# RISK DISTRIBUTIONS UNDERLYING INSURANCE CHARGES IN THE RETROSPECTIVE RATING PLAN

BY

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Anyone who has reflected in even a non-technical way on retrospective rating in casualty insurance recognizes that there is or should be a connection between the setting of the minimum and maximum premiums and insurance charges of retrospective rating plans and the way the risks are expected to "stack up" as to losses. Perhaps the first thought that occurs is that the minimum and maximum premium limits must be so selected relative to each other that the redundancy of premium from risks turning out very well but paying the minimum premium is expected to be just enough to offset the deficiency from risks turning out badly and not paying their way, as seen retrospectively, because of the maximum limit on premium. There is no theoretical reason why they should not be so selected, and perhaps originally it was intended to select them thus, but this notion has to be modified when net insurance charges are encountered, as they usually are, and it is seen that the redundancy on the one hand does not generally offset exactly the deficiency on the other. It must then be concluded that the minimum premium risks are not sufficient in point of number, low loss ratio or the two combined to offset the relative number, high loss ratio or the two combined of the maximum premium risks. Put in a more statistical style, it is readily recognized that proper insurance charges are dependent on the way the risks are expected to distribute themselves by loss ratio prospectively, in those intervals of the possible range in loss ratio from zero to infinity, where, under the terms of the retrospective rating agreement, the minimum or maximum premium is indicated.

## Excess Ratio Charts and Tables

Our non-technical observer takes it for granted that in plans "tailor-made" for individual risks covering several lines of insurance in one agreement, analogy and judgment enter into the setting of premium limits and insurance charges. Risk plans often reflect special requirements of the assured and may even be frankly one-sided, compared to standard premium, to meet unusual hazard conditions. If, however, he has delved[1] into the make-up of

* This study was begun jointly by J. W. Wieder, Jr., a student of the Society, and the writer, but Mr. Wieder was called into the Service before its completion.

[1] The best exposition is S. D. Pinney's paper, "The Retrospective Rating Plan for Workmen's Compensation Risks," P.C.A.S. XXIV, pp. 291-359.

one of the formal plans such as the Retrospective Rating Plan of the National Council on Compensation Insurance, where he has a right to expect the insurance charges would be based on valid statistics, he has found that they are so indeed, but the basic data are derived charts or tables of "excess pure premium ratios" and not the fundamental risk distributions, and, in fact, the risk tables he might have expected to be available for the various sizes of risk are not in existence. He has found the table of excess ratios well adapted to the calculation of the insurance charge for the insurance granted by the provision of a maximum premium. If he has tried to understand entirely the calculation whereby the credit arising from a minimum premium is derived from this same chart or table of excess ratios, and has succeeded, he has demonstrated some degree of actuarial talent. It should be said at this point that there are very good reasons, which will be investigated later herein, for the use, hitherto, of excess ratios rather than risk distributions as basic data for the insurance charges of retrospective rating plans.

In this paper, only insurance charges arising from maximum premiums and modified by the effect of minimum premiums are considered. There are also to be found in formal plans and individual risk plans charges for limitations directly on losses, and hence on derived premium, eliminating from consideration in the plan losses in excess of certain limits per case or per accident, such charges having been usually handled as increments to the other insurance charge, which is first determined on the basis that individual cases and accidents are not limited in cost beyond the limitations that inhere in the standard coverage of the line in question. In the New York compensation plan, since the adoption of a separate New York excess ratio chart in 1941, the per claim limit has been worked into the chart itself by increments on what would have been the excess ratios without loss limitations so that no further account need be taken of the per claim limit in calculating insurance charges.[2] Numbers of rating plans, encountered or proposed to be encountered in casualty insurance, ostensibly with few or no features in common with the prototypal workmen's compensation retrospective rating plan beyond the fact that the final premium as to the individual assured is determined with reference to the experience actually had with his policy or policies involved in the plan, can be transformed to show insurance charges arising from maximum premiums and perhaps modified by minimum premiums, thus coming within the scope of this paper.[3]

It is planned to show herein that any table or chart of excess pure premium ratios as developed in connection with retrospective rating implies corresponding risk distributions which can be worked out fairly accurately from

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[2] See Paul Dorweiler, "On Graduating Excess Pure Premium Ratios," P.C.A.S. XXVIII, pp. 138-142.

[3] See T. O. Carlson, "An Actuarial Analysis of Retrospective Rating," P.C.A.S. XXVIII, pp. 283-284.

the excess ratio table; and there is presented a risk distribution table based on the chart of the Compensation Insurance Rating Board of New York issued in May 1941, and subsequent table of readings therefrom, entitled "New York Workmen's Compensation Excess Pure Premium Ratios," for premium sizes from $5,000 to $500,000. The New York chart, as mentioned in the last paragraph, includes increments for a limitation per claim to $10,000. A modified table without the increment for the claim limit has been prepared and used as the basis of insurance charges in a proposed workmen's compensation rating program submitted by the National Bureau of Casualty and Surety Underwriters to various compensation rate-making bodies in June 1942. The table of excess pure premium ratios submitted by the Bureau is the one from which the risk distributions here presented have been worked out and is shown as Exhibit I, herewith.

Generally accepted excess pure premium ratio charts or tables are found only in the compensation line. Tentative charts on other lines can probably be found in some offices. It might be said in passing that a chart for one line may be used in another if the dispersion of losses is not considered too widely different. An adjustment for the difference in permissible loss ratio is accomplished by the device given in Mr. Carlson's previously cited paper, P.C.A.S. XXVIII, on page 319. When both maximum and minimum premiums are involved, even the difference due to dispersion is overcome considerably. Suppose the non-charted line is considered to have its loss ratios dispersed more, that is, not grouped so closely about the permissible (average) loss ratio as the charted line. In that case, the charge for losses over the maximum loss ratio as figured from the chart with the adjustment mentioned might be considered too low for the non-charted line. The credit for the minimum, however, will also be too low and hence the difference or net charge may well be more nearly correct than the two items composing it.

Excess pure premium ratio charts as developed for retrospective rating (they can be used for any kind of aggregate stop loss insurance) show, for various sizes of risk as measured by standard premium and for all loss ratios in an interval of practical usefulness, the expected or average ratio to total losses of losses in excess of any selected loss ratio. Up to this point the reference sources of excess ratios have been called charts and tables somewhat indiscriminately. By "chart" we have meant the original graphical presentation of the ratios. The most frequently used portion of the National chart for compensation presently effective in states other than New York, is reproduced in P.C.A.S. XXIV, page 353, in Mr. Pinney's paper, "The Retrospective Rating Plan for Workmen's Compensation Risks." The newer chart of the Compensation Insurance Rating Board of New York, adjusted to be on a comparable basis by omitting the additional New York retrospective limitation of $10,000 per claim, is shown in P.C.A.S. XXVIII, page 152, in Mr.

Dorweiler's paper, "On Graduating Excess Pure Premium Ratios." In using the term "table" we have had in mind compilations of readings from the charts such as the one presented as Exhibit I, herewith. The entries of Exhibit I may be compared with the chart mentioned above, found in P.C.A.S. XXVIII, page 152.

Corresponding to any selected maximum or minimum premiums there are calculated "selected" loss ratios which indicate the limits of the assured's participation in losses. Thus, under the National Council Plan, a risk of \$25,000 standard premium has a minimum premium of 60%, a maximum of 140%, and a basic charge of 30%. Say also that the risk is in a state with a loss conversion factor of 1.12. When the loss ratio is approximately 26.8%, the minimum premium is earned.

$$26.8\% \times 1.12 + 30\% = 60\%$$

When the actual loss ratio falls below 26.8% the assured, by paying the minimum premium, may be said to be paying for a 26.8% loss ratio, despite his actual experience being better. On the other hand, when the loss ratio is 98.2%, the maximum premium is earned.

$$98.2\% \times 1.12 + 30\% = 140\%$$

In case the actual loss ratio exceeds this figure the assured does not pay retrospectively for more than the selected loss ratio of 98.2%, no matter how much higher his loss ratio may be. Mr. Pinney's above cited paper, P.C.A.S. XXIV, gives in full in the appendix, middle of page 341 et seq., the process of calculation of the charge and credit for the \$25,000 risk size from the data of the chart, which process we have here followed only to the point of determining the selected loss ratios for entering the chart. The selected loss ratios by which the present charts or tables are entered would also be the reference points for calculations based on risk distributions. In the example above we should get from the corresponding risk table the relative number of risks having loss ratios over 98.2% and their average loss ratio and the relative number of those under 26.8% and their average loss ratio.

It is not claimed here that risk distribution tables will be better than excess ratio tables for purposes of retrospective plans. It is our thought that fairly well defined risk distributions are implicitly involved in the accepted excess ratio tables and that it is of some value and interest to work them out and examine them. We have found them useful in connection with various questions arising on insurance charges and credits, particularly credits, and in the analysis of unusual retrospective rating propositions, such as one that involved two formulas, one applying when the risk's actual loss ratio is below the permissible, the other when it is above. It is our thought also that the risk distributions implied should be compared with *a priori* notions thereof. Among other things, should they really be unimodal or are several modes in each size of risk appropriate? They also may be compared with standards of risk credibility in experience rating.

## DEDUCTION OF RISK DISTRIBUTIONS FROM EXCESS RATIOS

It happens that there is a relatively simple relation between the distributions and the excess ratios. When the excess ratios are the given quantities, as in our investigation, we find the second differences of these at the successive loss ratios and multiply them by the permissible loss ratio to arrive at the items of the distribution. Conversely, when we have the observed distribution given, two summing operations will enable us to pass to the excess ratios. An example of the derivation of the implied risk distribution from the excess ratios for the $25,000 premium size follows. We start with excess ratios which we have expanded to five decimal places from the original three place ones (the expansion and the need for it will be discussed later) and work out the few first entries in a frequency distribution by performing the operations indicated in the column headings. The permissible loss ratio is 59.8%.

| (1) Selected Loss Ratio $x$ | (2) Graduated Excess Ratio $u_x$ | (3) $-\Delta u_{x-1}$ | (4) Ogive $-59{,}800\,\Delta u_{x-1}$ | (5) Number of Risks $\Delta$ Col. (4) | (6) Final Selected Number | (7) Excess Ratio Based on (6) |
|---|---|---|---|---|---|---|
| 0 | 1.00000 | — | (1 000) | — | — | 1.000 |
| .01 | .98327 | .01673 | 1 000 | — | — | .983 |
| .02 | .96654 | .01673 | 1 000 | — | — | .967 |
| .03 | .94981 | .01673 | 1 000 | 1 | — | .950 |
| .04 | .93310 | .01671 | 999 | — | — | .933 |
| .05 | .91640 | .01670 | 999 | 2 | 1 | .916 |
| .06 | .89972 | .01668 | 997 | 1 | 2 | .900 |
| .07 | .88307 | .01665 | 996 | 3 | 2 | .883 |
| .08 | .86646 | .01661 | 993 | 2 | 3 | .866 |
| .09 | .84989 | .01657 | 991 | 4 | 4 | .850 |
| .10 | .83339 | .01650 | 987 | .. | .. | ... |
| .. | ... | ... | ... | .. | .. | ... |
| .. | ... | ... | ... | .. | .. | ... |
| .. | ... | ... | ... | .. | .. | ... |

In the calculation, it is convenient, before passing to the second difference or frequency distribution, to multiply by the permissible loss ratio times some power of 10 to give an ogive or cumulation of the risks by number mounting up to some convenient power of 10: we chose 1,000 risks. The derivation of the method in the above finite difference form is found in Appendix A. Mr. Stefan Peters has indicated it in terms of infinitesimal calculus in his discussion, P.C.A.S. XXVIII, pp. 588-590, of Mr. Dorweiler's previously cited paper.

