ABSTRACT OF THE DISCUSSION OF PAPERS READ AT THE PREVIOUS MEETING

IS THE RATE MAKING PLAN THE CHIEF TROUBLE WITH COMPENSATION INSURANCE?—WINFIELD W. GREENE

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WRITTEN DISCUSSION

MR. RAINARD B. ROBBINS:

To a life insurance actuary, Mr. Greene's suggestion that workmen's compensation carriers may take lessons from life insurance companies seems quite hopeful, possibly in part because it tickles his pride. The experiences of life insurance companies in recent years with permanent total disability and group insurance would seem, however, to require some modification of Mr. Greene's statement that the life companies are free from serious controversy as to scale of premium rates. While it may be only fair to disregard permanent total disability with the thought that Mr. Greene may not have had it in mind, it would seem that group insurance qualifies for careful consideration.

In many respects group life insurance, as we know it in practice, resembles workmen's compensation insurance more than does any other part of a life insurance company's business. It is usually conducted on a one year renewable term plan and deals with groups of insured lives rather than with individual lives. It is doubtless true that there has been no major controversy in life insurance circles for a long time as to premium rates for life insurance policies on other than the renewable term plan. There has been comparatively little interest in term insurance when sold to individuals, except as this interest was for a time artificially stimulated by unfortunate methods of measuring legal expense limits. But when, in group insurance, there developed an interest in renewable term insurance on large collections of lives, at once the situation was entirely different. Reserve laws established in the infancy of life insurance in this country have resulted in a minimum basis for life insurance premiums for policies issued on other than a one year term basis. Any company which attempted to undercut this minimum found very quickly

that the consequent surplus drain in setting up the deficient premium reserve was severe indeed. But for insurance on the one year term plan, the deficient premium reserve was of little consequence.

Group insurance was very attractive from the beginning, both to the home office and to the more venturesome salesmen. But there was no serious penalty for insufficient rates. Within five years of the time this type of business became popular, the rate war had become so severe that the companies asked state officials to step in and, in addition, the competing companies formed an organization among themselves for the purpose of bringing order out of what was rapidly becoming chaos. This is the only form of life insurance in which we have direct rate regulation to any considerable extent and it seems to be the form of life insurance most like workmen's compensation insurance. It seems inevitable that rate supervision will be necessary in any form of insurance involving one year term contracts.

It is, of course, true that no attempt has been made to put a maximum limit on group life premiums. Competition seems to take care of this quite successfully. It would seem that a maximum limit on premiums is apt to appear in any form of insurance which is compulsory or practically so.

It is interesting to note that Mr. Greene's suggestion that stock companies arrange to pay dividends on workmen's compensation insurance parallels a practice in group life insurance. While minimum premium rates for group life insurance are fixed, the nonparticipating companies may, if they so desire, make premium reductions at the end of a policy year, based on experience of the risk during the year, and may make such reductions retroactive to the beginning of the policy year.

MR. CLARENCE W. HOBBS:

One who has long been accustomed to the atmosphere of Boston reads with peculiar appreciation the quotation from Browning with which Mr. Greene prefaces his essay.* But Mr. Greene is far too modest. This is not the defiant and dauntless

* *Editor's Note*—Mr. Greene introduced the reading of his paper with the following quotation:

"Dauntless the slug-horn to my lips I set,
And blew . . ."

bugle blast of a military gentleman about to be put on the spot. Rather, in view of the fact that it has aroused a keen interest in many quarters and figures somewhat prominently in the present investigation wherein the National Convention of Insurance Commissioners seeks to compel all and sundry to lay the entire matter of compensation insurance before its high judgment seat, it seems more appropriate to quote.

"Tuba mirum spargens sonum
Per sepulchra regionum
Coget omnes ante thronum."

and indeed his description of the compensation situation has a distinct "*Dies Irae*" flavor. One cannot well deny the cogency of his facts—an underwriting loss on the part of stock carriers in each and every year of the past decade, aggregating over a hundred millions and averaging over 10% of premium income: a profound insecurity, present and future: two conflicting groups of carriers fundamentally at loggerheads, whose present truce on rating policies can in the nature of things be of no long duration: a high rate of turnover in business: an expense ratio which appeals to many as unreasonably high, and which serves as the effective cause of the tendency of large assureds to flee stock insurance for non-stock insurance or "self-insurance": with the threat of monopolistic state insurance in the background. Some exception may be taken to his statement "Rate making bodies and the various state authorities having jurisdiction over premium rates have frequently been in disagreement, and time and again rate increases requested by the carriers' organizations have been denied in whole or in part." Of a few states that is true: but by and large the attitude of rate making authorities has been by no means captious, and in the crisis that has existed during the past three years has been distinctly helpful. The difficulties with the rating program of 1932 cannot justly be laid at the door of supervising officials. The deficit of the stock carriers would not have been materially reduced had each and every rating proposal been approved *in toto*. But all must agree with Mr. Greene's statement "No business is healthy when the major part of it is conducted at a loss".

The situation is unhealthy to the degree that it has led to a portentous brood of empirical remedies advocated as sufficient to

effect a cure. Mr. Greene has dealt with some of these very ably: and merely the feeling that, after all, these are important only as symptoms prevents an appreciative discussion thereof. It is not the symptoms, but the cause which is of major consequence.

Why do the stock carriers lose money on compensation insurance? That is the question of questions. Mr. Greene's view that it is not primarily the fault of the rating system is much appreciated by those who have been connected with the development of that system, and whose attempts to shift the burden of blame to other shoulders might in the eyes of the sufferers mark them as alibi-hunters. The rating system is not perfect. As it stands today, it is the result of a continuous line of experimentation. An endeavor has been made to preserve a foundation of actuarial soundness, and to plug up the leaks as rapidly as possible. It started with a system of pitching rate levels on the average of the three latest policy years. The rapidly mounting loss ratios have impelled the adoption of a one year basis. It started with an experience basis pure and simple. It has now introduced one element of projection, i.e., the projection of medical loss ratios. It started on the basis of ratios of losses to payrolls. The huge discrepancy between theoretical and actual loss ratios has led to the adoption of a system which substantially pitches the rate at the level indicated by actual loss ratios. It started with no contingency loading: it now has introduced such a loading. In a normal time, the system could hardly fail to produce reasonably satisfactory results: but neither this nor any other system could operate successfully under conditions where increased, depression-bred malingering and depression-bred wage scale reductions send loss ratios upward a deal faster than rates can possibly be stepped up.

The real difficulty with the rating system is, as Mr. Greene points out, that it is under the necessity of attempting to achieve the impossible. It serves two sets of carriers, one of which operates at an expense cost twenty points below the other, and at a loss cost which in times of depression seems somewhat lower also. The stock group, selling on commission, have the assistance of a numerous and far-flung force of solicitors, which gathers in all business which can be won merely by solicitation. To the purchaser of insurance who weighs the cost, the twenty percent. differential is a deal of an inducement. It is met in part by argu-

ments of service and advantage, the merits of which need not be discussed. But it is met only in part. The stock carriers have been under necessity of keeping the rate level very close to the bare necessities. They have used an expense loading less than their actual expense ratios, and only lately have been convinced of the need of a safety margin. Even in normal times a certain aggregate loss was inevitable: and while some companies could and did emerge with a modest underwriting profit, the majority of the business habitually went in the red. Under abnormal conditions, practically everybody lost money. They have consistently needed a higher rate: and yet saw no way to produce that higher rate without increasing the differential and expedite the flight of the more desirable lines of business. In all this, the rating organizations have been between the devil and the deep sea, and their position has been none too comfortable.

This difficulty has been often pointed out: and Mr. Greene's paper is mainly valuable not as a critique, but as coming forth with a concrete suggestion of a remedy. Naturally this is the vulnerable point. So long as one sticks to criticism merely, one is on safe ground. But the moment he offers his remedy, then he exposes himself to the assaults of all upon whose toes he has stepped. And unfortunately there is no remedy for the situation that does not involve a relentless treading on some very tender corns.

Briefly, Mr. Greene's remedy is to take a leaf out of the book of life insurance. He is charmed, as many another has been with the solid actuarial character of life insurance. Life insurance is not without its woes, and the failure of life insurance companies of substantial size has occurred during the current year somewhat more frequently than is at all pleasant. But the woes of life insurance companies are connected almost exclusively with their investments and with the economic crisis which has rendered it necessary for them to finance extensive loans and cash surrenders. Their rate structure is as solid as rating structure well can be, and produces a substantial underwriting profit year in and year out. It has attained to a standard which insures soundness, and at the same time has preserved a competitive element. It can adjust its rate schedules and its dividend schedules, and regulate its expense loading with relative freedom.

Now the policy of the stock casualty companies has been to

avoid competition by the adoption of a single set of rates, binding on all carriers, officially at least. Mr. Greene is probably correct to a degree in saying that the attempt even when backed up by laws, merely means that the competitive element is introduced surreptitiously in the form of cutting corners on the manual, misclassifications and playing tricks with payroll audits. Such practices certainly are not unknown, and it is a singular fact that certain companies who clamor most loudly as to the inadequacy of rates are in the van also in competitive practices. One major part of Mr. Greene's plan is to introduce the competitive element, either by having all stock carriers go on the participating plan, at a rate level sufficient to make the payments of dividends a possibility: or to have two rate levels, one a high, participating rate, the other a lower non-participating rate, and to give stock carriers an option as to writing on a participating or non-participating basis.

Mr. Greene expresses the belief that supervising officials would not be likely to criticise a rate as excessive if all carriers were on the participating plan. But he has also his idea for preventing too lofty an increase in present rating schedules, and at the same time for wiping out the differential between stock and non-stock carriers in point of expense, namely by cutting down acquisition cost. Another idea, looking in the same direction is, that carriers should withdraw from states wherein they have no prospect of obtaining sufficient volume to carry proper service organizations, and it was pretty plainly intimated that carriers having a small or widely scattered compensation business had better leave the field entirely.

These points merit at least a comment. It is probably true that the crux of the whole difficulty is the existence of a disparity in net cost between stock and non-stock carriers, so great that many of the bigger risks feel they cannot afford to place their insurance with stock carriers at the full manual rate. It is true also that the system of loading premiums for expense is calculated to cause the larger risks to bear more than their share of the expense burden in respect to all items of expense which do not vary strictly or generally in accordance with premium volume. It is true furthermore that the commission scale, uniformly applied, produces in the case of the larger risks sums away and beyond the fair value of any service rendered by the agent or

the broker. On the other hand, at the lower end of the scale you get a class of business where the expense allowances contained in the rates are less than the actual cost to the company, and where the commission scale produces returns to agents and brokers insufficient to compensate them for the cost of doing the business. To cut acquisition cost may suffice to reduce the differential between stock and non-stock carriers to a point where stock carriers can compete for the larger risks on an even basis: but even if you abolished the acquisition cost entirely you would not completely wipe out the differential. So severe a dealing with the commission scale is unlikely unless the carriers are really at the point of indicating they do not want compensation business and will carry it only as pure accommodation without paying any commission at all, or the most nominal of commissions only. That is one way of settling the matter.

Another would be to permit stock carriers to quote to assureds who apply direct to the home office a rate ex-commission, paying commissions only to agents who solicit and procure business. But either method involves direct antagonism with the agency force. A third method, that of graded commissions and graded expense loadings, is a far more scientific way of approaching the matter, and it is a great pity that it was forced into issue last year. It is not too late, to be sure, for the special committee of the National Convention of Insurance Commissioners to consider it, but there are so many adverse decisions that it starts in with a severe handicap. I do not know how great a reduction Mr. Greene had in mind. It would have, in any event to be substantial in order to accomplish the desired result: and at the moment, the National Association of Insurance Agents is probably not prepared to go beyond a reduction in top commissions from 17.5% to 15%: an amount which would hardly meet the situation. To be sure, this was coupled with a proposal to step up minimum premiums to $50.00. This would result in a huge increase in premiums on the smaller risks, enabling a reduction on the larger risks which would perhaps make for a better competitive situation: but it may be pretty confidently assumed that supervisory officials generally would desire an ample justification of so severe a dealing with the smaller risks.

As to the matter of the stock carriers going participating, that is an idea which is worth considering, surely. But Mr. Greene's

endeavor to show that there is a real analogy between life insurance and compensation insurance is none too convincing. Life insurance has a sort of Rock of Ages atmosphere, suggesting that you are dealing not with a mere business but with the eternal verities: and this is due in part at least to the fact that in so far as their underwriting refers to the actual mortality experience it is dealing with an element which varies but slowly, and which for a long series of years has varied steadily downward, particularly on the younger lives. It is an element moreover which does not follow state lines, though it does follow climatic lines. Companies can and do make rates which are changed only at long intervals, and which hold good all over the United States. To go no further, nothing can be more unlike the compensation situation. The compensation rate basis is afflicted by a variety of violent variables: legislatures, courts, administrative officials and the economic cycle. Rates must be changed year by year, and even that is not sufficient to cope with the extreme mutability of the rate basis. And there must be a different rate level for each separate state. This has its bearing upon Mr. Greene's proposal as to all companies going participating, and the results he thinks will follow.

You have today a situation where rates are fairly close to adequate in some states, very far from adequate in others. Assume all companies are on a participating basis, and seek a rate with a goodly margin, saying that it makes no difference what the rate is, since the surplus will be returned in dividends. Will not this or that supervisory official say, well, how do you mean to declare dividends? Do you mean to declare a flat rate all over the United States? In that case, in my state, when rates are adequate, these increased rates mean assessment of my policyholders to pay dividends to policyholders in states where rates are way below what they ought to be. There are but two answers to that —either hold the rates down to the bare limit of adequacy, or declare dividends upon the experience in the particular state: and any mutual carrier will tell you that such a policy is extremely difficult if not impracticable. Indeed, if you look at the experience of particular companies in particular states over a series of years, you will see a variation in loss ratio that is highly fortuitous and that requires to be levelled out for dividend purposes, either by averaging for the entire nation or averaging over

a series of years and you cannot do the latter without being accused of holding out on your policyholders. The proposal to regulate dividends in accordance with the state's own experience is of course nothing new, though fortunately for the moment of rather limited application. With all companies on a participating basis, it could not longer be kept in the background.

