LEGAL LIMITS OF WEEKLY COMPENSATION IN THEIR BEARING ON RATEMAKING FOR WORKMEN'S COMPENSATION INSURANCE

BY

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In nearly all of the workmen's compensation laws it is provided that the weekly indemnity for disability and the weekly rate of indemnity in the case of specific provisions for permanent disability is a percentage of the earnings of the injured employe, subject to certain arbitrary maximum and minimum limits. The effect of these limits on the rate of compensation paid the injured workers has been discussed from the social and economic point of view by a number of writers in different places and I shall not give consideration to these aspects of the matter in this paper. My purpose is to present the subject from the standpoint of the maker of rates for workmen's compensation insurance.

It must be obvious that were the compensation a fixed percentage of the wages without the introduction of these arbitrary limits, fluctuations in the earnings of injured employes (assuming other conditions remained unchanged) would have no effect upon the rate for compensation insurance if that rate is applicable to the total payroll of the employer.* Where, however, these arbitrary limits intervene, fluctuations in wage levels have an important bearing on the cost of compensation in relation to payroll. That this is so, is rather generally understood among underwriters and those engaged in the business, but my personal contact with many company representatives has led me to the view that the precise reason for this effect and the proper way of measuring it is not generally understood or appreciated.

A simple illustration, at the outset, of the way these statutory limits work will probably help to clear the ground and make further discussion more readily understood. Let us take, for example, the compensation law of the State of New York in which it is provided that the rate of weekly indemnity for total

*As the medical benefit is independent of wages, important changes in wage scales, if doctors' fees and hospital charges did not parallel them, might call for some slight variation in rates on account of wage changes.

disability is to be 66⅔% of the average weekly earnings of the injured worker, subject to a maximum limit of $20.00 per week and a minimum limit of $8.00 per week. Under this provision, if the injured worker's weekly earnings lie between $12.00 and $30.00, compensation will be precisely 66⅔% of his actual earnings; but if his earnings are less than $12.00 per week, the indemnity provided will be more than 66⅔% of the earnings, and if his earnings are more than $30.00 per week, the compensation will be less than 66⅔% of his earnings. In any given industry or establishment there are, of course, employes earning different rates of wages and the average rate of compensation which will be payable to such employes in the case of injury will depend upon the proportion of the employes in the various wage grades. If there is a considerable proportion earning more than $30.00 per week, the average rate of compensation will drop considerably below 66⅔%, and where the industry is such that there is a considerable proportion of workers earning less than $12.00 per week that is not offset by an equal or greater proportion earning more than $30.00 per week, then the average rate of compensation in the event of injury would be something more than 66⅔%.

Let us now consider a particular establishment and assume that the general level of wages is such that the average rate of compensation is slightly under 66⅔% and that an abrupt rise has taken place in the general level of wages. This will throw a larger proportion of employes into the class earning more than $30.00 per week upon which the upper limit of $20.00 is operative. It will also tend to throw employes from the group below $12.00 per week into the group above and diminish the number of those whose rate of compensation is more than 66⅔%. Both these changes tend to lower the average rate of compensation as a percentage of wages. The actual amount of money payable will, of course, be greater, but its increase will be less than the increase in average wages. On the contrary, a drop in the level of wages has directly the opposite effect and tends to raise the average percentage rate of weekly compensation.

As long as premiums are based on the total payroll of the assured, it is not possible to make compensation insurance rates precisely reflect rapid changes of this kind because to attempt to do so would require continuous, almost daily, recision of outstanding contracts to the great confusion and annoyance of

insurer and assured alike. It is, however, both necessary and desirable to recognize changes of this kind when they are of the "long swing" character, and it becomes a matter of importance in the making of compensation rates to give proper consideration to the wage levels under which the experience used in making the rates has been accumulated and that at which it is probable the rates to be made will be used. The rates must be properly keyed to the latter.

It is obvious that the exposure to losses arising out of industrial accidents is a function of, if not directly proportional to, the number of persons employed by the assured and the length of time they work. These two quantities are fluctuating quantities and not easily ascertainable and it was therefore a custom of employers' liability insurance prior to the introduction of the workmen's compensation principle to base the premium upon the employers' payroll as an approximate measure of the exposure covered by the insurance. Under employers' liability insurance there was not the direct relation between the wages of the employe and the amount he received in the event of injury that obtains under workmen's compensation laws and as the rates for employers' liability insurance were not fixed by the precise methods used for workmen's compensation insurance, the question of nice adjustments such as are here considered did not come up.

When the Massachusetts Workmen's Compensation Act came into effect, however, Mr. S. H. Wolfe, Consulting Actuary of the Massachusetts Employes Insurance Association noted this effect of the statutory limits of weekly compensation and recognized the further fact that in the same industrial classification there might be employers paying different scales of wages and upon whose risk cost the statutory limits would have different effects. He therefore devised a system of premium determination for that company which was based upon the principle of adjusting the payroll to reflect the effect of these limits and then applying a uniform rate to such adjusted payroll.* Theoretically this proposal followed sound lines, but at the time it was put forward, it provoked a sharp controversy with those who had

*In actual practice there were other features such as the basing of that part of the premium required for medical on the number of employes. The student desiring further details will find them in the "The Standard" and other insurance journals in 1912.

had extensive experience in employers' liability insurance. The scheme was tried for about six months by the Association but found unworkable in practice for a company seeking business in competition with other carriers who quoted a flat rate on the total payroll in each classification. The company was not able to quote a prospective assured a definite rate which he could apply to his total payroll, and in order to estimate his premium, it was necessary to obtain a considerable amount of information not readily revealed by his general account books. Similar information in the same detail had to be obtained before adjusting the premium on audit after the close of the policy year. After the data were obtained from the assured there were extensive calculations necessary before he could be billed. The Association, therefore, abandoned this scheme, after such trial, in favor of a flat rate applicable to the assured's total payroll by manual classification.

Under the early compensation laws the statutory limits were such, in relation to the percentage of wages provided as compensation and the average wage then earned, that they had no marked effect upon the average rate of wages payable as compensation and for practical purposes in most cases might have been ignored. With the increase of wages, however, which resulted from the industrial boom following the outbreak of the war, the limits had an increasing effect in depressing the effective percentage of wages paid as compensation. This led to a volume of legislation amending the compensation acts to bring the limits more nearly into line with the average wages and statutory percentage allowed as compensation. Even as yet, however, they have not generally been brought to the point where the effective percentage of wages payable as compensation substantially equals the statutory limits, nor have the efforts to amend the laws and increase the benefits noticeably diminished.

In some cases the limits have been changed for some of the benefits and the old limits have been allowed to remain for certain of the other benefits, or if they have been changed have not been brought to the same level. Under such circumstances the intricacy of detail involved in an attempted adjustment of payroll to be used as the basis of premium so as to allow for the effect of limits becomes even greater. I think it may be taken as demonstrated that it is practically impossible to make provision in this way for the effect of limits. It, therefore, becomes necessary to make provision through the manual rate.

Where the manual rate is made from past experience, it may possibly happen that, through the relationship found to exist between the wage level and limits in effect at the time the experience was accumulated and those in effect at the time the rates are to be used, it is possible to use the past experience without adjustment on this point, but rarely will this be so. I cannot recall a case. But, of course, the recent past has been one of rapidly changing wage levels. Generally there will have been a change in the wage level, or a change in the statutory limits, or both, between the time the experience has been accumulated and the time the rates will probably be used.

In this case, an adjustment must be made. This adjustment may be made directly upon the pure premium or by operating separately upon the two parts of the pure premium fraction—the payrolls and the losses. If the former method of procedure is followed, then the modifying factor will be the ratio of the effective percentage rate of compensation payable at the time the rates are to be used to the effective rate of compensation payable during the period whose experience is used. If the experience of more than one policy year is used, it will usually be necessary to use different factors for each of the several policy years. If the latter method is to be followed, then the modification factor for losses will be the ratio of the average effective monetary rate of compensation payable at the time the rates are to be used, to the average effective monetary rate of compensation payable on the accidents of the policy year whose experience is used, and the corresponding modification factor for the denominator (payroll) is the ratio of the anticipated average wage for the time when the rates become effective to the actual average wage of the experience data.

