# LOSS RESERVE ADEQUACY TESTING: A COMPREHENSIVE, SYSTEMATIC APPROACH

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While specific guidelines for reserve adequacy testing may be established and specific examples of an actuarial approach to the testing of loss reserves may be offered for particular situations, loss reserving cannot be reduced to a purely mechanical process or to a "cookbook" of rules and methods. The utilization and interpretation of insurance statistics requires an intimate knowledge of the insurance business as well as the actuary's ability to quantify complex phenomena which are not readily measurable. As in the case of ratemaking, while certain general methods are widely accepted, actuarial judgment is required at many critical junctures to assure that reserve projections are neither distorted nor biased. That judgment is specifically required in such decisions as:

1. Ascertaining the optimal combination of the kinds of loss statistics to be used in a reserve analysis,
2. Assessing the impact of changes in company operations and procedures on the loss statistics to be utilized in a reserve analysis,
3. Adjusting the loss data for the influences of known and quantifiable events,
4. Evaluating the strengths and weaknesses of various reserving methods, and
5. Making the final selection of estimates.

Throughout the entire process of testing the adequacy of loss reserves, the actuary's expertise must be called upon in tailoring the methodology to the characteristics of the insurer's book of business, the specific data available, and to recent changes in company operations and procedures.

The purpose of this paper is to present what we believe to be some essential guidelines for any comprehensive and systematic approach to testing the adequacy of loss reserves. In this paper, these guidelines will often be illustrated by a specific example of an actuarial approach (among many) to a particular problem. Within the framework of these guidelines, however, much latitude exists for the development of a wide variety of actuarially sound approaches to loss reserving. These guidelines, and the sections devoted to discussion of them, are as follows:

1. A thorough understanding of the existing data base and the *trends* and *changes* underlying that data base is a prerequisite to the application of actuarially sound reserving methods. Familiarity with the underwriting, claims, data processing and accounting operations within a company, and knowledge of *changes* in the operations and procedures of these departments which have occurred during the experience period, are essential to the accurate interpretation and evaluation of various reserving methods. Comprehension of key developments and trends in the legal, regulatory and socio-economic environment in which an insurer operates is also a prerequisite to the formulation of accurate reserve estimates. (Section I).

2. Where possible, loss data which has been relatively unaffected by changes in company procedures and operations should be utilized in testing loss reserves. The possibility of subdividing or combining the data in order to increase its homogeneity or to minimize the distorting effects of underlying or procedural changes on the data should be fully explored. The quality and reliability of the various kinds of available data should also influence the choice of the forms of data to be analyzed. (Section II).

3. Whenever the impact of changes in company procedures or operations on loss data can be isolated or reasonably quantified, adjustment of the data may be advisable before applying various reserving methods. Whenever possible, the underlying assumptions of each method should be tested statistically. It may be possible to adjust the historical data so that the underlying assumptions of a method are more nearly satisfied. New projections may then be computed. (Section III).

4. No single reserving method can possibly produce the best estimates in all situations. Every reserving method is based on certain underlying assumptions, which may or may not be satisfied in a given situation.

Thus, several methods should be applied. Where possible, these should include:[1]

A. Projections of incurred losses,

B. Projections of paid losses,

C. Projections of ultimate reported claims and ultimate losses per ultimate reported claim,

D. Estimates of the number and average amount of outstanding losses, and

E. Loss ratio estimates.

Wherever appropriate, the concepts of credibility, regression analysis and data smoothing should be incorporated into the actuarial methods utilized. The methods applied should range from those which are highly stable (i.e., representative of the average of experience over several years) to those which are highly responsive to trends and to more recent experience. The actuary must then decide which methods provide the appropriate balance between stability and responsiveness in accordance with the credibility of the data and whether or not past trends may be expected to continue into the future. (Section IV).

5. In determining which methods are believed to be the best in a given situation, the following procedures should be implemented: (Section V).

   A. Whenever regression analysis has been incorporated into a method, some measure of goodness of fit (such as the coefficient of determination)[2] should be noted in evaluating the appropriateness of that method's projections. Additionally, the possibility that seasonal variations or cycles have been mistaken for trends should be carefully explored.

[1] Ruth Salzmann, "Estimated Liabilities for Losses and Loss Adjustment Expenses," Chapter 3, *Property-Liability Insurance Accounting*, ed. Robert W. Strain, The Merritt Company, Santa Monica, California, 1974; and David Skurnick, "A Survey of Loss Reserving Methods," PCAS, Vol. LX (1973), p. 16.

[2] G. G. C. Parker and E. L. Segura, "How to Get a Better Forecast," *Harvard Business Review*, March-April 1971, p. 99; and D. L. McLagan, "A Non-Econometrician's Guide to Econometrics," *Business Economics*, Vol. VIII, No. 3, May 1973, p. 38.

B. Whenever sufficient loss history is available, each method should be tested retrospectively to determine its historical record of accuracy and freedom from bias in projecting future paid losses. The projections of each method should then be adjusted for any detectable bias.

C. Significant differences between the projections of the various methods should be explained, where possible, in terms of changes in company procedures and operations. The convergence of the projections of several methods after the data has been adjusted for changes or trends in company procedures and operations (see Section III) may serve to considerably narrow the range of reasonable reserve estimates.

D. In making the final selections, the actuary must attach judgmental credibilities to basic as well as sophisticated methods as applied to both unadjusted and adjusted data. These judgmental credibilities should be based upon an evaluation of the relative strengths and weaknesses of each method in the context of the data to which it is applied.