If the result is, not to secure a rate level with an ample safety margin which with anything like a normal experience would permit the declaration of participating dividends, but a rate in close relation to the bare necessities, then Mr. Greene's suggestion loses most of its value. There is no advantage in going on the participating basis unless there are going to be dividends. Also, there is no possibility of a non-participating rate lower than the participating rate. Furthermore his idea of making the business more stable by compelling an insured who changes his insurance carrier to forfeit his dividends depends entirely on the dividend privilege being of substantial value. If acquisition cost could be vigorously slashed, it is of course possible that, in view of a general writing of insurance on a participating basis, the supervising officials would countenance a continuance of present rate levels: but I must admit the prospect of a very substantial slash seem rather dubious. It is not in accord with the present views of the agents certainly: and while carriers are disconsolate enough over the compensation situation to envision rash actions aplenty, it may fairly be doubted if they are prepared to deal with the situation in a way calculated to disrupt their selling organization, which is, after all, their main competitive asset, offset though it may be by very substantial competitive liabilities.

Mr. Greene's suggestion as to withdrawal of companies from unprofitable areas, or withdrawal from the compensation business entirely in case of a small or widely scattered line of compensation, is eminently sound. The compensation business has suffered not a little, in common with casualty lines generally by the ingress of many new companies. That has now been replaced by a notable egress. The incorporation of new casualty companies has come substantially to a standstill, and those already in the field have been thinned out considerably. Those left have been forced to trim their sails and mind their step: and some have already followed Mr. Greene's advice. It is a sound development. The casualty field was overcrowded, and the eager efforts

of certain carriers to gain a larger place in the sun, and the inexpert essays of others, not only spoiled the game for everybody, but has through a series of resultant failures, created a very painful situation, the repercussions of which will be felt for some time to come. It will be some time before a spirit of cheery optimism as to the possibilities of exploiting the casualty field is again rampant: and a spirit of caution on the part of underwriters with respect to workmen's compensation is beginning to be painfully felt by the selling organizations.

As yet, these organizations have been moved to criticism rather than helpful cooperation and have rather obstinately declined to look the facts in the face, seeking a remedy anywhere and everywhere rather than in the quarter which touches their own personal interests. That is, to be sure, an attitude natural enough: but one which cannot be much longer maintained. The carriers cannot perennially wrestle with an impossible situation, nor can the supervisory officials or the state legislatures afford to neglect a situation which threatens the integrity of the major fraction of the compensation business, and all the third party interests which depend thereon. Mr. Greene's paper is of great value in that he has not feared to lay his finger on the crux of the problem: and his panacea, despite the foregoing comments, is deserving of praise as a logical and well-thought-out plan of dealing with the situation.

MR. THOMAS F. TARBELL:

Mr. Greene is to be congratulated on his able presentation of some of the underlying factors contributing to the unsatisfactory situation in workmen's compensation insurance. Answering the specific question raised, the conclusion of the writer, in the light of the record of the past, is that the rate making plan has been the chief trouble with compensation insurance. This general conclusion might be somewhat modified to the effect that the chief trouble during the experience period under consideration was the failure of the rate making plan to reflect the rising loss cost trend of the compensation business.

During the period in question—calendar years 1923 to 1931 inclusive—the loss ratio for the non-participating and participating carriers combined was 67.1%. The excess loss ratio over the provision in the rates was the major source of underwriting loss.

Had rates for the period been pitched on the average to a level which would have produced a loss ratio of 60%, it would have required an increase in the average collectible rate level of 11.8%. Had this increased rate level been obtained, the non-participating carriers would have suffered an underwriting loss of .8%. This loss is arrived at by assuming the same acquisition cost ratio as was actually experienced, namely 18.0%, and a tax ratio of 2.5%, which is substantially accurate. It further assumes that all other expenses would have been the same in amount as actually experienced. Upon these assumptions the loss ratio for the non-participating carriers would have been 60.7% and the expense ratio 40.1%. The development of the foregoing results is shown in the following exhibit:

NON-PARTICIPATING COMPANIES

	Actual			On Basis of Adequate Rate Level for All Companies
Earned Premiums.		$1,182,752,407	x 1.118	$1,322,317,000
Incurred Losses...		803,342,898	x 1.000	803,342,898
Acquisition	18%	212,895,000	18%	238,017,000
Taxes	2.5%	29,569,000	2.5%	33,058,000
Other Expenses ..		258,704,860	x 1.000	258,704,860
Total Expenses ...		501,168,860		529,779,860
Total Losses and Expenses		1,304,511,758		1,333,122,758
Loss Ratio		67.9%		60.7%
Expense Ratio....		42.4		40.1
Loss and Expense Ratio		110.3		100.8
Und. Loss— Amount		121,759,351		10,805,758
Und. Loss—%		10.3%		0.8%

It might further be argued that the acquisition cost ratio of 18% would have been slightly less, on the theory that in case of branch office companies the "other acquisition" cost would have been the same in amount as actually experienced. The difference, however, would be very small and this refinement has not been introduced.

The subject of rate levels has recently been given extensive study as evidenced by papers presented and discussed at this meeting, and it is hopeful that out of this will come a better method, possibly not an ideal one, of determining rate levels.

It is my conclusion that the chief trouble with workmen's compensation has been the rate level and that the problems of

compensation would not have been as acute as they are today had the rate making plan produced an adequate over-all rate level and that if the problem of rate level is satisfactorily solved the main difficulty will disappear. However, there are unquestionably other troubles or at least unsatisfactory conditions of consequence involved in the workmen's compensation insurance business.

Before departing from the subject of adequate rate levels, it should be pointed out that the situation in this respect has been acute, not only from the standpoint of the companies but from the standpoint of supervising officials in those states where control of rates is exercised. One substantial reason for the rate inadequacies in the past has been the increasing claim mindedness of employees, increasing liberalities in benefits provided by statute and increasing liberality in claim awards. Increasing claim cost has unquestionably resulted in some instances in a reluctance on the part of supervising authorities to grant requested and needed increases and caused them to have a distorted viewpoint on the question of reasonableness, looking at it more from the standpoint of "what the traffic would bear" rather than from the rate requirements of the insurance carriers. One important problem of compensation insurance is to arouse public opinion, particularly amongst those who are most seriously affected—the employers—to curb the forces previously mentioned, tending to increase claim costs and accordingly insurance premiums. While the employer is primarily interested in these adverse factors, the subject is one which should be of interest to the public in general, since the cost must be shifted to the ultimate consumer.

The writer agrees with Mr. Greene that such solutions as "abandonment" and "necessary evil" do not furnish a satisfactory answer. Abandonment should be considered only as a last resort. Of course, if adequate rates cannot be obtained then abandonment and probably state funds will eventually result. However, the writer believes in the ability of private enterprise to solve the problems of compensation insurance. He believes that the interests of everyone concerned, the assured, the employee and the public will be best served if compensation insurance continues to be the subject of private enterprise. The "necessary evil" theory is clearly demonstrated to be impracticable by Mr. Greene.

If every company endeavors to avoid all except the most desirable risks, adequate facilities will not be available to the assured, with the result that the states will be forced to establish state funds. This theory, however, furnishes some food for thought along the lines of minimizing the expense of administering compensation insurance. There is unquestionably a considerable duplication of expense at the present time in that each competing company must maintain complete organizations for inspection and payroll audit service. There is a question if too great emphasis has not been placed in the past upon the inspection service of the individual carriers and if the management expense of the business could not be reduced if inspection (except for those risks which require special service) and payroll audit were functions carried on by central bureaus or agencies operated by the companies. There are other collateral advantages that might result from such centralization of such functions.

A comparison of conditions and practices between life insurance and workmen's compensation insurance is interesting, but it does not furnish any important clews or suggestions for solving the fundamental compensation problems. This is due, of course, to the fact that the motives affecting both supply and demand for these two kinds of insurance are entirely different. Life insurance is purely voluntary insurance, whereas compensation insurance is compulsory. Life insurance is independent of any other form, its nearest running mate being accident and health, but compensation insurance, in addition to being compulsory as respects the assured, is only one of several major casualty insurance lines and as a practical matter is forced upon the casualty company.

The question may well be raised as to the proper function of the state in controlling the operations of compensation insurance. Since compensation insurance is imposed upon industry by statute, the majority of the states have assumed it to be their duty not only to insure the solvency of licensed carriers, but to regulate and control the rates charged the assured and have enacted legislation to accomplish this. It is obvious from past history that the state has placed the greater emphasis on the regulation and control of rates. Under the statutes of most states the criteria for compensation rates are reasonableness, adequacy and non-discrimination. In the past the emphasis has unquestionably

been placed upon reasonableness with a preponderance of weight, as previously mentioned, upon the interest of the assured.

The main function of the state as respects life insurance companies has been to insure the solvency of the companies. The state requires that a life company must maintain sufficient reserves to liquidate its unmatured and unliquidated policy liabilities. It further provides that if a company elects to charge a premium which is less than the net level premium necessary to mature the contract in accordance with the legally prescribed rates of mortality and interest, it must carry a reserve equal to the present value of the difference between such net (pure) premium and the premium charged. The liabilities of a life insurance company are easily valued.

The liabilities of a casualty insurance company are not so simple of valuation. The incurred but unliquidated compensation claims, which constitute one of the most important liabilities of a casualty company are valued according to the present value of the unpaid amounts of incurred claims, but the annual statement is made up in such a way that it is impossible for the state to determine definitely whether or not these claim reserves or the reserves for other important lines are adequate without an examination of the records of the company. It may be appropriate to inquire what the results would be if the state relinquished its function as respects the regulation and control of rates and devoted its efforts to the assuring of the financial condition and solvency of admitted carriers. Would not the state be better performing its functions if it amended the annual statement to provide a better check on the adequacy of a company's loss reserves or if this is unattainable to devote its efforts to more frequent examinations of the company's loss liabilities?

It is not possible to give a definite answer to this question. However, let us consider the experience in the so-called "open" or non-regulated states. The compensation results in such states have been somewhat more satisfactory than in the closed states from the standpoint of the companies and in general there has been no complaint upon the part of assureds that they have not been fairly dealt with by the companies in the matter of rates. This also has an apparent bearing upon the point raised by Mr. Greene regarding competition between stock companies in respect to the assureds' net premium cost. The writer is one of those

old-fashioned individuals who, in spite of current noble experiments, believes that reasonable competition is good for any business and to the advantage of the purchaser of the competitive product.

One fundamental weakness present in the rating of compensation risks is a failure to adequately differentiate between risks of varying sizes. The experience and statistics show that there is a material differential in expense by size of risk due to the fact that a part of the expenses are flat charges independent of premium size, a part properly a function of payroll and a further part a function of premium, the expense ratio varying indirectly with the size of risk. On the loss cost side it has been demonstrated that the loss ratio usually varies inversely with size of risk. While attempts have been made to remedy this inequity by means of the adoption of expense and loss constants, the inequities have been only partially corrected.

Mr. Greene believes that stock companies should be permitted to pay dividends to policyholders. The writer sees no reason why stock companies should not pay dividends to policyholders if they choose and are permitted to operate upon the participating plan. It has been done in life insurance for many years and is being done today in casualty insurance. The writer is not particularly concerned with this feature as respects the differential between participating and non-participating carriers. So long as participating carriers operate upon a comparatively low acquisition cost, a substantial differential, other things being equal, will exist in their favor.

The principle underlying the paying of dividends to policyholders by stock companies appeals to the writer from another standpoint. Compensation insurance differs in another respect from life insurance (excluding Group insurance) in that the latter involves few problems of the size of risk nature either from the expense or experience standpoints. While there is somewhat the same variation of expense by size, the mortality under life insurance policies generally varies directly (rather than inversely) with size of policy and accordingly the operation of the two factors of expense and mortality tends to produce compensating results by risk size.

Further, in life insurance there is no occasion to recognize any variation in experience between risks of the same class. These

factors justify the paying of a uniform scale of dividends by an individual company. It is a fair question to ask if the dividend principle could not be incorporated in compensation insurance to supplement if not supplant the present experience rating plan and thus modify the risk premium retroactively rather than prospectively. Such a method would have the desirable advantage of modifying the rate or charge for a given period according to the actual experience during the period rather than according to the expected experience.

Upon the question of commission the writer feels that compensation insurance should receive substantially the same treatment by the stock carriers as other lines of insurance. There is no sound reason for any material variation in practice. Commissions are paid upon insurance contracts on the theory that the agent producing the risk is entitled to the commission as a recompense for the time and effort that he devotes to securing the risk, the service that he performs for the assured and for the company in providing it with the data necessary to prepare and issue the policy. Commissions on compensation insurance are lower than for most other lines of insurance. The question is whether or not the rate or return to the agent is a fair one.

The majority of compensation policies are comparatively small policies and it must be conceded that upon 90% to 95% of the cases the agent earns his compensation. There are, however, a comparatively few large risks where the premium is such that the commission rate provides the agent with a recompense out of proportion to the time and effort expended in performing his functions. This point is, of course, a controversial one and probably would not be admitted by the majority of agents. It is obvious, however, that the only source of any substantial saving on acquisition is in connection with large risks. Efforts to grade commissions by size of risk have been made in the recent past and undoubtedly will continue to be made and may eventually be successful and result in some lowering of the expense ratio.

The credit problem is an important one. It is not peculiar to compensation insurance, but applies to all casualty insurance. The writer in his views on this point is probably influenced by his early training in the life insurance field where the initial premiums must be paid or there is no coverage and renewal premiums must be paid within thirty days. In most cases the life com-

panies take no chances in extending a grace period of thirty days for renewal premiums, since they are protected by the equity or reserve on the contract. In casualty insurance, however, there is no such equity or reserve and the companies are unprotected. While reliable statistics are lacking, there seems to be little doubt but that waived premiums on all casualty insurance result in an appreciable depression of collectible premium levels. It is generally argued that credit is a feature of all lines of business and the question is asked why insurance should be any exception. An answer to this is that credit in practically all lines of business is extended not generally to any and everyone—but with discrimination. This fact seems to have been somewhat lost sight of in casualty insurance.

