ABSTRACT OF THE DISCUSSION OF PAPERS READ AT THE PREVIOUS MEETING

CONTINGENCY LOADINGS — NEW YORK WORKMEN'S COMPENSATION INSURANCE

JAMES M. CAHILL

VOLUME XXVI, PAGE 12

WRITTEN DISCUSSION

MR. A. N. MATTHEWS:

Mr. Cahill has covered, in his usual thorough manner, this very important element of the New York Workmen's Compensation rate-making procedure and there is little that can be added to his complete exposition. The contingency loading, which has played a very important part in transforming the compensation business from a most unprofitable basis to a fairly respectable line, will be entirely eliminated as far as New York is concerned in the revision effective July 1, 1940. It is hoped that it will be many years before the need for this loading again arises.

The adjustment for interest discount shown in Table 6 is calculated on the basis of an interest rate of 3½% on the mean losses valued with credit for interest discount. Mr. Cahill states that this rate is proper even though the companies may not currently be earning as high a rate of interest, because the tables used to value the outstanding losses are calculated at 3½%. This is correct if only the effect of the interest discount on the incurred losses is taken into account. If the companies cannot earn sufficient interest on these reserves to maintain them, however, it is necessary to obtain the deficiency from surplus funds. It might well be argued that the drain on surplus necessary to maintain the reserves should be added to the underwriting loss or deducted from the profit for each calendar year. The same result could be accomplished by calculating the adjustment for interest discount at the average return rate of interest for the latest calendar year. Related to this is the matter of whether or not the mortality element in the tables used is producing redundant reserves. It is

possible that the savings as claims are liquidated (particularly permanent total claims) will largely off-set the losses that will be caused by the use of an interest rate higher than that currently realized.

The resolution relative to the contingency loading which was adopted by the National Convention of Insurance Commissioners included a paragraph to the effect that the accumulation of underwriting results should not continue indefinitely "and that it shall be terminated as to old balances after a reasonable period, viz. 5 years." Mr. Cahill is very decidedly of the opinion that old balances should not be terminated. As a matter of fact it would be difficult to justify the elimination of these balances. If the balance were to be eliminated at a time when either a net underwriting profit or a net underwriting loss is shown the contingency loading would not have served its function of producing a balance of profits and losses over a period of years. If a net loss were to be eliminated the insurance companies would be penalized and if a profit were to be eliminated the policyholders would feel that they have just cause for complaint. Of course the accumulated balance is automatically eliminated whenever the balance changes from a loss to a profit or vice versa.

In at least two states large profit balances have stimulated requests for the inclusion of negative contingency loadings in the rates. The California accumulated profit at the end of 1937 amounted to $5,708,590 or 17.3% of the 1937 earned premium. At the time of the January 1, 1939 rate revision two California insurance carriers advocated a contingency loading of minus 5%. Similarly, the Minnesota experience at the end of 1938 showed an accumulated profit of $3,199,992 or 48.7% of the 1938 earned premium. The Associated General Contractors of Minnesota has proposed the use of a contingency loading of minus 5% in the rates for that state. Incidentally, a group of employers in Minnesota unsuccessfully sued for the retroactive elimination of the contingency loading which was included in the rates for 1936 and 1937 and the return of that portion of the premium which resulted from the use of the contingency loading.

At various times in the past certain company executives have held to the belief that compensation loss ratios run in cycles the phases of which are opposite to those of the so-called business cycle. The following countrywide compensation loss ratios for all

stock companies licensed in New York tend to disprove this theory:

Calendar Year	Loss Ratio	Calendar Year	Loss Ratio	Calendar Year	Loss Ratio
1923	67.6%	1929	68.3%	1935	60.4%
1924	71.6	1930	68.9	1936	58.5
1925	67.4	1931	73.3	1937	53.0
1926	67.3	1932	71.4	1938	50.7
1927	65.2	1933	73.4	1939	54.9
1928	63.8	1934	61.9		

