THE MULTI-SPLIT EXPERIENCE RATING PLAN IN NEW YORK

BY

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I. Introduction

The so-called Multi-Split Experience Rating Plan for workmen's compensation risks has been in effect in the State of New York since July 1, 1941. The chief purpose of this paper is not to outline the features of the plan itself, which was ably done in Mr. Smick's paper entitled "Merit Rating —The Proposed Multi-Split Experience Rating Plan" (P.C.A.S. XXVI, p. 84), but rather to show how it operates in this state and to discuss some of the problems presented by its introduction.

It is the general understanding that the plan, being new, is to be tried out for a year or so without material change. Then, such developments as appear desirable in the light of a year's experience with it are to be incorporated. Some suggestions will be made herein which it is felt will improve the operation of the plan in this state and might well be studied for consideration in other states.

Throughout this paper, no mention will be made of risks written wholly or partially on an ex-medical basis, since a slightly modified, but parallel, procedure is employed in rating such risks.

II. Elements of the Plan

(a) *Actual Losses*

As anyone who is familiar with this plan is aware, the division of actual losses into normal (called primary in this plan) and excess is on the multi-split principle.

The plan provided that the initial value, or point where discounting of losses begins, shall be such that 90% of the number of compensable cases shall have total losses (indemnity plus medical) below this point. Since New York experience shows that this point is approximately $1,000, New York falls into the

largest, or $500 group. Theoretically, this means that each loss is divided into sections of $500, the primary value being equal to the full value of the first section, plus ⅔ of the second section, plus ⅘ of the third section, and so on. The maximum primary loss is, therefore, $1500, since that is the limit of the geometric series—

$$\$500\ (1 + \tfrac{2}{3} + (\tfrac{2}{3})^2 + \cdots\cdots).$$

In actual practice, it is desirable to have primary values easily obtainable from a table. This table has been prepared showing primary losses at $10 intervals from $500 to $1500 with an actual loss group to which each of these primary losses corresponds. The primary loss is actually obtained by means of the formula

$$Ap = \frac{\$500\ (1 - r^n)}{1 - r} = \$1500\ [1 - (\tfrac{2}{3})^n] \text{ where } n = \frac{\text{Actual Loss}}{\$500}$$

which gives exact values only for even multiples of $500. The values in between even multiples of $500 are slightly higher than those produced by the theory, since the above formula produces an exponential curve, while the theory would produce a series of straight lines meeting the curve at each multiple of $500. The formula method produces a smooth curve which is preferable, in actual practice, to the "straight lines" method.

(b) *Expected Losses*

Expected losses, on the other hand, are obtained by extending the actual payrolls for the three latest available policy years at expected loss rates. The split between primary and excess expected losses is determined by applying *"D"* ratios to the expected losses class by class.

(c) *"D" Ratios*

The statewide *"D"* ratios calculated in 1941 using experience for the first reporting of policy year 1938 and the second reporting of policy year 1937 were .274 for Serious, 1.197 for Non-Serious, and .303 for Medical. The average New York *"D"* ratio (called *"d"*) was .643. It is contemplated that three policy years will be used in calculating the 1942 *"D"* ratios.

The classification *"D"* ratios were obtained by weighting these statewide *"D"* ratios by the selected partial pure premiums on the new rate level for each classification.

(d) *Expected Loss Rates*

The expected loss rates were obtained for each classification by applying factors to the 1941 manual rate (less catastrophe loading). These factors vary by industry group and policy year, and are obtained by taking the reciprocal of the product of the following:

(1) Law amendment factor
(2) Loss development factor
(3) Rate level projection factor
(4) Security funds factor (1.012)
(5) Factor for expenses of the Labor Department (1.045 on indemnity losses; averages 1.029)
(6) Expense loading ($1.0 \div .605 = 1.653$)

Since risk losses are used in rating without modification, it is desirable to have expected losses on a comparable basis. Therefore, it is necessary to unload the current manual rate by the above factors which have been used in its calculation.

Under this plan, for risks effective from July 1, 1941 to December 31, 1941, only policy year 1939-first reportings, policy year 1938-second reportings, and policy year 1937-third reportings may be used in rating. Expected loss rates for these three policy years were calculated and printed in Table II of the plan. However, for risks effective from January 1, 1942 to June 30, 1942, the following reports may be used: policy year 1940-first reportings, policy year 1939-second reportings, policy year 1938-third reportings, and also some policy year 1937-third reportings. (See Section III, Rule 2 of the New York Experience Rating Plan.) This necessitates calculation of expected loss rates for policy year 1940 and recalculation of policy year 1939 and policy year 1938 on the later report basis, with added cost for printing revised pages for the Plan. Unfortunately, although the same reportings (without policy year 1937) will be used in ratings from July 1, 1942 to December 31, 1942, the expected loss rates for these ratings will have to be based on the new 1942 manual rates effective July 1st, and therefore will have to be recalculated for that period. This means semi-annual calculation and printing of expected loss rates.

Several proposals have been advanced with a view toward sim-

plifying expected loss rates in order to have only one annual printing.

One plan contemplates calculating expected loss rates on a "fiscal year" basis. This is done by averaging the development factors which are applied to each policy year for the various reportings. For instance, for ratings effective from July to December 1941, the second report of policy year 1938 is used. In calculating expected loss rates for policy year 1938, the development factor from second to ultimate must be removed. Likewise, for January to June 1942 ratings, the third report of policy year 1938 is used, and the development factor from third to ultimate must be taken out in calculating expected loss rates. On a "fiscal year" basis, we would use the average of these two development factors and use the same expected loss rate for the whole July-June year. Likewise for policy year 1939, the average of development factors (a) from first to ultimate and (b) from second to ultimate would be used in calculating a single expected loss rate for policy year 1939. While this plan is not yet in effect, the National Council Actuarial Committee has adopted the following procedure with regard to recalculation of expected loss rates for January 1942:

"When the average of the ratios of

$$(1)\ \frac{\text{Average development from 3rd to 4th}}{\text{Average development from 2nd to 4th}}$$

$$\text{and}\ (2)\ \frac{\text{Average development from 2nd to 4th}}{\text{Average development from 1st to 4th}}$$

is not over 1%, the expected loss rates for policy year 1940 will be the same as for policy year 1939 and it will not be necessary to recalculate the expected loss rates for the earlier years."

Another proposal suggests calculating a single expected loss rate for all policy years to be effective during the "fiscal" year. To make this calculation would mean averaging various factors for the following six reportings:

1937 — third report
1938 — second report, third report
1939 — first report, second report
1940 — first report

Law amendment factors would vary by policy year, and development factors would vary by report. In each case, the six appropriate factors could be selected and averaged. In the case of projection factors, it so happens that unity is used in three of the six due to lack of experience. The other factors for the earlier years may vary so widely by industry group as to cast doubt on their true worth. Three alternatives are presented: (a) to use 1.000 in all cases, (b) to use an average projection factor for each industry group, and (c) to use an average projection factor for all industry groups combined.

