Efficient Estimators Through Data Segmentation

ABSTRACT

Statutory Statement of Accounting Principles 55 of the NAIC states that "management shall record its best estimate of its liability for unpaid claims, unpaid losses and loss/claim adjustment expenses". One of the considerations in analyzing the reasonability of these estimates will be the amount of variability in the estimates. Variability of reserve estimates can be reduced through data segmentation.

From a statistical standpoint, among unbiased estimators, an efficient estimator is defined as an estimator with the same mean as another estimator, but a lower variance. The best estimator would have the same mean as other estimators, but the least variance overall.

The efficiency of various reserve estimators (alternative data segmentations) can be compared by projecting the variance of the one year runoff of the loss and loss/claim adjustment expense data for each data segmentation alternative. A model has been developed to compare the efficiency of various reserve estimators and determine the impact of data segmentation on the variability of loss reserving estimates.

Efficient Estimators Through Data Segmentation

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OVERVIEW

Statutory Statement of Accounting Principles 55 of the NAIC states that “management shall record its best estimate of its liability for unpaid claims, unpaid losses and loss/claim adjustment expenses”. One of the considerations in analyzing the reasonability of the loss or loss/claim adjustment expense reserve estimates will be the amount of variability in the estimate. Variability of reserve estimates can be reduced through data segmentation.

GENERAL RESERVE ANALYSIS CONSIDERATIONS

An estimate of ultimate losses and loss adjustment expenses is made by projecting future development on the reserving data. A particular set of assumptions used in projecting the future development is commonly referred to as a projection method. A reserve analysis normally utilizes multiple projection methods.

Two considerations underlie each reserve analysis, bias and efficiency.

If the incurred to date and ultimate losses for a given line of business are represented by the random variables K and L, a projection method (or reserve estimator) that is a function of incurred losses, R(K), is unbiased if the expected value of the projected ultimate losses, E(R(K)), is equal to the expected value of the actual ultimate losses, E(L) [1].

Efficiency among unbiased estimators, can be calculated as Var(R(K)-L) with lower variances implying more efficient estimators.

DATA SEGMENTATION AND EFFICIENT ESTIMATORS

When a reserve analysis is performed on a book of business, the data is normally split into a number of reserving segments with each segment reviewed separately. A reserving segment is defined as a subdivision of the book of business being analyzed. A scenario refers to the data segmentation chosen for the reserve analysis. The data segments and projection methods used form an estimator of the true ultimate losses and loss adjustment expenses for the book of business being reviewed. The ultimate losses and loss adjustment expense selections in the reserve analysis for the book of business being reviewed are normally the reserving actuary's best estimate of the true ultimate losses and loss adjustment expenses for the book of business being reviewed.

An efficient estimator is defined as an estimator with the same mean as another estimator, but a lower variance. The best estimator would have the same mean as other estimators, but the least variance overall.

A data segmentation model has been developed that will measure the amount of variability in reserving estimates. The model also allows comparison of the efficiencies of a number of different estimators (alternative data segmentations). More efficient estimators will result in more stable reserve estimates.

DATA SEGMENTATION CONSIDERATIONS

Data segmentation decisions are governed by both actuarial and economic considerations.

Some of the actuarial considerations (as stated in the Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves) in determining data segmentation are volume, credibility, homogeneity and similarity with respect to emergence, development and statistical patterns. Segmentation decisions resulting from applying these criterion are often in conflict.

Economic considerations arise when the costs of performing analyses in various levels of detail are compared to the associated benefits. A more refined analysis costs more, but should have less variability.

A measurement tool is needed to evaluate the implications of various segmentation alternatives.

INITIAL DATA SEGMENTATION THOUGHTS

Data segmentation is an art as well as a science. The possibilities are limited only by the number of available variables in the reserving data and the creativity of the actuary performing the reserve analysis.

Sound knowledge of the business to be analyzed is needed with respect to operational characteristics (marketing, underwriting, claims handling) to perform a thorough reserve analysis. The segmentation model is a tool that augments, rather than creates or replaces, sound business judgment.

Use of the data segmentation model requires two steps to be performed. The first step is selecting the data segmentation variables to be analyzed. The next step is to determine which scenarios should be examined.

INITIAL DATA SEGMENTATION VARIABLE SELECTIONS

The variables selected can be based on one of the following:

1. Historical Precedent

   The segmentation variables used in the last reserve analysis (and possibly one or two more that make sense to be examined from a business standpoint) are selected.

2. Business judgment

   Based on the reserving actuary's knowledge of the business and the reserving data available, segmentation variables are selected.

3. Factor analysis

   Factor analysis is a multivariate statistical analysis technique. It is well suited to this task because its primary use is to summarize and reduce large volumes of data. Its advantage over the prior selection criterion is that it can analyze many more variables more effectively. It has disadvantages over the other methods in that the results, although probably preferable from a statistical standpoint, can often be difficult to interpret and explain to others. Factor analysis results should not attempt to serve as a substitute for applying sound business judgment.

The initial data segmentation variable selections of this paper are the results of applying either historical precedent or business judgment. The application of factor analysis is beyond the scope of this paper.

SELECTING THE SCENARIOS TO BE ANALYZED

The scenarios to be analyzed depend on the business issues to be addressed. A reserving actuary analyzing a General Liability book may have selected sub-line of business (Products vs. Non-Products) and coverage (Bodily Injury vs. Property Damage) as segmentation variables.

Four groups of data are available:

1. Products Bodily Injury
2. Non-Products Bodily Injury
3. Products Property Damage
4. Non-Products Property Damage

and two analyses are performed on the book (Products vs. Non-Products).

Three scenarios may be analyzed to focus on the business issue of whether Products and Non-Products should continue to be analyzed separately, analyzed separately and also subdivided into Bodily Injury and Property Damage components, or analyzed on a combined basis in one segment:

1. Continue performing two analyses (Products vs. Non-Products).

2. Perform four analyses (Products Bodily Injury, Non-Products Bodily Injury, Products Property Damage, Non-Products Property Damage).