Whether the modification factor is determined for application to pure premiums or to losses, it is necessary for its determination to have two* tables showing the distribution of wages in each period around the average wage for that period. Heretofore it has been the practice to collect such a distribution from the records of wages received by injured employes during the experience period and from a like record of wages received by injured employes during a period thought to be representative of

*It is shown later in this paper how a single standard distribution may be used, but this has to be used twice corresponding to different averages.

the period when the rates are to become effective. The most convenient distribution table of this kind proceeds by regular intervals (usually of $1.00) and records the number of employes out of a given total receiving wages lying within each particular interval.

In Table I following is given a distribution covering 3,092 cases with an average wage of $22.46, collected during the 1920 national revision of rates and based upon accidents occurring in Massachusetts during the latter half of calendar year 1919.

TABLE I.

TYPICAL WAGE DISTRIBUTION

Actual Wages (1)	Assumed Average Wages (2)	Number of Cases (3)	Total Wages (2) x (3)
$4.01 to 5.00	$4.50	11	$49.50
5.01 " 6.00	5.50	3	16.50
6.01 " 7.00	6.50	9	58.50
7.01 " 8.00	7.50	15	112.50
8.01 " 9.00	8.50	30	255.00
9.01 " 10.00	9.50	58	551.00
10.01 " 11.00	10.50	24	252.00
11.01 " 12.00	11.50	71	816.50
12.01 " 13.00	12.50	67	837.50
13.01 " 14.00	13.50	67	904.50
14.01 " 15.00	14.50	113	1638.50
15.01 " 16.00	15.50	71	1100.50
16.01 " 17.00	16.50	107	1765.50
17.01 " 18.00	17.50	216	3780.00
18.01 " 19.00	18.50	114	2109.00
19.01 " 20.00	19.50	217	4231.50
20.01 " 21.00	20.50	212	4346.00
21.01 " 22.00	21.50	155	3332.50
22.01 " 23.00	22.50	131	2947.50
23.01 " 24.00	23.50	191	4488.50
24.01 " 25.00	24.50	249	6100.50
25.01 " 26.00	25.50	99	2524.50
26.01 " 27.00	26.50	98	2597.00
27.01 " 28.00	27.50	110	3025.00
28.01 " 29.00	28.50	74	2109.00
29.01 " 30.00	29.50	169	4985.50
30.01 " 31.00	30.50	93	2836.50
31.01 " 32.00	31.50	39	1228.50
32.01 " 33.00	32.50	51	1657.50
33.01 " 34.00	33.50	21	703.50

TABLE I. (Continued)

Actual Wages (1)	Assumed Average Wages (2)	Number of Cases (3)	Total Wages (2) x (3)
$34.01 to 35.00	$34.50	59	$2035.50
35.01 " 36.00	35.50	31	1100.50
36.01 " 37.00	36.50	11	401.50
37.01 " 38.00	37.50	15	562.50
38.01 " 39.00	38.50	4	154.00
39.01 " 40.00	39.50	39	1540.50
40.01 " 41.00	40.50	3	121.50
41.01 " 42.00	41.50	6	249.00
42.01 " 43.00	42.50	1	42.50
43.01 " 44.00	43.50	3	130.50
44.01 " 45.00	44.50	9	400.50
45.01 " 46.00	45.50	5	227.50
46.01 " 47.00	46.50	..	..
47.01 " 48.00	47.50	2	95.00
48.01 " 49.00	48.50	..	..
49.01 " 50.00	49.50	9	445.50
50.01 " 51.00	50.50	1	50.50
51.01 " 52.00	51.50	1	51.50
52.01 " 53.00	52.50	..	..
53.01 " 54.00	53.50	1	53.50
54.01 " 55.00	54.50	1	54.50
59.01 " 60.00	59.50	2	119.00
62.01 " 63.00	62.50	2	125.00
69.01 " 70.00	69.50	1	69.50
70.01 " 71.00	70.50	1	70.50
Average Wage	22.46	3092	69461.00

The use of this table may be illustrated by determining the effect of the Massachusetts statutory limits of $4.00 and $10.00 on fatal and dismemberment cases under the 1919 law, the nominal percentage of wage payable in such cases being 66⅔%. To get at the effect of these limits, we first find from the table the number of cases where the wages were less than $6.00 (the lower limit of $4.00 being ⅔ of $6.00). In this tabulation there are 14 such cases which compensated at $4.00 per week would require $56.00 of compensation. Next we consider the cases where the limits do not apply. There are 454 such cases where the wages lie between $6.00 and $15.00. On these cases the statutory rate of 66⅔% is payable. Summing the figures in column (4) for this section of the table, the total weekly wage of this group is found to be $5,426, 66⅔% of which is $3,617.

We next consider the cases to which the upper limit applies. There are 2,624 such cases with wages over $15.00 which would be compensated at $10.00 per week and which would call for $26,240 weekly compensation. The total weekly cost of compensation on all the 3,092 cases in this distribution would be $29,913 = ($56 + $3,617 + $26,240). The total wages in this distribution are $69,461, two-thirds of which would be $46,307. The ratio of the compensation above, $29,913, to this figure is .646 which is the limit factor applicable to the valuation of this law at this wage level in terms of weeks' wages without limits. This factor shows that the rate of weekly compensation under this wage distribution and these limits, instead of being 66⅔% of wages, would be actually 43.09%. The latter figure might have been derived independently by directly comparing $29,913 of weekly compensation with the total corresponding wage of $69,461.

If we were deriving rates for this benefit and wage level from experience data, and the laws under which the data were accumulated were the same in other respects than the limits, or if it were the same law in all respects but under different wage conditions, the estimated value of compensation for a given number of cases in weeks' wages without limits* would be the same for both cases and the pure premium modification factor would be the ratio of two limit factors each calculated as in our illustration. The loss modification factor (if payrolls and losses are to be modified separately) would be found by applying the effective rate of compensation in each period to the average wage of that period, thereby determining the effective monetary rate of weekly compensation and comparing these two values. In this case the corresponding payroll factor would be the ratio of the average wages.

In working up any set of data for use in making rates for different conditions it is necessary (if there is no difference in law to be considered) to work out such a modification factor for each set of limits in the law and apply it to the pure premiums (or losses)† for each benefit subject to those limits.

In some statutes there is a maximum limit of total compensa-

*i. e., time lost multiplied by the statutory percentage rate.

†The factor applicable to payroll is independent of the limits and does not change when they do.

tion which becomes operative on long term benefits such as are given for total permanent disability and to dependents in fatal cases. If this is less than the total payable at the maximum weekly rate for the maximum term of benefit (if limited) then there must be a more complicated calculation to make allowance for this limitation in determining the factors. In such cases it has usually been more convenient to calculate the total amount payable for the entire number of cases in the distribution and reduce to terms of weekly wages for comparison by dividing by the average wage.

The method of procedure for determining the effect of limits of weekly indemnity outlined above is very laborious and requires the collection of a typical wage distribution for each of the periods between which comparison is to be made. This it is not always possible to do—at least, not satisfactorily.* For this reason, I recently set out to see if it were not possible to find a typical distribution which may be applied to any average wage and represent the most probable actual distribution within a reasonable margin of error.

In seeking this typical distribution, I first had a number of actual distributions transformed into terms of percentage departure from the average wage. I then had histograms of these distributions plotted in superposition on the same chart with their means at the same point. After roughly smoothing out irregularities in individual distributions there was apparently sufficient similarity in the form of the several distributions to indicate that a composite distribution might be substituted for each of them without introducing serious error. I therefore had a new chart of the same form prepared including several more distributions and a freehand curve drawn through it so as to represent as nearly as could be determined the composite indications of each of the histograms. Using this curve, I then had the number of squares falling between 2½% above and 2½% below the mean value counted and the number of squares for 5% intervals, determined in the same way, for the remainder of

*For example, it may be entirely possible to estimate from mass data with considerable precision what the average weekly wage will be at a certain future time, although it would be quite impossible to secure a wage distribution which would be recognized as corresponding to this average wage.

the curve. Dividing these by the total indicated area, we got a new standard percentage distribution.

