ABSTRACT OF THE DISCUSSION OF PAPERS READ AT THE PREVIOUS MEETING

INCURRED BUT NOT REPORTED CLAIM RESERVES—THOMAS F. TARBELL
VOLUME XX, PAGE 275

WRITTEN DISCUSSION

MR. H. O. VAN TUYL:

The determination of adequate reserves for claims incurred but not reported is a problem that is worthy of increased attention and the exposition of the subject by Mr. Tarbell should prove of real value to all who have the task of establishing these reserves. He has set forth the essential nature of the problem and has explained in detail the formulæ used by him in computing these reserves. Under this procedure, the incurred but not reported losses of the previous year as developed for a period of eleven months are taken as a base and these amounts are modified by the application of two factors, one to reflect exposure and accident frequency, and the other average claim costs. I believe the method as outlined is thoroughly sound. What I shall have to say will therefore be in the nature of supplemental comment.

That there is at present no uniformity in the methods followed by casualty companies in arriving at these reserves is quite evident. A review of the statements published in Part III of the New York Insurance Department annual report discloses a decided variation in the amounts shown as a reserve for incurred but not reported claims. The reserve for a particular line of insurance in one company may be five times the amount shown by another company with the same approximate volume of exposure. It is hardly possible that each of these reserves is a correct measure of the latent liability for the particular company. It should be kept in mind, however, that the practice of companies differs considerably in the matter of recording claims on

the Home Office records as having been reported during the current year. Where a company holds open its claim register to record every claim that was reported to December 31st to any representative of the company anywhere in the field, a smaller reserve for incurred but not reported claims would be necessary than in the case of a company which used the date reported to the Home Office as the determining date. Then again some companies may include among their known claims all notices received up to the time of compiling final lists of claims where such notices concern claims incurred in the year of the statement. This would of course have the effect of reducing the volume of losses for which a special bulk reserve would be required.

Whatever the procedure followed by a particular company, when once it has been established, it is normally continued from year to year and the data gathered for one or more previous years is applicable as a guide for the future. It is the problem of the individual company therefore to estimate on the basis of its own past experience the probable future cost of claims already incurred but which have not been already provided for through individual estimates because their existence is as yet unknown.

Of the three factors named by Mr. Tarbell as affecting this reserve, the volume of exposure is of the greatest importance. For most lines of insurance the accident or claim frequency and the average claim cost should not change sufficiently in the period of one year seriously to affect the calculation. Except where the volume of exposure is very large, the variations in these factors in a single company can well be ignored. As respects the factor of exposure it is obvious that the greater the volume of risk as measured by premiums in force the greater the number of claims. It is also true that the greater number of incurred but not reported claims will be found to have occurred in the period just preceding the end of the year and that the number of these claims becomes less in each preceding period. To be exact, a sliding scale of weights should be assigned to the premiums in force during each of the months of the year and we would then have an accurate measure of exposure for the purpose in view. It would then be found that the premiums in force during the last two or three months of the year exercise a preponderating influence on the final results and it is for this reason that the volume of premiums in force at the close of the year has been found in actual

practice to be a quite satisfactory basis on which to determine the reserve.

In the case of fidelity and surety only have definite regulations been promulgated for the computation of this reserve. The minimum reserves required by the New York Insurance Department are based upon premiums in force and if the method is satisfactory in the case of the bonding lines, which are subject to considerable variation in incidence of claims and in claims cost, it should afford an even more reliable basis for most other lines.

One reason why the writer favors the use of premiums in force as a basis arises from the fact that he has used this method for several years in determining these reserves. By means of punch cards an exact record was obtained of all claim payments and all claim reserves on claims reported subsequent to the year in which the loss actually occurred. Tabulations of losses paid were made each month and of losses outstanding at the end of each quarter. The accumulated figures at the end of the third quarter furnished nine months' development of incurred losses. By relating these incurred losses to the premiums in force at the end of the previous year percentages were obtained which in turn were applied to the premiums in force at the end of the current year to arrive at a reserve for these incurred but not reported claims. It would be quite in order to use a longer period of development or to modify these percentages to reflect a fully developed cost but for most lines it is safe to ignore claims reported more than nine months after the occurrence of the event producing the claim.

The great advantage of this method is its simplicity and the ease with which the computation can be verified. It satisfies the requirement of reflecting the change in volume of risks exposed to loss and as respects changes in loss cost this is given effect to a certain degree. Any change in the cost of the incurred losses generally will affect the cost of the previous year's subsequently reported claims and so automatically affect the percentage indicating the relationship of the losses to the premiums in force.

If all companies followed a similar procedure in the recording of claims at the close of the year it would seem to be entirely feasible to adopt uniform percentages for use by all companies even as is now done in the case of the bonding lines. Such standard percentages could be revised from time to time as the

data furnished by the companies disclosed the advisability of a change.

MR. ROBERT S. HULL:

Every one who has to struggle with the problem of setting up proper claim reserves is indebted to Mr. Tarbell for his presentation of the subject of Incurred But Not Reported Claim Reserves. Every company must make its estimates, but the methods used have in many cases been somewhat less than scientific. Mr. Tarbell's formula provides an excellent basis for computation subject to the limitations which he sets forth in his paper. The comments that follow are rather by way of suggestion than criticism.

Mr. Tarbell's formula is dependent upon the validity of certain factors as a guide to what may be expected for the coming year. It seems possible that for a company doing a moderate volume of business other factors based on a broader time spread of experience might produce more dependable results.

