THE COMPENSATION EXPERIENCE RATING PLAN — A CURRENT REVIEW

BY

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More than twenty years ago, Mr. Perryman contributed his tremendous work on "Experience Rating Plan Credibilities." (Proceedings XXIV) Shortly thereafter, Mr. Smick described the new plan, comparing it to the old and providing the various formulae used, and then in 1941 Mr. Johnson set down some oddities observed in calculating the original New York values, suggesting improvements. Subsequent to this wealth of material, our Proceedings contain nothing, evidence of a job well done and of little trouble with the plan's application.

But, like many of our rating processes, the plan contains its nuisance quota of constant values changing in significance as dollars lose theirs.

The loss discounting formula continues to assign $1,500 maximum primary to cases that now average double or triple the average costs of twenty years ago, and certain credibility values have become upset as they are allied to loss discounting. As discounting may be restored to some semblance of the originally intended level, the credibility values will need examination for possibile revision.

If the plan is to be adjusted materially, the event might also serve as the occasion for simplification towards facilitating mechanical ratings. Some considerable success in that direction already has been achieved despite the relatively cumbersome tabular requirements, but improvement appears quite possible and highly desirable.

Analytical study surrounding these questions has provided a subcommittee of the National Council Actuarial Committee with many happy hours and this work continues. Undoubtedly, considering an almost twenty-year omission in our Proceedings, the subject must be of interest to many Society members, and by writing now, discussion provided by our forum may contribute to final action or at least to understanding of solutions finally adopted.

The writer, therefore, intends this approach:

(a) A conception of the logic and development of what amounts to a dual modification formula;

(b) Brief developments of the important underlying formulae, as these are convenient here for discussion of departures, with the suggestion that Mr. Perryman's paper is a "must" for completely general analyses and thorough foundation;

(c) Some critical inspections of how the credibility structure has been operating, with particular emphasis upon those values appearing most susceptible to simplification; and

(d) Suggestions for changes appearing most logical, and though these include benefit of committee discussion, it should be understood they are not necessarily the committee's intended adoption.

1. *Some Basic Concepts—A Dual Credibility System*

Most people readily understand manual classification rates as average measures of loss exposure per $100 of payroll for classified portions of individual risks, and since the classification system is as fine as can be reasonably expected, such rates logically could be applied without further ado if there were no means of measuring the extent to which individual risks do not fit the average contemplated by the class rates. Thus the usual explanation of risk experience rating modification includes the term, "better or worse than the average." We need not disturb this comfortable familiarity, but for our present purpose it seems best to tuck it away as over-simplification and proceed more analytically.

The manual rate includes a gross expense factor, this addition to the pure loss rate being intended to provide the expenses necessary for handling risks with premiums under the $1,000 size, above which premium discounts operate to reduce the expense loadings for larger risks. Removing the gross expense portion leaves the rate for losses —the "pure premium"— and after minor adjustments for differences in cost levels between current period and the older rating period, this is the basis of the "expected losses" which will be used in determining risk modifications. Thus the manual rates serve to establish the point of reference for actual risk losses, a self-correcting feature: redundancy in rates promotes credit modifications of those rates, inadequacies promote debits; to the risk large enough to receive 100 per cent credibility, when $E = S$, the "self-rating" point, manual rates are substantially unimportant.

A general expression for the simplest form of rating formula is

$$M = \frac{AZ + E\ (1 - Z)}{E} \text{ where}$$

A = Actual Losses
E = Expected Losses
Z = Credibility Factor, less than or equal to 100%.

In such simple form, using total actual losses which may suffer extreme variation if no restrictions are imposed, the criteria by which the system of Z or credibility values are set up must recognize the practical undesirability of substantial variations caused by chance severe losses, or their absence, and rating effectiveness is less than it could be if loss experiences might be used under a system of recognizing that an incurred loss involves two sets of influences: Those bringing about its occurrence, and those determining its amount.