While it is true that the loss ratio was at its maximum in 1933 when the business cycle was at the bottom, it is equally true that in 1929 at the crest of the business cycle, the compensation loss ratio was close to the top. It is reasonable to anticipate that in the future with the contingency loading available the compensation loss ratios will run in cycles to a much greater extent than in the past, since as soon as the loss ratio has been unfavorable for a few years the contingency loading in the rates will tend to correct the situation. After a period of favorable experience as in the case of the last few years, the automatic elimination of the contingency loading and the effect of the favorable experience on the pure premiums will decrease the rate level to a point where the loss ratios will no longer show a substantial margin of profit. If the contingency loading procedure were to be modified to provide for negative loadings as has been advocated, it is probable that following a period of favorable loss ratios the rates would be reduced to an inadequate rate level and very unfavorable loss ratios would result. A company writing the compensation business for the first time at this period would be at a great disadvantage since it would not have had an opportunity to accumulate a reserve during the profitable period.

Since the contingency loading is zero when the accumulated profit is 2½% and 5 points when the accumulated loss is 2½% of the earned premium for the latest calendar year, it would appear at a casual glance that the companies are guaranteed an underwriting profit of 2½%. The following example will show that this is not the case: Assume a state with a rate level which produces exactly the permissible loss ratio each year and with no accumulated balance at the end of a particular year. The following year a 2½ points contingency loading will be included in the rates,

which in turn will produce an underwriting profit of 2½%. After this profit has been realized the contingency loading will not longer be used. Thus it is seen that the companies will have accumulated a profit of 2½% of only one year's premium over a period of a considerable number of years.

It is interesting to review the present situation as respects the contingency loading. For New York the accumulated profit at the end of 1939 amounted to $12,777,229 or 15.2% of the 1939 earned premium. Since the contingency loading will again become effective when the accumulated profit becomes less than 2½% of the annual premium, there is in New York an accumulated profit of approximately $10,670,000 or 12.7% of an annual premium to be absorbed before the contingency loading will again be used. Most other states show profit balances which appear to be very substantial when related to the earned premium of a single year. These profits appear small however when compared with the underwriting losses suffered by the companies during the thirteen year period from 1923 to 1935 inclusive.

MR. KENDRICK STOKE:

Mr. Cahill's paper is a recording of the latest development in one step of our rate-making procedure. Since he records only the latest chapter in the history of this subject, he moves smoothly from the contingency loading in use to July 1st, 1938, through the questions raised regarding its continued use, to the amendments agreed upon. I like the author's style but wish he had recorded in more detail the reasoning which preceded the conclusions arrived at. Although the subject of contingency loading was being studied in 1924 and quite possibly earlier, nowhere can I find light on certain questions which keep recurring in my mind. I seem to be in the class of a certain radio comedian of our times who also has trouble with things which keep "whizzing by." So if you will bear with me, we will confine ourselves to the question of interest and be into our subject.

In Table 4 the author presents an illustration showing reserve inadequacies indicated for each of a series of years in the case of one annuitant. Assuming a reliable mortality table, the repeated deficiencies are offset in part by reserves released when others in the group drop out. The interest discount remains to plague us, however, and the payment of the present value of awards into the

Aggregate Trust Fund is no solution. In his paper presented at the May 1938 meeting, Mr. Hipp called attention to an operating loss in this Fund for 1937, and with the almost negligible return currently received from short term Government securities we may expect deficits to continue for a time; and if they do continue, the carriers will find the problem tossed back in their laps —probably as a loading on present values to make up the deficiency.

Since the New York Compensation Act and Special Bulletin No. 190 of the New York State Department of Labor are not available to me at the moment, let us use the general annuity formula here:

$$a_x = \frac{v\, l_{x+1} + v^2\, l_{x+2} + v^3\, l_{x+3} \cdots\cdots \text{etc.}}{l_x}$$

Why have we retained the discount factor in evaluating our case reserves? Probably because the life insurance companies have always considered interest in their reserve computations, but I don't believe this is a good enough reason. Workmen's Compensation is a form of social insurance designed to relieve disabled workmen (or their dependents in fatal cases) of financial distress resulting from industrial accidents. Our first concern, then, should be to ensure the continuance of compensation benefits and what better way is there than plenty of reserves? We have been leaning more and more towards a statistical approach to our problems, so let us leave the life actuary here and listen to the statistician. We find that over a period of years he has accumulated quite a volume of statistical data and knows much concerning losses paid and their "development." Using the information he supplies, we begin:

Required Reserves = Ultimate Cost minus Losses Paid.