E. A final check which should be applied to the selected estimates for the most recent accident years or quarters is a review of the loss ratios, pure premiums, frequencies and severities by accident period which result from those selections. The reasonableness of such statistics, when compared with those of immediately prior accident periods, may increase confidence in the reserve estimation process or raise questions which must be more thoroughly investigated before a conclusion is reached.

## I. GATHERING DATA AND SEARCHING FOR PROBLEM AREAS

The first part of gathering information is the review of all available sources of data which may be reasonably utilized in a reserve analysis. It is, of course, unlikely (and unnecessary) that any given company will have all the data in the detail prescribed in Appendix A.

The reconciliation of the most important source documents and tabulations for a reserve analysis with the Annual Statement or with other public documents or audited data is a necessary and often instructive exercise.

Frequently an inability to reconcile reveals an unsuspected missing piece of the book of business which will also require analysis. Additionally, a review of the available data, with an eye to spotting significant shifts, changes, and seeming irregularities, can raise many questions. When such questions are directed to top management as well as underwriters, claims and data processing personnel, and accountants, they can yield invaluable insights into the interpretation of the history of losses which often could never have been obtained through the most sophisticated statistical analyses.

Another integral part of a reserve analysis is the development of a deeper understanding of changes in company operations which have occurred during the experience period. Such changes frequently result in distortions in the loss history that the actuary is analyzing in his attempts to forecast future developments and trends. The actuary must concern himself seriously with the task of determining the nature of such changes and the extent to which such changes have affected the data under analysis. To do this, the actuary should engage in discussions with the most knowledgeable members of management within the underwriting, claims, data processing, and accounting departments and with the actuaries specializing in ratemaking.

Appendix B provides a sampling of the kinds of questions which can be directed to the management of the various departments in an effort to pinpoint problem areas and to more accurately interpret the loss data and reserve projections. Throughout the course of these discussions, the unquestioning acceptance of opinions should naturally be avoided. Wherever possible, supplementary data should be sought to support and to quantify (or to counter) the opinions expressed.

## II. TREATING PROBLEM AREAS THROUGH DATA SELECTION AND REARRANGEMENT

Appendix C provides a sampling of the types of problems which can seriously affect the consistency of loss data or cause subsequent losses to develop in ways markedly different from past patterns. To consistently and effectively deal with such problems, a systematic and analytic approach is often helpful. The following questions provide an outline of one such approach:

1. What type of event or trend could potentially cause data problems?

2. What is the expected impact of this problem on each of the available forms of data? On each of the proposed reserve methods?

3. Will this problem result in shifts in the loss data between *successive* accident years? Calendar years? Report years? Policy years? Years of development? Are such shifts observable in the loss data?

4. Is the problem serious enough to warrant further attention?

5. Will the problem be so serious as to render past history irrelevant in predicting future developments?

6. What forms of data and actuarial methods will be substantially unaffected by this problem? How can these be used in a reserve analysis? Can the available data be subdivided or reorganized to isolate the problem?

7. Does there exist supplementary data which accurately quantifies the magnitude of the impact of the problem?

Essentially, there are two stages in this analysis. In the first, the nature of the problem is defined. Its impact is estimated and, whenever possible, accurately quantified. In the second stage, the search for solutions, one of two general approaches is followed:

1. Utilization of data and actuarial methods which are relatively unaffected by the problem.

2. Accurate quantification of the impact of the problem and the application of adjustments to the data before utilizing the various reserve methods.

The first approach will be discussed in this section and the second in Section III.

Two primary means may be employed in obtaining data which is relatively unaffected by a given problem. The first is the selection of substitute types or forms of data. Examples of this would include the following:

1. Utilization of earned exposures in place of claim counts when count data is of questionable accuracy or there has been a major change in the definition of claim count.

2. Substitution of policy year data for accident year data when there has been a significant change in policy limits or deductibles between successive policy years.

3. Substitution of report year data for accident year data when there has been a dramatic shift in the social or legal climate which causes claim severity to more closely correlate with the report date than with the accident date.

4. Substitution of accident quarter for accident year data when the rate of growth of earned exposures changes markedly, causing distortions in development factors due to significant shifts in the average accident date within each exposure period.

The second means of obtaining relatively unaffected data is that of subdividing the loss experience into more homogeneous groups of exposures and/or types of claims. This is particularly desirable whenever there have been major changes in the composition of business by state, subline, class, territory or size of risk. However, it may not be advisable if it results in a marked decline in the credibility of each new block of experience.

The subdivision of loss experience into more homogeneous types of claims is particularly important whenever the types of claims in the experience are widely heterogeneous or a particular procedural change impacts only a few types of claims. While it may be possible to recompile loss experience based on types of claims (e.g., property versus liability losses under multi-peril policies), it is sometimes more expedient to use certain characteristics of various types of claims to segregate their loss experience fairly effectively. Such characteristics include the lag between accident date and settlement date or adjuster's estimates of incurred losses. For example, in homeowners multi-peril, claims closed within the first two years of development are primarily property claims while those closed after the first two years are primarily liability claims. This observation suggests that claims closing after the first two years of development should be analyzed separately from those which closed within the first two development years.

