A Suggestion for a Modified Form of Amortization with a Brief Memorandum of the Applicability of that Principle to the Bonds of Miscellaneous Companies.

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As long as bonds are bought above or below par the properly managed insurance company must have recourse to some form of amortization or accumulation. Unfortunately the present method, although it is the one used in text books and prescribed by the laws of some of the states (see § 18 of the Insurance Law of the State of New York), is based upon hypotheses which never exist and in consequence produce theoretical results only. I am sure that this thought must have occurred many times to those who have considered the question of investment accounts, and the object of this paper is merely to suggest a form of relief for insurance companies.

To demonstrate the incorrectness of the present method, let me briefly review the operations now followed in preparing a schedule of amortization for a bond bought above par.

After examining the bond to determine its nominal rate of interest (i) and the number of years it will have to run to maturity, we attempt to find the effective interest rate (i') by locating its purchase price in a book of bond values; this book is constructed as follows:

TABLE I.

	i.		
	First Period.	Second Period.	Third Period.
i'	$P+\frac{Pi-Pi'}{1+i'}$	$P+\frac{Pi-Pi'}{1+i'}+\frac{Pi-Pi'}{(1+i')^2}$	$P+\frac{Pi-Pi'}{1+i'}+\frac{Pi-Pi'}{(1+i')^2}+\frac{Pi-Pi'}{(1+i')^3}$

In which P is the principal of the bond.

This is the mathematical basis of a bond value book, part of a a typical page of which will be recognized in the following form:

VALUES, TO THE NEAREST CENT, OF A BOND FOR $1,000,000 AT 4 PER CENT. INTEREST, PAYABLE SEMI-ANNUALLY.

Net Income.	½ Year.	1 Year.	1½ Years.	2 Years.	2½ Years.
2.50	$1,007,407.41	$1,014,723.37	$1,021,949.00	$1,029,085.43	$1,036,133.76

After locating the proper nominal rate at the top of the page we enter the column corresponding to the length of time the bond has to run; when we find the purchase price, we will have obtained the effective interest rate, which will be the figure appearing in the column at the extreme left of the page. In actual practice the method is not quite so simple for two reasons:

(*a*) Unless the bond is bought on a coupon date its maturity period will not be a multiple of six months.

(*b*) It is unusual to have the purchase price agree exactly with the table.

It is necessary, therefore, to make an approximation to allow for these variations from the normal. This having been done a schedule is prepared upon the assumption that the investor will receive only the effective rate (i') upon the investment and the balance of each payment received from the debtor is to be used to extinguish the premium. Indicating the amortization factor by A, we have

$$A = Pi - Pi'.$$

In other words, the wiping out of the premium will require us to use some part of each coupon payment, the amount depending upon three factors—the principal, the nominal rate and the effective rate—the first two of which may be termed fixed quantities as they are unaffected by the price at which the security is bought. The value of i', however, fluctuates inversely to the premium, for as the latter becomes greater i' becomes smaller.

Let us now consider three lots of bonds all purchased at the same time, having the same par value—$1,000,000—and bearing 4 per cent., 4½ per cent. and 5 per cent. respectively, interest payable semi-annually and all maturing in ten years. The purchase price and the consequent effective interest rates are shown in the following schedule:

TABLE II.

Lot.	Purchase Price.	Effective Interest Rate.
1	$1,131,944.87	2.50%
2	1,128,764.79	3.00%
3	1,125,646.61	3.50%

On each coupon date $20,000 will be clipped from lot 1, $22,500 from lot 2 and $25,000 from lot 3, which amounts will be divided into two parts, one being carried to the interest earnings and the other set aside to accumulate at its own particular i' rate so that at the maturity date it will extinguish part of the premium. This division will be as follows:

TABLE III.

(1) Lot.	(2) Interest Factor.	(3) Amortization Factor.	(4) Effective Interest Rate
1	$14,149.32	$5,850.68	2.50%
2	16,931.47	5,568.53	3.00%
3	19,698.81	5,301.19	3.50%

It must be borne in mind that all of the amounts shown in column (3) are received at the same moment, that they must be reinvested the instant they are received and the amount from each lot must be used to purchase securities which will yield its effective rate shown in column (4). If all of those conditions are not complied with, the premiums on the different lots will not be extinguished scientifically.

In actual practice it is impossible to reinvest immediately and it is absurd to consider that each different instalment of A will be so invested as to produce a different i'. Both are purely theoretical assumptions.