Relatively high credibilities might be assigned to occurrences as measured against an average frequency reference, and lesser credibilities might be used in evaluating severity, this logic possibly leading to a conclusion that two modifications be established and somehow combined through an equitable weighing process. Exploration of what these weights might be leads to understanding of the split type of loss treatment, the splitting of actual case losses between "primary," the early loss dollars, and "excess," the losses accumulating from continued payments related to severity—and the parallel splitting of expected losses to primary and excess references. The coincidence with a pure occurrence type of modification can be seen by letting the primary loss definition be the first dollar only of each case, which would actually amount to a one-for-one case count.

But using only the first dollar of each loss would make no distinction at all between cases. Conceivably this might be overcome by setting up several loss categories, perhaps by type such as medical only, temporaries, etc., or by size, but we must always remember the need for an expected average point of reference paralleling the treatment of actual cases, and complicated treatments have to be avoided.

By placing equal value upon all loss dollars up to a specific amount per case, and lesser value or weight upon successive loss dollars over such amount, it is seen that recognition is given to both the frequency or occurrence element and to the distinction between types of cases as gauged by case amounts. In the original design, $500 ($300 or $400 in some lower-cost states) was taken as the specific point below which loss dollars would be treated equally. To the next $500 of each case, a weight of two-thirds, instead of one, represents decreased emphasis upon these dollars as frequency indicators, while the remaining one-third of each dollar is placed to the excess side, as distinguished from primary, to be used for severity indications as risk size permits. Of the next $500, two-thirds squared, or four-ninths, are assigned to primary and so on, all part of the operation of the complete formula for splitting any loss over $500 in amount to the primary and excess portions:

$$\text{Primary, } A_p = \text{Initial } 500 + \frac{2}{3}(500) + \left(\frac{2}{3}\right)^2(500) + \left(\frac{2}{3}\right)^3(500) + \ldots\ldots + \left(\frac{2}{3}\right)^{n-1}(500) + \left(\frac{2}{3}\right)^n(R) \ldots\ldots\ldots (1)$$

where R is the remainder after (n — 1) intervals of $500 beyond the initial $500.

Selection of the $500 point must originally have been assigned to judgment, as well as the practical consideration that only 10 per cent of cases at that time would be above $500 and need discounting. The previous plan used a split of $1,000 for indemnity and $100 for medical, so the principle and experience with it was not entirely new, and the 1940 plan could be termed a refinement in using multiple splits and applying these to indemnity and medical combined. And as the treatment of actual losses must be reflected in like treatment of expected losses, the discounting method may be selected with some degree of freedom, limited by judgment upon the answers to essentially two questions:

First, will the amount of losses included within the initial value, and within the additional primary portions of successive increments, comprise a sufficiently large portion of total losses for the application of a separate primary credibility system, such that frequency indications given by the primary portion will provide reasonable but not extreme effects upon risk modifications, and

Second, will the discounting formula provide a reasonable limit to the amount of primary loss added to the rating by a single case, bearing in mind that the complement of low primary definition must be larger excess loss and lower excess credibilities.

These questions are somewhat allied, of course, and really boil down to the one of proportions deemed most reasonable and practical for application of a dual modification formula. We may write the modification formula as follows, wherein the dual modification system and the weighting by primary and excess is clear:

$$M = (Z_p \times \frac{A_p}{E_p} + 1 - Z_p)\frac{E_p}{E} + (Z_e \times \frac{A_e}{E_e} + 1 - Z_e)\frac{E_e}{E} \quad \ldots (2)$$

in which the subscripts p and e designate primary and excess portions of actual and expected losses, and Z_p and Z_e designate primary and excess credibilities, derived as equivalent to credits for clear experience;

$$Z_p = \frac{E}{E_p + WE_e + B} \quad \ldots (3)$$

$$Z_e = WZ_p \quad \ldots (4)$$

W and B will be defined below, but for the moment we may note that by substituting the credibility expressions (3) and (4) for Z_p and Z_e in equation (2), the formula used directly in the rating form may be obtained:

$$M = \frac{A_p + WA_e + B}{E_p + WE_e + B} \quad \ldots (5)$$

II. *Derivation of K_e, K, and Q*

The expected loss size at which, or below which, excess losses are not included is termed the Q point. At this point and below W = O so the modification becomes

$$M = \frac{A_p + B}{E_p + B} \text{ or}$$

since $B = K$ when $E < Q$,

$$M = \frac{A_p + K}{E_p + K}$$

Inclusion of excess losses begins above this Q point according to the values of

$$W = \frac{E - Q}{S - Q}$$

Wherein $S =$ the self-rating expected loss size at which, of course, $W = 1$.