3. Perform one analysis (all lines and coverages combined).

Alternatively, four scenarios may be analyzed to focus on all possible combinations of the potential segmentation variables selected:

1. Combine all lines and coverages (perform one analysis).

2. Analyze based on line of business (perform two analyses).

3. Analyze based on coverage (perform two analyses).

4. Analyze based on line of business and coverage (perform four analyses).

Lastly, all possible subsets of the four groupings of data could be analyzed. This would require fifteen scenarios to be examined. A list of the 15 scenarios is given in **Exhibit 1**. Although it would be a more complete and thorough data segmentation analysis, it is discouraged because as the number of segmentation variables grows, an analysis of this nature can get unwieldy very quickly.

EXPLORATORY ANALYSIS

Exploratory analysis with respect to the volume of the business and the stability of the development patterns should be done before the scenarios to be analyzed are selected. This analysis consists of generating loss development triangles for each data segment and examining the growth and loss development patterns for each data segment.

Segments with similar growth patterns or loss development patterns may result in more stable reserve indications if combined rather than kept separate due to the increased volume of the data. Segments with stable loss development patterns, but different growth rates should not be combined in a reserve analysis due to the potential for a biased analysis to result. The scenarios should be modified based on the results of the exploratory analysis to avoid a biased analysis.

MODEL ASSUMPTIONS

The following simplifying assumptions were made with respect to the design of the data segmentation model:

1. The estimated variance of the one year runoff is an appropriate measure of the variability in loss projection estimates. Estimating the variance of the total runoff would give a more appropriate measure, however, this would be more difficult to model. It is assumed that a one year runoff model would give the same conclusions with respect to data segmentation as a total runoff model would.

2. That the expected mean and variance of each future age-to-age loss development factor is equal to the sample means and variances of the past age-to-age loss development factors.

Tests of these assumptions and possible modifications to the model will be discussed later in this paper.

CONSTRUCTING THE MODEL AND ESTIMATING THE VARIANCE IN THE ONE YEAR RUNOFF

Preliminary Calculations

1. Cumulative incurred loss, paid loss, incurred loss and allocated loss adjustment expenses, or paid loss and allocated loss adjustment expense triangles are generated for each data segment. The remainder of this paper uses incurred loss triangles and assumes that no tail is needed after the most mature evaluation age shown in the triangle.

2. Age-to-age loss development factors are calculated.

3. The expected mean and variance of the age-to-age loss development factors are estimated as the sample mean and variance of the age-to-age loss development factors.

4. Covariances of the one year runoff for each pair of reserving segments are calculated. Note that if loss processes of two reserving segments are independent, then the covariance will be equal to zero.

Estimating the Mean and Variance of the One Year Runoff

The variance of the one year runoff is the measure used to compare the efficiencies of various data segmentation scenarios. The mean one year runoff is not explicitly needed to estimate the variance of the one year runoff. Its estimation, however, does aid in determining what modifications to the data segmentation definitions would best reduce the overall estimated variance of the one year runoff, thus it is calculated for use as a diagnostic statistic.

For a given reserving segment *i* and accident year *j*, the mean and variance of the one year runoff, $E(R_{ij})$ and $Var(R_{ij})$ are calculated using the following formulas:

$$E(R_{ij}) = IL_{ijy} * (LDF_y - 1)$$

$$Var(R_{ij}) = (IL_{ijy})^2 * (Var(LDF_y - 1))$$

$$= (IL_{ijy})^2 * (Var(LDF_y))$$

where:

y is the evaluation age

$\mathbf{LDF_y}$ is the loss development factor from evaluation age y to evaluation age y + 1

IL_{ijy} is the incurred loss for reserving segment *i* for accident year *j* at evaluation age y

For reserving segment *i* in total (all years combined), the mean and variance of the one year runoff for a given , $E(S_i)$ and $Var(S_i)$ are calculated using the following formulas:

$$E(S_i) = \sum_{j=1}^{n} E(R_{ij})$$

$$Var(S_i) = \sum_{j=1}^{n} Var(R_{ij})$$

where **n** is the number of accident years included in the analysis.

It is assumed that loss processes for a given reserving segment and accident year are independent of each other.

For a data segmentation scenario consisting of *k* reserving segments, the mean and variance of the one year runoff, E(T) and Var(T) are calculated using the following formulas:

$$E(T) = \sum_{i=1}^{k} E(S_i)$$

$$Var(S_i) = \sum_{j=1}^{n} Var(R_{ij})$$

$$Var(T) = \sum_{i=1}^{k} Var(S_i) + 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} Cov(S_i, S_j)$$

If the loss processes for the reserving segments are independent, the covariance terms can be eliminated, and the calculation is simplified considerably.

The estimated variance of the one year runoff for each scenario is compared. The scenario with the smallest variance estimate is the most preferable segmentation of the data from a statistical standpoint assuming that the estimators are unbiased. Differences in the variance estimates between scenarios can be compared to judge the impact of alternative data segmentations on the estimated variance of the one year runoff.

The same conclusions with respect to data segmentation will be reached if the estimated standard deviations of the one year runoff are compared. Similar, but possibly not identical, conclusions will be made if the coefficients of variation are compared because the mean one year runoff for all scenarios will not be exactly equal.

Diagnostics and Further Analysis

Examining the components of the calculation of the mean and variance of the 1 year runoff can give good insight into which segments should possibly be subdivided to reduce the estimated 1 year runoff variance.

From a materiality standpoint, data segments with large variances should possibly be subdivided as they provide the greatest contribution to the estimated 1 year runoff variance. The variance contribution of a particular data segment, however, is influenced by the size of the segment, as well as the variance of the loss development factors.

