POSSIBLE VALUES FOR RETROSPECTIVE RATING PLANS

BY

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This paper is in a sense complementary to the paper "An Actuarial Analysis of Retrospective Rating," by Thomas O. Carlson, P.C.A.S., XXVIII, p. 283: it is less abstractly mathematical and gives arithmetical examples.

The notation is basically the same as that given in Carlson's paper: however, in this paper all dollar values (except P) will be in terms (ratios) of P, the standard premium, e.g.: if G, the maximum premium, is 125% of the standard premium P \$10,000, G in Carlson's notation means the maximum premium \$12,500 while in mine it means the maximum premium ratio 1.25. In both notations, however, P means \$10,000.

We shall find it convenient to use the following additional symbols:

(i) $Lo = L + Lp$, where L is any loss ratio, e.g.:

$$H'o = H' + H'p, \; G'o = G' + G'p$$

(ii) $'L = K$ where $K' = L$, e.g. if E is the expected loss ratio and $X = B + CE$ then $X' = E$ and we will write $'E = X$. Thus $'G' = G$, $'H' = H$.

The symbols* required for this paper are listed below for convenience of reference:

P = standard premium
C = loss conversion factor
Γ = maximum value of C
B = basic premium ratio
H = minimum premium ratio
G = maximum premium ratio
H' = allowable loss ratio in the minimum premium
G' = allowable loss ratio in the maximum premium
R = final retrospective premium ratio (to P)
Rv = average retrospective premium ratio (to P)
E = expected loss ratio
$'E$ = final retrospective premium ratio (to P) if actual loss ratio is E
K, X are constants

* See Note 1 of Appendix III, with particular reference to the meaning of the symbols B andC.

If L is any loss ratio (for a premium size P)

$$Lx = \text{ratio to total losses of losses in excess of } L \text{ (for the premium size } P)$$

$$Lp = \text{ratio to } P \text{ of losses in excess of } L \text{ (for the premium size } P)$$

$$= ELx$$

$$Lo = L + Lp$$

The Retrospective Plans dealt with in this paper (as in Carlson's) will be linear, i.e. of the type in which the final premium R depends linearly on the loss ratio L actually experienced (subject, however, to a maximum G and a minimum H) i.e.

$$R = B + CL \text{ but } R \not< H \text{ and } \not> G$$

C will always be positive and $B \leqslant H \leqslant G$. L of course is also positive.

The average retrospective premium, Rv, or in other words the expected value of R, is easily seen to be

$$Rv = H + C\ (H'p - G'p)$$

for in every case the minimum H is payable and in addition C times the losses in excess of a loss ratio of H' and not in excess of a loss ratio of G' (see also Carlson's paper).

For the purpose of this paper, which is to study the possible variations in the retrospective rating values, the question is "Given Rv what are the possible retrospective rating values (i.e. C, B, H, G, etc.)?" and it doesn't matter at all how Rv has been determined so long as we know its value for any value of P. Thus in this paper we are not directly concerned with Rv except that it is assumed to be a definite function of P. However, we have in mind that Rv is determined in practice by giving effect to certain reductions in the expense loadings of the standard premium: these reductions are usually made in the acquisition, general administration and payroll audit items in such a way as to reduce these loadings as the premium size increases: this is generally termed an expense gradation by size of risk. Usually in retrospective plans a small portion of the reductions in loadings is kept back to provide for contingencies. Some further remarks on this subject will be found in Appendix III where the actual data of an example are given. For our present purpose, however, all we require to know is that

$$H + C\ (H'p - G'p) = Rv, \text{ a determinate function of } P$$

In the original construction and presentation of the (Compensation) Retrospective Plan the loss conversion factor C was computed as the ratio of losses loaded for claim expense and taxes to losses and used at that value, so that in the original plan there was "full" reimbursement (to the carrier) of losses and related expenses. In these original plans, therefore, C was fixed (or practically so—actually the computed value was usually rounded to an even percentage and the difference carried to B). The introduction later of the concept of credibility or partial reimbursement or the idea of letting C be variable (for example a function of the size of risk) gave greater flexibility and a wider range of practical plans, and a broader view of the real structure of retrospective rating. To take one instance, the explanation of the modus operandi of the original retrospective rating plan was considerably complicated by the necessity of explaining the calculation of the loss conversion factor and then the rationale of the basic factor which "contained" the remaining expenses: (an additional, and from our point of view irrelevant, complexity arose out of the assumed need of balancing out, by including an extra item in the basic premium for the excess or deficiency caused by rounding the loss conversion factor to say the nearest whole percent). Under the newer concept, of C being allowed to vary, C can be any positive value subject to a practical limit discussed below, and all we are concerned with is that our rating values shall satisfy

$$H + C\,(H'p - G'p) = \text{the given value of } Rv$$

without having to decide which part of the premium comes from C (or rather $C - 1$) and which from B: thus actually if C is less than unity then from the older point of view some of the losses are provided for in B and only some by C, but from the newer point of view it is not necessary to go into that question.

A varying "credibility" C thus can be used as a key providing much more flexible formulas and easier construction of retrospective plans. Let us consider it for a minute or two. Theoretically or arithmetically it could take any positive value (I suppose it could even be negative) but for practical reasons its range must be restricted. If it were considerably in excess of unity, say three, the insured would pay a premium that would increase, within the limits of the minimum and maximum premium, three times as fast as the losses and he would save money by paying losses himself. Insureds would be entitled to and would take a very low premium when losses were light and would be charged a very high premium for heavy losses but would be able to, and tempted to, find ways of avoiding at least some part of the high charge. Such an "option against the carrier" is unsound and thus the value of C must be restricted so that the increase in premium for increase in losses is justifiable. Two limitations have been used (i) a limit of unity as in the premium return plan: this is of course quite safe and (ii) a limit of unity plus the loading for claim expenses and taxes, the reason being that these can quite reasonably be regarded as "belonging" to the losses: this is the usual basis of C and limits it to a value of about 1.20 in Compensation insurance: such a value is reasonably "safe" but nevertheless there is still some temptation for an insured to pay some minor losses without reporting them and thereby save about 20% of them: it is obvious that the arguments, sometimes put forward, that in C (usually called the loss conversion factor) should be included also loadings for acquisition cost and other expense elements, will not hold water. The values of C in use at present are already high enough.

My personal view is that unity is the limit to be preferred but in view of the widespread plans with higher limits, we will deal with an upper limit, which we will call Γ, equal approximately to unity increased by loadings for claim expenses and taxes.

This paper was originally drafted a year or two ago when interest was being manifested in the more flexible plans that a variable C renders possible. Unfortunately, from many points of view (including that set out above), the original retrospective plan with a fixed loss conversion factor proved to be so strongly entrenched that immediate practical interest in variable credibility plans has vanished: the new Compensation plans brought out last year retain the old concept of loss conversion factor. The only example of a varying credibility plan in general use is the Premium Return Plan in Utah.

Nevertheless in this, a professional, paper we will cover the whole field including the possibility of partial credibility, i.e., a C between 0 and Γ.

The basic formulas are our fundamental equation

$$Rv = H + C\,(H'p - G'p)$$

and the relations

$$H = R + CH'$$
$$G = B + CG'$$

from these we readily obtain

$$Rv = B + C\,(H'o - G'p)$$
$$Rv = G + C\,(H'o - G'o)$$

These are our basic formulas which we find it convenient to write in the form

$$H = B + CH' \qquad (1)$$
$$G = B + CG' \qquad (2)$$
$$B = Rv - C\,(H'o - G'p) \qquad (3)$$
$$H = Rv - C\,(H'p - G'p) \qquad (4)$$
$$G = Rv + C\,(G'o - H'o) \qquad (5)$$

Note that in the last three the terms in brackets are all essentially positive: for as $H' < G'$, then $H'o \geqslant H'p > G'p$ and $G'o > H'o$. Note also that (as it should be) $B \leqslant H \leqslant Rv \leqslant G$.

Of these five equations only three are independent; e.g., from the first three the last two can be derived.

We have thus six variables, C, B, H, G, H' and G' for each value of P and three independent relations, and we wish to investigate the interrelationship of these six variables, e.g., possible sets of consistent values and possible and practical plans that can be made from such possible values.

Note that when we state we have six variables we assume (i) that the values of Rv have been set for every value of P, i.e., Rv is a known function of P: (ii) that a table of excess pure premium rates has been settled on, so that $H'p$, $H'o$ are "known functions" of H' (and $G'p$, $G'o$ of G') or in other words that for every value of P we can determine $H'p$ (or $H'o$) for every H' and vice versa, and the same for the functions of G'.

We could proceed by constructing a series of exhibits or tables showing for each of a series of selected values of P the various possible typical combinations of the variables. This would produce a very voluminous and unwieldy set of exhibits where we could not "see the wood for the trees," unless we are careful to go about the job very systematically. I learned this by experience. At one time I had such a set of tables constructed and found them very difficult to analyze. So I set about devising methods of selection and classification.

However, let us suppose for the moment that we have a complete list of the possible combinations of the values of the quantities P, C, B, H, G, H' and G'. We know that for any fixed value of P only three of the remaining six quantities are independent, there being three independent relations connecting them.

If we assume another relation between the quantities B, H, G, H', G' we would have left only two independent variables for each value of P and could chart on a two dimensional graph or diagram the remaining possibilities for that value of P. For the additional relation to be assumed we could take any equation involving C, B, H, G, H' and G': the simplest type of such equation would be to put the value of one of the quantities equal to a constant. Then we could put $G = 1$ say and make a series of charts for a selected set of values of P: then if we wanted we could do the same for $G = 1.2$, $G = 1.4$, etc.

In this way as we shall see we can get considerable insight into the range of possibilities of retrospective plans. By making different assumptions, e.g., first $G = 1$, $G = 1.2$, etc., and then say $H = .25$, $H = .5$, and so on, we can study successively what we will call different "aspects" of possible plans and see what sort of results follow.

Let us suppose then we first take the "aspect" $G = 1$ (perhaps intending later to make charts for other values of G or because we are, or think we will be, interested only in plans with $G = 1$).

