AN ANALYSIS OF PREPAYMENT DISCOUNTS

BY

ROBERT J. MYERS

In many lines of fire and casualty insurance it is customary to issue policies for periods longer than one year to be paid for by a single premium at the date of issue. The usual periods are 3 and 5 years with the single premium for the former being 2½ times the annual premium and that for the 5-year period, 4 times the annual premium. Looking at it superficially, the policyholder will probably believe that the 5-year plan is more to his advantage since a discount of 20% is given as contrasted with only 16⅔% for the 3-year plan. However, as will be shown subsequently, this conclusion in most instances is not valid when interest is taken into consideration (as should probably be done in all private insurance matters).

The problem of proper prepayment discounts can be considered either from the policyholder's viewpoint or from the insurance company's viewpoint. As far as the latter is concerned, the problem should be analyzed in terms of the three basic elements of insurance—interest, expense, and risk. Under prepayment the company can earn interest on the excess funds available—a portion or all of which should be credited to the policyholder. In respect to expense there are appreciable savings under prepayment since only one premium is collected rather than several. Probably it may safely be said that the absolute cost in dollars for collection of a prepayment premium is the same as that for collection of an annual premium (exclusive of agent's commission in each case); for each of the renewal years there is a savings to the company equal to the cost of premium collection. The same consideration also applies in respect to dividends when the policy is participating, since such disbursements are made only once under prepayment (at the end of the period) rather than annually. Also it is quite likely that general expenses are somewhat lower under prepayment policies, since persistency is probably improved thereunder. Finally, under prepayment the company for some types

of policies (especially fire) has a slightly smaller amount at risk, since the coverage over the remainder of the period is reduced by the amount of any loss without any refund in premium, whereas under an annual premium basis the premiums in the years following the loss could be decreased or the coverage brought back to its original amount. Unlike life insurance the factor of increase in cost with duration probably is of no significance in casualty insurance, so that the average annual risk under a 3- or 5-year term policy is the same as under a 1-year policy.

It may be assumed that from the policyholder's viewpoint, in determining which of the three premium payment plans is best, the only cost element to be taken into consideration is the effective interest rate which he earns from the prepayment discount.[1] Also under prepayment plans he is saved the nuisance of making small premium payments each year.

First, consider the question as to the policyholder's "effective interest rate" under the two prepayment plans as compared to the annual premium plan for various dividend rates. When participating policies are involved, an additional element is introduced. Dividends on annual policies are, of course, paid at the end of each year, whereas under the prepayment plans they are not payable until the end of the period. This has an appreciable influence on the effective interest rate under prepayment, since it may be considered that the company retains a portion of the policyholder's dividends throughout the longer period without paying interest thereon. The effective annual compound rate of interest may be obtained by solving for i in the following equations which equate the annual cost under the two prepayment plans to that under the annual premium basis:

$$\frac{2.5\,(1 - Kv^3)}{a_{\overline{3}|}} = 1 - Kv$$

$$\frac{4\,(1 - Kv^5)}{a_{\overline{5}|}} = 1 - Kv$$

where K is the dividend rate and the annual premium is \$1.

[1] Since under such policies the individual probability of loss is relatively small, it is reasonable for the policyholder to neglect to consider the cost to him of "forfeiture" of a portion of his prepayment premium when a loss occurs.

The solution is best achieved by transforming these equations into polynomial equations in terms of i as follows:

$$3i^3 + (2K + 7)i^2 + (6K + 3)i + (K - 1) = 0$$
$$3i^5 + (K + 14)i^4 + 5(K + 5)i^3 + 10(K + 2)i^2 + 5(2K + 1)i + (K - 1) = 0.$$