Another effective means of accomplishing the segregation of claims into more homogeneous groups is the analysis of loss experience by separate size of loss categories or separate layers of loss.[3] Examples of this technique which have long been used in ratemaking are the separation of basic limits from total limits experience[4] and the determination of catastrophe loadings.[5] Similar procedures should also be employed in reserve analyses whenever large claims comprise a significant portion of total losses. An important refinement which should be a part of any size of loss analysis is that the definitions of each category should be adjusted for inflation over each successive accident year, as shown below.

| Size of loss or layer of loss | Accident year | Months of Development | | |
|---|---|---|---|---|
| | | 12 | 24 | 36 |
| $ 1– 99 | 1974 | x | x | x |
| 1– 109 | 1975 | x | x | |
| 1– 120 | 1976 | x | | |
| $ 100– 999 | 1974 | x | x | x |
| 110– 1,099 | 1975 | x | x | |
| 121– 1,209 | 1976 | x | | |
| $ 1,000– 9,999 | 1974 | x | x | x |
| 1,100–10,999 | 1975 | x | x | |
| 1,210–12,099 | 1976 | x | | |
| $10,000 & over | 1974 | x | x | x |
| 11,000 & over | 1975 | x | x | |
| 12,100 & over | 1976 | x | | |

One problem which is susceptible to the size of loss approach is that of shifts in emphasis by the claims department on priorities in settling large versus small claims. Such a shift can cause major distortions in the loss

[3] Ruth Salzmann, "Rating by Layer of Insurance," PCAS, Vol. L (1963), p. 15; David R. Bickerstaff, "Automobile Collision Deductibles and Repair Cost Groups: The Lognormal Model," PCAS, Vol. LIX (1972), p. 68; Robert J. Finger, "Estimating Pure Premiums By Layer—An Approach," PCAS, Vol. LXIII (1976), p. 34; and Charles A. Hachemeister, "Breaking Down the Loss Reserving Process."

[4] Jeffrey T. Lange, "The Interpretation of Liability Increased Limits Statistics," PCAS, Vol. LVI (1969), p. 163.

[5] Michael A. Walters, "Homeowners Insurance Ratemaking," PCAS, Vol. LXI (1974), p. 15.

projections of nearly all reserving methods. This problem may be adequately dealt with by analyzing loss history separately by size of loss category. Within each size of loss category, paid losses should be examined at equal percentiles of claims closed. (See Section III).

The analysis of loss experience by size of loss categories may also be quite effective in handling the problem of changes in the claims procedures for very small or trivial claims. For example, when guidelines for the establishment of a claim file for very small claims are changed, such a change may result in noticeable distortions in claim count data. These distortions may adversely affect frequency and severity projections for either rate-making or reserving purposes. By defining several size of loss categories so that the experience for the very small claims is isolated, such distortions in count data can be adequately treated.

## III. TREATING PROBLEM AREAS THROUGH DATA ADJUSTMENT

Whenever reformulations of the format of the data base will not yield satisfactory solutions to problems such as those enumerated in Appendix C, the primary alternative is the accurate quantification of the extent of the problem and the application of adjustments to the loss experience before utilizing it to estimate reserves. The existence of supplementary data which can accurately quantify the magnitude of the change should be fully explored in communications with other departments. In general, the nature of the problem and the kind of supplementary data available will often suggest the types of data adjustments to be made. The two most common problems encountered in reserve analyses are treated specifically in the remainder of this section.

### *Detecting Changes in the Adequacy Level of Case Reserves and Reducing the Impact of Such Changes on Incurred Loss Projections*

The sensitivity of projections of ultimate losses based on incurred loss development factors to changes in the adequacy level of case reserves increases significantly for the long-tail lines. To illustrate this sensitivity and to indicate a general method for significantly reducing the distortions created by changing case reserve adequacy, an example from medical malpractice will be explained in this subsection.

The development of incurred losses for the eight most recent accident years and projections of ultimate losses based on average development factors is displayed in Exhibit A. Before utilizing the incurred projections derived in Exhibit A for reserving purposes, the primary underlying assumption of the incurred loss development method should be tested. Has the adequacy level of case reserves remained relatively constant during the experience period[6]? Several approaches may be taken in testing this hypothesis, but only one will be discussed here. In this approach, severity trends derived from changes in case reserves per open claim (Exhibit B) for each separate year of development are compared with severity trends in paid losses per closed claim (Exhibit C) for each separate year of development as well as over successive calendar years. The severity trends obtained from the fitting of exponential curves to the case reserves per open claim from Exhibit B range from + 27.6% to 34.2%, with the exception of such averages at 12 months of development. In contrast, the severity trends derived from the array of paid losses per closed claim in Exhibit C range from + 6.7% to 14.3%. Furthermore, the traditional approach of estimating the severity trend from the fitting of an exponential curve to calendar year paid losses per closed claim produces a trend of + 15.0% (with a coefficient of determination of .9793).

In the above example, no evidence was found which supported the notion that the severity trend for paid losses was inaccurate, and the indicated severity trend of + 15.0% was close to that experienced by many malpractice carriers. Thus, severity trends on the order of + 30% derived from changes in case reserves per open claim were rejected as unreasonable and the + 15% severity trend was selected as being representative of the underlying trend. The + 15% severity trend was thus used as the basis for adjusting the magnitude of case reserves in past years to their approximate value under the assumption that they are at the same relative adequacy level as the case reserves as of December 31, 1976. Working separately within each column of the array shown in Exhibit B, the value of case reserves per open claim as of December 31, 1976, was selected as the basis for readjusting the case reserves for past years. The year-end 1976 average case reserve was reduced by 15% per year for each year of development separately to obtain estimates of adjusted case reserves per open claim. Each adjusted average reserve estimate was then multiplied by the corresponding number of open claims (Exhibit D) to obtain an estimate of case reserves for some past year which is on approximately the same adequacy level as the year-end

[6] W. H. Fisher and E. P. Lester, "Loss Reserve Testing in a Changing Environment," PCAS, Vol. LXII (1975).