From a homogeneity standpoint, data segments with large coefficients of variation should possibly be subdivided because these cells show the least homogeneity. The coefficient of variation is not influenced by the size of the segment.

EXAMPLE #1 - STANDARD COMMERCIAL GENERAL LIABILITY

Many possible segmentation variables exist for General Liability. Among them are:

1. Bodily Injury/Property Damage
2. Products/Non-Products
3. Account Size
4. State
5. Branch
6. Coverage Form (occurrence vs. Claims-made)
7. Monoline vs. Package
8. Industry Group
9. Corporate business unit responsible for the business
10. Claims office

In this particular example, the book of business is monoline occurrence. The segmentation variables chosen for analysis are Account Size (large and small) , Bodily Injury/Property Damage and Products/Non-Products.

Eight scenarios will be analyzed to focus on all possible combinations of the potential segmentation variables selected:

Scenario	Segment based on	Number of Reserving Segments
1	• No Segmentation	1
2	• Account Size	2
3	• Bodily Injury/Property Damage	2
4	• Products/Non-Products	2
5	• Account Size • Bodily Injury/Property Damage	4
6	• Account Size • Products/Non-Products	4
7	• Bodily Injury/Property Damage • Products/Non-Products	4
8	• Account Size • Bodily Injury/Property Damage • Products/Non-Products	8

Exhibit 2 shows the incurred loss triangles used in the analysis.

Exhibit 3 and 4 show the underlying data and the calculation of the one year runoff variance estimate for Scenario 1.

Exhibit 5 provides summary statistics for all eight scenarios sorted by the estimated variance of the one year runoff. The results indicate the following:

1. Data segmentation has a significant impact on the variability of reserving estimates. The estimated variance of the one year runoff was over three times higher for the segmentation producing the greatest variance than the segmentation producing the least variance.

2. More subdivisions of the reserving data do not necessarily produce more stable reserving estimates. In fact, segmenting the data by Bodily Injury/Property Damage, Products/Non-Products, and account size resulted in a variance equal to approximately 2.8 times the variance estimate obtained when all available data is combined into one reserving segment. Accompanying the increase in variance resulting from performing a more "refined" reserve analysis is the increase in actuarial resources needed to analyze eight reserving segments rather than one.

The larger variance estimates when the data is subdivided into smaller reserving segments are driven by the fact the age-to-age loss development factor variances for the eight individual reserving segments are much larger than the comparable age-to-age loss development factor variances that result when all reserving segments are combined. This is shown in **Exhibit 6**. As data is subdivided into more homogenous reserving segments, it is expected that age-to-age loss development factor variances will decrease due to the increase in homogeneity, and increase due to the decrease in volume potentially making development patterns less stable.

Considerations other than the statistical indications or efficient and effective use of actuarial resources can govern the data segmentation decisions. These considerations will be discussed later in this paper.

These results and conclusions, however, could change if the mix of business, volume, emergence patterns or development patterns of the reserving segments change over time. The remaining examples examine the impact that changes in the mix of business can have on the data segmentation indications.

EXAMPLES #2 AND #3 - STABLE MIX OF BUSINESS

In these examples, data is available for two reserving segments. Example #2 analyzes Workers Compensation. The reserving segments are two different states. The first state is longer tailed than the second. Example #3 analyzes General Liability with the reserving segments being Products and Non-Products.

For both examples, the issue is whether the reserving segments should be reviewed separately, or on a combined basis. The incurred losses at the end of the second development year for each reserving segment are equal to $100,000 for each reserving segment and accident year.

The incurred loss triangles are shown in **Exhibits 7 and 9**. The segmentation model results are summarized in **Exhibit 8** and indicate that combining the reserving segments results in a variance comparable to that obtained by analyzing the reserving segments separately. Results of this nature are to be expected for a stable book of business.

EXAMPLES #4 AND #5 - CHANGING MIX OF BUSINESS

In these examples, the development patterns used are the same as those in Examples #2 and #3. As in the prior examples, the incurred losses at the second evaluation of the first accident year were equal to $100,000. In subsequent accident years, however, the incurred losses increase by 5% annually for the General Liability Non-Products and Workers Compensation State #1 reserving segments (the longer-tailed state) and decrease 5% annually for the other reserving segments. Examples #4 and #5 analyze Workers Compensation and General Liability, respectively.

The incurred loss triangles are shown in **Exhibits 10 and 11**. The segmentation model results are summarized in **Exhibit 8** and indicate that separating the reserving segments lowers the variance by approximately 33% for Workers Compensation and approximately 88% for General Liability.

Data segmentation has a significant impact on the variance in reserving estimates in these examples. The indications with respect to whether to combine various reserving segments or not, however, can change as the mix of business changes.

ADDITIONAL CONSIDERATIONS IN DATA SEGMENTATION

Many additional considerations exist with respect to data segmentation. Among them are the following:

1. The ability to satisfy external reporting requirements.

   Schedule P has requirements with respect to reporting reserving data. If reserve analyses are not done at a Schedule P level of detail, or a finer level of detail than that required by Schedule P, a mechanism to report credible data on this basis would have to be developed.

2. The ability to impact or monitor pricing and operational strategy.

   Reserving actuaries often report loss ratio and profitability indications to business units or the pricing actuaries to aid in profitability monitoring and the development of operational strategy. If reserve analyses are not done at a meaningful level of detail, the potential value of the reserve analysis with respect to comparisons to pricing analyses and developing operational strategy could be greatly diminished.

3. The ability to quickly recognize and react to changes in business conditions.

   The ability to recognize, monitor and react to the impacts of changing business conditions can vary greatly depending on how the reserving data is segmented.

VARYING THE CRITICAL MODEL ASSUMPTIONS

The critical model assumptions, and potential revisions to the model if these assumptions are modified, are summarized below:

1. The appropriateness of using the estimate of the variance of the one year runoff rather than the total runoff to guide segmentation decisions

   The total runoff could be modeled by assuming that each future age-to-age loss development factor is equal to the mean of the age-to-age loss development factors (actual or projected) for all prior accident years. The difference in the indications regarding segmentation could be examined for materiality and an appropriate decision made on the proper form of the model to be used for future analyses.