The long term contract suggested by Mr. Greene has certain advantages and is worthy of consideration. It would save the maintaining of certain records in the home offices and branches, cut down the expenses of printing policies and permit other minor economies. It would, however, be necessary to issue annually a renewal certificate containing some of the declarations portion of the policy and possibly the statutory state endorsement, so that the actual saving probably would not be very material. There may be some advantages in the other points mentioned in Mr. Greene's paper—discouraging shopping around and switching—but it is doubtful if this applies to the general run of policies.

AUTHOR'S REVIEW OF DISCUSSIONS (INCLUDING COMMENTS UPON THE PAPERS PRESENTED BY MR. SENIOR AND MR. PERRYMAN)

MR. WINFIELD W. GREENE:

Since the above paper was presented, there has been much intensive study of the compensation problem upon the part not only of individuals but also of committees representative of state authority, of carriers and of producers. Indeed at the present moment the entire matter is under consideration by a committee representing the stock companies, and a special committee of the National Convention of Insurance Commissioners is now awaiting presentation of the stock carriers' plan. In view of the

many plans and suggestions which have been advanced during the past few months and particularly in view of the fact that this subject is now receiving attention by parties most vitally interested, it seems unnecessary to elaborate this discussion unduly, and I shall therefore confine myself to fundamentals.

The three members of the Society who have discussed my paper do not any of them support the analogy which I endeavored to draw between life insurance and compensation insurance. However, the aptness or inaptness of the analogy is not the major consideration. It appears evident that underwriting matters are better in hand in the life business than they are in the compensation insurance field; and this fact warrants study of the possibility of improving conditions in the latter field through the adoption, or adaptation, of viewpoints and methods which have long stood the older branch of insurance in good stead. And such study is still indicated, even though in group insurance the life companies have to some extent fallen afoul of certain difficulties which are more characteristic of the casualty branch than of the life business.

Controverting my own conclusions, Mr. Tarbell maintains that the *rate making plan has been the chief trouble with compensation insurance,* and to substantiate his point asserts that if during the nine years ended with 1931 the rate level had been 11.8% higher than it was, the stock companies, as a group, would have sustained but a trifling underwriting loss. Assuming the present "non-participating" plan of operating, and the present scale of acquisition costs, it is a matter of speculation just how much the stock companies would have been benefited by such higher rate levels during the period cited. Surely the current adverse selection against the non-participating companies would have been greatly intensified, the trend towards self-insurance would have been accelerated, and the position of the participating carriers would have been strengthened materially. Indeed, it seems likely that although with the higher rates the underwriting results of the stock companies might have been slightly bettered, their competitive position would have been correspondingly weakened. And there remains the question whether, still assuming the present operating plan, rates as high as those mentioned by Mr. Tarbell could have been "sold" to the supervising authorities.

As Mr. Hobbs says, the determination of dividends to policyholders according to the experience of individual states is impracticable, but does this constitute any obstacle to the participating plan? Even in the case of life insurance, it is undoubtedly true that the experience of any given carrier for a stated period varies considerably as between states, and yet life insurance companies are not required to make the individual state the unit for dividend computation purposes. The adoption of the participating plan, coupled with a substantial reduction in commission cost, would enable the stock companies to compete legitimately in point of net cost both with the other classes of carriers and with self-insurance. In view of the public benefits obviously accruing from such an improved situation and also considering the long-established precedent in respect of participating life insurance, I fail to see why the supervising authorities should insist upon determining policyholders' dividends according to state lines. As for supervising officials, if any, who are shortsighted enough to play "dog in the manger" regarding rate levels, the ultimate remedy lies in litigation carried, if necessary, all the way to the highest court of the land. However, it is my own feeling that the fundamental reforms just mentioned would go far to overcome any objections that presently exist to the approval of ample rate levels. Certainly, the steps recommended would remove all merit from such objections and thereby improve the outlook of the stock carriers in any court action which might be required to protect not merely the rights of the companies but also the interests of the great body of their policyholders located in states where compensation insurance rates are adequate.

The papers presented by Mr. Senior and Mr. Perryman are concerned with the rate level problem, which, as I see it, remains a difficult problem only as long as the present operating plan of the stock companies is continued.

The main proposal embodied in these papers is that, from and after a certain date, in the determination of the rate level the insurance buyers are to be not only charged with *expected losses* (as indicated by the latest calendar year experience) but also charged with the *deficit* (or credited with the *surplus*) which develops in respect to the loss element of premium income.

It would appear self-evident that the loss element in the premium rates for compensation insurance should be adequate over

a period. If the non-participating plan is continued, the test of "reasonableness" must also be applied. Inadequacy might have been avoided during the past decade by the employment of a sufficiently substantial contingency loading, but under the non-participating plan such a solution would have been temporary only as any loading big enough to have kept the loss ratio down to 60% would at once become unnecessary and unwarranted in the event of a decline or even a levelling off in pure premium cost. The interest attaching to the proposals of Messrs. Senior and Perryman lies in the forthright attempt which these gentlemen have made to develop a plan which will automatically maintain a close balance between accumulated incurred losses and the loss fund developed from premium income. This is a difficult assignment, and, in this writer's opinion an avoidable one, but nevertheless these authors are to be commended for the frankness and thoroughness with which they have approached their task.

Mr. Senior's paper is a stimulating and suggestive exposition of his viewpoint, but Mr. Perryman has, I think, worked out the better technical solution of the above outlined problem. Mr. Perryman's formula, though founded upon a rather lengthy mathematical investigation, is essentially quite simple. I feel that the importance of his proposal warrants its restatement in non-mathematical terms. The plan, as I understand it, amounts to a comparison of the loss fund provided by the latest calendar year's premium income (re-computed upon basis of the "latest rates", i.e., those in effect on the date when experience is reviewed) with the sum of the following:

(1) The losses which actually occurred in the calendar year just past.

(2) The deficit which had developed by the end of such preceding calendar year.

(3) The deficit anticipated respecting premiums earned, and to be earned, after the close of such year, upon business which will not be affected by the proposed new scale of rates. (It is contemplated that experience will be reviewed in April or May, with the intention of establishing new rates effective July 1st. If this procedure is carried out in 1934, premiums earned in 1934 on 1933 business, also all premiums on business written in the first half of 1934, will not be affected by the rate scale established as at July 1st, 1934.)

If the sum of (1), (2), and (3) exceeds 60% of the latest calendar year's premiums (re-figured at "latest" rates), the rate level is to be increased proportionately. If such sum is less than said 60%, a corresponding rate level reduction is indicated. Of course, a surplus developed to the end of the preceding calendar year, or a surplus indicated in respect of the committed business not affected by the new rates will operate in the above formula as a credit to the insurance buyers, rather than a charge.

If I have correctly interpreted Mr. Perryman's proposal, it would appear that his plan embodies a direct and logical solution of the rate level problem as he envisions it. However, I am not prepared to admit that the tests which he presents based on experience in the State of New York indicate that the practical results of his plan would be satisfactory.

When the experience of only the latest calendar year is used, his method produces tremendously unstable rate levels. Referring to the appendix to Mr. Perryman's paper, we find that when making rates to become effective July 1st, 1928 he would arrive at a 10% increase. The next year he would make a 7% reduction, followed by a 7% increase to take effect July 1st, 1930. A 3% reduction would then ensue in 1931, followed by a 27% increase to take effect July 1st, 1932; and a 4% decrease then follows to become effective July 1st, 1933. I am sure that these rapidly alternating fluctuations in rate level would be most disturbing to everyone concerned. When the two latest calendar years of experience are employed, instead of only the single latest year, the broader base cuts down the rate level changes considerably. Whether one or two years are used, however, Mr. Perryman's method indicates the following for the years 1925-32:

1—Yearly rate level changes.

2—Some accumulated deficit in respect of the loss element of premium income, though much less than was actually experienced.

3—Considerably higher rate levels than actually prevailed.

The last two of the above points bring us squarely back to the question whether the compensation problem can ever be solved by dealing with rate levels alone. Personally, I believe not. Under the non-participating plan, there is certainly a limit to the rate level that can be sold to the supervising authorities. Further, as long as the present plan continues, the higher the

rates the greater the disparity between the stock company loss ratio and the average loss ratio. For both these reasons, I feel that the examples submitted by Mr. Perryman are no warranty whatever that, given his method in conjunction with the present operating plan, the stock carriers would have enjoyed a loss ratio as low as the one he indicates for the period 1925-32.

It is only fair that a critic should be required to submit his own remedy. In my opinion the following are the essentials of a program to make the compensation business pay its own way:

(1) Acquisition cost must be drastically reduced. Such reduction should not be uniform for risks of all sizes but should be graduated according to size of risk as measured in premium; and this graduation should be reflected in the premium collected from the assured.

(2) The participating principle should be introduced. By the participating plan I do not mean a retrospective experience rating plan concerned with the individual risk. Nor do I mean a retrospective readjustment of rate level which is to apply to the business of all companies. Instead, I mean the practice of giving the compensation policyholders of a given carrier, as a group, irrespective of state lines, and after due allowance for profits to stockholders, the benefit of the lowest cost which the company management can achieve both as to expense and as to loss ratio. This benefit would be conveyed to the policyholders in the form of a dividend proportionate to the total earned premium of the individual assured. It would not affect the producer's commission, which would be computed upon the gross premium (before dividend).

(3) The third essential step is the adoption and approval of premium rates which are adequate beyond peradventure; in fact, so "adequate" that there will be a considerable margin of safety in them for any company of reasonably efficient management.

The program which I have in mind would, I believe, have the following advantages:

(1) Rate levels, once established upon an abundantly adequate plane, could be left alone for at least three years at a time, possibly for five years or more. The benefits of such a situation require no comment.

(2) Rating organizations, relieved of the turmoil incident to yearly rate level changes, could concentrate upon really constructive study of such problems as the following:
   (a) *Relativity of rates,* both inter and intra classification. The schedule rating plan is being done away

with. The fact remains that in some classifications all risks do not present the same hazard, as the outstanding loss producing causes of the class present themselves in varying proportions. This condition is recognized in the Chemical Rating plan, and undoubtedly there are other classes to which a similar plan might properly apply.

(b) *Cooperative activity of the companies* in respect of such things as accident prevention, payroll auditing, claims handling, and medical and hospital service. The National Council on Compensation Insurance has already engaged in study and experiment in connection with certain of these matters. There is reason to believe that a number of them can be done, either wholly or in part, much more economically and on the average more efficiently on behalf of the companies as a group.

(3) The underwriting drain upon the stock companies would be stopped and increases or fluctuations in compensation loss cost would not longer be a source of anxiety to company management because of the substantial safety margin in the rates. The companies could ride through a period of adverse experience merely by making the necessary adjustment in the scale of policyholders' dividends, as has recently been done by the life companies.

(4) The interests of the assured would be amply safeguarded, as competition would eliminate companies not equipped to render satisfactory service at a reasonable net premium cost.

It is most devoutly to be hoped that the stock companies will not be beguiled into compromise by the glimmer of the expected business revival which is still "just around the corner", but instead that, whatever specific plan is adopted, they will move promptly and courageously toward a thorough correction of the fundamental ills of the compensation business.

CALCULATION OF THE COST OF UNEMPLOYMENT BENEFITS (WITH PARTICULAR REFERENCE TO OHIO AND PENNSYLVANIA)—
CLARENCE A. KULP
VOLUME XIX, PAGE 268

WRITTEN DISCUSSION

MR. I. M. RUBINOW:

I am extremely gratified by Professor Kulp's paper—first for a perhaps somewhat personal reason—as evidence that he has taken quite seriously my actuarial attempt undertaken after many years of abstention from active participation in actuarial practice. As I still value my professional standing as an actuary after the many attempts of the gentlemen of the Ohio Chamber of Commerce to destroy it, I am under obligation to Professor Kulp. And secondly it is no less gratifying to find him agreeing almost altogether both with my method and my results. Many of Professor Kulp's criticisms are quite valid—though I find myself unable to accept all of them—and helpful. I shall touch upon them briefly presently. I find myself particularly unable to follow the reasoning that because of certain difficulties now met with in the effort to compute rates the "reserve" type of legislation is to be preferred to the insurance type. But if one can at all interpret legislation tendencies—the choice between these two types will not be made by actuaries nor based primarily on actuarial considerations.

The field of possible discussions is considerably limited by Professor Kulp's ready acceptance of my formula

$$\frac{U \times B \times (1 - W - L) \times (1 - M)}{E}$$

and it may be unnecessary to argue as to the complete accuracy of the statement that "there is nothing actuarially new or radical" in this formula. There really is no place for "radicalism" or "conservatism" in any actuarial computation, but Professor Kulp's assurance may be helpful to the formula.

But there is at least that much that is new in the formula —its "symbols". I ought to know because I devised them. The formula itself is but a summation of my efforts. It was not evolved until *after* the work had to be done—and not so much

for the benefit of the members of the Ohio Commission (who honestly admitted that it was all Greek to them and were willing to trust my computations and to accept my final results) as for the special benefit of those members of our (or your) profession who might be called upon to meet the same problems in other states.

Perhaps it will only be fair to point out here that notwithstanding Professor Kulp's generous endorsement of this formula, it is by far not complete. It is extremely simplified. Only the main factors—rate of benefits, working period, duration of benefits and maximum limits—are taken into consideration. There are a great many other factors—largely qualifying conditions—contained in the Ohio Commission's bill, and many others in bills of other states, which should (and eventually may) be taken into consideration in a general universally applicable formula. They were omitted for purely practical reasons because no factual material on these factors was available, because their inclusion at this time would unnecessarily complicate matters and be confusing certainly to the legislator and even actuary. The simplicity of the formula is provisional, temporary, just as the original compensation rate formulae devised in 1915, with the aid of the Rubinow Standard Accident Table and the flat law differentials were temporary. My older colleagues will remember it; the younger members of the Casualty Actuarial Society were probably required to study the history of these rates in preparation for their Society examinations. The present compensation formulae are very much complicated (whether or not they are more accurate) but the development of 20 years cannot be used as indictment of the earlier steps. It was very largely dependent upon them.

