PREMIUMS AND RESERVES FOR DEFERRED PAYMENT PROTECTION

BY

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As deferred payment protection was clearly described before this Society by Mr. R. O. Davidson at the Fall meeting in 1926 (PROCEEDINGS, Vol. XIII, Page 80) brief reference only will be made to the nature of the benefits.

The contract is made between the creditor and the insurance company. Its purpose is to prevent the creditor from suffering any financial loss due to the accidental death or to the disability of any debtor who has purchased an article under one of the established time payment plans.

Accordingly these benefits are divisible into two main parts:

1. Accidental Death. In event of this contingency the liability of the insurance company is the sum of all monthly notes outstanding but not due at time of death.

2. Disability. In event the purchaser becomes totally disabled by either accident or disease the insurance company pays the creditor a proportionate amount of the monthly notes for the period of disability.

In this connection it should be noted that the amount paid does not depend upon whether the notes fell due during the period of total disability. For example, the debtor or purchaser may be ill for a period of one and one-half months. The amount to be paid by the insurance company (on the retroactive basis) would be equal to one and one-half monthly payments regardless of whether one note or two notes fell due during the period of disability.

Calculations are here made on the following two bases:

A. Non-retroactive—No payments are made for the first fourteen days of any disability.

B. Retroactive—Payments are made from the first day of disability but no disability of less than fifteen days' duration is covered.

Some time ago the question of rates on this plan of insurance was referred to the Committee of Five on Statistics of the Bureau of Personal Accident and Health Underwriters of which committee the writer is a member. The premium rates produced

here differ only slightly from those produced by the Committee. These differences are of a theoretical nature and will be referred to in due course.

In preparing any set of premiums the basic data upon which the premiums are based is extremely important. In the matter of Deferred Payment Protection, sufficient statistics are not available to determine what the correct pure premium should be.

Perhaps at the present time we could do no better than to use the available experience under accident and health policies of the Bureau of Personal Accident and Health Underwriters.

In 1921 the Committee of Five on Statistics collected data for the computation of accident premiums. Using the Ordinary classification the results gave a pure premium of $.00116 for each $1.00 of accidental death benefits and of $.2370 for each $1.00 of weekly indemnity with coverage beginning from the first day of disability and with fifty-two weeks' maximum claim. This, therefore, would give a rate of $.0547 for each $1.00 per month.

For the health weekly indemnity the report of this same Committee to the Bureau covering the combined health experience for the years 1921 to 1923 inclusive is used. The policy form 02 providing fifty-two weeks' limit with full indemnity for total disability irrespective of house confinement shows a premium of $.70 for each $1.00 of weekly indemnity or $.1615 for each $1.00 of monthly income.

From the above we obtain an annual combined accident and health premium of $.2162 for a monthly indemnity of $1.00 with fifty-two weeks' limit or a monthly premium of $.0180. The monthly premium for accident principal sum coverage of $1.00 is $.0001.

Taking as a unit a monthly income of $1.00, it will be noted that the last month, the last two months, etc., of any term are exactly equivalent to the corresponding periods of any other term. Therefore, by computing the premiums separately for the first month of each term where the terms run from one month up, the premium for any period desired may be obtained by summation.

While the calculation of any premium on a theoretically correct basis will involve a discount of the expected payments from the date payment is to be made to the date the premium is to be collected, the nature of the data did not seem to warrant such a

refinement. The amount of this discount would be appreciable in the case of an eleven year term but as such a policy is approaching a non-cancellable accident and health policy care must be exercised not to adopt premiums that may prove to be inadequate. Consequently, the effect of interest has been disregarded except in the calculation of single premiums for terms over two years.

When interest is disregarded the premium for any term is the same proportion of a year's premium as the actual days disability expected from the given term is to the total provided by the full year's premium.

In Table I, which is taken from the report of the Committee of Five referred to above, the percentage of liability due to the first day, the first two days, the first three days, etc., is shown. This table is used for both accident and health.