   Use of a one year runoff to guide segmentation decisions has two inherent advantages:

   a) It lends itself more easily to a retrospective test of how well the selected segmentation worked relative to other alternatives.

   b) It is easier to model

2. The expected mean and variance of each future age-to-age loss development factor is equal to the sample means and variances of the past age-to-age loss development factors.

Operational changes within a given corporation can cause nonrandom movement in the magnitude of the age-to-age loss development factors over time. Examples of such changes would be processing changes, changes in available limits or changes in claims handling procedures.

In these cases, estimating future age-to-age loss development factors as the mean of prior age-to-age loss development factors is not reasonable. Possibilities to correct this include estimating the future age-to-age loss development factors as a weighted average of the prior loss development factors or, analytically or judgmentally adjusting prior loss development factors to mitigate the impacts of the operational changes. However, this could be a very difficult and time-consuming adjustment to make. Some of these types of adjustments are discussed in Berquist/Sherman [2]. The one year runoff variance estimations could then be performed using the adjusted age-to-age loss development factors in place of the unadjusted factors.

OTHER CONSIDERATIONS

The model utilizes incurred loss triangles and inherently assumes that variability in the reserve estimate will be comparable to the variability of the incurred loss runoff. In many cases, the incurred loss development method is not the method of choice for reserve analyses, and other methods with much less variability are used. This would make the model a poor tool for determining segmentation for these lines of business. An example would be excess coverage.

There is a potential for small volume cells to unduly influence the indications of the segmentation model. If no losses are reported at the first evaluation for a particular segment and accident year, the age-to-age loss development factor is undefined, the variance in the age-to-age loss development factors cannot be calculated unless the cell is excluded, and thus the model becomes inoperable.

If some accident years of a particular segment have small volume relative to other accident years, presumably the age-to-age loss development factors for the small volume cells would vary more from the true mean age-to-age loss development factor than the age-to-age loss development factors from the higher volume cells would. Giving all age-to-age loss development factors equal weight in the calculation of the variance of the loss development factors will result in the variance estimate of the age-to-age loss development factors being overstated for a rapidly growing book, and understated for a rapidly contracting book. This problem does not surface, however, if all accident years have comparable volume. This area deserves further analysis and investigation.

CONCLUSIONS

1. The variability of loss and allocated loss adjustment expense reserving estimates as required by Statement 55 resulting from the application of traditional loss development methods (paid loss or incurred loss including or excluding allocated expenses) can be measured using the data segmentation model to project the one year runoff.

The estimated variance of the one year runoff can be estimated for a number of reserving scenarios and the results of a number of scenarios compared. The scenario with the smallest variance estimate among the unbiased estimators is the most preferable segmentation of the data from a statistical standpoint. Differences in the variance estimates between scenarios can be compared to judge the impact of alternative data segmentations on the estimated variance of the one year runoff.

2. Data segmentation can have a significant impact on the variability of loss reserving estimates when the mix of business changes.

3. More subdivisions of the reserving data do not necessarily produce more stable reserving estimates.

4. Changes in the mix of business can impact the indications resulting from the segmentation model.

5. The segmentation model as developed, along with sound business judgment, should serve well to guide the actuary's decision of how to segment their data for reserving purposes. Using the techniques as developed in this paper, the most efficient estimate available through data segmentation and the use of standard development techniques can be defined and will assist the actuary in projecting the best estimate reserves.

BIBLIOGRAPHY

1. Stanard, James N., "A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques", PCAS LXXII, 1985

2. Berquist, J.R.; and Sherman, R.E., "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach," PCAS LXIV, 1977

EXHIBIT 1 - DATA SEGMENTATION STUDY-

ALL POSSIBLE SUBSET LISTING FOR FOUR SEGMENTS

# of Reserving Segments	**Scenario #**	**Groups included in Reserving Segment #1**	**Groups included in Reserving Segment #2**	**Groups included in Reserving Segment #3**	**Groups included in Reserving Segment #4**
1	1	1,2,3,4			
2	2	1,2,3	4		
	3	1,2,4	3		
	4	1,3,4	2		
	5	2,3,4	1		
	6	1,2	3,4		
	7	1,3	2,4		
	8	1,4	2,3		
3	9	1,2	3	4	
	10	1,3	2	4	
	11	1,4	2	3	
	12	2,3	1	4	
	13	2,4	1	3	
	14	3,4	1	2	
4	15	1	2	3	4

EXHIBIT 2 - DATA SEGMENTATION STUDY - HYPOTHETICAL DATA -

INCURRED LOSS DEVELOPMENT TRIANGLES (000'S)

Large Risks, Products, Bodily Injury

Development Year \ Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	2	114	399	528	200	348	180	228	511	233	281	457	422	656
2	3	13	606	613	1294	1480	1713	529	764	876	371	1219	694	1662	
3	31	158	765	2034	5943	2339	6028	1841	1107	1450	826	1358	988		
4	253	394	913	2304	6367	2721	6708	2919	1477	1816	1378	2585			
5	488	944	1433	2540	6354	4822	7519	2920	3841	2831	1475				
6	2453	2434	1597	2313	6648	4607	6032	3932	3635	3251					
7	3813	2611	1609	2109	6818	4600	5783	3437	3323						
8	4018	2563	1959	2032	6719	4728	5484	3528							
9	3961	2645	1952	2228	6431	4702	5774								
10	4072	2711	1952	2307	6460	4740									
11	4093	3062	2151	2280	6619										
12	4117	2670	2149	2485											
13	4115	2673	2137												
14	4115	2760													
15	4203														

EXHIBIT 2 (CONTINUED)