Now, for a given value of P, we are left with C, B, H, H' and G' and three independent relationships of these and can graph the possibilities in terms of any two of these quantities. Suppose we decide to use G' and H' (which choice has the advantage of showing the effective range in terms of loss experience) then each separate pair of values for G' and H' gives definite values for C, B and H. We can therefore make a graph where the $x\,y$ coordinates are G' and H' and show on it the loci of C for different values (say .5, 1 and Γ) and the loci of H for say .25, .5, etc., and the loci of B for say 0, .25, etc.

Before proceeding it is necessary to remember that while we can in general use any two of our "variables" C, B, H, G, H' and G' as our independent variables for our charts, we can use any particular pair only if the additional relationship assumed does not (i) fix one of the pair—for example obviously if we assume $G = 1$ we cannot use G as a coordinate of the graphs, or (ii) give us a definite equation connecting the pair, e.g., if the relationship were $H + G =$ constant we could not use H and G (but we could use say H and B). In other words each of the two coordinates must be a variable and the two must be mutually independent. Usually it will be quite clear which quantities cannot be used as coordinates but occasionally the "aspect" being examined will involve an implied equation between two of the quantities: thus if $H' = 0$, it is obvious that H' cannot be used; but in addition the $H' = 0$ leads to $H = B$: again, if L is a given loss ratio the "aspect" '$L =$ constant leads to $B + LC =$ constant and therefore the pair B and C cannot be used in this instance.

Of the quantities C, B, H, G, H' and G' the pairs G and H, G' and H', and B and C seem to be logical combinations. I will usually choose G' and H' for the reason stated above: however, in investigating relationships such as $H' = 0$ (equivalent to $B = H$, the "no specified minimum" plan) this pair cannot of course be used: I find B and C suitable for this case.

Before going on let us review the proposed procedure: we started with *seven* variable quantities, namely the standard premium P and the six ratios C, B, H, G, H' and G', with three independent equations connecting them: this is equivalent to four independent variables in terms of which the other three can be expressed (this assumes that Rv is a determinate function of P, and $H'p$, $H'o$ are determinate functions of P, H', and $G'p$, $G'o$ of P, G'): we reduce the four independent variables to two by (i) using a selected series of values of P and making our calculations separately for each such value and (ii) by assuming another relationship between the other six quantities, giving a particular "aspect." We can then draw for each value of P a graph of the "aspect" using as coordinates any pair of remaining independent varying quantities, indicating on it the graph for the other varying quantities.

We have so far assumed that we have a complete list of possible combinations; that is to say that for a given P we can determine by inspection the value of all of C, B, H, G, H' and G' from the values of any three of them. Theoretically we have such a list, since from our equations (1)-(5) we can calculate the other three from the values of any three (see Appendix II), but as a practical matter of course we don't have such a list. What we are going to do now is to construct a restricted list that we can use for rapidly examining different "aspects." This will prove much more satisfactory than having to stop and calculate values for each different aspect we want to look at.

The first thing is to consider and restrict the field, that is the arithmetical range of the variables.

Consider the possibilities for a particular fixed value of P. Of C, B, H, G, H' and G' any three can be taken as the independent variables: let us take C, H' and G' and consider these as coordinates in ordinary three dimensional space. Any set of values for C, H' and G' give a point in space, and any point in space gives a set of values. All possible points form a solid some of the boundaries of which reach to infinity. We want to confine our "points" to a region or volume of space that will give reasonable and practical values. First H', G' must both be positive and G' cannot be less than H'. Also C must be positive and less than Γ (in accordance with our discussion above). In addition we should put some upper limits on H' and G' since values of say 2 or 3 for such quantities are scarcely practical. For the purpose of this paper I took $H' = .60$ and $G' = 1.20$ as upper limits. These limits confine the "points" to a finite volume: but some of the points in it may have values of G between Rv and 1 and some of the points may have values of B less than zero: now while it is not impossible to have practical plans with $B < 0$ or $G < 1$, such plans would look rather bizarre and accordingly I find it desirable to make the additional limitations that $B \geqslant 0$ and $G \geqslant 1$ (these conditions have a further advantage: they establish a lower limit for C). Accordingly the limits are:

$$H' \geqslant 0 \text{ and } \leqslant .60 \qquad G' \leqslant 1.20 \qquad C \leqslant \Gamma$$
$$B \geqslant 0 \qquad G \geqslant 1$$

and our points are confined to a finite volume—a polyhedron with faces (not necessarily flat) expressed by the limits. In passing it may be noted that these limits prevent G from being too large and it is not necessary to adopt a formal upper limit for G.

The next step is to find the "vertex" points of this polyhedron: these are the points where three of the limit conditions hold. The simplest way is to try all the possible combinations of these three at a time (there are sixteen such) and rule out those which produce answers violating another of the limitations. (We could determine the possible cases from theoretical considerations but it is quicker to try the sixteen possible cases.)

We will illustrate this process by giving it for a particular case, the one on which are based all the examples and graphs given in this paper. This is, briefly, a compensation retrospective plan for a "40%" state, with the expense gradation underlying the *A*, *B* and *C* retrospective plans introduced in 1943. Values are given for $P = 5{,}000$, $25{,}000$ and $100{,}000$: these three premium sizes give a comprehensive view of the range of retrospectively rated risks.

Details of the expense gradation, contingency loading and excess pure premium table used are given in Appendix III and details of the working out of the tables are given in Appendix IV with the complete tables.

It is found, in this instance, that for each of the three values of *P*, there are six vertex points, namely the intersections of the "plane" $H' = 0$ and the "plane" $H' = .6$ with the "line" ($C = \ulcorner$, $G = 1$) the "line" ($C = \ulcorner$, $G' = 1.2$) and the "line" ($G = 1$, $G' = 1.2$), namely

Table of Vertex Points

C	B	H	G	H′	G′
		$P = 5{,}000$			
1.162**	.643	.643	1.**	.0**	.307
1.162**	.198	.895	1.**	.6**	.690
1.162**	.100	.797	1.494	.6**	1.2**
1.162**	.366	.366	1.760	.0**	1.2**
.105	.875	.875	1.**	.0**	1.2**
.153	.821	.910	1.**	.6**	1.2**
		$P = 25{,}000$			
1.162**	.355	.355	1.**	.0**	.555
1.162**	.102	.799	1.**	.6**	.773
1.162**	.031	.728	1.425	.6**	1.2**
1.162**	.194	.194	1.589	.0**	1.2**
.214	.743	.743	1.**	.0**	1.2**
.277	.676	.838	1.**	.6**	1.2**
		$P = 100{,}000$			
1.162**	.217	.217	1.**	.0**	.674
1.162**	.099	.796	1.**	.6**	.776
1.162**	.069	.766	1.464	.6**	1.2**
1.162**	.160	.160	1.555	.0**	1.2**
.239	.712	.712	1.**	.0**	1.2**
.276	.678	.839	1.**	.6**	1.2**

where the double asterisk denotes one of the limiting conditions. Note that the limit $B = 0$ does not come into play. 1.162 is of course the value of $\ulcorner$.

Looking at these points we see that the minimum and maximum values of the quantities are

	$P = 5{,}000$		$P = 25{,}000$		$P = 100{,}000$	
	Min.	*Max.*	*Min.*	*Max.*	*Min.*	*Max.*
C	.105	1.162	.214	1.162	.239	1.162
B	.100	.875	.031	.743	.069	.712
H	.366	.910	.194	.838	.160	.839
G	1.000	1.760	1.000	1.589	1.000	1.555
H'	.0	.6	.0	.6	.0	.6
G'	.307	1.2	.555	1.2	.674	1.2

and we note that the following "selected values" will cover the range of values:

C	.167,	.333,	.500,	.667,	.833,	1.000,	1.162
B	.1,	.303,	.5,	.7			
H	.303,	.5,	.7,	.9			
G	1.0,	1.2,	1.4,	1.6			
H'	.0,	.2,	.4,	.6			
G'	.4,	.6,	.8,	1.0,	1.2		

(*Note*: The value .303 was taken for B and H instead of .3 because $B = .303$ satisfies the condition $'E = 1$ for $C = \Gamma$: in other words for $C = \Gamma$, $B = .303$ gives points for which the final retrospective premium is the standard when the actual loss ratio equals the expected.)

So what we do is to make a table of all possible points for which the values of C, B, etc., are such that (a) all are within our limitations and (b) three of them are selected values. The details of how we do this and complete tables for the three values of P are given in Appendix IV.

Now we have our restricted list and can use it to examine very quickly various "aspects" of the possibilities. Thus we can take $C = 1.162$ or $C = 1$, etc.

or $B =$ .1 or .303, etc.
or $H =$ .303 or .50, etc.
or $G = 1$ or 1.20 or 1.40, etc.

and pick out the values belonging to this "aspect." These values can be graphed, separately, for each value of P, using any independent pair of the other variables as plotting coordinates. As mentioned above, I usually use H' and G' when available. On the diagrams we can then show the loci for the various selected values of the remaining three quantities and other information of interest. The examples given below will make this clear.

If we want to study some "aspect" which is not given by our table (e.g. $'E = 1$) we will have to make some additional calculations (see Appendix IV) but often it will be sufficient to interpolate in the main table.

It was my intention to give diagrams for the "aspects"

(i) $C = \ulcorner$ (the usual retrospective plan)
(ii) $G = 1$ (the "no penalty" plan)
(iii) $B = .303$ (a constant basic)
(iv) $H = .5$ (a constant minimum)
(v) $H' = 0$ (the "no specified minimum" plan)
(vi) $'E = 1$ (a plan where an actual loss ratio equal to the expected produces the standard premium)

For all these, except (v), the independent or plotting coordinates taken were to be G' and H'; for (v) they were to be B and C.