For $E > Q$, as excess losses are included, the K value must be gradually eliminated so that self-rating may be accomplished, and the modification formula then could be

$$M = \frac{A_p + WA_e + K(1 - W)}{E_p + WE_e + K(1 - W)} \quad \text{(6)}$$

so $M = \dfrac{A}{E}$ when $E = S$, $W = 1$

However, Z_p may be greater than unity in

$$Z_p = \frac{E}{E_p + WE_e + K(1 - W)}$$

if $K < E_e$. Above the Q point, therefore, a value K_e is substituted for K, and to ensure that $K_e > E_e$ at any risk size, values of K_e are obtained from the linear function proceeding from the point $K_e = K$, $E = Q$, to the point $K_e = gS$, $E = S$, wherein g is a maximum anticipated excess ratio $E_e \div E$. If g is indeed the maximum excess ratio for any E, it follows that at any point below S, $K_e > E_e$.

Deriving the function for K_e by equating slopes

$$\frac{K_e - K}{E - Q} = \frac{gS - K}{S - Q} \quad \text{(7)}$$

$$K_e = K + \frac{(E - Q)\ (gS - K)}{(S - Q)}$$

$$K_e = K + W\ (gS - K), \text{ since } W = \frac{E - Q}{S - Q}$$

$$K_e = K\ (1 - W) + WgS \quad \text{(8)}$$

Substituting K_e for K in (6), and letting $B = K_e (1 - W)$, yields the familiar form (5).

The K value used without alteration when $E < Q$, and which is part of K_e when $E > Q$, is based upon a judgment decision that a minimum ratable risk be debited a maximum 25 per cent for a maximum loss. The maximum ratable loss is twice the "Average Death and Permanent Total Value", which at the minimum risk size can be used only at the primary maximum of $1,500. An expression for minimum 3-year expected losses is 3PL, in which P = annual minimum subject premium, now $500, and L = the permissible loss ratio. Since only expected primary, E_p, will be used in the modification, 3PL is multiplied by the statewide ratio, D, of primary to total losses, to construct an approximate modification for the minimum risk,

$$M = \frac{A_p + K}{E_p + K} = \frac{A_p + K}{3PLD + K} \quad \text{(9)}$$

Letting A_p take on a $1,500 addition:

$$M + \triangle M = \frac{A_p + 1500 + K}{3\,PLD + K}$$

$$\triangle M = \frac{1500}{3\,PLD + K} = .25, \text{ the maximum intended debit}$$

Solving,

$$K = 4 \times 1500 - 3PLD \quad \text{(10)}$$

For example, if L and D each are approximately .60, and P = 500, then K = 5,460, rounded to $5,500, a K value common to most states, since variations in L and D by state have small effect.

The derivation of expression (8) for K_e would not be valid if K itself did not meet the requirement $K > Qg$. Thus we have a condition which should be met by a selection of Q, otherwise not restricted in theory, that $Q < \frac{K}{g}$. In practice, Q has been set at $Q = \frac{K}{D}$, and since D, the average statewide primary ratio, is still well over .40, Q values are well below those required by $Q < \frac{K}{g}$, as g is allowed to retain its original 1939 value of .40. But we may note for the moment that a g of .40 is now much too low for many risks, and Q values now do not fit the true requirement

$$Q < K \div \frac{E_e}{E}$$

so that the many risks with excess ratios $E_e \div E$ higher than D are receiving primary credibilities greater than 100 per cent.

As Z_p may exceed unity at $E = Q$ for certain risks, the same illogical result carries over into areas involving K_e, when $E > Q$.

This difficulty with g as a fixed value is of course referred to by Mr. Perryman, and later pointed to by Mr. Johnson shortly after the plan's New York introduction as needing revision as D ratios decline with inflated loss costs. If complete assurance is demanded that Z_p never exceed unity, g should now be practically double .40 in many states. A further discussion of g will follow.