Large Risks, Nonproducts, Bodily Injury

Development Year \ Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2091	2783	2371	1343	2441	2164	1093	649	670	4959	4369	3147	3338	4487	6212
2	4452	7170	4085	2766	3856	4735	2481	1738	1588	10580	9618	6760	6450	8005	
3	6257	8280	5160	3912	5078	7570	4046	2403	2331	13993	13551	8975	9020		
4	7287	9182	6457	4945	6422	10047	5136	2943	3454	18578	15180	12722			
5	8756	9263	7071	4941	7127	11345	6003	5435	4462	19487	16299				
6	9435	9598	7167	5015	7802	11271	6206	5069	4501	20387					
7	9548	10387	7970	4828	7497	11773	6894	5779	4379						
8	9369	8564	7541	5198	8051	11703	6976	5680							
9	9254	8688	7607	5065	8089	11681	7072								
10	9358	8227	7614	5137	8371	11775									
11	9465	8227	7722	5147	8458										
12	9827	7872	7803	5136											
13	9452	8084	8096												
14	9518	8161													
15	9679														

EXHIBIT 2 (CONTINUED)

Large Risks, Products, Property Damage

Development Year \ Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	2	7	83	167	400	170	146	79	124	117	275	47	142	207
2	2	3	28	693	299	587	375	214	160	164	141	162	92	146	
3	3	21	56	813	2701	723	374	321	158	506	264	361	291		
4	31	205	82	810	2780	482	774	233	146	541	294	574			
5	231	272	96	1072	2665	477	690	205	801	525	505				
6	300	989	85	1463	2659	424	924	205	812	728					
7	903	1064	55	1296	2585	371	3732	216	107						
8	985	1105	55	1293	2618	371	3736	216							
9	1047	1070	55	1391	2661	371	3744								
10	1004	1051	55	1407	2657	371									
11	984	1080	55	1373	2828										
12	1017	1096	55	1373											
13	1032	1088	55												
14	1023	1140													
15	1065														

EXHIBIT 2 (CONTINUED)

Large Risks, Nonproducts, Property Damage

Development Year \ Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	393	446	429	353	601	436	328	138	283	385	691	690	664	740	277
2	648	584	614	682	1280	485	434	268	326	544	699	953	915	659	
3	1017	597	862	698	1438	591	433	450	374	785	647	1267	1034		
4	910	747	890	712	2048	702	452	499	521	781	657	1303			
5	980	740	863	716	2097	863	1031	4542	842	1115	667				
6	1100	835	861	702	1964	860	1066	6540	940	1120					
7	1079	791	867	734	1953	867	1083	6588	1115						
8	1012	951	1149	806	2048	1385	1099	6609							
9	1002	882	1170	785	2043	1556	1245								
10	1006	900	1223	797	2038	982									
11	1005	692	1220	784	2037										
12	1005	601	1220	775											
13	1017	601	1219												
14	1021	595													
15	1025														

EXHIBIT 2 (CONTINUED)

Small Risks, Products, Bodily Injury

Development Year \ Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	2	86	238	181	335	638	672	838	935	839	730	449	405	656
2	2	10	151	479	424	1413	1200	1005	1733	2339	1973	1630	1213	1871	
3	23	65	148	1023	811	1740	2319	1547	3126	3332	4087	2609	3542		
4	98	167	139	1150	1026	2047	2822	2118	4138	4752	6925	4344			
5	202	347	209	1113	1262	2700	3081	2319	6343	5595	7417				
6	397	496	215	1322	1320	2665	3649	2694	6142	5555					
7	543	553	231	1458	1599	2959	3947	2699	6136						
8	589	667	397	1548	1999	3472	4252	2771							
9	704	729	403	1894	2100	3265	4428								
10	794	831	418	1995	1951	3205									
11	888	1022	350	1624	1988										
12	1078	1065	350	1902											
13	1112	946	398												
14	981	1018													
15	1072														

EXHIBIT 2 (CONTINUED)

Small Risks, Nonproducts, Bodily Injury

Development Year	Accident Year 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1774	1425	1563	1935	2007	1836	2469	2010	2283	3690	4698	5973	6226	5637	5578
2	3173	2959	3445	3616	4441	4045	4758	3191	5150	8426	10113	13035	12111	14875	
3	7145	5078	5385	5519	6379	6716	6394	5228	7733	14705	16514	20865	23781		
4	5664	6253	6852	6791	7741	8791	7842	6787	12241	18506	20402	29431			
5	6738	9954	7617	7631	7813	9279	9522	7541	15708	21052	22301				
6	7014	6110	7926	7762	8173	9892	10995	9044	16239	24140					
7	7107	6441	7884	9479	8041	10034	11286	9492	18511						
8	7637	6498	8124	9621	8622	10550	11611	10464							
9	7516	6698	8157	10524	8238	10396	11651								
10	7671	6804	8197	10528	9025	10421									
11	8262	6878	8667	10655	9410										
12	8281	6994	8836	10664											
13	8290	6973	8673												
14	8315	6820													
15	8585														

EXHIBIT 2 (CONTINUED)

Small Risks, Products, Property Damage

	Accident Year														
Development Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	2	2	27	77	216	241	442	404	463	282	611	432	925	236	687
2	2	4	76	240	289	346	621	924	550	479	1282	781	1389	865	
3	8	27	40	488	412	351	667	1045	774	718	1521	959	1785		
4	40	118	36	663	486	386	856	1237	1144	1069	2067	1392			
5	135	189	106	951	585	682	1269	1182	1185	1617	2381				
6	204	268	103	1014	708	702	1304	1155	1186	1321					
7	296	348	262	969	767	790	1982	1184	1159						
8	386	406	224	955	790	759	2061	1498							
9	466	430	261	969	796	734	2010								
10	476	449	259	969	768	724									
11	496	458	260	969	772										
12	505	460	249	969											
13	507	454	277												
14	497	454													
15	505														