It was my intention to give eighteen diagrams (each of the above six assumptions for each of the three values of P) showing the lines for various values of C, B, G, H, etc., as applicable; also to show the loci for $'E = 1$ and $'E = Rv$ and the areas where $'E < Rv$, $'E > Rv$ and < 1, and $'E > 1$ (i.e. the areas where the expected loss ratio produce a final retrospective premium less than Rv, between Rv and 1, and greater than 1): but owing to limitations of time (my time to draw the diagrams) and cost (the cost of reproducing the diagrams) I am giving the diagrams only for the first "aspect," that is $C = \ulcorner$ (the present "standard" plan).

These diagrams are accordingly given: they are mostly self-explanatory so that only a brief description of them, as follows, is required:

For the assumption or aspect $C = \ulcorner = 1.162$ it is clear (from the list of "vertex" points) that the points all fall in the area bounded by $H' = 0$, $H' = .6$, $G' = 1.2$ and $G = 1$. Plotted as functions of H' and G' three of these boundary lines are straight lines and the fourth ($G = 1$) is curved. The boundaries meet at four intersections. Taking as an example the diagram for $P = 25{,}000$, these points are $H', G' = (0, .555)$ $(0, 1.2)$ $(.6, 1.2)$ and $(.6, .773)$. The minimum and maximum values of B are .031 and .355 and the lines for $B = .1$, $B = .303$ are shown. The minimum and maximum values of H are .194 and .799 and the lines for $H = .303$, $H = .5$ and $H = .7$ are shown. The minimum and maximum values of G are 1 and 1.589 and the lines for $G = 1$, $G = 1.20$ and $G = 1.40$ are shown. All the values for these come from the table. The lines $B = .1$, $B = .303$ are the same as $'E = .797$ and $'E = 1$ respectively, since $C = 1.162$. In addition there is shown the locus for $'E = Rv$: additional values were calculated for this, which is equivalent (for $C = 1.162$) to $B = .170$. The areas are shaded to show where $'E$ is less than Rv, where $'E$ is between Rv and unity and where $'E$ is greater than unity. The diagrams for $P = 5{,}000$ and $P = 100{,}000$ were similarly constructed. (On the $P = 100{,}000$ diagram there is no locus for $'E = 1$ because in this instance $'E$ is always less than one.)

Now just what does such a diagram indicate? Take for example that for $P = 25{,}000$. It shows the possible values (within our chosen limits) for $P = 25{,}000$, $C = 1.162$, $Rv = .8668$. Any point on the diagram (not outside the area) signifies as follows: take for instance $H' = .262$, $G' = 1.036$. For this $G = 1.4$, $H = .5$ and B is between .1 and .303 (actually .196 on reference to the table in the appendix, or $B = G - 1.162\,G'$ gives the value): $'E$ is between Rv or .867 and 1 (actually $B + 1.162\,E$ gives .893). This means the set of values $C = 1.162$, $B = .196$, $H = .5$, $G = 1.4$, $H' = .262$, $G' = 1.036$ satisfies the basic equations or in other words, with our assumptions as to excess pure premium tables and gradation of expenses and contingency loading, $C = 1.162$, $B = .176$, $H = .5$, $G = 1.4$ are possible retrospective rating values and with these the minimum is reached at a loss ratio (H') of .262 and the maximum at (G') 1.036: an actual loss ratio equal to the expected of .6 gives a retrospective premium of .893.

The diagram, however, has more value than this: it gives us a birdseye view of the possibilities under the "aspect" $C = 1.162$. If, for instance, we want G to be less than 1.4, we are confined to the area below the line $G = 1.4$. If we want $'E$ to be less than Rv we must take values from the right-hand shaded part of the area. It also shows at a glance impossible combinations: thus, if we want $H = .5$ or less, we cannot have $'E$ less than Rv.

Of course to fix definite values (for $P = 25{,}000$) for use in a plan we need two more conditions, e.g. $G = 1.2$, $H = .7$ gives the point $H' = .476$, $G' = .906$, and $H' = 0$, $G = 1.4$ gives the point $H' = 0$, $G' = 1.021$.

By studying these diagrams for the various values of P and others based on other "aspects" we can see what kinds of retrospective plans we can construct. (Regarding the actual construction of plans see the remarks made later on.) Similar charts for various other aspects can readily be made from the given tables: I regret I could not give more of them here: the procedure for drawing them is the same as for those for $C = \lceil\!\!\!\!-$. Thus the first step for a diagram for $P = 25{,}000$ showing the possibilities for $H' = .5$ is to pick out the boundaries and their intersections. The table at once shows the boundaries to be

$$H' = 0,\ G' = 1.2,\ C = 1.162,\ \text{and}\ G = 1$$

and the intersections to be

C	B	H	G	H'	G'
1.162**	.354	.5*	1**	.125	.556
1.162**	.175	.5*	1.570	.280	1.2**
.760	.5*	.5*	1**	.0**	.660
.634	.5*	.5*	1.261	.0**	1.2**

The diagram is to be completed as before.

The analysis of possible combinations is of course only the first part of the complete work of analyzing retrospective rating and constructing retrospective rating plans. The second part is the actual construction of plans and necessitates still further selection or, using the language employed above, the making of still more assumptions, i.e. the imposing of further restrictions. Thus we may decide we are interested in plans with $C = \Gamma$ (as we seem to be at present) and want to construct plans (with $C = \Gamma$) with the further condition $G = 1$ ("no penalty"). This reduces our independent variables (including P) to two, namely P, B, H, G' and H', with three relations say (6), (7) and (10), in which $C = 1.162$ and $G = 1$. We can thus construct a (single) diagram giving all the possibilities for all values of P. We can use as plotting coordinates any two *independent* values (not e.g. B and G' for $B + CG' = 1$). It is logical to use P as one and as the other either B or H or G' or H' according to the features of the plan in which we are most interested. It was my intention to give some examples of this second part of the problem but this will have to be postponed to a later paper: there are many practical points to be considered including the problem of keeping the values in proper relation for compensation insurance in different states and possibly as between different lines of insurance. Thus, therefore, considerations of time and space make it impossible to tackle this task now. In the meanwhile, however, I wanted to put before the other members of our profession the results I had obtained and which I have explained in this paper.

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APPENDIX I

Solution of the Basic Equations

In order to make the arithmetical calculations we must be in a position to determine all the variables C, B, H, G, G' and H' given either any three of them or any three independent relations between them. The first is the most usual problem that arises and can be systematically solved as follows (it is assumed that Rv is known and that we have tables of Lp and Lo).

We have the five equations

$$H = B + C\,H' \qquad (1)$$
$$G = B + C\,G' \qquad (2)$$
$$B = Rv - C\,(H'o - G'p) \qquad (3)$$
$$H = Rv - C\,(H'p - G'p) \qquad (4)$$
$$G = Rv + C\,(G'o - H'o) \qquad (5)$$

I. Given C and any two others:

(i) Given C and B, G

From equation (2) we find G' and then from (3) we find H' and then from (1) we find H.

(ii) Given C and B, H

From (1) we get H' and from (3) we get G' and from (2) we get G.

(iii) Given C and H, G

$G' - H' = (G - H)/C$: then by trial we find values of G', H' differing by this amount and satisfying (5).
Then B comes from (1).

(iv) Given C and B, G'

From (3) we get H' and from (1) and (2) we get H, G.

(v) Given C and B, H'

From (3) we get G' and from (1) and (2) we get H, G.

(vi) Given C and G, G'

From (5) we get H' and from (2) we get B, and then from (1) we get H.

(vii) Given C and H, H'

From (4) we get G' and from (1) we get B, and then from (2) we get G.

(viii) Given C and G, H'

From (5) we get G' and then B from (2) and then H from (1).

(ix) Given C and H, G'

From (4) we get H' and then B from (1) and then G from (2).

(x) Given C and G', H'

(3) gives us B and (1) and (2) give H and G.

II. If C is not one of the given quantities:

(xi) Given B, G, H

Trial values of C give values of $G' = (G - B)/C$ and $H' = (H - B)/C$ to satisfy (4).

(xii) Given B, G, G'

We get C from (2) and then H' and H as in (i).

(xiii) Given B, H, H'

We get C from (1) and then G' and G as in (ii).

(xiv) Given B, G, H'

Eliminate C from (2) and (5) and find G' by trial and then C (or trial values of C give G' from (2) to satisfy (5)). Then H from (1).

(xv) Given B, H, G'

Eliminate C from (1) and (4) and find H' by trial and then C (or trial values of C give H' from (1) to satisfy (4)). Then G from (2).

(xvi) Given H, G, G'

Eliminate C from (4) and (5) and find H' by trial and then C (or trial values of C give H' from (4) to satisfy (5)). Then B from (1) or (2).

(xvii) Given H, G, H'

Eliminate C from (4) and (5) and find G' by trial and then C (or trial values of C give G' from (4) to satisfy (5)). Then B from (1) or (2).

(xviii) Given B, G', H'

Get C from (3) and then H, G from (1) and (2).

(xix) Given H, G', H'

Get C from (4) and then B, G from (1) and (2).

(xx) Given G, G', H'

Get C from (5) and then B, H from (1) and (2).

If we are given some relation or relations between the quantities we must solve the equations in the easiest manner. For instance, if we are given that $'E$ is equal to a given value X (which is equivalent to $X = B + C\,E$) and are given in addition

(a) the value of C (or B) we get B (or C) immediately from $X = B + C\,E$ and proceed as above.

(b) the values of G', H', or $G\,G'$, or $H\,H'$, we can get C immediately from $X - C\,E = Rv - C\,(H'o - G'p)$ or $G - C\,G' = X - C\,E$, or $H - C\,H' = X - C\,E$, respectively: and then proceed as above.

(c) the values of G, H', we use trial values of C to get G' from $G = Rv + C\,(G'o - H'o)$ to satisfy $X - C\,E = Rv - C\,(H'o - G'p)$ and so on: similarly, if given H, G'.

(d) the values of G, H, we use trial values of B which give values of C and thus values of G' from $C\,G' = G - B$ and H' from $C\,H' = H - B$ to satisfy $G - Rv = C\,(G'o - H'o)$.