This "single loss rate" plan would not only eliminate semi-annual recalculation, but would greatly simplify both the printing of Table II of the Experience Rating Plan and the work in connection with the actual rating itself. This plan was discussed by the Actuarial Committee of the Compensation Insurance Rating Board, who felt that as long as rate levels maintain an even keel and there are no substantial law amendments, it might be an advantageous one. It was felt that it might be advisable to have the data worked up each time for the Committee's review with complete refinement by policy year, and, when practicable, the factors may be averaged to permit the use of a single expected loss rate for all policy years for each classification.

A third plan suggests using the expected loss portion of the actual policy year manual rate for each policy year. The use or elimination of loss development factors might be a problem, but law amendment and projection factors could be omitted. This plan would probably work well for those classifications which show slight manual rate changes from year to year, but it is not likely to succeed where sudden changes occur. However, this method would enable a single expected loss rate for each policy year to be used without change, in three consecutive ratings. Each year, the expected loss rates for one additional policy year would have to be calculated.

(e) *1941 Rating Values—New York*

An Average Death and P.T. Value of $9000 was adopted. The plan provides that the Self-Rating point (S) shall be twenty times the Average Death and P.T. Value, which is $180,000. At that time, the plan provided that the Q point should be 1/10 of S.

However, the Actuarial Committee felt that a lower value would be more advisable and adopted \$12,000, which is 1/15th of \$180,000. Subsequent study has made even lower Q point desirable. (See Section IV below.)

K was determined by the formula $4m - 3ILd$, where

m = maximum primary loss = \$1500
I = initial value = 500
L = expected loss ratio = .581
d = average "D" ratio = .643, and was rounded to \$5400.

$K_e = K + W(gS - K)$, where g is the maximum value of $\frac{E_e}{E}$, taken arbitrarily as .4. (See Section IV below.) Therefore, K_e is equal to $5400 + W(72000 - 5400) = 5400 + 66600W$.

Since $B = K_e(1-W)$, $B = (1-W)(5400 + 66600W)$. B was calculated for each of the 99 values of W from .01 to .99.

(f) *Credibility and Modification Formulae*

The National Council Actuarial Committee has adopted the following formulae for credibility under the Experience Rating Plan:

For Risks with Total Expected Losses:	*Above the Q Point*	*Below the Q Point*
Primary Credibility (Z_p) =	$\frac{E}{E_p + B + WE_e}$	$\frac{E}{E_p + K}$
Excess Credibility (Z_e) =	WZ_p	0
Average Credibility (Z) =	$\frac{E_p + WE_e}{E_p + B + WE_e}$	$\frac{E_p}{E_p + K}$

That Z is an average of Z_p and Z_e, weighted by E_p and E_e respectively, can easily be shown.

The following formula gives the Experience Modification for risks of any size:

$$\text{Modification} = \frac{A_p Z_p + E_p(1-Z_p) + A_e Z_e + E_e(1-Z_e)}{E} \quad \text{(I)}$$

Above the Q point, $Z_p = \frac{E}{E_p + B + WE_e}$, and $Z_e = WZ_p$, and by

substitution in Formula I, the modification becomes $\frac{A_p + B + WA_e}{E_p + B + WE_e}$, which is the form used in actual rating.

For risks below the Q point, $Z_e = 0$, and Formula I reduces to

$$\text{Modification} = \frac{A_p Z_p + E_p (1 - Z_p) + E_e}{E} \qquad \text{(II)}$$

Now, since $Z_p = \frac{E}{E_p + K}$, Formula II can be reduced to

Modification $= \frac{A_p + K}{E_p + K}$, which is the form used in rating.

Some misunderstanding has arisen due to the *erroneous* assumption that for risks *below the Q point* the excess portion of both actual and expected losses is disregarded in rating. That this is *not* the case, can be seen from Formula II, which has been incorrectly shown at times without the E_e term in the numerator. What actually happens is that actual primary losses modified by Z_p are taken, plus expected primary losses modified by $(1 - Z_p)$, plus the entire expected excess losses, the total being compared with total expected losses to obtain the modification. Only the *actual* excess losses are disregarded.

III. Behavior of Credibility*

In any experience rating plan, credibility should be a function of the size of the risk in such a manner that it *gradually* increases from zero at a given point to unity at another point known as the self-rating point. Under this plan, the "primary" credibility and "average" credibility should be zero at size of risk zero and rise gradually (but not at the same rate) until they reach unity at the self-rating point (S). The "excess" credibility should be zero for all risks below the Q point and then rise gradually until it also reaches unity at S. Obviously, the general equations for these credibilities satisfy these conditions.

Unfortunately, it was found advisable in the practical application of the plan not to use formulae to determine W and B values for each risk above the Q point, but to limit W to 99 different values (from .01 to .99), each of which has a corresponding B

* See "Experience Rating Credibilities," by Francis S. Perryman (P.C.A.S. XXIV, p. 60).

value. Each of these pairs of values is then applied to a certain group of expected losses, the distance from Q to S being divided into 99 equal groups of expected losses. For purposes of simplification, this works out quite nicely. The value of E is looked up in a table from which W and B can be read off immediately and entered on the Rating form.

But, upon examining the results, we find that the credibilities produced by this tabular method behave quite strangely. Z_p, below the Q point, starts at zero and rises gradually (and smoothly) until it reaches a certain value at the Q point. Above the Q point, we must use tabular values for W and B. Immediately we introduce into the denominator a B which is enough greater than K to cause a sudden drop in our primary credibility. Z_p will increase gradually within each group, but will drop considerably in passing from one group to the next. In a test made using $D = .62$, it was found that Z_p gradually rose until it reached .935 at the Q point, but dropped to .889 at 12,001, rose again to .941 at the end of the "$W = .01$" group, and then dropped to .902 at the beginning of the next group. Obviously, a situation where Z_p changes by .013 between initial group points, but rises as much as .053 within one group is undesirable.

Z_e, which is the product of W and Z_p, behaves in quite a different manner. At the Q point, $Z_e = 0$. At 12,001, it suddenly becomes .009 and remains at .009 throughout the first group. When we pass from the first to the second group, Z_e jumps from .009 to .018. Here we have no overlapping as in Z_p, but rather discontinuity, which is perhaps just as undesirable.

Z, the average credibility, which is the weighted average of Z_p and Z_e, combines the bad features of both. For small values of W, Z overlaps from group to group, but for high values of W, it becomes discontinuous.