EXHIBIT 2 (CONTINUED)

Small Risks, Nonproducts, Property Damage

Development Year \ Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	919	838	1410	1366	3315	1683	1540	1680	2047	1999	2615	3423	4587	4298	4829
2	1283	1434	2240	1925	3841	2182	2529	2316	2656	3388	3858	5424	7520	6299	
3	1279	1719	2601	1965	3651	2617	2849	3101	2752	4804	4382	7050	7517		
4	1541	1669	2897	2260	3967	2667	3003	3355	2902	6067	4834	6841			
5	1804	1731	2936	2287	3965	2642	3110	3903	3145	6140	5329				
6	1789	1872	3014	2408	4056	2734	3259	4677	3334	6130					
7	1820	1969	2895	2293	4016	2458	4328	5033	3347						
8	1901	2177	2958	2431	4066	2635	4278	4887							
9	2058	2322	3045	2443	4193	2707	4366								
10	2052	2402	3087	2681	4191	2730									
11	2090	2495	3305	3213	4201										
12	2090	2558	3264	2993											
13	2165	2668	3266												
14	2153	2710													
15	2186														

EXHIBIT 3 - DATA SEGMENTATION STUDY - HYPOTHETICAL DATA -

INCURRED LOSS DEVELOPMENT TRIANGLES (000'S)

Scenario 1 - Combine All Available Data

Development Year	Accident Year 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	5185	5500	6007	5794	9456	7295	7028	5879	6891	12885	14173	14951	16693	16367	19102
2	9565	12177	11245	11014	15724	15273	14111	10185	12927	26796	28055	29964	30384	34382	
3	15763	15945	15017	16452	26413	22647	23110	15936	18355	40293	41792	43444	47958		
4	15824	18735	18266	19635	30837	27843	27593	20091	26023	52110	51737	59192			
5	19334	23440	20331	21251	31868	32810	32225	28047	36327	58362	56374				
6	22692	22602	20968	21999	33330	33155	33435	33316	36789	62632					
7	25109	24164	21773	23166	33276	33852	39035	34428	38077						
8	25897	22931	22407	23884	34913	35603	39497	35653							
9	26008	23464	22650	25299	34551	35412	40290								
10	26433	23375	22805	25821	35461	34948									
11	27283	23914	23730	26045	36313										
12	27920	23316	23926	26297											
13	27690	23487	24121												
14	27623	23658													
15	2832														

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EXHIBIT 3 (CONTINUED)

Indicated Age to Age Loss Development Factors

	Accident Year													
Age -to- Age	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1 - 2	1.8447	2.2140	1.8720	1.9009	1.6629	2.0936	2.0078	1.7324	1.8759	2.0796	1.9795	2.0041	1.8202	2.1007
2 - 3	1.6480	1.3094	1.3354	1.4937	1.6798	1.4828	1.6377	1.5647	1.4199	1.5037	1.4896	1.4499	1.5784	
3 - 4	1.0039	1.1750	1.2164	1.1935	1.1675	1.2294	1.1940	1.2607	1.4178	1.2933	1.2380	1.3625		
4 - 5	1.2218	1.2511	1.1131	1.0823	1.0334	1.1784	1.1679	1.3960	1.3960	1.1200	1.0896			
5 - 6	1.1737	0.9642	1.0313	1.0352	1.0459	1.0105	1.0375	1.1879	1.0127	1.0732				
6 - 7	1.1065	1.0691	1.0384	1.0530	0.9984	1.0210	1.1675	1.0334	1.0350					
7 - 8	1.0314	0.9490	1.0291	1.0310	1.0492	1.0517	1.0118	1.0356						
8 - 9	1.0043	1.0232	1.0108	1.0592	0.9896	0.9946	1.0201							
9 - 10	1.0163	0.9962	1.0068	1.0206	1.0263	0.9869								
10 - 11	1.0322	1.0231	1.0406	1.0087	1.0240									
11 - 12	1.0233	0.9750	1.0083	1.0097										
12 - 13	0.9918	1.0073	1.0082											
13 - 14	0.9976	1.0073												
14 - 15	1.0252													

EXHIBIT 4 - DATA SEGMENTATION STUDY- HYPOTHETICAL DATA -

ESTIMATION OF MEAN, VARIANCE AND COEFFICIENT OF VARIATION

FOR SCENARIO 1

Accident Year	Incurred Loss Current Diagonal	Squared Incurred Loss	Mean Age-to-Age Loss Development Factor	Mean Age-to-Age Loss Development Factor - 1.0	Estimated Mean 1-Year Runoff	Variance of Age-to-Age Loss Development Factor	Estimated 1-Year Runoff Variance
1	28,320	802,022,400	1.0000	0.0000	0	0.0000	0
2	23,658	559,700,964	1.0252	0.0252	597	0.0000	0
3	24,121	581,822,641	1.0024	0.0024	59	0.0000	13,687
4	26,297	691,532,209	1.0024	0.0024	64	0.0001	39,318
5	36,313	1,318,633,969	1.0041	0.0041	148	0.0003	417,361
6	34,948	1,221,362,704	1.0257	0.0257	898	0.0001	137,323
7	40,290	1,623,284,100	1.0089	0.0089	358	0.0002	310,206
8	35,653	1,271,136,409	1.0146	0.0146	519	0.0005	588,419
9	38,077	1,449,857,929	1.0236	0.0236	899	0.0009	1,348,885
10	62,632	3,922,767,424	1.0580	0.0580	3,635	0.0023	9,122,836
11	56,374	3,178,027,876	1.0572	0.0572	3,225	0.0045	14,416,117
12	59,192	3,503,692,864	1.1863	0.1863	11,029	0.0134	46,850,774
13	47,958	2,299,969,764	1.2293	0.2293	10,998	0.0099	22,750,761
14	34,382	1,182,121,924	1.5072	0.5072	17,437	0.0120	14,152,134
15	19,102	364,886,404	1.9420	0.9420	17,995	0.0220	8,022,048
Totals					67,859		118,169,870