This distribution I had applied to the determination of the effective limits under a number of different compensation laws and at different wage levels and compared the result with the factor produced by calculating direct from the wage distribution used in getting the average wage in question. Some of these tests placed a very severe strain on the use of the standard distribution, since among other factors calculated was the factor corresponding to limits of $4.00 and $10.00 on a two-thirds rate with average wages of $39.10. Even in this case the error was less than 2%, and, in general, the average error of 13 tests was 1¼%. These tests indicated that a standard distribution could be used satisfactorily in lieu of an actual distribution for practically any limit factor calculation required. In the test used, the average wages ranged from $19.00 to $39.00 and the limits from $4.00 and $10.00 on a 66⅔% rate in one case or $5.00 and $11.00 on a 50% rate in another to $3.00 and $18.00 on a 60% rate.

I found, however, that the labor of passing from a percentage distribution to a distribution in dollars for the purpose of calculating the limit factor was much greater than the calculation from an actual distribution already collected in dollars and almost offset the advantage gained by not having to call for such a distribution. I have since found a way of overcoming this difficulty. I did not apply it to this first distribution, but as will be apparent when the method is described, it could equally well be used with this one as with the distribution finally adopted.

Satisfactory as these results appeared to be, it seemed to me that a distribution determined by graphic methods must always contain a large element of the personal equation and be open to attack on that ground. It seemed that it would be better, if possible, to find some way of constructing the standard distribution by a mechanical method which would not involve so large an element of personal judgment.*

I therefore set out anew and selected ten wage distributions all of which were based upon a large number of cases and which represented a wide variety of conditions both as respects average

*The personal element, however, is not entirely eliminated since a number of distributions must be selected from which to derive the standard.

wage and the extent of dispersion around the average. Most of these distributions were obtained either from material collected by the National Council for use in the 1920 rate revision or from Schedule Z. I had, however, learned from Mr. S. B. Perkins, Actuary of the Compensation and Liability Department of the Travelers Insurance Company, that he had recently made a number of distributions from accidents reported to his company by New York risks and I obtained copies of these distributions from him and selected, for use in constructing this composite, four wage distributions, three of which came from the latter part of 1920 and one from the early part of 1919. In this way I increased the range of average wages represented in my basic data and the variation in the distribution of the several distributions around their means.

I then had each of these distributions transformed into the form of percentage departures from the mean and had their moments calculated by the summation method. Next I had the weighted averages of these moments determined, using as the weights for each distribution the number of cases involved and, after applying Sheppard's corrections, adopted these as the moments for the new curve.

I am not aware that this method of amalgamating material has heretofore been used. The method of moments has, of course, been frequently used for determining the constants of curves for graduating mortality and other statistical data to produce smooth and regular curves and there is nothing new in applying it to this problem unless it be in this use of it to amalgamate and get a composite of a large amount of basic data. I see no reason why the method should not be available for such a purpose. The distributions used both in monetary and percentage form and their actual moments as calculated are shown in Appendix A.

The weighted averages of these moments were:

ν_1	1.000
ν_2	48.504
ν_3	271.876
ν_4	10,450.566

After applying Sheppard's adjustment, the final values are:

μ_1	1.000
μ_2	48.421
μ_3	271.876
μ_4	10,426.343

(I doubt whether application of Sheppard's adjustment was really necessary and, as will be noted, it had little effect on the value of the moments.) From these values I determined:

β_1	.651
β_2	4.447

I first tried out the construction of a Pierson frequency curve from these moments but found that the values indicated the transcendental (Type IV) curve which could only be integrated to get the distribution by some mechanical process and was therefore unsatisfactory. I then turned to the method described by Mr. Carver in his paper in *Proceedings* VI, page 52, and attempted the construction of the distribution by this method which proved highly satisfactory. The values of the several constants in his formula worked out as:

C_1	34.542
C_2	794.365
C_3	53.948
C_4	847.313

This gave as the finite difference equation for determining the curve:

$$\frac{y_x + 1}{y_x} = \frac{x^2 + 34.542\,x + 794.365}{x^2 + 53.948\,x + 847.313}$$

The following column heads indicate the work sheet for constructing the distribution:

(1)	(2)	(3)	(4)	(5)	(6)
x	x^2	$C_1 x$	(2) + (3)	(4) + C_2	$C_3 x$

(7)	(8)	(9)
(2) + (6)	(7) + C_4	$\frac{(5)}{(8)} = \frac{y_x + 1}{y_x}$

As Mr. Carver explains in his paper, this equation may be used for graduating any particular section of a general curve and starting with a particular value of x and an assumed corresponding value of y, the remaining values may be determined by successive multiplication, if the value of x is the lowest value in the section to be dealt with, or by division if it is the highest, or it is possible to begin with the mean value and work in both directions.

In working out this distribution we considered the value of x to be zero for the mean and y to be 1000, carrying the calculation of the successive values of y to the point where they became less

than one on either side. We then reduced the numbers so that the total distribution summed to 100.00. The basic data were grouped in 5 point intervals of percentages and therefore a unit difference in the value of x corresponds to a difference in the wage equal to 5% of the average.† As will appear from the table in the Appendix B, tests of this distribution proved quite as satisfactory as the earlier one I had obtained and indeed slightly more so.

The next problem undertaken was to find some means of reducing the mechanical work of calculating limit factors from such a standard distribution in percentage form. For that purpose certain auxiliary columns were calculated. The following table shows, in column (2), the distribution obtained as above described and in columns (3), (4) and (5), the auxiliary figures.

TABLE II.
STANDARD WAGE DISTRIBUTION
LESS THAN AVERAGE
Limit has a Positive (+) Effect on Average.*

(1) % Deviation from Average	(2) Distribution	(3) (1) x (2)	(4) Σ (2)	(5) Σ (3)
− 90	.02	1.80	.02	1.80
85	.04	3.40	.06	5.20
80	.08	6.40	.14	11.60
75	.18	13.50	.32	25.10
70	.33	23.10	.65	48.20
65	.59	38.36	1.24	86.55
60	.96	57.60	2.20	144.15
55	1.46	80.30	3.66	224.45
50	2.09	104.50	5.75	328.95
45	2.81	126.45	8.56	455.40
40	3.58	143.20	12.14	598.60
35	4.34	151.90	16.48	750.50
30	5.04	151.20	21.52	901.70
25	5.61	140.25	27.13	1,041.95
20	6.02	120.40	33.15	1,162.35
15	6.24	93.60	39.39	1,255.95
10	6.29	62.90	45.68	1,318.85
5	6.17	30.85	51.85	1,349.70

*Meaning of this note will appear in the following work.

†Theoretically, Mr. Carver's formula graduates unit values and where an area is used as in this case as though it were an ordinate, the use of a quadrature formula as he has indicated in his paper might be necessary. The rate of change in the distribution we had was not so rapid that in my judgment such further adjustment was necessary.

GREATER THAN AVERAGE
Limit has a Negative (−) Effect on Average*

(1) % Deviation from Average	(2) Distribution	(3) (1) x (2)	(4) Σ (2)	(5) Σ (3)
170	.04	6.80	.04	6.80
160	.02	3.20	.06	10.00
155	.02	3.10	.08	13.10
150	.03	4.50	.11	17.60
145	.03	4.35	.14	21.95
140	.04	5.60	.18	27.55
135	.05	6.75	.23	34.30
130	.06	7.80	.29	42.10
125	.08	10.00	.37	52.10
120	.10	12.00	.47	64.10
115	.12	13.80	.59	77.90
110	.15	16.50	.74	94.40
105	.18	18.90	.92	113.30
100	.22	22.00	1.14	135.30
95	.28	26.60	1.42	161.90
90	.34	30.60	1.76	192.50
85	.42	35.70	2.18	228.20
80	.51	40.80	2.69	269.00
75	.63	47.25	3.32	316.25
70	.77	53.90	4.09	370.15
65	.93	60.45	5.02	430.60
60	1.13	67.80	6.15	498.40
55	1.36	74.80	7.51	573.20
50	1.63	81.50	9.14	654.70
45	1.95	87.75	11.09	742.45
40	2.30	92.00	13.39	834.45
35	2.70	94.50	16.09	928.95
30	3.14	94.20	19.23	1,023.15
25	3.62	90.50	22.85	1,113.65
20	4.12	82.40	26.97	1,196.05
15	4.62	69.30	31.59	1,265.35
10	5.10	51.00	36.69	1,316.35
5	5.55	27.75	42.24	1,344.10

*Meaning of this note will appear in the following work.