1976 case reserves. Each recomputed reserve was then added to the corresponding amount of cumulative paid losses (Exhibit E) to obtain a hypothetical history of incurred losses (Exhibit F) based on a relatively constant level of adequacy. The incurred projections obtained by again accepting the arithmetic mean of the development factors for each respective column are shown in the last column of Exhibit F.

In this example, the historical values of cumulative paid losses were adjusted to reduce the impact of increases in the rate of settlement of claims. Exhibit G provides a comparison of the reserve estimates derived from the incurred and the paid projections, both before and after the above adjustments. The aggregate difference between the paid and incurred estimates of loss reserves was reduced by 80% by applying the above adjustments, and apparent overstatements in those estimates were markedly reduced by these adjustments.

*Detecting Changes in the Rate of Settlement of Claims and Adjusting Paid Losses for Such Changes*

The importance of recognizing the impact of shifts in the rate of settlement of claims upon historical paid loss data has received previous attention in the Proceedings.[7] In this section a specific numerical method for making adjustments for changes in settlement rates will be described in detail. Exhibit H displays the accident year history of cumulative paid losses for automobile B.I. liability which will be adjusted for changing settlement rates. Exhibits I and J show the corresponding history of cumulative closed and cumulative reported claims. For each accident year, the ultimate claims disposed ratios contained in Exhibit K were derived by dividing the cumulative closed claims in Exhibit I by the projected ultimate number of reported claims in Exhibit J. Close examination of each column of claims disposed ratios for trends should reveal any persistent shifts in settlement rates. Caution should be exercised in this analysis and the impact of any procedural changes within the company should be particularly noted in terms of their influence on the claim count data from which these ratios were derived. In general, however, the absence of trend within the columns of Exhibit K indicates that no adjustment to the paid loss history in Exhibit

[7] David Skurnick, Discussion of "Loss Reserve Testing: A Report Year Approach" (W. H. Fisher and J. T. Lange, PCAS, Vol. LX (1973), p. 189), PCAS, Vol. LXI (1974), p. 73.

H would be recommended before analysis of such data by various actuarial methods.

Skurnick[8] has described a general approach to be taken in making adjustments for changing settlement rates. However, data in the format that Skurnick prescribes is frequently not available from many companies. A few minor modifications of Skurnick's approach, however, yields a more general method which can be applied to loss data maintained by most companies.

The first step in this process is the identification of a mathematical curve which closely approximates the relationship between the cumulative number of closed claims (X) and cumulative paid losses (Y). In the case of the automobile B.I. data in Exhibits H and I, a curve of the form $Y = ae^{bX}$ fits exceptionally well. As Exhibit L indicates, the coefficient of determination of this curve, when fitted to the loss data from Exhibits H and I for accident year 1969, is .99573. This coefficient increases to .99821 when the first point is dropped. Of course, a different curve may be required for a different company or line of business and it may be that no simple mathematical function reasonably describes the above relationship. In that event, generalized numerical methods, such as Lagrange's formula[9], may be applied in the interpolation process.

Since the exponential curve ($Y = ae^{bX}$) very closely approximates the relationship between cumulative closed claims and cumulative paid losses in our example, it may be used as the basis for exponential interpolation in applying adjustments for shifting claims disposed ratios. First, a representative claims disposed ratio was selected for each year of development. Selection of the claims disposed ratios for the latest calendar year of the experience (1976) possesses some key advantages. First, it eliminates the need for extrapolation into the future in making adjustments, and second, it leaves the most recent values of cumulative paid losses for each accident year unadjusted. However, some adjustments may be necessary in the event that these selected ratios do not progress upward in a smooth fashion from lower to higher years of development.

The claims disposed ratios for calendar year 1976 appear as the column headings in Exhibit M. These ratios are then applied to the projected

[8] *Ibid*, p. 83.

[9] Stephen G. Kellison, *Fundamentals of Numerical Analysis*, Richard D. Irwin, Inc., Homewood, Illinois, 1975, pp. 100-102.

ultimate number of reported claims for each accident year to obtain the number of cumulative closed claims which would be equivalent to the indicated claims disposed ratio for that year of development and accident year. For example, for accident year 1969, a selected claims disposed ratio of 88.55% for 36 months of development is equivalent to 6,926 cumulative closed claims. Since the coefficient of determination (.99821) of the exponential curve is exceptionally high, interpolation by means of only a two point curve fit seems appropriate. In order to approximate the value of cumulative paid losses which corresponds to 6,926 cumulative closed claims, the exponential curve ($Y = ae^{bX}$) is fitted to the two points (6,616, $5,398,000) and (7,192, $7,496,000) for accident year 1969 (from Exhibits H and I). The resultant approximation of $6,441,000, as well as other similarly derived estimates, are shown in Exhibit N. These adjusted estimates of cumulative paid losses may then be analyzed by the methods described in Section IV (or by other suitable mathematical procedures) to derive a set of reserve estimates.