Why should we not assume a condition which will harmonize with the actual situation and take cognizance of the fact that all of the A items received at the same time will be invested to earn the same rate of interest, irrespective of their origin. The present method does not give correct results and requires the preparation of a special amortization schedule for each lot of bonds purchased. (The present conditions of the bond market whereby good securities earning high rates of interest are selling below par, are materially increasing the difficulty in the preparation of the schedules.) Why not adopt some method which will involve no more serious error and which will permit the use of a standard table applicable to all bond issues without regard to their nominal interest rates and dealing only with the purchase prices. That would be accomplished if we extinguished the premium of a bond by means of an annuity certain, the rate of which (i_x) could be agreed upon and the rent of which (r) would be a substitute for A and therefore

$$Pi' = Pi - r.$$

The plan is feasible if we can assign a proper value to i_x. If a company's average interest rate remain constant year after year, the solution would be simple, but even in the best managed companies there is an annual variation due to change in the supply of investment capital in the money market or a change in the form of securities purchased or a change in some of the many other factors entering into the management of the investment department. However, by taking the average interest rate earned during the past five years, conservatively adjusted with an arbitrary factor to allow for the rising or falling tendency, we can obtain i_x sufficiently correct for all practical purposes.

After fixing upon i_x the next step would be to construct a table of the amount of an annuity of 1 for the various n periods by using the formula

$$\frac{(1+i_x)^n - 1}{i_x}.$$

Let us assume an average annual earning power of $4\frac{1}{2}$ per cent. A specimen table constructed by means of the formula just given will be

TABLE IV.

AMOUNT OF AN ANNUITY CERTAIN FOR VARIOUS PERIODS AT $4\frac{1}{2}$ PER CENT. PER ANNUM, PAYABLE SEMI-ANNUALLY, SEMI-ANNUAL RENT $1.

Period.		Period.		Period.		Period.	
½ yr.	1.	13 yrs.	34.81731628	25½ yrs.	93.79966416	38 yrs.	196.67350941
1 "	2.0225	13½ "	36.60070590	26 "	96.91015661	38½ "	202.09866337
1½ yrs.	3.06800625	14 "	38.42422178	26½ "	100.09063513	39 "	207.64588329
2 "	4.13703639	14½ "	40.28876677	27 "	103.34267442	39½ "	213.31791567
2½ "	5.23011971	15 "	42.19526402	27½ "	106.66788460	40 "	219.11756877
3 "	6.34779740	15½ "	44.14465746	28 "	110.06791200	40½ "	225.04771407
3½ "	7.49062284	16 "	46.13791226	28½ "	113.54444002	41 "	231.11128763
4 "	8.65916186	16½ "	48.17601528	29 "	117.09918992	41½ "	237.31129160
4½ "	9.85399300	17 "	50.25997563	29½ "	120.73392169	42 "	243.65079560
5 "	11.07570784	17½ "	52.39082508	30 "	124.45043493	42½ "	250.13293857
5½ "	12.32491127	18 "	54.56961864	30½ "	128.25056972	43 "	256.76092967
6 "	13.60222177	18½ "	56.79743506	31 "	132.13620754	43½ "	263.53805069
6½ "	14.90827176	19 "	59.07537735	31½ "	136.10927221	44 "	270.46765674
7 "	16.24370788	19½ "	61.40457334	32 "	140.17173083	44½ "	277.55317902
7½ "	17.60919130	20 "	63.78617624	32½ "	144.32559477	45 "	284.79812555
8 "	19.00539811	20½ "	66.22136521	33 "	148.57292066	45½ "	293.20608337
8½ "	20.43301957	21 "	68.71134592	33½ "	152.91581137	46 "	299.78072025
9 "	21.89276251	21½ "	71.25735121	34 "	157.35641713	46½ "	307.52578645
9½ "	23.38534966	22 "	73.86064161	34½ "	161.89693651	47 "	315.44511665
10 "	24.91152003	22½ "	76.52250605	35 "	166.53961758	47½ "	323.54263177
10½ "	26.47202923	23 "	79.24426243	35½ "	171.28675898	48 "	331.82234099
11 "	28.06764989	23½ "	82.02725834	36 "	176.14071106	48½ "	340.28834366
11½ "	29.69917201	24 "	84.87287165	36½ "	181.10387705	49 "	348.94483139
12 "	31.36740338	24½ "	87.78251126	37 "	186.17871429	49½ "	357.79609010
12½ "	33.07316996	25 "	90.75761776	37½ "	191.36773536	50 "	366.84650213

The foregoing table and similar ones for the various interest rates which will be required have been worked out and may be found in standard text-books.