Since this equation gives a result greater than the total of the tabular reserves, we start looking for voluntaries to add and one of them is an "Interest Reserve"; we have now reversed ourselves and added the discount back into the reserves. Let us experiment with the tabular reserves making them a summation of future payments without consideration of interest. Our annuity formula becomes

$$a'_x = \frac{l_{x+1} + l_{x+2} + l_{x+3} \cdots\cdots \text{etc.}}{l_x}$$

A similar adjustment would be made in the other formulas omitting interest but retaining our measures of the contingencies, death, remarriage, and attainment of the non-compensable age by minor dependents. This would give us terminal or ultimate values to be used prospectively. What advantages accrue from such a change?

First, loss reserves would then approach their proper values without resorting to this legerdemain of now it's in the reserves and now it isn't.

Second, Schedule "P", parts 2 and 5a would then have more indicative value as measures of reserve adequacy.

Third, with the recent spread between assumed and realized interest rates, it is difficult to earn enough to cover these recurring reserve deficiencies. Since increasing the rate of return is apt to lead to unsound investment practices, we will find it easier to avoid this pitfall when interest earnings are no longer required to maintain reserves.

Fourth, the claims turned over to the New York Aggregate Trust Fund would carry with them an adequate payment. There would be no necessity for a supplemental deficit loading and conceivably no need for an administrative loading when the rent on capital stages a comeback.

Fifth, it is manifestly impossible to keep these annuity tables abreast the gyrations in the interest rate. Furthermore, their periodic recalculation is laborious and costly, but having established tabular values into which no discount factors entered, they would remain fixed, barring a marked change in the death or remarriage rates.

Sixth, this would eliminate some of the adjustments necessary in our rate-making process for it strikes at the *raison d'etre* of our contingency factor.

There are two rather patent objections to such a departure which should be mentioned:

First, what shall be the amount paid in case of a lump sum payment or lump sum settlement? In general, industrial commissions appear to be discouraging this practice but, when permitted because of facts in an individual case, the payments could be discounted as they are now.

Second, what is to be done where some part of the interest earned on loss reserves is to be eliminated in our rate-making calculations? I have inferred, perhaps erroneously, that this adjustment made by the New York Board was dictated by a set of mutable circumstances and not caused by any fundamental objection to interest on loss reserves for sociological reasons. In any case, there seems to be no insurmountable

barrier here—just follow a method like that given by Mr. Cahill in his Table No. 6 but substitute for the tabular rate of 3½% the net realized interest rate.

Iconoclastic perhaps, but what do outsiders think? A learned man of laws might deliver a telling counterblow by paraphrasing the cover quotation on No. 50 of our *Proceedings*:

"The jurisprudence of every nation will show that, when law becomes a science and a system, it ceases to be justice."

AUTHOR'S REVIEW OF DISCUSSIONS

MR. JAMES M. CAHILL:

Mr. Matthews and Mr. Stoke have prepared very interesting discussions of this paper. As might be anticipated, they have commented at some length on the adjustment for interest discount which was adopted concurrently with the New York July 1, 1939 rate revision. This was the most important of the several changes adopted in the method of computation of the indicated underwriting profit or loss.

Without arguing the merits of whether interest discount should be reflected in determining incurred losses and what rate of interest may properly be used in these calculations, I wish to emphasize again that the sole purpose of the change introduced in New York in the method of computing the calendar year underwriting profit or loss was to make the method consistent with the other steps of the rate-making procedure. In determining the rate level and also the classification rate relativity, the experience is developed to the equivalent of sixty months and the incurred losses are equal to the sum of the paid losses and the outstanding losses as of the valuation date. Most of such outstanding losses will represent the unpaid portion of awards on long term cases which are to be valued on the basis of tables incorporating an interest discount rate of 3.5% for claims with date of accident prior to July 1, 1939 and 3% for claims with date of accident July 1, 1939 and thereafter. If the adjustment outlined in my paper had not been introduced, there would be a basic difference in the two sets of experience data and, as a long term matter, there would inevitably be a tendency for an underwriting loss to be indicated by the accumulated results compiled from the Casualty Experience Exhibit. To make the principles underlying the computation of the calendar year under-

writing profit or loss consistent with those underlying the rate-making procedure, the adjustment outlined in the paper was adopted.