The deduction of the frequency distributions from the excess ratios for successive loss ratios depends on knowledge of two statistics, the excess ratios themselves, which are given, and the average loss ratio. In the use of an excess ratio chart or table in retrospective rating, it is assumed that the average loss ratio for any size of risk in the table is equal to the permissible loss ratio. The excess ratios in Exhibit I are assumed to be keyed so that every risk size has an average loss ratio of 59.8%, which was the basic permissible loss ratio for New York at the time of constructing them. While considerable effort was made to have the underlying data correctly adjusted to 59.8%, it is unlikely that they were always within the usual error limits of a three-place figure.[4] Also the underlying ungraduated excess ratios were necessarily worked out for broad groupings of risks by size and only at intervals of loss ratio, to get stability of data and keep the work within bounds, and then extended over the whole range by various processes, including mathematical and a final graphical graduation. It would not be surprising, therefore, if the keying to 59.8% is in reality only approximate in the final result. It would still be correct to use the permissible loss ratio in deriving the implied frequency distributions instead of the unknown approximations, because this accords with the application of the excess ratio table.

Having established that the two necessary statistics are at hand, the average loss ratio and the excess ratios per loss ratio at successive loss ratios, the method still works only in theory because of the shape in which the excess ratios are available. In taking out second differences, the error due to confining the excess ratios to three places of decimals (and the charts do not justify an attempt to read off the values to any more places) is enormously magnified. In fact, when the attempt is made to establish the frequency distributions in this way, taking out second differences and multiplying them by 59.8% and an appropriate power of 10, and the results are graphed, they are unrecognizable as frequency distributions, excepting possibly graphs for the largest sizes of risk. The frequent negative entries suggest some oscillating data are being recorded rather than frequency distributions.

The first differences when plotted, however, constitute ogives, or cumulative frequency distributions of the risks having loss ratios equal to and over the various loss ratios, which yield a good deal of significance. Smoothing or graduation is obviously required, but is also obviously quite feasible, which latter seems hardly the case for the second difference distribution.

Our first approach to the problem, suggested by such graphs, was to attempt graduation of the first differences or ogives by means of the Whittaker-Henderson Graduation Formula A. The results were not unsatis-

[4] See Mr. Peters' cited discussion, P.C.A.S. XXVIII, at top of page 589, or Compensation Insurance Rating Board staff memorandum May 20, 1941.

factory and were retained for some sizes of risk, but the final method adopted was to graduate the excess ratios directly by the same formula, which we have referred to earlier as a process of expanding the ratios from three decimal places to five. In this procedure there appeared to be several advantages. The end conditions gave less trouble, and the relative goodness of fit was under observation throughout the process of graduation, the test of goodness of fit in this case being not how the final distribution measures up to the observed one, because, as we have seen, the graduated one would be expected to be unrecognizably different, but how the excess ratios based upon the final distribution compare with the original. To check the assumption that the distributions produced were approximately those implied by the excess ratios and did not just happen to work more or less fortuitously, graphic comparisons of the distributions, graduated and ungraduated, in ogive form, were made. The graduation process is described in more detail in Appendix B.

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In Exhibit II are shown the resulting distributions for all sizes of risks on the bases of 1,000 risks in each size. The general shape of each of these distributions when plotted on graph paper seems not to do violence to our preconception of what they should be. The relationship between the distributions for different sizes is rather satisfactory. The median and the primary mode properly move downward from about permissible loss ratio in the $500,000 premium size toward the zero side as one passes across the sizes downward to the $5,000 size. On the whole, the excess ratios of Exhibit I stand scrutiny from the angle of their implied loss ratio distributions reasonably well. Graphs of the distributions for representative sizes of risk are shown in Exhibit III. (Page 94.)

There is a tendency in all sizes for subsidiary modes or near-modes to appear. Our *a priori* opinion on this point was that the distributions should be unimodal but there are arguments for multiple modes. Perhaps there should be a mode for the risks with normal losses only and others for those with excess losses. For instance, there might be a slight mode for the occasional death case in the $5,000 size somewhere above 150% loss ratio. This is speculation in an area not very much explored at the present time.

As an example of the possible practical use of the results we give in Exhibit II, and also of its approximate equivalence to Exhibit I, we return to the example of the $25,000 risk in Connecticut under the National Council Compensation Retrospective Rating Plan and work it through first as in Mr. Pinney's cited paper, P.C.A.S. XXIV, middle of page 341, et seq., substituting the New York chart by means of our table of readings in Exhibit I for the National chart. We first adjust our selected loss ratios of 26.8% and 98.2% for entry to the table:

$$26.8\% \times \frac{59.8}{62.5} = 25.6\% \qquad 98.2\% \times \frac{59.8}{62.5} = 94.0\%$$

It should perhaps be recalled that the present accepted method of adjusting from the state permissible loss ratio (Connecticut 62.5%) to the chart underlying permissible ratio in referring to the chart or table was not in use at the time of Mr. Pinney's paper and so a corresponding adjustment to the above is not made there. In the former method, however, the chart permissible ratio was used as the factor to convert the excess ratio to an insurance charge in terms of premium, where, in the present method, the state permissible is used. The former calculation may be considered as approximately on the chart level; the present is more exactly on the particular state level throughout.

Reference to Exhibit I gives the excess pure premium ratio for selected loss ratio 94.0% as .079. Expressed in terms of the risk premium this becomes $.079 \times .625 = .049$. Similarly for a 25.6% loss ratio limitation, the excess ratio is .593. Therefore, the ratio to total losses of losses falling below the 25.6% limitation equals $1.000 - .593 = .407$. Related to premium this becomes $.407 \times .625 = .254$. The indicated credit is then $.268 - .254$ or .014 of the risk premium. The net insurance charge becomes $.049 - .014$ or .035 of the risk premium compared to .044 under the former chart and method of calculation.

Next we work out the net insurance charge from Exhibit II, the risk table, and we observe that 149 risks of a thousand will have loss ratios of 95% and above, averaging 125.7%. Also we find that there are 147 risks of 25% loss ratio and below. Their average loss ratio is 17.1%. Keying both these figures back to the 62.5% level, they become

$$125.7\% \times \frac{625}{598} = 131.4\%; \qquad 17.1\% \times \frac{625}{598} = 17.9\%$$

The average loss of $131.4\% - 98.2\%$ on 149 of a thousand cases necessitates an insurance charge of $33.2\% \times .149$ or .049. The offsetting credit to this is $26.8\% - 17.9\%$ or 8.9% realized in 147 cases of a thousand, amounting to $8.9\% \times .147$ or .013. The net insurance charge turns out to be $.049 - .013$ or .036, compared to .035, computed from Exhibit I.

## Note on Producing Graduated Excess Ratios

Basically, the reverse of the process outlined above for passing from excess ratios to risk distributions by loss ratio underlies the working out of the excess ratios from risk experiences. The work has usually been done from an accounting point of view with excess loss calculations rather than from a statistical one, and the ease with which loss ratio distributions may be used with a double summation has seemingly been overlooked. It should be mentioned, however, that in the earliest discussion of excess ratios per loss ratio appearing in the *Proceedings* of the Society, in Mr. Dorweiler's paper "Observations on Making Rules for Excess Compensation Insurance," the work proceeded directly from loss ratio groupings, see P.C.A.S. XIII, page 174-175.

Through the courtesy of the Compensation Insurance Rating Board we were furnished actual risk distributions by loss ratio in certain size of risk groups, for one policy year, 1939, first reports, available in connection with the Board's present study of variations by industry. It was remarkably easy to sum these up twice and produce the excess ratio chart shown as Exhibit IV. The adjustment or keying to permissible loss ratio in each premium size group was done last while graphing the excess ratios produced. Instead of graphing each one at the loss ratio associated with it, it was graphed at the point to which that loss ratio was adjusted by a flat factor: 59.8%/(premium size group loss ratio).

Exhibit IV (Page 95) is introduced in this paper because of one or two interesting features. In the first place, the results are untenable when we compare the different sizes of risk, which was to be expected because of paucity of data (one year). On the other hand, each curve is in itself satisfactorily smooth without graduation of the underlying data. This suggests the point that the process of taking out the excess ratios is a graduating process, in fact, a double unsymmetrical summation graduation of the underlying distributions, and will give smooth results on rather limited data. This is equally true for any legitimate method of deriving the excess ratios as, of course, all methods should give at least approximately the same results.