Mr. Tarbell takes as his starting point "the amount of incurred but not reported claims of the preceding year developed down to the end of (or for the first eleven months of) the current year, modified, if necessary, by a factor to project such claims to an ultimate basis." For a company having a relatively small volume of business, it may be that the claims which chanced to be included in the incurred but not reported of the previous year were insufficient in number to produce a dependable average, i.e., that they might as Mr. Tarbell suggests include a disproportionate share of major claims which would make the results for that year not typical of a normal year. It is possible that a more dependable factor would be the number of incurred but not reported claims developed in the preceding year.

Another factor which Mr. Tarbell uses is the average incurred cost per notice, computed for the last three months of the current and of the preceding year. Mr. Tarbell suggests a longer period than three months for the test, depending on the volume. For workmen's compensation, and liability lines, it is doubtful whether experience so recent as the last quarter or even for the last half of the current calendar year would be dependable for the great majority of companies.

With these considerations in mind, it is suggested that for a

smaller company a more simple formula would produce substantially as dependable results. For example:

$$\text{Reserve} = \frac{N^{y}_{10\text{-}11\text{-}12}}{N^{y-1}_{10\text{-}11\text{-}12}} \times N^{y-1}_{1} \times C$$

When $N =$ Number of notices
$C =$ Average incurred cost per notice
$N_1 =$ Number of incurred but not reported claims
y Designates the current calendar year
$y - 1$ Designates the previous calendar year
Subscripts designate calendar months

In this case C will be the latest dependable average ascertainable from the company's experience. It is assumed that the number of incurred but not reported losses may be expected to bear the same relation to the number of notices in the past three months that the number of such notices for the past year bore to the corresponding period of the previous year. This formula has the advantage that it may be applied with slight modification to any month in the year.

Another method which has been followed with reasonably good results is to record the average lag in reporting losses. With a sufficient volume of business and in the absence of exceptional conditions, it will be found that the percentage of notices reported in the month in which they occurred will be fairly constant. Similarly the percentage reported by the end of the next succeeding month and at the end of the second succeeding month will be reasonably dependable.

The following formula may be used:

$$\frac{\frac{N^{a}_{12}}{R_1} + \frac{N^{a}_{11}}{R_2} + \frac{N^{a}_{10}}{R_3} - (N^{a}_{12} + N^{a}_{11} + N^{a}_{10})}{R_3} \times C$$

Where N^a represents the number of notices with date of accident in the month denoted by the subscript reported to December 31st; R represents the average accumulated percentage of such notices received to December 31st and R_3 is less than unity.

MR. W. P. COMSTOCK:

In his paper entitled, "Incurred But Not Reported Claim Reserves," Mr. Tarbell has made another valuable contribution to our *Proceedings*. He does not claim that he has reached an

entirely complete solution to the problem, but states that he has obtained fairly satisfactory results from the application of his formula.

In a paper entitled, "A Method of Testing Loss Reserves," which I presented before the Society in November, 1930, I called attention to the fact that the bulk of incurred but not reported losses was developed under the workmen's compensation, automobile liability and other liability lines. I do not believe that adequate reserves can be determined on these lines unless the incurred but not reported liability is taken into account. Some companies set up a special voluntary additional reserve in Schedule "P" to take care of this feature as well as any other cause, known or unknown, which may tend to produce an inadequacy.

Mr. Tarbell lists as current factors affecting past experience the following:

(1) Comparative volume of exposure
(2) Comparative accident frequency
(3) Comparative average notice or claim costs

However, he does not directly use the comparative volume of experience in his formula, stating, "The comparative number of notices reflects not only change in volume of business, but change in accident frequency. The trend in claim cost or claim severity is reflected in the average notice cost." I think he is correct in not attempting to use directly the volume of exposure.

The general formula which Mr. Tarbell proposes is as follows:

$$\text{Reserve} = \frac{N^{y}_{10\text{-}11\text{-}12}}{N^{y-1}_{10\text{-}11\text{-}12}} \times \frac{C^{y-1}_{10\text{-}11\text{-}12}}{C^{y}_{10\text{-}11\text{-}12}} \times I^{y-1}_{(1)\text{-}(12)}$$

In which N = Number of notices
C = Average incurred cost per notice
I = Amount of incurred but not reported claims
y Designates the current calendar year
$(y-1)$ Designates the previous calendar year, and subscripts designate calendar months.

Any one attempting to use the formula should note that the last factor $I^{y-1}_{(1)\text{-}(12)}$ represents losses incurred in year "$(y-1)$" but reported in year "y". In other words the superscript designates the previous calendar year and the subscript applies to the current calendar year. It would not be correct to base a formula

upon the total losses incurred as shown on a company's statement for the reason that total losses would include increases and decreases on old claims.

The following results were obtained by application of Mr. Tarbell's formula to the three major lines of business written by the two companies with which the writer is associated.

RATIO OF ACTUAL TO EXPECTED LOSSES INCURRED BUT NOT REPORTED USING TARBELL'S FORMULA

Company A

Calendar Year	Workmen's Compensation %	Automobile Liability %	Other Liability %
1929	107.7	91.3	69.4
1930	85.4	108.6	96.8
1931	97.9	77.0	112.0
1932	109.2	123.8	139.8
1933	108.9	159.4	81.0

Company B

1932	113.0	96.0	48.0
1933	88.0	105.0	132.0

The results for 1933 are based upon actual losses incurred but not reported as developed to September 30, 1934 and hence are not as reliable as they would be were developments available for 12 months. Percentages greater than 100 indicate that use of the formula would have produced inadequate reserves while percentages less than 100 indicate that use of the formula would have produced redundant reserves. The result of using a full year's notices is shown below.