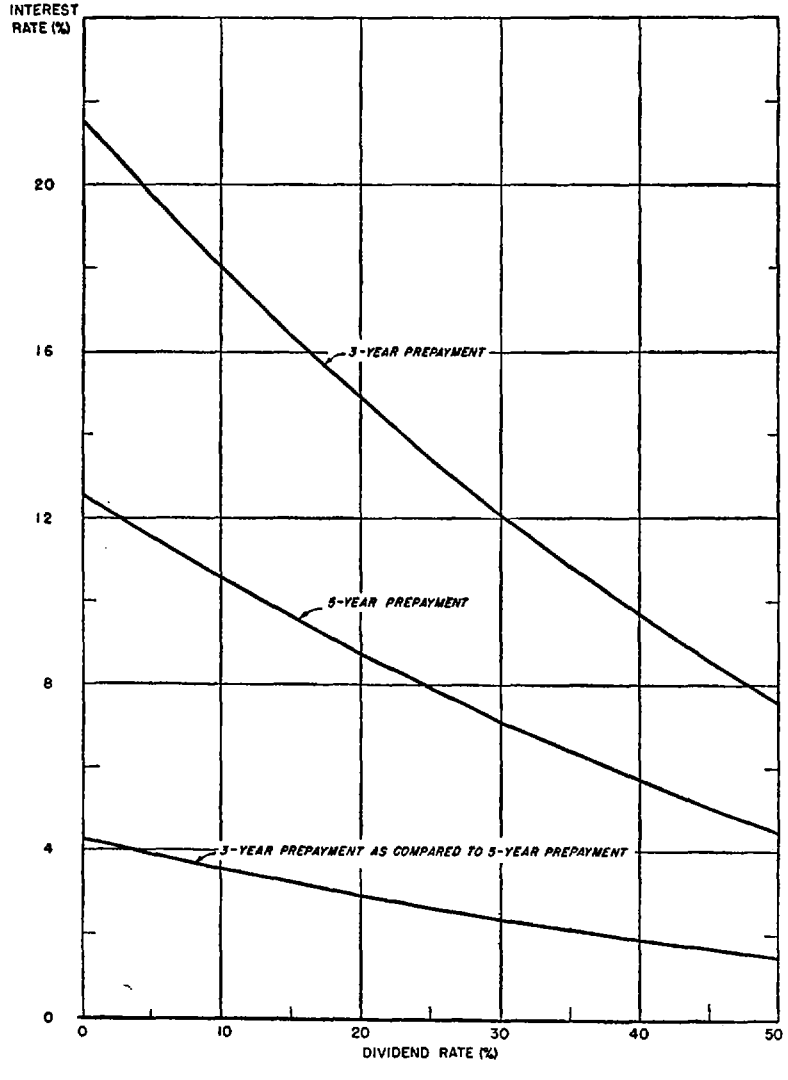
For various values of K these equations may then be solved by successive approximation with results for i being obtained to any desired number of decimal points. Calculations were made for 11 uniformly spaced values of K running from zero (for non-participating policies) to 50%, which is probably a reasonable maximum for dividend rates in casualty insurance. The resulting figures are shown on the chart appearing on page 11.

Under the 3-year prepayment plan the purchaser of a non-participating policy nets an effective interest rate of about 21½% by paying in advance rather than annually.[2] The effective earned interest rate for participating policies is somewhat less and becomes more so as the dividend rate increases (this does not necessarily mean that non-participating policies are the lowest in cost, but rather that the policyholder in such a company obtains a relatively better interest return from a prepayment plan). Where the dividend rate is as high as 50%, the interest return under the 3-year prepayment plan is only about 7½%. Under the 5-year prepayment plan the interest rate earned is appreciably smaller; for non-participating policies the rate is only about 12½%, while for participating policies it is even less, decreasing to only about 4½% at a 50% dividend rate. Thus, the policyholder will usually find it advantageous to pay his premiums under either of the prepayment plans rather than annually, since the effective interest rate earned is so high. There seems to be an inequity between the 5-year prepayment plan and the 3-year one, since the effective interest rate under the latter is about 70% larger, whereas the savings to the company should be greater for longer periods of prepayment.

---

[2] It should, of course, be recognized that a major portion of this large "interest rate" represents relative savings effected by the company because of prepayment.

EQUIVALENT EFFECTIVE ANNUAL INTEREST RATE EARNED UNDER PREPAYMENT DISCOUNTS



Another procedure is to consider the case of a policyholder who has decided to take a prepayment plan, but is undecided as to whether to take the 3-year or 5-year plan. The comparison of the two upper curves in the chart would seem to indicate that he should always take the 3-year plan, but this depends upon the effective interest rate which he thinks is applicable to his surplus money. The effective interest rate for which the two plans are equally "good buys" may be determined by equating the annual costs of the two plans as follows:

$$\frac{2.5\ (1 - Kv^3)}{a_{\overline{3}|}} = \frac{4\ (1 - Kv^5)}{a_{\overline{5}|}}$$

which may be solved for i by successive approximation. The resulting figures appear on the chart as the lowest line. For non-participating policies the resulting interest rate is about 4¼%, decreasing for participating insurance until for a 50% dividend rate it is about 1½%. This indicates that for non-participating policies the 5-year plan is preferable if the policyholder believes that money is worth less than 4¼%, whereas the 3-year plan is better if money is worth more than 4¼%.