The use of these columns will perhaps be best understood, if we briefly review the remarks earlier in this paper as to the effect of limits and the nature of the limit factor. The limit factor, from one point of view, is the ratio of the effective rate of compensation to the statutory rate of compensation. If, however, we look at the matter from the earlier point of view presented and assume that notwithstanding the practical difficulties pointed out, instead of applying the effective rate to the entire payroll, we so adjust the payroll that the effective rate will be equal to the nominal rate, it can easily be proven that the ratio of the adjusted payroll to the actual total payroll

will be the same as the ratio of the effective compensation rate to the nominal statutory rate. We may, therefore, calculate the limit factor by ascertaining what adjustment would be required in a total payroll distributed around its average wage in accordance with this standard distribution.

Let us now assume for illustration that our average wage in a particular problem is $24.00. The statutory rate of compensation is 66⅔% and the minimum limit is $8.00. $8.00 being 66⅔% of $12.00, it would become necessary in the adjustment of the payroll to treat all employes receiving $12.00 or less as though receiving $12.00. Since $12.00 is 50% of $24.00, this limit is therefore a departure of 50% from the average, and we find in column (4) of the above table (in upper half) that 5.75% of the employes are affected by this lower limit. Column (3) gives the product of the departure by the frequency of cases having that departure and the items in column (5) are the sum of the items in column (3). Therefore, we may find the average percentage deviation of actual wages for those in this class from the average of all wages by dividing the figure in column (5) corresponding to 50% (328.95) by the figure in column (4)—5.75. This gives an average departure of 57% for which we must substitute the departure of 50%. The product of the difference between these average departures by the percentage of the cases affected (5.75) gives the correction for the lower limit. As a matter of mechanics, however, it is probably simpler to multiply the figure in column (4)—5.75—by the substitute departure corresponding to the limit, in this case 50% giving $287.50. Subtraction of this figure from the figure corresponding to 50% in column (5) will also give the correction and the result, if the calculations are both carried to the same decimal place, will be identical. The difference between these two figures taken positively gives the correction on the average wage corresponding to this substitution of limit value. In this connection, it should be borne in mind that the total distribution is 100 and that the results in all cases are divisible by 100.

In actual practice it will be necessary to interpolate between the tabulated values of the deviation and I have tested out to determine whether or not second differences need be considered. I have found that for all practical purposes an interpolation by first differences will be sufficient and since columns (2) and (3)

respectively are from the nature of the construction of columns (4) and (5) the first differences of the later columns, we have all the required material in this table.

The calculation of the modification for the upper limit follows precisely the same lines.

Where the lower limit is a fixed amount "or wages" this calculation does not give the exact results but slightly over-estimates the effect of the lower limit. I first worked out a modification of the process to take account of the "or wages" clause but it greatly complicates the work even when certain approximations are used. I have, therefore, had tests made of the effect of this "or wages" clause in about its most important case, that is when the minimum limit is as high as $8.00, and find the effect is practically negligible so that I have issued instructions for my office that in the calculation of limit factors, this clause may be ignored.

In one or two cases we have found the wage corresponding to the upper limit falls below the average wage indications for the jurisdictions whose law we were valuing. This case somewhat complicates the formula but the principle above outlined can be followed through and we have actually tested out the results and found them correct.

In order to facilitate calculations in this office, of limit factors from this distribution, we have had work sheets multigraphed in which the data may be inserted from the wage facts, the terms of the law, and these standard tables and on actual tests with the use of these sheets we find our calculations reduced to a minimum of time and effort.

In Appendix C is a calculation on the basis of this distribution and using this calculation form, of the limit calculated earlier in this paper ($4.00 and $10.00 on a 66⅔% rate) from an actual wage distribution whose average is the same as that used in the first illustration. You will note that in this case the upper limit lies below the average wage and the use of the columns is somewhat complicated but the form has been drawn to facilitate this use. The principle is the same as heretofore explained.

By these methods I have had made a set of test calculations of limit factors for comparison with calculations made on actual wage distribution. The results of these test calculations are shown in the table in Appendix B.

A review of this table will indicate that the largest error in the limit factor was only a trifle more than 2% and that in only one other case was the error so large as 1½%. These errors are within the limit of accuracy of the basic assumption in such work and the method, therefore, may, I think, be accepted as practically sufficient. It will enable us to go ahead with the determination of the effect of statutory limits when we are able only to obtain an average wage from mass figures without having the detail figures necessary for an actual distribution. If it is desired to obtain the average effective compensation in the monetary terms rather than as a percentage of the wages, this may easily be found by applying the statutory provision to the average wage and then applying the limit factor calculated in this way to that amount. The result will be the effective rate of compensation in monetary terms.

In a few states there is a maximum limit on the total compensation payable in fatal and permanent total disability cases which is less than the product of the maximum limit of weekly indemnity and the maximum term fixed by the act. To estimate the effect of the various limits in these cases with accuracy would require an actual wage distribution, the determination of the present value of the compensation corresponding to each wage level and the comparison of the weighted average of these present values with the present value for the statutory percentage rate and term. This is a very laborious calculation.

Fatal and permanent total disability benefits rarely represent more than 15% to 25% of the total pure premiums and an error of 3% or 4% in this factor would be less than 1% of the total. The approximation method described below has been found by test not to show greater error than this and often to lie within 1% or 2% of a direct estimate, an admissible error since we must assume averages to measure the mortality and cannot hope for higher accuracy. The method is based on the fact that the only difference in the value of the benefit after the maximum amount is reached is due to the greater or less effect of discount.

In calculating the effect of these limits I first find the lowest rate of weekly compensation which gives the maximum aggregate compensation prescribed for the benefit and calculate the limit factor by the method described above using this as the upper limit. I next find the present value in terms of weeks' com-

pensation and note the effect of discount. Then I find by use of my standard distribution the correction in the average wage resulting from excluding all cases below the upper limit above set and the corresponding rate of compensation substituting the statutory weekly maximum if this is less. Next I find the term required to pay out the maximum at this rate, the present value and the effect of discount. The difference between the effect of discount at this rate and at the lower rate above found, I multiply by the proportion of the distribution lying above the first limit, thus in effect distributing it over the whole. The resulting figure I apply as a correction of the limit first calculated.

This description of the method sounds laborious, but the following example shows that it is fairly simple:

Under the Massachusetts law in effect in 1918, the benefit is two-thirds wages with limits of $5.00 and $14.00 and a maximum period prescribed for permanent total disability of 500 weeks. The maximum absolute amount payable, however, is $4,000. It is desired to find the limit factor when the average wage is $22.46. It will first be noted that 500 weeks at $8.00 equals the maximum of $4,000, and therefore, this absolute limit except for the effect of discount, is equivalent to an $8.00 weekly limit. The limit factor on that basis is 51.55%. The commuted value of 500 weeks' wages is 404.53, and the effect of discount and mortality is, therefore, 19.9%; 7.68% of the total cases in our standard distribution would at this average wage have less than $12.00 wages. The effect of eliminating these cases from the standard distribution is to raise the average wage on the remainder of the distribution 4.1% or to $23.38 of which two-thirds is $15.59. This exceeds the statutory maximum of $14.00. Using, therefore, $14.00 we find $4,000 consumed in 286 weeks, of which the commuted value is 253.14. The effect of discount is, therefore, 11.49%. The present value, therefore, of compensation on these cases is 7.61% greater than on cases running the full term. Spreading this over the entire distribution, that is multiplying by .9232* leaves 7.03% as the correction. The product of the basic limit factor, 51.55 times 1.073, gives 55.17% as the final limit factor.

Acknowledgment is made of the assistance rendered by Mr. C. W. Graham and Mr. Kendrick Stoke of the staff of the National Council on Workmen's Compensation Insurance in carrying through the details of this investigation.

*.9232 = 1 − .0768.

APPENDIX A—PART I

BASIC DISTRIBUTIONS USED IN CONSTRUCTING STANDARD WAGE DISTRIBUTIONS

(a) LIST OF SOURCES OF DISTRIBUTIONS.