RATIO OF ACTUAL TO EXPECTED LOSSES INCURRED BUT NOT REPORTED USING NOTICES RECEIVED DURING 12 MONTHS' PERIOD

Company A

Calendar Year	Workmen's Compensation %	Automobile Liability %	Other Liability %
1928	69.2	124.4	186.5
1929	115.9	108.6	77.9
1930	86.7	92.1	70.7
1931	107.1	82.0	140.0
1932	101.1	121.3	132.8
1933	136.6	134.5	66.0

Company B

1931	94.0	99.6	95.0
1932	169.4	107.1	90.1
1933	56.3	87.5	119.7

The use of a longer period did not have the steadying effect which I thought might result. In fact a greater departure from actual results is noticeable when a full year's notices are used. As a last test I eliminated December notices from the calculation as the number of December notices is likely to fluctuate from year to year due to early or late closing of the claim records.

RATIO OF ACTUAL TO EXPECTED LOSSES INCURRED BUT NOT REPORTED USING OCTOBER AND NOVEMBER NOTICES

Company A

Calendar Year	Workmen's Compensation %	Automobile Liability %	Other Liability %
1929	109.3	100.5	74.7
1930	93.1	108.1	93.5
1931	91.7	82.0	106.4
1932	114.6	114.9	154.0
1933	111.3	158.2	73.0

Company B

1932	172.3	126.5	105.9
1933	50.5	103.0	129.7

On the whole I believe that the results in the last table are the best. Perhaps the inclusion of September notices would have produced better results. Application of the theory of credibility, as Mr. Tarbell suggests, might make the fluctuations less violent. Elimination of abnormal cases is a desirable refinement. With a little more study it would be possible, by use of Mr. Tarbell's formula, to arrive at a fairly close prediction as to the ultimate incurred but not reported loss to be expected.

AUTHOR'S REVIEW OF DISCUSSIONS

MR. THOMAS F. TARBELL:

The author feels that the interest in this subject as evidenced by the thoughtful discussions has well repaid his modest effort to stimulate interest in a rather important phase of casualty reserves.

The discussion of Mr. Hull points out desirable, if not necessary, modifications of the suggested formula to fit the conditions of a company having a comparatively small volume of business in a particular line; the use of what might be termed an average

"normal" value per notice and the extension of the "experience" period, both in the line of overcoming the possible effects of chance fluctuation. The indicated formula would become

$$\frac{N^{Y}_{(12-n+1)\text{----}11\text{-}12}}{N^{Y-1}_{(12-n+1)\text{----}11\text{-}12}} \times N_I^{Y-1} \times C$$

Where $n =$ the number of months' data to be used
and $C =$ a "normal" average value

The formula suggested by Mr. Hull for determining the number of cases to be reserved for, based upon the average lag in the reportings of a particular accident months' notices is a useful one and has been used rather extensively by the author's company in the past, particularly for the automobile property lines and plate glass. The formula as given is a modification, usually sufficiently accurate for practical purposes, of the general formula:

$$\sum_0^n N^a \left(\frac{1}{R_n} - 1\right) \times C$$

This formula, using a somewhat different notation, together with an example of its practical application will be found in the author's chapter on Reserves in "Casualty Insurance Principles" —Michelbacher.

Mr. Van Tuyl draws attention to the variations in the amounts shown as reserve for incurred but not reported claims. This variation is no doubt due in part to lack of uniformity of definition. In the case of The Travelers we divide the so-called Incurred but not Reported Reserve into two parts for annual statement purposes; one part is designated "Transit" and represents the reserve for claims reported to the field claim offices prior to December 31st but not so reported to the Home Office, the other part is designated "Incurred But Not Reported" and represents cases reported to the field claim offices subsequent to December 31st. The transit portion is included in column (1), "Adjusted or in process of adjustment", page 5 of the annual statement and the balance in column (5), "Incurred But Not Reported". The factors necessary for the division of the aggregate reserve are obtained from the punch cards briefly described in the original paper—last paragraph, page 276. The report year referred to designates the calendar year of report to the field claim office. With the exception of Burglary, Boiler and Machinery losses in

excess of $5,000 which may be reported subsequent to December 31st, we do not include any specific cases in our incurred but not reported reserve.

Mr. Van Tuyl also brings up the point that the volume of exposure is probably the most important factor influencing the incurred but not reported reserve and except where the volume of exposure is very large, the variations in claim frequency and average claim cost can be ignored. The method which he advocates would be expressed in formula form as follows:

$$\frac{PF_{12}^{y}}{PF_{12}^{y-1}} \times I_{(1)-(9)}^{y-1}$$

Where PF_{12} = Premiums in force at end of year.

This formula should produce satisfactory results for most lines under normal conditions, but the author prefers the more refined formula for the major lines, compensation, liability and the automobile lines, particularly as conditions respecting claim frequency and claim severity for liability and the automobile lines have not been normal in recent years and probably will continue to exhibit somewhat abnormal characteristics in the immediate future. Further such a method does not reflect changes in rate or premium levels. The formula has a serious deficiency, at least theoretically when applied to the compensation line, since the premiums in force are predicated in large part upon estimated advance premiums which tend to be overstated in a period of decreasing industrial activity and depressed in a period of increasing industrial activity.