III. *Alternative Discounting Formulae*

Summing formula (1) to infinity, the maximum primary loss is $1,500. Probably this limit, and the rapidity with which it is approached, has operated most strongly to accelerate the decrease in D ratios as case costs increased, and also has been the source of most of the discomfiture felt by practical underwriters as they observe the small use of today's high cost cases in a majority of ratings.

A few average death and permanent total values used in 1940 were $1,930 in Georgia, $3,800 in Massachusetts, $3,830 in Michigan, $6,800 in New York. In the atmosphere of those cost levels, $1,500 maximum primary must have appeared quite adequate. The $500 as initial value and subsequent split points is somewhat less disturbing, although increase to at least $750 would appear to be a minimum step in this connection. However, as there is an increase in the splitting points $750, $1,500, $2,250, etc., at each of which a new discounting ratio must apply, there is an instinctive concern over the discontinuity of this type of function. There seems to be no serious reason why the successive split points be determined as multiples of the initial value. Ideally, the successive additions to primary, for successive equal increments of cost, should be continuously decreasing for increments chosen as small as we please. Suppose then the expression for a primary equivalent of any given loss size, over the initial value I, were

$$A_p = I + a_1r_1 + a_2r_2 + \dots \qquad (11)$$

in which the increments $a \to o$, and $r_2 < r_1$, etc.

This may be written $A_p = I + (A - I)\, r$ . . . . (12)
where A = Actual case cost, and r is a function of A and is the average discounting ratio as successively decreasing discount ratios have been applied to the small increments a, i.e., $r = \dfrac{a_1r_1 + a_2r_2 + \dots}{A - I}$.

Equation (12), although expressing an ideal type of discounting function, does not appear useful in practice. But the precise nature of the relations between successive ratios r_n need not concern us if we can determine an expression for the average r compatible with a selected practical expression for A_p, so long as the selected expression approaches the limit of the maximum desired primary for an infinitely large loss, A, and also if $A_p = A$ when $A = I$. These restrictions are observed in the quite simple and usable

$$A_p = \frac{A}{A + C} \times \text{Maximum Primary} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

Substituting I for A_p and A, in (13),

$$I = \frac{I}{I + C} \times \text{Maximum Primary}$$

so Maximum Primary $= C + I$, and (13) becomes

$$A_p = \frac{A}{A + C} \times (C + I)$$

Equating to (12)

$$I + (A - I)r = \frac{A}{A + C} \times (C + I)$$

$$(A - I)r = \frac{A(C + I) - I(A + C)}{A + C}$$

$$r = \frac{AC - IC}{(A + C)\,(A - I)}$$

$$r = \frac{C}{A + C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Substituting this expression for r in equation (12) yields, of course, equation (13).

It may be noted that formula (13), although demonstrated above to be a refinement of the present multi-split discounting system, has a logic of its own, as it expresses application of a credibility factor, $\frac{A}{A + C}$, to a maximum primary.

The desirability of a formula of this type in the light of mechanical application is obvious, not only in individual risk rating but also in the mass treatment of losses necessary for periodic revisions of classification D ratios. In addition, facility is gained for occasionally adjusting the formula for new initial values and maximum primaries. Probably a reasonable shift at the moment, without incurring too much risk disturbance, would be to a \$750 initial and a maximum of \$3,750, so that the constant C could be the nicely round figure of 3,000.

$$A_p = \frac{A}{A + 3000} \times 3750 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

Thus a $750 loss would be $750 primary. A future increase in initial value is easily accommodated by an increase in maximum primary, or conceivably a decrease in the constant C, so that Maximum Primary $- C = I$. Hence the discounting emphasis may be shifted easily without any concern over split-type ratios of the two-thirds variety. If greater primary for smaller cases becomes desirable, the initial value increase may be accommodated by a corresponding decrease in C; or if higher primary for large cases is wanted, the maximum primary and the constant C are increased by the same amount. Adjustment to current cost conditions in such an important part of risk rating is highly desirable, and the suggested formula enables corresponding adjustment in class D ratios quite easily.