It would perhaps be permissible and possibly advantageous to overlook the strange behaviour of credibility under this plan, especially in view of the fact that credibility does not appear on the rating blank per se, but rather as a mysterious unknown quantity hidden behind the scenes. Over a period of time these variations would average out so that the over all results would be satisfactory. Unfortunately, that this fluctuation will give unfavorable results is apparent from the following example: Let us take a risk

with expected losses slightly below either the Q point or one of the other group limits. The actual losses are such that a credit is produced by the Experience Rating Plan. Now, if we increase the expected losses by an amount which is great enough to push the risk over into the lower part of the next higher group, we should expect the risk (with the same actual losses and greater expected losses) to produce a lower modification. This, however, is not the case. The effect of moving into the next group, with the corresponding drop in primary credibility, causes the risk to produce a higher modification. This is because less weight is given to the good experience and a smaller credit results.

It frequently happens that a risk will have a rerating on the basis of a final audit which produced slightly higher payrolls. Imagine the assured's astonishment to find that increased expected losses can produce a higher modification with the same actual losses. It is admitted that this situation is not unique to this plan, and was known to happen under certain conditions in the old Experience Rating Plan, but this hardly seems to justify it if it can be avoided.

This strange situation might be unavoidable if it were true that the formulae for W and B are too complicated for simplification, and could only be feasibly obtained from tables. It is the writer's opinion that this is not the case and he proposes the following method of obtaining W and B values for each individual risk above the Q point:

(1) Determine W to four decimal places from the following formula:

$$W = \frac{E - Q}{S - Q}$$

This is a single operation. Q and $S - Q$ are constants, and $E - Q$ can be obtained at sight.

(2) Compute B from the following formula:

$$B = (1 - W)\,[K + (gS - K)\,W].$$

This consists of two operations. $(gS - K)$ is a constant. Multiply $(gS - K)$ by W and add K. Then multiply this result by the complement of W to obtain B.

By using these values of B and W in the experience rating forms, more consistent and reliable results will be obtained. It is the writer's opinion that the additional work entailed will be negligible, since (a) the above operations will replace the work performed in looking up B and W in the tables, and (b) only one rating in fifteen is above the Q point.

IV. Primary Credibility Greater than Unity

It was discovered that certain risks with low "D" ratios could produce a primary credibility greater than unity for certain sizes of expected losses. The absurdity of this situation can be shown by the following example: The general formula for the risk experience modification is:

$$\text{Modification} = \frac{A_p Z_p + E_p (1 - Z_p) + A_e Z_e + E_e (1 - Z_e)}{E}$$

If we consider that the numerator of this expression consists of two parts—a weighted average of A_p and E_p, and a weighted average of A_e and E_e, we see that in the case where Z_p exceeds unity, the left hand term will not lie between A_p and E_p (as a good weighted average should) but will be greater or less than both values as A_p is greater or less than E_p. Obviously, this will give distorted results in actual practice.

$$Z_p = \frac{E}{E_p + 5400} = \frac{E_p + E_e}{E_p + 5400} \text{ for risks below the } Q \text{ point.}$$

This obviously is greater than unity when E_e is greater than 5400. This situation is not consistent with the fundamentals of the plan, and should be remedied.

The K value of 5400 is determined by m, I, L and d. Of these, only "d" may vary, but even if we set $d = .000$, K would equal 6000 and Z_p could still be greater than unity under certain conditions. Therefore, it seems that the fault must lie elsewhere.

The Q value of 12,000 apparently is too high. If the Q point were lower, Z_p could not reach unity for risks below it, and we would use the other formula for risks above it. The National Council Actuarial Committee has recommended that Q be determined by the ratio of K/d. In New York, this would be $\frac{5400}{.643}$

or 8398, which would be rounded to 8500. This seems to be a much better value, since Z_p could only be greater than unity below the Q point for $D = .36$ or less. (See below.)

For risks above the Q point, it would still be possible to find Z_p greater than unity, even with the lower Q point. Since Z_p is dependent to a certain extent on B, it would appear that we can eliminate this trouble by increasing B. The formula for B is

$$B = (1 - W)\, K_e = (1 - W)\, [K + W\, (gS + K)]$$

"g" is defined as the maximum value of $\frac{E_e}{E}$ and was taken arbitrarily at .4 on the recommendation of the National Council Actuarial Committee. A study of the Experience Rating Plan Values in Table II shows that this value is much too low for New York. Of the 668 classifications for which "D" ratios are shown, 287 (or 43%) are less than .60 which corresponds to $g = .4$. Of these 287, only 17 are less than .47, these 17 comprising the nine "explosives" classes, four "aircraft" classes and four others. The writer would recommend disregarding the "D" ratios which are smaller than .47 and adopting .53 as the correct value of "g" for New York. Then B becomes

$$(1 - W)\, (5400 + 90000\, W).$$

Tests show that Z_p will not exceed unity using these B values under the proposed "non-grouping" method outlined in Section III, although it is quite likely that Z_p would still fluctuate enough under the present "grouping" plan to exceed unity at some points under certain conditions.

V. Experience Rating Statistics

Certain data to be used in rate making, such as average credibility and average off-balance by industry group, are readily obtainable from the experience rating statistics.

One writer, in a discussion of Mr. Smick's paper, pointed out the necessity of obtaining the average off-balance of the experience rating plan from these statistics and worked out some unnecessarily complicated formulae for doing so, using the wrong weighting process. Actually, it is a comparatively simple matter. The risk modification and risk average credibility are weighted

by the total expected losses for the risk and punched. Then the average modification and average credibility can be obtained for any group of risks by the formula:

$$\text{Average modification} = \frac{\Sigma E \times \text{Mod.}}{\Sigma E}$$

$$\text{Average credibility} = \frac{\Sigma E \times Z}{\Sigma E}$$

Not necessary from a rate making standpoint, but quite useful from a practical point of view is the average primary credibility. The primary credibility for each risk is weighted by the primary expected losses and punched. We can get the average primary credibility for any group of risks from:

$$\text{Primary credibility} = \frac{\Sigma E_p \times Z_p}{\Sigma E_p}$$

Three additional calculations are necessary to prepare the experience rating sheet for punching. The product of total expected losses and modification is obvious. For risks below the Q point, it is true that

$$E \times Z = E_p \times Z_p = E_p \times \frac{E}{E_p + K} = E \times \frac{E_p}{E_p + K}$$

A table of values of $\frac{E_p}{E_p + K}$ has been prepared, so that all that is necessary is to look up E_p in the table and then multiply the corresponding value of $\frac{E_p}{E_p + K}$ by E. This value is punched for both $E \times Z$ and $E_p \times Z_p$.