With today's low interest rates it appears that in most cases there is little to choose from between the two plans as far as the policyholder is concerned, since funds invested in defense bonds or savings banks earn only 2 to 3% interest. However, the companies might well introduce a larger differential in favor of the 5-year plan, inasmuch as greater economies should be achieved thereunder. Just as the effective interest rate on the 3-year plan makes it a favorable buy as compared to the 1-year plan, so from the policyholder's standpoint the 5-year plan should be more favorable than the 3-year one. It might well be contended that the 3-year plan is now on too favorable a basis and should be changed so as to allow somewhat less discount.

If it be assumed that the discount under the 5-year plan should be changed so as to produce interest returns to the insured comparable with those under the present 3-year plan, then such a policy might be sold for 3½ annual premiums[3] (or, expressed in

[3] These rounded values which produce roughly equal interest rates under the two plans were obtained by trial and error process.

another way, at a discount of 30% over the total of five annual premiums). The effective interest rate on this basis is 21.9% for a non-participating policy, decreasing to 8.1% for a participating policy with a 50% dividend rate, or slightly greater than the rates for the present 3-year plan (21.5% and 7.6% respectively). On the other hand, if the discount for the 3-year plan is to be reduced so as to be comparable with the present 5-year plan discount, the single premium might be 2.7 times the annual premium[3] (or in other words, a discount of 10% on three times the annual premium). The effective interest rate on this basis is 11.6% for a non-participating policy, decreasing to 4.0% for a participating policy with a 50% dividend rate, or slightly less than the interest rates of the present 5-year plan (12.6% and 4.5% respectively).

From the company's viewpoint, one procedure for determining the proper size for prepayment discounts is to make certain assumptions as to interest and savings in expense. Thus the policyholder should be given interest on his advanced funds and a portion of the savings effected. As a specific case, let it be assumed that the cost of collection of an annual premium (exclusive of commissions and taxes) is 10% of the average premium. Under a prepayment plan the cost of collection of the single premium can be assumed to be of the same absolute size. In other words, if the average annual premium is assumed to be $20, then the cost of collection is $2 per year. On the other hand, for prepayment the cost of collection for the average single premium is still assumed to be only $2 at the beginning of the period with no other expenses of collection assessed during the period. Where dividends are paid there are similar savings, since only one dividend is paid for the whole period rather than one each year as under the annual premium basis.

Interest rates of 3% and 6% have been used in the calculations. The former rate is that which should be granted to policyholders if it is decided to pay the current "market rate." The 6% rate might be given if it is desired to make the "investment" an attractive one for policyholders, with the thought that although the company cannot earn this amount on its excess funds, the small differential will be more than made up by improved persistency.

Using these interest and expense assumptions, the prepayment

discounts may readily be calculated from the following formulas which equate the annual net costs (taking into account only collection expenses):

$$\frac{D\,(1 - Kv^3) - .1}{a_{\overline{3}|}} = (1 - Kv) - .1$$

$$\frac{D\,(1 - Kv^5) - .1}{a_{\overline{5}|}} = (1 - Kv) - .1$$

where D is the multiple of the annual premium which gives the single premium.

The results of the computations are shown in the following table for three selected dividend rates:

Dividend Rate	3-Year Prepayment		5-Year Prepayment	
	3% Interest	6% Interest	3% Interest	6% Interest
None	2.72	2.65	4.34	4.12
25%	2.61	2.51	4.08	3.77
50%	2.41	2.26	3.62	3.22

Under the assumptions made the current prepayment discount for the 3-year plan appears to be reasonably consistent, but that for the 5-year plan is not quite large enough. A single premium of about 3¾ annual premiums for the 5-year plan would be comparable with the single premium of 2½ times the annual premium for the 3-year plan.

This paper has shown that it is decidedly in the interest of the policyholder to take a prepayment plan when available. The analysis has also indicated that the discounts currently given for the two periods are not entirely equitable in comparison with each other. Perhaps the best course of action indicated is that the discount on the 5-year plan should be increased to 25% (as compared to the present 20%), while that on the 3-year plan should remain at the present 16⅔%. Of course, the two present discounts have become almost traditional so that practical considerations might in this case far outweigh any theoretical ones.