It becomes quite clear that while the idea of producing finished graduated excess ratios by means of graduated basic frequency distributions in each size or size group of risks makes a very strong appeal because of its theoretically satisfying and attractive quality, yet, in a practical way, the greatest need for graduation methods is across the sizes to line them up consistently with each other, bringing in the weight of the whole experience of all sizes. In graduating across the sizes, the indicated excess ratios appear to be the most convenient basic data. Thus, it would seem Mr. Dorweiler's graduation method[5] made the proper frontal attack. As Mr. Peters says in the discussion "An ideal graduation method would reflect both kinds of relationship" and it is to be hoped the method he outlines will receive further attention. He also says, "The excess pure premium ratios are so closely linked with the distributions of risks of a given premium size by size of loss ratio and, ultimately, with the basic concepts of accident frequency and severity that it is desirable that these relationships be reflected in the graduation method or

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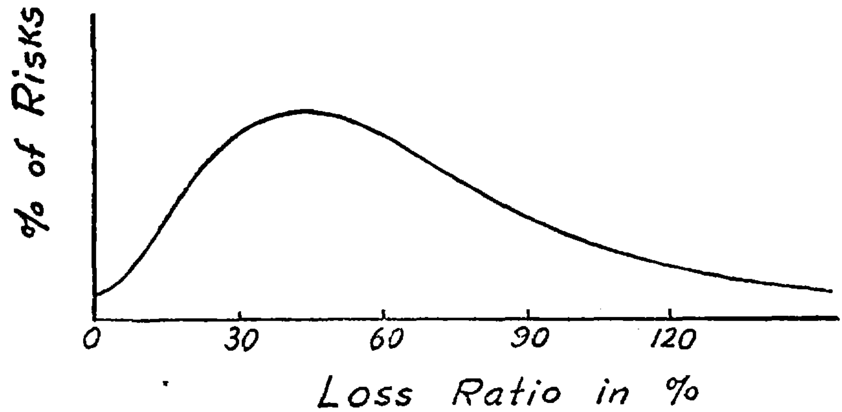
[5] P.C.A.S. XXVIII, pp. 132-157, "On Graduating Excess Pure Premium Ratios," Discussion pp. 586-590.

be used to test its accuracy." One of the objects of this paper is partially to enable the test to be made at least against *a priori* notions of the risk distributions.

## Appendix A

*Relation of Excess Ratios and Risk Distributions*

Assume the risks of one premium size to be arranged in a frequency distribution by size of loss ratio and that the average loss ratio is .60. Thus,




Let $x$ = loss ratio, figured to nearest whole per cent, and regarded as a whole number.

Let $y$ = the % of risks by number at each loss ratio.

Let $s$ = a certain "selected" loss ratio.

Let $\rho_s$ = the losses in excess of the selected loss ratio $s$, compared to all losses, or excess pure premium ratio per loss ratio $s$.

Then total losses of all risks = 100% $\times$ 60 = 6000%

$$\text{or} \quad \sum_{0}^{x=\infty} x\,y = 6000\%$$

The losses within the certain selected loss ratio $s$, compared to all losses, or

$$1-\rho_s = \frac{\sum_{0}^{x=s-1} x\,y + s\sum_{s}^{x=\infty} y}{\sum_{0}^{x=\infty} x\,y}$$

Then

$$\rho_s = \frac{\sum_{0}^{x=\infty} x\,y - \sum_{0}^{x=s-1} x\,y - s\sum_{s}^{x=\infty} y}{6000}$$

$$= \frac{\sum_{s}^{x=\infty} x\,y - s\sum_{s}^{x=\infty} y}{6000} \qquad (1a)$$

When $s+1$ is the selected loss ratio, we have excess ratio per loss ratio $s+1$,

$$\rho_{s+1} = \frac{\sum_{s+1}^{x=\infty} x\, y - (s+1) \sum_{s+1}^{x=\infty} y}{6000}$$

$$= \frac{\sum_{s}^{x=\infty} x\, y - s\, y_s = (s+1)\left[\sum_{s}^{x=\infty} y - y_s\right]}{6000}$$

$$= \rho_s + \frac{-s\, y_s - \sum_{s}^{x=\infty} y + (s+1)\, y_s}{6000}$$

$$= \rho_s + \frac{y_s - \sum_{s}^{x=\infty} y}{6000}$$

$$= \rho_s - \frac{\sum_{s+1}^{x=\infty} y}{6000}$$

so that

$$6000\,[\rho_s - \rho_{s+1}] = \sum_{s+1}^{x=\infty} y$$

or

$$-\,6000\, \Delta\, \rho_s = \sum_{s+1}^{x=\infty} y \qquad (2a)$$

From the last expression, it is evident that an ogive of the frequency distribution can be formed by differencing of successive excess ratios per loss ratio. If we difference the successive excess ratios twice we should derive the number of risks at each loss ratio, thus

$$6000\, \Delta^2 \rho_s = \sum_{s+1}^{x=\infty} y - \sum_{s+2}^{x=\infty} y = y_{s+1} \qquad (3a)$$

where $y_{s+1}$ = the per cent number of risks at loss ratio $s+1$.
It will be noted that the number of risks at loss ratio zero cannot be established in this way, which might be expected, since the risks without losses are not involved in ratios of excess losses to total losses. The per cent at zero will be 100% minus the sum per cent of all other risks.

We write the expressions (1a), (2a) and (3a), above, in general form by substituting $\bar{x}$, average loss ratio, for the specific 60% above, and we have

$$100\,\bar{x}\,\rho_s = \sum_{s}^{x=\infty} x\,y - s \sum_{s}^{x=\infty} y \qquad (1)$$

$$-100\,\bar{x}\,\Delta\,\rho_s = \sum_{s+1}^{x=\infty} y \qquad (2)$$

$$100\,\bar{x}\,\Delta^2 \rho_s = \sum_{s+1}^{x=\infty} y - \sum_{s+2}^{x=\infty} y = y_{s+1} \qquad (3)$$

As the $\rho$'s are always given in decimal form and the $x$'s and $s$ are taken as whole numbers, the $y$'s or number of risks in the right hand sides above will be in terms of per cent of all risks or decimals dependent on whether $\bar{x}$ is taken in per cent or ratio form.

We have treated the loss ratios as though they were always exactly expressible in units, 0, $\cdots$ $s-1$, $s$, $s+1$, etc. Since they are not actually a discrete series, we take them to the nearest unit, so that if we are using .01 as the unit, as we do in working from the excess ratio tables, we have the number of risks at loss ratio $s = 43$ say are all those with loss ratios .425 to .4349.

## Appendix B

### *Notes on the Graduation*

There appears not to have been an occasion for the Whittaker-Henderson Graduation Formula A to be mentioned in the *Proceedings*, so that perhaps a short introductory statement about it should be made before describing the specific application. This formula is more than a formula as the term is usually understood, it is a whole system of graduation, and one would expect it should be fully as useful in casualty insurance as it has been in life insurance, where there is a considerable literature on it.[1] If we let $u''_x$ be the general term of the ungraduated series and $u_x$ the corresponding term of the graduated series, the system proceeds on the basis that

$$\Sigma\,(\Delta^z u_x)^2 + k\,\Sigma\,(u_x - u''_x)^2$$

shall be made a minimum,[2] where $z$ and $k$ are constants to be selected. The first term $\Sigma\,(\Delta^z u_x)^2$ is the measure of smoothness and there is freedom in choosing $z$, the order of differences to be minimized. The other term is the

[1] The method is described and the literature outlined in Hugh H. Wolfenden, "The Fundamental Principles of Mathematical Statistics," under the heading, "Graduation by the Difference — Equation Method." For a working reference, the paper by Charles A. Spoerl, "Whittaker-Henderson Graduation Formula A," in *Transactions* of the Actuarial Society of America, Vol. XXXVIII, pp. 403-462, should be consulted.

[2] There is also an elaboration with more terms, the "mixed difference" case.

measure of closeness of fit, and a choice between relative smoothness and relative closeness of fit is made through the constant $k$, as the larger the $k$, the more the emphasis thrown on the "fit" term. As $k$ approaches zero, a least-squares fitting of a polynomial of degree $z-1$ is approached and as $k$ gets very large, a state of no graduation at all. In between there tends to be for each selection of $k$ a series of successive polynomials of order $z-1$ fitted to the data. As a practical matter $z$ is 2 or 3 and $k$ is a number less than one. Generally speaking the smaller $z$ is and the larger $k$ is, the easier the graduation will be, and the easiest graduation that will reasonably fulfill the requirements of the specific job in hand has considerable claim to be called the best.

In the working out of the series $\{u_x\}$ from the ungraduated series $\{u''_x\}$, an intermediate series is first constructed, $\{u'_x\}$, from $\{u''_x\}$ and then $\{u_x\}$ from $\{u'_x\}$ by means of a formula whose form depends on $z$ and whose coefficients numerically depend on $k$. The graduation can also be done directly in a linear compound form. The first method seemed the more feasible in this case because of the many points to be graduated. By trial, $z=2$ appeared to give satisfactory values in general. This gives the graduation formulae:

$$u'_x = \frac{2n\,(n+2)}{(n+1)(n+2)}\,u'_{x-1} - \frac{n\,(n+1)}{(n+1)(n+2)}\,u'_{x-2} + \frac{2}{(n+1)(n+2)}\,u''_{x+n}$$

$$u_x = \frac{2n\,(n+2)}{(n+1)(n+2)}\,u_{x+1} - \frac{n\,(n+1)}{(n+1)(n+2)}\,u_{x+2} + \frac{2}{(n+1)(n+2)}\,u'_{x-n}$$

where $n$ is a constant fulfilling the condition $k = \dfrac{4}{n\,(n+1)^2(n+2)}$.

The biggest difficulty in a Whittaker-Henderson Formula A graduation is to get the right start. In this case, referring to the formula, it is seen that the calculation of each term in the intermediate series $\{u'_x\}$ depends on knowing two before it, so there must be two to start with. A seemingly satisfactory device was hit upon in meeting a special condition of this graduation, which is discussed in the next paragraph. The same situation occurs when "turning the corner" and the $\{u_x\}$ series is to be developed from the $\{u'_x\}$ one in reverse order, but the rules of the method provide for this situation.

The series of excess ratios was regarded as a series of observations which missed the true values as any other observations would, although they are results of a graphic graduation, with one exception, the excess ratio at selected loss ratio zero is definitely unity. To insure the final value being unity, the series was extended to negative loss ratios, making it symmetrical about the point: excess ratio $=1$ and selected loss ratio $=0$. The graduation method automatically turns up the right answer for this point under these conditions as the linear compound alternative form of it suggests. By starting out rather far back on the appendage to the series, an error in the start gets worn off before the part that matters is reached. Two successive terms of the $u'_x$ column were assumed to be equal to the corresponding terms in the $u''_x$ column and set down and the start made. For the larger sizes of risk it was not necessary to use this device as the curves became practically identical with the line: selected loss ratio $= .598$ $(1 -$ excess ratio), considerably to the right of the point, and $\{u''_x\}$, $\{u'_x\}$, and $\{u_x\}$ all merge into it.

Various values of $n$ were used. This constant varying inversely to $k$, a large value of $n$ goes with relative smoothness and a small one with relative closeness of fit. This shows up in the last term of the working formulae which reach out farther ahead along the series to be smoothed when $n$ is large than when it is taken small. It was decided that on the one hand $n$ should be at least large enough to graduate out all negative risks or actual dips in the observed ogive, that is the first differences must be a descending series. On the other hand, $n$ should not be so large as to iron out what appear to be characteristic modulations in the ogive or to produce a series departing more than an occasional two units in the third place from the given excess ratios, which might be inaccurate to that extent, due to faulty drafting of, or reading off from, the excess ratio chart. It was thought it was in order to attempt to have the greatest smoothness consistent with these rules.