Mr. Stoke has stressed the desirability of eliminating the interest discount element entirely and has emphasized the fact that an interest rate of 3.5% is much too high for current conditions. It might be well to point out again that the New York paid losses during the first sixty months development of a policy year do not reflect the element of interest discount except insofar as the paid losses include the present value of long term claims paid into the Aggregate Trust Fund by stock and mutual carriers. In this connection, it is pertinent to review the results for policy year 1935 at six months development as taken from the Loss Ratio Data Report:

WORKMEN'S COMPENSATION — NEW YORK
LOSS RATIO DATA
POLICY YEAR 1935 AS OF DECEMBER 31, 1939
(60 MONTHS' DEVELOPMENT)

Kind of Loss	Paid Losses	Incurred Losses	Ratio of Paid to Incurred (2) ÷ (3)
(1)	(2)	(3)	(4)
Indemnity	$17,684,239	$22,528,708	78.5%
Medical	10,100,583	10,453,559	96.6
TOTAL	$27,784,822	$32,982,267	84.2%

It will be noted that the paid losses at this stage of development amount to approximately 84% of the estimated incurred losses as of the same valuation date. Obviously, the interest discount element applies to only a minor proportion of the total losses as used in the rate-making procedure in New York.

A further point is that in computing this adjustment for the July 1, 1939 rate revision we were dealing with policy years 1914-1933. The present assets of the carriers doing business during these years undoubtedly include many investments which were made during this period when it was possible to obtain a better yield than can be obtained today. Not all bonds issued years ago have been called or refunded. This point was cogently dealt with by Mr. Tarbell in the informal discussion contained in page 379 of Volume XXV of the *Proceedings*.

Mr. Flynn's paper in Volume XIV on "Interest Earnings as a Factor in Casualty Insurance Rate Making" covered in some de-

tail the subject of interest discount in the case of workmen's compensation insurance. Mr. Flynn explained the extent to which this element is reflected in the making of New York compensation rates. To my knowledge, no argument has ever been made for determining rates in New York on the basis of the terminal values of all claims rather than on the basis of paid losses plus reserves reflecting interest discount beyond a specified valuation date in the case of long term claims.

It is true that in most other states terminal values are used in the rate-making procedure. The laws of most other states are far less liberal than the New York Law, however, and only in comparatively few laws is there a provision for life pension awards for certain types of claims.

From a practical standpoint, let us analyze what the effect would be if we were to eliminate the element of interest discount from the New York rate-making procedure. It is estimated that an increase in rate level of somewhat more than 5% would be required by such a change. The effect on the average Death & Permanent Total value employed in experience rating would be much more substantial and would amount to an increase of 35% or more. The average D. & P. T. value is now $8,100 and this increase would raise it to $11,000 or more. There would be a consequent reduction in the average credibility allowed to experience rated risks because of the necessary adjustment in the rating values.

Mr. Matthews has given an excellent explanation of the fallacy of modifying the contingency loading procedure to provide for negative loadings when a substantial underwriting profit is indicated by the accumulation. Compensation experience moves in cycles. Following a period of favorable loss ratios, it is quite likely under our rate-making procedure that rates will be reduced to an inadequate level and that unfavorable loss ratios will result. This tendency would be accentuated by the use of negative loadings. The 1939 amendment of the contingency loading resolution in New York was for the purpose of introducing a further element of stability in the rate structure, thereby avoiding wide swings in rate level because of one element. This theory appears sound. To introduce a provision for the use of negative loadings would be entirely inconsistent with the principles followed in New York and would unquestionably prove very unsatisfactory in actual practice.