## IV. APPLYING A VARIETY OF RESERVING METHODS

In this section, some of the methods of projection frequently utilized in reserve analyses will be described. The specific methods presented in this section are representative of those which we are currently utilizing and serve only as examples of what we believe are acceptable procedures. These methods will, of course, undergo refinement as continuing advances are made in actuarial science.

In this example, the data analyzed by these methods is the unadjusted automobile B.I. data introduced in Section III. This data is in the form of paid losses per ultimate reported claim. Projections of the ultimate number of reported claims were first derived from an analysis of the historical development of cumulative reported claims contained in Exhibit J. These estimates are shown in the last column of Exhibit J by accident year. For each individual accident year, cumulative paid losses at the end of each year of development (Exhibit H) were then divided by the projected ultimate number of reported claims. The resultant averages are shown in the upper portion of Exhibit O. For each accident year, two sets of averages are shown above the diagonal. The first is that of development year paid losses per ultimate claim, while the second is that of cumulative paid losses per ultimate claim. This array of averages was then analyzed by six projection methods and the resultant projections are shown in the lower triangle of

Exhibit O. These projections are displayed in clusters of six for ease of comparison, with the Method I estimate at the top, the Method II estimate next, and so forth. Usually an estimate is selected for each cluster of estimates or a particular method is selected and its estimates are totalled.

The methods described in this section are not limited in application to accident period data such as paid losses per ultimate claim. They may also be applied to report or policy period loss history (either paid or incurred) which is in a triangular form. While Methods I, II and V do not require that the loss data be divided by claim counts or exposures, Methods III, IV and VI do require this (unless the volume of business over the experience period has been changing at a constant rate)[10]. Each of these methods may also be applied to accident period arrays of reported claims or reported claims per earned exposure.

For the purpose of describing these methods, mathematical notation will be introduced in order to shorten the narrative. The following matrices will frequently be mentioned in this section:

A — Paid losses per ultimate claim

C — Cumulative paid losses per ultimate claim

D — Development factors of cumulative paid losses

A and C are (m) x (n) matrices ($m \geq n$) while D is ($m - 1$) x ($n - 1$). The A and C matrices represent loss data for m exposure periods over n periods of development:

PAID LOSS MATRIX

| Exposure Period | Development Period 1 | 2 | | | | n |
|---|---|---|---|---|---|---|
| 1 | $a_{1,1}$ | $a_{1,2}$ | • | • | • | $a_{1,n}$ |
| 2 | $a_{2,1}$ | $a_{2,2}$ | • | | | $a_{2,n}$ |
| • | • | • | | • | | • |
| • | • | • | | | • | • |
| • | • | • | | | | • |
| m-1 | $a_{m-1,1}$ | $a_{m-1,2}$ | • | • | • | $a_{m-1,n}$ |
| m | $a_{m,1}$ | $a_{m,2}$ | • | • | • | $a_{m,n}$ |

where $a_{i,j} = 0$ if $i + j \geq m + 2$.

[10] Methods III, IV and VI are primarily based on the application of estimated trend factors to loss statistics which have been divided by some measure of the volume of business or of claims. Such statistics would include claim frequency or severity, pure premium or paid losses per ultimate claim, but not incurred losses or paid losses.

The matrix D of development factors of cumulative paid losses is identical to that of development factors of the matrix C since a constant divisor is used for each accident year in deriving C from the array of cumulative paid losses.

In general terms, the six methods differ in terms of the data from which trend factors are estimated, the statistical technique utilized in estimating the trend factors and the data to which the estimated trend factors are applied in making projections. The following table summarizes these differences:

| Method | Data from which Trend Factors are Estimated | Technique for Estimating Trend Factors | Data to which Trend Factors are Applied |
|---|---|---|---|
| | Projections of Paid Loss Development Factors | | |
| I | D | Linear regression | D |
| II | D | Weighted average | D |
| V | D-1 | Adjusted exponential | D-1 |
| | Estimates of Claim Cost Growth Rates | | |
| III | C | Exponential curve fit | A |
| IV | C | Adjusted exponential | A |
| VI | A | Adjusted exponential | A |

As this table indicates, Methods I, II and V are based upon projections of paid loss development factors. These methods differ only with respect to the statistical technique which is applied to the paid loss development factors (Exhibit P) in order to project the factors shown in Exhibit Q. In Method I, linear regression projections are determined separately for each year of development. In Method II, a weighted average of the development factors is computed for each column (development year) of Exhibit P. As can be seen from Exhibit Q, this weighted average is assumed to be constant for each year of development. In Method V, an adjusted exponential projection technique is applied. An exponential growth rate (trend factor) is first determined for each column of the array D-1 (1.0 is subtracted from each factor shown in Exhibit P). A weighted average of these growth rates is then obtained for the entire matrix. This weighted average is then "credibility weighted" with the initially determined growth rate of each column to determine the adjusted growth rate for that column. This adjusted growth rate is then utilized in projecting the development factors shown in the Method V section of Exhibit Q. For each of Methods I, II, and V, the development factors shown in Exhibit Q are applied successively to the corresponding average of cumulative paid losses per ultimate claim to estimate such cumulative averages for each future development period. These cumulative averages are then de-cumulated to obtain the estimates shown in Exhibit O.

The computations utilized in the development of estimates by Methods III, IV and VI all involve the following steps:

1. Estimation of claim cost growth rates for each year of development.

2. Utilization of the estimated growth rates to increase historical values of paid losses per ultimate claim during a given year of development to the estimated calendar year 1977 claim cost level.