The author feels that this feature of reserve determination is necessarily an individual company problem and that uniform percentages of premiums in force to be used by all companies as suggested by Mr. Van Tuyl would produce irrational results for individual companies. In addition to the indicated defects of the method as applicable to compensation it should be pointed out that such a method assumes that all companies will have substantially the same loss ratio for each line of business. It would also be predicated upon a uniform practice of reporting notices to the Home Office by the field offices and there is unquestionably considerable variation in such practices, some companies reporting all notices to the Home Office immediately upon report of accident or loss while others wait for varying

periods of time to permit furnishing the Home Office with more complete details of the accident or loss and a more reliable estimate of the cost.

Mr. Comstock's practical tests of the author's suggested formula as applied to the business of the companies with which he is associated are of interest and, of course, pertinent as the value of any such formula is measured by the dependability of the results produced. In the cases of Mr. Comstock's tests the results are admittedly none too satisfactory. There are two conditions which may explain this situation. The factors of accident frequency and accident severity are both subject to chance fluctuation and while so far no attempt has been made to apply the theory of credibility or possibly modifying these factors through an application of experience rating principles casual tests indicate that a rather substantial volume of exposure is required to produce a reasonably high degree of credibility.

The human element is also involved to considerable degree since the factors C^v and C^{v-1} (or A^v and A^{v-1}) depend for accuracy on the ability of adjusters to place relatively correct initial estimated reserves on new cases—that is, the estimates from period to period must correctly reflect trends in claim costs. If estimates on new cases are inadequate at the end of the current year in relation to the reserves at the end of the previous year the result will be to depress the ratio $\frac{C^v}{C^{v-1}}$ and consequently produce too low a reserve. Conversely if the reserves at the end of the current year are relatively redundant the result will be too high a reserve. In case of The Travelers tests similar to Mr. Comstock's show results which are reasonably satisfactory. These tests which are based upon the original formula embracing three months notices show the following ratios of actual to expected incurred but not reported reserves.

Calendar Year of Reserve	Compensation	Auto Liability	Other Liability	Total
1932	112.8	105.1	67.3	100.8
1933	89.9	95.1	123.8	95.5
Total	101.5	100.2	84.3	98.3

While the results for Other Liability are somewhat wide of the mark for both years the results for the other lines, which involve more substantial volumes of exposure, are in our opinion quite

satisfactory. We believe that an undue pessimism, in the minds of our adjusters, as to the trend of Other Liability claims at the end of 1932, accounts in large part for the rather wide variations in the reserves for this line at the end of 1932 and 1933. It is interesting to note that for the three major lines combined the differences between the actual and expected reserves amount to only .8% at the end of 1932 and 4.5% at the end of 1933.

We are continuing to give study to this subject, particularly on the problem of some modification of the basic data by projection methods or weighting of the individual month's experience to reflect the effect of loss cost trends which the present formula necessarily does not fully reflect. If the results of such study are likely to prove of general interest they will be submitted to the Society.

SUGGESTIONS FOR A STANDARD SYSTEM OF NOTATION FOR CASUALTY ACTUARIAL WORK—THOMAS O. CARLSON

VOLUME XX, PAGE 264

WRITTEN DISCUSSION

MR. J. J. SMICK:

At the last meeting of the Society, a paper by Mr. Carlson containing suggestions for a system of notation was presented. This paper was predicated on the premise that "casualty actuarial science has progressed by now to a point where a standardized system of notation in formulas, applicable as far as possible to all casualty lines is feasible and desirable". A system of notation chiefly centering about basic and delimiting terms and in general following the criteria of (1) simplicity, (2) universality, that is, applicability to all casualty lines, and (3) foundation upon symbols already generally accepted, is presented to the Society for its consideration with the suggestion that the Society establish an acceptable system of standard notation.

For that portion of the Society's membership which is engaged in the technical phases of the work, in the actual calculation of formulæ, in the preparation and analysis of memoranda and papers, the use of the same symbol to represent the same term

would greatly facilitate the exchange and understanding of ideas. As Mr. Carlson points out, it is a great help to be able to recognize at a glance the symbols used in technical problems. But that merely requires uniformity of procedure and not necessarily a fixed and therefore relatively unchangeable system of notation, or at most it requires a bare minimum of standard symbols. Casualty actuarial science has progressed and in the course of its progress formulæ have been established, modified, abandoned and replaced. Thus in experience rating the credibility formula was evolved by steps from an earlier form until it became $Z = \frac{P}{P + K}$ and is now used infrequently, having in turn been replaced by tables of credibility values, usually called Table "E". Another example of a term fallen into disuse is the wage factor which, during the recent emergency rate making program for compensation insurance, suffered a temporary revival, but has again been abandoned. Only a few of the many symbols and formulæ in use in the past remain and most of those in vogue in previous years have served their purpose and are now rarely, if ever, used. Mr. Carlson has also pointed out a few instances where changing conditions have removed the necessity of retaining symbols. The adoption of a standard system of notation would serve to give many terms a permanence which they do not deserve and thus prevent the use of convenient symbols which might be reassigned for later use. The alternative is to adopt Mr. Carlson's suggestion and maintain a committee, whose duties shall be constantly to revise any adopted system, approving new symbols and deleting old ones. Unless this is done we might soon have an outmoded and unwieldy notational system on our hands.