It should be borne in mind that "4½ per cent." in the heading of the table is the i_x we will use to extinguish the premium. Its application to the three lots of bonds previously referred to is as follows:

Lot 1—Premium to be extinguished	$131,944.87
Amount of semi-annual rent of $1 from table (20 periods)	24.91152003
Semi-annual rent of an annuity whose amount is $131,944.87	5,296.54
Therefore the semi-annual coupon payments of $20,000 will be divided into	
Interest factor	14,703.46
Amortization factor	5,296.54
Lot 2—Premium to be extinguished	$128,764.79
Amount of semi-annual rent of $1 from table (20 periods)	24.91152003
Semi-annual rent of an annuity whose amount is $128,764.79	5,168.88
Therefore the semi-annual coupon payments of $22,500 will be divided into	
Interest factor	17,331.12
Amortization factor	5,168.88
Lot 3—Premium to be extinguished	$125,646.61
Amount of semi-annual rent of $1 from table (20 periods)	24.91152003
Semi-annual rent of an annuity whose amount is $125,646.61	5,043.71
Therefore the semi-annual coupon payments of $25,000 will be divided into	
Interest factor	19,956.29
Amortization factor	5,043.71

It will be noted in the above that we have used the same factor for the amount of the annuity—24.91152003—for all of the lots, notwithstanding that the bonds bore different interest rates. A comparison of the results of the present method of amortization and those obtained by the use of the proposed method is shown in the following table:

TABLE V.

Lot.	Semi-annual Interest Earnings.		Semi-annual Amortization Factor.	
	Present Plan.	Proposed Plan.	Present Plan.	Proposed Plan.
1	$14,149.32	$14,703.46	$5,850.68	$5,296.54
2	16,931.47	17,331.12	5,568.53	5,168.88
3	19,698.81	19,956.29	5,301.19	5,043.71

Instead of using, therefore, a bond value book with values applicable only to certain interest rates, we need have only one table which will answer for all rates and which will give more accurate results as they will be logical and based upon assumptions representing the actual investment conditions.

It is unnecessary to burden this suggestion with any examples of bonds purchased below par for the same method will be followed, but it would appear to me that the proposed method will be more nearly correct in that the discount factor will be accumulated not at an artificially high rate of interest caused by unusual conditions, but at a rate which more nearly conforms to actual conditions.

It will be apparent that this suggestion will not affect the actual interest earnings of the company, but since it deals with a readjustment in the distribution as between effective interest factors and amortization (or accumulation) factors, it will have a bearing upon the rate used in dividend assumptions and similar calculations.

To summarize: The advantages of the proposed method are

First—It requires the use of only one table, which is applicable to all bonds, irrespective of their nominal interest rates.

Second—It overcomes the difficulties due to the inability of finding bond tables for high nominal rates at prices below par. Some of the earlier publications of bond tables did give such values, but used only two or three decimal places, the resulting error being considerable on large purchases or sales.

Third—It eliminates the illogical assumption that interest installments received at the same time are so invested as to earn different rates of interest.

Fourth—It permits of an easy adjustment if at any future time it should be deemed advisable to use a different i_x.

I fully realize that objections may be urged properly against this proposed plan, but it seems to me that they are outweighed by the advantages. It is interesting to note the discussion by Mr. L. P. Orr of the very excellent paper on the amortization of bonds by Mr. Henry Moir, which appears in the *Transactions* of the Faculty of Actuaries, Vol. VII, p. 171. In discussing the advantages of the amortization plan and the market-value plan, Mr. Orr stated at page 198:

"Now, Sir, I do not wish my remarks to be regarded as unduly commending or depreciating either Method of Valuation. Each has its merits and its defects. Indeed I have long ago come to the

conclusion, that up to the present time, no method has been put forward that is not open to objection: and I almost doubt the ability of our profession to produce one that will receive the general approval of Actuaries throughout the world. The Amortization Plan has rather gone out of fashion in the United Kingdom. On the other hand, the Market Price Method is being ousted in the United States, and is not in favor in Australia or South Africa; while in Canada, although it is the legal basis, the Amortisation Plan is used for certain Bonds, and one large Company recently announced its completion of the adoption of that plan to its investments."

II.

It may be correctly stated, I think, that the amortization plan is of prime importance to life insurance companies, for their contracts are entered into for a long period of years and it is difficult to conceive of a situation which would require their securities to be converted into cash upon short notice. That the question of investments, however, is an important one to the so-called "miscellaneous" companies as well as to life insurance companies, is well illustrated by some tables which were prepared by Miss Katherine M. O'Leary, of the Massachusetts Insurance Department. Table VI shows the investment profit as compared with the underwriting profit or loss for the year 1915 exhibited by the statements filed by ten of the largest companies with insurance departments:

TABLE VI.

Company.	Profit from Investments.	Profit from Underwriting.	Loss from Underwriting.
A.....	$521,864.64	$254,013.13	
B.....	395,572.74	294,606.69	
C.....	209,581.91	361,074.90	
D.....	304,112.00	297,656.65	
E.....	109,678.06		$231,585.69
F.....	270,097.85	193,683.75	
G.....	257,258.85		54,640.51
H.....	152,703.37	200,832.54	
I.....	138,846.88	128,602.45	
J.....	216,406.91	72,522.27	
	$2,576,123.21	$1,802,992.38	$286,226.20

Since it is apparent that "miscellaneous" companies relied upon their securities for so large a percentage of their profits, it is interesting to note how the investments of those companies were distributed:

TABLE VII.