3. Estimation of paid losses per ultimate claim during calendar year 1977 for the given year of development by computing a weighted average of past paid losses per ultimate claim on the estimated 1977 claim cost level.

4. Estimation of paid losses per ultimate claim during calendar years beyond 1977 by successive applications of the estimated claim cost growth rate to the estimate for calendar year 1977.

The calculations required by the second and third steps above are shown in Exhibit R for Methods III, IV and VI. The calculations shown are only those required to compute the estimates of paid losses per ultimate claim for accident year 1974 during calendar year 1977 (the fourth year of development). These calculations differ only in terms of the trend factors in Column 2 which are used to increase past paid losses per ultimate claim to the 1977 cost level. In this example, square weights are utilized but many other weighting procedures, as deemed appropriate, may be used here.

The calculations required by the fourth step of Methods III, IV and VI are illustrated in Exhibit S. For each method and year of development, a constant growth rate is successively applied to the estimate of that method for calendar year 1977.

In the above discussion, a broad framework within which the various methods may be viewed has been described. The remainder of this section contains a more precise, mathematical description of those methods.

*Method I—Linear Regression Estimates of Paid Loss Development Factor*

In this method, a linear least squares trend line is fitted to each column of D which contains three or more development factors. These fitted regression lines are then used to project down each column to provide estimates of the future development factors. Let D denote the array that includes all the projections, then

$$D = \begin{vmatrix} d_{1,1} & d_{1,2} & \cdot & \cdot & \cdot & d_{1,n-1} \\ d_{2,1} & d_{2,2} & \cdot & \cdot & \cdot & d_{2,n-1} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ d_{m-1,1} & \hat{d}_{m-1,2} & \cdot & \cdot & \cdot & \hat{d}_{m-1,n-1} \\ d_{m,1} & \hat{d}_{m,2} & \cdot & \cdot & \cdot & \hat{d}_{m,n-1} \end{vmatrix}$$

where each $\hat{d}_{i,j}$ denotes a linear least squares estimate (if $m \geq n + 2$).

If $m = n$, the last two columns are taken to be

$\hat{d}_{i,n-1} = d_{1,n-1}, i = 2, 3, \cdots, m$ and

$\hat{d}_{i,n-2} = (d_{1,n-2} + d_{2,n-2})/2, i = 3, 4, \cdots, m.$

If $m = n + 1$, the last column is defined by

$\hat{d}_{i,n-1} = (d_{1,n-1} + d_{2,n-1})/2, i = 3, 4, \cdots, m.$

In both of these special cases, the remaining $\hat{d}_{i,j}$'s are the linear least squares estimates.

The Method I estimates of average paid losses per ultimate claim can then be calculated as follows, for $1 \leq k \leq m$, $1 \leq j \leq n$:

$$\hat{c}_{k,j} = \begin{cases} c_{k,j} & \text{if } k + j < m + 2 \\ c_{k,m+1-k} \times \prod_{l=m+1-k}^{j-1} \hat{d}_{k,l} & \text{if } k + j \geq m + 2 \end{cases}$$

$$\text{and} \quad \hat{a}_{i,j} = \begin{cases} \hat{c}_{i,j} - \hat{c}_{i,j-1} & i + j \geq m + 2 \\ a_{i,j} & i + j < m + 2 \end{cases}$$

$\hat{A}_I$, the array of Method I estimates is then:

$$\hat{A}_I = \left| \begin{array}{ccccccc} a_{1,1} & a_{1,2} & \bullet & \bullet & \bullet & a_{1,n} \\ a_{2,1} & a_{2,2} & \bullet & \bullet & \bullet & a_{2,n} \\ \bullet & \bullet & \bullet & & & \bullet \\ \bullet & \bullet & & \bullet & & \bullet \\ \bullet & \bullet & & & \bullet & \bullet \\ a_{m-1,1} & a_{m-1,2} & \bullet & \bullet & \bullet & \hat{a}_{m-1,n} \\ a_{m,1} & \hat{a}_{m,2} & \bullet & \bullet & \bullet & \hat{a}_{m,n} \end{array} \right|$$

*Method II—Weighted Average Estimates of Paid Loss Development Factors*

A set of weights, $W = \{w_1, w_2, \cdots, w_{m-1}\}$, is first selected. These selections may be based upon such factors as the credibility of the data and/or a general assessment of the comparative relevance of newer (versus older) experience to future developments. In general, with data of low credibility and large random variations, weights of relatively equal magnitude should be used. With data of full credibility greater weight should generally be assigned to more recent experience simply on the grounds that it is likely to be more relevant to future developments.

For the jth column of D, the weighted average estimate is

$$\hat{d}_{i,j} = \frac{\sum_{k=1}^{m-j} d_{k,j} w_{j+k-1}}{\sum_{k=1}^{m-j} w_{j+k-1}}, i + j > m.$$

Thus, all of the projected development factors for any given column (i.e., period of development) are identical. The notational representation for D is the same as that for Method I. The $\hat{c}_{i,j}$'s and hence $\hat{A}_{II}$ are then calculated according to the same equations as those for Method I.

