ON THE USE OF SYNTHETIC RISKS IN DETERMINING PURE PREMIUM EXCESS RATIOS FOR LARGE COMPENSATION AND LIABILITY RISKS

BY

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The pure premium ratio which denotes the ratio of the aggregate of risks' losses in excess of a specified loss ratio to the total aggregate losses depends upon the size of the risks considered. There is a dearth of material in the larger premium sizes for determining such pure premium ratios. The premium volume may be large when viewed as an aggregate but the data are rather limited when viewed as to the number of units—the individual risks—involved. This is not strange, for risks falling into the higher premium size groups do not exist in large numbers. Even if the experience of every risk of larger size could be secured there still would be a scarcity of material. This scarcity suggests recourse to other possible sources of material.

Concept of Actual and Synthetic Risks

The common conception of a risk is rather indefinite. A risk is perhaps most often considered as the total of the insured operations of an individual in a line for a specified term, usually a year. Should the term be doubled, then immediately the size of the risk, measured in premium, would be doubled, or should the term be increased n-fold the size of the risk would also be increased n-fold. Obviously, one way to obtain larger risks would be to extend the term. Or, instead of extending the term, two or more consecutive terms could be united and thus composite risks of various sizes could be constructed synthetically. However, the form of the material used in this paper precludes following this process. The data are not given out in such form that it is possible to identify and unite consecutive terms of the same assured.

If a combination of terms of the same assured is not possible the next recourse might be to various combinations of separate but comparable risks. The available material is in such form that it is possible to unite risks in the same premium size groups for the same policy year or to unite risks in the same premium size group but in two different policy years. Risks may be combined to build larger synthetic risks by adding premiums and losses and determining the resulting loss ratios. By continuing such combinations a number of large synthetic risks may be built and utilized for studying the behavior of loss ratios and determining pure premium ratios for losses in excess of specified loss ratios for the very large premium size groups where now little or no material is available.

Source of Material Used

In this study such synthetic risks were built up from printer tabulator lists of premiums and losses for individual experience rated compensation risks of the Compensation Inspection and Rating Board of New York. These records listed the premiums and losses arranged in groups by size of premium for policy years 1931 and 1932. For policy year 1931 all risks having annual premiums of $5,000 or more were given, arranged within the size groups according to experience rating modification. For policy year 1932 all risks having annual premiums of $2,500 or more were given, arranged without definite order within the size groups. With respect to factors affecting the size of loss ratios, the risks were arranged at random within each size group for each policy year.

Starting with the lowest premium size group in each year, the adjacent risks were combined in pairs consecutively by adding premiums and losses. The composite risks, each of whose elements came from the group $2,500-$5,000, fell into group $5,000-$10,000. These synthetic risks in the $5,000-$10,000 groups were similarly combined in pairs making new risks falling into the $10,000-$20,000 premium size group. The actual risks of the $5,000-$10,000 group were combined in pairs into synthetic risks also falling into the $10,000-$20,000 premium size group. The process was continued to the point where there were but 25 synthetic risks falling into the premium size group $640,000-$1,280,000. When combining risks in pairs it is convenient to select as group limits such values that the upper limits of the groups are double the lower. Under such a selection the limits of synthetic risks combined from a given size group are co-terminal with the limits of the next larger group of actual risks.

The process of forming synthetic risks may be readily observed by following the risks on a line from left to right in the table below. The number in parentheses at the left on each line is the number of actual risks in the premium size group shown in the heading and the policy year indicated in the left column. On the same line moving to the right are shown the number of synthetic risks derived from pairing the risks in the preceding column. As will be noted from the table, each policy year and size group was treated separately in combining into higher groups, and odd risks left over in the process of pairing were dropped. The reason for the separate treatment of policy years and groups will be discussed under the heading "Adjustment of Premium and Industrial Cost Level."