As an illustration of the meaning of this table, it will be noted that opposite 7 days 31.48 % is shown. This means that if the disability benefits were restricted to a maximum of one week instead of 52 weeks the pure premium required for such a policy would be 31.48% of the amount required for the 52 weeks' limit policy.

TABLE I

Number of Days	Percentage of Total Year's Claims Included in Number of Days Shown	Number of Days	Percentage of Total Year's Claims Included in Number of Days Shown
1	4.88%	16	53.14%
2	9.72	17	54.65
3	14.47	18	56.06
4	19.05	19	57.40
5	23.41	20	58.69
6	27.55	21	59.94
7	31.48	22	61.01
8	34.77	23	62.04
9	37.80	24	63.03
10	40.61	25	63.98
11	43.14	26	64.90
12	45.49	27	65.79
13	47.70	28	66.65
14	49.81	29	67.39
15	51.53	30	68.11

The pure disability premium for a one month term would, therefore, be obtained by multiplying the monthly premium for a 52 weeks' limit policy by the average of the percentages in Table I. This average is found to be 46.81%.

By a similar method we construct Table II showing the percentage of claims of 52 weeks' limit which were actually incurred within the number of months indicated, no claim being carried beyond the end of the term.

TABLE II

PERCENTAGE OF 52 WEEKS' COVERAGE INCURRED WITHIN TERM UNDER CLAIMS INCURRED DURING FIRST MONTH OF TERM

Term in Months	Average Percentage
1	46.81%
2	76.11
3	84.50
4	88.64
5	91.31
6	93.26
7	94.81
8	96.08
9	97.14
10	98.07
11	98.91
12	99.67

In developing the rates for terms of more than twelve months it is necessary to find a measure for the claims that may be expected to extend beyond 52 weeks. For the purpose of projecting these claims we have used Table VIII in Mr. Cammack's paper entitled "Premiums and Reserves for Non-Cancellable Accident and Health Policies" (PROCEEDINGS, Vol. VII, Page 298) using age 40 as an average age.

In using this table the ratios of claims lasting 2, 3, etc., years to those lasting at least one year were obtained, these ratios being shown in Col. 2 of Table III below.

As the experience which we have been using shows 273 claims disabled for a full year the ratios obtained as above are applied to this number in order to get the number of claims that may be expected to last for the term of years indicated. These results are shown in Col. 3 of Table III.

TABLE III

Disabilities Lasting More Than One Year		
(1) Term in Years	(2) Percentage of One Year Claims Running for Term	(3 Number of Claims 273 × Col. (2)
1	100 %	273.
2	48.3	131.9
3	39.8	108.7
4	33.9	92.5
5	29.8	81.4
6	27.7	75.5
7	25.0	68.3
8	22.9	62.5
9	21.2	57.9
10	19.1	52.1
11	16.6	45.3

It was felt that single premiums should be prepared for each term of months from 1 to 24 inclusive. In order to do this it is necessary to further analyze the claims in Table III according to their duration by individual months. To do this the claims for one year and two years were interpolated using constant first differences. If the basic data were known to be accurate no doubt a better method for obtaining the values at intervening months would be either to plot the known values on graphic paper interpolating directly from the graph for the intervening values or to develop a frequency curve that would yield the number of claims direct by the substitution of the term. Using constant first differences as indicated above the number of claims by successive months are found as shown in Table IV below.

TABLE IV

Period of Dis. Term in Months	No. of Claims	Period of Dis. Term in Months	No. of Claims
12	273	19	190.7
13	261.2	20	178.9
14	249.5	21	167.2
15	237.7	22	155.4
16	226.0	23	143.7
17	214.2	24	131.9
18	202.5		

To obtain the multiplying factor for terms from 12 to 24 months let us consider a term of T months where T is some number between 12 and 24. The premium for the first month of this term will be considered in three separate parts: First, the amount of

claim that can be incurred in a month and last for one year or less; Second, the amount of claim that can be more than one year in duration but not more than $(T-1)$ months; Third, the amount of claim that can be experienced in the final or Tth month of the term.

As the basic data furnishes a measure of the first part it remains only to find the ratio by which the last two parts increase the coverage of one year's limit.