In general an integral value of $n$ was used in calculating the distributions given and varied between 3 and 8. A very convenient value to work with was $n = 3$, for the coefficients become 1.5, .6, and .1. Choice of other $n$'s involves recurring decimal coefficients or divisions by the common denominator $(n + 1)\ (n + 2)$. With $n = 3$, it was unnecessary to move the carriage of the calculating machine as at the conclusion of the preceding operation with the formula

$$u'_x = 1.5\, u'_{x-1} - .6\, u'_{x-2} + .1\, u''_{x+3}$$

$1.0\, u'_{x-1}$ is already in the machine and it is only necessary to add $.5\, u'_{x-1} - .6\, u'_{x-2} + .1\, u''_{x+3}$. There was, therefore, an inclination to run through with $n = 3$ and then, if sufficient smoothness was not attained, to use the results as observed values and run over again with $n = 3$, which results in a fractional $n$, about 4.3 in value. One more repetition makes $n$ about 5.3.

While the above method of graduating was generally used, namely applying the second difference formula to the excess ratios, some of the results given in Exhibit II, as was stated in the paper, were derived from our first method of graduating the indicated ogives formed from the first differences of the excess ratios keyed to a thousand-risk total by applying the factor 59,800. Another exception was that for the two largest sizes of risk, where the range in the second differences is greatest, the final selection was based on a graduation with $z$ taken as 3, that is, a third difference formula graduation.

It has already been said that the graduation of the excess ratios was done in order to expand the ratios from three decimal places to five. The three-place ratios, while they were readings from smoothly turning graduated curves, were treated as though they were rough observations of the true five-place values. It was necessary to have five decimal places in the graduated excess ratios in order to produce second differences with enough significant figures to give distributions of 1000 risks in each size. This is readily seen from the table on Page 100. In the graduations that were done on the ogives, the situation was like the usual graduation problem, as what was required was the production of smooth curves from indications which when plotted presented jagged lines with, however, the inherent trends more or less evident.

## EXHIBIT I

### NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk

Total Losses

No Limitation on Individual Losses

| Loss Ratio | Premium Size | | | | | | |
|---|---|---|---|---|---|---|---|
| | $5,000 | $7,500 | $10,000 | $15,000 | $20,000 | $25,000 | $50,000 |
| .01 | .983 | .983 | .983 | .983 | .983 | .983 | .983 |
| .02 | .967 | .967 | .967 | .967 | .967 | .967 | .967 |
| .03 | .950 | .950 | .950 | .950 | .950 | .950 | .950 |
| .04 | .934 | .933 | .933 | .933 | .933 | .933 | .933 |
| .05 | .918 | .917 | .917 | .917 | .917 | .917 | .916 |
| .06 | .903 | .901 | .901 | .901 | .901 | .901 | .900 |
| .07 | .888 | .886 | .885 | .885 | .884 | .884 | .883 |
| .08 | .872 | .870 | .868 | .868 | .867 | .867 | .866 |
| .09 | .857 | .854 | .852 | .851 | .850 | .850 | .849 |
| .10 | .843 | .838 | .836 | .835 | .834 | .834 | .833 |
| .11 | .829 | .823 | .820 | .818 | .817 | .817 | .816 |
| .12 | .815 | .808 | .805 | .802 | .801 | .800 | .799 |
| .13 | .801 | .793 | .789 | .786 | .785 | .784 | .783 |
| .14 | .788 | .779 | .774 | .770 | .768 | .767 | .766 |
| .15 | .775 | .765 | .759 | .754 | .752 | .751 | .749 |
| .16 | .762 | .751 | .745 | .739 | .736 | .735 | .732 |
| .17 | .748 | .737 | .730 | .724 | .721 | .720 | .716 |
| .18 | .736 | .724 | .716 | .710 | .707 | .705 | .700 |
| .19 | .724 | .711 | .703 | .696 | .693 | .690 | .684 |
| .20 | .712 | .698 | .689 | .682 | .679 | .676 | .669 |
| .21 | .700 | .686 | .676 | .668 | .665 | .661 | .653 |
| .22 | .688 | .673 | .663 | .654 | .651 | .646 | .637 |
| .23 | .677 | .662 | .650 | .640 | .637 | .631 | .621 |
| .24 | .666 | .650 | .638 | .627 | .623 | .616 | .605 |
| .25 | .654 | .638 | .625 | .613 | .609 | .601 | .589 |
| .26 | .643 | .626 | .613 | .600 | .595 | .587 | .574 |
| .27 | .632 | .614 | .601 | .587 | .581 | .573 | .558 |
| .28 | .621 | .603 | .590 | .575 | .568 | .560 | .543 |
| .29 | .611 | .593 | .579 | .562 | .554 | .546 | .528 |
| .30 | .602 | .583 | .569 | .550 | .541 | .532 | .513 |
| .31 | .592 | .572 | .558 | .538 | .528 | .519 | .498 |
| .32 | .583 | .562 | .547 | .526 | .515 | .505 | .483 |
| .33 | .574 | .552 | .536 | .514 | .502 | .492 | .468 |
| .34 | .565 | .542 | .526 | .503 | .489 | .479 | .454 |
| .35 | .557 | .533 | .516 | .492 | .477 | .467 | .440 |
| .36 | .548 | .523 | .506 | .481 | .465 | .454 | .426 |
| .37 | .540 | .514 | .496 | .470 | .453 | .441 | .413 |
| .38 | .531 | .504 | .486 | .459 | .441 | .429 | .400 |
| .39 | .522 | .494 | .476 | .448 | .429 | .417 | .387 |
| .40 | .514 | .485 | .467 | .438 | .418 | .405 | .374 |
| .41 | .506 | .477 | .458 | .428 | .408 | .394 | .362 |
| .42 | .498 | .468 | .448 | .418 | .398 | .383 | .350 |
| .43 | .490 | .460 | .439 | .408 | .387 | .372 | .339 |
| .44 | .482 | .451 | .430 | .399 | .377 | .361 | .326 |
| .45 | .475 | .443 | .421 | .389 | .367 | .351 | .316 |

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk

Total Losses

No Limitation on Individual Losses

| Loss Ratio | Premium Size | | | | | | |
|---|---|---|---|---|---|---|---|
| | $5,000 | $7,500 | $10,000 | $15,000 | $20,000 | $25,000 | $50,000 |
| .46 | .468 | .436 | .413 | .380 | .358 | .341 | .305 |
| .47 | .460 | .428 | .404 | .371 | .349 | .332 | .294 |
| .48 | .452 | .420 | .396 | .362 | .340 | .323 | .284 |
| .49 | .445 | .412 | .388 | .354 | .332 | .314 | .274 |
| .50 | .438 | .404 | .380 | .346 | .324 | .306 | .264 |
| .51 | .431 | .397 | .373 | .338 | .316 | .298 | .255 |
| .52 | .424 | .390 | .366 | .330 | .308 | .290 | .245 |
| .53 | .418 | .383 | .359 | .323 | .301 | .282 | .236 |
| .54 | .411 | .376 | .352 | .316 | .294 | .275 | .227 |
| .55 | .405 | .369 | .345 | .309 | .287 | .268 | .218 |
| .56 | .399 | .363 | .338 | .302 | .280 | .261 | .210 |
| .57 | .394 | .357 | .332 | .295 | .273 | .254 | .202 |
| .58 | .388 | .350 | .325 | .288 | .266 | .247 | .194 |
| .59 | .383 | .344 | .319 | .282 | .260 | .240 | .186 |
| .60 | .378 | .338 | .313 | .276 | .253 | .233 | .179 |
| .61 | .373 | .332 | .307 | .270 | .247 | .227 | .172 |
| .62 | .368 | .327 | .301 | .264 | .241 | .220 | .164 |
| .63 | .363 | .321 | .295 | .258 | .235 | .213 | .158 |
| .64 | .359 | .316 | .289 | .252 | .229 | .207 | .151 |
| .65 | .354 | .311 | .283 | .246 | .223 | .201 | .145 |
| .66 | .349 | .306 | .278 | .240 | .217 | .195 | .138 |
| .67 | .344 | .301 | .272 | .234 | .211 | .189 | .133 |
| .68 | .340 | .296 | .267 | .229 | .205 | .184 | .127 |
| .69 | .336 | .291 | .262 | .224 | .200 | .178 | .122 |
| .70 | .332 | .287 | .257 | .219 | .194 | .173 | .116 |
| .71 | .328 | .283 | .252 | .214 | .189 | .168 | .112 |
| .72 | .324 | .278 | .247 | .209 | .184 | .162 | .107 |
| .73 | .320 | .273 | .242 | .204 | .178 | .157 | .102 |
| .74 | .316 | .269 | .238 | .200 | .173 | .152 | .097 |
| .75 | .312 | .265 | .233 | .195 | .168 | .148 | .093 |
| .76 | .308 | .261 | .229 | .190 | .163 | .143 | .088 |
| .77 | .304 | .257 | .225 | .186 | .159 | .138 | .084 |
| .78 | .300 | .253 | .221 | .181 | .154 | .133 | .081 |
| .79 | .297 | .249 | .217 | .177 | .150 | .129 | .077 |
| .80 | .293 | .245 | .213 | .173 | .146 | .125 | .074 |
| .81 | .290 | .241 | .209 | .168 | .141 | .121 | .070 |
| .82 | .287 | .238 | .206 | .164 | .137 | .117 | .067 |
| .83 | .284 | .234 | .202 | .160 | .133 | .113 | .064 |
| .84 | .280 | .230 | .198 | .156 | .130 | .110 | .061 |
| .85 | .277 | .227 | .195 | .152 | .126 | .106 | .058 |
| .86 | .274 | .224 | .192 | .149 | .123 | .102 | .055 |
| .87 | .271 | .221 | .189 | .145 | .119 | .099 | .052 |
| .88 | .268 | .218 | .186 | .142 | .116 | .096 | .050 |
| .89 | .265 | .214 | .182 | .138 | .113 | .093 | .047 |
| .90 | .262 | .211 | .179 | .135 | .110 | .090 | .045 |