For risks above the Q point, separate calculations of $E \times Z$ and $E_p \times Z_p$ are necessary. Since E_p, WE_e, and $E_p + B + WE_e$ are given in the lower left hand corner of the rating form, it is a fairly easy matter to compute

$$\frac{E_p + WE_e}{E_p + B + WE_e} \quad \text{and} \quad \frac{E_p}{E_p + B + WE_e}$$

and then multiply each of these values by E to obtain the necessary values for punching.

The punch card on p. 27 is used for compiling these statistics, any information which does not appear on the experience rating form being coded in before punching.

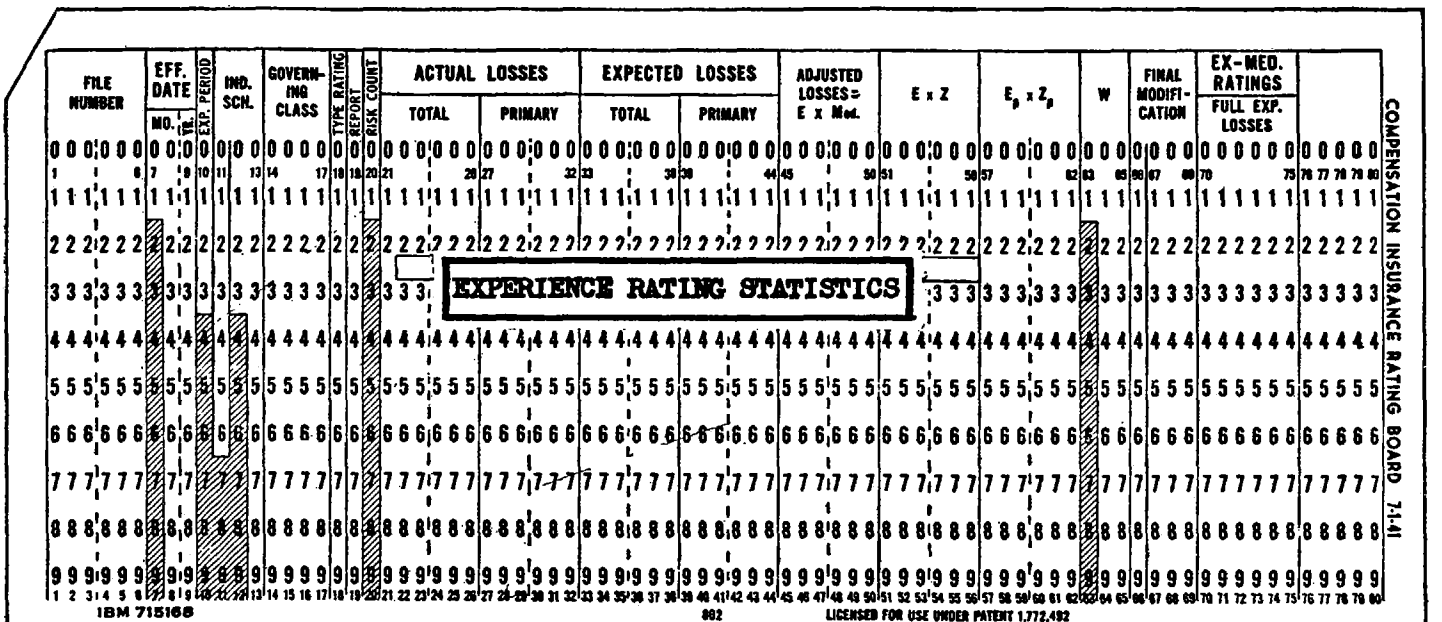
FILE NUMBER
EFF. DATE
MO.
YR.
EXP. PERIOD
IND. SCH.
GOVERN-ING CLASS
TYPE RATING
REPORT
RISK COUNT
ACTUAL LOSSES
TOTAL
PRIMARY
EXPECTED LOSSES
TOTAL
PRIMARY
ADJUSTED LOSSES = E x Mod.
E x Z
$E_p \times Z_p$
W
FINAL MODIFI-CATION
EX-MED. RATINGS
FULL EXP. LOSSES
EXPERIENCE RATING STATISTICS
COMPENSATION INSURANCE RATING BOARD 7-1-41
IBM 715168
802
LICENSED FOR USE UNDER PATENT 1,772,492

Table I shows the statistics for certain July 1941 ratings. This somewhat detailed tabulation is produced here more from the point of view of showing what information is available than from any practical standpoint. It should be kept in mind that Industry Schedule, Industry Group, and Governing Class are shown on the punch card, so that data for any specific group of risks is readily available.

Each individual *"W"* value has been shown in order to follow the trend of the three credibility values shown in columns 14, 15, and 16.

VI. Calculation of "D" Ratios

In the calculation of statutory-medical coverage *"D"* ratios, it is necessary to have distributions of losses by size of total loss for major permanent partial, minor permanent partial, temporary, and non-compensable medical cases. Since total losses are not shown on the experience card, and consequently are not punched, somewhat of a problem is presented. Revision of the Unit Statistical Plan has been suggested, to have indemnity and total losses reported for each claim, medical being obtained by subtraction, when needed. Since this is the only place where total losses are more desirable than medical, this suggestion seems impracticable. In Schedule *"Z"* and practically all other cases, it is preferable to have indemnity and medical losses shown; at least, as rate making is currently practiced. It would be impossible to make any study of medical costs if medical losses were not shown on the cards.

This problem has been handled in New York in the following manner: The punch cards for which total losses are needed are picked out, as follows:

> Only major, minor and temporary cases; not ex-medical coverage; whose total losses could exceed $500. (It is possible by sorting on the hundreds column of both indemnity and medical to eliminate a great majority of the cards whose total loss is less than $500.) These cards are then cross-footed. That is, indemnity and medical are added together and punched in a vacant field of the card by an automatic machine. With the cards in this condition, it is a simple matter to obtain the desired distributions by size of total loss.

These cards are also utilized in calculating the Average Death & P.T. Value, since it is necessary to reflect the extent to which this value acts as a limiting value for cases other than Death or Permanent Total.

VII. Actual Savings Under the New Plan

It is estimated that the amount of time saved in carrying out actual rating procedure is between 25 and 30 percent. As the data in the 1940 ratings is on the old basis it is now necessary to obtain all information for the 1941 ratings directly from the experience cards, but an even greater saving in time can be expected a year from now, when the raters will have a previous rating made under the new plan to follow.