The number of days claims due to the second part will obviously be found from the following formula:

$$\frac{\text{No. claims duration 12 months} + \text{No. claims duration } (T-1) \text{ months}}{2} \left[(T-1) - 12 \right] \frac{365}{12}$$

The number of days due to the third part is found approximately by the formula

$$\left[\frac{\text{No. claims duration } (T-1)}{3} + \frac{\text{No. claims duration } T}{6} \right] \frac{365}{12}$$

If it be assumed that the duration of claims is a continuous function varying by moments of time rather than by a unit of one day the application of integral calculus will show the above formula to be exact. When the three results are combined and the ratio taken to the number of days disability within one year the result is the factor by which the premium for one year's limit must be multiplied. These results are shown in Col. 2 of Table V for terms from 13 to 24 months.

LOADING

In computing these premiums it has been assumed that the cost of Acquisition would be 15% and that other expenses, H. O. Administration, Taxes and Claim Expense would total 23½% giving a total loading of 38½% of the gross premium.

Without doubt, the cost of handling business covering short terms only with the resulting small premiums is proportionately much greater than that of handling business of longer terms and higher premiums. Accordingly I have added in an arbitrary flat amount of 50c. for each $100 of monthly income or ½c. for each $1.00 of monthly income.

The Committee of Five in their rates made some allowance for this extra expense on claims of shorter term by assuming that a claim incurred any time during a month could continue for the same length of time as if incurred on the first day of that month.

By taking the Ordinary classification for the accident benefits allowance is made for the impossibility of selecting the risks individually. In the case of health weekly indemnity, however, the data is based on all risks combined and it would seem that some little margin should probably be allowed for adverse experience. Accordingly an additional loading factor of 5% of the gross premium has been inserted. This, therefore, gives a total adjusting factor of 43.5%.

The calculation of the premiums on this basis is shown in Table V.

Column III of Table V is found by deducting from Column II the percentage of one year's claims included within the first fourteen days. In a term of one month, because of the short length of time that any claim may run which has been incurred in the last half of the month this amount is only 40.90%. After the first month the amount is 49.81%.

It will be noted that the rates are slightly higher than those prepared by the Committee of Five. This is because the margin for the flat expense allowed in these calculations is somewhat greater than that allowed by the Committee.

The adjustment for the retroactive feature is made as follows: In the report of the Committee 60,478 claims have been analyzed according to duration. The total days disability suffered under these claims during the year was 1,239,333.

Of the 60,478 claims 4,798 lasted exactly fourteen days and 21,332 lasted for a period of 15 days or longer. If we made an adjustment for those claims only which lasted 15 days or longer this would add 298,648 days which is 24.1% of a full year's claims. This would call for an increase in the premium of $.0521. Without doubt, however, many claimants who return to work at the end of 14 days or even less would be inclined to present claim for at least 15 days if that meant that they would get 15 days' coverage while if they returned to work at the end of 14 days they would get nothing. This would be especially true in many cases where the recovery of the claimant is so gradual that a day or so more or less could not be considered as fraudulent. It is,

TABLE V

CALCULATION OF PREMIUMS ON ACCIDENT AND HEALTH DEFERRED PAYMENT PROTECTION BY INDIVIDUAL MONTHS OF EXPOSURE BASED ON INDEMNITY OF $1.00 PER MONTH

