year } j \text{ for all accident years} \\ &= \sum_{i=1}^{j} f_i \, I_{j+1-i} \quad (5) \\ \text{so that, if } j=n,\ P_j &= \sum_{i=1}^{n-1} f_i \, I_{n+1-i} + (1 - \sum_{i=1}^{n-1} f_i) \, I_1 \end{aligned}$$

Introducing loss level indices facilitates solving equation (5). We define indices as follows:

$$\begin{aligned} g_j &= \text{Index for accident year } j \text{ loss level} \\ &= \frac{I_j}{I_1} \quad (6) \\ \text{so that } g_1 &= 1.000 \\ g_j &= g_2^{(j-1)} \quad \text{for uniform growth} \end{aligned}$$

Equation (5) can now be rewritten as:

$$P_j = \sum_{i=1}^{j} f_i \, g_{j+1-i} \, I_1 \qquad \text{if } j<n$$

$$P_j = I_1 + \sum_{i=1}^{n-1} f_i \, (g_{n-i}-1) \, I_1 \qquad \text{if } j=n$$

In order to generate n linear equations in n unknowns, we introduce a variable equal to the reciprocal of incurred losses.

$$R_j = \frac{1}{I_j} \qquad (8)$$
$$= \text{Reciprocal of incurred loss for accident year } j$$

The resulting n linear equations are:

$$0 = -P_j R_1 + \sum_{i=1}^{j} f_i \, g_{j+1-i} \qquad \text{if } j<n$$

$$-1 = -P_j R_1 + \sum_{i=1}^{n-1} f_i \, (g_{n-i}-1) \qquad \text{if } j=n$$

Thus, the algebraic reserving method solves the n equations

$$\begin{aligned}
0 &= -P_1 R_1 + 1 f_1 + 0 f_2 + 0 f_3 + \ldots + 0 f_{n-1} \\
0 &= -P_2 R_1 + g_2 f_1 + 1 f_2 + 0 f_3 + \ldots + 0 f_{n-1} \\
0 &= -P_3 R_1 + g_3 f_1 + g_2 f_2 + 1 f_3 + \ldots + 0 f_{n-1} \qquad (10) \\
\ldots &= \ldots \\
-1 &= -P_n R_1 + (g_n-1) f_1 + (g_{n-1}-1) f_2 + (g_{n-3}-1) f_3 + \ldots + (g_2-1) f_{n-1}
\end{aligned}$$

for $R_1, f_1, \ldots, f_{n-1}$.

Examples

In the attached exhibits, data for private passenger automobile liability/medical from pages 63 and 79 of the 1993 edition of Best's Aggregates & Averages is used to illustrate the algebraic method. For convenience, loss and allocated loss adjustment expense is called "loss" in this discussion.

Exhibit I presents a link ratio approach to establish a benchmark for comparison to the results of the algebraic method. Weighted three point average development factors are employed. Other link ratio calculations are possible, but only one is used for comparison purposes in this paper. Exhibit I-1 presents raw data. Exhibit I-2 derives development factors. Exhibit I-3 derives reserve estimates using the development pattern from Exhibit I-2.

Exhibit II derives values for use in subsequent algebraic method calculations. Exhibit II-1 derives calendar year paid loss as if accident year 1983 were the first year of a program. Exhibit II-2 uses earned premiums to estimate loss level indices. Distinct indices by year are used in Exhibit III and a rough average annual growth rate is used in Exhibit IV.

Exhibit III applies the algebraic method using distinct indices by year. Exhibit III-1 presents the matrix defining the simultaneous equations. Exhibit III-2 presents the inverted matrix and the estimated parameters R_1, f_1, ..., f_{n-1}. Exhibit III-3 compares the paid amounts based on the parameters to the actual paid amounts by accident year component as well as by calendar year total. Exhibit III-4 adjusts the development pattern for negative values and derives corresponding reserve estimates.