Company.	(a) Government, State or Municipal Bonds.	(b) R. R. Bonds Running 10 Years or Less.	(c) All Other R. R. Bonds.	(d) Miscellaneous Bonds.	(e) Stocks.	(f) Mortgages.	(g) Total Investments.
A.....	$ 699.858	$359,380	$4,191,985	$387,350	$3,690,762		$9,329,335
B.....	1,645,614	292,000	1,798,250	444,550	1,806,141	$268,977	6,255,532
C.....	4,049,296	787,680	450,280	307,500	504,726	22,750	6,122,232
D.....	4,030,831	619,400	2,504,000	315,000			7,469,231
E.....	2,069,591	200,050	1,351,617	457,025	325,507	142,343	4,546,133
F.....	1,418,373	216,728	1,473,610	163,435	721,050	1,448,245	5,441,441
G.....	3,953,448	32,500	171,890	409,900		130,650	4,698,388
H.....	1,093,292	452,860	2,183,370	241,500		4,500	3,975,522
I.....	1,583,710	156,870	1,637,221	63,980			3,441,781
J.....	827,760	1,077,650	2,796,830	223,000		123,000	5,048,240
	$21,371,773	$4,195,118	$18,559,053	$3,013,240	$7,048,186	$2,140,465	$56,327,835

This table indicates how large a percentage of the investments are in securities which would ordinarily be subject to some method of amortization, and we must consider, therefore, the advisability of so treating the investments of companies issuing policies of insurance, the nature of which may require the payments of large sums upon short notice. In some states amortization for miscellaneous companies is sanctioned by statute, and the pertinence of this inquiry is emphasized. In the event of a catastrophe it would become necessary to effect a quick liquidation of assets if the companies did not protect themselves in some way; this protection is secured in different ways. Some companies guard against the catastrophe hazard by reinsuring part of each risk: others reinsure only when the amount at risk exceeds a limit which they have established as a safety point: others are members of a mutual "reinsurance pool" to which each subscriber contributes a percentage of the premiums received and the pool agrees to assume the liability for any payments in excess of a stated loss which may result from a single contingency. This last method has been in successful operation for some time and the number of demands made upon the funds of those "pools," with the operations of which I am familiar, has been surprisingly small.

The grouping of the different forms of "miscellaneous" business may be summarized as follows:

Group 1. Consisting of those branches which are subject to catastrophes.

Group 2. Those branches which are free from that possibility.

Group 1.	*Group 2.*
Personal accident.	Health.
Workmen's compensation.	Liability (policy containing limits).
Plate glass.	Fidelity.
Workmen's collective.	Burglary and theft.
Unlimited liability.	Automobile and teams property damage.
	Surety.

It will be noted that credit insurance has been omitted from both of the foregoing groups, for while credit policies contain a maximum limit of liability and therefore the element of catastrophe is absent from *each* contract, a severe financial depression may produce a "catastrophe" hazard in that a large number of policies will develop claims which in normal times would not exceed the initial loss limit.

The catastrophe hazard in personal accident insurance is illustrated by a recent occurrence when all of the members of a fraternal organization bound for a convention in a distant city were insured against travel accidents by an insurance company; unfortunately the train was wrecked and a number were killed.

Workmen's collective policies may be considered as group accident insurance policies, and as such are subject to the possibility of single accidents involving a number of claims.

In considering the advisability of applying the principle of amortization to the bonds of a miscellaneous company it should be borne in mind that no company transacts only one of the lines shown in groups 1 and 2; multiple lines are the rule and in consequence the danger to a company of extinction by a catastrophe is minimized greatly.

The possibility of requiring funds for reinsurance purposes presents the only other factor which would require sudden conversion of assets into cash, but even this situation might be met if in cases of reinsurance the company assuming the risks would be permitted to carry the assets given as the reinsurance consideration at the same amortized figures used by the ceding company. That this method would be practicable was demonstrated in the case of a merger of two companies recently in a western state; at the request of the insurance commissioner I prepared the schedule of assets and liabilities and took the position that although the merged

company was a new corporate entity, no logical reason existed for considering that the assets had been sold; the new corporation, therefore, was permitted to carry the bonds at the same figures as its two constituent parts did before the amalgamation.

Although the principle of amortization has heretofore been applied in a very limited degree to the assets of miscellaneous companies, I am of the opinion that with existing safeguards against catastrophes, we may safely amortize the bonds of such companies.