I am in complete agreement with Mr. Carlson in his desire to obtain more uniformity in the use of symbols, but I do not believe it wise for the Society to establish a standard system of notation. It would seem to me sufficient for the purpose of obtaining uniformity, to have occasional papers presented on the subject in which analysis of customary and criticism of poor or ambiguous terminology can be made. In this way we could always keep abreast of the subject, could keep in the foreground the terms and symbols important in the principal subjects of discussion at the time, and allow little used terms and outmoded formulæ to gradually fade out of the picture. At the same time

the membership of the Society would be able to use these papers as a reference and would see the symbols others are or may be using and will be free to choose or reject the suggestions, in accordance with their own needs.

If we are prepared to adopt a system of notation, then Mr. Carlson's suggestions deserve our most serious consideration. He has very wisely concentrated on those symbols which have been used in the past and for which there is great likelihood to be need in the future. He has pointed out a number of instances where the formulæ are of restricted use, or where contemplated changes will lessen the need for considering and retaining symbols. Mr. Carlson does not directly mention it, but I presume that he implies that by the establishment of a committee due precaution will be maintained to keep the system up to date and anticipate symbols for lines which may shortly become important. A good many of the symbols that Mr. Carlson suggests and which now mainly apply to or are derived from workmen's compensation insurance will in all probability be equally applicable in case unemployment insurance becomes important.

The actual notation presented is simple and convenient to use. Approximately a score of symbols are sufficient to cover the most important formulæ now in use. The device of using EP to represent expected losses and nq the number of claims tends to simplify matters and keep the number of symbols at a minimum. If the EP is to represent expected losses, then P, the premium, will have to be defined, that is, distinguished as between earned premium and premium at current rates. It seems to me that the expression for payroll, $100n$, might have been included in the list and probably replace the symbol for the constant in the experience rating credibility formula, K, which is not used to any great extent. Another possible substitution might be made for the off-balance factor, B, which although still in use in the states of North Carolina, Texas and Wisconsin, could advantageously be replaced by a symbol representing the factor introduced to correct for the off-balance of rating plans. But such changes are largely a matter of individual preference which may be influenced by the amount of use to which the symbols are put by the individual.

The paper is really a practical contribution to the literature of the Society and regardless of whether or not any standard system

of notation is adopted, the symbols presented should be used wherever possible and to as great an extent as is practical in order to obtain greater uniformity in technical terms.

MR. N. M. VALERIUS:

"The editors of several mathematical journals have agreed upon the following suggestions. . . . In typewritten formulas, . . . 0 means zero. For capital *O* backspace and overprint period *O*." Thus would they avoid the dilemma Mr. Carlson's memorandist contrived. They have another suggestion for a predicament of the same kind that has at some time bothered the reviewer. "1 means one. Backspace and overprint *l* for ell".

The subject of the paper, fortunately occasioned by this unfortunate dilemma, is deserving of attention and I believe that Mr. Carlson's effort will have actual results, if not to the full extent that a standard set of symbols should be adopted by the Society. The conservative basis of Mr. Carlson's suggestions insures a degree of success for them. He is attempting to establish as a code the most appropriate or the most widely recognized symbols previously used for each of the various actuarial notions, in accordance with his criterion "(3) foundation upon symbols already generally accepted and used", meanwhile requiring that such symbols shall pass inspection in the light of "(1) simplicity and (2) universality, that is, applicability to all casualty lines". There are other criteria not so formally insisted upon—availability for the office typewriter and agreement with mathematical usage. Incidentally, the use of mathematical relations has avoided ambiguities in past usage and the setting up of several additional terms, e.g., the selection of EP for expected losses, being expected loss ratio $\times$ premium, and $1.0 - E$ for expense loading. This last might be written into the List of Basic Symbols by the members for convenience of reference.

The innovations are few and well considered and generally supply specific lacks in past usage. Certain substitute symbols have been suggested where those in use conflict with the criteria. The thorough-going insistence on the basic distinction between elementary and delimiting symbols is, of course, new and entails some new symbols as C_L and C_E for loss and expense constants.

One desideratum, that the symbol be suggestive of its meaning, is not mentioned, although it is acknowledged in fact. In this connection, it might be said, only English-speaking actuaries and American usages are considered, but with the present almost complete lack of international relations in casualty insurance technic nothing more is practicable.

I have no deep-seated disagreements with the choices of symbols to record. It seems possibly unnecessary, when L is defined as "Actual losses", to have a subscript a for actual unmodified losses, i.e., $L = L_a$, or, further, L might possibly have been defined as Losses, with sub i (incurred) for actual, or no sub-symbol, sub m for modified, and sub a for adjusted, thus saving the basic concept, losses, from appearing twice in the list of basic symbols, in what might be looked upon as two delimited senses, as L and A. This is, in fact, the only such duplication except loss ratio and expected loss ratio, but expected loss ratio is of so very frequent occurrence and so fundamental a notion it is in a class by itself. There would be another advantage in this, that if experience rating plans were being considered where the comparison between the risk experience and classification experience is made on a premium or pure premium basis instead of loss basis, P_a and p_a would be available symbols. Plan D in compensation had an item, "Adjusted Premium". The burglary plan's experience comparison is on a premium basis, though the concept of adjusted premium is not at present used since the calculation is in terms of deviation and the final modification is a deviation, equivalent to $1 - M$ in the standard notation. It might be convenient to describe the essential equation of that plan as $M = P_a/P$, in line with the notation of other plans.