Negative values might be attributable to several causes (e.g., influence of particular large claims, shifts in development patterns over time). Consideration of alternative possible adjustments will vary with available data and reserving context, and is, therefore, beyond the scope of this paper.

Exhibits IV are organized identically to Exhibits III. The difference is that a uniform annual change in loss level is used in lieu of individual annual indices.

Following Exhibits IV are four graphs. Graph 1 presents the three cumulative development patterns fit using the above techniques. Graph 2 presents the same development patterns on an interval basis. Graph 3 compares reserves estimates by accident year. Graph 4 presents the components of accident year losses using the three methods.

Conclusion

For the data used in the example, the algebraic method presented above produced reserve estimates quite close (within 10%) to reserve estimates based on a link ratio method. Therefore, it might prove useful in situations in which detailed data is unavailable. In particular, it might prove useful in reserving situations for which only calendar year paid loss data is available.

For the example, the method required elimination of some negative values from the development pattern. Also, the algebraic reserving method is quite sensitive to the selection of loss level indices. Therefore, although it can prove useful in particular

situations, it is not well suited to use as an algorithm without professional scrutiny by a casualty actuary.

Link Ratios - Amounts* by Maturity

Year	1	2	3	4	5	6	7	8	9	10
1983	6,336,136	12,087,849	14,818,118	16,318,063	17,116,020	17,537,521	17,709,879	17,801,459	17,869,922	17,914,459
1984	7,115,534	13,753,038	16,936,543	18,711,542	19,694,316	20,188,931	20,439,114	20,585,826	20,663,474	xx
1985	7,816,829	15,437,173	19,146,806	21,281,679	22,406,157	22,974,802	23,280,050	23,433,065	xx	xx
1986	8,701,618	17,269,667	21,541,709	23,930,252	25,211,125	25,858,777	26,169,144	xx	xx	xx
1987	9,697,467	19,400,683	24,147,746	26,805,181	28,826,913	28,910,978	xx	xx	xx	xx
1988	10,916,881	21,763,312	26,980,241	29,807,907	31,280,692	xx	xx	xx	xx	xx
1989	12,051,811	24,101,585	29,805,169	32,850,611	xx	xx	xx	xx	xx	xx
1990	13,320,847	26,043,240	32,085,138	xx	xx	xx	xx	xx	xx	xx
1991	13,320,110	25,851,612	xx	xx	xx	xx	xx	xx	xx	xx
1992	14,400,031	xx	xx	xx	xx	xx	xx	xx	xx	xx
Total	103,677,266	175,708,159	185,461,470	169,705,235	144,535,223	115,471,009	87,598,187	61,820,350	38,533,396	17,914,459
3 Pt Num	75,996,437	88,870,548	89,463,699	85,318,730	77,744,557	69,888,308	61,820,350	38,533,396	17,914,459	xx
3 Pt Den	38,692,768	71,908,137	80,933,156	80,543,340	76,444,195	69,022,510	61,429,043	38,387,285	17,869,922	xx