Non-Retroactive

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Term in Mos.	% of 1st Yrs. Claims from 1st day within term	Adj. for 2 weeks out	Single Dis. Prem. for 1st month of term: Col. (3) by .0180	Acc. Death Prem. for 1st month of term	Total Single Prem. for 1st month of term: Col. (4) plus Col. (5)	Col. (6) adjusted for flat expense of $.0050	Gross Single Prem. for 1st mo. of term Col. (7) / .565	Gross Rate: Sum of Col. (8)	Comm. of Five Rates
1	46.81	5.91	.0011	.0001	.0012	.0062	.0110	.0110	.0060
2	76.11	26.30	.0047	.0002	.0049	.0049	.0087	.0197	.0165
3	84.50	34.69	.0062	.0003	.0065	.0065	.0115	.0312	.0288
4	88.64	38.83	.0070	.0004	.0074	.0074	.0131	.0443	.0425
5	91.31	41.50	.0075	.0005	.0080	.0080	.0142	.0585	.0570
6	93.26	43.45	.0078	.0006	.0084	.0084	.0149	.0734	.0722
7	94.81	45.00	.0081	.0007	.0088	.0088	.0156	.0890	.0880
8	96.08	46.27	.0083	.0008	.0091	.0091	.0161	.1051	.1042
9	97.14	47.33	.0085	.0009	.0094	.0094	.0166	.1217	.1211
10	98.07	48.26	.0087	.0010	.0097	.0097	.0172	.1389	.1384
11	98.91	49.10	.0088	.0011	.0099	.0099	.0175	.1564	.1561
12	99.67	49.86	.0090	.0012	.0102	.0102	.0181	.1745	.1742
13	100.33	50.52	.0091	.0013	.0104	.0104	.0184	.1929	.1926
14	100.97	51.16	.0092	.0014	.0106	.0106	.0188	.2117	.2113
15	101.58	51.77	.0093	.0015	.0108	.0108	.0191	.2308	.2304
16	102.17	52.36	.0094	.0016	.0110	.0110	.0195	.2503	.2499
17	102.72	52.91	.0095	.0017	.0112	.0112	.0198	.2701	.2697
18	103.25	53.44	.0096	.0018	.0114	.0114	.0202	.2903	.2899
19	103.74	53.93	.0097	.0019	.0116	.0116	.0205	.3108	.3104
20	104.21	54.40	.0098	.0020	.0118	.0118	.0209	.3317	.3313
21	104.65	54.84	.0099	.0021	.0120	.0120	.0212	.3529	.3526
22	105.06	55.25	.0099	.0022	.0121	.0121	.0214	.3743	.3740
23	105.44	55.63	.0100	.0023	.0123	.0123	.0218	.3961	.3957
24	105.79	55.98	.0101	.0024	.0125	.0125	.0221	.4182	.4179

of course, impossible to say to what extent the claims would be increased by this adverse selection. If all claims that lasted 14 days were extended to 15 this would further increase the claims by 71,970 days which would call for a pure premium of $.0126. No doubt, this would be too great an adjustment. If then we arbitrarily take $.0075 the results would probably be reasonably close. This will give a total adjustment of premium of $.0596 which is an increase on the monthly basis of $.0050.

Since any claim incurred during the last 14 days of the last month of any term would receive no indemnity under the non-retroactive basis and under the retroactive would receive indemnity for not more than the number of days from date of disability to the end of the term only, provided disability lasted for at least 15 days the adjustment to be added for the last month of the term is $.0035 instead of $.0050.

The rates on the retroactive basis are constructed as shown in Table VI.

While the resulting rates are greater for all terms than those produced by the Committee, they are in nearly every instance less than 1% greater.

For terms over two years both single and annual premiums are calculated. As a claim incurred in the first year has the possibility of involving one year's payments more than a claim incurred in the second year it follows that the annual premiums are of a decreasing nature. It would be possible to compute level annual premiums but this would either involve negative reserves or a policy so drawn as to limit the liability in the earlier years to an amount not greater than the average liability for each year. Such a procedure would appear to involve more complications than the use of decreasing annual premiums.

In the following table interest is disregarded in the calculation of annual premiums but the single premiums are obtained by discounting the annual premiums with interest at 4% per annum. The factors in Column 2 are found in the same general way as those in Column 2 of Table V.

In Tables VII and VIII the annual premiums are given in the reverse order in which they are paid. That is, the first premium is paid when the remaining term is one year, the second premium when the remaining term is two years, etc.