EXHIBIT 4 (CONTINUED)

Summary Statistics

Mean	Variance	Standard Deviation	Coefficient of Variation
67,859	118,169,870	10,871	0.1602

EXHIBIT 5 - DATA SEGMENTATION STUDY - HYPOTHETICAL DATA -

SCENARIO VARIANCE SUMMARY

Scenario	Segment Based on	Estimated 1 year Runoff Variance	Estimated 1 year Runoff Standard Deviation	Estimated 1 year Runoff Coefficient of Variation	# of Reserving Segments
3	• Bodily Injury/Property Damage	107,911,751	10,388	0.1517	2
1	• Combine all available data	118,169,869	10,871	0.1602	1
5	• Account Size • Bodily Injury/Property Damage	121,589,647	11,027	0.1569	4
2	• Account Size	130,938,426	11,443	0.1665	2
6	• Account Size • Products/Non-Products	296,161,860	17,209	0.2128	4
4	• Products/Non-Products	330,180,642	18,171	0.2231	2
8	• Account Size • Bodily Injury/Property Damage • Products/Non-Products	332,828,739	18,244	0.2158	8
7	• Bodily Injury/Property Damage • Products/Non-Products	365,441,708	19,117	0.2271	4

EXHIBIT 6 - DATA SEGMENTATION STUDY -

AGE-TO-AGE LOSS DEVELOPMENT FACTOR VARIANCES

Age-to-Age	Large Risks				Small Risks				All Segments Combined
	Products	Non-Products	Products	Non-Products	Products	Non-Products	Products	Non-Products	
	Bodily Injury		Property Damage		Bodily Injury		Property Damage		
1 - 2	3.6052	0.0870	3.5706	0.1234	1.3124	0.0614	0.5638	0.0262	0.0220
2 - 3	11.5551	0.0149	5.6559	0.0510	7.9607	0.0508	2.6047	0.0193	0.0120
3 - 4	3.6042	0.0113	11.1096	0.0248	0.7697	0.0296	1.6554	0.0070	0.0099
4 - 5	0.2940	0.0512	4.4616	5.2206	0.1351	0.0225	0.5508	0.0038	0.0134
5 - 6	1.5463	0.0018	0.6009	0.0187	0.0835	0.0233	0.0415	0.0031	0.0045
6 - 7	0.0372	0.0044	1.4087	0.0040	0.0114	0.0053	0.2206	0.0135	0.0023
7 - 8	0.0064	0.0055	0.0009	0.0414	0.0437	0.0009	0.0212	0.0018	0.0009
8 - 9	0.0019	0.0002	0.0012	0.0051	0.0086	0.0017	0.0076	0.0006	0.0005
9 - 10	0.0002	0.0007	0.0003	0.0209	0.0056	0.0011	0.0006	0.0012	0.0002
10 - 11	0.0031	0.0000	0.0011	0.0082	0.0256	0.0007	0.0002	0.0049	0.0001
11 - 12	0.0061	0.0009	0.0002	0.0031	0.0078	0.0001	0.0005	0.0012	0.0003
12 - 13	0.0000	0.0011	0.0001	0.0000	0.0104	0.0001	0.0031	0.0003	0.0001
13 - 14	0.0003	0.0000	0.0008	0.0000	0.0094	0.0002	0.0001	0.0001	0.0000
14 - 15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

EXHIBIT 7 - DATA SEGMENTATION STUDY - HYPOTHETICAL DATA -

INCURRED LOSS DEVELOPMENT TRIANGLES (000'S)

Workers Compensation, State #1, Stable Mix of Business

Development Year	Accident Year 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	64	63	66	65	63	65	64	64	63	63	62	64	64	62	62
2	100	100	100	100	100	100	100	100	100	100	100	100	100	100	
3	126	122	127	124	126	130	126	123	124	125	124	125	125		
4	147	137	147	145	144	152	148	139	146	145	143	142			
5	168	150	166	159	157	168	165	151	160	159	160				
6	179	160	179	171	168	182	175	164	171	171					
7	189	165	189	180	177	190	184	171	182						
8	195	171	195	186	184	197	191	177							
9	201	175	200	191	189	202	196								
10	205	179	204	194	193	206									
11	208	181	206	197	195										
12	210	182	208	199											
13	211	184	210												
14	212	185													
15	213														

EXHIBIT 7 (CONTINUED)

Workers Compensation, State #2, Stable Mix of Business

	Accident Year														
Development Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	79	77	77	79	77	76	78	79	76	75	77	75	77	77	77
2	100	100	100	100	100	100	100	100	100	100	100	100	100	100	
3	108	111	108	107	105	108	109	106	110	108	109	108	108		
4	112	115	109	113	107	113	113	110	114	113	116	109			
5	115	117	113	120	108	117	117	112	115	117	117				
6	117	118	114	122	109	120	121	114	119	120					
7	117	119	116	120	111	121	123	115	119						
8	118	119	117	122	111	122	125	116							
9	119	119	117	122	111	122	125								
10	119	120	118	122	112	123									
11	120	120	118	122	112										
12	120	120	118	122											
13	120	120	118												
14	120	121													
15	120														

EXHIBIT 8 - DATA SEGMENTATION STUDY - HYPOTHETICAL DATA -

SCENARIO VARIANCE SUMMARY

Example #	Line of Business	Mix of Business	Description	Estimated 1 year Runoff Variance	1 year Runoff Standard Deviation	1 year Runoff Coefficient of Variation
2	Workers Compensation	Stable	Combine reserving segments	37.5	6.1	0.0337
			Review reserving segments separately	36.7	6.1	0.0334
3	General Liability	Stable	Combine reserving segments	522.5	22.9	0.0108
			Review reserving segments separately	487.4	22.1	0.0104
4	Workers Compensation	Changing	Combine reserving segments	111.1	10.5	0.0444
			Review reserving segments separately	74.3	8.6	0.0328
5	General Liability	Changing	Combine reserving segments	2,532.5	50.3	0.0273
			Review reserving segments separately	311.8	17.7	0.0108