MERIT RATING — THE PROPOSED MULTI-SPLIT EXPERIENCE RATING PLAN AND THE PRESENT EXPERIENCE RATING PLAN

J. J. SMICK

VOLUME XXVI, PAGE 84

WRITTEN DISCUSSION

MR. MARK KORMES:

When I was asked by Mr. Constable, our vice-president, some three weeks ago to write a discussion of this paper, I did not even have an opportunity to see it. Nevertheless, being somewhat familiar with the subject, I agreed and shortly received some forty-six pages into which there were condensed the results of studies extending for a period of more than two years. If I add that the report of the Actuarial Committee to which Mr. Smick refers comprises no less than one hundred fifty-one pages, then I believe I will have established an airtight alibi for touching only lightly upon some of the aspects of the plan.

In my discussion I will follow the general pattern of the paper, first giving some attention to general considerations and then turning to technical and actuarial aspects of the proposed plan.

I cannot resist the temptation to recollect with relish the occasion when a big executive of a small company was denied a change in classification for a risk by the Classification and Rating Committee of the Rating Board. Upon being told that the Experience Rating Plan will take care of the good experience of the risk, he became red in the face and waving his arms violently exclaimed: "Don't talk to me experience rating, I know it backwards. Why, I even get it on toast for breakfast!"

Now that we have educated the company executives, the underwriters, brokers and some of the assureds to the point where they have some understanding of the workings of the plan and have sold them the idea of the scientific soundness of the plan, we are ready to scrap the entire structure and substitute a new one.

At the outset let me emphasize that I am not opposing the multi-split plan. On the contrary, I am in favor of its introduction, but I feel that any new plan must meet the test of comparison and prove that it actually accomplishes what it is purported to accomplish. I say this because I know from practical experi-

ence that if the plan were introduced, the underwriters, brokers and assureds would insist on a comparison with the results under the previous plan.

Mr. Smick enumerates several elements with respect to which the multi-split plan produces more satisfactory results. I agree that the plan offers greater responsiveness and flexibility and that it possesses further inherent possibilities for development. I cannot, however, entirely subscribe to its greater simplicity. As far as the simplicity of the rating procedure is concerned, the multi-split rating plan is vastly superior to the present plan, but as respects the explanation of the various elements it cannot claim that degree of simplicity. True, loss modification factors, loss splits and payroll factors are eliminated, but the "expected loss rate" and the "D" ratio will not be as easy to explain as it may appear. While more accurate than the present payroll factors, the "expected loss rates" will be just as obscure to the general public as the payroll factors. The "D" ratios will most probably defy any attempt at explanation. Moreover, the "D" ratios are calculated in a somewhat similar manner to the calculation of the excess ratios at the present time. It is still questionable whether the distribution of losses by size of loss for individual classifications follows the pattern of such distribution for the business as a whole (see Exhibits IV to VI inclusive). This problem in my opinion requires further study and a very interesting paper could be written on the subject.

It is claimed for the multi-split plan that it places greater emphasis on frequency and lesser on severity. It is questionable, however, whether it gives such greater emphasis in comparison with the present plan. To illustrate the point I have taken the "Illustrative example No. 1" from Mr. Smick's paper and calculated the corresponding results under the present plan. This involved several assumptions as respects the size of the losses under $400 and as respects the payrolls for the years 1932 and 1933. The losses under $400 were considered to be all normal and the payrolls were taken at $150,000 for the early years. For the early years the actual adjusted losses were taken equal to the expected losses. The present plan modification was then calculated to be a charge of 25.6% which compared with the multi-split plan charge of 23.7% gives the latter an edge. The question was then raised, "What was the effect of a single additional loss

of $50, $100, $200, $400, $1,000, $2,400 and $5,500 on the modification of the risk or, of course, the reverse, the reduction in losses by such a claim?" The table given below shows the results of such calculations.