Moreover, it is estimated that there is a saving of one-quarter in number of rating sheets used under the new plan. A majority of the small risks were "one-sheet" risks under the old plan and are still "one-sheet" risks under the new. There is a considerable saving, however, in the number of sheets used on larger risks. Where formerly it was necessary to list individually each medical loss greater than $100, it is now necessary to list only cases whose total losses exceed $500. At the same time, while the payrolls for each separate policy period had to be listed under the old plan, they are now summarized by classification and policy year. One risk, in particular, which used fifty sheets in the 1940 rating was rated under the new plan on only three sheets.

VIII. Test of Proposed Changes in the Plan—Table II

Two proposals have been offered in this paper which it is believed will enable the plan to work better in the State of New York. In Section III, it was suggested that the W and B values be calculated individually for each risk above the Q point, and in Section IV, dropping the Q point to 8500 and using "g" $= .53$ were suggested.

In order to determine the effect of making these changes in the plan, the 136 risks with expected losses above the Q point shown in Table I were recalculated in two ways: first eliminating the grouping method and calculating W and B for each risk (Proposed Method I), and second, using this method of calculating W and B,

but dropping the Q point to 8500 and using "g" = .53 (Proposed Method II). In the latter test, the 58 risks between 8500 and 12,000 expected losses were also recalculated, since they would be affected by this proposal.

These risks are listed in Table II by size of expected losses, showing the experience modification and primary credibility for each risk under the present plan and each of the proposals. A summary shows the overall effect of making these changes.

Proposed Method I. This method, which eliminates the tabular values of B and W, has the effect of smoothing out the values of Z_p. However, since Z_p is dependent upon both E and the risk D ratio, this result is not apparent upon inspection.

If we were to calculate Z_p for each of these risks using a constant D ratio, the result would be a smooth curve, with Z_p increasing as E increases. The variations of the Z_p's from a smooth curve which are caused by variations in the D ratios which are actually used can be expressed by the following formula, where Z_p is calculated for a given size of risk using a D ratio of "D", and Z'_p is calculated for the same risk size using a D ratio of "$D + x$":

$$\frac{1}{Z'_p} - \frac{1}{Z_p} = (1 - W)\,x \quad \text{(See footnote for derivation)}$$

This inverse relationship can be readily seen in Table I, where high D ratios produce low credibilities, and low D ratios produce high credibilities which sometimes even exceed unity.

The writer does not recommend this proposal in itself, but only as it is combined with the changing of values in Proposed Method II. The test was made separately, however, to determine the individual effect of each of the proposals.

NOTE:

Derivation: $Z_p = \dfrac{E}{E_p + B + WE_e}$; $E_p = D \cdot E$; $E_e = (1 - D)\,E$

$$\frac{1}{Z_p} = \frac{D \cdot E + B + WE\,(1 - D)}{E}$$

$$\frac{1}{Z'_p} = \frac{(D + x)\,E + B + WE\,(1 - D - x)}{E}$$

$$\frac{1}{Z'_p} - \frac{1}{Z_p} = \frac{xE - WEx}{E} = x\,(1 - W).$$

Proposed Method II. We can see by inspection of Table II, that there are no longer any credibilities greater than unity, which was one of the desired results of using this method.

This method has the general effect of reducing the primary credibility and as a result the experience modifications are brought closer to unity. The overall change in average modification for risks of all sizes is only .003, however, from .959 to .962.

TABLE I—PART I

WORKMEN'S COMPENSATION—NEW YORK

PRELIMINARY STUDY OF EXPERIENCE RATINGS FOR JULY 1941 FOR RISKS WRITTEN ON A STATUTORY MEDICAL COVERAGE BASIS

Size of Expected Losses	"W"	"B"	No. of Risks	Actual Losses			Expected Losses		
				Total "A"	Primary "A_p"	Ratio (6)+(5)	Total "E"	Primary "E_p"	Average "D" Ratio (9)+(8)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0— 999	.00	5,400	249	166,761	117,057	.702	222,212	137,764	.620
1,000— 1,999	.00	5,400	788	1,136,916	699,928	.616	1,110,521	686,071	.618
2,000— 2,999	.00	5,400	308	658,238	442,737	.673	757,085	468,471	.619
3,000— 3,999	.00	5,400	145	407,155	278,488	.684	506,987	310,623	.613
4,000— 4,999	.00	5,400	85	353,616	234,410	.663	380,956	234,994	.617
5,000— 5,999	.00	5,400	81	420,694	263,245	.626	445,730	274,470	.616
6,000— 6,999	.00	5,400	48	307,603	191,843	.624	310,134	190,897	.616
7,000— 7,999	.00	5,400	36	218,365	154,199	.706	270,247	164,284	.608
8,000— 8,999	.00	5,400	25	134,260	95,645	.712	212,222	130,586	.615
9,000— 9,999	.00	5,400	18	176,139	111,143	.631	170,323	101,957	.599
10,000— 10,999	.00	5,400	13	112,044	79,167	.707	135,793	84,875	.625
11,000— 12,000	.00	5,400	14	121,314	86,595	.714	160,827	98,818	.614
12,001— 13,697	.01	6,005	18	207,831	122,483	.589	230,178	144,329	.627
13,698— 15,394	.02	6,597	14	199,082	109,941	.552	202,598	125,004	.617
15,395— 17,091	.03	7,176	12	178,786	108,698	.608	191,136	114,846	.601
17,092— 18,788	.04	7,741	7	101,253	70,004	.692	125,267	77,821	.621
18,789— 20,485	.05	8,294	7	127,649	77,514	.607	137,423	85,440	.622
20,486— 22,182	.06	8,832	11	212,850	118,867	.558	233,758	141,411	.605
22,183— 23,879	.07	9,358	7	146,183	102,380	.700	159,800	104,890	.656
23,880— 25,576	.08	9,870	10	260,707	169,968	.652	248,183	153,187	.617
25,577— 27,273	.09	10,369	4	115,056	89,061	.774	104,464	63,863	.611
27,274— 28,970	.10	10,854	3	101,263	61,099	.603	85,784	52,670	.614
28,971— 30,667	.11	11,326	3	124,919	82,016	.657	87,964	54,574	.620
30,668— 32,364	.12	11,785	2	55,354	34,258	.619	63,526	37,288	.587
32,365— 34,061	.13	12,230	1	34,659	31,214	.901	33,930	22,382	.660
34,062— 35,758	.14	12,663	7	195,651	124,251	.635	244,364	152,992	.626
35,759— 37,455	.15	13,082	3	118,499	59,301	.500	109,387	62,841	.574
37,456— 39,152	.16	13,487	1	32,606	20,949	.642	38,698	25,352	.655
42,546— 44,242	.19	14,624	3	123,991	79,055	.638	129,772	82,936	.639
45,940— 47,636	.21	15,315	3	135,269	83,483	.617	140,268	89,563	.639
47,637— 49,333	.22	15,641	2	37,676	34,075	.904	97,979	57,277	.585
49,334— 51,030	.23	15,953	1	35,775	17,458	.488	50,048	32,531	.650
51,031— 52,727	.24	16,252	1	20,353	15,167	.745	51,483	31,117	.604
52,728— 54,424	.25	16,538	1	32,687	22,592	.691	54,040	32,229	.596
54,425— 56,121	.26	16,810	1	52,602	34,103	.648	54,792	36,690	.670
56,122— 57,818	.27	17,069	2	105,004	68,570	.653	113,405	69,793	.615
62,910— 64,606	.31	17,972	1	68,080	46,676	.686	64,396	39,252	.610
64,607— 66,303	.32	18,164	1	55,166	32,484	.589	65,399	39,716	.607
74,789— 76,485	.38	19,039	1	74,406	51,940	.698	75,072	45,209	.602
76,486— 78,182	.39	19,138	1	73,580	37,825	.514	77,951	46,370	.595
83,274— 84,970	.43	19,402	1	52,600	42,925	.816	84,651	51,368	.607
95,153— 96,849	.50	19,350	1	75,165	39,820	.530	95,432	59,959	.628
103,637—105,333	.55	18,914	1	105,437	53,409	.507	104,801	68,465	.653
110,425—112,121	.59	18,325	1	152,286	71,145	.467	110,846	70,916	.640
147,759—149,455	.81	11,276	2	269,723	167,815	.622	298,035	197,698	.663
156,243—157,939	.86	8,775	1	104,389	49,554	.475	157,316	94,851	.603
180,000 & Over	1.00	—	1	218,952	152,909	.698	276,094	183,941	.666
0— 12,000	—	—	1,810	4,213,105	2,754,457	.654	4,683,037	2,883,810	.616
12,000 & Over	—	—	136	4,005,489	2,483,009	.620	4,398,240	2,748,771	.625
TOTAL	—	—	1,946	8,218,594	5,237,466	.637	9,081,277	5,632,581	.620