Professor Kulp's valuable study has been undertaken in connection with plans for unemployment insurance in Pennsylvania. While I am familiar in a general way with the statistical sources of that state—my former home—I have not made any thorough search for necessary data in that state. I am sure that Professor Kulp's opinion may be accepted that Pennsylvania's statistical material is very much more deficient than that of the State of Ohio. Much of the material which proved to be exceedingly useful in my Ohio computations, is lacking in Pennsylvania, and while I am not prepared to say that with such material as is available in

Pennsylvania and a greater dose of actuarial ingenuity, it might not have been impossible even in Pennsylvania to make some provisional and tentative computations (more provisional and tentative than they were in Ohio) yet it simply wasn't my problem. And it may be that in the early days of unemployment insurance—when (or if) it actually comes—it will be simpler and safer to adopt for the time being the computations of other states with better statistical experience. There will be "nothing new or radical" in that, because it was done very extensively in workmen's compensation insurance for the last fifteen or twenty years and must be very largely done now, because many of the lesser industrial states cannot expect to base their probability computations in individual industries on the insignificant experience within the state itself.

It is, however, towards the specific Ohio data and the methods of their computation that Professor Kulp's detailed criticisms are directed, and it is that part of his valuable study that requires most careful analysis. Professor Kulp divides the formula itself, the method of its application, and the statistical material upon which my calculations are based, into three groups: (1) those about which there is no question or at least no important question; (2) those in which the result of (my) inaccuracy is towards redundancy or safety; and (3) those which are not only questionable but tend to produce deficient premiums.

Under the first group Professor Kulp places the values of U (rate of unemployment), B (rate of benefit) and M (effect of maximum benefits). "B" is of course an assumed value. The method of computation of M is indeed simple. Yet again, the older compensation actuaries may be reminded, the method for computing M, used in the Ohio report, was my own method used for the same purpose in computing workmen's compensation law differentials in 1914 and following years. A simple method—almost as simple as the famous story of Columbus' egg.

Since Professor Kulp accepts the (my) value of U—namely 13.5%—it may be ungrateful to bring it up again. But personally I believe that the value of 13.5% is redundant and this offers an additional margin of safety. And since this is the very basic figure of all subsequent computations, affecting them in a double way (because exaggeration of the value of U also reduces the value of E—the divisor—and further raises the value of the final

result)—this point deserves to be dwelt upon at some length. To begin with, at best the employment figures of Ohio must be *deficient* to some *extent*. They are collected largely towards the end of the year. Establishments which have been abandoned during the year may not render any report and may be difficult to follow up. There is an indication of that in the larger variation between the figures for December of one year and that for June of next year than in the average variations of any other two consecutive months.

But there is an even more important reason to believe that the value of 13.5% is higher than any long time series would show. It covers the nine years 1923-1931, of which two years, 1930 and 1931, were very bad. It is true that if the year 1932 were added, the average would have been even worse. But two catastrophic years out of 9 is a much larger average than the economic history of the last 40 years would indicate. The crises of 1929-1933 cannot be compared with any other crises subsequent to 1898. Ideally the entire larger cycle, 1893-1929 or 1898-1933 should be used as a basis for determining the long-time value of U. In the long run I feel that the figure of 13.5% will prove redundant and furnish a margin of safety.

Redundant factors. In so far as the criticism of the formula's value is directed at its possible deficiency and therefore the unsafety of the premiums calculated, one might be justified in failing to reply to all of Professor Kulp's criticisms on the basis of apparent redundancy, for any influence toward safety. Yet, of course, the actuary is (or should be) objectively interested in computing a rate as closely accurate as possible.

Under this head of redundancy Professor Kulp's criticisms are directed—very properly—to the value of $1 - W - L$; which it must be admitted, is the weakest part of the formula—or more accurately speaking, not of the formula itself, but of the statistical foundations for computing the value of the formula *at this time*. The value of $1 - W - L$ means this: at any one time all the unemployed may be roughly divided into three groups: (1) those who recently lost their jobs and are not yet receiving their benefits (W); (2) those who have been out of work for some time, had received their full benefit and are again left without their benefit (L); and (3) the remainder who are entitled to the benefit and are presumably receiving it ($1 - W - L$). (It must

be added parenthetically that in so far as various provisions of the act may deny the benefits to certain groups, strikers, boycotted, discharged for cause, leaving their employment on their free will, refusing suitable employment, etc.—these groups should be excluded from the value of $1 - W - L$ and the value of $(1 - W - L)$ thus further reduced. But no reliable data are available, no reasonable estimate appeared possible, and therefore no account was taken of these factors—a decided factor of redundancy, and therefore of safety.)

Of this formula $1 - W - L$, Professor Kulp makes two criticisms,—one decidedly valid, and the other, in my conviction, due to his misunderstanding of my statistical reasoning.

The valid criticism obviously is that the values of W and L were computed on the basis of U. S. Census data as to the (elapsed or past) duration of unemployment *on a single census day*. The minor criticisms that (1) extensive interpolation has been necessary to convert the broad duration classes into weekly duration classes and (2) that certain occupations excluded from the Ohio bill are included in the table of duration, may both be disregarded as trivial. Since over 75% of all wage workers would be within the law—the difference between a table of distribution limited to them and the actual table including all employees could not be very great—and at any rate this limited table *was not available*. And as to the propriety of extensive interpolations for the purpose of smoothing a distribution curve, I feel certain that both actuaries and statisticians will admit that when necessary, interpolations are legitimate.

To be sure, the presumption that the distribution curve for one day in April 1930 is the characteristic, typical, average distribution, is a bold one. But what was the actuary to do? For the whole State of Ohio no other curves presented themselves. There were other curves in Censuses of 1890 and 1900—too old to be used now. There were some interesting sample data for other communities (e. g., for instance a very interesting series for Buffalo) but that would have raised the equally difficult problem whether data of *one city* in a different state should be used as a basis of Ohio computations.

Of course the boldness of this presumption was frankly admitted in my report. *But better data were not available.* And there is that much to be said in its defense. In the beginning of

a depression the value of W is likely to be larger, and the value of L lower; as the depression is prolonged, L may grow in value and W become smaller. The combined value of $W + L$ is therefore likely to fluctuate less than the value of either one of these two quantities, their fluctuations at least partly compensating for each other.

Still there is no gainsaying the fact that the value of $(W + L)$ and therefore of $(1 - W - L)$ may be subject to violent fluctuations. But in which direction are they likely to be? Provisionally with such data as I had, and W being based upon a 3 weeks' waiting period and L upon a benefit period of 16 weeks, $1 - W - L$ had a value of 49.7, or approximately 50%. This means that on the average about one-half of the unemployed would be receiving benefits. There is this additional reason to believe this figure to be fairly representative. The census of the unemployed was taken in April 1930. The average ratio of unemployment in Ohio for 1930 was 17.3. In April 1930 it was a little over 13%, so that the average unemployment rate of 13.5% is closely approximated by the April 1930 Census. But whether it does or not, it was the best material that could be had on the distribution by duration.

But when Professor Kulp proceeds further with his analysis and claims that my method of computing $1 - W - L$ is wrong in principle, because "there occurs a curious shift in reasoning from a concept of unemployment as a number of weeks to one as the number of men unemployed", when he emphasizes that "*the computation of W and L values are all made on the number of persons unemployed, not on the number of weeks lost*" and that "the result is a considerable overstatement of compensable unemployment because *from the total of all unemployment in weeks are deducted W and L values which are in number of workers*" (italics are all Professor Kulp's) I can only express my deep regret that he has not sufficiently carefully followed my statistical reasoning. I did not commit the unpardonable arithmetical error of deducting workers from weeks.

In the formula, 1 stands for total quantity of unemployment or unity or 100%. W and L are discounts from 100% in terms of percentages. I was therefore subtracting percentages from 100% and not workers from weeks.

Now, what Professor Kulp failed to remember is that while

"weeks of unemployment" are compensated for, it is the living unemployed who are being compensated or not compensated. To say that *on the average* half of the unemployed are compensated and half are not is exactly the same as to say that half of the time lost is being compensated for and half isn't.

That a complete record of all cases of unemployment, with the exact duration of each case would have been desirable, goes without saying. If such a record were available it might have been possible to construct a standard unemployment table akin to my standard accident table, showing the normal distribution of 100,000 accidents. But obviously no such data exists in America. They do not exist in England; and a table of this character could only be constructed after the unemployed have gone back to work (or injured persons died or recovered). But it would be subject to violent fluctuation between boom and depression years. I was fully aware of the fact that the April 1930 data show only the duration of the unemployment *up to that date,* that the same cases would continue for a variable time. But the method used was not only a short cut, it was the only possible method and bound to be as accurate as the more complicated one.

The most difficult criticism to overcome would be evidence that a serious factor of cost was disregarded and the rate therefore would be decidedly deficient. Fortunately as against all redundancies, or factors of safety, Professor Kulp only mentions one factor of deficiency, namely, he charges failure to include the cost of compensating for partial unemployment. However, this charge is only partially justified.

In arriving at the unemployed ratio (U) the Census data of the partially unemployed were added to those total unemployed—because the Census definition of partial unemployed (those not employed *but having jobs*) was not at all satisfactory.

It is true, however, that on the Ohio data of employment no differentiation is made between those who had a full time job and those who only did part time work. It is not unlikely, in fact it is probable that an (unknown) percentage suffered from part time unemployment. But what was one to do if better data were not available?

Thus there is here one factor of deficiency against a number of factors of redundancy, one factor which might tend to make the rate insufficient as against all the factors of safety. The

danger to the stability of rates, is in my conviction, not a very serious one. And for this reason: the general tendency for shortening of hours, particularly during times of depression, has been accepted. The shorter working week—38, 35 or even 32 hours—may be in one respect another name for the share-the-work movement or part time work—but once the standard working day and working week is reduced, the opportunities for part time unemployment (as compared with the standard working week) is considerably reduced also. When the standard was 54, anything less than 38 (30) hours would give claim to compensation. But with a standard week of 32 hours—the hours would have to fall below 25 (20) to create the claim—and the cost of partial unemployment thus considerably reduced.

To sum up: Professor Kulp accepts the formula and certain data, his criticism of remaining data is partly unjustified and due to misunderstanding; in so far as it is justified, the explanation must be sought in the inevitable limitations of available sources.

Where I must differ fundamentally with Professor Kulp is in the appraisal of the immediate importance of those statistical deficiencies and the practical conclusions to be drawn from them.

As a theorist rather than practitioner of insurance, Professor Kulp, I believe, overestimates the importance of absolute accuracy or the opportunities for such accuracy in this or any other branch of insurance. He objects, for instance, to the Ohio plan being entitled "insurance" because "it lacks that quality of certainty that is the first characteristic of the modern insurance institution". There is no absolute "quality of certainty" in any insurance institution (not even life) though there are varying "degrees of certainty". The appearance of mathematical accuracy, resulting from mathematical complexities, may make a good selling point, but actuaries, and certainly casualty actuaries know better than to claim it among themselves. What does exist is an honest effort to achieve as high a degree of certainty as is possible. In that respect unemployment insurance, being in a preparatory stage only, is no different from any other insurance field in its early and preparatory stages.

This argument by reference to other fields, Professor Kulp is inclined to reject. "This," he says, "is to argue that all risks are equally uncertain." But that is not the intent of the argument—only—"that all risks have some (perhaps unequal) degree of

uncertainty." It is not necessary to minimize, but neither should one exaggerate the "difficulties of rate making for an insurance with neither precedence nor experience to guide it." What is the claim of the actuarial profession to a scientific standing, if it is to be limited to precedence and experience—and forego its right to exercise invention, imagination and judgment.

To be just, Professor Kulp does not "decide permanently against insurance." He has not joined the unsavory chorus whose refrain is "Unemployment is not insurable." He suggests an "ideal approach" through a system of unemployment reserves which would serve as a perfect basis for collection of data, but would stop short of insurance only by omission of contractual obligation. I have no intention to raise at this place the controversy as to the respective merits of the Wisconsin (reserve) and the Ohio (insurance) plan. The controversy is much more than an actuarial question. The discussion of the question as to what should come first, insurance or statistics, may appear as baffling as the old question as to respective ages of the chicken and the egg. But, assuming my own preference for the Ohio plan, I must point out that

1. The preparatory actuarial work for unemployment insurance has been advanced much more than the preparatory work for compensation insurance 20 years ago.

2. That it is a comparatively simple matter to provide for necessary statistics when you have insurance, but statistics alone does not lead to insurance automatically and to offer a statsitical plan in response to the growing demand for unemployment insurance is to offer a stone in response to demands for bread.

3. And finally that the Wisconsin (reserve) plan would fail even in furnishing the necessary statistics. Only a contractual obligation to give and receive compensation makes complete and accurate statistics possible.

The actuarial profession is confronted by a very grave responsibility, and it is presented with a great opportunity. The situation is not unlike that of twenty years ago in regard to workmen's compensation. With the difference that before 1914 there was no Casualty Actuarial Society and no casualty actuarial profession.

A year ago the Ohio Chamber of Commerce sent out a questionnaire to actuaries, asking their opinion as to whether "unemployment is insurable". The Chamber of Commerce claims that

out of more than 100 actuaries only two (including the writer as one) answered in the affirmative, and all the others in the negative. However, a large number of actuaries honestly admitted that they had given hardly any thought to the problem and knew little about it. That was but natural considering how recent are the serious discussions of the problem in this country. Yet I feel quite hopeful, nay almost certain, that in response to a demand from the public and the state, enough casualty actuaries will be found to do the necessary work as soon as the necessary legislation is passed, and that American casualty actuarial science as represented in this Society, will not be found unwilling or unable to render this important public service and help solve one of the gravest problems of modern industrial civilization.