APPENDIX II

Conversion of Excess Pure Premium Table from One Expected Loss Ratio to Another

If we have a table of "excess pure premium ratios" showing some or all of Lx*, Lp, Lo for loss ratios L and such table was constructed on the assumption of an expected loss ratio of E, then if we wish to use the table for a different expected loss ratio say $\overline{E}$ we proceed as follows:

(i) we make the assumption that the excess pure premium ratio for a loss ratio L, if the expected loss ratio is $\overline{E}$, is obtained by entering the E table with a loss ratio of L multiplied by $E/\overline{E}$, i.e.

$$Lx = \text{the tabular value of } \left(\frac{LE}{\overline{E}}\right) x$$

(ii) Put $\frac{E}{\overline{E}} = k$ and put $\overline{L} = k\,L$ and write "tab. val." for "the tabular value of"

Then $Lx =$ tab. val. $\overline{L}x$

$Lp = \overline{E}\,Lx =$ (tab. val. $\overline{L}p)/k$

$Lo = L + Lp =$ (tab. val. $\overline{L}o)/k$

(iii) Now if we put $\overline{C} = \frac{C}{k}$ and $\overline{H}' = k\,H'$, $\overline{G}' = k\,G'$ our working equations become

$$H = B + \overline{C}\,\overline{H}' \qquad G = B + \overline{C}\,\overline{G}'$$
$$B = Rv - \overline{C}\,((\text{tab. val. } \overline{H}'o) - (\text{tab. val. } \overline{G}'p))$$
$$H = Rv - \overline{C}\,((\text{tab. val. } \overline{H}'p) - (\text{tab. val. } \overline{G}'p))$$
$$G = Rv + \overline{C}\,((\text{tab. val. } \overline{G}'o) - (\text{tab. val. } \overline{H}'o))$$

so that all we have to do is

(iv) Convert C to $\overline{C}$ by *dividing* by k. Convert H', G', and all loss ratios to $\overline{H}'$, $\overline{G}'$, etc., by *multiplying* by k. Work out whatever problem we are concerned with in terms of B, G, H, $\overline{G}'$, $\overline{H}'$ and $\overline{C}$ and then reconvert $\overline{C}$ to C by *multiplying* by k and reconvert $\overline{H}'$, $\overline{G}'$ and all loss ratios to H', G', etc. by *dividing* by k.

(v) The conversion and reconversion of C, H', G', etc., can be done by inspection by a simple "conversion table" if the difference between E and $\overline{E}$ is small, as it is usually. For example, I used this method in my calculations and had to convert from a loss ratio of .598 (the expected loss ratio in the excess pure premium table) to .600 (the expected loss ratio assumed in my examples). I worked to three decimals both in loss ratios and credibilities (loss conversion factors), e.g. $H' = .398$, $C = 1.165$ and accordingly the difference be-

* We do not need Lx for calculations according to the methods of this paper.

tween .600 and .598 was 2 "points". So I constructed a table which gives the ranges of values for which the difference between X and kX is less than ½ point, greater than ½ point but less than 1½ points and so on; and then for the first range we take the difference as 0 points, for the second as 1 point, and so on.

The complete table (up to the largest value, 1.2, I need) follows:

Loss Ratio ($E = .600$) *LCF* ($E = .598$)		Loss Ratio ($E = .598$) *LCF* ($E = .600$)
.000		.000
	$-0+$	
.150		.150
	$-1+$	
.450		.449
	$-2+$	
.750		.748
	$-3+$	
1.050		1.047
	$-4+$	
1.350		1.346

(Note: In critical cases descend.)

This is used as follows: (i) Given $H' = .398$, what is $\overline{H}'$? In the left-hand column headed "Loss Ratio ($E = .600$)" .398 is between .150 and .450 and we find the direction -1 meaning to subtract 1 point. So $\overline{H}' = .397$. (ii) Given $\overline{G}' = 1.047$, what is G'? In the right-hand column we find 1.047 and bearing in mind the note "In critical cases descend" we take the direction $+4$ and so $G' = 1.051$. (iii) If $C = 1.162$, what is $\overline{C}$? In the *right*-hand column (headed "*LCF* ($E = .600$)" we find the direction $+4$ so $\overline{C} = 1.166$.

In making the calculations for the examples in Appendix IV we have to convert from .598 to .600—that is why I gave this particular conversion table.

APPENDIX III

Description of the Arithmetical Example

All the numerical examples and calculations in this paper are based on the following underlying data, namely

(i) Compensation insurance.

(ii) The excess pure premium table (based on unlimited losses) constructed on New York experience in 1941*. This is keyed to a loss ratio of .598.

* See "Risk Distributions Underlying Insurance Charges in the Retrospective Rating Plan," by Nels M. Valerius—P.C.A.S. XXIX, p. 96. The table is given on pp. 111-117.

(iii) A so-called "40%" state with the following make-up of the standard premium dollar:

Losses	.600
Loss Expense	.080
Inspection	.025
General Administration and Payroll Audit	.095
Acquisition	.175
Taxes	.025
	1.000

(Note: I could have saved some of the arithmetical work by assuming a hypothetical state with an expected loss ratio of .598, the same as in the excess pure premium table. The reason I did not is that I made some of the calculations for a 60% expected loss ratio state a year or two ago, using the unlimited New York excess pure premium table (then the latest available) and it was easier to finish the arithmetical work on this basis rather than recalculate all the values I already had. Also, the "40% state" is usually selected for illustration purposes. Anyhow the conversion from one expected loss ratio to another is not so burdensome particularly if the difference in expected loss ratios is small. For the method used see Appendix I).

(iv) There is an expense gradation equal to savings in acquisition of 5 points in acquisition from 1000 to 5000 of standard premium and of 10 points from 5,000 to 100000, and savings in General Administration and Payroll Audit expense of 5.4 points on all standard premium over 1000. This is the expense gradation underlying the new retrospective plans introduced in 1943.

(v) A flat contingency loading of .01 is included. My examples will thus be for a "40% state" with the expense gradation and contingency loading underlying the new retrospective plans introduced in 1943.

Thus the net reduction on account of the gradation and contingency loading, expressed as usual as a ratio of the standard premium P, is for $5000 \leqslant P \leqslant 100000$ (the range covered by my examples)

$$\frac{1}{.975}\left(\frac{.154\,(P-5000)+416}{P} - .01\right)$$

and this equals $1 - Rv$.

Accordingly $Rv = .8523 + \dfrac{363}{P}$.

We will be interested only in the values of Rv for $P = 5000$, 25000 and 100000, for which $Rv = .9249$, .8668 and .8559 respectively.

The value of Γ is $\dfrac{.600+.080}{.600}\,\dfrac{1}{.975} = 1.162.$

Note 1: In this paper I have used throughout the older terminology and notation wherein the tax loading is incorporated in all the factors: the new *A, B, C* Plans, however, (and the Comprehensive Rating Plan for War Projects) have a separate "tax multiplier". Care must be taken in all cases to see just what the effect of this is on the values of the retrospective rating factors and on the notation.

The difference between the terminology used in this paper and in the *A, B, C* Plans is that if the tax multiplier in the *A, B, C* Plans is called $1 + T$, then in the notation of this paper

(a) the basic premium ratio in the *A, B, C* Plans is $\frac{B}{1+T}$

(b) the loss conversion factor in the *A, B, C* Plans is $\frac{C}{1+T}$

so that to apply the formulas of this paper the basic premium ratio and the loss conversion factors of the *A, B, C* Plans must be multiplied by the tax multiplier.

Note 2: Contingency loadings can be incorporated in several ways: (a) flat, say 1% of the standard premium, i.e. $= .01$ (remember this means $.01\,P$); or (b) a percentage say 1½% of the retrospectively variable part of the formula, i.e. $.015\,(R - H)$; or (c) a percentage say 2½% of C, i.e. $.025\,C$ ($= .025\,CP$): and so on, including of course a combination of one or more of these methods.

Thus if for a certain standard premium size P there is to be allowed a savings of 10% (of P) on account of the gradation of the expense loading the retrospective formulas will be as follows according to the three examples given above of contingency loadings:—

(a) flat contingency loading of 1%

$$Rv = 1 - .10 + .01 = .91$$
$$.91 = H + C\,(Hp - Gp)$$

(b) contingency loading of 1½% of the retrospectively variable part of the final premium.

$$Rv = 1 = .10 + .015\,(Rv - H)$$
$$Rv = \frac{.9 - .015\,H}{.985} = H + C\,(Hp - Gp)$$
$$\text{or } .9 = H + .985\,C\,(Hp - Gp)$$

(c) contingency loading of 2½% of C

$$Rv = 1 - .10 + .025\,C$$
$$.9 = H + C\,(Hp - Gp - .025)$$

The example given in this Appendix has a contingency loading of type (a), i.e. flat.

APPENDIX IV

Calculation of Table Points

To calculate the table of values for possible combinations of select values of the quantities C, B, H, G, H', G' for a given value of P, I proceeded as follows:

(i) I constructed a table of Lp, Lo for each value of P (we do not need Lx). This shows Lp and Lo for each 1% of loss ratio from 0 to 120%. I found it convenient to work with C, B, H', G' to three decimals (e.g. $C = .895$, $G = 1.181$, $H' = .592$) and found that in these circumstances it was advisable to have Lp and Lo to four decimals, so that proper interpolations could be made. Excerpts from the table for $P = 25000$ are given.