TABLE I—PART II

WORKMEN'S COMPENSATION—NEW YORK

PRELIMINARY STUDY OF EXPERIENCE RATINGS FOR JULY 1941 FOR RISKS WRITTEN ON A STATUTORY MEDICAL COVERAGE BASIS

Size of Expected Losses	"E x Z"	"$E_p \times Z_p$"	"$E_e \times Z_e$" (11) − (12)	Credibility: Primary "Z_p" (12) + (9)	Credibility: Excess "Z_e" (13) / (8) − (9)	Credibility: Average "Z" (11) + (8)	"E x Mod."	Average Modification (17) + (8)
(1)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0— 999	20,774	20,774	—	.151	.000	.093	218,932	.985
1,000— 1,999	158,844	158,844	—	.232	.000	.143	1,111,879	1.001
2,000— 2,999	167,856	167,856	—	.358	.000	.222	748,462	.989
3,000— 3,999	144,397	144,397	—	.465	.000	.285	491,538	.970
4,000— 4,999	129,120	129,120	—	.549	.000	.339	380,801	1.000
5,000— 5,999	171,841	171,841	—	.626	.000	.386	438,965	.985
6,000— 6,999	131,491	131,491	—	.689	.000	.424	311,057	1.003
7,000— 7,999	123,654	123,654	—	.753	.000	.458	263,101	.974
8,000— 8,999	104,271	104,271	—	.798	.000	.491	184,365	.869
9,000— 9,999	87,104	87,104	—	.854	.000	.511	178,475	1.048
10,000— 10,999	74,295	74,295	—	.875	.000	.547	130,636	.962
11,000— 12,000	91,127	91,127	—	.922	.000	.567	149,797	.931
12,001— 13,697	131,768	131,009	759	.908	.009	.572	211,061	.917
13,698— 15,394	116,931	115,494	1,437	.924	.019	.577	188,516	.930
15,395— 17,091	109,994	107,826	2,168	.939	.028	.575	184,553	.966
17,092— 18,788	74,530	72,760	1,770	.935	.037	.595	116,943	.934
18,789— 20,485	82,714	80,237	2,477	.939	.048	.602	129,329	.945
20,486— 22,182	140,623	135,310	5,313	.957	.058	.602	212,425	.909
22,183— 23,879	99,626	96,088	3,538	.916	.064	.623	156,400	.979
23,880— 25,576	153,724	146,475	7,249	.956	.076	.619	263,894	1.063
25,577— 27,273	64,718	61,220	3,498	.959	.086	.620	127,498	1.220
27,274— 28,970	54,218	51,015	3,203	.969	.097	.632	94,405	1.100
28,971— 30,667	55,506	51,988	3,518	.953	.105	.631	115,320	1.311
30,668— 32,364	40,149	37,004	3,145	.992	.120	.632	59,862	.942
32,365— 34,061	22,428	21,037	1,391	.940	.120	.661	41,225	1.215
34,062— 35,758	158,811	146,276	12,535	.956	.137	.650	213,920	.875
35,759— 37,455	69,872	62,766	7,106	.999	.153	.639	107,777	.985
37,456— 39,152	25,966	23,954	2,012	.945	.151	.671	34,286	.886
42,546— 44,242	87,814	79,252	8,562	.956	.183	.677	125,388	.966
45,940— 47,636	96,129	85,884	10,245	.959	.202	.685	136,052	.970
47,637— 49,333	66,529	57,516	9,013	1.004	.221	.679	66,479	.679
49,334— 51,030	34,833	30,980	3,853	.952	.220	.696	35,834	.716
51,031— 52,727	35,472	30,632	4,840	.984	.238	.689	32,177	.625
52,728— 54,424	37,558	32,100	5,458	.996	.250	.695	41,503	.768
54,425— 56,121	38,957	34,519	4,438	.941	.245	.711	52,436	.957
56,122— 57,818	79,950	68,382	11,568	.980	.265	.705	110,079	.971
62,910— 64,606	46,623	38,895	7,728	.991	.307	.724	70,578	1.096
64,607— 66,303	47,414	39,305	8,109	.990	.316	.725	57,290	.876
74,789— 76,485	56,154	44,893	11,261	.993	.377	.748	78,976	1.052
76,486— 78,182	58,775	46,459	12,316	1.002	.390	.754	71,013	.911
83,274— 84,970	65,351	51,129	14,222	.995	.415	.772	66,112	.781
95,153— 96,849	76,441	58,977	17,464	.984	.492	.801	75,582	.792
103,637—105,333	86,356	66,863	19,493	.977	.536	.824	98,513	.940
110,425—112,121	92,889	69,722	23,167	.983	.580	.838	135,010	1.218
147,759—149,455	275,733	195,442	80,311	.989	.800	.925	269,811	.905
156,243—157,939	148,506	94,862	53,644	1.000	.859	.944	105,402	.670
180,000 & Over	276,094	183,941	92,153	1.000	1.000	1.000	218,952	.793
0— 12,000	1,404,774	1,404,774	—	.487	.000	.300	4,608,008	.984
12,000 & Over	3,109,156	2,650,212	458,964	.964	.278	.707	4,105,101	.933
TOTAL	4,513,930	4,054,986	458,964	.720	.133	.497	8,713,109	.959