Policy Year	Lower Limit of Premium Size Groups in Thousands								
	2.5	5	10	20	40	80	160	320	640
1931	..	..	..	..	..	..	..	..	..
1932	(959)	479	239	119	59	29	14	7	3
1931		(538)	269	134	67	33	16	8	4
1932		(465)	232	116	58	29	14	7	3
1931			(227)	113	56	28	14	7	3
1932			(170)	85	42	21	10	5	2
1931				(99)	49	24	12	6	3
1932				(78)	39	19	9	4	2
1931					(34)	17	8	4	2
1932					(32)	16	8	4	2
1931						(8)	4	2	1
1932						(6)	3	1	..
1931							(3)	1	..
1932							(2)	1	..
Total Actual	959	1,003	397	177	66	14	5	..	..
Total Synthetic		479	740	567	370	216	112	57	25
Grand Total	959	1,482	1,137	744	436	230	117	57	25

Method Used in Calculating Pure Premium Ratios

The method used in deriving pure premium ratios is substantially the one described in a previous paper in the *Proceedings,* Volume XIII, pp. 163-7, and Table IV, pp. 174-5. The method involves relatively simple operations, produces reasonably accurate results, facilitates combinations of experience from various sources, and permits the graduation of frequency distributions of risks by loss ratio size. The decisive factor in selecting the method was the possibility of using for comparisons results of prior studies based upon this method.

The disadvantages of the method consist of the uniform weighting of all risks within a given premium-size group and the use of the mid-points of the class interval of the loss ratio groups into which the risks have been divided as the loss ratio for all risks of the group. This latter feature, however, is not an essential part of the method. These disadvantages make an offsetting correction necessary when the experience of a group is keyed to a specific loss ratio. Recently in connection with Retrospective Experience Rating, Mr. S. D. Pinney and Mr. Mark Kormes have used methods which are more accurate though somewhat more laborious, not involving the disadvantages mentioned. The results from the three methods show close agreement.

Dispersion of Risk Loss Ratios

The causes of dispersion and skewness in the loss ratios of risks of a premium size group when these loss ratios are considered as a frequency distribution may for convenience in discussion be divided into these three kinds:—

Accidental Dispersion
Lack of Homogeneity within Classifications
Variations in Industrial Cost Level

Accidental Dispersion

It is the accidental deviations of the loss ratios of risks that are of primary concern here. It is known that the frequency distribution of risk loss ratios changes in form with the size of the risk. The distribution* is of a form that is high at the extreme left, then

* Where abscissas represent size of loss ratios and ordinates, the number of risks.

descends very sharply and extends to the right as a long low flat curve for very small risks. It changes to other forms which tend somewhat toward the normal curve for very large risks. These accidental dispersions are inherent. They are the most important factor in the study of pure premium ratios for excess insurance per loss ratio for small and medium risks.

Lack of Homogeneity in Classifications

The hazards and conditions of individual risks vary almost endlessly. The attempt to assign risks to a limited number of classifications composed of equally hazardous risks can only partly succeed. For the risks when arrayed according to hazard vary gradually whereas the classifications on account of their limited number must have abrupt differences in the pure premiums which represent the average hazards. Even with the application of experience rating it is vain to believe that complete justice has been served in each case. The best that can be said is that there is no more reason for believing that individual rates are too high than that they are too low. If the experience rated rate is either too high or too low, it will tend to decrease or increase the risk loss ratio and thus introduce a deviation. In view of the refinements made in assigning classifications, calculating manual rates, and determining special rates, it is not believed that the lack of homogeneity of risks within classifications is comparable in degree to the other two divisions as a cause of dispersions of loss ratios. Deviations arising from a lack of homogeneity are not biased. The caution needed is not regarding the use of the experience for determining pure premium ratios but rather regarding the propriety of applying the pure premium ratios to every risk in the classification as the measure of its probable deviation.

Variations in Industrial Cost Level

Wage level, accident frequency, accident severity, medical cost and claim consciousness, major factors entering into the cost of compensation coverage, are continually changing. If a premium level for a specific term is predicated on a definite set of factors which determine what will be called the industrial cost level, and then during the term the factors change so that a different industrial cost level prevails there will be definite responses in the risk loss ratios. If the industrial cost level is lower than the cost level predicated in the premium level there will be general decreases in loss ratios. Similarly, if the industrial cost is above that predicated in the premium level there will be increases in the loss ratios.