COMPARISON OF THE RESULTS UNDER THE MULTI-SPLIT PLAN WITH THOSE UNDER PRESENT PLAN

Basis	Modification Under the		Charge for Additional Losses	
	Present Plan	Multi-split Plan	Present Plan	Multi-split Plan
The same experience	1.256	1.237	..	..
Additional loss of:				
$ 50 Indemnity	1.260	1.243	.4%	.6%
100 "	1.264	1.249	.8	1.2
200 "	1.272	1.261	1.6	2.4
200 "				
200 Medical	1.283	1.284	2.7	4.7
400 Indemnity	1.288	1.284	3.2	4.7
1,000 "	1.335	1.327	7.9	9.0
2,000 "				
400 Medical	1.375	1.367	11.9	13.0
4,000 Indemnity				
1,500 Medical	1.438	1.379	18.2	14.2

It appears from the above that a single loss has a far greater effect under the multi-split rating plan than under the present plan and that the benefit of discounting the losses does not accrue until the loss reaches a substantial sum. The risk in question produces an annual premium of approximately $4,000. Thus a $50 claim will cost the assured $16 under the present plan and $24 under the multi-split plan, and for ten such cases the assured will pay $240 under the multi-split plan as against $160 under the present plan. Of course, the reduction in the experience charge or the increase in the experience credit will be greater under the multi-split plan than under the present plan. It may be therefore argued that the multi-split plan offers a greater incentive toward accident prevention.

Far be it from me to base my conclusions on a single example. I believe that similar tests should be conducted on a number of risks with various premium sizes, particularly smaller risks where the possibilities of effective accident prevention are rather limited. The example, however, has brought out the fact that the concept of greater emphasis on frequency is a relative one. Perhaps the

real solution lies in the recognition of the fact that credibility should be expressed as a function of two variables, frequency and severity. True, this will make our formulae still more complicated, but we need not fear complications if our results will meet the criteria which we propound in advance.

The simplicity of the rating procedure will tend to reduce the mechanical work of experience rating in the various rating organizations. On the other hand the work required in the calculation of rating values will be considerably increased as is hinted by Mr. Smick in connection with the calculation of the "D" ratios.

Now let us turn to some theoretical aspects of the multi-split plan. The modification formula is given as

$$M = \frac{A_p + B + WA_e}{E_p + B + WE_e} \tag{1}$$

In the calculation of loss constants the off-balance of the rating plan plays a very important part. Let us examine what changes will be necessary under the multi-split plan. The experience rating data will have to be punched to produce the following amounts:

$$\Sigma A_p, \Sigma A, \Sigma E_p, \Sigma E, \Sigma WE_e, \Sigma (A_p + B + WA_e), \Sigma (E_p + B + WE_e)$$

the sum to extend over all rated risks. It will become apparent from the following why all of the above information is necessary. In the first place we must establish the average off-balance produced by the plan. We have for the off-balance, b:

$$1 - b = M_a = \frac{\Sigma (A_p + B + WA_e)}{\Sigma (E_p + B + WE_e)} \tag{2}$$

In order to represent (2) in the form of equation (1) let us consider that we can obtain the average value of W from

$$W_a = \frac{\Sigma WE_e}{\Sigma E_e} \tag{3}$$

This value of W_a will permit us to find the corresponding value of E and B_a. It can be found from the definitions of W and B that

$$E = W_a (S - Q) + Q \tag{4}$$

$$\text{and } B_a = [K + (gS - K) W_a] (1 - W_a) \tag{5}$$

In actual practice it may be just as accurate to read of the values of E and B_a from the tables for W.

Now the average discount of Expected Losses of D^E can be obtained from

$$\frac{\Sigma E_p}{\Sigma E} = D^E \tag{6}$$

and the average discount of Actual losses D^A from

$$\frac{\Sigma A_p}{\Sigma A} = D^A \tag{7}$$

We then have $M_a = \dfrac{AD^A + B_a + W_a A\,(1 - D^A)}{ED^E + B_a + W_a E\,(1 - D^E)}$ (8)

In the expression (8) there is only one unknown element, A. Solving for A we obtain

$$A = \frac{M_a\,[ED^E + B_a + W_a E\,(1 - D^E)] - B_a}{D^A + W_a\,(1 - D^A)} \tag{9}$$