TABLE II

WORKMEN'S COMPENSATION—NEW YORK

PRELIMINARY STUDY OF EXPERIENCE RATINGS FOR JULY 1941 FOR RISKS WRITTEN ON A STATUTORY MEDICAL COVERAGE BASIS

Amount of Expected Losses	Present Method		Proposed Method I		Proposed Method II		Amount of Expected Losses	Present Method		Proposed Method I		Proposed Method II	
	Modif.	Z_p	Modif.	Z_p	Modif.	Z_p		Modif.	Z_p	Modif.	Z_p	Modif.	Z_p
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
8,514	.936	.843	Same as		.936	.842	11,534	.675	.904			.705	.806
8,622	.663	.811	Present		.665	.807	11,612	1.073	.911			1.061	.810
8,718	.787	.777	Method		.789	.770	11,677	1.111	.934			1.099	.827
8,718	1.033	.856	up to		1.032	.846	11,688	1.039	.884			1.031	.787
8,758	.846	.782	$12,000		.848	.773	11,812	1.191	.985			1.167	.863
8,773	.783	.781			.785	.772	11,843	1.214	.942			1.199	.829
8,808	.696	.860			.700	.847	11,956	.817	.933			.834	.821
8,831	.854	.802			.856	.790	11,961	.716	.956			.745	.837
8,841	.839	.770			.841	.758	12,059	.712	.781	.701	.813	.731	.727
8,844	.945	.819			.945	.807	12,291	.698	.895	.688	.931	.721	.818
8,872	.781	.806			.785	.792	12,319	1.388	.908	1.404	.945	1.354	.829
8,897	1.140	.824			1.137	.810	12,382	1.202	.869	1.204	.901	1.192	.796
8,907	1.079	.823			1.079	.806	12,449	.623	.910	.612	.942	.653	.828
9,039	1.938	.847			1.916	.827	12,512	.797	.878	.792	.907	.812	.800
9,066	.922	.922			.925	.895	12,515	1.071	.902	1.068	.932	1.072	.820
9,096	.812	.813			.816	.791	12,588	.802	.908	.798	.937	.817	.824
9,165	1.429	.809			1.416	.785	12,598	.654	.911	.645	.939	.682	.825
9,171	.750	.863			.756	.836	12,650	1.109	.886	1.111	.912	1.103	.805
9,192	.735	.804			.743	.780	12,698	.770	.928	.764	.954	.792	.836
9,275	.818	.868			.823	.838	12,998	.933	.943	.932	.962	.941	.843
9,319	.712	.783			.722	.756	13,142	.811	.911	.809	.925	.829	.815
9,484	1.260	.849			1.247	.812	13,189	.583	.897	.578	.908	.622	.803
9,569	.964	.866			.964	.826	13,267	1.084	.925	1.085	.935	1.072	.823
9,574	1.352	.898			1.338	.853	13,300	1.090	.899	1.090	.908	1.091	.803
9,600	.955	.877			.955	.834	13,526	1.239	1.017	1.240	1.021	1.206	.888
9,647	1.178	.863			1.170	.820	13,695	.915	1.016	.915	1.016	.928	.885
9,649	.922	.833			.931	.794	13,733	.909	.794	.904	.823	.917	.737
9,765	1.208	.868			1.195	.821	13,736	.560	.983	.544	1.031	.595	.896
9,836	1.096	.902			1.094	.848	13,778	1.075	.893	1.075	.929	1.073	.820
9,895	.922	.882			.924	.829	13,940	1.155	.955	1.155	.992	1.148	.868
9,981	.903	.867			.906	.814	13,995	.655	.882	.642	.912	.687	.807
10,032	1.028	.847			1.026	.796	14,160	.895	.992	.892	.911	.905	.806
10,142	1.192	.863			1.181	.806	14,415	.904	.939	.904	.963	.909	.846
10,148	1.083	.841			1.079	.786	14,551	.989	.927	.988	.945	.992	.833
10,256	1.373	.871			1.347	.809	14,862	.946	.964	.947	.975	.947	.856
10,322	1.000	.924			.997	.852	14,921	1.179	.941	1.180	.952	1.161	.839
10,333	.936	.856			.939	.794	15,029	1.112	.906	1.111	.913	1.108	.809
10,436	.753	.867			.775	.801	15,036	1.143	.932	1.144	.938	1.124	.829
10,505	1.015	.891			1.014	.819	15,209	.729	1.009	.728	1.013	.760	.885
10,589	.686	.884			.709	.812	15,233	.757	.974	.756	.977	.787	.858
10,704	1.032	.897			1.026	.820	15,421	1.145	.920	1.153	.956	1.130	.843
10,727	.938	.865			.940	.793	15,436	1.197	.923	1.207	.959	1.178	.845
10,742	.753	.906			.771	.826	15,485	1.175	.930	1.182	.965	1.159	.849
10,857	.754	.875			.772	.799	15,510	1.142	.931	1.145	.965	1.130	.850
11,014	.679	.903			.711	.817	15,579	.945	.969	.945	1.005	.948	.880
11,024	.710	.910			.733	.822	15,677	1.015	.960	1.017	.991	1.011	.870
11,079	.671	.920			.699	.829	15,778	.834	.899	.829	.923	.850	.818
11,177	.894	.900			.901	.810	16,027	.729	.935	.723	.957	.757	.844
11,203	1.235	.916			1.209	.823	16,136	.823	.938	.819	.957	.842	.845
11,247	.986	.919			.984	.824	16,418	.981	.868	.981	.878	.980	.784