It is rare that the premium level is just right. Over a period of years there will be generally some high and some low levels even though they average out for the period. These variations from the proper premium level produce biased deviations in the risk loss ratios; biased in the sense that all loss ratios deviate in the same direction from what they would be if the premium level were such as to produce the permissible loss ratio. If proper adjustment of premium level is not made in deriving pure premium ratios serious errors may be introduced unless the period of the experience is long enough to include industrial cost levels and premium levels that will approximately reproduce the expected average.

Relative Effect on Pure Premium Ratios

It is difficult to give the relative importance of the first and third of the above divisions of the causes of dispersion and skewness with regard to their effect on pure premium ratios. The relative importance changes with the size of both the selected risk loss ratio and the risk premium. For all but the large risks, accidental dispersion is generally the dominant factor. But even in moderate sized risks when the selected loss ratio is small (e.g., .10 or less) the variations in industrial cost level is more important than accidental dispersion. As the risks increase in size of premium, the effect of variation in industrial cost level increases and in large risks becomes dominant.

Adjustment of Premium and Industrial Cost Level

There may be some question as to what is the proper premium level to use in determining pure premium ratios. Should the actual premiums in effect during the period the experience was developed be used or should some adjustments be made? It would seem that the premium used in determining the pure premium ratio should preferably be on the same basis as that which will be used in the application of the pure premium ratios, or the rates derived from them, in insuring excess losses per loss ratio. Generally, this would be the collected premium, the premium at manual rates modified by schedule and/or experience rating, or what has recently been termed "standard" premium. The standard premium basis would be most desirable if the experience extended over several rate levels so as to approach an average experience. Since the data used in this paper came from only two policy years and since the loss level deviated on the same side of the permissible in both it has been deemed better to adjust the premium level by premium size groups to the levels producing the permissible loss ratio by groups.* The permissible loss ratio has been taken at .60 as representative of general conditions.

It would seem reasonable to have used a premium level adjusted to produce the permissible loss ratio for all risks of the groups used for each policy year as a unit rather than for each premium size group separately. This would be desirable if the volume were large enough. It was decided to use the other method on the basis that with an adequate experience rating plan there should be no known premium size group that would be expected to deviate far from the loss ratio level of the whole. A comparison of the results of the adopted method and the method mentioned in this paragraph may be made from Tables III and III′.

If the pure premium ratios for selected excess loss ratios have been determined for a definite premium level and charted as in Chart I it is relatively simple to use the same chart to obtain pure premium ratios for any selected excess loss ratios on other premium levels and industrial cost levels.

Pure Premium Ratios

Pure premium ratios for compensation losses in excess of various selected ratios were calculated upon the basis of all available data. The actual data for New York risks for policy years 1931 and 1932 were used for each premium size group. The data for each size group except the lowest were augmented by using syn-

* It will be noted from the preceding tabulation, page 79, that this procedure when combined with successive pairing of risks results ultimately in a single synthetic risk for each premium size group for a given policy year which, aside from the effect of dropping odd risks in pairing, will have the permissible loss ratio. This lessens the value of higher premium groups containing "ultimate" or "near ultimate" synthetic risks.

thetic risks derived from the lower premium size groups. The resultant pure premium ratios are shown in Table II. In the four lower premium groups the pure premium ratios for losses in excess of the selected loss ratios of .55, .65, and .75 were interpolated using third differences. For the five higher premium size groups these ratios were determined directly from the data by using a more detailed procedure in the calculation.

The pure premium ratios for various selected excess loss ratios are plotted against the size of annual premium for the risk from the data in Table II and the results are shown in Chart I. Upon examining the chart and on reflection it becomes obvious that curves for the various selected excess loss ratios approach definite horizontal lines as asymptotes when the risks become indefinitely large. In general, the asymptote for all curves for selected excess loss ratios in excess of the permissible "*E*" is the line $(ppr) = 0$, where (ppr) represents the pure premium ratios. For any selected excess loss ratio r, less than permissible "*E*," the curve has as its asymptote the line $(ppr) = 1 - r/E$. It will be noted from Table II and Chart I that the data for the larger risks have reached the limits represented by the asymptotes for the lower selected excess loss ratios.