Having in this manner expressed the average off-balance in form (1) let us from now on use for the off-balance the form

$$b = 1 - M = 1 - \frac{A_p + B + WA_e}{E_p + B + WE_e} \tag{10}$$

In order to eliminate the offsetting adjustment in rates, a_1, we must divide the expected losses by a_1. Since, however, both W and B are functions of the expected losses we will obtain

$$b_1 = 1 - \frac{A_p + B_1 + W_1 A_e}{\dfrac{E_p}{a_1} + B_1 + W_1 \dfrac{E_e}{a_1}}$$

$$\text{or } b_1 = 1 - \frac{(A_p + B_1 + W_1 A_e)\,a_1}{E_p + B_1 a_1 + W_1 E_e} \tag{11}$$

where by simple calculations

$$W_1 = \frac{E - a_1 Q}{a_1\,(S - Q)} \tag{11a}$$

$$\text{and } B_1 = [K + (gS - K)\,W_1]\,(1 - W_1) \tag{11b}$$

Of course, it may be found best in practice to find both W and B from tables for the value of $\dfrac{E}{a_1}$.

Now the offsetting adjustment in the new rates, a_2, will again effect the off-balance (as well as the values W and B) we have

$$\text{in effect } b_2 = 1 - \frac{(A_p + B_2 + W_2 A_e)\, a_1}{E_p a_2 + B_2 a_1 + W_2 E_e a_2} \tag{12}$$

$$\text{where } W_2 = \frac{E a_2 - a_1 Q}{a_1 (S - Q)} \tag{12a}$$

$$\text{and } B_2 = [K + (gS - K) W_2] (1 - W_2) \tag{12b}$$

Unfortunately we cannot use the tables since a_2 is an unknown value and must satisfy the equation

$$a_2 - a_2 k b_2 = e \tag{13}$$

where k is the proportion of premium over \$500 subject to rating and e is given by

$$e = 1 - \frac{\text{Excess of Premium over permissible loss ratio (Risks over \$500)}}{\text{Total Premium at Manual Rates (Risks over \$500)}}$$

We must therefore solve simultaneously equations (12) and (13). Since B_2 is quadratic in W_2 and therefore in a_2 and since from (13)

$$b_2 = \frac{a_2 - e}{k a_2} \tag{14}$$

a substitution in (12) will lead to a cubic equation:

$$\alpha a_2^3 + \beta a_2^2 \gamma + a_2 + \delta = 0. \tag{15}$$

Where the coefficients α, β, γ and δ can be calculated from the known values of E, K, g, S, Q, a_1, A_p, A_e, E_p, E_e, k and e. The expressions are rather complicated and are omitted in order to conserve the space.

It is natural to ask the question why should there be any off-balance under the multi-split plan. If it were decided to make the plan balance the situation would be simplified considerably. We would have

$$b = 0 \tag{16}$$

$$\text{and therefore } a = e \tag{17}$$

and this would eliminate the whole question of off-balance in connection with the loss constant calculation. The difficulty lies, however, in the fact that e may represent quite a substantial reduction in rates (10% or even more) which in turn would result in a sizable increase in loss constants. Still the so much desired simplification would be attained, the off-setting adjustments would be reduction factors in all cases and the equalization

of small risk loss ratio and large risk loss ratio just as accurate as under the present procedure.

Mr. Smick by writing his paper has performed a valuable service not only to the membership of the Society but also to the public at large. The wealth of new ideas presented in this paper, its clear and readable form, will no doubt stimulate a great deal of thought and discussion. When the plan is put into operation (which I sincerely hope) the impetus for further research and improvements will and must always come from the acid test of the actual results.

AUTHOR'S REVIEW OF DISCUSSION

MR. J. J. SMICK:

In some respects the paper read before the Society at the November, 1939, meeting was not exactly an impartial presentation of the Multi-Split Experience Rating Plan. It was an obvious effort to influence the adoption of a plan which to me seems superior to the existing experience rating plan. I presented the new plan in as favorable a manner as possible. It was my belief that, in view of the rather strong sentiment prevailing in some quarters against its adoption, that ample criticism would be forthcoming in the discussions.

Mr. Kormes has taken advantage to criticize certain features of the plan. He has, however, treated it so gently that I am somewhat disappointed. A rating plan which, on the basis of present indications is about to supplant one which with some modifications has been in effect since 1923 certainly should be closely analyzed and all weaknesses publicized. If it is a worthwhile plan it should be able to withstand much rougher treatment than that accorded it by Mr. Kormes.