Standard Premium 25,000

L	Lp	Lo	L	Lp	Lo	L	Lp	Lo
.00	.5980	.5980	.40	.2422	.6422	.80	.0748	.8748
.01	.5878	.5978	.41	.2356	.6456	.81	.0724	.8824
. .	. .	. .	. .	. .	. .	. .	. .	. .
.39	.2494	.6394	.79	.0771	.8671	1.19	.0215	1.2115
.40	.2422	.6422	.80	.0748	.8748	1.20	.0209	1.2209

(ii) It must be remembered that we are working with an expected loss ratio of 60%, whereas the table was keyed to a 59.8% loss ratio. It is accordingly necessary to convert all loss ratios and values of C in accordance with the conversion table in Appendix II and then reconvert at the end when the calculations have been made. We want to construct tables for $P = 5000$, 25000 and 100000 for which $Rv = .9249$, .8668 and .8559 respectively. What we want to do is to find all possible combinations of selected values of any three of C, B, H, G, H' and G' and the values of the other three quantities for these possible combinations, all subject to the limiting conditions that

$$C \geqslant 1.162,\ B \geqslant 0,\ G \geqslant 1,\ H' \geqslant 0 \text{ and } \leqslant .60,\ G' \leqslant 1.20$$

Remember that given values of any 3 of the 6 quantities we can calculate the values of the other three, but of course the calculated values may be impossible or may be outside the limits we have set. In fact, our problem is solely to determine possible combinations. To put it another way, we want to find all possible "points" (C, B, H, G, H', G') in which *three* of the quantities each has one of the selected values given in the paper and repeated below and in which *all* of the quantities are within the limits just set forth, e.g. the point ($C = 1.162$, $B = .20$, $H = .60$, $G = 1.217$, $H' = .581$, $G' = .717$) if it exists—that is, if the values satisfy our fundamental equations, is a value we are trying to find because in it we have $C = 1.162$, $B = .20$, $H = .60$, which are selected values, and the other three quantities, G, H' and G', are within the limits set: on the other hand the "point" ($C = 1$, $B = 0$, $H = .828$, $G = 1$, $H' = .828$, $G' = 1.021$) even if it "exists" is not a point we are looking for because H' is greater than the limit .60: also the "point" ($C = 1$, $B = 0$, $H' = .528$, $G = 1.010$, $H' = .528$, $G' = 1.010$) even if it "exists" is not a point we are interested in finding for there are not in it three quantities with selected values.

(iii) The first thing to do is to find the most extreme cases (which I call "vertex" points). These are those "points" for which hold three of the limiting equations

$$C = 1.162,\ B = 0,\ G = 1,\ H' = 0,\ H' = .6,\ G' = 1.2$$

The simplest way is to try all possible combinations of these three at a time, ruling out those which produce answers violating another of the limitations (or we can determine the possible cases by a theoretical investigation but it is simpler to try the sixteen possible cases). We find that with the data we are using the following six "points" exist for all three of our values of P:

C	B	H	G	H'	G'
1.162			1	0	
1.162			1	.6	
1.162				.6	1.2
1.162				0	1.2
			1	0	1.2
			1	.6	1.2

The limiting condition $B = 0$ does not come into play, B being positive for all points otherwise possible. We then finally choose the selected values as explained in the paper where it will be recalled we determined on the following (including the limiting values)

C	.167	.333	.500	.667	.833	1.000	1.162
B	.1		.303		.5	.7	
H	.303		.5		.7	.9	
G	1.0		1.2		1.4	1.6	
†H'	0		.2		.4	.6	
†G'	.4		.6		.8	1.0	1.2

† Note, however, as explained below, that in the tables printed we have omitted the points determined by the non-limiting values of H' and G'.

(iv) The rest of the work is purely arithmetical: we will give details for $P = 25000$.

We set out the table of six vertex points in full, putting a double asterisk against the three limiting values in each set:

$P = 25000$ — Vertex Points

C	B	H	G	H'	G'
1.162**	.355	.355	1**	0**	.555
1.162**	.102	.799	1**	.6**	.773
1.162**	.031	.728	1.425	.6**	1.2**
1.162**	.194	.194	1.589	0**	1.2**
.214	.743	.743	1**	0**	1.2**
.277	.676	.838	1**	.6**	1.2**

We now set out the table of "edge points" namely those combinations for which two of the limiting conditions hold. We do this by taking all combinations of two limiting values and each of these pairs we find in either two or more of the sets of vertex points: e.g. for $C = 1.162$, $G = 1$ we find this combination in the first two vertex points above: going from one of these points to the other we see that B varies from .102 to .355 (passing through the selected value .303) H varies from .355 to .799 (through .5 and .7) H' varies from 0 to .6 (through .2 and .4) and G' varies from .555 to .773 (through .6). So we get the edge points

$C = 1.162$** $\quad G = 1$** $\quad B = .303$

$C = 1.162$** $\quad G = 1$** $\quad H = .5$

etc.

We set these out in the table of edge points and for each point calculate the values of the other quantities: we mark the "limiting" values (e.g. $C = 1.162$, $B = 1$) with a double asterisk as before, and the other selected value (e.g. $B = .303$) with a single asterisk. We do this for all the possible pairs of limit values. The table of edge points follows:

$P = 25000$ — Edge Points

C	B	H	G	H'	G'
1.162**	.303*	.686	1**	.330	.6*
1.162**	.354	.5*	1**	.125	.556
1.162**	.293	.7*	1**	.350	.608
1.162**	.342	.574	1**	.2*	.568
1.162**	.268	.733	1**	.4*	.630
1.162**	.303*	.303*	1.090	0**	.677

etc. (See table at end of Appendix)

We now examine the table of edge points to determine the "face points", namely those for which *one* of the limiting conditions hold. We do this in a manner similar to that by which we got the edge points from the vertex points: we take all possible combinations of one limiting value (with a double asterisk) and a non-limiting value (with one asterisk); if such a combination occurs in the edge point table it occurs twice. Thus the combination $C = 1.166$**, $B = .1$* occurs twice—once with $H = .797$, $G = 1.006$, $H' = .6$**, $G' = .780$, and again with $H = .674$, $G = 1.495$, $H' = .494$, $G' = 1.2$** and so gives face points with $H = .7$* and $G = 1.2$* and $G = 1.4$* and $G' = .8$* and $G' = 1.0$*. Note that each "face point" will be found twice by this method and should not be duplicated in our table: thus $C = 1.162$**, $B = .1$*, $H = .7$* arises not only as above but also from the two occurrences of $C = 1.162$**, $H = .7$* in the edge points. We thus construct the table of "face points"

$P = 25000$ — Face Points

C	B	H	G	H'	G'
1.162**	.1*	.7*	1.132	.517	1.061
1.162**	.1*	.728	1.2*	.541	.946
1.162**	.1*	.689	1.4*	.507	1.119
1.162**	.1*	.787	1.029	.561	.8*

etc.

Now for the final step: from the table of face points we get, in a similar manner, the "interior points", namely those for which none of the limiting conditions hold. We take all combinations of two non-limiting values (i.e. with one asterisk): if one such occurs once in the table of face points it appears twice and from the two occurrences we determine interior points. We thus construct the following table of interior points, taking care to put each point in only once (they will occur three times):

$P = 25000$ — Interior Points

C	B	H	G	H'	G'
1.*	.303*	.5*	1.285	.197	.982
1.*	.303*	.7*	1.087	.398	.785
1.*	.303*	.598	1.2*	.295	.896

etc.

The tables for $P = 5000$, $P = 25000$, $P = 100000$, given at the end of the Appendix, give the values of all the points found as above except that (to save space) I have omitted those where one, or two, of the selected values fixing the point are $H' = .2$ or .4, or $G' = .4$, .6, .8 or 1.0. This cuts almost in half the number of points to be tabulated and yet does not take away a great deal from the usefulness of the tables, since in most cases H' and G' will be used as plotting coordinates.

If we need additional values, such as say for $'E = \text{a constant}$, we can either calculate the additional values by combining the new condition with the limiting values and so on or can often get close enough values by interpolating in the tables already calculated.

We give the values for $C = \ulcorner$, $'E = Rv$. Note that for this $B = Rv - \ulcorner E$, a fixed value: also $H'o - G'p = E$.

Table for $C = \ulcorner$ $\quad 'E = Rv$

$P = 5000$

C	B	H	G	H'	G'
1.162**	.228	.894	1**	.573	.664
1.162**	.228	.712	1.623	.416	1.2**
1.162**	.228	.820	1.2*	.510	.837
1.162**	.228	.762	1.4*	.460	1.008
1.162**	.228	.717	1.6*	.420	1.181

$P = 25000$

C	B	H	G	H'	G'
1.162**	.170	.786	1.**	.530	.715
1.162**	.170	.524	1.565	.305	1.2**
1.162**	.170	.7*	1.159	.457	.851
1.162**	.170	.680	1.2*	.438	.886
1.162**	.170	.586	1.4*	.358	1.059

$P = 100000$

C	B	H	G	H'	G'
1.162**	.159	.748	1.**	.507	.724
1.162**	.159	.389	1.554	.198	1.200**
1.162**	.159	.5*	1.068	.468	.783
1.162**	.159	.7*	1.381	.293	1.052
1.162**	.159	.623	1.2*	.399	.896
1.162**	.159	.466	1.4*	.264	1.068