MR. W. R. WILLIAMSON:

Mr. Kulp's paper is essentially based upon the third and fourth sentences of his first paragraph:—"The computation of the cost of a system of unemployment benefits is not a particularly difficult one, provided the actuary is satisfied of the dependability of his raw materials. It is on this ground of statistical inadequacy that most criticism of cost calculations is based."

There is one additional factor of great importance and that is the applicability of the raw materials to the problem under consideration. Even given an assurance that the data are accurate, nevertheless in such a thing as unemployment benefits the very provision of benefits is now pretty well recognized as affecting the chances of unemployment, even as the existence of a permanent-total-disability provision does seem to affect disability itself.

In all insurances past experience fails to indicate adequately future trends. Carefully compiled experience is most desirable but is never duplicated. So I would differ from Mr. Kulp in the assumption that when the raw materials are dependable the computation of the cost of a system is not a particularly difficult one. I still believe that it is extremely difficult, even as Mr. Kulp still believes that workmen's compensation rates are extremely complicated. This, however, is no real difference of opinion. It is simply a change of expression because Mr. Kulp clearly implies that difficulties are present.

The intricacy of the factors determining cost I have outlined

in a discussion on Unemployment Insurance before the Connecticut Chapter of the American Statistical Association, as follows:—

"FACTORS DETERMINING COST.

"*A. Unemployment Causes.*

"Before considering cost factors the causes back of these cost factors must be briefly reviewed. Completeness of survey will be served by the following analysis:

"(a) The individual and his innate capacities—his training and experience and his discipline—factors that determine his availability for various types of work and his capacity to meet with some adequacy the requirements which go with various jobs.

"(b) The employer—his position as a manufacturer or organization; his personnel department and its efficiency in initially selecting employees; his standards of employee administration; his position in the manufacturing world; his ability to read market signs and to organize his employees most efficiently; the extent of his advance planning of work; his method of handling the periods of rush and of slack work; the flexibility which he has built up in his organization so as to permit transfers of employees from a department ahead of schedule to one behind schedule; his adaptability as an employer, his planning of fill-in jobs during times of rush work for the periods of slackness.

"(c) Society as represented by other employers, the buying public or consumers, and the government in all its phases. This factor includes what seem to be the cycles of progress, with the alternation of speeding up and slowing down; the general technical advance in the development of machinery to replace men; the shifting attitude towards work in respect to that large group of individuals who might be either dependents or self-supporting workmen, including adolescents, unmarried women and married women; the variation in immigration and emigration; other elements affecting the labor supply, such as falling birth rate and reduced mortality at the lower ages; the force of advertising in working changes in buying habits and in shifting of patronage from one line of commodities to another; the still greater subject of distribution generally, including not only advertising but wholesale and retail machinery and individual selling; the forces back of the conflicting evidence called 'confidence' and 'optimism' on the one hand, 'fear' and 'pessimism' on the other; the bal-

ance, in the mass, made up of the balances in the individual cases, between income and outgo, spending and saving; the effect upon production, on manufacturing and the public of uncurbed competition on the one hand and trade associations and general planning on the other, including also, possibly, associations for the direct purpose of limiting production—or stated in another way—complete individualism versus state socialism of operation; and finally, the political effects of legislation, taxes, tariffs, and even government management of public utilities and government competition along manufacturing or production lines.

"The chance that a man may get a job and once holding the position is able to keep it depends upon all of the above mentioned considerations and many more. The too simple solution which boils down the causes of unemployment to a single element states the basis in such a way that corrective treatment is voluntarily impossible. The subjective extension of employment difficulties include individual stupidity, employer selfishness, capitalism, anarchy, socialistic trends. These extensions mean a great deal as to the point of view of the person making them but they offer little basis for constructive treatment. Until there is more knowledge than now exists as to the elimination of individual stupidity or cupidity with the retention of all of the driving forces necessary for personal and mass advance; until we know how to replace with better the intricate imperfect machinery which capitalism has built up; until we know how to substitute for anarchy an ordered usefulness, belief in the essential simplicity of the problem will give us but little aid in its solution.

"The chances of unemployment have their roots in our diverse modern civilization in the accumulation of unorganized individual and mass efforts. In certain occupations, over certain periods of time the danger of unemployment is very slight. In the period when railroads were expanding rapidly, a man trained along specific lines of railway work believed, and had reason to believe, that he practically had a lifetime job. More recently the electrical industry has been in a similar enviable position due to the steady growth in the use of its products. Technical advances, improvements and simplification could roughly be assumed to make unnecessary large expansion in personnel, but rarely necessitated laying off workmen. On the other hand in the textile industry shifting styles have, at different times, given popularity to woolen, to cotton, to silk or to the more recent products like rayon and the simplicity of the operations has

permitted transfer of manufacturing operations from one location to a widely different one. The tariff has had its effect on woolen manufacture. The change in women's styles has reduced the consumption of certain types of cloth and it has not been possible in the textile industry to make steadiness of work even probable for its employees. A report of the New York Department of Labor* traces employment month by month from January '21 to December '30. As a basis for its index numbers the average situation for the years 1925, 1926 and 1927 were treated as 100, the number of employees, the total amount of payroll and the average weekly earnings being compared with this standard. Employment as a whole in all industries shows a range from a high of 114.6 in 1923 to a low of 77.5 in 1930. Payrolls as a whole range from a high of 108.4 in May 1923 to a low of 73.7 in December 1930 and average weekly earnings per capita from a high of 105.2 in March 1929 to a low of 94.7 in November 1930. This report does not show turnover—the balancing of lay-offs and withdrawals against new employees —but even cursory examination indicates a large falling off in employment from 1923 down to 1930. The yearly averages show a general downward trend over the period of the last eight years, but there is no suitable basis evident in this data from which to determine the chance of lay-off or the necessity of paying unemployment benefits.

"The individual industries, of course, have shown a much wider range of variation. Among these are brick, tile and pottery; lime, cement and plaster; silverware and jewelry; cooking, heating and ventilating apparatus; saw and planing mills, pianos and other musical instruments; cotton goods; men's furnishings and tobacco. In the tobacco industry the number of employees ranges from 160 in 1921 down to 57 in 1930 and to only 48 in December 1930. Less than one-third of the 1921 employment appears to be shown in the year 1930, though the average weekly earnings per capita are definitely higher in 1930 for those who were employed than they were in 1921.

"*B. Cost Factors.*

"Certain factors influencing the cost can be set down as follows:

" 1. Formula of benefit—

(a) The waiting periods from date of employment of the employee to the beginning of insurance.

* Department of Labor of the State of New York, Division of Statistics and Information, Bulletin No. 171, The Course of Factory Employment in New York State from 1921 to 1930 inclusive.

" 2. Waiting period from the employee's last date of employment to the date when benefits accrue.
" 3. The duration of benefits.
" 4. The supplemental benefits, such as right to return the unused premium.
" 5. The possibility of added benefits because of the presence of dependents.
" 6. The extent of a delay in making insurance effective so as to permit accumulation of reserves.
" 7. The forms of unemployment excluded from benefits.
" 8. The distribution of employees by age, sex, duration of service, occupation, industrial classification and geographical location.
" 9. Position of the insurance year in the general employment cycle.
"10. Necessary expenses with particular reference to the probable high loss adjustment expense.
"11. Probable interest earnings in the accumulation of reserves.
"12. A recognition of the effect upon unemployment itself of the establishment of unemployment insurance or reserves.
"13. Extent of the factor which in Life Insurance is called selection, resulting in a low early apparent loss rate followed by a more normal loss rate.
"14. Extent of contribution which might be assessed against premium income for prevention work corresponding to the outlay in Compensation Insurance for the work of Safety Engineers and Inspectors.
"15. Expense of research.

"These cost factors are not completely comprehensive. They ignore, for example, certain highly objectionable practices which have developed in certain other lines of returning apparent surpluses because of delay in reaching ultimate claim rates. It ignores adequate recognition of the dangers contemplated in all of the commission reports of building up reserves to a certain point and then arbitrarily terminating further accumulation at that point. It does introduce very briefly the actuarial problems."

Mr. Kulp weighs rather carefully certain underestimates and certain overestimates in the Ohio use of the formula when he says, "There is apparently no up-trend against which the actuary must guard." One is inclined to refer him to the voluminous

discussions on technological unemployment, the implications of which discussions are that there is a definite up-trend.

For a considerable time to come the basis of unemployment insurance will undoubtedly be the process of comparing a guess with an inference. It is probably desirable, as Mr. Kulp concludes, to administer unemployment reserves first and from the observations derived from the administration of those reserves to build up statistics which will be of value should unemployment ultimately lend itself to insurance methods.

AUTHOR'S REVIEW OF DISCUSSIONS

MR. CLARENCE A. KULP:

I find myself so nearly in agreement with Mr. Williamson that extended rebuttal on his discussion would be a superfluity. He is concerned not only with the factors that would determine the unemployment compensation premium, but with the diversity and complexity of the base causes of unemployment itself. These last are for the most part difficult even to define, not to say analyze and measure. They range from causes as relatively objective as technological change to those as imponderable as "the individual and his innate capacities", "individual stupidity, employer selfishness, capitalism, anarchy, socialistic trends". There is nothing objectionable in a list such as this. Similar summaries of unemployment causes are found in every book describing unemployment and unemployment compensation. I do question whether there is in this list much of value for the actuary, aside from giving him the general background indispensable for all rate making. The workmen's compensation actuary likewise must keep in the back of his mind a hundred intangible forces and influences all of which to a greater or less degree affect the validity of his methods and results. But unless he can express them objectively there is no place for them in his formulae.

With direct cost factors Mr. Williamson is on firmer ground, and insofar as these are capable of measurement from available data are already a part of the Rubinow formula. By their nature direct information on some, as on the cost of preventive activities, must await the actual operation of a scheme. It will be

years if ever before the actuary will find out the values of other cost factors Mr. Williamson would like to know: for example, "12. A recognition of the effect upon unemployment itself of the establishment of unemployment insurance or reserves", and "13. Extent of the factor which in life insurance is called selection, resulting in a low apparent loss rate followed by a more normal loss rate". Even if ultimately we are able to measure the shifting weights of these and a hundred other variables, it is still true, as Mr. Williamson himself points out in his third paragraph, that all you have is a mass of statistical history, which may or may not be duplicated in the future.

Mr. Williamson also objects that the primary requisite I have set up for rate making: statistical adequacy, is not wide enough. So stated and bare of its immediate context I quite agree: the term would need more precise definition. But in the very same paragraph, I had thought to make the meaning of "adequacy" clear by adding the phrases "dependability of raw materials" and "reliable data". "Adequacy" must mean much more than the mere massing of data properly classified, and this wider meaning runs through the entire paper, recurring particularly in its second last paragraph. Mr. Williamson's main fear is that data merely sufficient in quantity may not be reliable for indicating the future. Specifically he is afraid that perhaps unemployment benefits themselves will increase the risk. In this country we worry about this much more than they do, for instance, in England where they have had unemployment insurance since 1911.

Finally Mr. Williamson believes I am overbrash in saying, on the authority of the only data we have in this country, that "there is apparently no uptrend against which the actuary must guard". This also is a statement built on the past, and it may have to be revised. It is much too soon to say now what will be the effect of this, our most terrible depression, on the long-time unemployment rate. Even if we add the abnormally bad experience since 1926 (Douglas' latest year) and ignore the inevitable recovery to come, the average unemployment rate from 1897 to 1933 inclusive (on admittedly defective data) rises only to about 10 per cent. This compares with the Paul Douglas 8 per cent. for 1897-1926, and the 13.5 per cent. of Rubinow for Ohio for 1923-1931. The question of technological change cannot be ignored—at least in one's general thinking if not in his rate

making—and illustrates perfectly the difficulties, not to say the futilities of trying to map and measure every cost factor in advance. Compared with this the problems of workmen's compensation rate making appear very simple. What will happen with technological change is bound up absolutely but inscrutably with what will happen with government, business and human relations. Two reflections occur on this point. The world has stirred itself up before, and undoubtedly will be stirred again at the speed of technological change. And even agreeing that the *rate* of this change has recently been sped up faster than ever before, we may still be able to devise appropriate rate making correction factors as we have for other insurances. For years employers' liability rate formulae have included a factor familiarly known as "business growing worse". Workmen's compensation rates also have included factors for uptrend. With unemployment compensation as with these, as the times change the rate maker will change his formulae with them.

With Dr. Rubinow I have even less to disagree. There are a number of questions of technique, but these are few, partly because of the brilliance of Dr. Rubinow's attack and partly because he and I were in constant correspondence during the writing of my paper. On the most important point at issue, that of the values for: 1 –W– L, I fear we must continue to disagree. I have once more gone over his original computations and his discussion in the present number of the *Proceedings*. Once again I have taken the data in his Table VI and like Dr. Rubinow have found that the percentage of the total number of *unemployed persons* covered under a 13 week benefit and 2 week waiting period is 44.7. Again I have found that the percentage of the total number of *weeks lost* falling within the same conditions is not 44.7 but less than 23. The I –W– L value of 44.7 is therefore redundant since the effective scope of a plan depends not on the number of persons unemployed but on the number of weeks lost by them. As for my ready acceptance of his formula, it would have been unfair to expect now details that can only come from actual experience, the formula is plainly provisional, and the essential factors have caused the actuary trouble enough. Finally I hasten to assure Dr. Rubinow that he cannot be considered ungrateful for his discussion of the possible redundancy of his assumption of a 13.5 unemployment rate. I have (p. 273) called

this a "relatively high level" and have elsewhere written that I consider it much too high for the entire country and over a longer period.

As Dr. Rubinow says, our fundamental difference is in the degree of credibility we place on the results of his formula, and it is precisely here that further debate is of least use. I am particularly unable to understand why the reserve plan would not provide the mass of uniform statistics necessary for a true system of insurance. It would certainly not result in exactly the same payments to individuals as an insurance scheme, but it would furnish all the information necessary to calculate the costs and benefits of insurance: unemployment rates, durations, average number of spells per year, administrative expense and so on. Like Dr. Rubinow I consider the unemployment risk a challenge to the imagination of casualty actuaries, and I predict that within 10 years this Society will have assumed the task of unemployment compensation rate making as one of its principal tasks.