1. New York 1919 (from 1920 Revision). Average wage $25.77. Number of cases 10,771. All groups combined.
2. California 1919 (from 1920 Revision). Average wage $27.88. Number of cases 13,853. All groups combined.
3. Massachusetts 1918 (from Schedule Z). Average wage $22.15. Number of cases 801. All groups combined.
4. Massachusetts 1919 (from Schedule Z). Average wage $26.11. Number of cases 938. All groups combined.
5. Massachusetts 1920 (from Schedule Z). Average wage $28.75. Number of cases 975. All groups combined.
6. New York 1920 (calendar year 1921, July-December). From Traveler's data. Average wage $26.14. Number of cases 3,741. Group I—Manufacturing.
7. New York 1921 (calendar year 1921, July-December). From Traveler's data. Average wage $33.05. Number of cases 1,570. Group II—Contracting.
8. New York 1921 (calendar year 1921, July-December). From Traveler's data. Average wage $26.34. Number of cases 2,680. Group V—All other.
9. New York 1919 (calendar year 1919, January-June). From Traveler's data. Average wage $21.44. Number of cases 669. Group V—All other.
10. Tennessee 1919 (from 1920 Revision). Average wage $19.06. Number of cases 495. All groups combined.

(b) ACTUAL DISTRIBUTIONS BY KEY NUMBERS

DISTRIBUTION No. 1.

In Dollars as Reported. In Percentage Form as Grouped

(1) Wages in Dollars	(2) Number of Cases	(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases
4.01- 5.00	6	44.01-45.00	31	− 85	.0
5.01- 6.00	14	45.01-46.00	11	− 80	.1
6.01- 7.00	4	46.01-47.00	59	− 75	.1
7.01- 8.00	41	47.01-48.00	44	− 70	.5
8.01- 9.00	53	48.01-49.00	10	− 65	.8
9.01-10.00	71	49.01-50.00	29	− 60	.9
10.01-11.00	75	50.01-51.00	8	− 55	1.8
11.01-12.00	181	51.01-52.00	4	− 50	1.2
12.01-13.00	77	52.01-53.00	..	− 45	2.5
13.01-14.00	122	53.01-54.00	6	− 40	2.0
14.01-15.00	254	54.01-55.00	2	− 35	3.8
15.01-16.00	146	55.01-56.00	2	− 30	5.5
16.01-17.00	192	56.01-57.00	3	− 25	4.4
17.01-18.00	616	57.01-58.00	4	− 20	6.4
18.01-19.00	323	58.01-59.00	..	− 15	4.6
19.01-20.00	374	59.01-60.00	9	− 10	8.3
20.01-21.00	575	60.01-61.00	..	− 5	5.7
21.01-22.00	446	61.01-62.00	1	0	4.9
22.01-23.00	294	62.01-63.00	1	+ 5	3.0
23.01-24.00	911	63.01-64.00	..	10	6.5
24.01-25.00	411	64.01-65.00	..	15	14.7
25.01-26.00	470	65.01-66.00	4	20	4.1
26.01-27.00	278	66.01-67.00	..	25	3.3
27.01-28.00	224	67.01-68.00	..	30	1.9
28.01-29.00	630	68.01-69.00	2	35	2.4
29.01-30.00	1494	69.01-70.00	2	40	2.2
30.01-31.00	316	70.01-71.00	1	45	1.4
31.01-32.00	373	71.01-72.00	1	50	1.7
32.01-33.00	226	72.01-73.00	..	55	1.2
33.01-34.00	142	73.01-74.00	1	60	1.0
34.01-35.00	211	74.01-75.00	..	65	.4
35.01-36.00	184	75.01-76.00	1	70	.5
36.01-37.00	184	76.01-77.00	..	75	.2
37.01-38.00	101	77.01-78.00	..	80	.6
38.01-39.00	152	78.01-79.00	..	85	.4
39.01-40.00	101	79.01-80.00	1	90	.2
40.01-41.00	99			95	.2
41.01-42.00	79			100	.1
42.01-43.00	30			Over	
43.01-44.00	54			100	.4
Average Wages 25.77		Total No. of Cases 10,771		Total 99.9	

DISTRIBUTION No. 2.

In Dollars as Reported				In Percentage Form as Grouped			
(1) Wages in Dollars	(2) Number of Cases	(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases
4.01– 5.00	17	39.01–40.00	135	−85	.1	90	.3
5.01– 6.00	10	40.01–41.00	67	−80	.1	95	.2
6.01– 7.00	10	41.01–42.00	322	−75	.2	100	.3
7.01– 8.00	28	42.01–43.00	76	−70	.4	105	.1
8.01– 9.00	48	43.01–44.00	184	−65	1.0	110	.1
9.01–10.00	97	44.01–45.00	96	−60	1.4	115	.2
10.01–11.00	75	45.01–46.00	77	−55	1.3	120	.1
11.01–12.00	196	46.01–47.00	25	−50	2.4	125	.3
12.01–13.00	90	47.01–48.00	190	−45	1.6	130	.0
13.01–14.00	206	48.01–49.00	48	−40	2.7	135	.1
14.01–15.00	291	49.01–50.00	87	−35	3.5	140	.1
15.01–16.00	103	50.01–51.00	30	−30	4.2	145	.0
16.01–17.00	182	51.01–52.00	13	−25	6.1	150	.3
17.01–18.00	471	52.01–53.00	12	−20	5.2	over	
18.01–19.00	259	53.01–54.00	44	−15	10.9	150	.2
19.01–20.00	391	54.01–55.00	16	−10	5.6		
20.01–21.00	690	55.01–56.00	44	− 5	7.2		
21.01–22.00	484	56.01–57.00	5	0	5.1		
22.01–23.00	533	57.01–58.00	7	+ 5	9.2		
23.01–24.00	1229	58.01–59.00	3	10	3.1		
24.01–25.00	688	59.01–60.00	22	15	3.6		
25.01–26.00	449	60.01–61.00	13	20	2.3		
26.01–27.00	797	61.01–62.00	7	25	4.5		
27.01–28.00	658	62.01–63.00	39	30	2.8		
28.01–29.00	308	Over 63.00	97	35	2.2		
29.01–30.00	1146			40	2.0		
30.01–31.00	296			45	1.0		
31.01–32.00	340			50	2.4		
32.01–33.00	401			55	1.5		
33.01–34.00	135			60	1.0		
34.01–35.00	449			65	.5		
35.01–36.00	508			70	1.5		
36.01–37.00	136			75	.6		
37.01–38.00	220			80	.5		
38.01–39.00	253			85	.1		
Average Wages 27.88		Total No. of Cases 13,853		Total 100.1			

DISTRIBUTION No. 3.

In Dollars as Reported				In Percentage Form as Grouped.			
(1) Wages in Dollars	(2) Number of Cases	(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases
5.01– 6.00	3	40.01–41.00	..	−75	.5	100	.6
6.01– 7.00	6	41.01–42.00	2	−70	.8	105	.1
7.01– 8.00	3	42.01–43.00	..	−65	.6	110	..
8.01– 9.00	9	43.01–44.00	3	−60	1.6	115	..
9.01–10.00	16	44.01–45.00	5	−55	1.6	120	.4
10.01–11.00	8	45.01–46.00	..	−50	1.8	125	.1
11.01–12.00	16	46.01–47.00	..	−45	1.9	130	.1
12.01–13.00	12	47.01–48.00	..	−40	2.6	135	..
13.01–14.00	22	48.01–49.00	2	−35	4.5	140	..
14.01–15.00	34	49.01–50.00	2	−30	3.5	145	.1
15.01–16.00	24	50.01–51.00	..	−25	4.5		
16.01–17.00	28	51.01–52.00	1	−20	7.2		
17.01–18.00	61	52.01–53.00	..	−15	5.2		
18.01–19.00	26	53.01–54.00	..	−10	9.5		
19.01–20.00	61	54.01–55.00	1	− 5	7.6		
20.01–21.00	77			0	4.0		
21.01–22.00	37			+ 5	6.4		
22.01–23.00	24			10	7.7		
23.01–24.00	54			15	2.4		
24.01–25.00	56			20	2.9		
25.01–26.00	14			25	2.6		
26.01–27.00	22			30	3.9		
27.01–28.00	18			35	3.9		
28.01–29.00	21			40	1.4		
29.01–30.00	42			45	1.8		
30.01–31.00	9			50	1.3		
31.01–32.00	11			55	2.4		
32.01–33.00	14			60	2.1		
33.01–34.00	7			65	.4		
34.01–35.00	19			70	.3		
35.01–36.00	15			75	.5		
36.01–37.00	3			80	.9		
37.01–38.00	2			85	.1		
38.01–39.00	1			90	.1		
39.01–40.00	10			95	.3		
Average Wages 22.15		Total No. of Cases 801		Total 100.2			

DISTRIBUTION No. 4.