TABLE II—*Continued*

WORKMEN'S COMPENSATION—NEW YORK

PRELIMINARY STUDY OF EXPERIENCE RATINGS FOR JULY 1941
FOR RISKS WRITTEN ON A STATUTORY MEDICAL COVERAGE BASIS

Amount of Expected Losses	Present Method		Proposed Method I		Proposed Method II		Amount of Expected Losses	Present Method		Proposed Method I		Proposed Method II	
	Modif.	Z_p	Modif.	Z_p	Modif.	Z_p		Modif.	Z_p	Modif.	Z_p	Modif.	Z_p
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
16,787	.928	.951	.927	.956	.943	.845	28,829	1.095	.988	1.095	.988	1.081	.882
16,882	.715	1.081	.714	1.085	.751	.942	28,978	1.024	.912	1.025	.926	1.022	.833
17,153	1.011	.928	1.013	.958	1.009	.847	29,141	1.071	.994	1.071	1.011	1.065	.900
17,304	.841	.947	.839	.977	.852	.861	29,845	1.824	.958	1.895	.966	1.746	.865
17,950	1.109	.928	1.136	.942	1.119	.835	31,558	1.048	.988	1.049	.995	1.043	.890
18,008	.969	.902	.967	.914	.976	.817	31,968	.838	.996	.838	.999	.853	.893
18,065	1.144	.955	1.147	.968	1.125	.868	33,930	1.215	.940	1.215	.939	1.190	.847
18,321	.829	.971	.827	.980	.846	.865	34,230	.857	.929	.857	.940	.868	.848
18,466	.641	.919	.640	.922	.675	.821	34,414	1.090	.964	1.091	.974	1.082	.875
18,936	1.136	.943	1.138	.969	1.126	.857	34,736	.929	.942	.929	.949	.935	.855
19,020	.821	.899	.818	.922	.835	.821	35,051	.828	1.093	.826	1.100	.847	.974
19,445	1.172	.918	1.175	.934	1.155	.831	35,209	.733	1.000+	.732	1.004	.762	.900
19,887	1.206	1.000+	1.210	1.011	1.178	.890	35,354	.809	.856	.809	.857	.824	.782
19,940	.911	.904	.909	.911	.925	.814	35,370	.887	.970	.887	.971	.894	.874
20,027	.766	.913	.764	.918	.795	.819	36,081	.897	.934	.898	.942	.905	.851
20,168	.616	1.016	.615	1.021	.653	.898	36,121	.943	1.091	.942	1.104	.951	.978
20,548	.819	.930	.817	.955	.833	.848	37,185	1.112	.997	1.112	.997	1.106	.898
20,550	.749	.993	.742	1.021	.773	.899	38,698	.886	.945	.886	.946	.896	.856
20,918	.866	.954	.866	.973	.876	.863	43,111	.901	.943	.901	.948	.909	.861
20,957	.794	.939	.790	.957	.814	.851	43,112	.768	.979	.768	.986	.788	.891
21,237	.739	.910	.737	.922	.761	.823	43,549	1.227	.946	1.228	.949	1.208	.862
21,248	.664	.965	.659	.978	.697	.867	46,088	.838	.940	.837	.947	.866	.862
21,313	1.455	.946	1.459	.959	1.417	.852	47,075	1.514	.997	1.514	.998	1.473	.904
21,540	1.101	1.030	1.103	1.041	1.086	.915	47,105	.751	.943	.741	.932	.770	.861
21,569	1.062	.922	1.063	.929	1.054	.829	48,814	.677	1.013	.695	1.043	.702	.919
21,718	.681	.969	.679	.974	.713	.865	49,165	.680	.995	.677	.990	.704	.903
22,160	1.048	.987	1.048	.985	1.059	.873	50,048	.716	.952	.710	.949	.741	.872
22,390	.742	.918	.739	.937	.762	.836	51,483	.625	.984	.635	1.006	.654	.901
22,592	1.047	.934	1.047	.951	1.043	.847	54,040	.768	.996	.778	1.009	.786	.908
22,753	.961	.919	.960	.933	.966	.834	54,792	.957	.941	.943	.932	.961	.866
22,765	1.003	.916	1.004	.929	1.000	.830	56,587	1.133	.969	1.152	.989	1.124	.890
22,921	1.185	.840	1.186	.849	1.173	.767	56,818	.809	.991	.817	1.003	.821	.907
23,072	.841	.939	.840	.949	.854	.846	64,396	1.096	.991	1.127	1.017	1.088	.909
23,307	1.067	.963	1.069	.969	1.056	.862	65,399	.876	.990	.880	.996	.911	.885
24,029	.959	.951	.959	.970	.961	.863	75,072	1.052	.993	1.052	.996	1.048	.932
24,320	.601	.964	.597	.981	.636	.872	77,951	.911	1.002	.911	1.000+	.918	.927
24,474	1.492	.955	1.495	.968	1.452	.869	84,651	.781	.995	.781	.994	.793	.927
24,672	1.028	.964	1.030	.974	1.024	.867	95,432	.792	.984	.792	.984	.804	.926
24,801	1.267	.931	1.268	.939	1.245	.840	104,801	.940	.977	.941	.976	.945	.924
24,860	.909	.928	.909	.935	.914	.837	110,846	1.218	.983	1.217	.983	1.209	.935
25,098	1.189	.988	1.190	.994	1.165	.883	148,897	1.151	.990	1.150	.989	1.148	.967
25,206	1.007	.947	1.008	.949	1.004	.848	149,138	.660	.987	.660	.987	.667	.965
25,359	1.113	.966	1.113	.966	1.106	.862	157,136	.670	1.000+	.670	.998	.676	.983
25,364	1.060	.973	1.061	.975	1.053	.868	276,094	.793	1.000	.793	1.000	.793	1.000
25,875	1.302	.948	1.308	.963	1.274	.860							
26,012	1.163	.982	1.167	.997	1.145	.886							
26,126	.666	.945	.664	.958	.694	.856							
26,451	1.745	.960	1.751	.968	1.671	.864							
28,227	.728	.968	.728	.976	.751	.872							
28,728	1.472	.950	1.472	.951	1.436	.853							

TABLE II—Continued

WORKMEN'S COMPENSATION—NEW YORK

PRELIMINARY STUDY OF EXPERIENCE RATINGS FOR JULY 1941

FOR RISKS WRITTEN ON A STATUTORY MEDICAL COVERAGE BASIS

SUMMARY

Size of Expected Losses	Type of Risk	No. of Risks	Present Method		Proposed Method I		Proposed Method II	
			Average Modification	Average Z_p	Average Modification	Average Z_p	Average Modification	Average Z_p
0— 8,499	All Risks	1,752	.987	.433	Same as Present Method		.987	.433
8,500—12,000	Credits	36	.818	.862			.833	.810
	Charges	22	1.190	.881			1.179	.817
	All Risks	58	.961	.869			.966	.813
0—12,000	All Risks	1,810	.984	.487			.985	.480
12,001 & Over	Credits	79	.791	.966	.790	.973	.805	.897
	Charges	57	1.180	.960	1.184	.970	1.166	.880
	All Risks	136	.933	.964	.935	.972	.937	.891
Grand Total	All Risks	1,946	.959	.720	.960	.724	.962	.681