* 1993 Edition of Best's "Aggregates & Averages" Page 79

Link Ratios - Development

Year	1	2	3	4	5	6	7	8	9	10
1983	1.908	1.226	1.101	1.049	1.025	1.010	1.005	1.004	1.002	xx
1984	1.933	1.231	1.105	1.053	1.025	1.012	1.007	1.004	xx	xx
1985	1.975	1.240	1.112	1.053	1.025	1.013	1.007	xx	xx	xx
1986	1.985	1.247	1.111	1.054	1.026	1.012	xx	xx	xx	xx
1987	2.001	1.245	1.110	1.075	1.003	xx	xx	xx	xx	xx
1988	1.994	1.240	1.105	1.049	xx	xx	xx	xx	xx	xx
1989	2.000	1.237	1.102	xx	xx	xx	xx	xx	xx	xx
1990	1.955	1.232	xx	xx	xx	xx	xx	xx	xx	xx
1991	1.941	xx	xx	xx	xx	xx	xx	xx	xx	xx
1992	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
Average	1.966	1.237	1.106	1.055	1.021	1.012	1.006	1.004	1.002	xx
3 Pt Avg	1.965	1.236	1.106	1.059	1.018	1.013	1.006	1.004	1.002	xx
3 Pt Wtd	1.964	1.236	1.105	1.059	1.017	1.013	1.006	1.004	1.002	xx
Selected	1.964	1.236	1.105	1.059	1.017	1.013	1.006	1.004	1.002	xx
Cum Factor	2.964	1.509	1.221	1.105	1.043	1.025	1.013	1.006	1.002	1.000
Cum %	33.74%	66.26%	81.89%	90.52%	95.89%	97.52%	98.74%	99.37%	99.75%	100.00%
Int %	33.74%	32.52%	15.63%	8.63%	5.37%	1.63%	1.22%	0.63%	0.38%	0.25%

Link Ratios - Reserve Estimates

Year	Paid Loss	Factor	Est Inc	Est Res
1983	17,914,459	1.000	17,914,459	0
1984	20,663,474	1.002	20,714,973	51,499
1985	23,433,065	1.006	23,580,881	147,816
1986	26,169,144	1.013	26,501,970	332,826
1987	28,910,978	1.025	29,645,938	734,960
1988	31,280,692	1.043	32,621,524	1,340,832
1989	32,850,611	1.105	36,289,927	3,439,316
1990	32,085,138	1.221	39,180,226	7,095,088
1991	25,851,612	1.509	39,014,898	13,163,286
1992	14,400,031	2.964	42,684,451	28,284,420
Total	253,559,204	1.215	308,149,247	54,590,043

Calendar Year Paid Losses by Accident Year Components

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	6,336,136	5,751,713	2,730,269	1,499,945	797,957	421,501	172,358	91,580	68,463	46,537
1984	xx	7,115,534	6,637,504	3,183,505	1,774,999	982,774	494,615	250,183	146,712	77,648
1985	xx	xx	7,816,829	7,620,344	3,709,633	2,134,873	1,124,478	568,645	305,248	153,015
1986	xx	xx	xx	8,701,618	8,568,049	4,272,042	2,388,543	1,280,873	647,652	310,367
1987	xx	xx	xx	xx	9,697,467	9,703,216	4,747,063	2,657,435	2,021,732	84,065
1988	xx	xx	xx	xx	xx	10,916,881	10,846,431	5,216,929	2,827,666	1,472,785
1989	xx	xx	xx	xx	xx	xx	12,051,811	12,049,774	5,703,586	3,045,442
1990	xx	xx	xx	xx	xx	xx	xx	13,320,847	12,722,393	6,041,898
1991	xx	xx	xx	xx	xx	xx	xx	xx	13,320,110	12,531,502
1992	xx	xx	xx	xx	xx	xx	xx	xx	xx	14,400,031
Total	6,336,136	12,867,247	17,184,602	21,005,412	24,548,105	28,431,287	31,825,299	35,436,266	37,763,560	38,161,290

Thus,

P1 = 6,336,136
P2 = 12,867,247
P3 = 17,184,602
P4 = 21,005,412
P5 = 24,548,105
P6 = 28,431,287
P7 = 31,825,299
P8 = 35,436,266
P9 = 37,763,560
P10 = 38,161,290

Loss Level Indices based on Earned Premiums

Loss level indices by year

Year	Earned Premium	Index
1983	22,382,780	1.0000
1984	23,968,070	1.0708
1985	26,608,441	1.1888
1986	31,360,994	1.4011
1987	35,801,570	1.5995
1988	39,732,848	1.7752
1989	43,038,375	1.9228
1990	46,899,296	2.0953
1991	50,069,836	2.2370
1992	54,197,133	2.4214