*Method III—Exponential Curve Fit Estimates of Claim Cost Growth Rates*

In Method III, a growth rate $\beta_j$, for the jth development period is computed by fitting an exponential curve $y = \alpha_j e^{\beta_j x}$ to the jth column of the matrix C. Once all $\beta_j$'s have been estimated, then each element of A is adjusted to current cost level by applying the appropriate power of $\gamma_j = e^{\beta_j}$ to $a_{i,j}$:

$$A' = \begin{vmatrix} \gamma_1^{m-1} \cdot a_{1,1} & \gamma_2^{m-2} \cdot a_{1,2} & \cdot & \cdot & \cdot & \gamma_n^{m-n} \cdot a_{1,n} \\ \gamma_1^{m-2} \cdot a_{2,1} & \gamma_2^{m-3} \cdot a_{2,2} & \cdot & \cdot & \cdot & \gamma_n^{m-n-1} \cdot a_{2,n} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \gamma_1^{2} \cdot a_{m-2,1} & \gamma_2 \cdot a_{m-2,2} & \cdot & \cdot & \cdot & 0 \\ \gamma_1 \cdot a_{m-1,1} & a_{m-1,2} & \cdot & \cdot & \cdot & 0 \\ a_{m,1} & 0 & \cdot & \cdot & \cdot & 0 \end{vmatrix}$$

A weighted average $\hat{a}_j$, of the adjusted $a_{i,j}$'s, $\{\gamma_j^{m-i-j+1} \cdot a_{i,j};\ i = 1, \cdots, m - j + 1\}$ is then computed for each column:

$$\hat{a}_j = \frac{\sum_{i=1}^{m-j+1} w_i \gamma_j^{m-i-j+1} a_{i,j}}{\sum_{i=1}^{m-j+1} w_i}$$

Each $\hat{a}_j$ is then projected into the future by applying the appropriate power of $\gamma_j$ to increase $\hat{a}_j$ from current cost level to expected future cost levels:

$$\hat{A}_{III} = \begin{vmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdot & \cdot & \cdot & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdot & \cdot & \cdot & a_{2,n} \\ \cdot & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot & \cdot \\ a_{m-1,1} & a_{m-1,2} & \gamma_3 \cdot \hat{a}_3 & \cdot & \cdot & \cdot & \gamma_n^{n-2} \cdot \hat{a}_n \\ a_{m,1} & \gamma_2 \cdot \hat{a}_2 & \gamma_3^{2} \cdot \hat{a}_3 & \cdot & \cdot & \cdot & \gamma_n^{n-1} \cdot \hat{a}_n \end{vmatrix}$$

*Method IV—Adjusted Exponential Estimates of Claim Cost Growth Rates*

Method IV is a modification of Method III. In Method IV, each $\gamma_j$ derived in Method III is adjusted to $\gamma_j'$ before it is applied in computing $\hat{a}_j$ and $\hat{a}_{i,j}$. These adjustments are made by the credibility-type formula

$$\gamma_j' = [w_j\gamma_j + (w_1 - w_j)\gamma]/w_1$$

$$\text{where } \gamma = \sum_{j=1}^{n} w_j\gamma_j \Big/ \sum_{j=1}^{n} w_j \cdot$$

and $w_j$ is taken to be $(m + 1 - j)^2$, the square of the number of historical averages in the jth column of A. The determination of $\{\gamma_j', j = 1, \cdots, n\}$

from $\{\gamma_j, j = 1, \cdots, n\}$ is illustrated in Exhibit T. In that exhibit, each $\gamma_j$ is referred to as an initial growth rate, $\gamma(1.0797)$ is the overall growth rate and each $\gamma_j'$ is an adjusted growth rate. Each $\gamma_j'$ may thus be viewed as the credibility weighted average of the initial growth rate, $\gamma_j$, and the overall growth rate, $\gamma$. In terms of Exhibit T, this equation becomes

$$\gamma_j' = \frac{\{(\text{Col. } 2 \times \text{Col. } 3) + (\Sigma\text{Col. } 4) \times ((64/203) - \text{Col. } 3)\}}{(64/203)}$$

In Method IV estimates are then calculated from the adjusted $a_{i,j}$'s, $\{(\gamma_j')^{m-i-j+1} \cdot a_{i,j};\ i = 1, \cdots, m - j + 1\}$ in the same way as for Method III.

*Method V—Adjusted Exponential Estimates of Paid Loss Development Factors*

The adjusted exponential projection technique utilized in Method V is completely analogous to that described for Method IV. The only differences between these methods are summarized in the first table of this section. In Method IV, $\{\gamma_j', j = 1, \cdots, n\}$ is derived from the matrix C, but in Method V it is derived from D-1. In Method V, $\{\gamma_j', j = 1, \cdots, n\}$ is applied to D-1, instead of to A (Method IV). The matrix D-1 is obtained by subtracting 1.0 from each nonzero element of D.

*Method VI—Adjusted Exponential Estimates of Claim Cost Growth Rates*

Method VI differs from Method IV only with respect to the matrix from which the set of initial growth rates, $\{\gamma_j\}$, is derived. For Method IV, the $\gamma_j$'s are derived from the matrix C, while for Method VI, they are derived from A. Both of these methods are applied to the matrix A in determining their estimates. Thus, Method VI is more responsive to the growth rates of paid losses per ultimate claim for higher years of development.

***************

The six methods described in this section provide an example of a group of methods which comprises a range of varying degrees of stability versus responsiveness. If these methods were ranked from the most stable to the most responsive, they would probably appear in this order: II, IV, III, I, VI, and V. Additionally, three of these methods are based on the development factor hypothesis, while the other three are based on a growth rate hypothesis. Stated in another way, Methods I, II and V are based upon projections across each row of the triangular array of loss data by means of estimated development factors. On the other hand, Methods III, IV and VI are based on projections down each column of the triangular array by means of estimated claims cost growth rates.