AN AMERICAN REMARRIAGE TABLE—WILLIAM F. ROEBER AND RALPH M. MARSHALL

VOLUME XIX, PAGE 279

WRITTEN DISCUSSION

MR. WARD VAN BUREN HART:

The authors of this paper have given in general a clear and complete description of the thorough work done by the Society's Committee. On a few points, however, further comment seems appropriate.

After carefully collecting and tabulating the material, the Committee prepared remarriage rates at each age by using a five year moving average. The rates so obtained are referred to both in the text and in the Exhibit headings as "ungraduated rates". A purist might object to such nomenclature on the ground that the use of the five year moving average was actually a preliminary graduation. This particularly shows up in Exhibit IX, which shows a total number of remarriages amounting to 5,759, whereas Exhibit VIII shows 1,187. This is due to the fact that most of the remarriages, because of the method used, appear five times in Exhibit IX, but only once in Exhibit VIII. Exhibit IX, there-

fore, shows an appearance of closer conformity between the expected remarriages and the actual number than is actually the case. The use of the moving average cannot be criticized, as it is approximately correct for all practical purposes, even though not rigorously correct. In fact, it is an excellent expedient for removing most of the irregularities. This comment is given, however, for the use of the student who is studying the paper as an example of a typical statistical problem.

The Committee then proceeded to graduate rates of remarriage by age irrespective of year of widowhood. Graduation is in some respects more of an art than a science, and the Committee used good judgment in trying one method after another until satisfactory results were obtained. The fit of the table as finally adopted seems very good.

In obtaining rates by year of widowhood the decision to assume a constant ratio for each year of widowhood seems wise. Anyone who has tried to graduate even crudely a function of two variables will appreciate the bewildering complexity of the problem. In fact, in this particular case the Committee could have been criticized for introducing greater refinements than they did. The trend lines shown on page 318 are reasonably uniform in slope except for that of the fourth year, which particular year is based upon a relatively small volume of data. To have attempted to obtain a separate graduation for each year of widowhood would very probably have introduced accidental fluctuations into the graduated figures.

In the final paragraph of the paper the authors refer to the table as being based "upon an adequate volume of American experience". The word, "adequate", is largely relative. By omitting several refinements the table may very correctly be said to be an adequate table. If further refinements were desired, additional volume would undoubtedly be necessary, as the authors state.

A few comments on the characteristics of the table may be in order. It is quite obvious that year of widowhood does have a greater effect upon the remarriage rate than age, as was anticipated. Its effect upon annuity values is not so marked, since the ultimate rates appear in a great many terms of the annuity formula, whereas the remarriage rate in the first five years of widowhood appears only once for each year. Because of the

importance of the ultimate rate it might be desirable at some time to have a statistical test of the appropriateness of considering the sixth year rates as the ultimate rates.

The question of using a Makeham graduation in order to facilitate joint life calculations may very possibly have been considered by the Committee. If it is desired to preserve the separate rates by year of widowhood, it would not be possible to use the principle of uniform seniority when the combined mortality and remarriage table is combined with a mortality table upon a second life depending upon age only. Although Mr. Dawson and Mr. Olifiers (T. A. S. A. Volume XV, page 306) regraduated the Dutch remarriage table by Makeham's law in order to calculate joint annuities, Mr. Woodward (T. A. S. A. Volume XXIV, page 423) did not use the principle of uniform seniority in calculating formulas for the new tables required by the 1922 Amendment to the New York Workman's Compensation Law. If reasonably close approximations can be secured for annuities on more than one life by merely changing the interest rate, it would seem undesirable to try to Makehamize the remarriage tables with the inevitable distortion that is rather apt to occur, particularly since the mortality among the widows, not to mention the remarriage rate, may very possibly not be subject to graduation by Makeham's law, if the indication of most modern mortality studies is any guide.

On the cover of the *Proceedings* in which this paper appears is the quotation, "Thought once awakened does not again slumber". As the authors intimate, this investigation should not be considered a finished product, but as the commencement of a series of researches into the subject.

MR. FRANCIS S. PERRYMAN:

It has been a great pleasure to me to review this paper even though it has taken considerable time and labor. The arduous work of the Remarriage Committee has produced an extremely valuable contribution to casualty actuarial knowledge and this has been ably supplemented and extended by the extremely lucid and interesting presentation made by the authors of the paper. The actuarial world is indeed indebted to the members of the Remarriage Committee and to the authors of the paper under discussion. It is my hope that as a reward for these services the

Society will ask these gentlemen, or such of them as are available, to resume their labors at some not too future date in order to give us the benefit of the compilation and examination of any further data then available.

When I first set out to review this paper I imagined that it would be no light task. On looking into it more closely, however, I realized that the methods used in the compilation, graduation, and presentation of the Table were so clearly explained that the reviewing was considerably lightened. As a result I was able to devote a little more time to thinking about some of the fundamentals involved and it is mainly round two of the basic decisions made by the Committee that this discussion will be centered. (These points are first the choosing of the experience to be regarded as "ultimate" and second the question of the mortality rates to be used.) The collection and compilation of the data was done with great care and eminently satisfactory results, as is evident from the account given in the paper of the steps taken to this end.

While it is, of course, no fault of the Committee, it is a pity that more complete information was not available regarding the older cases so that all experience relating to policy years prior to 1921 had to be excluded. It is unfortunate that this step had to be taken because, as will be emphasized below, the result is that there is little or no information available for constructing what is the most vital part of the Remarriage Table, namely, the "ultimate" experience, that is, that relating to widows whose husbands have died more than a few years ago. Presumably it was not possible to collect the experience on the cases of policy years prior to 1921 that were outstanding at say the beginning of 1921 and trace this experience through to 1929 or 1930, for no doubt had this course been possible the Committee would have followed it. Presumably the next time the remarriage experience is compiled, if this is not postponed too far into the future, it will be possible to pick up the new experience from the time the present study terminated, and in this way arrive at an experience based more solidly on a longer period than the present one. The reasons for the lack of the experience on the older cases being unfortunate can be summarized as follows:—

1. As a result of the study of the experience actually available, which was that by year of widowhood for the first six years

of widowhood, considerable and consistent differentials were noticed between the experience by year of widowhood and thus a select table was compiled showing the experience by years of entry for the first six years only. However, no experience was available as to the rates of remarriage after the first six years of widowhood, so the Committee decided to assume that the experience of the sixth year represented the ultimate experience. There were good grounds for doing this. After the first year which, as would be expected, showed a very low remarriage rate, the remarriage rates showed a decrease each year, from high second year values. The Committee presumably, therefore, assumed that this decrease could not be continued indefinitely with the increase in time elapsed since widowhood, and decided, therefore, to take the sixth year experience as representing the ultimate rates of remarriage.

Now it is beyond question that the experience does show these decreases in remarriage rates from the second year of widowhood to the sixth, although it is not so evident just why this decrease in trend should be continued so long. Arguing from the point of view of what one might expect, it is difficult to see why at most after two or three years of widowhood the remarriage rate should not depend entirely on the age attained and be independent of the period of widowhood. In this connection it is to be remembered that when the experience is looked at divided by years of widowhood there are only 250, 186, 81 and 36 remarriages respectively.

Now one of the principal uses of remarriage tables is the computation of reserves and commutation values and for these purposes the part most frequently used is the ultimate portion; furthermore, even when the select portion is being used the ultimate portion has a very large effect on the values brought out. The graduation given in the paper, taking the sixth year (the lowest) as the ultimate, really gives for the ultimate portion of the table the lowest possible number of remarriages which the experience is capable of developing. Furthermore, the experience for the seventh, eighth and ninth years of widowhood as shown in Exhibit III of the paper, indicates higher remarriage rates than those for the sixth year. There are about twenty actual remarriages for these three years of widowhood to six or seven expected, using the sixth year graduated rates. (These figures are taken from a comparison of actual and expected given later in this discussion). Now it is admitted that the experience on these seventh, eighth and ninth years of widowhood is very scanty and is not particularly depend-

able. Nevertheless the disparity between the actual and expected remarriages is very great.

The arguments under this heading are thus very greatly in favor of a collection at some not too future date of further information regarding the ultimate rates of remarriage.

2. From a practical point of view it is awkward to have to use a select table with more than a few years of selection. The modern tendency in life and annuity tables is to cut down the select portion as much as possible without undue distortion of the facts. This is especially so in the case of tables of annuitants' mortality which are in many respects analogous to remarriage tables as regards their uses. It would certainly be more practical if the remarriage facts could be fitted to a select table with only one or two years of selection, provided always that such a shortened period of selection gives a proper representation of the underlying facts. It is admitted that this consideration is purely concerned with labor-saving but consider the bulkiness of Tables I to XI of the paper. With only one year of selection these would be reduced to one-third.

In conformity with the above reasons I was led to consider whether the experience shown in the paper was capable of being developed in the form of a select table with a shorter period of selection than six years, and which would therefore have a larger volume of data available for the ultimate portion. After a few experiments I came to the conclusion that very little more distortion would be introduced by taking just one year of selection and combining all the experience for all years of widowhood other than the first. The remarriage rates brought out on this basis are given below but first of all it seems necessary to make a few remarks regarding the methods used in the paper in graduating the experience and which were followed in many respects in my regraduation. The methods are perfectly sound, although at a first glance the manner of attack was a trifle confusing to me. What was done was to graduate the rates for ages at entry, combining the first six years of widowhood, and then make (flat) differentials for the various years of widowhood, the sixth year being taken as indicative of the ultimate experience. The more usual procedure is to graduate the aggregate (total) or truncated (i.e., excluding the years of selection) experience by ages attained and then make differentials for the select period by number of years elapsed since entry. However, the method used produces the

desired result. Then in order to give "more volume," five-year moving averages were used: this procedure is usually expressed as to give "more smoothness," and is really the first step of a mechanical (or summation) graduation. This five-year summation requires an adjustment for the slight distortion introduced. In the carrying out of the graduation in the paper this adjustment could have been made in the differentials for the year of widowhood but was not: the error on account of this is nevertheless not serious.

The graduation made by the Committee using the third degree parabola is remarkably successful, especially considering the not very large volume of experience. It is a little surprising that such an algebraic curve involving only four constants can give such a close approximation to the remarriage rates for the long range of nearly sixty years. From the point of view of pure theory a more elegant solution of the problem would have been an exponential curve approaching zero asymptotically about ages 75 to 80, but the difficulties met by the Remarriage Committee and by Mr. Olifiers with the Dutch Remarriage Table indicate the impracticability of such a fitting and the parabolic graduation used gives to my mind a better representation of the experience than would result from other methods, such as a mechanical or summation formula, on account of the small volume of the data. A desirable refinement would have been the adjustment of the values at the older ages to approach zero smoothly. The table given in the paper stops abruptly in the ultimate portion at age 78 with a remarriage rate of .0015. The final values of the remarriage rates r_x^r are given to four decimal places, but the mortality q_x values combined with the remarriage rates are taken to five decimal places, and I think it would perhaps have been more consistent either to take out the remarriage rates to five places or to cut the q_x values down to four places. These, however, are minor points.

To come back now to the rates I determined from the experience, combining all years of widowhood except the first, I could have used an average differential for years of widowhood 2 to 6 inclusive, along the lines of the procedure set forth in Exhibits VIII and IX of the paper (this would have been about 1.152), applied it to the Committee's graduated average rates and considered the rates so brought out as belonging to attained ages

equal to age at entry plus 3; but since what I wanted was a series of rates for attained ages, it seemed more direct to put together this experience (which is easily done from Exhibit III) and graduate the values so obtained. I decided at the same time to regraduate by itself also the experience for the first year of widowhood, thus arriving at graduated values quite independent of the graduation in the paper. Benefiting by the experience of the Remarriage Committee I graduated both the select (first year) and the ultimate (next five years) experience by third degree parabolas, using the exposures as weights and fitting by the method of least squares just as was done by the Committee. However, in order to save labor I first of all grouped the experience in five-year age groups. The graduation made by the Remarriage Committee showed that the experience is closely represented by such third degree parabolas and accordingly the two graduations made by me were very successful. The final rates brought out are shown in Table A appended.

A correction was incorporated in these final rates for the double summation in fives applied to the original data, namely the one involved in the tabulation of Exhibit III and the one introduced by me in using five-year age groups. Also an adjustment was made from ages 70 upwards to run the rates smoothly into zero at about age 80. From these rates combined with the United States life mortality as in the paper, complete tables of l'_x, etc., as in Tables I to XII of the paper, were compiled and specimen values are given later. At this point emphasis should be laid on the different interpretations of the original data underlying the two tables, the official ones given in the paper and the alternative one prepared by me. The former gives to the ultimate portion of the experience probably the lowest possible values for the remarriage rates, while the latter gives the highest possible. It will be observed that the select values are very similar to the Committee's values while the ultimate are somewhat like those of the Committee's for $r'_{[x-2]+2}$.

The following table gives a comparison of the actual remarriages by age groups and by years of widowhood, the expected remarriages by the Committee's official graduation and my alternative one. Such a comparison (for the official table) would have been a useful addition to the exhibits in the paper, as it gives a valuable check on the graduation.