Loss level assuming uniform annual rate

1	Earned premium for 1992	54,197,133
2	Earned premium for 1983	22,382,780
3	Ratio: (1)/(2)	2.4214
4	g2 = Ninth root of (3)	1.1032
5	Indices by year	
	1983	1.0000
	1984	1.1032
	1985	1.2172
	1986	1.3428
	1987	1.4815
	1988	1.6344
	1989	1.8032
	1990	1.9894
	1991	2.1948
	1992	2.4214

Algebraic Method using Yearly Indices - Simultaneous Equation Matrix

Coefficients of R1	f1	f2	f3	f4	f5	f6	f7	f8	f9	=	Value
-6336136	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-12867247	1.0708	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-17184602	1.1888	1.0708	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-21005412	1.4011	1.1888	1.0708	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-24548105	1.5995	1.4011	1.1888	1.0708	1.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-28431287	1.7752	1.5995	1.4011	1.1888	1.0708	1.0000	0.0000	0.0000	0.0000	=	0.0000
-31825299	1.9228	1.7752	1.5995	1.4011	1.1888	1.0708	1.0000	0.0000	0.0000	=	0.0000
-35436266	2.0953	1.9228	1.7752	1.5995	1.4011	1.1888	1.0708	1.0000	0.0000	=	0.0000
-37763560	2.2370	2.0953	1.9228	1.7752	1.5995	1.4011	1.1888	1.0708	1.0000	=	0.0000
-38161290	1.4214	1.2370	1.0953	0.9228	0.7752	0.5995	0.4011	0.1888	0.0708	=	-1.0000

Algebraic Method using Yearly Indices - Inverse Matrix and Solutions

R1	f1	f2	f3	f4	f5	f6	f7	f8	f9
0.0000000038	0.0000000010	0.0000000035	0.0000000029	0.0000000066	0.0000000099	0.0000000124	0.0000000072	0.0000000045	-0.0000000634
1.0263871066	0.0064850583	0.0219796232	0.0182600681	0.0415287133	0.0626983083	0.0786896308	0.0453516440	0.0284386579	-0.4015266761
-1.0474160274	1.0062253031	0.0210992423	0.0175286718	0.0398653051	0.0601869643	0.0755377637	0.0435351105	0.0272995642	-0.3854437605
-0.0300395396	-1.0676134154	1.0108894151	0.0090466274	0.0205746657	0.0310627667	0.0389853843	0.0224686690	0.0140894295	-0.1989292822
-0.0771176366	-0.0405499867	-1.0654993606	1.0044254940	0.0100648624	0.0151955068	0.0190711497	0.0109913846	0.0068923681	-0.0973136520
0.0418089213	-0.0825008529	-0.0403348031	-1.0693418180	1.0033761961	0.0050972392	0.0063972997	0.0036869922	0.0023120024	-0.0326432651
0.0553379117	0.0405217265	-0.0806710901	-0.0401635819	-1.0663729923	1.0067234513	0.0084382803	0.0048632823	0.0030496187	-0.0430577017
0.0592292651	0.0530628298	0.0409788867	-0.0820705381	-0.0399439834	-1.0675384980	1.0041263906	0.0023781863	0.0014912894	-0.0210555812
-0.0071439575	0.0585140748	0.0546330919	0.0414133519	-0.0794186593	-0.0366723564	-1.0639871140	1.0039416802	0.0024717096	-0.0348981783
0.0359820107	-0.0099568210	0.0556006888	0.0507706570	0.0353866483	-0.0897308763	-0.0505340641	-1.0756746687	0.9969597474	0.0429254613

R1	=	6.337091E-08
f1	=	40.15%
f2	=	38.54%
f3	=	19.89%
f4	=	9.73%
f5	=	3.26%
f6	=	4.31%
f7	=	2.11%
f8	=	3.49%
f9	=	-4.29%
f10	=	-17.19%