In Dollars as Reported				In Percentage Form as Grouped			
(1) Wages in Dollars	(2) Number of Cases	(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases
7.01– 8.00	3	42.01–43.00	3	−75	.1	100	.1
8.01– 9.00	1	43.01–44.00	9	−70	.2	Over	
9.01–10.00	3	44.01–45.00	4	−65	.3	100	.6
10.01–11.00	4	45.01–46.00	4	−60	.6		
11.01–12.00	8	46.01–47.00	1	−55	1.0		
12.01–13.00	6	47.01–48.00	1	−50	1.4		
13.01–14.00	13	48.01–49.00	1	−45	2.9		
14.01–15.00	23	49.01–50.00	6	−40	2.4		
15.01–16.00	18	50.01–51.00	1	−35	3.9		
16.01–17.00	14	51.01–52.00	1	−30	3.3		
17.01–18.00	44	52.01–53.00	1	−25	9.0		
18.01–19.00	15	55.01–56.00	1	−20	6.8		
19.01–20.00	68	56.01–57.00	1	−15	4.3		
20.01–21.00	62	57.01–58.00	1	−10	9.7		
21.01–22.00	31	59.01–60.00	1	− 5	8.9		
22.01–23.00	31	74.01–75.00	2	0	5.7		
23.01–24.00	74			+ 5	5.7		
24.01–25.00	79			10	5.1		
25.01–26.00	34			15	4.6		
26.01–27.00	46			20	3.4		
27.01–28.00	40			25	3.3		
28.01–29.00	25			30	3.1		
29.01–30.00	67			35	3.8		
30.01–31.00	16			40	1.6		
31.01–32.00	17			45	1.5		
32.01–33.00	26			50	2.0		
33.01–34.00	18			55	1.1		
34.01–35.00	26			60	.5		
35.01–36.00	29			65	1.0		
36.01–37.00	10			70	.6		
37.01–38.00	10			75	.4		
38.01–39.00	11			80	.2		
39.01–40.00	17			85	.1		
40.01–41.00	6			90	.6		
41.01–42.00	5			95	.2		
Average Wages 26.11		Total No. of Cases 938		Total 100.0			

DISTRIBUTION. No 5.

In Dollars as Reported				In Percentage Form as Grouped			
(1) Wages in Dollars	(2) Number of Cases	(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases
4.01– 5.00	1	44.01–45.00	55	−85	.1	115	..
5.01– 6.00	2	45.01–46.00	5	−80	.2	120	.1
6.01– 7.00	..	49.01–50.00	11	−75	.1	125	.1
7.01– 8.00	1	52.01–53.00	8	−70	.1	130	..
8.01– 9.00	1	53.01–54.00	1	−65	.3	135	..
9.01–10.00	1	54.01–55.00	1	−60	1.0	140	.1
10.01–11.00	3	55.01–56.00	1	−55	1.2	Over	
11.01–12.00	8	56.01–57.00	2	−50	2.0	140	.5
12.01–13.00	6	59.01–60.00	4	−45	2.4		
13.01–14.00	11	63.01–64.00	1	−40	2.9		
14.01–15.00	13	64.01–65.00	1	−35	3.5		
15.01–16.00	18	69.01–70.00	1	−30	4.7		
16.01–17.00	13	79.01–80.00	2	−25	5.0		
17.01–18.00	23	82.01–83.00	1	−20	7.6		
18.01–19.00	15	89.01–90.00	1	−15	10.9		
19.01–20.00	44	99.01–100.00	1	−10	6.4		
20.01–21.00	24			− 5	6.5		
21.01–22.00	37			0	6.3		
22.01–23.00	28			+ 5	5.9		
23.01–24.00	75			10	4.1		
24.01–25.00	78			15	3.0		
25.01–26.00	43			20	5.3		
26.01–27.00	44			25	2.8		
27.01–28.00	44			30	1.7		
28.01–29.00	29			35	3.1		
29.01–30.00	70			40	2.6		
30.01–31.00	22			45	.5		
31.01–32.00	25			50	.2		
32.01–33.00	38			55	5.7		
33.01–34.00	5			60	.4		
34.01–35.00	45			65	..		
35.01–36.00	27			70	.6		
36.01–37.00	9			75	.5		
37.01–38.00	13			80	.4		
38.01–39.00	7			85	.5		
39.01–40.00	46			90	.1		
40.01–41.00	3			95	.3		
41.01–42.00	3			100	..		
42.01–43.00	4			105	.3		
43.01–44.00	..			110	.1		
Average Wages 28.75		Total No. of Cases 975		Total 100.1			

DISTRIBUTION No. 6.

In Dollars as Reported				In Percentage Form as Grouped			
(1) Wages in Dollars	(2) Number of Cases	(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases
2.01– 3.00	1	42.01–43.00	24	−90	.1	110	.5
3.01– 4.00	2	43.01–44.00	20	−85	.0	115	.1
4.01– 5.00	1	44.01–45.00	40	−80	.1	120	.1
5.01– 6.00	1	45.01–46.00	35	−75	.2	125	.0
6.01– 7.00	6	46.01–47.00	7	−70	.7	130	.5
7.01– 8.00	16	47.01–48.00	12	−65	.8	Over	
8.01– 9.00	20	48.01–49.00	12	−60	1.3	130	.5
9.01–10.00	31	49.01–50.00	34	−55	2.3		
10.01–11.00	35	50.01–51.00	32	−50	2.2		
11.01–12.00	56	51.01–52.00	4	−45	4.0		
12.01–13.00	67	52.01–53.00	4	−40	3.4		
13.01–14.00	82	53.01–54.00	7	−35	4.4		
14.01–15.00	112	54.01–55.00	10	−30	6.4		
15.01–16.00	100	55.01–56.00	8	−25	3.3		
16.01–17.00	104	56.01–57.00	2	−20	7.4		
17.01–18.00	162	57.01–58.00	2	−15	5.2		
18.01–19.00	176	58.01–59.00	1	−10	6.4		
19.01–20.00	164	59.01–60.00	10	− 5	7.5		
20.01–21.00	156	60.01–61.00	10	0	4.0		
21.01–22.00	141	61.01–62.00	2	+ 5	4.2		
22.01–23.00	132	62.01–63.00	1	10	4.2		
23.01–24.00	179	63.01–64.00	..	15	6.0		
24.01–25.00	226	64.01–65.00	..	20	2.8		
25.01–26.00	161	65.01–66.00	1	25	3.2		
26.01–27.00	112	66.01–67.00	2	30	2.1		
27.01–28.00	122	68.01–69.00	1	35	3.6		
28.01–29.00	126	69.01–70.00	1	40	2.1		
29.01–30.00	156	70.01–71.00	1	45	.9		
30.01–31.00	138	71.01–72.00	1	50	1.6		
31.01–32.00	79	74.01–75.00	2	55	1.5		
32.01–33.00	88	75.01–76.00	2	60	.9		
33.01–34.00	72	77.01–78.00	1	65	.7		
34.01–35.00	79	80.01–81.00	1	70	1.3		
35.01–36.00	99	90.01–91.00	1	75	.9		
36.01–37.00	62	98.01–99.00	1	80	.3		
37.01–38.00	32	99.01–100.00	1	85	.5		
38.01–39.00	32			90	1.4		
39.01–40.00	50			95	.4		
40.01–41.00	44			100	.2		
41.01–42.00	26			105	.2		
Average Wages 26.14		Total No. of Cases 3,741		Total 100.4			

DISTRIBUTION No. 7.