Perhaps the choice of v to represent the decimal portion of the rate to be proportional to the total rate could better have been g. The letter v as an interest function is a fundamental term. The notion of *graded* expenses would be as good as *variable* expenses, the letters f and g (the two kinds of expenses with this notation) occur together in the alphabet, and the only paper in the *Proceedings* dealing with deductible and excess coverages, Dorweiler—Vol. XIII, gives g the claim of previous use.

It would have been desirable if Mr. Carlson had suggested a standard way to delimit the symbols with respect to time periods and territories or other experience limits involved. Occasionally

such delimiting symbols are useful, for instance, where calculations are made on several experience periods and the results compared for selection between them.

Possibly certain of the symbols might be given a wider definition. F seems to be intended as the symbol for any factor to correct, adjust, or modify, whereas its listing as "Correction factor (for general purposes)" might imply the narrower meaning correction factor has, that of a final reconciliating adjustment. The two expense loading symbols should be considered available in other connections than deductible average and excess insurance.

Certain of the symbols will not be of one meaning in casualty practice, when brought together with the accident and health symbols, and annuity symbols, but there would be no confusion in use. For instance, E, v, and $1 + a$ would have other meanings in those connections.

I note two errors in the lists of symbols. M for merit-rating decimal modification of a rate should be added to the list of basic symbols. Incidentally, $1 - M$ is referred to as discount in the discussion. It is, of course, sometimes negative and not discount. The final symbol of the paper should be $L_{n:m}$ not $L_{n=m}$.

The brevity of Mr. Carlson's paper belies the amount of careful research and discrimination which a review of the problem reveals. It is seen, on reflection, that certain seeming omissions, as, for instance, a reserve symbology, must await further crystallization of ideas. I believe the members of the Society will be pleased to use the symbols suggested. Perhaps the adoption of an official standard notation should wait upon a period of tentative uniformity on this basis.

There is in the records of the International Actuarial Congresses a resolution voted unanimously on May 19, 1898 "That a Universal Notation be adopted, not only for Life Assurance, but for all other branches of assurance;". It was probably the main thought of the resolution to include accident and health insurance. Some of the casualty lines of great present importance were hardly in existence anywhere at that time. It may be hoped the result of Mr. Carlson's paper will be a fulfillment, in some measure, of the letter of the resolution of 1898.

AUTHOR'S REVIEW OF DISCUSSIONS

MR. THOMAS O. CARLSON:

Mr. Smick expresses doubt as to the advisability of establishing a standard system of notation, and suggests instead the presentation of "occasional papers on the subject in which analysis of customary and criticism of poor or ambiguous terminology can be made". The author has little sympathy with such a suggestion for several reasons. His paper was written in the hope that present conflict and ambiguity along this front would be eliminated. Additional papers by this or that member expressing individual and almost certainly dissenting views, without a standardizing body of any kind, could only add to the present confusion and uncertainty. Furthermore a discussion of two or three symbols is hardly a subject worthy of the distinction of being presented as a paper. The author also feels that Mr. Smick over-emphasizes the effect of time on symbols. Most of the symbols discussed in the paper are for fundamental terms which have always been part and parcel of casualty actuarial work and will continue to be so; many terms that could have been included were excluded because the author doubted their permanence. It was pointed out that the individual memorandist may have to use his own special symbols for the more uncommon terms.

The suggestion of Mr. Smick to include the payroll symbol in any final listing is a good one and the author also agrees that B may not be the best symbol for the off-balance factor. He is open to any suggestions for a satisfactory substitute, barring the use of O.

Mr. Valerius also has suggested certain additions and changes which are desirable, such as the inclusion of the expense loading symbol in the table, the use of g instead of v for the functional expense loading, and the use of L for losses, with subscript m for modified and subscript a for adjusted. Note should also be made of the two errors he lists; $1 - M$ should be referred to as "discount or charge", and the subscripts in the final symbol in the table should be in small letters.

The author disagrees, however, with Mr. Valerius' proposal to include delimiting symbols with respect to time periods and territories. If a system is too heavily laden it will not be used:

this is one reason why Mr. Perkins' earlier suggestions along the same line have not had greater influence. Such symbols are needed very infrequently, and where they are needed the individual author could use whatever symbols are most convenient for him, explaining them as he introduces them.

The author still feels it would be desirable for the Society to sanction some table of the important symbols, not necessarily the one proposed, but one agreed upon by a group selected for that purpose. This group or committee could receive suggestions for additions, deletions and changes, and decide upon them without any necessity of airing pros and cons through papers. Such action would constitute a step in the direction of facilitating discussion and reading in connection with the problems of casualty actuarial science.