Test discounting of losses for several states under formula (15) reveals very close to a 10-point addition to discounting ratios obtained from the present formula. There happens to be so little variation, in fact, between states and between classes that the considerable problem of switching during a transitionary period, as it must involve all states, could be substantially reduced by addition of 10 points to all current D ratios.

IV. *Effect of New Discounting Upon the Constant K*

The K value derivation has been shown to depend upon maximum primary and a judgment decision as to maximum debit for a single large case. Adoption of a formula such as (15) — and for purposes of discussion formula (15) will be used from here on — means a substantial increase from the present $1,500, and this increase will vary according to each state's Accident Limitation equal to twice the average cost of death and permanent total cases. If the initial and present 25 per cent debit criteria is to be continued, K values will become quite high, seriously decreasing the complementary credit available to the smaller risks, as credibility will be reduced.

Deriving some K values through formula (10);

State Accident Limitation	*Equivalent Primary*	*Approximate New K* (4M-800)	*Present K*
20,000	3261	12,200	5500
30,000	3409	12,800	"
40,000	3488	13,200	"
50,000	3538	13,400	"
60,000	3571	13,500	"

It becomes questionable that the old 25% criteria should be retained, as it requires a variation in K hardly consistent with practical considerations: (1) The State Accident Limitation, dependent upon benefits for relatively rare cases, is only occasionally a guide to benefits for most cases; (2) a criteria set according to the very special case of a minimum rating, though providing a convenient statement, is not highly germane to the determination of credibility values having important application through the whole range of risk sizes; (3) credits now provided to no-loss small risks, substantial in number, would be seriously reduced, a transition problem.

The following table of primary values, present and proposed formulae, for a few case sizes, shows the change in primary dollars, through the range of predominately probable cases, is quite a different matter from the increase over present $1500 primary occasioned by the larger cases.

Case Cost	*Primary Values Present*	*Primary Values Proposed*	*Case Cost*	*Primary Values Present*	*Primary Values Proposed*
750	680	750	7,500	1,500	2,679
1,000	830	938	10,000	1,500	2,885
2,000	1,200	1,500	20,000	1,500	3,261
3,000	1,370	1,875	30,000	1,500	3,409
4,000	1,440	2,143	40,000	1,500	3,488
5,000	1,470	2,344	50,000	1,500	3,538

Consideration of these and other factors suggests judgment be applied to the end result rather than to only extreme conditions. We might, therefore, contemplate the modification effect, with several suggested K values, for occurrence of several case sizes; and because an increase in minimum ratable size could be a logical possibility and nothing is gained here by tossing that question around, whether it should be $750, $1,000 or higher, examination at a $1,000 subject premium size is convenient:

Using (9), $M = \frac{A_p + K}{3PLD + K}$, approximately $= \frac{A_p + K}{1{,}080 + K}$,

and $\Delta M = \frac{\Delta A_p}{1{,}080 + K}$:

(1) Case Size	*(2)* Present Primary	*(3)* Present ΔM, $K = 5{,}500$	*(4)* New Primary	*(5)* New ΔM, for Several K Values: $K = 7{,}500$ $\Delta A_p \div 8{,}580$	*(6)* $K = 10{,}000$ $\Delta A_p \div 11{,}080$	*(7)* $K = 12{,}500$ $\Delta A_p \div 13{,}580$
750	680	10%	750	9%	7%	6%
1,000	830	13	938	11	8	7
2,000	1,200	18	1,500	17	14	11
3,000	1,370	21	1,875	22	17	14
4,000	1,440	22	2,143	25	19	16
5,000	1,470	22	2,344	27	21	17
10,000	1,500	23	2,885	34	26	21
20,000	1,500	23	3,261	38	29	24
50,000	1,500	23	3,538	41	32	26

Comparing column (3) to (5), (6) and (7) provides a broad view of what might happen debit-wise. Confining this view to the range of case sizes up to \$5,000 which will predominate, a new K value of \$7,500 appears most fitting to present concepts. Also, an important consideration must be the credit available for no losses, a common circumstance for the \$1,000 subject premium risks:

Using $1 - \frac{K}{1080 + K}$, this is 16% presently, and the new credits would be 13% for $K = 7{,}500$, 9% for $K = 10{,}000$ or 8% if K were 12,500. Again, $K = 7{,}500$ provides the closest approach.