In Dollars as Reported				In Percentage Form as Grouped			
(1) Wages in Dollars	(2) Number of Cases	(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per-cent. of Cases
5.01– 6.00	1	40.01–41.00	27	−80	.2	95	1.2
6.01– 7.00	1	41.01–42.00	22	−75	.1	Over	
7.01– 8.00	1	42.01–43.00	15	−70	.1	95	.6
8.01– 9.00	2	43.01–44.00	38	−65	.4		
9.01–10.00	1	44.01–45.00	40	−60	.7		
10.01–11.00	2	45.01–46.00	14	−55	1.9		
11.01–12.00	3	46.01–47.00	7	−50	2.4		
12.01–13.00	4	47.01–48.00	13	−45	7.3		
13.01–14.00	8	48.01–49.00	16	−40	4.7		
14.01–15.00	16	49.01–50.00	67	−35	8.7		
15.01–16.00	16	50.01–51.00	64	−30	4.8		
16.01–17.00	23	51.01–52.00	4	−25	5.4		
17.01–18.00	60	52.01–53.00	2	−20	4.1		
18.01–19.00	68	53.01–54.00	8	−15	4.2		
19.01–20.00	44	54.01–55.00	24	−10	6.4		
20.01–21.00	54	55.01–56.00	26	− 5	1.5		
21.01–22.00	82	56.01–57.00	7	0	3.3		
22.01–23.00	56	57.01–58.00	2	+ 5	2.4		
23.01–24.00	54	58.01–59.00	3	10	4.1		
24.01–25.00	60	59.01–60.00	4	15	4.7		
25.01–26.00	32	60.01–61.00	3	20	5.4		
26.01–27.00	40	61.01–62.00	1	25	2.4		
27.01–28.00	45	62.01–63.00	2	30	2.4		
28.01–29.00	35	63.01–64.00	11	35	3.9		
29.01–30.00	55	64.01–65.00	10	40	.8		
30.01–31.00	50	65.01–66.00	1	45	1.7		
31.01–32.00	14	67.01–68.00	2	50	7.6		
32.01–33.00	26	68.01–69.00	3	55	1.2		
33.01–34.00	30	69.01–70.00	1	60	.4		
34.01–35.00	22	70.01–72.00	1	65	2.6		
35.01–36.00	42	74.01–75.00	1	70	1.3		
36.01–37.00	35	75.01–76.00	2	75	.3		
37.01–38.00	26			80	.5		
38.01–39.00	64			85	.1		
39.01–40.00	57			90	.4		
Average Wages 33.05		Total No. of Cases 1,570		Total 100.2			

DISTRIBUTION No. 8.

In Dollars as Reported				In Percentage Form as Grouped			
(1) Wages in Dollars	(2) Number of Cases	(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases
3.01– 4.00	2	38.01–39.00	38	−85	.1	90	.8
4.01– 5.00	2	39.01–40.00	34	−80	.3	95	.1
5.01– 6.00	8	40.01–41.00	30	−75	.6	100	..
6.01– 7.00	13	41.01–42.00	13	−70	.9	105	.4
7.01– 8.00	16	42.01–43.00	8	−65	1.3	110	.3
8.01– 9.00	20	43.01–44.00	10	−60	1.5	115	..
9.01–10.00	28	44.01–45.00	19	−55	2.6	120	..
10.01–11.00	30	45.01–46.00	12	−50	1.8	125	.1
11.01–12.00	43	46.01–47.00	2	−45	3.5	130	.3
12.01–13.00	46	47.01–48.00	10	−40	2.3	135	.1
13.01–14.00	42	48.01–49.00	10	−35	3.2	140	..
14.01–15.00	70	49.01–50.00	12	−30	5.5	145	.1
15.01–16.00	60	50.01–51.00	12	−25	6.0	150	..
16.01–17.00	36	51.01–52.00	2	−20	4.9	Over	
17.01–18.00	102	52.01–53.00	2	−15	4.3	150	.7
18.01–19.00	107	53.01–54.00	4	−10	5.4		
19.01–20.00	102	54.01–55.00	8	− 5	7.3		
20.01–21.00	123	55.01–56.00	5	0	2.8		
21.01–22.00	100	56.01–57.00	1	+ 5	4.6		
22.01–23.00	82	59.01–60.00	5	10	9.9		
23.01–24.00	98	60.01–61.00	6	15	7.4		
24.01–25.00	154	61.01–62.00	2	20	2.0		
25.01–26.00	106	62.01–63.00	1	25	2.0		
26.01–27.00	62	64.01–65.00	2	30	3.1		
27.01–28.00	91	65.01–66.00	2	35	4.8		
28.01–29.00	161	68.01–69.00	2	40	.9		
29.01–30.00	210	69.01–70.00	2	45	2.4		
30.01–31.00	130	71.01–72.00	1	50	1.7		
31.01–32.00	43	74.01–75.00	4	55	1.1		
32.01–33.00	36	75.01–76.00	4	60	.5		
33.01–34.00	32	79.01–80.00	1	65	.5		
34.01–35.00	92	81.01–82.00	1	70	.9		
35.01–36.00	102	89.01–90.00	1	75	.2		
36.01–37.00	26	98.01–99.00	2	80	.4		
37.01–38.00	36	99.01–100.00	1	85	.4		
Average Wages 26.34		Total No. of Cases 2,680		Total 100.0			

DISTRIBUTION No. 9.

In Dollars as Reported				In Percentage Form as Grouped			
(1) Wages in Dollars	(2) Number of Cases	(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases
3.01– 4.00	1	38.01–39.00	4	−85	.1	90	.3
4.01– 5.00	1	39.01–40.00	3	−80	..	95	.3
5.01– 6.00	4	40.01–41.00	2	−75	.6	100	.2
6.01– 7.00	5	41.01–42.00	2	−70	.9	105	.2
7.01– 8.00	5	42.01–43.00	2	−65	.7	110	.2
8.01– 9.00	6	43.01–44.00	1	−60	.9	115	..
9.01–10.00	10	44.01–45.00	1	−55	2.1	120	.2
10.01–11.00	13	47.01–48.00	1	−50	1.6	125	..
11.01–12.00	23	49.01–50.00	2	−45	5.1	130	.2
12.01–13.00	25	50.01–51.00	1	−40	2.5	135	.3
13.01–14.00	14	60.01–61.00	1	−35	2.4	Over	
14.01–15.00	24	70.01–71.00	1	−30	5.2	135	.5
15.01–16.00	26	75.01–76.00	1	−25	3.0		
16.01–17.00	22			−20	4.9		
17.01–18.00	39			−15	7.3		
18.01–19.00	40			−10	5.4		
19.01–20.00	36			− 5	6.4		
20.01–21.00	40			0	7.2		
21.01–22.00	45			+ 5	5.5		
22.01–23.00	36			10	4.8		
23.01–24.00	27			15	6.6		
24.01–25.00	41			20	4.2		
25.01–26.00	31			25	2.1		
26.01–27.00	15			30	3.9		
27.01–28.00	19			35	3.9		
28.01–29.00	25			40	3.4		
29.01–30.00	24			45	1.3		
30.01–31.00	16			50	.6		
31.01–32.00	6			55	.7		
32.01–33.00	4			60	.7		
33.01–34.00	4			65	1.3		
34.01–35.00	6			70	.7		
35.01–36.00	8			75	.3		
36.01–37.00	4			80	.6		
37.01–38.00	2			85	.6		
Average Wages 21.44		Total No. of Cases 669		Total 99.9			

DISTRIBUTION No. 10.

In Dollars as Reported			In Percentage Form as Grouped		
(1) Wages in Dollars	(2) Number of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases	(3) Per cent. Variation from Ave. Wage	(4) Per cent. of Cases
5.01– 6.00	3	−75	.2	125	.2
6.01– 7.00	1	−70	.4	130	..
7.01– 8.00	4	−65	.4	135	.2
8.01– 9.00	4	−60	.8	Over	
9.01– 10.00	5	−55	.8	135	.8
10.01– 11.00	6	−50	1.0		
11.01– 12.00	23	−45	1.4		
12.01– 13.00	10	−40	4.2		
13.01– 14.00	13	−35	2.0		
14.01– 15.00	65	−30	2.4		
15.01– 16.00	19	−25	12.3		
16.01– 17.00	45	−20	4.4		
17.01– 18.00	91	−15	8.1		
18.01– 19.00	19	−10	16.6		
19.01– 20.00	28	− 5	6.3		
20.01– 21.00	20	0	5.1		
21.01– 22.00	12	+ 5	4.2		
22.01– 23.00	10	10	2.8		
23.01– 24.00	15	15	2.0		
24.01– 25.00	34	20	2.6		
25.01– 26.00	12	25	5.1		
26.01– 27.00	6	30	4.2		
27.01– 28.00	7	35	1.8		
28.01– 29.00	2	40	1.2		
29.01– 30.00	13	45	1.0		
30.01– 31.00	1	50	1.2		
31.01– 32.00	2	55	1.6		
32.01– 33.00	5	60	.4		
33.01– 34.00	1	65	.4		
34.01– 35.00	6	70	.8		
35.01– 36.00	2	75	.4		
37.01– 38.00	2	80	1.0		
39.01– 40.00	2	85	.4		
42.01– 43.00	1	90	..		
43.01– 44.00	1	95	.4		
45.01– 46.00	1	100	..		
49.01– 50.00	1	105	.4		
59.01– 60.00	1	110	..		
64.01– 65.00	1	115	..		
100.01–101.00	1	120	.2		
Average Wages 19.06	Total No. of Cases 495		Total 99.7		