VALUATION OF INVESTMENTS—JOSEPH J. MAGRATH
VOLUME XX, PAGE 281

WRITTEN DISCUSSION

MR. FREDERICK RICHARDSON:

This is a valuable and timely contribution to our *Proceedings*. The facts have been ably presented by Mr. Magrath and the implications to be drawn therefrom have been left mainly to his readers. He has in my opinion completely justified the action of the National Convention of Insurance Commissioners in seeking to solve the problem of valuation by easy stages even though the solution rested upon a theory of market values that is open to criticism. The line of greatest safety was not along the lines of a narrow conservatism. It seldom is. As for the amortization of bonds it was an innovation, at least for Fire and Casualty Companies, and entirely warranted in the circumstances; but further studies are called for before it can be permanently adopted, such as studies of probable speeds of liquidation of liabilities in relation to income, and probability and effect of catastrophic losses. He has not attempted to lay down hard and fast rules for the future. Naturally there crops out of this paper the question of the regulation of investments which might well be the subject of

another thesis. There is no reason to believe that the matter has been settled now the crisis is over. We have to admit that similar emergencies will arise again, and although the method of meeting them is becoming stereotyped, there is need of settled practices that will lessen the necessity for exceptional treatment in the future.

The interest income on reserve funds is a function of the underwriting process, and the highest standards of investment practice are logically called for where the standard of underwriting is lowest. However, we know that this is most unlikely to obtain, as looseness of practice in one field usually goes with looseness in the other. We have for practical purposes to assume a single standard that will protect the public against the weakest organizations. It would be little use trying to apply a sliding scale as ultimately the best companies would not be satisfied with anything but the highest standard of investment practice for themselves. Of course, there is bound to be some difference of opinion regarding suitable statutory requirements. We have heard from Mr. Fortington (Vol. XII, page 294), and Mr. Tarbell (Vol. XIII, page 110), on the subject of desirable investments for Casualty Companies, but though times have changed it can be stated that the main assumptions of that discussion have been borne out, particularly those of Mr. Tarbell.

After a long period of expansion it would seem that the field of high-class bond investment is contracting. Along with this there is a growing demand for the more desirable securities from approved standards of safety. We are not only entering a market where the growing funds of Life Insurance Companies play a tremendous part but we have to compete with savings banks, trustees, industrial concerns and endowed institutions of all kinds, to say nothing of the wealthy income tax dodger. We are definitely in a low yield era when all of our instincts will be crying out for the widest field of investment to support interest earnings. If underwriting profits go up, well and good; if not then our troubles are not over by any means. At present America has a surfeit of money. It is estimated that over five billion dollars are waiting employment and earning nothing. A marked improvement in railroading and in big business generally would ease the situation and make it more tolerable. It would have the effect of reconditioning depressed railroad bonds and at the same time

bring about a dearer money market which would reduce the present phenomenally high prices for all high grade securities. The administration in Washington is interested in keeping money cheap until the needs of the national government are satisfied. This stage may be reached quite early or quite late. Who knows?

But when the Insurance Commissioners meet in December they might as well collate the statutory requirements of their respective States in reference to the investments of Fire and Casualty Companies, as they will have to make up their minds regarding what should be done to prevent a repetition of the worst features of investment depreciation in the past as they affected the insurance business. In Great Britain, where there is very little State regulation, it is customary to carry all securities at book value and to set up funds for depreciation. By this method boom values are flattened out and depressed values are covered wholly or partly by reserves. I mention these practices for what they are worth without wishing to argue their suitability for American conditions. However, I am inclined to believe that contraction of the field of investment is not so imperative as conservative valuations plus stabilization of funds. The subject of investment in stocks might be handled quantitively as well as qualitively. The better control of security markets and investment issues of all kinds now in process of development should lessen the need for contraction. Greater stabilization of prices and an increasing demonstration of values are bound to result, and this will make for greater safety over the entire range.

CORRECTION OF CERTAIN DEFICIENCIES IN THE EXPERIENCE RATING PLAN BY THE SO-CALLED "ACCOUNT CURRENT" METHOD—WRITTEN DISCUSSION—MARK KORMES

VOLUME XX, PAGE 350

WRITTEN DISCUSSION

MR. J. M. CAHILL:

Mr. Kormes' review of the writer's original discussion of this paper contains several statements with which the writer does not agree and hence has led to a continuation of the discussion. Mr. Kormes states that the credit off-balance of the compensation experience rating plan has been steadily increasing in New York, probably largely because of loss underestimates. He considers

that the insurance companies are short-sighted in refusing to recognize this situation and estimates that the annual loss in premium volume resulting from the underestimating of losses on rated risks is in the neighborhood of $1,000,000. These statements deserve further analysis and comment.

In New York it is not contemplated that the compensation experience rating plan will be in balance. The compensation rate making structure in New York provides for the determination of a manual level which when reduced by a certain expected credit off-balance will reproduce a selected collectible level for rated business. In other words, the rate making structure is such that a sizeable credit off-balance is expected and, unless the realized off-balance exceeds the expected off-balance, the companies suffer no actual loss of premium on rated business. A comparison of the actual off-balance determined from experience rating statistics with the expected off-balance on rated business for the three most recent policy years is given below:

Policy Year	Actual Off-Balance	Expected Off-Balance
1931	7.3% Credit	9.3% Credit
1932	6.9 "	8.0 "
1933	8.3 "	8.0 "

It appears from this exhibit that in recent years the companies have not suffered any loss in premium from an excessive credit off-balance on rated business. The actual result for policy year 1933 is slightly in excess of the expected, but this tendency was probably corrected at the July 1, 1934 rate revision when the actual off-balance data for policy year 1933 were used as the basic figures in calculating the expected off-balance in terms of the revised manual rates.