Results from Synthetic and Actual Risks

It would be expected that the use of synthetic risks introduces a stabilizing effect into the risk loss ratios as compared with actual risk loss ratios. The homogeneity within a risk tends to reproduce variations that are not accidental. Any inherent traits or qualities that produce results in one part of the term will have the same tendency in the rest of the term. In a synthetic risk formed by uniting two risks selected entirely at random a particular characteristic of one part that produces a definite variation will on the average be partly neutralized by the effect of the risk forming the other part which fails to have the particular characteristic. The lack of homogeneity in the classifications is a cause of variations in the loss ratios of the risks in the classification. When these risks are combined into synthetic risks the different traits producing the lack of homogeneity become blended, thus lessening the effect of heterogeneity in classifications.

In Table III and Table III′ are given, by premium size groups, a comparison of pure premium ratios produced for synthetic risks derived from lower size groups with the pure premium ratios for the actual risks. These ratios must be accepted with reservation for the number of risks with large loss ratios in a premium size group is small and a few additional losses in particular risks may be reflected in a perceptible difference in the pure premium ratios.

In Appraisement of Method

Assuming that further tests will confirm those in this paper, it must be admitted that the results are not of a quality demanded for calling the method a complete success, nor are they such that the method can be considered a total failure. Not much has been added to what has already been known or believed with regard to the behavior of large Compensation risks as to excess losses per loss ratio, though some confirmation has been given to existing beliefs. However, in the determination of excess ratios per loss ratio for other lines of coverage where there is a scarcity even of medium size risks and an even greater scarcity of large size risks the method may have positive value in determining first approximations of the ratios. The method also may be useful in getting approximations to compensation or liability pure premium ratios for selected excess loss ratios when these are used jointly with per case and/or per accident limits.

TABLE II

Pure Premium Ratios for Excess Losses per Loss Ratio

Table showing pure premium ratios—ratios of losses in excess of selected risk loss ratios to total losses—for various selected loss ratios, by premium size groups. Data from New York Board risks in Table I.

Selected Risk Loss Ratio	a Lower Limit of Premium Size Group; Upper Limit is Double Lower b Average Risk Premium of Group—Synthetic and Actual Combined c Number of Risks—Synthetic and Actual Combined								
a b c	$2,500 $3,049 959	$5,000 $6,217 1,482	$10,000 $12,290 1,137	$20,000 $24,824 744	$40,000 $49,369 436	$80,000 $97,078 230	$160,000 $193,024 117	$320,000 $386,304 57	$640,000 $775,957 25
00%	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10	.848	.840	.835	.834	.833	.833	.833	.833	.833
20	.722	.698	.681	.672	.668	.667	.667	.667	.667
30	.620	.580	.545	.524	.507	.502	.500	.500	.500
40	.537	.484	.434	.396	.364	.342	.336	.333	.333
50	.468	.405	.344	.290	.247	.206	.183	.172	.167
55	.438	.372	.306	.247	.199	.151	.120	.100	.089
60	.411	.341	.272	.209	.157	.108	.071	.046	.030
65	.385	.314	.242	.177	.122	.074	.040	.016	.005
70	.362	.289	.215	.150	.094	.050	.023	.005	.001
75	.341	.267	.190	.126	.071	.033	.012	.003	.000
80	.320	.246	.168	.106	.054	.021	.006	.001	
90	.285	.210	.132	.075	.031	.011	.003	.000	
100	.255	.180	.104	.054	.019	.006	.001		
110	.229	.156	.083	.039	.011	.004	.000		
120	.208	.135	.066	.029	.007	.002			
130	.190	.118	.053	.022	.004	.001			
150	.163	.090	.034	.013	.001	.000			
175	.137	.067	.023	.007	.000				
200	.117	.051	.017	.004					
300	.062	.016	.004	.000					
500	.028	.004	.000						
700	.019	.002							
1000	.013	.000							
1500	.004								
2000	.000								

Note: In assigning risks to premium size groups the actual premiums for the individual risks were used. The original risk assignments were not changed with later adjustments of premium levels. The individual risk loss ratios and the average premiums for the groups were recalculated with changes in premium level. As a result there have been brought together in the premium groups of the Table risks from different sources having adjusted premium limits somewhat at variance from one another and from the Table limits.