TABLE OF POINTS

$P = 5{,}000$

$Rv = .9249$

C	B	H	G	H'	G'
		VERTEX POINTS			
1.162**	.643	.643	1.**	.0**	.307
1.162**	.198	.895	1.**	.6**	.690
1.162**	.100	.797	1.494	.6**	1.2**
1.162**	.366	.366	1.760	.0**	1.2**
.105	.875	.875	1.**	.0**	1.2**
.153	.821	.910	1.**	.6**	1.2**
		EDGE POINTS			
1.162**	.303*	.882	1.**	.499	.600
1.162**	.5*	.845	1.**	.297	.430
1.162**	.640	.7*	1.**	.052	.310
1.162**	.5*	.5*	1.172	.0**	.578
1.162**	.488	.488	1.2*	.0**	.613
1.162**	.423	.423	1.4*	.0**	.841
1.162**	.386	.386	1.6*	.0**	1.044
1.162**	.146	.843	1.2*	.6**	.907
1.162**	.113	.810	1.4*	.6**	1.108
1.162**	.1*	.797	1.494	.6**	1.200
1.162**	.1*	.797	1.494	.600	1.2**
1.162**	.303*	.627	1.698	.279	1.2**
1.162**	.352	.5*	1.747	.127	1.2**
1.162**	.240	.7*	1.635	.396	1.2**
1.162**	.205	.730	1.6*	.452	1.2**
1.*	.670	.670	1.**	.0**	.330
.833*	.699	.699	1.**	.0**	.361
.667*	.728	.728	1.**	.0**	.407
.5*	.789	.789	1.**	.0**	.421
.333*	.801	.801	1.**	.0**	.598
.167*	.851	.851	1.**	.0**	.889
.828	.7*	.7*	1.**	.0**	.362
1.*	.297	.897	1.**	.6**	.703
.833*	.398	.898	1.**	.6**	.723
.667*	.498	.898	1.**	.6**	.753
.5*	.599	.899	1.**	.6**	.802
.333*	.702	.902	1.**	.6**	.894
.167*	.807	.907	1.**	.6**	1.154
.990	.303*	.897	1.**	.6**	.704
.664	.5*	.898	1.**	.6**	.754
.337	.7*	.902	1.**	.6**	.891
.483	.610	.9*	1.**	.6**	.807
.122	.854	.9*	1.**	.375	1.2**
1.*	.443	.443	1.643	.0**	1.2**
.833*	.523	.523	1.523	.0**	1.2**
.667*	.603	.603	1.403	.0**	1.2**
.5*	.683	.683	1.283	.0**	1.2**
.333*	.764	.764	1.164	.0**	1.2**
.167*	.841	.841	1.042	.0**	1.2**
.881	.5*	.5*	1.556	.0**	1.2**
.466	.7*	.7*	1.259	.0**	1.2**
.384	.740	.740	1.2*	.0**	1.2**
.662	.605	.605	1.4*	.0**	1.2**
.941	.471	.471	1.6*	.0**	1.2**
1.*	.215	.815	1.415	.6**	1.2**
.833*	.334	.834	1.334	.6**	1.2**
.667*	.452	.852	1.252	.6**	1.2**
.5*	.570	.870	1.170	.6**	1.2**
.333*	.688	.888	1.088	.6**	1.2**

C	B	H	G	H'	G'
		EDGE POINTS (*Cont'd.*)			
.167*	.806	.900	1.006	.6**	1.2**
1.162	.1*	.797	1.494	.6**	1.2**
.876	.303*	.829	1.354	.6**	1.2**
.599	.5*	.859	1.218	.6**	1.2**
.317	.7*	.890	1.080	.6**	1.2**
.226	.764	.9*	1.036	.6**	1.2**
.561	.526	.863	1.2*	.6**	1.2**
.970	.237	.816	1.4*	.6**	1.2**
		FACE POINTS			
1.162**	.303*	.7*	1.443	.341	.981
1.162**	.303*	.787	1.2*	.416	.772
1.162**	.303*	.712	1.4*	.352	.944
1.162**	.303*	.654	1.6*	.302	1.116
1.162**	.5*	.7*	1.106	.173	.522
1.162**	.487	.5*	1.2*	.011	.613
1.162**	.420	.5*	1.4*	.069	.844
1.162**	.376	.5*	1.6*	.106	1.053
1.162**	.422	.7*	1.2*	.239	.669
1.162**	.319	.7*	1.4*	.328	.930
1.162**	.250	.7*	1.6*	.387	1.162
1.*	.303*	.896	1.**	.593	.697
1.*	.5*	.857	1.**	.357	.499
1.*	.669	.7*	1.**	.031	.331
.833*	.5*	.877	1.**	.453	.600
.833*	.699	.7*	1.**	.001	.361
.667*	.5*	.897	1.**	.596	.750
.667*	.7*	.821	1.**	.182	.449
.5*	.7*	.868	1.**	.335	.600
.333*	.712	.9*	1.**	.565	.863
.167*	.827	.9*	1.**	.432	1.033
.350	.7*	.9*	1.**	.571	.856
1.*	.5*	.5*	1.307	.0**	.807
1.*	.533	.533	1.2*	.0**	.667
1.*	.479	.479	1.4*	.0**	.921
1.*	.448	.448	1.6*	.0**	1.153
.833*	.583	.583	1.2*	.0**	.740
.833*	.540	.540	1.4*	.0**	1.032
.667*	.7*	.7*	1.034	.0**	.502
.667*	.637	.637	1.2*	.0**	.845
.667*	.604	.604	1.4*	.0**	1.194
.5*	.7*	.7*	1.172	.0**	.944
.5*	.695	.695	1.2*	.0**	1.008
1.116	.5*	.5*	1.2*	.0**	.627
.942	.5*	.5*	1.4*	.0**	.955
.487	.7*	.7*	1.2*	.0**	1.025
1.*	.247	.847	1.2*	.6**	.953
1.*	.217	.817	1.4*	.6**	1.183
.833*	.352	.852	1.2*	.6**	1.017
.667*	.459	.859	1.2*	.6**	1.112
.333*	.7*	.900	1.012	.6**	.933
.333*	.700	.9*	1.012	.6**	.933
.911	.303*	.850	1.2*	.6**	.984
.602	.5*	.861	1.2*	.6**	1.163
.333	.7*	.9*	1.012	.6**	.933
1.*	.303*	.759	1.502	.457	1.2**
1.*	.441	.5*	1.640	.059	1.2**

TABLE OF POINTS

$P = 5{,}000$—(*Cont'd.*)

$Rv = .9249$

FACE POINTS (*Cont'd.*)

C	B	H	G	H'	G'
1.*	.365	.7*	1.565	.335	1.2**
1.*	.400	.647	1.6*	.247	1.2**
.833*	.5*	.667	1.501	.200	1.2**
.833*	.484	.7*	1.484	.259	1.2**
.833*	.400	.765	1.4*	.438	1.2**
.667*	.5*	.817	1.300	.476	1.2**
.667*	.587	.7*	1.387	.170	1.2**
.667*	.600	.662	1.4*	.092	1.2**
.5*	.683	.7*	1.284	.033	1.2**
.5*	.400	.654	1.2*	.508	1.2**
.333*	.7*	.883	1.100	.549	1.2**

INTERIOR POINTS

C	B	H	G	H'	G'
1.*	.303*	.832	1.2*	.529	.897
1.*	.303*	.782	1.4*	.480	1.098
1.*	.5*	.7*	1.188	.200	.688
1.*	.5*	.690	1.2*	.190	.700
1.*	.479	.5*	1.4*	.021	.921
1.*	.446	.5*	1.6*	.054	1.155
1.*	.492	.7*	1.2*	.208	.708
1.*	.414	.7*	1.4*	.286	.986
.833*	.5*	.7*	1.413	.240	1.096
.833*	.5*	.783	1.2*	.339	.840

FACE POINTS (*Cont'd.*)

C	B	H	G	H'	G'
.167*	.817	.9*	1.018	.494	1.2**
1.081	.303*	.7*	1.601	.367	1.2**
.914	.303*	.812	1.4*	.356	1.2**
1.080	.303*	.699	1.6*	.366	1.2**
.810	.5*	.7*	1.472	.247	1.2**
.750	.5*	.762	1.4*	.349	1.2**
.417	.7*	.821	1.2*	.290	1.2**
.772	.476	.5*	1.6*	.031	1.2**
.687	.575	.7*	1.4*	.182	1.2**
1.082	.301	.7*	1.6*	.368	1.2**

INTERIOR POINTS (*Cont'd.*)

C	B	H	G	H'	G'
.833*	.5*	.705	1.4*	.246	1.081
.833*	.563	.7*	1.2*	.165	.765
.833*	.503	.7*	1.4*	.238	1.078
.667*	.5*	.844	1.2*	.516	1.050
.667*	.611	.7*	1.2*	.103	.853
.5*	.695	.7*	1.2*	.009	1.008
1.082	.303*	.7*	1.2*	.368	1.199
.984	.5*	.7*	1.2*	.203	.712
.838	.5*	.7*	1.4*	.239	1.075

$P = 25{,}000$

$Rv = .8668$

VERTEX POINTS

C	B	H	G	H'	G'
1.162**	.355	.355	1.**	.0**	.555
1.162**	.102	.799	1.**	.6**	.773
1.162**	.031	.728	1.425	.6**	1.2**
1.162**	.194	.194	1.589	.0**	1.2**
.214	.743	.743	1.**	.0**	1.2**
.277	.676	.838	1.**	.6**	1.2**

EDGE POINTS

C	B	H	G	H'	G'
1.162**	.303*	.686	1.**	.330	.600
1.162**	.354	.5*	1.**	.125	.556
1.162**	.293	.7*	1.**	.350	.608
1.162**	.303*	.303*	1.090	.0**	.677
1.162**	.251	.251	1.2*	.0**	.815
1.162**	.213	.213	1.4*	.0**	1.021
1.162**	.1*	.797	1.006	.6**	.780
1.162**	.054	.751	1.2*	.6**	.986
1.162**	.032	.730	1.4*	.6**	1.177
1.162**	.1*	.674	1.495	.494	1.2**
1.162**	.193	.303*	1.588	.094	1.2**
1.162**	.175	.5*	1.570	.280	1.2**
1.162**	.069	.7*	1.464	.543	1.2**
1.*	.411	.411	1.**	.0**	.589
.833*	.473	.473	1.**	.0**	.632
.667*	.538	.538	1.**	.0**	.693
.5*	.606	.606	1.**	.0**	.788
.333*	.694	.694	1.**	.0**	.918
.760	.5*	.5*	1.**	.0**	.660
.297	.7*	.7*	1.**	.0**	1.009

EDGE POINTS (*Cont'd.*)