Mr. Kormes is correct in his statement that in the past there has been no general provision in New York for the development of the losses used in experience rating to an ultimate basis except insofar as this may have been recognized in that the payroll modification factors have been calculated to discount the average cost at new rates to the level of the experience period. Since the rate making structure contemplated a definite credit off-balance which was determined on the basis of the latest available experience rating statistics at the time of each rate revision, however, it follows that there was no need for the introduction of such

loss development factors. In order that the actual off-balance should reproduce the expected off-balance, all other conditions being equal, it was essential that the method of calculating the factors of the experience rating plan should remain unchanged.

At the July 1, 1934 rate revision in New York, however, loss development factors were definitely included in the experience rating plan. The effect of the factors is included in the payroll modification factors and not in the loss modification factors, but as a practical matter it makes no difference whether the loadings are applied to the actual losses or whether the reciprocals of the loading factors are included in the payroll modification factors. The effect is exactly identical with either method.* A factor of 1.031 was adopted for policy year 1932 in order to provide specifically for the ultimate development of losses underestimated in the first reporting. This factor was calculated on the basis of the development of loss ratio data experience beyond 24 months. This experience has shown a less substantial development, however, than has the experience reported under the Unit Statistical Plan. In comparing the loss ratios for the earlier years with the adopted rate level loss ratio, the actual undeveloped losses of policy year 1931 on a first reporting basis, of policy year 1930 on a second reporting basis, and of policy year 1929 on a third reporting basis were used. In the experience rating calculation, the second reporting for policy year 1931, the third for policy year 1930 and the fourth for policy year 1929 are actually being used. From the foregoing, it is obvious that the factors being currently employed in experience rating New York risks include sizeable loadings for loss underestimates.

It is the writer's opinion that there is theoretically no need for the inclusion of loss development factors in the experience rating plan because the New York compensation rate making method contemplates a definite off-balance. If, in the interest of conservatism, it is considered desirable to give definite recognition to the fact that losses are underestimated on the average in the earlier reportings of experience, the logical method of making this adjustment is to include proper average loadings in the experience rating plan along the general lines followed at the July 1, 1934 rate revision. It is obvious that if the "account

* "Recent Developments in Workmen's Compensation Insurance Rate Making," by W. F. Roeber, *Proceedings* Casualty Actuarial Society, Volume XV, page 230.

current" method were introduced, it would be necessary to make such adjustments in the existing rating structure in order to avoid doubling up on this phase that the companies would not benefit in the least from the introduction of this method. The effect of the elimination of existing factors would offset any benefit in premium derived from the "account current" method.

Even if it were logical to assume that the companies would benefit in any degree from the introduction of the "account current" method, Mr. Kormes' estimate of an increase in annual premium volume of approximately $1,000,000 is seriously in error. This estimate was apparently derived as follows:

Where: $50,000,000 = annual premium volume.

.70 = proportion of premium volume which represents rated business.

.55 = average credibility of rated business.

.05 = estimated loss underestimate of first reports under Unit Statistical Plan.

$$\$50,000,000 \times .70 \times .55 \times .05 = \$962,500$$

Mr. Kormes has apparently assumed that the loading of 5% which he considers to be necessary to develop first reporting losses to an ultimate basis is likewise required in the case of the earlier years where the losses used in experience rating are actually on a second, third and fourth reporting basis respectively.

If this calculation is made correctly giving recognition to the trend plan and using the loss development factors employed by the National Council in preparing the New York classification experience for the July 1, 1934 rate revision, the corresponding premium indication is only $539,000:

Unit Reporting	Trend Weights	Indicated Loading for Subsequent Development	Product (2) × (3)
(1)	(2)	(3)	(4)
4th	.25	.000	.000
3rd	.50	.010	.005
2nd	.75	.027	.020
1st	1.00	.045	.045
Total	2.50		.070

$$\frac{.070}{2.50} = .028 \text{ average indicated loading}$$

$$\$50,000,000 \times .70 \times .55 \times .028 = \$539,000$$

The New York size of risk tabulations compiled from successive reportings indicate that the experience of rated risks develops no more adversely than that of non-rated risks. For this reason, it seems proper to use in this calculation the loss development factors employed in determining manual rates. Even if a 5% loading were used in connection with the losses of the first reporting with a proportionate increase for the earlier years, Mr. Kormes' estimate would still be over 60% too high. This calculation has been included merely to point out that Mr. Kormes did not take into account the fact that smaller loadings than 5% would be in order for the early years. It in no way modifies the statement made previously that the introduction of the "account current" method would not serve to increase the total premium volume.

Mr. Kormes' footnote referring to the introduction of a flat factor of 1.05 to be applied to the actual losses through the medium of the loss modification factors is entirely irrelevant to this discussion. This factor is an integral part of the rate level calculation and, whether it had been adopted or not, the selected collectible rate levels for both the rated and the non-rated groups of business would have been identically reproduced. The adoption of this factor served to decrease the expected off-balance with the result that a lower manual level was adopted than would otherwise have been the case. The adoption of a lower manual level necessarily meant that higher loss constants were indicated for non-rated risks than would have been the case if the 1.05 factor had not been adopted. The effect of adopting this factor was to decrease the manual rate level by more than 4% and to increase the loss constants by more than 35% on the average as compared with what would otherwise have been adopted. The collectible rate level for either rated business or non-rated business was not changed by the inclusion of this factor.

Finally, let the writer conclude by stating that it was not his intent to imply a chronological order of events when he used the word "following" at the top of page 347. Unfortunately, however, he used the word "following" with the meaning of "employing".