ABSTRACT OF THE DISCUSSION OF PAPERS READ

AT THE PREVIOUS MEETING

NON-RANDOM ACCIDENT DISTRIBUTION AND THE POISSON SERIES.

JOHN CARLTON
Volume XXXII Page 21
Written Discussion
F. S. PERRYMAN

This is a neat little paper dealing with a subject which at first sight may not look too important but yet is one that is worth investigating, not only for its own sake, that is to say to judge the importance of the effect of the alteration in the variances and probabilities under the conditions cited, but also worthwhile because it points the way in which such questions should be investigated. In this respect the paper is a model one. Mr. Carleton sets out the data and premises clearly and succintly and then makes the appropriate mathematical investigation, gives numerical examples and draws conclusions. Students and younger members of the Society, not to mention older ones, can benefit greatly from the subject matter of the paper and its treatment.

It is very timely to have presented such an example of restricted randomness among events whose probabilities it is desired to measure. Text books, both on "classic" statistics and on "modern" statistics, and papers in the Proceedings and other technical publications, particularly recent ones, giving accounts of modern theories and applying them to insurance, are always careful to stress that the events must be independent or random or unconnected, etc., but rarely do they give very practical examples of statistics or events that are not random, etc., and how to deal with them if they do occur. This last statement is not universally true of course and from time to time we do come across discussions of non-independent events. Taking an example at random, I remember a good discussion of independence in connection with events coming within the scope of the Poisson formula: this discussion you will find in Mr. Satterthwaite's paper in the PCAS Vol. XXIX. Nevertheless it is quite welcome to have further examples of non-random or dependent events and so I am glad that Mr. Carleton chose to give this phase of our theory the spotlight of a paper to itself.

The mathematics of the probabilities of the events postulated in the paper are treated quite satisfactorily from first principles. The double, triple and multiple integrals required may look rather formidable to some but after all they do represent the operations needed to obtain the probabilities. Mr. Carleton attains quite a neat formula which can be expressed in words as follows. Using

the notation of the paper, which is that the accident expectation
is such that in a unit period the average number of accidents would
be   a   but the conditions assumed are such that after each accident
a period of   b   elapses during which no further accidents are possi-
ble: then "the probability of having not more than   n   accidents
in a unit period under such conditions is the same as the probab-
ility of having not more than   n   accidents in a period of unity
less   n   times   b   if the accident expectation were not suspended
for the period of   b   after each accident".   This formula of
course only applies if   n   times   b   does not exceed unity as it
is impossible to have more than   n   accidents if   nb   exceeds unity.
It may be mentioned here, for those who do not wholly trust the
multiple integrals which were used to develop Mr. Carleton's for-
mula, that it is quite possible to check the formula by mathemat-
ical induction, i.e. assuming it to be true for   n   -   1   and then
proving it is true for   n .   To do this it is only necessary to
use single integrals.   It is also worthy of note, and this time by
the more mathematically minded ones, that while Mr. Carleton's
formula is quite a simple one, considering the underlying assump-
tions, it naturally applies only when it is assumed, as Mr. Carle-
ton does, that at the beginning of the period it is known that no
catastrophes have occurred in the immediate past.   The mathematical
formula to express the probabilities if it is unknown whether such
catastrophes have recently taken place is considerably more compli-
cated.   However, the simple formula is obviously all that is need-
ed in practice.   The fact that the probabilities considered by Mr.
Carleton can be expressed by means of straight-forward Poisson
terms greatly simplifies any calculations that have to be made
and I assume that Mr. Carleton utilized this fact in the calcula-
tion of the arithmetical examples which he gives in his paper.
Using the tables of Poisson distributions, given in the well-known
"Tables for Statisticians and Biometricians" edited by Karl Pear-
son I checked these calculations as a matter of interest and ar-
rived at the same results as in the paper, except in one instance
where the deviation was minor.

The paper concludes by giving an example of the application of
the arithmetical results.   In this connection it must be borne in
mind that the example suggested, namely that of a Workmen's Com-
pensation risk of such a size that an average of 1.8 serious acci-
dents occur each year, must be a single risk and not an aggrega-
tion of Compensation risks, say for instance of one industry, for
by the nature of the assumptions in the paper the occurrence of a
catastrophe must operate so as to eliminate or suspend the prob-
ability of a repetition.   This condition is much more likely of
realization in a single large risk than it would be in an industry
as a whole.