C	B	H	G	H'	G'
1.*	.203	.803	1.**	.6**	.798
.833*	.307	.807	1.**	.6**	.833
.667*	.411	.811	1.**	.6**	.883
.5*	.519	.819	1.**	.6**	.962
.333*	.630	.830	1.**	.6**	1.113
.841	.303*	.806	1.**	.6**	.830
.523	.5*	.814	1.**	.6**	.948
.250	.7*	.827	1.**	.508	1.2**
1.*	.288	.288	1.488	.0**	1.2**
.833*	.384	.384	1.384	.0**	1.2**
.667*	.481	.481	1.281	.0**	1.2**
.5*	.577	.577	1.178	.0**	1.2**
.333*	.675	.675	1.074	.0**	1.2**
.973	.303*	.303*	1.470	.0**	1.2**
.634	.5*	.5*	1.261	.0**	1.2**
.288	.7*	.7*	1.045	.0**	1.2**
.536	.556	.556	1.2*	.0**	1.2**
.858	.370	.370	1.4*	.0**	1.2**
1.*	.147	.747	1.347	.6**	1.2**
.833*	.267	.767	1.267	.6**	1.2**
.667*	.387	.787	1.187	.6**	1.2**
.5*	.507	.807	1.107	.6**	1.2**
.333*	.623	.823	1.023	.6**	1.2**
1.065	.1*	.739	1.379	.6**	1.2**
.783	.303*	.773	1.243	.6**	1.2**
.510	.5*	.806	1.112	.6**	1.2**
.694	.368	.784	1.2*	.6**	1.2**
1.109	.069	.734	1.4*	.6**	1.2**

TABLE OF POINTS

$P = 25{,}000$—(*Cont'd.*)

$Rv = .8668$

Face Points

C	*B*	*H*	*G*	*H'*	*G'*
1.162**	.1*	.7*	1.332	.517	1.061
1.162**	.1*	.728	1.2*	.541	.946
1.162**	.1*	.689	1.4*	.507	1.119
1.162**	.303*	.5*	1.068	.170	.658
1.162**	.241	.303*	1.2*	.043	.825
1.162**	.212	.303*	1.4*	.078	1.019
1.162**	.240	.5*	1.2*	.224	.826
1.162**	.196	.5*	1.4*	.262	1.036
1.162**	.147	.7*	1.2*	.476	.906
1.162**	.084	.7*	1.4*	.531	1.133
1.*	.303*	.771	1.**	.469	.697
1.*	.412	.5*	1.**	.088	.589
1.*	.369	.7*	1.**	.331	.631
.833*	.474	.5*	1.**	.032	.632
.833*	.447	.7*	1.**	.304	.664
.667*	.5*	.756	1.**	.383	.750
.667*	.448	.488	1.2*	.0**	1.068
.5*	.603	.7*	1.**	.195	.795
.333*	.682	.7*	1.**	.054	.956
1.142	.303*	.7*	1.**	.348	.610
.718	.5*	.7*	1.**	.252	.696
1.*	.303*	.303*	1.336	.0**	1.033
1.*	.326	.326	1.2*	.0**	.874
1.*	.296	.296	1.4*	.0**	1.105
.833*	.405	.405	1.2*	.0**	.954
.667*	.5*	.5*	1.119	.0**	.929
.667*	.448	.488	1.2*	.0**	1.068
1.051	.303*	.303*	1.2*	.0**	.853
.986	.303*	.303*	1.4*	.0**	1.113
.643	.5*	.5*	1.2*	.0**	1.089
1.*	.162	.762	1.2*	.6**	1.038
.833*	.303*	.803	1.013	.6**	.852
.833*	.273	.773	1.2*	.6**	1.133
1.093	.1*	.756	1.2*	.6**	1.007
.790	.303*	.777	1.2*	.6**	1.137
1.*	.288	.303*	1.488	.015	1.2**
1.*	.278	.5*	1.478	.222	1.2**
1.*	.208	.7*	1.408	.493	1.2**
1.*	.200	.707	1.4*	.507	1.2**

Face Points (*Cont'd.*)

C	*B*	*H*	*G*	*H'*	*G'*
.833*	.303*	.741	1.303	.526	1.2**
.833*	.385	.5*	1.385	.138	1.2**
.833*	.351	.7*	1.351	.420	1.2**
.667*	.480	.5*	1.280	.029	1.2**
.667*	.459	.7*	1.259	.362	1.2**
.667*	.400	.778	1.2*	.567	1.2**
.5*	.571	.7*	1.171	.258	1.2**
.333*	.674	.7*	1.074	.077	1.2**
1.128	.1*	.7*	1.454	.532	1.2**
1.083	.1*	.728	1.4*	.580	1.2**
.963	.303*	.5*	1.459	.205	1.2**
.880	.303*	.7*	1.359	.452	1.2**
.914	.303*	.650	1.4*	.379	1.2**
.606	.5*	.7*	1.228	.330	1.2**
.583	.5*	.743	1.2*	.418	1.2**
.860	.368	.5*	1.4*	.154	1.2**
.555	.534	.7*	1.2*	.300	1.2**
.981	.222	.7*	1.4*	.487	1.2**

Interior Points

C	*B*	*H*	*G*	*H'*	*G'*
1.*	.303*	.5*	1.285	.197	.982
1.*	.303*	.7*	1.087	.397	.785
1.*	.303*	.598	1.2*	.295	.896
1.*	.296	.303*	1.4*	.007	1.105
1.*	.322	.5*	1.2*	.178	.878
1.*	.288	.5*	1.4*	.213	1.113
1.*	.254	.7*	1.2*	.445	.946
1.*	.208	.7*	1.4*	.492	1.192
.833*	.303*	.756	1.2*	.544	1.076
.833*	.404	.5*	1.2*	.115	.955
.833*	.361	.7*	1.2*	.407	1.007
.667*	.5*	.7*	1.075	.300	.863
.667*	.488	.5*	1.2*	.019	1.069
.667*	.465	.7*	1.2*	.326	1.103
1.143	.1*	.7*	1.4*	.524	1.137
1.036	.303*	.5*	1.2*	.191	.866
.968	.303*	.5*	1.4*	.202	1.126
.924	.303*	.7*	1.2*	.427	.971
.611	.5*	.7*	1.2*	.328	1.146

TABLE OF POINTS

$P = 100{,}000$

$Rv = .8559$

Vertex Points

1.162**	.217	.217	1.**	.0**	.674
1.162**	.099	.796	1.**	.6**	.776
1.162**	.069	.766	1.464	.6**	1.2**
1.162**	.160	.160	1.555	.0**	1.2**
.239	.712	.712	1.**	.0**	1.2**
.276	.678	.839	1.**	.6**	1.2**

Edge Points

1.162**	.1*	.796	1.**	.599	.775
1.162**	.215	.303*	1.**	.075	.675
1.162**	.213	.5*	1.**	.247	.677
1.162**	.183	.7*	1.**	.448	.703
1.162**	.175	.175	1.2*	.0**	.882
1.162**	.166	.166	1.4*	.0**	1.066

TABLE OF POINTS

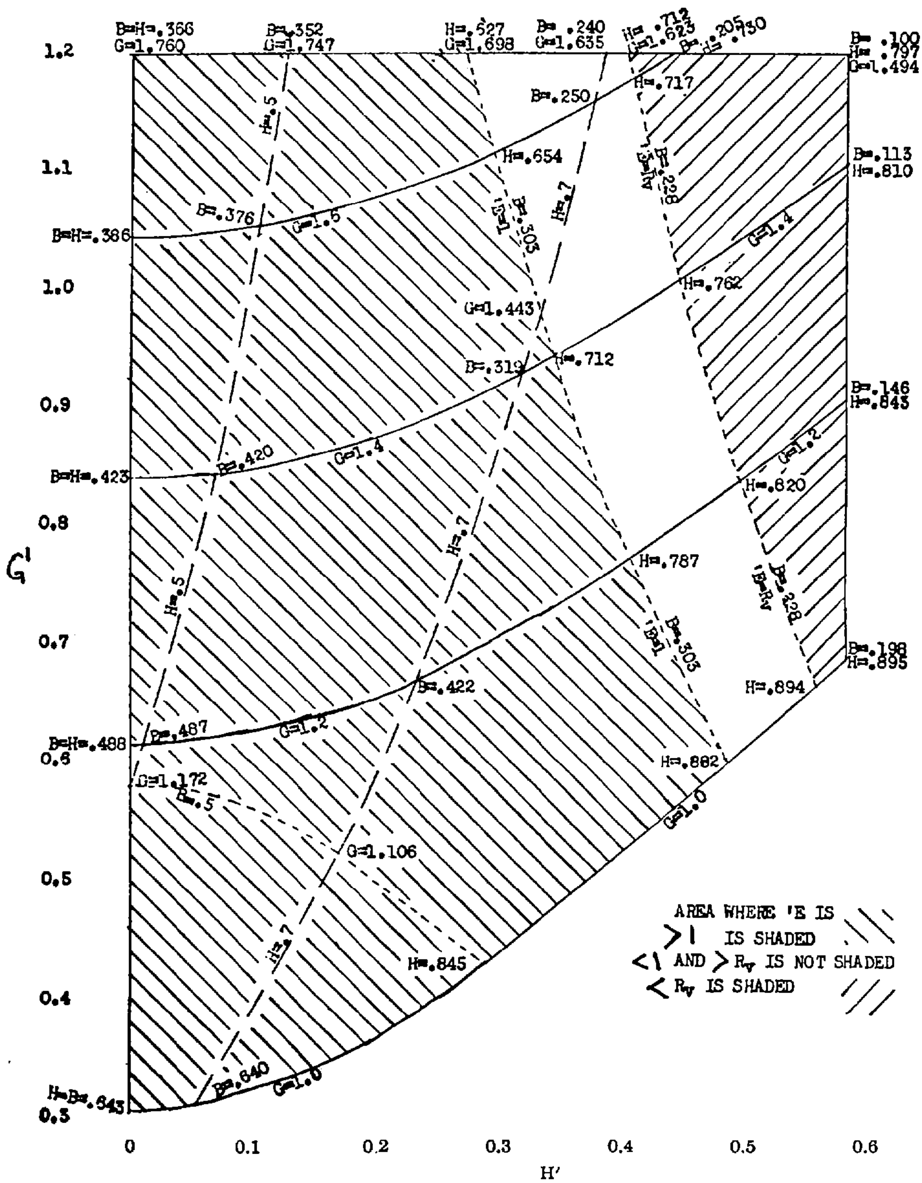
$P = 100{,}000$ (*Cont'd.*)