Algebraic Method using Yearly Indices - Estimated Calendar Year Paid Amounts by Accident Year Components

Year	Est Inc	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	15,780,112	6,336,136	6,082,346	3,139,126	1,535,620	515,114	679,455	332,259	550,697	(677,369)	(2,713,274)
1984	16,897,760	xx	6,784,901	6,513,136	3,361,459	1,644,383	551,598	727,579	355,792	589,701	(725,344)
1985	18,759,251	xx	xx	7,532,340	7,230,636	3,731,764	1,825,531	612,363	807,730	394,987	654,664
1986	22,109,854	xx	xx	xx	8,877,696	8,522,105	4,398,297	2,151,591	721,738	952,000	465,536
1987	25,240,510	xx	xx	xx	xx	10,134,738	9,728,797	5,021,077	2,456,246	823,933	1,086,798
1988	28,012,106	xx	xx	xx	xx	xx	11,247,608	10,797,091	5,572,428	2,725,960	914,407
1989	30,342,540	xx	xx	xx	xx	xx	xx	12,183,339	11,695,343	6,036,020	2,952,743
1990	33,064,532	xx	xx	xx	xx	xx	xx	xx	13,276,292	12,744,518	6,577,504
1991	35,299,799	xx	xx	xx	xx	xx	xx	xx	xx	14,173,811	13,606,087
1992	38,209,590	xx	xx	xx	xx	xx	xx	xx	xx	xx	15,342,170
Total	xx	6,336,136	12,867,247	17,184,602	21,005,412	24,548,105	28,431,287	31,825,299	35,436,266	37,763,560	38,161,290

Differences - Estimated less Actual

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	0	330633	408857	35675	-282843	257954	159901	459117	-745832	-2757811
1984	xx	-330633	-124368	177954	-130616	-431176	232964	105609	442989	-802992
1985	xx	xx	-284489	-389708	22131	-309342	-512115	239085	89739	501649
1986	xx	xx	xx	176078	-45944	126255	-236952	-559135	304348	155169
1987	xx	xx	xx	xx	437271	25581	274014	-201189	-1197799	1002733
1988	xx	xx	xx	xx	xx	330727	-49340	355499	-101706	-558378
1989	xx	xx	xx	xx	xx	xx	131528	-354431	332436	-92699
1990	xx	xx	xx	xx	xx	xx	xx	-44555	22125	535606
1991	xx	xx	xx	xx	xx	xx	xx	xx	853701	1074585
1992	xx	xx	xx	xx	xx	xx	xx	xx	xx	942139
Total	0	0	-0	-0	-0	0	-0	0	-0	0

Algebraic Method using Yearly Indices - Development Pattern and Reserve Estimates

Maturity	Pattern	Neg Val	Mod Patt	Cum Patt
1	40.15%	0.00%	33.05%	33.05%
2	38.54%	0.00%	31.73%	64.78%
3	19.89%	0.00%	16.37%	81.15%
4	9.73%	0.00%	8.01%	89.16%
5	3.26%	0.00%	2.69%	91.85%
6	4.31%	0.00%	3.54%	95.39%
7	2.11%	0.00%	1.73%	97.13%
8	3.49%	0.00%	2.87%	100.00%
9	-4.29%	4.29%	0.00%	100.00%
10	-17.19%	17.19%	0.00%	100.00%
Total	100.00%	21.49%	100.00%	xx

Year	Est Inc	% Unpaid	Est Unpaid
1983	15,780,112	0.00%	0
1984	16,897,760	0.00%	0
1985	18,759,251	0.00%	0
1986	22,109,854	2.87%	635,125
1987	25,240,510	4.61%	1,162,514
1988	28,012,106	8.15%	2,282,980
1989	30,342,540	10.84%	3,288,208
1990	33,064,532	18.85%	6,231,732
1991	35,299,799	35.22%	12,433,203
1992	38,209,590	66.95%	25,580,919
Total	263,716,053	19.57%	51,614,681