APPENDIX A. PART II-A

WEIGHTED AVERAGE OF MOMENTS OF ALL DISTRIBUTIONS

Weighting of ν_2

(Column 1) Dist. of Wage Serial No.	(Column 2) ν_2	(Column 3) Number of Cases	(Column 4) (2) x (3)
1	38.0186	10,771	409,498.3406
2	51.3220	13,853	710,963.6660
3	47.8390	801	38,319.0390
4	39.0044	938	36,586.1272
5	46.0310	975	44,880.2250
6	59.5380	3,741	222,731.6580
7	58.1040	1,570	91,223.2800
8	59.0250	2,680	158,187.0000
9	52.0720	669	34,836.1680
10	46.1080	495	22,823.4600
Grand Total....	..	36,493	1,770,048.9638

$$\text{Ave. } \nu_2 = \frac{1,770,048.9638}{36,493}$$

$$= 48.504$$

APPENDIX A. PART II-B

WEIGHTED AVERAGE OF MOMENTS OF ALL DISTRIBUTION

Weighting of ν_3

(Column 1) Dist. of Wage Serial No.	(Column 2) ν_3	(Column 3) Number of Cases	(Column 4) (2) x (3)
1	86.1575	10,771	928,002.4325
2	364.8115	13,853	5,053,733.7095
3	223.9770	801	179,405.5770
4	147.5683	938	138,419.0654
5	302.3160	975	294,758.1000
6	401.3390	3,741	1,501,409.1990
7	216.4130	1,570	339,768.4100
8	408.9470	2,680	1,095,977.9600
9	270.5410	669	180,991.9290
10	422.4620	495	209,118.6900
Grand Total....	..	36,493	9,921,585.0724

$$\text{Ave. } \nu_3 = \frac{9,921,585.0724}{36,493}$$

$$= 271.8764$$

APPENDIX A. PART II-C

Weighted Average of Moments of All Distributions

Weighting of ν_4

(Column 1) Dist. of Wage Serial No.	(Column 2) ν_4	(Column 3) Number of Cases	(Column 4) (2) x (3)
1	4,988.7795	10,771	53,734,143.9945
2	13,151.0647	13,853	182,181,699.2891
3	8,986.7840	801	7,198,413.9840
4	5,839.8423	938	5,477,772.0774
5	10,235.4420	975	9,979,555.9500
6	13,232.1390	3,741	49,501,431.9990
7	8,038.4450	1,570	12,620,358.6500
8	17,377.2270	2,680	46,570,968.3600
9	11,764.9120	669	7,870,726.1280
10	12,600.8480	495	6,237,419.7600
Grand Total....	..	36,493	381,372,490.1920

$$\text{Ave. } \nu_4 = \frac{381,372,490.1920}{36,493}$$

$$= 10,450.566$$

APPENDIX B

RESULTS OF TESTS ON COMPOSITE WAGE DISTRIBUTION CONSTRUCTED BY FORMULA

No. of Tests (1)	State (2)	Policy Year (3)	Industrial Group (4)	Was Data Used in Curve (5)	Average Wage (6)	Legal Per cent. of Comp. (7)	Compensation Limits (8)	Limit Fact. on Actual Dist. (9)	Limit Fact. on Stand. Dist. (10)	Differential (11)	Per cent. Variation from Actual (12)
1	Wisc.	1919	All	No	22.47	65	6.83–16.90	.938	.926	−.012	−1.28
2	Tenn.	1919	All	Yes	19.06	50	5.00–11.00	.916	.927	+.011	+1.20
3	(a) N. Y.	1920 (July-Dec.)	Cont.	No	39.10	66⅔	8.00–20.00	.744	.728	−.016	−2.15
3	(b) "	" " "	"	"	"	"	4.00–10.00	.383	.383	.000	.00
3	(c) "	" " "	"	"	"	60	3.00–18.00	.740	.728	−.012	−1.62
4	(a) "	1919 (Jan.-June)	A. O.	Yes	21.44	66⅔	8.00–20.00	.976	.978	+.002	+.20
4	(b) "	" " "	"	"	"	"	4.00–10.00	.668	.675	+.007	+1.05
4	(c) "	" " "	"	"	"	60	3.00–18.00	.965	.970	+.005	+.52
5	(a) "	1919 (July-Dec.)	Mfg.	No	30.03	66⅔	8.00–20.00	.876	.867	−.010	−1.14
5	(b) "	" " "	"	"	"	"	4.00–10.00	.496	.499	+.003	+.61
5	(c) "	" " "	"	"	"	60	3.00–18.00	.876	.866	−.010	−1.14
6	(a) "	1921 (Jan.-June)	A. O.	No	25.88	66⅔	8.00–20.00	.939	.926	−.013	−1.38
6	(b) "	" " "	"	"	"	"	4.00–10.00	.573	.571	−.002	−.35
6	(c) "	" " "	"	"	"	60	3.00–18.00	.934	.924	−.011	−1.18

APPENDIX C

LIMIT FACTORS FOR THE STATE OF MASSACHUSETTS

OLD FATAL—PRESENT SPECIFIC DISMEMBERMENT

Average Wage $22.46 No. of cases
Compensation rate 66⅔% Limits $4 and $ 10
Wages at which limits become effective are $6 and $15
(a) $6 represents a—73.29 % deviation from average wage.
(b) $15 " "—33.21 % " " " "
Interpolation formula $f(x') = f(x) - s/t\,\Delta f(x)$

A. FOR THE LOWER LIMIT

1. s	= 3.29	2. s/t	= .658
3. From (Col. 4) $f(x)$ =	.65	4. From (Col. 5) $f(x)$	= 48.20
(Col. 2) (2)		(Col. 3) (2)	
5. $s/t\,\Delta f(x)$ =	.33	6. $s/t\,\Delta f(x)$ =	23.10
× .658	= .22	× .658	= 15.20
7. $f(x')$	= .43	8. $f(x')$	= 33.00
9. (a) × (7) = 73.29 × .43 =		.....	= 31.51
Effect of lower limit =		.....	+ 1.49

B. FOR THE UPPER LIMIT WHEN THE WAGE AT WHICH IT BECOMES EFFECTIVE IS GREATER THAN THE AVERAGE WAGE.

1. s	=	2. s/t	=
3. From (Col. 4) $f(x)$	=	4. From (Col. 5) $f(x)$	=
(Col 2) (2)		(Col. 3) (2)	
5. $s/t\,\Delta f(x) = x$	=	6. $s/t\,\Delta f(x)\,x$	=
7. $f(x')$	=	8. $f(x')$	=
9. (b) × (7)	=	.....	=

FOR THE UPPER LIMIT WHEN THE WAGE AT WHICH IT BECOMES EFFECTIVE FALLS BELOW THE AVERAGE WAGE

1.	s	= 3.21	2.	s/t	=	.642
3.	From (Col. 4) $f(x)$ (Col. 2) (2)	= 21.52				
4.	$s/t\,\Delta f(x)$ = 5.04 × .642	= 3.24				
5.	$f(x')$	= 18.28 from 51.85 = 33.57				
6.	= (5) + 5.91* + 42.24*	= 81.72				
7.	= (6) × (b) =	= 81.72 × (−33.21) = − 2,713.92				
3.	Total above average	= 1,344.10*				
4.	Total below average	= 1,349.70*				
5.	From (Col. 5) $f(x)$ (Col. 3) (2)	= 901.70				
6.	$s/t\,\Delta f(x)$ = 151.20 × .642	= 97.07				
7.	$f(x')$	= 804.63				
8.	(4) − (7)	= 545.07				
9.	(3) − (8)	= 799.03				
10.	Net effect of upper limit	= 3,512.95				
A	Effect of lower limit	= + 1.49				
	Net effect of limits	= − 3,511.46				

Limit factor = 100 − 3511.46/100 = 64.89%

*Fixed value printed in form.