TABLE VI

CALCULATION OF PREMIUMS ON ACCIDENT AND HEALTH DEFERRED PAYMENT PROTECTION BY INDIVIDUAL MONTHS OF EXPOSURE BASED ON INDEMNITY OF $1.00 PER MONTH

Retroactive

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Term in Mos.	Single Pure Prem. for 1st Mo. of Term Non-Retroactive Table V Col. (6)	Adjust. for Retroactive	Col. (2) + Col. (3)	Col. (4) adjusted for flat exp. of $.0050	Gross Single Prem. for 1st Mos. of Terms Col. (5) / .565	Gross Rates sum of Col. (6)	Comm. of Five Rates
1	.0012	.0035	.0047	.0097	.0172	.0172	.0145
2	.0049	.0050	.0099	.0099	.0175	.0347	.0335
3	.0065	.0050	.0115	.0115	.0204	.0551	.0543
4	.0074	.0050	.0124	.0124	.0219	.0770	.0765
5	.0080	.0050	.0130	.0130	.0230	.1000	.0995
6	.0084	.0050	.0134	.0134	.0237	.1237	.1232
7	.0088	.0050	.0138	.0138	.0244	.1481	.1474
8	.0091	.0050	.0141	.0141	.0250	.1731	.1722
9	.0094	.0050	.0144	.0144	.0255	.1986	.1975
10	.0097	.0050	.0147	.0147	.0260	.2246	.2234
11	.0099	.0050	.0149	.0149	.0264	.2510	.2496
12	.0102	.0050	.0152	.0152	.0269	.2779	.2761
13	.0104	.0050	.0154	.0154	.0273	.3052	.3030
14	.0106	.0050	.0156	.0156	.0276	.3328	.3303
15	.0108	.0050	.0158	.0158	.0280	.3608	.3579
16	.0110	.0050	.0160	.0160	.0283	.3891	.3858
17	.0112	.0050	.0162	.0162	.0287	.4178	.4142
18	.0114	.0050	.0164	.0164	.0290	.4468	.4428
19	.0116	.0050	.0166	.0166	.0294	.4762	.4719
20	.0118	.0050	.0168	.0168	.0297	.5059	.5012
21	.0120	.0050	.0170	.0170	.0301	.5360	.5310
22	.0121	.0050	.0171	.0171	.0303	.5663	.5609
23	.0123	.0050	.0173	.0173	.0306	.5969	.5911
24	.0125	.0050	.0175	.0175	.0310	.6279	.6218

TABLE VII
ANNUAL AND SINGLE PREMIUMS BASED ON INDEMNITY OF $1.00 PER MONTH
Non-Retroactive

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Term in Years	% of 1st yrs. Claims from 1st day within term	Col. (2) less 49.81%	Single Dis. Prem. for 1st Yr. of Term: Col. (3) by .2162	Acc. Death Prem. for 1st yr. of term	Total Single Prem. for 1st yr. of term: Col. (4) plus Col. (5)	Gross Annual Prem. Col. (6) / .565	Comm. of Five Rates	Gross Single Prem. Col. (7) Disc. @ 4%	Comm. of Five Rates
1	For calculations see Table V					.1745	.1742	.1745	.1742
2	For calculations see Table V					.2437	.2437	.4115	.4112
3	107.79	57.98	.1254	.0354	.1608	.2846	.2846	.6803	.6800
4	111.03	61.22	.1324	.0493	.1817	.3216	.3216	.9757	.9754
5	113.78	63.97	.1383	.0632	.2015	.3566	.3566	1.2947	1.2944
6	116.20	66.39	.1435	.0771	.2206	.3904	.3904	1.6353	1.6350
7	118.42	68.61	.1483	.0911	.2394	.4237	.4238	1.9960	1.9959
8	120.44	70.63	.1527	.1050	.2577	.4561	.4561	2.3753	2.3752
9	122.29	72.48	.1567	.1189	.2756	.4878	.4877	2.7717	2.7715
10	123.99	74.18	.1604	.1328	.2932	.5189	.5190	3.1839	3.1838
11	125.51	75.70	.1637	.1467	.3104	.5494	.5494	3.6107	3.6106

Both the single and annual premiums are practically identical with those of the Committee of Five.