Algebraic Method using Uniform Annual Rate - Simultaneous Equation Matrix

Coefficients of R1	f1	f2	f3	f4	f5	f6	f7	f8	f9	=	Value
-6336136	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-12867247	1.1032	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-17184602	1.2172	1.1032	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-21005412	1.3428	1.2172	1.1032	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-24548105	1.4815	1.3428	1.2172	1.1032	1.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-28431287	1.6344	1.4815	1.3428	1.2172	1.1032	1.0000	0.0000	0.0000	0.0000	=	0.0000
-31825299	1.8032	1.6344	1.4815	1.3428	1.2172	1.1032	1.0000	0.0000	0.0000	=	0.0000
-35436266	1.9894	1.8032	1.6344	1.4815	1.3428	1.2172	1.1032	1.0000	0.0000	=	0.0000
-37763560	2.1948	1.9894	1.8032	1.6344	1.4815	1.3428	1.2172	1.1032	1.0000	=	0.0000
-38161290	1.4214	1.1948	0.9894	0.8032	0.6344	0.4815	0.3428	0.2172	0.1032	=	-1.0000

Algebraic Method using Uniform Annual Rate - Inverse Matrix and Solutions

R1	f1	f2	f3	f4	f5	f6	f7	f8	f9		Solution
0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	-0.0000000628	R1 =	6.280758E-08
1.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	-0.3979573429	f1 =	39.80%
-1.0651383684	1.0381107459	0.0381107459	0.0381107459	0.0381107459	0.0381107459	0.0381107459	0.0381107459	0.0381107459	-0.3691145068	f2 =	36.91%
0.0193819998	-1.0838671145	1.0193819998	0.0193819998	0.0193819998	0.0193819998	0.0193819998	0.0193819998	0.0193819998	-0.1877207365	f3 =	18.77%
0.0132712953	0.0132712953	-1.0899778190	1.0132712953	0.0132712953	0.0132712953	0.0132712953	0.0132712953	0.0132712953	-0.1285366503	f4 =	12.85%
0.0089095215	0.0089095215	0.0089095215	-1.0943395928	1.0089095215	0.0089095215	0.0089095215	0.0089095215	0.0089095215	-0.0862915054	f5 =	8.63%
0.0087455143	0.0087455143	0.0087455143	0.0087455143	-1.0945036000	1.0087455143	0.0087455143	0.0087455143	0.0087455143	-0.0847030444	f6 =	8.47%
0.0029733371	0.0029733371	0.0029733371	0.0029733371	0.0029733371	-1.1002757772	1.0029733371	0.0029733371	0.0029733371	-0.0287977006	f7 =	2.88%
0.0021077831	0.0021077831	0.0021077831	0.0021077831	0.0021077831	0.0021077831	-1.1011613312	1.0021077831	0.0021077831	-0.0204145390	f8 =	2.04%
-0.0086343461	-0.0086343461	-0.0086343461	-0.0086343461	-0.0086343461	-0.0086343461	-0.0086343461	-1.1118834604	0.9913656539	0.0836263452	f9 =	-8.36%
										f10 =	-21.99%

Algebraic Method using Uniform Annual Rate - Estimated Calendar Year Paid Amounts by Accident Year Components