CHART I. *Showing pure premium ratios for losses in excess of selected loss ratios, by size of premium. From risks in Table I and data in Table II. Figures over curves represent selected loss ratios.*

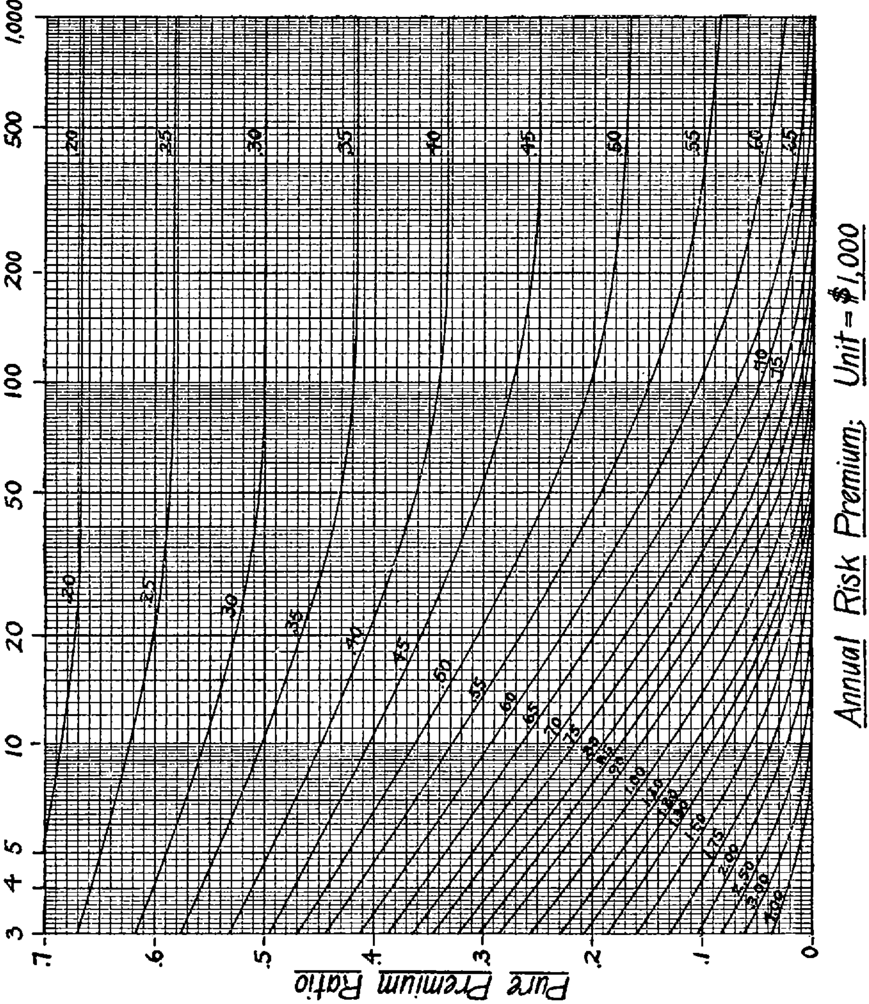

TABLE III

Pure Premium Ratios

Comparison of Pure Premium Ratios of Actual and Synthetic Risks for four premium size groups. Premium level adjusted by Policy Years to 60% loss ratio for each premium size group.

Selected Risk Loss Ratio	PREMIUM SIZE GROUP							
	$5000–$9999		$10000–$19999		$20000–$39999		$40000–$79999	
	Actual	Synthetic	Actual	Synthetic	Actual	Synthetic	Actual	Synthetic
0%	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10	.841	.838	.838	.834	.835	.833	.833	.833
20	.700	.693	.686	.678	.677	.670	.673	.667
30	.585	.571	.554	.541	.535	.521	.529	.505
40	.491	.469	.445	.427	.409	.392	.408	.361
50	.415	.386	.356	.337	.305	.286	.304	.243
60	.352	.319	.285	.265	.223	.205	.213	.152
70	.299	.267	.229	.207	.163	.146	.141	.088
80	.256	.225	.181	.162	.119	.102	.090	.050
90	.219	.191	.144	.126	.087	.071	.052	.029
100	.188	.165	.115	.098	.065	.050	.026	.018
110	.162	.143	.094	.077	.048	.036	.013	.011
120	.140	.126	.077	.060	.035	.027	.006	.007
130	.120	.113	.062	.047	.026	.021	.004	.004
140	.104	.101	.050	.038	.019	.016	.001	.002
150	.090	.091	.040	.031	.015	.013	.000	.001
175	.064	.072	.025	.022	.008	.007		.000
200	.046	.060	.016	.017	.005	.004		
300	.010	.030	.002	.006	.000	.000		
400	.002	.018	.000	.001				
500	.000	.012		.000				
Number of Risks	1,003	479	397	740	177	567	66	370
Average Premium	$6,276	$6,099	$12,020	$12,435	$25,579	$24,571	$47,255	$49,076

See note under Table II.