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses

No Limitation on Individual Losses

| Loss Ratio | Premium Size | | | | | | |
|---|---|---|---|---|---|---|---|
| | $5,000 | $7,500 | $10,000 | $15,000 | $20,000 | $25,000 | $50,000 |
| .91 | .259 | .208 | .176 | .132 | .107 | .087 | .043 |
| .92 | .257 | .206 | .174 | .130 | .105 | .085 | .041 |
| .93 | .254 | .203 | .171 | .127 | .102 | .082 | .039 |
| .94 | .251 | .200 | .168 | .124 | .099 | .079 | .037 |
| .95 | .249 | .198 | .165 | .121 | .096 | .077 | .036 |
| .96 | .247 | .195 | .162 | .118 | .094 | .074 | .034 |
| .97 | .244 | .192 | .159 | .116 | .092 | .072 | .032 |
| .98 | .242 | .190 | .157 | .114 | .089 | .069 | .030 |
| .99 | .239 | .187 | .155 | .112 | .087 | .067 | .029 |
| 1.00 | .237 | .185 | .153 | .109 | .085 | .065 | .027 |
| 1.01 | .234 | .182 | .150 | .107 | .083 | .064 | .026 |
| 1.02 | .232 | .180 | .148 | .104 | .081 | .062 | .025 |
| 1.03 | .230 | .178 | .146 | .102 | .078 | .060 | .024 |
| 1.04 | .228 | .176 | .144 | .100 | .077 | .058 | .023 |
| 1.05 | .225 | .173 | .141 | .098 | .075 | .056 | .022 |
| 1.06 | .223 | .171 | .139 | .096 | .073 | .054 | .021 |
| 1.07 | .221 | .169 | .137 | .094 | .071 | .053 | .020 |
| 1.08 | .219 | .167 | .135 | .092 | .069 | .051 | .019 |
| 1.09 | .217 | .165 | .133 | .090 | .067 | .050 | .019 |
| 1.10 | .215 | .163 | .131 | .088 | .066 | .048 | .018 |
| 1.11 | .213 | .161 | .129 | .086 | .064 | .047 | .017 |
| 1.12 | .211 | .159 | .128 | .084 | .063 | .046 | .016 |
| 1.13 | .209 | .157 | .125 | .082 | .060 | .044 | .015 |
| 1.14 | .206 | .154 | .123 | .081 | .059 | .043 | .015 |
| 1.15 | .204 | .152 | .121 | .079 | .057 | .041 | .014 |
| 1.16 | .202 | .150 | .119 | .077 | .056 | .040 | .014 |
| 1.17 | .201 | .149 | .118 | .075 | .054 | .038 | .013 |
| 1.18 | .199 | .147 | .116 | .074 | .052 | .037 | .013 |
| 1.19 | .197 | .145 | .114 | .072 | .051 | .036 | .012 |
| 1.20 | .195 | .143 | .112 | .071 | .049 | .035 | .011 |
| 1.21 | .193 | .141 | .110 | .069 | .048 | .034 | .011 |
| 1.22 | .191 | .139 | .108 | .068 | .047 | .033 | .010 |
| 1.23 | .189 | .137 | .107 | .066 | .045 | .031 | .010 |
| 1.24 | .187 | .135 | .106 | .065 | .044 | .030 | .009 |
| 1.25 | .185 | .133 | .103 | .063 | .042 | .029 | .009 |
| 1.26 | .183 | .132 | .102 | .062 | .041 | .028 | .008 |
| 1.27 | .181 | .130 | .099 | .061 | .040 | .027 | .008 |
| 1.28 | .179 | .128 | .097 | .059 | .039 | .027 | .007 |
| 1.29 | .177 | .126 | .096 | .058 | .038 | .026 | .007 |
| 1.30 | .175 | .124 | .094 | .057 | .037 | .025 | .007 |
| 1.31 | .173 | .122 | .093 | .055 | .036 | .024 | .006 |
| 1.32 | .171 | .120 | .092 | .054 | .035 | .023 | .006 |
| 1.33 | .170 | .119 | .090 | .053 | .034 | .023 | .006 |
| 1.34 | .168 | .117 | .088 | .051 | .033 | .022 | .005 |
| 1.35 | .166 | .115 | .087 | .050 | .032 | .021 | .005 |

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses

No Limitation on Individual Losses

| Loss Ratio | Premium Size | | | | | | |
|---|---|---|---|---|---|---|---|
| | $5,000 | $7,500 | $10,000 | $15,000 | $20,000 | $25,000 | $50,000 |
| 1.36 | .164 | .113 | .086 | .049 | .031 | .021 | .005 |
| 1.37 | .162 | .111 | .084 | .048 | .030 | .020 | .005 |
| 1.38 | .161 | .110 | .083 | .046 | .029 | .020 | .004 |
| 1.39 | .160 | .109 | .081 | .045 | .028 | .019 | .004 |
| 1.40 | .158 | .108 | .080 | .044 | .028 | .019 | .004 |
| 1.41 | .156 | .106 | .079 | .043 | .027 | .018 | .003 |
| 1.42 | .154 | .104 | .078 | .042 | .026 | .018 | .003 |
| 1.43 | .152 | .102 | .077 | .041 | .026 | .017 | .003 |
| 1.44 | .150 | .101 | .075 | .040 | .025 | .017 | .003 |
| 1.45 | .149 | .100 | .074 | .039 | .024 | .016 | .003 |
| 1.46 | .148 | .099 | .073 | .038 | .024 | .015 | .003 |
| 1.47 | .146 | .098 | .072 | .037 | .023 | .015 | .003 |
| 1.48 | .145 | .097 | .070 | .037 | .023 | .015 | .002 |
| 1.49 | .143 | .095 | .069 | .036 | .022 | .014 | .002 |
| 1.50 | .141 | .094 | .068 | .035 | .022 | .014 | .002 |
| 1.51 | .140 | .093 | .067 | .034 | .021 | .013 | .002 |
| 1.52 | .138 | .091 | .066 | .033 | .020 | .013 | .002 |
| 1.53 | .137 | .090 | .065 | .033 | .019 | .012 | .002 |
| 1.54 | .136 | .088 | .064 | .032 | .019 | .012 | .002 |
| 1.55 | .134 | .087 | .063 | .031 | .018 | .012 | .002 |
| 1.56 | .133 | .086 | .062 | .031 | .018 | .012 | .001 |
| 1.57 | .131 | .084 | .061 | .030 | .017 | .011 | .001 |
| 1.58 | .129 | .083 | .060 | .030 | .017 | .011 | .001 |
| 1.59 | .128 | .082 | .059 | .029 | .017 | .010 | .001 |
| 1.60 | .127 | .081 | .058 | .029 | .017 | .010 | .001 |
| 1.61 | .126 | .080 | .057 | .028 | .016 | .010 | .001 |
| 1.62 | .124 | .078 | .056 | .028 | .016 | .010 | 0 |
| 1.63 | .123 | .077 | .056 | .027 | .016 | .009 | 0 |
| 1.64 | .121 | .076 | .055 | .027 | .016 | .009 | 0 |
| 1.65 | .120 | .074 | .054 | .026 | .015 | .008 | 0 |
| 1.66 | .119 | .073 | .054 | .026 | .015 | .008 | 0 |
| 1.67 | .118 | .072 | .053 | .025 | .014 | .008 | 0 |
| 1.68 | .117 | .071 | .052 | .025 | .014 | .008 | 0 |
| 1.69 | .116 | .070 | .051 | .024 | .014 | .008 | 0 |
| 1.70 | .115 | .069 | .051 | .024 | .013 | .007 | 0 |
| 1.71 | .113 | .068 | .050 | .023 | .013 | .007 | 0 |
| 1.72 | .112 | .067 | .049 | .023 | .013 | .007 | 0 |
| 1.73 | .111 | .066 | .048 | .022 | .012 | .006 | 0 |
| 1.74 | .110 | .065 | .047 | .022 | .012 | .006 | 0 |
| 1.75 | .109 | .064 | .047 | .022 | .012 | .006 | 0 |
| 1.76 | .108 | .063 | .047 | .022 | .012 | .006 | 0 |
| 1.77 | .106 | .062 | .046 | .022 | .012 | .006 | 0 |
| 1.78 | .105 | .061 | .045 | .021 | .011 | .006 | 0 |
| 1.79 | .104 | .061 | .045 | .021 | .011 | .006 | 0 |
| 1.80 | .103 | .060 | .044 | .021 | .011 | .006 | 0 |

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk / Total Losses

No Limitation on Individual Losses

| Loss Ratio | Premium Size | | | | | | |
|---|---|---|---|---|---|---|---|
| | $75,000 | $100,000 | $150,000 | $200,000 | $300,000 | $400,000 | $500,000 |
| .01 | .983 | .983 | .983 | .983 | .983 | .983 | .983 |
| .02 | .967 | .967 | .967 | .967 | .967 | .967 | .967 |
| .03 | .950 | .950 | .950 | .950 | .950 | .950 | .950 |
| .04 | .933 | .933 | .933 | .933 | .933 | .933 | .933 |
| .05 | .916 | .916 | .916 | .916 | .916 | .916 | .916 |
| .06 | .900 | .900 | .900 | .900 | .900 | .900 | .900 |
| .07 | .883 | .883 | .883 | .883 | .883 | .883 | .883 |
| .08 | .866 | .866 | .866 | .866 | .866 | .866 | .866 |
| .09 | .849 | .849 | .849 | .849 | .849 | .849 | .849 |
| .10 | .833 | .833 | .833 | .833 | .833 | .833 | .833 |
| .11 | .816 | .816 | .816 | .816 | .816 | .816 | .816 |
| .12 | .799 | .799 | .799 | .799 | .799 | .799 | .799 |
| .13 | .783 | .783 | .783 | .783 | .783 | .783 | .783 |
| .14 | .766 | .766 | .766 | .766 | .766 | .766 | .766 |
| .15 | .749 | .749 | .749 | .749 | .749 | .749 | .749 |
| .16 | .732 | .732 | .732 | .732 | .732 | .732 | .732 |
| .17 | .716 | .716 | .716 | .716 | .716 | .716 | .716 |
| .18 | .699 | .699 | .699 | .699 | .699 | .699 | .699 |
| .19 | .683 | .683 | .683 | .682 | .682 | .682 | .682 |
| .20 | .668 | .668 | .667 | .666 | .666 | .666 | .666 |
| .21 | .652 | .652 | .650 | .649 | .649 | .649 | .649 |
| .22 | .636 | .635 | .633 | .632 | .632 | .632 | .632 |
| .23 | .619 | .618 | .616 | .615 | .615 | .615 | .615 |
| .24 | .603 | .602 | .600 | .599 | .599 | .599 | .599 |
| .25 | .587 | .586 | .584 | .583 | .582 | .582 | .582 |
| .26 | .571 | .570 | .568 | .566 | .565 | .565 | .565 |
| .27 | .555 | .554 | .552 | .549 | .548 | .548 | .548 |
| .28 | .539 | .538 | .536 | .533 | .532 | .532 | .532 |
| .29 | .523 | .521 | .519 | .516 | .515 | .515 | .515 |
| .30 | .507 | .505 | .503 | .500 | .499 | .498 | .498 |
| .31 | .492 | .490 | .487 | .484 | .483 | .482 | .482 |
| .32 | .477 | .475 | .471 | .467 | .466 | .465 | .465 |
| .33 | .461 | .459 | .454 | .450 | .449 | .448 | .448 |
| .34 | .446 | .443 | .438 | .433 | .432 | .431 | .431 |
| .35 | .431 | .428 | .422 | .417 | .416 | .415 | .415 |
| .36 | .416 | .412 | .406 | .401 | .400 | .398 | .398 |
| .37 | .402 | .397 | .391 | .386 | .384 | .382 | .381 |
| .38 | .388 | .382 | .375 | .370 | .368 | .366 | .365 |
| .39 | .374 | .367 | .360 | .355 | .352 | .350 | .348 |
| .40 | .360 | .353 | .346 | .340 | .336 | .334 | .332 |
| .41 | .347 | .338 | .331 | .326 | .321 | .318 | .316 |
| .42 | .334 | .325 | .316 | .311 | .306 | .302 | .300 |
| .43 | .322 | .311 | .301 | .295 | .290 | .287 | .285 |
| .44 | .309 | .298 | .287 | .280 | .274 | .271 | .269 |
| .45 | .297 | .285 | .273 | .265 | .260 | .255 | .254 |