Year	Est Inc	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	15,921,646	6,336,136	5,876,911	2,988,823	2,046,515	1,373,903	1,348,612	458,507	325,033	(1,331,469)	(3,501,324)
1984	17,565,542	xx	6,990,336	6,483,696	3,297,416	2,257,816	1,515,757	1,487,855	505,847	358,592	(1,468,942)
1985	19,379,169	xx	xx	7,712,082	7,153,132	3,637,872	2,490,933	1,672,258	1,641,475	558,075	395,617
1986	21,380,051	xx	xx	xx	8,508,348	7,891,687	4,013,479	2,748,120	1,844,917	1,810,955	615,696
1987	23,587,522	xx	xx	xx	xx	9,386,828	8,706,497	4,427,867	3,031,861	2,035,403	1,997,935
1988	26,022,913	xx	xx	xx	xx	xx	10,356,009	9,605,435	4,885,040	3,344,898	2,245,556
1989	28,709,755	xx	xx	xx	xx	xx	xx	11,425,258	10,597,187	5,389,416	3,690,256
1990	31,674,012	xx	xx	xx	xx	xx	xx	xx	12,604,906	11,691,337	5,945,869
1991	34,944,326	xx	xx	xx	xx	xx	xx	xx	xx	13,906,351	12,898,458
1992	38,552,297	xx	xx	xx	xx	xx	xx	xx	xx	xx	15,342,170
Total	xx	6,336,136	12,867,247	17,184,602	21,005,412	24,548,105	28,431,287	31,825,299	35,436,266	37,763,560	38,161,290

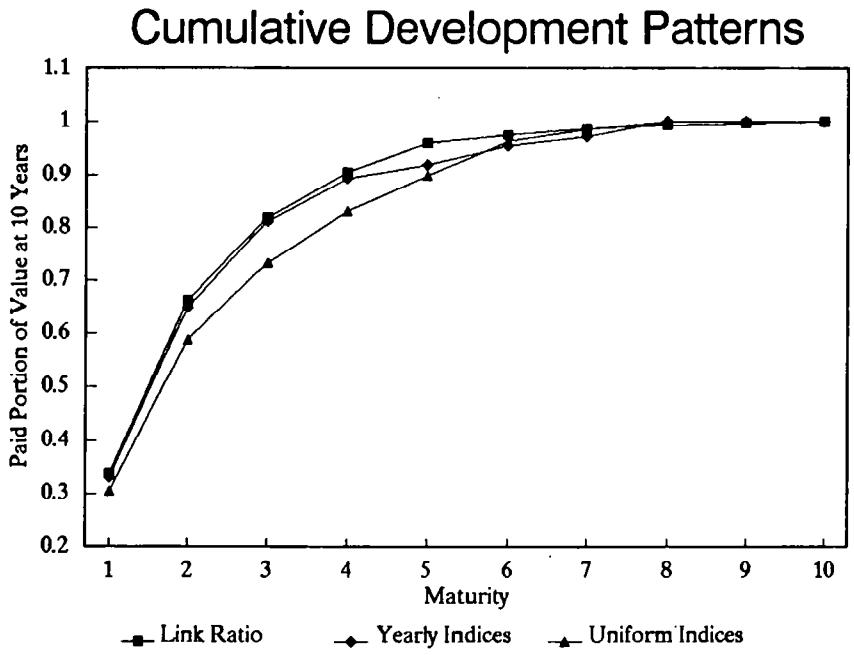
Differences - Estimated less Actual

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	0	5876911	2988823	2046515	1373903	1348612	458507	325033	-1331469	-3501324
1984	xx	6990335	6483696	3297416	2257816	1515757	1487855	505847	358593	-1468942
1985	xx	xx	7712081	7153132	3637872	2490933	1672258	1641475	558076	395617
1986	xx	xx	xx	8508347	7891687	4013479	2748120	1844917	1810955	615696
1987	xx	xx	xx	xx	9386827	8706497	4427867	3031861	2035403	1997935
1988	xx	xx	xx	xx	xx	10356008	9605435	4885040	3344898	2245556
1989	xx	xx	xx	xx	xx	xx	11425257	10597187	5389416	3690256
1990	xx	xx	xx	xx	xx	xx	xx	12604905	11691337	5945869
1991	xx	xx	xx	xx	xx	xx	xx	xx	13906351	12898458
1992	xx	xx	xx	xx	xx	xx	xx	xx	xx	15342170
Total	0	12867246	17184601	21005411	24548104	28431286	31825298	35436265	37763561	38161290