V. *Effect Upon Q Point*

Q has been shown to have only a maximum restriction $\frac{K}{g}$, g being selected as the maximum possible ratio $\frac{E_e}{E}$ for any risk. Using a K of 7,500 and assuming g at .75 which corresponds to D ratios of .25 for some classes in some states, the maximum $Q = 10{,}000$. It is interesting to see what happens to Z_p when total expected losses $= Q$, and the primary and excess portions are designated as Q_p and Q_e:

$$Z_p = \frac{Q}{Q_p + K} = \frac{Q}{Q_p + Qg} \quad \text{when } Q = \frac{K}{g}:$$

For a risk with $\frac{Q_e}{Q} = g$, $Z_p = 1$, and Z_p becomes less as $\frac{Q_e}{Q}$ decreases, which means that Z_p decreases as Q_p increases, apparently illogical. But expressing the modification as

$$M = Z_p\left(\frac{A_p - Q_p}{Q}\right) + 1$$

and letting $A_p = O$ and $Z_p = \frac{Q}{Q_p + Qg}$

$$M = \frac{Qg}{Q_p + Qg} = \frac{Qg}{Qg - Q_e + Q},$$

which becomes $\frac{Qg}{Q}$ when $Q_e = Qg$, so the selection of g is important in amount of credit, $1 - \frac{Qg}{Q}$, when $Q_e = Qg$, as well as being important to keep $Z_p < 1$.

As Q_e becomes less than Qg, if we use $\triangle Qg = Qg - Q_e$, $\triangle M = -M \left(\frac{\triangle Qg}{Q + \triangle Qg} \right)$ so that as the ratio $\frac{Q_e}{Q}$ become less than g, i.e., as $\frac{Q_p}{Q}$ increases, the credit for no losses increases, a logical end result despite the apparently illogical decrease in Z_p as the primary portion increases.

This demonstration has two purposes: to show the importance of g in determining credibility level, and to point up that although Z_p does move contrarily to the primary portion of a given E, the weighting effect of $\frac{E_p}{E}$ on the primary modification makes this less disturbing.

VI. *Avoiding Use of Maximum Excess Ratio g*

A selection of g is necessary to the tabular credibility values of B corresponding to values of $W = \frac{E - Q}{S - Q.}$. We have seen how important this value is in determining the credibility level, and some reflections upon this may be summarized as follows:

1. g must be kept up to date as the maximum, or Z_p will exceed unity.

2. If only one g is used for all states, this single selection is dependent then upon the highest cost state, and primary credibilities in lower cost states for equivalent expected loss sizes are less than if g were selected for each state, yet the maximum Q point must be much the same.

3. Depressed primary credibility in low-cost states, which also have lower self-rating points, results in a more rapid traverse of the distance from Q to self-rating.

4. Most important, the use of tabular B and W values, and the consequent need for g, results in its rigidity in substantial departure from the proper use of $\frac{E_e}{E}$ for each risk, and is required only because of the impracticability of constructing tabular values contemplating all possible variations of $\frac{E_e}{E}$.

In short, the rating plan is seriously encumbered by the conception of tabular B and W values. It is more complicated than it need be, a vital consideration as we should gear more and more of our procedures to mechanical processes. It becomes pertinent, therefore, to examine possibilities for individual risk determination of credibility values, so no tables are needed which are cumbersome to produce mechanically, and so no g selection would be required. But unless we can simplify our rating formula, individual risk calculations will remain less desirable than construction of tabular values.

Since Q may be as small as we please, the effect of low selection being only a reduction in primary credibility and an earlier use of excess losses, we may examine the result of eliminating Q entirely, with the reservation for the moment that we will later examine the possibility of setting some point E below which excess loss rating work will be avoided.

Instead of $W = \frac{E - Q}{S - Q}$, $W = \frac{E}{S}$, and g will be replaced by $\frac{E_e}{E}$.