COMPARISON OF ACTUAL AND EXPECTED REMARRIAGES

Age Group (Age Attained)	1st Year of Widowhood			2nd to 6th Years of Widowhood			7th, 8th, and 9th Years of Widowhood		
	Actual	Expected Official	Expected Altern.	Actual	Expected Official	Expected Altern.	Actual	Expected Official	Expected Altern.
Under 20	14	12	12	12	11	9	0	0	0
20 - 24	44	48	47	147	172	167	0	0	0
25 - 29	45	48	48	258	237	240	3	1	2
30 - 34	39	36	37	195	197	197	5	2	3
35 - 39	27	24	25	126	137	136	3	1	3
40 - 44	13	14	14	88	90	87	4	1	2
45 - 49	8	7	8	53	47	46	2	1	2
50 - 54	5	4	4	26	28	28	1	0	1
55 - 59	1	3	2	19	17	18	1	0	1
60 - 64	2	2	2	11	10	11	1	0	0
65 - 69	1	1	1	6	5	6	0	0	0
70 and over	0	0	0	1	2	1	0	0	0
TOTAL	199	199	200	942	953	946	20	6	14

	2nd Year of Widowhood			3rd Year of Widowhood			4th Year of Widowhood		
	Actual	Expected Official	Expected Altern.	Actual	Expected Official	Expected Altern.	Actual	Expected Official	Expected Altern.
Under 20	12	11	9	0	0	0	0	0	0
20 - 24	74	93	81	40	46	48	25	24	24
25 - 29	116	105	91	65	62	63	53	44	43
30 - 34	76	80	68	52	52	52	44	39	37
35 - 39	44	52	44	36	35	35	25	29	26
40 - 44	35	33	27	25	23	22	12	19	17
45 - 49	23	17	14	11	12	11	9	10	9
50 - 54	10	10	8	4	7	7	4	6	6
55 - 59	7	6	5	6	4	4	4	4	4
60 - 64	6	4	3	2	3	3	1	2	2
65 - 69	2	2	2	2	1	1	1	1	1
70 and over	0	1	1	1	0	0	0	1	0
TOTAL	405	414	353	244	245	246	178	179	169

	5th Year of Widowhood			6th Year of Widowhood		
	Actual	Expected Official	Expected Altern.	Actual	Expected Official	Expected Altern.
Under 20	0	0	0	0	0	0
20 - 24	7	7	11	1	2	3
25 - 29	17	18	27	7	8	16
30 - 34	14	18	25	9	8	15
35 - 39	12	14	19	9	7	12
40 - 44	13	10	13	3	5	8
45 - 49	8	5	7	2	3	5
50 - 54	5	3	4	3	2	3
55 - 59	2	2	3	0	1	2
60 - 64	0	1	2	2	0	1
65 - 69	1	1	1	0	0	1
70 and over	0	0	0	0	0	0
TOTAL	79	79	112	36	36	66

It will be seen that

(a) For the first year of widowhood the two graduations both fit about equally well.

(b) (i) For the second to sixth years of widowhood *combined* both the graduations are equally representative of the experience, but

(ii) taking the second to sixth years separately the official graduation gives a good fit for each year, while the alternative gives too few expected remarriages for the second and fourth and too many for the fifth and sixth.

(c) For the seventh, eighth and ninth years of widowhood both graduations give too low expected values, but the fit of the alternative graduation is much more credible than that of the official.

The results under (a) and (b) in the last paragraph, of course, are those that would be looked for in view of the assumptions made in the two graduations. The deficiency of the alternative graduation in the second year of widowhood is significant, but the differences for the other years are not so important. The results for the seventh, eighth and ninth years are interesting. However, in choosing between the two graduations, we have, it must be reiterated, no trustworthy clue to the experience for longer periods of widowhood: if when this is available it should show a continuation of the low rates of the fifth and sixth years, then the official graduation will be the closer, but if it should show a return to the comparatively higher rates of the third and fourth years, then the alternative graduation would be the closer, and the deviations for years 5 and 6 would not be of material importance. Perhaps somewhere in between the two graduations might lie the correct interpretation of the remarriage experience—say a graduation using two years of selection with all periods of widowhood longer than two years combined for the ultimate portion.

Before proceeding to compare further the results of the two graduations, it is necessary to discuss the question of the mortality which should be used in conjunction with the remarriage rates. The Remarriage Committee used the United States Life Table—1910—White Females, Original Registration States, and the reasons given for this are very plausible. It would seem reasonable at first sight to use such mortality rates in preference to, say, the table compiled from the experience of the Danish Female Beneficiaries under Survivorship Annuities, which table has here-

tofore been used in many jurisdictions, including New York. However, it seemed to me that, at any rate, an investigation should be made as to what information the experience put together by the Remarriage Committee could give on the subject of mortality.

The deaths were tabulated separately and while the actual number of deaths recorded was only about one-third of the number of remarriages (380 against 1,187), yet surely the mortality shown must have some significance. Accordingly I took out from Exhibit III for exposures and numbers of deaths for (a) the first year of widowhood, (b) the next five years, and (c) the next three years, all by attained ages. An adjustment had to be made to the exposures shown which were computed for the purpose of obtaining the remarriage rates. In the case of a remarriage a full year's exposure was properly given, but to obtain values of q_x, the probability of dying within the year, only a fractional exposure can be given, for it is possible for a widow to re-marry and afterwards die within the year, and such deaths are not recorded. Thus, assuming an even distribution of remarriages and deaths throughout the year, only half a year's exposure on the average can be given to a case where remarriage takes place. So from the exposures as given in Exhibit III, half the number of remarriages was deducted. (Note: the assumption regarding an even distribution of remarriages is probably not accurate for the first year of widowhood where most of the remarriages probably occur in the second half of the year.

The effect of the assumption made is, therefore, to increase slightly the mortality rates brought out for the first year of widowhood. For example, age 33—first year of widowhood shows in Exhibit III, tabulation 1, an exposure of 1444—remarriages 40 and deaths 8. Deducting half the 40 remarriages we get an exposure of 1424 to be compared with 8 deaths, giving a value for q_x of .00562, whereas if the remarriages on the average occurred nine months after widowhood the exposure for the deaths would be 1444 minus 30, or 1414, giving a value for q_x of .00566. The difference, it will be observed, is small enough to be negligible.)

The exposures thus prepared were multiplied by the values of q_x from the United States Life Table to get the expected deaths and the results were rather surprising. The number of expected deaths was much higher all the way through than the number of

actual deaths. Apparently the mortality experienced was much lighter than that shown by the United States Table used. The number of expected deaths was therefore computed, using the Danish Survivorship Annuitants rates and a much closer agreement with the actual deaths was shown. There is exhibited in the following table the number of actual deaths and the number expected by the United States Table and by the Danish by age groups and years of widowhood. (Allowance has been made for the fact that the data in Exhibit III is a running five-year summation.)

COMPARISON OF ACTUAL AND EXPECTED DEATHS

Age Group (Ages Attained)	1st Year of Widowhood			2nd to 6th (incl.) Years of Widowhood			7th to 9th (incl.) Years of Widowhood		
	Actual Deaths	Expected U. S.	Expected Danish	Actual Deaths	Expected U. S.	Expected Danish	Actual Deaths	Expected U. S.	Expected Danish
Under 28	4	8.8	9.8	12	16.8	19.0	0	0.1	0.1
28 - 37	14	18.2	16.4	31	43.8	39.4	1	0.7	0.7
38 - 47	25	22.0	18.0	47	63.0	51.2	2	1.6	1.3
48 - 57	25	25.4	20.6	61	84.8	64.2	3	2.3	1.7
58 - 67	20	28.8	20.8	73	97.6	70.6	9	3.8	2.8
68 and over	5	12.2	9.6	45	67.0	53.2	3	2.9	2.3
TOTAL	93	115.4	95.2	269	373.0	297.6	18	11.4	8.9

Age Group (Ages Attained)	All Years of Widowhood		
	Actual Deaths	Expected U. S.	Expected Danish
Under 28	16	25.7	28.9
28 - 37	46	62.7	56.5
38 - 47	74	86.6	70.5
48 - 57	89	112.5	86.5
58 - 67	102	130.2	94.2
68 and over	53	82.1	65.1
TOTAL	380	499.8	401.7

It will be seen that the ratio of actual to expected deaths for all ages and years of widowhood combined is 76% for the United States table and 95% for the Danish, and that on the whole the same general relationship is maintained in the various parts of the table. It would thus appear that the United States table considerably overestimates the mortality experience by the widows and that the Danish table runs much closer to the indications. Here again I must emphasize that (1) the experience is very meager, (2) we have practically only experience for the first six years of widowhood and no indication of that for subsequent

years, and (3) we have no experience at all for the older ages. After the 75th year the mortality is of increasing importance in the calculation of annuity values, but in this connection it must be observed that from age 75 and up there is not a great deal of difference between the United States and the Danish tables. Whether the extremely light mortality shown for the first few years of widowhood would be continued in later years must be a matter of conjecture until further information is available.

It is interesting to speculate on the possible cause of the low death rates; there is no "selection" of the widows in the same sense as there is of purchasers of annuities, but perhaps the receiving of a life pension operates to some extent in the same manner as does the receipt of an annuity purchased from an annuity institution, leading to an ordered and securer life with resultant lightened mortality. However, this discussion of the paper is already running to great lengths and I must curtail these speculations.

In any case I considered it advisable in the light of the results of this investigation into the mortality experience to consider the effect of combining the remarriage rates (both those of the official calculation and those of my alternative graduation) not only with the United States life mortality rate, but also with those in the Danish table. This I did, using the same method of first adjusting q_w *to* q'_x, as is demonstrated in the paper, which is, of course, the correct procedure, although it is interesting to note that such an adjustment was not made by Mr. Olifiers when calculating the Dutch Remarriage table as presented in Mr. Dawson's paper (see *Transactions*, Actuarial Society of America, Vol. XV, page 306). The table constructed by him was used by Mr. Woodward with a few unimportant modifications in preparing the official New York tables (see *Transactions*, Actuarial Society of America, Vol. XXIV, page 414), and consequently the annuity and other values based on it are to that extent not exactly comparable with values from tables including the proper adjustment. The neglect of this adjustment has the effect of slightly increasing the effect of the mortality but the error introduced would probably not be great. In connection with the adjustment of q_w the remarks made above about the distribution of remarriages in the first year of widowhood are pertinent here.

I also had a table constructed combining the Dutch remarriage

rates with the United States Life Mortality rates—this for comparative purposes. Thus, four combined remarriage and death tables had to be constructed and this involved considerable labor, but it is interesting to see how quickly such work can be done with modern calculating appliances. A complete set of the ultimate values for q'_x, p'_x, l'_x, m'_x, D'_x, N'_x, and M'_x I found could be constructed by one calculating machine operator in one day (including the necessary checks) and the addition of the select portion took about another half day. The complete tables are too voluminous, of course, to be included in this discussion, but there is given in Table B appended a comparison of

(a) the complete expectation of unremarried life
(b) the values of continuous annuities at 3½% interest payable during the continuance of unremarried life
(c) the expectation of remarriage
(d) the present value of an endowment of 1 at 3½% interest payable upon remarriage.

according to

(i) the official American Remarriage Table
(ii) the alternative Table prepared by me
(iii) the Dutch Remarriage Table

combined with mortality as shown by

(1) the United States Life Table—1910
(2) the Danish Annuitant Table

together with (for the purpose of comparison) the values of the complete expectations of life and the continuous annuity values at 3½% according to the two mortality tables uncombined with remarriage rates.

These values are given in the case of the Official and Alternative American Remarriage Tables both for "select" and "ultimate," i. e., "select" applying to the moment of widowhood and the ultimate to, for the official table, after five years of widowhood, and for the alternative table, after 1 year of widowhood.

These values are given for ages 20, 30, 40, 50, 60, 70, and 80. In symbols (a) is $(\Sigma l'_{x+1} \div l_x) + \frac{1}{2}$

(b) is $\overline{N}'_x \div D'_x$

(c) is $\Sigma m'_x \div l'_x$

(d) is $\overline{M}'_x \div D'_x$

It will be seen from this Table B that

(I) comparing the remarriage experiences (on the same mortality basis)
  (i) both for select and ultimate values
    (1) the American (official) is lighter than the alternative
    (2) the American (official) is lighter than the Dutch at the younger ages but heavier at the older
    (3) the American (alternative) is heavier than the Dutch
  (ii) as between select and ultimate values
    (1) for the American (official) the select is heavier than the ultimate
    (2) for the American (alternative) the select is lighter than the ultimate

(II) comparing the mortality experience (on the same remarriage basis)
  (i) the United States is heavier than the Danish but the difference at the older ages is not great.

"Lighter" remarriage experience has been taken above as being indicated by higher expectations of unremarried life and annuity values and by lower expectations of remarriage and endowment values; while "lighter" mortality is indicated by lower expectations of life and annuity values and higher expectations of remarriage and endowment values. These results are about what would be expected from the characteristics of the tables.

Since from some points of view and for some purposes the alternative graduation of the American Remarriage Table combined with the Danish Table mortality rates may be regarded as the more suitable combination to use, I give in Table C the values of p^r_x, l^r_x, m^r_x, D^r_x, $\overline{N}^r_x$, and $\overline{M}^r_x$ (at 3½%) for this table. The values given are the ultimate (after first year) as these will cover the majority of the requirements and the extension to select values is not difficult.

This concludes my remarks on the two points mentioned at the beginning of this discussion. As to the rest of the paper there is not a great deal on which it seems necessary to comment.

The investigation into possible differences by geographical divisions and that seeking to find out whether the remarriage rates were affected by the number of dependent children, shows to my mind remarkably small variations and it is rather fortunate, considering the limited volume of experience available, that we

can safely say that as far as our present knowledge goes it is not necessary to make any distinction between the remarriage rates geographically or on account of the number of children dependent on the widow. If, for instance, the investigation by number of children had shown marked variations which could not be neglected, then the task of determining differentials would have been very difficult because of the thinness of the experience thus divided.

The tables attached to the paper give all the basic functions and commutation columns needed. Owing to the five-year period of selection, the tables necessarily run to considerable bulk. There are twenty-four pages of tables tabulating eleven functions. These eleven functions include both N'_x and $\overline{N}'_x$, which seems unnecessary. The latter would have been quite sufficient. In place of this it would have been possible to give a table of $\bar{a}'_x$ and if it had been desired to make the tables even more useful a table of the value of an endowment of 1 payable on remarriage could have been included. The notes at the end of some of the tables are helpful, but in one or two cases are likely to be a little confusing to a person not particularly well versed in actuarial symbols.