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk / Total Losses

No Limitation on Individual Losses

| Loss Ratio | Premium Size | | | | | | |
|---|---|---|---|---|---|---|---|
| | $75,000 | $100,000 | $150,000 | $200,000 | $300,000 | $400,000 | $500,000 |
| .46 | .285 | .271 | .259 | .251 | .245 | .240 | .237 |
| .47 | .274 | .259 | .245 | .237 | .230 | .225 | .221 |
| .48 | .262 | .246 | .233 | .223 | .216 | .210 | .206 |
| .49 | .251 | .234 | .222 | .209 | .201 | .196 | .191 |
| .50 | .240 | .224 | .211 | .197 | .187 | .182 | .176 |
| .51 | .230 | .213 | .199 | .184 | .172 | .166 | .160 |
| .52 | .220 | .202 | .186 | .170 | .158 | .150 | .144 |
| .53 | .210 | .191 | .174 | .157 | .144 | .134 | .128 |
| .54 | .200 | .181 | .162 | .145 | .131 | .119 | .115 |
| .55 | .191 | .172 | .151 | .133 | .119 | .107 | .102 |
| .56 | .181 | .163 | .141 | .122 | .108 | .094 | .089 |
| .57 | .173 | .154 | .132 | .113 | .097 | .083 | .078 |
| .58 | .164 | .146 | .122 | .105 | .087 | .073 | .068 |
| .59 | .156 | .137 | .113 | .097 | .077 | .065 | .059 |
| .60 | .148 | .129 | .104 | .088 | .068 | .057 | .051 |
| .61 | .140 | .121 | .098 | .080 | .062 | .050 | .045 |
| .62 | .133 | .114 | .091 | .074 | .057 | .045 | .040 |
| .63 | .126 | .107 | .084 | .069 | .051 | .040 | .035 |
| .64 | .119 | .101 | .078 | .062 | .046 | .036 | .031 |
| .65 | .113 | .095 | .072 | .057 | .042 | .032 | .027 |
| .66 | .107 | .090 | .067 | .053 | .038 | .028 | .024 |
| .67 | .101 | .084 | .062 | .048 | .034 | .026 | .021 |
| .68 | .096 | .079 | .058 | .044 | .032 | .023 | .018 |
| .69 | .092 | .074 | .054 | .040 | .029 | .020 | .016 |
| .70 | .087 | .070 | .050 | .038 | .027 | .018 | .014 |
| .71 | .082 | .067 | .047 | .035 | .024 | .017 | .012 |
| .72 | .078 | .063 | .044 | .033 | .021 | .015 | .010 |
| .73 | .075 | .059 | .041 | .030 | .019 | .013 | .009 |
| .74 | .071 | .055 | .038 | .028 | .018 | .012 | .008 |
| .75 | .068 | .052 | .035 | .026 | .017 | .011 | .007 |
| .76 | .065 | .049 | .033 | .024 | .015 | .010 | .007 |
| .77 | .061 | .046 | .030 | .022 | .014 | .009 | .006 |
| .78 | .059 | .043 | .028 | .020 | .013 | .009 | .006 |
| .79 | .056 | .040 | .026 | .018 | .011 | .008 | .005 |
| .80 | .053 | .038 | .024 | .017 | .010 | .007 | .004 |
| .81 | .050 | .036 | .022 | .016 | .009 | .006 | .003 |
| .82 | .048 | .034 | .020 | .014 | .008 | .006 | .003 |
| .83 | .045 | .032 | .019 | .013 | .008 | .006 | .003 |
| .84 | .042 | .029 | .018 | .012 | .007 | .005 | .003 |
| .85 | .039 | .028 | .016 | .011 | .006 | .005 | .003 |
| .86 | .037 | .026 | .015 | .009 | .005 | .004 | .002 |
| .87 | .035 | .025 | .014 | .008 | .004 | .003 | .002 |
| .88 | .033 | .023 | .013 | .008 | .004 | .003 | .002 |
| .89 | .031 | .022 | .012 | .007 | .003 | .002 | .001 |
| .90 | .029 | .020 | .010 | .006 | .003 | .002 | .001 |

## EXHIBIT I (Cont'd)

## NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses
No Limitation on Individual Losses

| Loss Ratio | Premium Size | | | | | | |
|---|---|---|---|---|---|---|---|
| | $75,000 | $100,000 | $150,000 | $200,000 | $300,000 | $400,000 | $500,000 |
| .91 | .027 | .019 | .010 | .006 | .003 | .002 | .001 |
| .92 | .026 | .018 | .009 | .005 | .003 | .002 | .001 |
| .93 | .024 | .017 | .009 | .005 | .003 | .002 | .001 |
| .94 | .023 | .016 | .008 | .004 | .002 | .001 | 0 |
| .95 | .022 | .014 | .007 | .004 | .002 | .001 | 0 |
| .96 | .021 | .013 | .006 | .003 | .001 | 0 | 0 |
| .97 | .020 | .012 | .006 | .003 | .001 | 0 | 0 |
| .98 | .019 | .011 | .006 | .003 | .001 | 0 | 0 |
| .99 | .018 | .010 | .005 | .002 | 0 | 0 | 0 |
| 1.00 | .017 | .010 | .004 | .001 | 0 | 0 | 0 |
| 1.01 | .016 | .009 | .003 | 0 | 0 | 0 | 0 |
| 1.02 | .015 | .008 | .003 | 0 | 0 | 0 | 0 |
| 1.03 | .014 | .007 | .002 | 0 | 0 | 0 | 0 |
| 1.04 | .014 | .007 | .002 | 0 | 0 | 0 | 0 |
| 1.05 | .013 | .006 | .001 | 0 | 0 | 0 | 0 |
| 1.06 | .012 | .005 | .001 | 0 | 0 | 0 | 0 |
| 1.07 | .011 | .005 | .001 | 0 | 0 | 0 | 0 |
| 1.08 | .010 | .005 | .001 | 0 | 0 | 0 | 0 |
| 1.09 | .010 | .005 | .001 | 0 | 0 | 0 | 0 |
| 1.10 | .009 | .004 | .001 | 0 | 0 | 0 | 0 |
| 1.11 | .009 | .004 | .001 | 0 | 0 | 0 | 0 |
| 1.12 | .008 | .004 | .001 | 0 | 0 | 0 | 0 |
| 1.13 | .008 | .003 | .001 | 0 | 0 | 0 | 0 |
| 1.14 | .007 | .003 | 0 | 0 | 0 | 0 | 0 |
| 1.15 | .007 | .003 | 0 | 0 | 0 | 0 | 0 |
| 1.16 | .007 | .003 | 0 | 0 | 0 | 0 | 0 |
| 1.17 | .006 | .002 | 0 | 0 | 0 | 0 | 0 |

Compiled
in 1941

## EXHIBIT II

### Distribution of Risks by Loss Ratio
### Corresponding To Excess Ratios In Exhibit I
### Based on 1000 Risks in Each Premium Size