From (8), $K_e = K(1 - W) + WgS$

$$K_e = K\left(\frac{S - E}{S}\right) + \frac{E}{S}\frac{E_e}{E}S$$

$$K_e = K\left(\frac{S - E}{S}\right) + E_e \qquad (16)$$

This is eminently practical, and K_e is, of course, always greater than E_e, as previously shown to be required to insure $Z_p < 1$.

Now using $B = K_e(1 - W)$,

$$B = K\left(\frac{S - E}{S}\right)^2 + E_e\left(\frac{S - E}{S}\right) \qquad (17)$$

but we may question the necessity, bearing in mind the search for a simpler form, of squaring the less than unity coefficient of K. If we could set

$$B = K\left(\frac{S - E}{S}\right) + E_e\left(\frac{S - E}{S}\right), \qquad (18)$$

we would have a usable

$$B = (K + E_e)\left(\frac{S - E}{S}\right) \qquad (19)$$

and such adjustment appears permissible:

1. K_e would equal $K + E_e$, and this, of course, satisfies the condition that $K_e > E_e$.

2. When $E = S$, $W = 1.00$, and the requirement is satisfied that B be entirely removed at the self-rating point, so that $M = \frac{A}{E}$.

3. The modification formula becomes the same as one originally derived by Mr. Perryman as one of the possibilities (his formula (14), based upon $Z_p = \frac{E}{E + K}$) and apparently preferred by him at that time:

$$M = \frac{A_p + W\ (A_e - E_e) + E_e + K\ (1 - W)}{E + K\ (1 - W)} \quad \ldots\ldots\ldots (20)$$

Since $B = (K + E_e)\ (1 - W)$, this becomes

$$M = \frac{A_p + WA_e + B}{E_p + WE_e + B}$$

identical in form to the present, but differing in B to the result that B may be calculated readily for each risk and no table look-up would be necessary.

In arriving at (20) however, $W = \frac{E}{S}$, and a note was made that although this provided for no Q point, we might nevertheless adopt one as a judgment point below which excess losses would not be rated and some work thus would be avoided on the considerable number of risks for which excess indications could amount to very little.

We might accomplish much the same saving in work were we to decide upon a value of E below which the W value would not be used, as it would be of negligible size, but this creates smoothing problems and appears clumsy. It would seem better to continue the familiar concept of Q, so that $W = \frac{E - Q}{S - Q}$, and although this Q might be varied between states, as a practical matter since no substantial theory is involved and no substantial modification effects are involved either, a common point such as 10,000 might better be used. It is easy to see that not much is involved here. At the Q point the modification for clear experience is

$$M_1 = \frac{K + E_e}{K + E}, \text{ since } W = O$$

and if W had been continued as $\frac{E}{S}$, (no Q point),

$$M_2 = \frac{K + E_e\ (1 - W)}{K\ (1 - W) + E}$$

Since the W involved in both numerator and denominator of M_2 must be quite small at a reasonable Q such as 10,000, ($W = .05$ when $S = 200{,}000$, or .025 when $S = 400{,}000$), the modification M_2 will tend to be quite close to M_1, the variations being plus or minus, depending upon the relative values of K, E and E_e. Thus the Q point may be selected entirely upon consideration of what size of risk distribution indicates may be the rating work involved, and selection of an all-state point has much in its favor for interstate ratings.

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It is hard to say precisely when an accumulation of changing forces demands adapting action. Difficulties anticipated—risk disturbances, re-education, interstate rating complications as two plans temporarily may be in effect—make it a step not lightly undertaken. There is a good question that these difficulties may be lessened by proceeding gradually, confining the first step to a change in discounting formula, a corresponding upward adjustment in D ratios and K values and in the g value necessary to retention of the tabular system of B and W values. This step would be relatively easy, would restore validity to the plan's originally sound foundations, and is easily understandable as a counterpart of long-term inflation. Actually, this is something which should have been accomplished steadily over the years, and a worthy intent now may be to keep pace in future, as well as to eventually adopt formulae which will automatically help in this purpose and also contribute to reduction of rating work.