The fact that Mr. Kormes did not see fit to criticize the plan severely does not in any way detract from the value of the points he does bring out. These are few but are nonetheless well taken. The points he has chosen to discuss are:

1. The expected loss rates and "d" ratios.
2. The effect of a single claim on the rating.
3. The effect of the plan on the off-balance and loss constant calculations.

The details of the derivation of the "expected loss rates" are given in the paper. The resulting expected losses that will be obtained from an extension of payrolls by the expected loss rates should, except for the introduction of group rate level, give the same results as the present procedure. It should be no more difficult to explain the expected losses under the Multi-Split Plan than under the present plan. I think that a more serious cause of trouble may be the adopted procedure of changing the expected loss rates from year to year. It will be necessary to tell an assured that in the 1941 rating the expected loss rate for policy year 1939 was, let us say, .90 while in the 1942 rating, the rate may change to .85, thus materially increasing the charge and reducing the credit.

It seems to me that the expected loss rates once established should remain fixed throughout the rating period. The present procedure of keying the expected losses to the level of the current manual rates is in a large part due to a desire for a balanced plan. In practice the plan has never been in balance.

I believe it would be in the interests of the business to dispense with some theoretical niceties in order to obtain a simple and more workable plan. A large step in that direction would be to start with the manual rate actually charged for the year of coverage, remove the expense loading, and use the remainder as the expected loss rate. This procedure would accomplish the following:

1. It would divorce the calculation of the modification from the manual rates and enable ratings to be performed in an orderly manner without waiting for approval of any pending revisions. The carriers and the administrative bureaus would be freed of the pressure occasioned by holding up calculations until rates and rating values become available.
2. It would simplify an explanation of the rating procedure to the assured. All that would be required as an explanation would be the following. "On your 1939 policy the manual rate was $1.00. This rate allowed 40 cents for expenses and 60 cents for losses. We are comparing your actual losses with the expected losses. Subject to other elements that must be taken into account, if your actual losses are less than the expected you receive a credit, if they are greater you receive a charge. You can verify the 1.00 manual rate by referring to your previous ratings."
3. It would tend to correct certain deficiencies in the present

rate-making procedure. If for some reason the current manual rate is out of line it is difficult to show an assured that the experience rating plan gives him any relief. On the other hand under the proposed procedure, it could be explained that if the rates charged in the past have been out of line, the use of the past rate in the rating gives some relief. If the rate has been too high, the expected losses will be greater and the modification will result in either a smaller charge or greater credit. If the rate has been too low, the reverse will be true. Furthermore, if the current manual rate is attacked it can be pointed out that if the actual experience under the coming policy year is better than that contemplated by the rate, relief will be given when the experience is used in the rating.

The effect of a single claim on the rating, under the present plan and under the Multi-Split Plan has been analyzed by Mr. Kormes. His deductions although correct, do not present the entire picture. The Multi-Split Plan is a three-year plan while the present plan uses five years of experience, weighted to be sure. It is to be expected that the effect of any loss, whether discounted or not, will be greater on a three-year plan than on a five-year plan. Thus the effect of a claim under the Multi-Split Plan though greater, will be felt for only three years while the lesser charge under the five-year plan remains for a longer period. The discounting procedure allows the use of a shorter period by minimizing the charges arising from high cost cases.

The effect of the Multi-Split Plan on the loss constant and off-balance calculations is a subject on which little time has heretofore been spent. The present program of the Actuarial Committee of the National Council contemplates a rather exhaustive study of rate-making methods and I believe one of the first items will be a revision of the loss constant procedure. It may well be that when the Multi-Split Plan is adopted the formulae and procedures outlined by Mr. Kormes will be an excellent starting point for integrating the proposed studies with the changes required because of the Multi-Split Plan.

Needless to say, I am in complete agreement with Mr. Kormes that the plan offers almost an unlimited field for future study and experimentation. The suggestions he has thrown out so freely should be followed up. I hope he follows some of them himself and prepares another paper on the plan.