$Rv = .8559$

C	*B*	*H*	*G*	*H'*	*G'*
Edge Points (*Cont'd.*)					
1.162**	.076	.773	1.2*	.6**	.967
1.162**	.070	.767	1.4*	.6**	1.145
1.162**	.1*	.733	1.495	.545	1.2**
1.162**	.161	.303*	1.556	.122	1.2**
1.162**	.156	.5*	1.551	.296	1.2**
1.162**	.119	.7*	1.514	.500	1.2**
1.*	.298	.298	1.**	.0**	.702
.833*	.384	.384	1.**	.0**	.739
.667*	.471	.471	1.**	.0**	.793
.5*	.563	.563	1.**	.0**	.873
.333*	.658	.658	1.**	.0**	1.028
.991	.303*	.303*	1.**	.0**	.703
.615	.5*	.5*	1.**	.0**	.813
.261	.7*	.7*	1.**	.0**	1.151
1.*	.201	.801	1.**	.6**	.800
.833*	.307	.807	1.**	.6**	.832
.667*	.413	.813	1.**	.6**	.881
.5*	.521	.821	1.**	.6**	.958
.333*	.631	.831	1.**	.6**	1.109
1.161	.1*	.797	1.**	.6**	.777
.839	.303*	.806	1.**	.6**	.831
.532	.5*	.819	1.**	.6**	.939
.250	.7*	.796	1.**	.386	1.2**
1.*	.257	.257	1.457	.0**	1.2**
.833*	.357	.357	1.357	.0**	1.2**
.667*	.457	.457	1.257	.0**	1.2**
.5*	.557	.557	1.157	.0**	1.2**
.333*	.657	.657	1.057	.0**	1.2**
.923	.303*	.303*	1.411	.0**	1.2**
.594	.5*	.5*	1.213	.0**	1.2**
.260	.7*	.7*	1.012	.0**	1.2**
.579	.506	.506	1.2*	.0**	1.2**
.906	.312	.312	1.4*	.0**	1.2**
1.*	.179	.779	1.379	.6**	1.2**
.833*	.292	.792	1.292	.6**	1.2**
.667*	.404	.804	1.204	.6**	1.2**
.5*	.518	.818	1.118	.6**	1.2**
.333*	.630	.830	1.030	.6**	1.2**
1.116	.1*	.770	1.440	.6**	1.2**
.816	.303*	.793	1.283	.6**	1.2**
.525	.5*	.815	1.131	.6**	1.2**
.660	.408	.804	1.2*	.6**	1.2**
1.043	.150	.775	1.4*	.6**	1.2**
Face Points					
1.162**	.1*	.752	1.2*	.561	.946
1.162**	.1*	.737	1.4*	.548	1.119
1.162**	.175	.303*	1.2*	.110	.882
1.162**	.128	.303*	1.4*	.151	1.065
1.162**	.170	.5*	1.2*	.284	.886
1.162**	.158	.5*	1.4*	.294	1.069
1.162**	.137	.7*	1.2*	.485	.915
1.162**	.120	.7*	1.4*	.499	1.111
1.*	.297	.303*	1.**	.006	.704
1.*	.286	.5*	1.**	.214	.715
1.*	.267	.7*	1.**	.434	.734
.833*	.384	.5*	1.**	.139	.739
.833*	.372	.7*	1.**	.394	.754

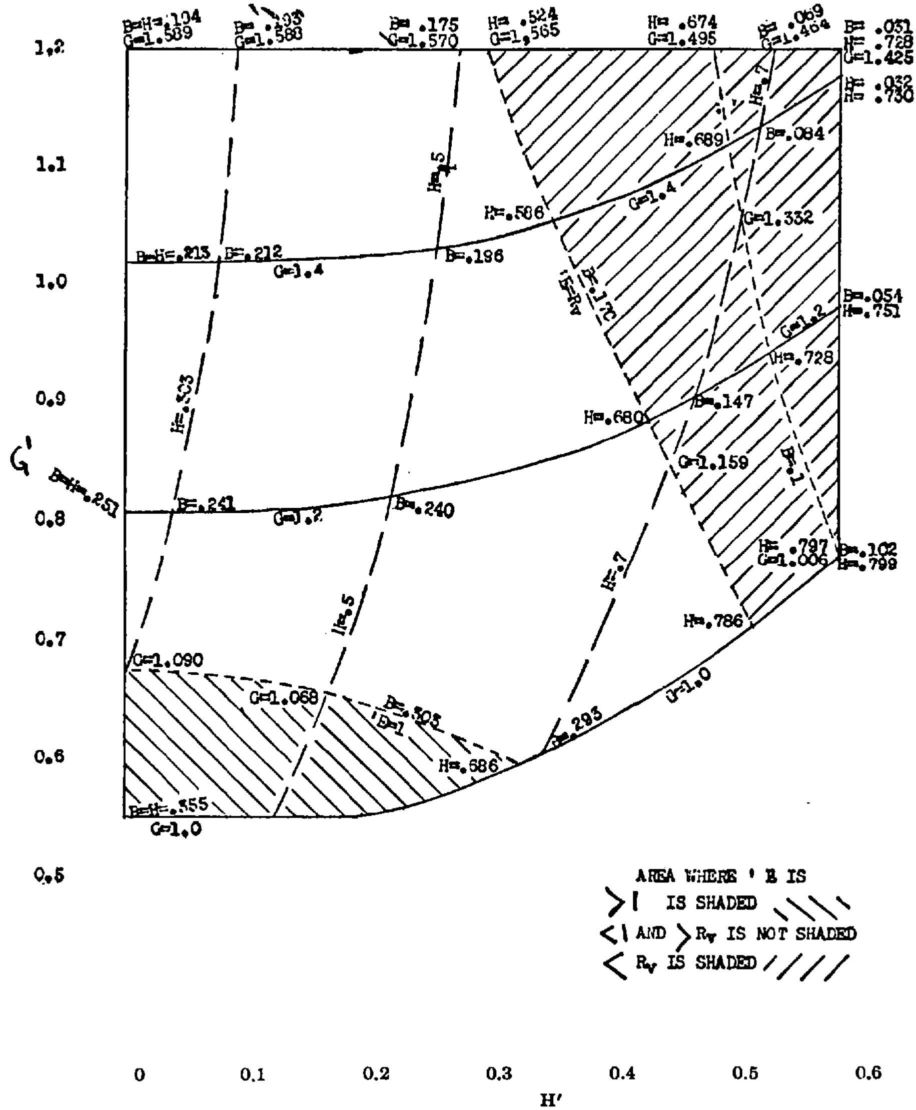
C	*B*	*H*	*G*	*H'*	*G'*
Face Points (*Cont'd.*)					
.667*	.470	.5*	1.**	.044	.794
.667*	.466	.7*	1.**	.351	.801
.5*	.561	.7*	1.**	.277	.877
.333*	.642	.7*	1.**	.174	1.075
.989	.303*	.5*	1.**	.200	.705
.956	.303*	.7*	1.**	.415	.729
.486	.5*	.7*	1.**	.329	.824
1.*	.266	.266	1.2*	.0**	.934
1.*	.258	.258	1.4*	.0**	1.143
.833*	.361	.361	1.2*	.0**	1.007
.667*	.464	.464	1.2*	.0**	1.114
.953	.303*	.303*	1.2*	.0**	.941
.924	.303*	.303*	1.4*	.0**	1.187
.594	.5*	.5*	1.2*	.0**	1.178
1.*	.183	.783	1.2*	.6**	1.017
.833*	.303*	.803	1.032	.6**	.875
.833*	.293	.793	1.2*	.6**	1.089
.667*	.405	.805	1.2*	.6**	1.193
1.127	.1*	.776	1.2*	.6**	1.089
1.117	.1*	.770	1.4*	.6**	1.164
.819	.303*	.794	1.2*	.6**	1.096
1.*	.258	.303*	1.457	.045	1.2**
1.*	.255	.5*	1.455	.245	1.2**
1.*	.231	.7*	1.430	.470	1.2**
1.*	.200	.756	1.4*	.556	1.2**
.833*	.303*	.780	1.303	.573	1.2**
.833*	.356	.5*	1.356	.173	1.2**
.833*	.342	.7*	1.342	.430	1.2**
.667*	.457	.5*	1.257	.065	1.2**
.667*	.450	.7*	1.250	.376	1.2**
.5*	.554	.7*	1.155	.291	1.2**
.333*	.656	.7*	1.056	.130	1.2**
.921	.303*	.5*	1.408	.214	1.2**
.892	.303*	.7*	1.373	.466	1.2**
.914	.303*	.596	1.4*	.320	1.2**
.587	.5*	.7*	1.204	.341	1.2**
.583	.5*	.725	1.2*	.386	1.2**
.907	.312	.5*	1.4*	.208	1.2**
.579	.505	.7*	1.2*	.336	1.2**
.942	.270	.7*	1.4*	.457	1.2**
Interior Points					
1.*	.266	.303*	1.2*	.037	.934
1.*	.258	.303*	1.4*	.045	1.143
1.*	.264	.5*	1.2*	.236	.936
1.*	.256	.5*	1.4*	.245	1.145
1.*	.240	.7*	1.2*	.460	.960
1.*	.232	.7*	1.4*	.469	1.169
.833*	.303*	.783	1.2*	.576	1.077
.833*	.361	.5*	1.2*	.168	1.007
.833*	.346	.7*	1.2*	.425	1.025
.667*	.457	.5*	1.2*	.064	1.114
.667*	.451	.7*	1.2*	.374	1.124
.930	.303*	.5*	1.2*	.212	.964
.920	.303*	.5*	1.4*	.215	1.193
.900	.303*	.7*	1.2*	.442	1.191
.588	.5*	.7*	1.2*	.340	.996

RETROSPECTIVE RATING PLAN RATIOS FOR
$P = 5{,}000 \quad C = \Gamma = 1.162 \quad Rv = .9249$



RETROSPECTIVE RATING PLAN RATIOS FOR

$P = 25{,}000 \quad C = \Gamma = 1.162 \quad Rv = .8668$



RETROSPECTIVE RATING PLAN RATIOS FOR

$P = 100{,}000 \quad C = \Gamma = 1.162 \quad Rv = .8559$