In Col. 9 of Table V and Col. 7 of Table VI it will be noted that the gross adjustment for making the rates retroactive for a term of one year is $.1034, while for the first year of a two year term it is $.1063. As the adjustment of any term of two years or more would be the same as the adjustment for the first year of the two year term the annual premiums on the retroactive basis are formed from Table VII by adding $.1034 to the premium for the term of one year and $.1063 to the annual premium for each term of more than one year.

The single premiums are formed by discounting the annual premiums in the same manner as given in Table VII.

In Table VIII the premiums produced are slightly greater than those of the Committee of Five. This difference is due to the fact that in the arbitrary adjustment used for the retroactive feature a slightly greater amount was used than was the case in the Committee of Five rates. No claim is made, therefore, that the rates of the Committee should be increased.

The annual premiums shown in Table VII and VIII make no provision for the waiver of any premiums falling due following disability other than the fact that provision is made for disability payments to the end of the term in event the insured remains totally disabled for the full period. If it is considered that any disability extending into the following year should include a provision for the waiver of a proportionate amount of the premium, then the part of the monthly indemnity premium which is being charged for the benefits beyond the end of the year should be increased by approximately the same percentage as the next annual premium bears to the amount of disability income provided for one year. As the creditor is paying the premium to the insurance company there is probably very little reason for the inclusion of a waiver of premium feature. However, if such premiums are desired, they can readily be computed from the foregoing tables.

Reserves

The unearned premium reserves carried in connection with commercial accident and health insurance is one-half of the premiums in force. In deferred payment insurance, the maximum

TABLE VIII

ANNUAL AND SINGLE PREMIUMS BASED ON INDEMNITY OF $1.00 PER MONTH

Retroactive

(1)	(2)	(3)	(4)	(5)	(6)
Term in Years	Gross Ann. Prem. Non-Retroactive	Gross Ann. Prem.	Comm. of Five Rates	Gross Single Prem. Col. (3) disc. @4%	Comm. of Five Rates
1	.1745	.2779	.2754	.2779	.2754
2	.2437	.3500	.3449	.6172	.6097
3	.2846	.3909	.3859	.9843	.9721
4	.3216	.4279	.4228	1.3743	1.3575
5	.3566	.4629	.4579	1.7843	1.7631
6	.3904	.4967	.4917	2.2123	2.1952
7	.4237	.5300	.5249	2.6571	2.6356
8	.4561	.5624	.5574	3.1172	3.0915
9	.4878	.5941	.5890	3.5913	3.5615
10	.5189	.6252	.6202	4.0782	4.0446
11	.5494	.6557	.6506	4.5769	4.5395

liability is of a decreasing nature. Consequently, the unearned premium at the middle of the term is less than one-half of the full premium. In the case of annual premiums or of single premiums for terms of less than one year, a valuation on the basis of one-half of the premiums in force at the end of the year would be very conservative, provided the business were evenly distributed throughout the year. When the commodity involved is one of a seasonal nature the business is not likely to be evenly distributed.

For reserves in connection with single premiums the business should be grouped according to the unexpired term. The full single premiums for such unexpired terms would produce the proper reserves.

Too much detail in connection with any phase of this business would require a greater margin for expenses than has been allowed in the calculation of the premiums.

Loss reserves would seem to involve the same general principles as regular accident and health policies.

When the Committee of Five on Statistics originally produced its premiums, no thought had been given to the matter of presenting the rates before this Society. Consequently, many of the theoretical factors that could have but little bearing on the rates were disregarded. A more theoretical study would seem to justify the action of the Committee in this regard.

It should not be assumed that the rates here produced would be sufficient for all classes of risks and all localities. Without doubt, a decided factor is that of age. If there is any reason to think that many of the risks to be covered are above 45 or 50, no doubt a further allowance should be made for this.

So also in cases where a substantial percentage of the risks are women, or are of an industrial grade, or where the commodity handled is sold largely to farmers, there might be a question of the adequacy of the rates. Many other questions will arise which must be considered by the underwriter.

In concluding this paper which is based so largely on the report of the Committee of Five, I wish to record the fact that their report was produced by the complete co-operation of the full Committee.