| Loss Ratio | Standard Premium Size in Thousands of Dollars | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5 | 7.5 | 10 | 15 | 20 | 25 | 50 | 75 | 100 | 150 | 200 | 300 | 400 | 500 |
| 0 | 6 | 3 | 2 | 1 | 1 | — | — | — | — | — | — | — | — | — |
| .01 | 8 | 4 | 2 | 1 | — | — | — | — | — | — | — | — | — | — |
| .02 | 10 | 5 | 3 | 1 | — | — | — | — | — | — | — | — | — | — |
| .03 | 12 | 6 | 3 | 1 | — | — | — | — | — | — | — | — | — | — |
| .04 | 14 | 8 | 3 | 2 | — | — | — | — | — | — | — | — | — | — |
| .05 | 15 | 9 | 5 | 2 | 1 | 1 | — | — | — | — | — | — | — | — |
| .06 | 16 | 10 | 6 | 3 | 2 | 2 | — | — | — | — | — | — | — | — |
| .07 | 16 | 11 | 7 | 4 | 3 | 2 | — | — | — | — | — | — | — | — |
| .08 | 16 | 12 | 8 | 5 | 3 | 3 | — | — | — | — | — | — | — | — |
| .09 | 16 | 12 | 9 | 6 | 6 | 4 | — | — | 1 | — | — | — | — | — |
| .10 | 16 | 14 | 11 | 8 | 7 | 5 | 1 | — | 1 | — | — | — | — | — |
| .11 | 16 | 15 | 12 | 9 | 8 | 6 | 2 | 1 | 1 | — | — | — | — | — |
| .12 | 15 | 15 | 12 | 10 | 10 | 7 | 2 | 1 | 2 | 1 | — | — | — | — |
| .13 | 15 | 15 | 14 | 11 | 12 | 8 | 4 | 2 | 2 | 1 | — | — | — | — |
| .14 | 15 | 15 | 14 | 12 | 12 | 9 | 4 | 2 | 2 | 1 | — | — | — | — |
| .15 | 14 | 15 | 15 | 14 | 14 | 9 | 5 | 3 | 2 | 1 | — | — | — | — |
| .16 | 14 | 16 | 16 | 14 | 14 | 10 | 5 | 3 | 2 | 1 | — | — | — | — |
| .17 | 13 | 16 | 16 | 14 | 13 | 9 | 5 | 3 | 2 | 1 | — | — | — | — |
| .18 | 13 | 16 | 16 | 14 | 13 | 9 | 5 | 3 | 2 | 1 | — | — | — | — |
| .19 | 13 | 15 | 16 | 13 | 12 | 9 | 5 | 3 | 2 | 2 | 1 | — | — | — |
| .20 | 13 | 15 | 17 | 13 | 11 | 9 | 5 | 3 | 2 | 2 | 1 | — | — | — |
| .21 | 13 | 15 | 16 | 13 | 9 | 9 | 5 | 3 | 2 | 2 | 1 | — | — | — |
| .22 | 13 | 14 | 16 | 13 | 8 | 9 | 5 | 3 | 2 | 2 | 1 | — | — | — |
| .23 | 13 | 14 | 16 | 12 | 8 | 9 | 6 | 3 | 3 | 2 | 1 | 1 | — | — |
| .24 | 13 | 13 | 16 | 12 | 6 | 9 | 6 | 4 | 3 | 2 | 1 | — | — | — |
| .25 | 14 | 13 | 16 | 12 | 7 | 9 | 6 | 5 | 3 | 2 | 1 | 1 | — | — |
| .26 | 14 | 13 | 15 | 12 | 6 | 9 | 7 | 5 | 3 | 2 | 1 | 1 | — | — |
| .27 | 15 | 12 | 14 | 12 | 7 | 9 | 8 | 5 | 3 | 2 | 1 | 1 | — | — |
| .28 | 15 | 12 | 13 | 12 | 7 | 9 | 9 | 6 | 3 | 2 | 2 | 2 | — | — |
| .29 | 14 | 11 | 12 | 12 | 7 | 9 | 9 | 6 | 4 | 2 | 2 | 2 | — | — |
| .30 | 14 | 11 | 11 | 12 | 9 | 10 | 10 | 8 | 5 | 2 | 3 | 2 | — | — |
| .31 | 13 | 10 | 10 | 12 | 9 | 11 | 11 | 9 | 6 | 3 | 3 | 2 | 1 | — |
| .32 | 12 | 10 | 10 | 11 | 11 | 11 | 12 | 10 | 7 | 4 | 6 | 3 | 2 | — |
| .33 | 11 | 10 | 9 | 11 | 11 | 11 | 13 | 11 | 7 | 5 | 7 | 4 | 4 | 1 |
| 34 | 10 | 10 | 9 | 11 | 12 | 11 | 14 | 12 | 8 | 7 | 8 | 5 | 5 | 2 |
| .35 | 9 | 10 | 8 | 11 | 14 | 13 | 15 | 14 | 9 | 8 | 9 | 6 | 6 | 3 |
| .36 | 9 | 10 | 8 | 11 | 14 | 13 | 16 | 14 | 10 | 9 | 9 | 6 | 7 | 5 |
| .37 | 7 | 10 | 8 | 11 | 16 | 14 | 16 | 15 | 11 | 10 | 9 | 7 | 7 | 6 |
| .38 | 7 | 10 | 8 | 11 | 16 | 15 | 16 | 15 | 12 | 12 | 9 | 7 | 7 | 6 |
| .39 | 7 | 10 | 8 | 12 | 17 | 16 | 17 | 16 | 13 | 12 | 9 | 7 | 7 | 7 |
| .40 | 7 | 9 | 9 | 12 | 17 | 17 | 17 | 16 | 14 | 12 | 9 | 7 | 7 | 6 |
| .41 | 7 | 9 | 9 | 12 | 18 | 17 | 17 | 16 | 15 | 14 | 9 | 7 | 7 | 6 |
| .42 | 7 | 9 | 10 | 12 | 18 | 18 | 17 | 16 | 16 | 15 | 9 | 7 | 7 | 5 |
| .43 | 7 | 9 | 10 | 13 | 18 | 18 | 17 | 17 | 17 | 15 | 9 | 7 | 6 | 4 |
| .44 | 7 | 9 | 11 | 13 | 18 | 19 | 17 | 17 | 18 | 16 | 9 | 7 | 5 | 3 |
| .45 | 7 | 9 | 11 | 14 | 18 | 19 | 17 | 17 | 18 | 17 | 10 | 7 | 3 | 2 |

EXHIBIT II (Cont'd)

| Loss Ratio | Standard Premium Size in Thousands of Dollars | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5 | 7.5 | 10 | 15 | 20 | 25 | 50 | 75 | 100 | 150 | 200 | 300 | 400 | 500 |
| .46 | 8 | 9 | 12 | 14 | 18 | 19 | 16 | 17 | 19 | 18 | 11 | 8 | 2 | 2 |
| .47 | 8 | 9 | 12 | 14 | 17 | 18 | 16 | 17 | 20 | 18 | 12 | 8 | 2 | 2 |
| .48 | 8 | 9 | 12 | 14 | 17 | 18 | 16 | 17 | 20 | 18 | 13 | 11 | 2 | 4 |
| .49 | 9 | 9 | 12 | 14 | 16 | 17 | 15 | 17 | 21 | 18 | 15 | 13 | 2 | 7 |
| .50 | 9 | 9 | 12 | 14 | 15 | 16 | 15 | 17 | 21 | 19 | 17 | 17 | 2 | 11 |
| .51 | 10 | 9 | 11 | 13 | 14 | 15 | 15 | 17 | 22 | 19 | 22 | 22 | 10 | 16 |
| .52 | 11 | 9 | 10 | 12 | 13 | 13 | 15 | 17 | 22 | 21 | 26 | 28 | 23 | 23 |
| .53 | 11 | 9 | 10 | 12 | 12 | 13 | 15 | 17 | 22 | 23 | 32 | 32 | 36 | 39 |
| .54 | 11 | 9 | 10 | 12 | 11 | 12 | 15 | 17 | 22 | 25 | 36 | 37 | 47 | 49 |
| .55 | 11 | 9 | 9 | 11 | 9 | 10 | 14 | 18 | 23 | 26 | 41 | 42 | 58 | 58 |
| .56 | 10 | 9 | 8 | 10 | 9 | 10 | 14 | 18 | 22 | 27 | 43 | 45 | 65 | 65 |
| .57 | 9 | 9 | 8 | 9 | 9 | 8 | 14 | 18 | 23 | 29 | 42 | 48 | 69 | 68 |
| .58 | 9 | 9 | 7 | 8 | 7 | 9 | 14 | 18 | 22 | 30 | 41 | 51 | 70 | 70 |
| .59 | 9 | 9 | 7 | 8 | 7 | 7 | 14 | 19 | 22 | 30 | 39 | 52 | 67 | 68 |
| .60 | 8 | 9 | 6 | 7 | 6 | 8 | 14 | 19 | 22 | 30 | 37 | 51 | 62 | 64 |
| .61 | 8 | 9 | 7 | 7 | 6 | 7 | 14 | 20 | 22 | 30 | 34 | 49 | 56 | 56 |
| .62 | 8 | 8 | 6 | 7 | 5 | 8 | 13 | 20 | 21 | 29 | 32 | 45 | 48 | 49 |
| .63 | 7 | 8 | 7 | 7 | 5 | 7 | 13 | 20 | 21 | 29 | 31 | 40 | 41 | 41 |
| .64 | 7 | 8 | 6 | 7 | 5 | 7 | 13 | 20 | 21 | 27 | 29 | 36 | 33 | 33 |
| .65 | 7 | 8 | 7 | 7 | 5 | 8 | 13 | 19 | 20 | 26 | 27 | 31 | 28 | 27 |
| .66 | 6 | 7 | 7 | 7 | 5 | 7 | 13 | 19 | 19 | 24 | 26 | 27 | 22 | 22 |
| .67 | 5 | 7 | 7 | 7 | 6 | 8 | 13 | 19 | 19 | 23 | 24 | 23 | 19 | 19 |
| .68 | 5 | 6 | 7 | 7 | 5 | 7 | 12 | 18 | 17 | 21 | 23 | 19 | 16 | 16 |
| .69 | 5 | 6 | 8 | 7 | 6 | 7 | 12 | 17 | 17 | 20 | 21 | 16 | 15 | 16 |
| .70 | 5 | 6 | 7 | 6 | 6 | 8 | 12 | 16 | 16 | 18 | 18 | 14 | 13 | 14 |
| .71 | 4 | 6 | 7 | 6 | 6 | 7 | 11 | 15 | 15 | 16 | 15 | 13 | 13 | 14 |
| .72 | 4 | 5 | 8 | 5 | 7 | 7 | 11 | 14 | 15 | 14 | 12 | 12 | 11 | 13 |
| .73 | 4 | 5 | 7 | 5 | 7 | 7 | 11 | 13 | 13 | 13 | 9 | 10 | 11 | 12 |
| .74 | 4 | 5 | 8 | 5 | 7 | 7 | 10 | 11 | 13 | 12 | 8 | 9 | 9 | 11 |
| .75 | 4 | 5 | 7 | 5 | 8 | 8 | 10 | 10 | 12 | 11 | 7 | 7 | 9 | 8 |
| .76 | 4 | 5 | 7 | 5 | 8 | 7 | 10 | 9 | 11 | 10 | 6 | 6 | 7 | 6 |
| .77 | 4 | 5 | 7 | 4 | 8 | 8 | 10 | 8 | 10 | 8 | 6 | 6 | 6 | 5 |
| .78 | 4 | 5 | 7 | 5 | 8 | 7 | 9 | 8 | 9 | 8 | 6 | 5 | 4 | 4 |
| .79 | 4 | 5 | 6 | 5 | 8 | 8 | 9 | 7 | 9 | 8 | 6 | 5 | 4 | 3 |
| .80 | 4 | 5 | 6 | 5 | 8 | 8 | 9 | 6 | 9 | 8 | 6 | 4 | 2 | 3 |
| .81 | 4 | 5 | 6 | 6 | 9 | 7 | 8 | 6 | 7 | 7 | 5 | 4 | 2 | 3 |
| .82 | 4 | 5 | 6 | 6 | 8 | 8 | 8 | 6 | 7 | 7 | 5 | 4 | 2 | 3 |
| .83 | 4 | 5 | 5 | 7 | 8 | 8 | 8 | 6 | 7 | 6 | 5 | 4 | 1 | 2 |
| .84 | 4 | 5 | 5 | 7 | 8 | 7 | 8 | 6 | 6 | 5 | 5 | 4 | 1 | 1 |
| .85 | 4 | 5 | 5 | 8 | 8 | 7 | 7 | 6 | 6 | 5 | 5 | 4 | 1 | 1 |
| .86 | 4 | 5 | 4 | 8 | 7 | 7 | 7 | 6 | 5 | 5 | 5 | 4 | 1 | — |
| .87 | 4 | 5 | 4 | 8 | 7 | 7 | 7 | 6 | 5 | 5 | 5 | 4 | 1 | 1 |
| .88 | 4 | 4 | 4 | 8 | 7 | 7 | 7 | 6 | 4 | 4 | 5 | 3 | 2 | — |
| .89 | 4 | 4 | 4 | 8 | 7 | 7 | 7 | 6 | 5 | 4 | 5 | 3 | 2 | — |
| .90 | 3 | 4 | 4 | 7 | 6 | 6 | 7 | 6 | 4 | 4 | 4 | 2 | 2 | 1 |