TABLE III

Pure Premium Ratios

Comparison of Pure Premium Ratios of Actual and Synthetic Risks for four premium size groups. Premium level adjusted by Policy Years to 60% loss ratio for the combined group $5,000-$79,999.

Selected Risk Loss Ratio	PREMIUM SIZE GROUP							
	$5000–$9999		$10000–$19999		$20000–$39999		$40000–$79999	
	Actual	Synthetic	Actual	Synthetic	Actual	Synthetic	Actual	Synthetic
0%	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10	.842	.839	.835	.835	.844	.834	.822	.836
20	.703	.695	.683	.680	.694	.671	.650	.672
30	.587	.573	.551	.543	.557	.520	.500	.512
40	.494	.472	.441	.429	.435	.390	.374	.368
50	.417	.389	.351	.339	.333	.284	.266	.250
60	.355	.322	.280	.268	.253	.204	.177	.157
70	.313	.270	.223	.210	.193	.145	.114	.093
80	.259	.227	.176	.164	.147	.103	.070	.053
90	.222	.195	.138	.128	.113	.072	.039	.031
100	.191	.169	.109	.099	.088	.050	.019	.064
110	.165	.147	.088	.078	.069	.036	.008	.013
120	.143	.130	.071	.060	.054	.027	.004	.009
130	.123	.117	.056	.047	.044	.021	.001	.006
140	.106	.105	.044	.037	.036	.016	.000	.004
150	.092	.096	.035	.031	.030	.013		.002
175	.066	.077	.021	.022	.022	.007		.001
200	.049	.064	.013	.017	.018	.004		.000
300	.012	.031	.000	.006	.004	.000		
400	.003	.017		.001	.000			
500	.000	.012		.000				
Number of Risks	1,003	479	397	740	177	567	66	370
Average Premium	$6,225	$6,039	$12,231	$12,327	$24,511	$24,579	$50,053	$49,117

See note under Table II.

TABLE I

Data of New York Experience Rated Risks Used

Premium Size Group	Policy Year	Number of Risks	Total Premium	Total Losses	Loss Ratio
$2500 to 4999	1931	....	....	....	....
	1932	959	$3,303,687	$1,754,614	.5311
	1931 & 32	959	3,303,687	1,754,614	.5311
$5000 to 9999	1931	538	3,737,494	2,147,467	.5746
	1932	465	3,227,297	1,629,134	.5048
	1931 & 32	1,003	6,964,791	3,776,601	.5422
$10000 to 19999	1931	227	3,090,109	1,635,912	.5294
	1932	170	2,318,709	1,227,148	.5292
	1931 & 32	397	5,408,818	2,863,060	.5293
$20000 to 39999	1931	99	2,720,551	1,519,407	.5585
	1932	78	2,113,820	1,197,100	.5663
	1931 & 32	177	4,834,371	2,716,507	.5619
$40000 to 79999	1931	34	1,956,128	1,004,324	.5134
	1932	32	1,729,889	866,958	.5012
	1931 & 32	66	3,686,017	1,871,282	.5077
$80000 to 159999	1931	8	809,539	329,236	.4067
	1932	6	569,857	254,451	.4465
	1931 & 32	14	1,379,396	583,687	.4231
$160000 to 319999	1931	3	584,196	351,157	.6011
	1932	2	454,450	195,990	.4313
	1931 & 32	5	1,038,646	547,147	.5268
$2500 to 319999	1931	909	12,898,017	6,987,503	.5418
	1932	1,712	13,717,709	7,125,395	.5194
	1931 & 32	2,621	26,615,726	14,112,898	.5302