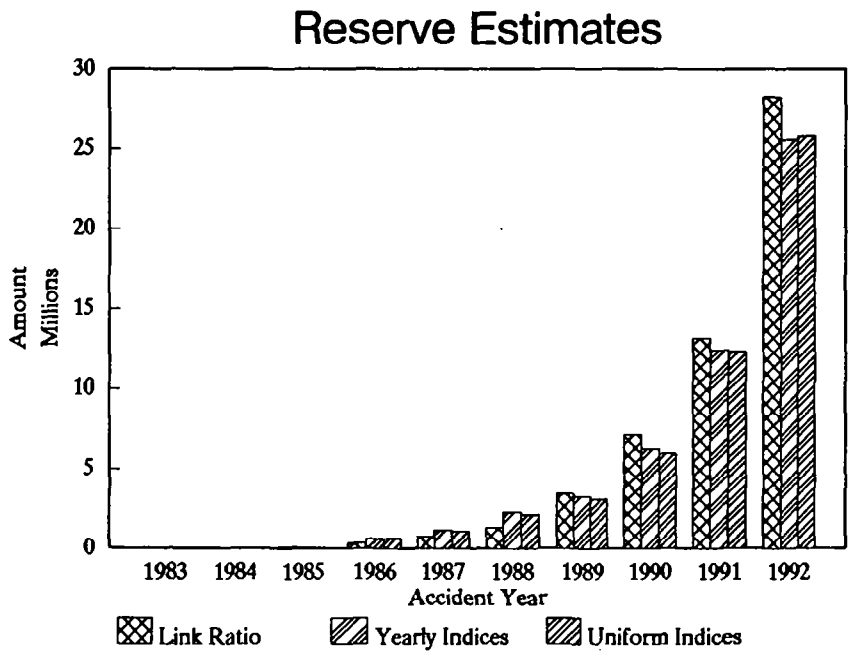
Algebraic Method using Uniform Annual Rate - Development Pattern and Reserve Estimates

Maturity	Pattern	Neg Val	Mod Patt	Cum Patt
1	39.80%	0.00%	30.53%	30.53%
2	36.91%	0.00%	28.32%	58.85%
3	18.77%	0.00%	14.40%	73.25%
4	12.85%	0.00%	9.86%	83.11%
5	8.63%	0.00%	6.62%	89.73%
6	8.47%	0.00%	6.50%	96.22%
7	2.88%	0.00%	2.21%	98.43%
8	2.04%	0.00%	1.57%	100.00%
9	-8.36%	8.36%	0.00%	100.00%
10	-21.99%	21.99%	0.00%	100.00%
Total	100.00%	30.35%	100.00%	xx

Year	Est Inc	% Unpaid	Est Unpaid
1983	15,921,646	0.00%	0
1984	17,565,542	0.00%	0
1985	19,379,169	0.00%	0
1986	21,380,051	2.87%	614,161
1987	23,587,522	4.61%	1,086,382
1988	26,022,913	8.15%	2,120,861
1989	28,709,755	10.84%	3,111,264
1990	31,674,012	18.85%	5,969,658
1991	34,944,326	35.22%	12,307,999
1992	38,552,297	66.95%	25,810,357
Total	257,737,233	19.80%	51,020,682


Cumulative Development Patterns
Paid Portion of Value at 10 Years
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
1
2
3
4
5
6
7
8
9
10
Maturity
Link Ratio
Yearly Indices
Uniform Indices

Development Patterns by Interval
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Portion Paid in Year
1
2
3
4
5
6
7
8
9
10
Maturity
Link Ratio
Yearly Indices
Uniform Indices


Reserve Estimates
30
25
20
15
10
5
0
Amount
Millions
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
Accident Year
Link Ratio
Yearly Indices
Uniform Indices

Incurred Loss Estimates

(Link ratio to left, yearly indices above, and uniform indices to right of accident year.)