In conclusion, let me repeat that the paper is very worthwhile

and the author is to be congratulated not only on the subject matter but also on the manner in which the material is presented. The paper is not only thorough but concise.

PURE PREMIUM TRENDS IN WORKMEN'S COMPENSATION

R. P. GODDARD
Volume XXXII, Page 48
Written Discussion
A. N. MATTHEWS

Mr. Goddard's study of Workmen's Compensation pure premium trends brings out the fact that, in general, loss costs related to payrolls in Massachusetts have been decreasing steadily since 1930. This downward trend will undoubtedly be terminated by the law amendments which became effective in October of 1945. The decreases in Compensation rates for most states during the past several years would indicate that the downward trend in pure premiums has been quite general. On the other hand, the New York pure premiums show no particular trend. During this period the trend of wages and of claim frequency in New York and Massachusetts has been very similar. In New York, however, the average claim costs have been steadily increasing, whereas in Massachusetts the average claim cost is nearly constant. It is apparent that in Massachusetts the upward trend in wages over the period has resulted in a downward trend in the pure premium and the average claim costs show no definite trend. On the other hand, in New York the upward trend in wages has been accompanied by an upward trend in average claim costs and the pure premiums show no definite trend.

The average claim cost for New York increased from \$268 in 1932 to \$417 in 1942. For Massachusetts the average claim cost was \$202 in 1932 and \$201 in 1942. An analysis of the average cost per claim by kind of injury shows very substantial increase in the average cost of each kind of injury in New York whereas the changes in Massachusetts have been relatively small. Furthermore, the proportion of serious claims in the experience increased from 2.5% to 3.0% in New York but decreased from 3.1% to 2.9% in Massachusetts.

This study shows that wages alone will not explain trends in pure premiums. Factors which tend to increase pure premiums are increasing liberality of Compensation Commissioners in awarding benefits, a tendency for injured employees to prolong the period of disability and increasing medical costs. On the other hand safety education and appliances designed to prevent accidents should have a tendency to reduce pure premiums. Of course, the effect of none of these items can be measured.

The obvious purpose of Mr. Goddard's study was to attempt to

obtain statistics which will reflect the trend in Workmen's Compensation costs currently and which will serve to cover the interval between the period over which experience is accumulated and the effective date of the rates based upon such experience. The manual rate revisions which are now in process to produce rates to become effective early in 1947 will use experience for no later than policy year 1944. In the interim, wartime restrictions on wages and prices have been lifted and both have increased to a very considerable extent. This inflationary trend tends to increase both payrolls and losses but the effect on pure premiums will probably vary by state and by industry and classification within states.

Although it is doubtful that any immediate use will be made of the phenomena brought to light in Mr. Goddard's paper, nevertheless, the work which he has so well presented should be continued and expanded in an ever continuing effect to obtain rates that will reflect as accurately as possible the trend of pure premiums. Thus only will the extended periods of excessive losses which have been the curse of the Compensation business be avoided.

VALUATION OF NON-CANCELLABLE ACCIDENT AND HEALTH

INSURANCE POLICIES

S. F. CONROD
Volume XXXII, Page 27
Written Discussion
JARVIS FARLEY

Mr. Conrod has performed a service by preparing these tables and setting forth in detail formulae for their use. Mr. Conrod warned that the Conference modification of Class 3 was designed as a minimum standard for reserves and not as a basis for premium computation. It might be well to point out that there is no inconsistency in this warning. The active life reserve depends not on the absolute level of the net one-year-term premium but upon the relationship which exists among the one-year-term premiums-- that is, on the slope of the line rather than on the level of the line. The Conference Committee was concerned with reserves, not with premiums, and found it expedient to limit its attention to the slope of the line rather than to the level of the line.

This warning does not necessarily limit the usefulness of Mr. Conrod's tables. It has been common in analyzing disability experience to express the results of the analysis in terms of a mod-

ification of Class 3--either a straight multiplication or one multiple of an initial disability period and a second multiple of the period following the initial period. Mr. Conrod's paper discusses this aspect (Multiple Reserve Standards, page), and by applying that concept the tables could be useful in premium calculations as well as in reserve calculations.