For instance, the note at the foot of Table XI shows how to calculate from the table the value of 1 payable upon remarriage, (a) at any time following husband's death, and (b) during n years following husband's death. The reference in this to "following husband's death" would necessitate the use of select values and in any case does not cover the use of the tables other than as at the moment of husband's death. A preferable phraseology would have been that $\overline{M}'_x$ divided by D'_x is the present value of 1 payable immediately upon remarriage at any future time, and similarly for the value of 1 payable during the next n years. These, however, are rather minor points. There is one misleading misprint on page 297, where the definition of $\overline{M}'_x$ should have been given as $\Sigma\, v^{x+\frac{1}{2}} \cdot m'_x$.

I do not want the authors of the paper to think that because in the course of this discussion I have devoted most of my space to a discussion of alternative hypotheses for the graduation of the remarriage rates and for the mortality basis I believe those used in the paper to be entirely fallacious. The Remarriage Committee in their deliberations no doubt considered the pros and cons of

these two points very much more fully than it was possible to report in the paper. In a discussion of a paper, it is often permissible to wander a little further afield than would be desirable in the formal presentation of a paper, and I believed it would perhaps be advisable in this discussion to present, so to speak, the other side's case in respect of the two points mentioned.

One of the strongest reasons for doing so was to emphasize the necessity for the Casualty Actuarial Society not to rest from its labors in the collection of remarriage data but to supplement at some future date the very valuable results already obtained. In any case as regards the paper itself, the authors were, of course, not solely or even necessarily primarily responsible for the decisions made on the two points in question. Their job was to present the results of the Remarriage Committee's work and in this they have succeeded admirably. Their account of the work of compilation and graduation of the experience is to me extremely lucid and very pleasurable reading. The clarity and continuity of their paper could be well copied by others.

TABLE A

AMERICAN REMARRIAGE RATES (ALTERNATIVE GRADUATION)

(Select Period — One Year)

Attained Age	Remarriage Rates		Attained Age	Remarriage Rates	
	Select	Ultimate		Select	Ultimate
	$r^{x}_{[x]}$	r^{r}_{x}		$r^{r}_{[x]}$	r^{r}_{x}
18	.06475	..	50	.00549	.01140
19	.06110	.14783			
20	.05759	.13861	51	.00503	.01055
			52	.00461	.00981
21	.05423	.12980	53	.00424	.00918
22	.05101	.12140	54	.00392	.00864
23	.04792	.11340	55	.00364	.00819
24	.04497	.10578			
25	.04215	.09853	56	.00339	.00782
			57	.00318	.00752
26	.03946	.09165	58	.00301	.00728
27	.03689	.08513	59	.00286	.00708
28	.03445	.07896	60	.00274	.00692
29	.03212	.07312			
30	.02991	.06761	61	.00264	.00679
			62	.00256	.00668
31	.02782	.06242	63	.00250	.00658
32	.02583	.05754	64	.00245	.00648
33	.02395	.05295	65	.00241	.00637
34	.02218	.04866			
35	.02050	.04464	66	.00238	.00624
			67	.00236	.00609
36	.01893	.04090	68	.00233	.00589
37	.01745	.03741	69	.00231	.00565
38	.01606	.03418	70	.00228	.00534
39	.01476	.03118			
40	.01355	.02842	71	.00218	.00497
			72	.00200	.00451
41	.01242	.02588	73	.00174	.00394
42	.01137	.02355	74	.00143	.00330
43	.01039	.02142	75	.00110	.00264
44	.00950	.01949			
45	.00867	.01774	76	.00077	.00198
			77	.00047	.00136
46	.00791	.01616	78	.00023	.00082
47	.00721	.01475	79	.00006	.00038
48	.00658	.01349	80	..	.00010
49	.00601	.01238			

TABLE B

COMPARISON OF MORTALITY AND REMARRIAGE VALUES ON VARIOUS TABLES

Mortality	U. S. Life 1910						Danish					
Remarriage	American Official		American Alternative		Dutch	(None)	American Official		American Alternative		Dutch	(None)
Age Attained	Sel.	Ult.	Sel.	Ult.	—	—	Sel.	Ult.	Sel.	Ult.	—	—
Complete Expectation of Unremarried Life												
20	18.17	22.43	13.20	12.11	13.78	44.88	18.90	23.86	13.59	12.46	14.17	47.34
30	24.41	26.82	21.45	20.63	21.23	36.96	26.04	28.62	22.76	21.89	22.55	39.75
40	24.73	25.65	23.55	23.20	24.75	29.26	26.87	27.88	25.50	25.12	26.94	31.92
50	20.31	20.63	19.77	19.66	21.03	21.74	22.48	22.83	21.83	21.71	23.36	24.12
60	14.39	14.55	14.19	14.13	14.86	14.92	16.11	16.30	15.87	15.81	16.69	16.73
70	9.20	9.26	9.21	9.18	9.38	9.38	10.14	10.22	10.16	10.13	10.35	10.35
80	..	5.35	..	5.35	5.35	5.35	..	5.55	..	5.55	5.55	5.55
Continuous Annuity (3½%) during Unremarried Life												
20	10.20	12.36	8.27	7.60	8.45	21.52	10.33	12.52	8.34	7.66	8.54	21.97
30	13.81	15.07	12.61	12.14	12.25	19.67	14.28	15.61	13.00	12.51	12.64	20.45
40	14.99	15.52	14.49	14.28	14.94	17.34	15.79	16.34	15.22	15.00	15.75	18.33
50	13.54	13.74	13.28	13.20	13.95	14.37	14.58	14.79	14.27	14.18	15.04	15.50
60	10.60	10.71	10.48	10.94	10.91	10.95	11.62	11.75	11.49	11.44	11.99	12.02
70	7.42	7.47	7.43	7.41	7.56	7.56	8.10	8.16	8.11	8.09	8.25	8.25
80	..	4.66	..	4.66	4.66	4.66	..	4.83	..	4.83	4.83	4.83
Expectation of Remarriage												
20	.676	.592	.808	.825	.771		.665	.583	.809	.825	.782	
30	.403	.339	.514	.533	.481		.401	.338	.522	.540	.491	
40	.193	.161	.259	.271	.174		.195	.172	.268	.279	.179	
50	.086	.071	.127	.132	.035		.089	.073	.135	.140	.036	
60	.046	.034	.066	.070	.003		.048	.036	.070	.074	.003	
70	.023	.015	.021	.024	.000		.023	.015	.022	.025	.000	
80	..	..	..	.000	.000		..	.000	..	.000	.000	
Endowment Payable on Remarriage (3½%)												
20	.551	.454	.649	.677	.638		.551	.454	.640	.669	.647	
30	.321	.256	.390	.415	.405		.323	.259	.396	.418	.410	
40	.152	.121	.191	.203	.150		.154	.123	.195	.207	.154	
50	.068	.053	.094	.099	.031		.070	.055	.098	.103	.032	
60	.038	.027	.053	.057	.003		.040	.028	.056	.060	.003	
70	.021	.013	.019	.022	.000		.021	.013	.020	.023	.000	
80	..	..	..	.000	.000		..	.000	..	.000	.000	

TABLE C

AMERICAN (ALTERNATIVE) AND DANISH — 3½% INTEREST

(Ultimate Values — After 1 Year of Widowhood)

Age Attained	p_x^r	l_x^r	m_x^r	D_x^r	$\overline{N}_x^r$	$\overline{M}_x^r$
20	.85642	672,163	93,168	337,806	2,587,859	226,078
21	.86519	575,654	74,720	279,521	2,279,196	180,053
22	.87354	498,050	60,463	233,660	2,022,605	144,389
23	.88149	435,067	49,337	197,209	1,807,171	118,894
24	.88900	383,507	40,567	167,959	1,624,587	96,911
25	.89626	340,961	33,595	144,275	1,468,470	79,447
26	.90308	305,590	28,007	124,935	1,333,865	65,743
27	.90955	275,972	23,493	109,011	1,216,892	54,220
28	.91566	251,010	19,820	95,798	1,114,487	45,098
29	.92144	229,840	16,806	84,752	1,024,212	37,663
30	.92687	211,784	14,319	75,454	944,109	31,571
31	.93199	196,296	12,253	67,571	872,597	26,557
32	.93679	182,946	10,527	60,846	808,388	22,411
33	.94129	171,382	9,075	55,072	750,429	18,970
34	.94648	161,320	7,851	50,086	697,850	16,104
35	.94940	152,686	6,816	45,802	649,906	13,708
36	.95276	144,960	5,929	42,014	605,998	11,698
37	.95638	138,112	5,167	38,676	565,653	10,009
38	.95947	132,088	4,515	35,738	528,446	8,587
39	.96231	126,734	3,952	33,134	494,010	7,386
40	.96491	121,957	3,466	30,803	462,042	6,370
41	.96726	117,678	3,046	28,717	432,282	5,510
42	.96938	113,825	2,681	26,837	404,505	4,780
43	.97129	110,340	2,363	25,136	378,518	4,159
44	.97296	107,172	2,089	23,589	354,156	3,630
45	.97443	104,274	1,850	22,175	331,274	3,178
46	.97571	101,608	1,642	20,877	309,748	2,792
47	.97677	99,140	1,462	19,681	289,469	2,460
48	.97765	96,837	1,306	18,574	270,341	2,175
49	.97834	94,673	1,172	17,545	252,282	1,929
50	.97886	92,622	1,056	16,584	235,217	1,715
51	.97918	90,664	957	15,684	219,083	1,529
52	.97935	88,776	871	14,838	203,822	1,367
53	.97934	86,943	798	14,040	189,383	1,224
54	.97917	85,147	736	13,285	175,721	1,097

TABLE C—Continued

Age Attained	p_x^r	l_x^r	m_x^r	D_x^r	$\bar{N}_x^r$	$\bar{M}_x^r$
55	.97884	83,373	683	12,568	162,794	984
56	.97834	81,609	638	11,886	150,567	883
57	.97768	79,841	600	11,235	139,007	791
58	.97684	78,058	568	10,613	128,083	709
59	.97585	76,250	540	10,017	117,768	633
60	.97469	74,409	515	9,445	108,037	563
61	.97336	72,526	492	8,895	98,867	499
62	.97184	70,594	472	8,365	90,237	440
63	.97014	68,606	451	7,854	82,127	385
64	.96824	66,557	431	7,362	74,519	334
65	.96613	64,443	411	6,887	67,395	287
66	.96380	62,260	389	6,429	60,737	244
67	.96123	60,006	365	5,987	54,529	204
68	.95841	57,678	340	5,560	48,755	169
69	.95530	55,279	312	5,149	43,401	137
70	.95190	52,808	282	4,752	38,450	108
71	.94818	50,268	250	4,372	33,888	82.8
72	.94410	47,663	215	4,005	29,700	61.4
73	.93963	44,999	177	3,653	25,871	43.3
74	.93472	42,282	140	3,316	22,386	29.5
75	.92924	39,522	104	2,995	19,230	18.8
76	.92312	36,725	73	2,689	16,389	11.0
77	.91627	33,902	46	2,398	13,845	5.8
78	.90857	31,063	25	2,123	11,585	2.6
79	.89995	28,223	11	1,864	9,591	0.9
80	.89028	25,399	3	1,621	7,849	0.2
81	.87945	22,612		1,394	6,341	
82	.86747	19,888		1,184	5,052	
83	.85436	17,252		992.3	3,964	
84	.84001	14,739		819.1	3,059	
85	.82435	12,381		664.8	2,316	

$l_{86}^r = 41438$

AUTHORS' REVIEW OF DISCUSSIONS

MR. WILLIAM F. ROEBER AND MR. RALPH M. MARSHALL:

The authors are gratified at the response which this paper has produced. Indeed, Mr. Perryman's very interesting discussion is worthy of being ranked as a full paper and is exactly the sort of thing that was hoped for. The authors, in presenting the underlying data, tried to give it in such complete form that experiments in various methods of combining the data could be carried out by those sufficiently interested in the subject.

It is unfortunate that adequate information regarding the remarriage rate beyond the sixth year of widowhood was not available. This, of course, is due to the fact that almost all of the states pay compensation for a limited time only and also because it was decided to eliminate the experience of the earlier period. One large company did identify their reports for the earlier period so that they could have been used. However, the total experience beyond the sixth year of widowhood, even after including the eligible cases from the earlier period, was much too small to warrant any credibility.

Referring to Exhibit VIII it is found that the remarriage rate by year of widowhood shows a decrease from the second year to the sixth. If the ratios shown in column (5) are plotted and a smooth curve drawn through the points, it will be seen that the remarriage rate is "flattening out". If the curve is extended to the seventh year a ratio of approximately .45 is obtained. Remarriage rates for the seventh year of widowhood calculated from this ratio correspond fairly closely with the ultimate rates given in the table for the corresponding attained age and it therefore seems logical to assume that the decrease from the sixth to the seventh year of widowhood was due to the increase in age only and that the effect of the select period had worn off. Admittedly, this assumption is based upon what would normally be expected and there is no mathematical proof regarding the shape of the curve beyond the sixth year; assuming that the recorded experience for the seventh and eighth year is too meager to have significance.

In any event it was apparently decided that it would be best to present the tables in as complete detail as possible, especially since the term of a large number of the benefit payments will lie

within the select period. If the tables come into general use and experience shows that a table with a shorter select period is more convenient and gives satisfactory results it would not be difficult to calculate such a table from the values given.

The members of the Remarriage Committee were in some doubt as to whether their task was not completed with the determination of the remarriage rates. However, in order to present the table in a practical shape so that the members of the Society could readily apply it to their own experience, it was decided to combine the remarriage rates with mortality rates and prepare commutation columns. The mortality rates for white females as obtained from the United States Life Tables for 1910 were selected for the purpose as being representative, in the opinion of the Committee, of conditions which would pertain to widow beneficiaries under the compensation act. The close agreement of the recorded deaths with the Danish Survivorship Table as brought out by Mr. Perryman is rather surprising. In view of the small number of recorded deaths, however, there must remain some doubt as to whether this has real significance or is merely fortuitous.

The authors do not believe that the American Remarriage Table as presented was intended to be in a final or irrevocable form. If any members of the Society have been applying these tables to their own experience, it is to be hoped that the Society will be favored with the results of such experiments in the near future.