EXHIBIT II (Cont'd)

| Loss Ratio | Standard Premium Size in Thousands of Dollars | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5 | 7.5 | 10 | 15 | 20 | 25 | 50 | 75 | 100 | 150 | 200 | 300 | 400 | 500 |
| .91 | 3 | 4 | 4 | 6 | 6 | 6 | 6 | 6 | 3 | 3 | 3 | 2 | 2 | 1 |
| .92 | 3 | 4 | 4 | 6 | 5 | 6 | 6 | 6 | 4 | 3 | 2 | 1 | 1 | 1 |
| .93 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 4 | 3 | 2 | 1 | 1 | 1 |
| .94 | 3 | 3 | 4 | 5 | 5 | 5 | 5 | 6 | 3 | 2 | 1 | 2 | 2 | 2 |
| .95 | 3 | 3 | 4 | 4 | 5 | 5 | 5 | 5 | 3 | 2 | 1 | 1 | 2 | 2 |
| .96 | 3 | 3 | 4 | 3 | 4 | 6 | 5 | 4 | 4 | 2 | 1 | 1 | 2 | 2 |
| .97 | 3 | 3 | 4 | 3 | 5 | 5 | 5 | 4 | 3 | 2 | 1 | 2 | 2 | 2 |
| .98 | 3 | 3 | 4 | 3 | 4 | 5 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 |
| .99 | 2 | 3 | 3 | 3 | 3 | 4 | 5 | 3 | 3 | 2 | 2 | 2 | 2 | 1 |
| 1.00 | 2 | 3 | 3 | 3 | 4 | 5 | 5 | 3 | 3 | 2 | 3 | 2 | 1 | |
| 1.01 | 2 | 2 | 3 | 3 | 3 | 4 | 5 | 3 | 4 | 2 | 3 | 1 | 2 | |
| 1.02 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 2 | 3 | 2 | 3 | 1 | 1 | |
| 1.03 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 2 | 2 | 2 | 3 | 1 | 1 | |
| 1.04 | 2 | 2 | 2 | 2 | 2 | 3 | 4 | 2 | 3 | 3 | 3 | 1 | | |
| 1.05 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 2 | 3 | 3 | 3 | 1 | | |
| 1.06 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | | |
| 1.07 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 2 | 3 | 2 | 1 | 1 | | |
| 1.08 | 2 | 2 | 2 | 3 | 2 | 2 | 3 | 2 | 2 | 2 | 1 | | | |
| 1.09 | 1 | 1 | 2 | 2 | 1 | 3 | 2 | 2 | 2 | 1 | 1 | | | |
| 1.10 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | | | | |
| 1.11 | — | 1 | 1 | 2 | 1 | 2 | 2 | 2 | 2 | 1 | | | | |
| 1.12 | 1 | 1 | 1 | 2 | 2 | 1 | 2 | 2 | 1 | 1 | | | | |
| 1.13 | — | 1 | 1 | 3 | 1 | 2 | 1 | 2 | 2 | 1 | | | | |
| 1.14 | 1 | 1 | 1 | 2 | 2 | 2 | 1 | 2 | 1 | 1 | | | | |
| 1.15 | — | 1 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 1 | | | | |
| 1.16 | — | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 1 | 1 | | | | |
| 1.17 | — | 1 | 1 | 2 | 2 | 2 | 1 | 2 | 1 | 1 | | | | |
| 1.18 | 1 | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | | | | |
| 1.19 | — | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | | | | |
| 1.20 | — | — | — | 2 | 2 | 2 | 1 | 1 | 1 | | | | | |
| 1.21 | — | — | 1 | 2 | 3 | 2 | 1 | 1 | — | | | | | |
| 1.22 | — | 1 | — | 1 | 2 | 2 | 1 | 1 | 1 | | | | | |
| 1.23 | 1 | 1 | 1 | 2 | 3 | 2 | 1 | 1 | — | | | | | |
| 1.24 | — | 1 | 1 | 1 | 3 | 3 | 1 | 1 | 1 | | | | | |
| 1.25 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | — | | | | | |
| 1.26 | 1 | 1 | 2 | 1 | 3 | 3 | 1 | 1 | 1 | | | | | |
| 1.27 | — | 1 | 2 | 1 | 2 | 2 | 1 | — | — | | | | | |
| 1.28 | 1 | 1 | 2 | 1 | 3 | 3 | 1 | — | — | | | | | |
| 1.29 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | — | — | | | | | |
| 1.30 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | — | 1 | | | | | |
| 1.31 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | — | — | | | | | |
| 1.32 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | — | — | | | | | |
| 1.33 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | — | — | | | | | |
| 1.34 | 2 | 2 | 2 | 1 | 2 | 2 | 1 | — | 1 | | | | | |
| 1.35 | 1 | 2 | 2 | 1 | 2 | 1 | 1 | — | — | | | | | |

EXHIBIT II (Cont'd)

| Loss Ratio | Standard Premium Size in Thousands of Dollars | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5 | 7.5 | 10 | 15 | 20 | 25 | 50 | 75 | 100 | 150 | 200 | 300 | 400 | 500 |
| 1.36 | 1 | 2 | 2 | 2 | 1 | 2 | 1 | — | — | | | | | |
| 1.37 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | — | — | | | | | |
| 1.38 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | — | 1 | | | | | |
| 1.39 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | — | — | | | | | |
| 1.40 | 1 | 2 | 1 | 2 | 2 | — | 1 | — | — | | | | | |
| 1.41 | 2 | 2 | 1 | 2 | 2 | 1 | 1 | — | — | | | | | |
| 1.42 | 1 | 2 | 1 | 2 | 1 | 1 | 1 | — | 1 | | | | | |
| 1.43 | 1 | 1 | 1 | 2 | 1 | — | 1 | — | — | | | | | |
| 1.44 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | — | — | | | | | |
| 1.45 | 1 | 1 | 1 | 2 | 1 | — | — | — | — | | | | | |
| 1.46 | 1 | 1 | 1 | 2 | 1 | 1 | — | — | — | | | | | |
| 1.47 | — | 1 | 1 | 2 | — | 1 | — | — | 1 | | | | | |
| 1.48 | 1 | 1 | 1 | 2 | 1 | 1 | — | 1 | | | | | | |
| 1.49 | 1 | 1 | 2 | 2 | — | — | — | 1 | | | | | | |
| 1.50 | 1 | — | 1 | 1 | 1 | 1 | — | 1 | | | | | | |
| 1.51 | 1 | — | 1 | 1 | 1 | 1 | — | 1 | | | | | | |
| 1.52 | 1 | — | 2 | 2 | 1 | 1 | — | 1 | | | | | | |
| 1.53 | 1 | — | 1 | 2 | 2 | — | — | 1 | | | | | | |
| 1.54 | 1 | — | 2 | 1 | 2 | 1 | — | 1 | | | | | | |
| 1.55 | 1 | — | 1 | 2 | 2 | 1 | — | 1 | | | | | | |
| 1.56 | 1 | — | 2 | 1 | 2 | — | — | 1 | | | | | | |
| 1.57 | 1 | 1 | 1 | 2 | 1 | — | — | 1 | | | | | | |
| 1.58 | 2 | 1 | 2 | 1 | 2 | 1 | — | | | | | | | |
| 1.59 | 1 | 1 | 1 | 1 | 1 | — | — | | | | | | | |
| 1.60 | 1 | 1 | 2 | 1 | 1 | 1 | — | | | | | | | |
| 1.61 | 1 | 1 | 2 | — | 1 | — | — | | | | | | | |
| 1.62 | 1 | 1 | 1 | 1 | — | — | 1 | | | | | | | |
| 1.63 | 2 | 1 | 1 | — | — | 1 | 1 | | | | | | | |
| 1.64 | 1 | 1 | 1 | 1 | — | 1 | 1 | | | | | | | |
| 1.65 | 1 | 1 | 1 | — | — | — | 1 | | | | | | | |
| 1.66 | 1 | 1 | 1 | 1 | — | 1 | 1 | | | | | | | |
| 1.67 | 1 | 2 | — | — | — | — | 1 | | | | | | | |
| 1.68 | 1 | 2 | 1 | 1 | — | 1 | 1 | | | | | | | |
| 1.69 | — | 2 | 1 | 1 | — | — | 1 | | | | | | | |
| 1.70 | 1 | 1 | 1 | 2 | 1 | 1 | | | | | | | | |
| 1.71 | 1 | 2 | — | 1 | — | 1 | | | | | | | | |
| 1.72 | — | 1 | 1 | 2 | 1 | 1 | | | | | | | | |
| 1.73 | 1 | 1 | — | 1 | 1 | 1 | | | | | | | | |
| 1.74 | — | 1 | 1 | 1 | 1 | 1 | | | | | | | | |
| 1.75 | 1 | 1 | 1 | 1 | 1 | 1 | | | | | | | | |
| 1.76 | — | 1 | — | 1 | 1 | 1 | | | | | | | | |
| 1.77 | — | 1 | 1 | 1 | 1 | 1 | | | | | | | | |
| 1.78 | 1 | 1 | — | — | 1 | — | | | | | | | | |
| 1.79 | — | 1 | — | — | 1 | — | | | | | | | | |
| 1.80 | —* | 1* | —* | —* | 1* | 1* | | | | | | | | |

*Also: $ 5,000 Size: 66 risks beyond 1.80, averaging 2.735
$ 7,500 Size: 50 risks beyond 1.80, averaging 2.515
$10,000 Size: 37 risks beyond 1.80, averaging 2.51
$15,000 Size: 16 risks beyond 1.80, averaging 2.58
$20,000 Size: 9 risks beyond 1.80, averaging 2.53
$25,000 Size: 6 risks beyond 1.80, averaging 2.34