EXHIBIT 9 - DATA SEGMENTATION STUDY - HYPOTHETICAL DATA -

INCURRED LOSS DEVELOPMENT TRIANGLES (000'S)

General Liability, Non-Products, Stable Mix of Business

	Accident Year														
Development Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	46	45	46	45	46	47	45	45	47	46	46	47	47	48	48
2	100	100	100	100	100	100	100	100	100	100	100	100	100	100	
3	151	155	160	156	152	158	156	157	157	159	156	152	155		
4	192	195	198	191	181	193	194	195	190	197	181	183			
5	210	213	220	210	198	212	214	216	212	218	202				
6	224	231	239	227	212	233	230	232	231	236					
7	240	242	254	240	223	248	243	245	243						
8	251	254	265	251	234	259	256	256							
9	260	262	273	260	242	268	265								
10	267	269	280	266	249	275									
11	272	274	286	271	253										
12	276	278	291	275											
13	279	282	294												
14	281	284													
15	283														

EXHIBIT 9 (CONTINUED)

General Liability, Products, Stable Mix of Business

	Accident Year														
Development Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	18	17	17	17	18	18	18	18	18	17	18	17	18	18	18
2	100	100	100	100	100	100	100	100	100	100	100	100	100	100	
3	278	270	276	286	282	289	287	284	288	282	278	281	279		
4	513	485	483	522	531	508	517	518	512	490	499	489			
5	697	675	674	734	730	712	719	708	698	672	699				
6	887	874	864	943	948	933	931	894	905	865					
7	1078	1065	1046	1154	1142	1132	1131	1093	1097						
8	1248	1232	1212	1342	1329	1313	1315	1261							
9	1404	1383	1363	1503	1489	1465	1479								
10	1536	1507	1484	1633	1623	1601									
11	1642	1615	1579	1741	1735										
12	1728	1695	1659	1833											
13	1795	1762	1722												
14	1846	1811													
15	1885														

EXHIBIT 10 - DATA SEGMENTATION STUDY - HYPOTHETICAL DATA -

INCURRED LOSS DEVELOPMENT TRIANGLES (000'S)

Workers Compensation, State #1, Changing Mix of Business

Development Year	Accident Year 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	64	66	72	75	77	83	85	90	93	98	101	109	115	117	122
2	100	105	110	116	122	128	134	141	148	155	163	171	180	189	
3	126	128	140	143	153	165	168	173	184	194	202	215	225		
4	147	144	162	168	175	194	198	196	215	225	233	242			
5	168	157	183	184	191	214	221	213	236	246	261				
6	179	168	197	198	204	232	235	231	252	265					
7	189	173	208	208	215	243	246	240	269						
8	195	180	215	215	224	251	256	249							
9	201	184	220	221	230	258	263								
10	205	188	225	225	234	263									
11	208	190	228	228	237										
12	210	192	230	230											
13	211	193	231												
14	212	194													
15	213														

EXHIBIT 10 (CONTINUED)

Workers Compensation, State #2, Changing Mix of Business

Development Year \ Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	79	73	69	68	62	59	57	55	50	48	46	43	42	40	38
2	100	95	90	86	81	77	74	70	66	63	60	57	54	51	
3	108	105	97	92	85	84	80	74	73	68	65	61	58		
4	112	110	99	97	87	87	83	77	75	71	70	62			
5	115	111	102	103	88	91	86	78	76	74	70				
6	117	112	103	104	89	93	89	80	79	76					
7	117	113	105	103	90	94	91	80	79						
8	118	113	106	104	90	94	92	81							
9	119	113	106	104	91	95	92								
10	119	114	106	105	91	95									
11	120	114	107	105	91										
12	120	114	107	105											
13	120	114	107												
14	120	115													
15	120														

EXHIBIT 11 - DATA SEGMENTATION STUDY - HYPOTHETICAL DATA -

INCURRED LOSS DEVELOPMENT TRIANGLES (000'S)

General Liability, Non-Products, Changing Mix of Business

Development Year \ Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	46	47	51	52	55	60	60	64	69	71	75	80	84	91	95
2	100	105	110	116	122	128	134	141	148	155	163	171	180	189	
3	151	163	177	180	185	201	209	221	232	247	255	260	279		
4	192	205	218	222	220	247	260	275	281	306	295	313			
5	210	223	243	243	241	271	286	304	313	337	328				
6	224	242	263	263	258	297	308	326	341	366					
7	240	255	280	278	271	316	326	345	359						
8	251	267	292	290	284	331	342	360							
9	260	276	301	301	294	342	355								
10	267	282	309	308	302	351									
11	272	288	315	313	308										
12	276	292	321	319											
13	279	296	324												
14	281	298													
15	283														

EXHIBIT 11 (CONTINUED)

General Liability, Products, Changing Mix of Business

Development Year \ Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	18	17	16	15	14	14	13	12	12	11	10	10	10	9	9
2	100	95	90	86	81	77	74	70	66	63	60	57	54	51	
3	278	256	249	245	230	224	211	198	191	178	166	160	151		
4	513	461	436	447	433	393	380	362	340	309	299	278			
5	697	641	608	629	595	551	528	494	463	424	419				
6	887	831	780	809	772	722	684	624	600	545					
7	1078	1012	944	990	930	876	832	764	728						
8	1248	1171	1094	1151	1083	1016	967	880							
9	1404	1314	1230	1289	1213	1134	1087								
10	1536	1432	1339	1400	1322	1239									
11	1642	1534	1425	1493	1413										
12	1728	1610	1498	1571											
13	1795	1674	1554												
14	1846	1720													
15	1885														