## V. ANALYZING THE PROJECTIONS OF THE VARIOUS METHODS AND DETERMINING SELECTIONS

Every method of estimating loss reserves is based on the general assumption that the future can in some way be extrapolated from the past. Each method is also based on various specific assumptions such as the consistent relative adequacy of case reserves (incurred projections) or consistent settlement rates (paid projections). To the extent that the underlying assumptions of a method are violated in a systematic and non-random manner, the projections of that method will likewise be systematically distorted. Thus, an evaluation of the extent to which the underlying assumptions of a method are violated should become a vital part of the process of making actuarial projections. When the actuary applies a variety of projection techniques and thereby obtains a range of reserve estimates, he is then faced with the task of making value judgments on the relative appropriateness of each method. The mere taking of an average of the initial estimates may not be a satisfactory approach, although this procedure has more merit than the blind acceptance of the projections of only one method. As the example of the medical malpractice estimates in Section III indicates, it may well happen that the actual range of reasonable (i.e., adjusted) estimates lies entirely outside the range of the initial estimates (see Exhibit G).

In terms of the automobile B.I. experience shown in Exhibits H through T, an examination of the primary underlying assumptions for projections of incurred and paid losses yielded the following observations:

First, the relative adequacy level of case reserves has increased significantly in the last four years, indicating that projections of incurred losses will most likely overestimate reserves.

Second, the rate of settlement of open claims has generally declined over the last eight years and has undergone a major drop between 1975 and 1976. Because of this decline, it may be expected that projections of paid losses will tend to underestimate loss reserves.

Application of the adjustments described in Section III to the automobile B.I. data results in a reduction of the incurred estimate of total loss reserves from $43.5 to $40.6 million and an increase in the paid projections from $35.3 to $42.5 million. These adjustments have thus resulted in reducing the difference between the incurred and the paid projections by 76%. A comparison of the initial and the adjusted loss reserve estimates is provided in Exhibit U.

*Retrospective Tests for Bias*

Retrospective tests for bias and accuracy can provide much information regarding the appropriateness of various methods in testing loss reserves. Such tests are, however, not infallible, for it may happen that the underlying changes in the data during the experience period (which caused a particular method to under or overestimate) may not continue to occur in the future. The actuary must therefore exercise judgment as to the validity of these tests as a measure of the accuracy and bias in future projections.

A method of estimating the accuracy and bias of each of the six methods described in Section IV is exemplified in Exhibit V. Method II projections are used in this illustration. The top section of Exhibit V displays some of the historical values of paid losses per ultimate claim from Exhibit O. For each of the averages contained in the top section, estimates of that average were developed by each of the six methods—based entirely on data from Exhibit O for calendar years prior to that of the given average. The estimates thereby developed by Method II are shown in the middle section of Exhibit V. The percentage deviations for Method II, are shown in the bottom section of Exhibit V. The average and median of these percentage deviations were then computed. The results for each method were as follows:

| Method | Average Deviation | Median Deviation |
|---|---|---|
| I | − 0.48% | − 0.48% |
| II | − 4.44 | − 4.95 |
| III | − 6.14 | − 7.16 |
| IV | − 6.64 | − 7.76 |
| V | + 2.16 | + 2.34 |
| VI | − 4.25 | − 4.89 |

Both the average and median deviations for each method (except V) are negative. This indicates that each of these methods has historically underestimated the actual values of paid losses per ultimate claim for the first calendar year subsequent to the known data. By way of comparison, the method of utilizing the arithmetic mean of the prior development factors would produce an average deviation of —6.28% and a median deviation of —8.02%. Thus, each of the above methods, with the possible exception of Method IV, would appear to be preferable to simply employing the mean of the development factors.

Under the hypothesis that the future projections of each method will have percentage deviations equal to the median deviation shown above, the projections of each method shown in Exhibit O may be adjusted for their expected bias. The median was selected instead of the mean since the latter can easily be distorted by extreme values. Since the formula for each percentage deviation is:

$$\text{Percentage Deviation} = \frac{100 \times (\text{Estimated Value} - \text{Actual Value})}{\text{Actual Value}}$$

a rearrangement of this equation becomes:

$$\text{Actual Value} = \frac{100}{100 + \text{Percentage Deviation}} \times \text{Estimated Value}$$

Thus, the adjustment factor for a projection of one year was taken to be the quantity, $\{100/(100 + \text{Percentage Deviation})\}$. As an approximation to the adjustment factor for a projection of N years, the quantity, $\{100/(100 + \text{Percentage Deviation})\}^N$, was used.

The coefficient of variation of the retrospectively adjusted estimates of the total reserve was 71% less than that of the initial estimates, indicating that the range between the various adjusted estimates is noticeably less than the range between the initial estimates. This observation tends to lend support to the appropriateness of the adjusted estimates.

*Selection of Estimates*

For each accident year, the selected estimate in Exhibit U is a weighted average of the various projections. The weights were selected on the basis of a judgmental assessment of the relative strengths and weaknesses of each method.

As a check of the reasonableness of the selected estimates in Exhibit U for the most recent accident years, the projections of ultimate losses corresponding to the selected reserve estimates, as well as the projections of the ultimate number of reported claims (Exhibit J), were translated into the resultant loss ratios, frequencies, severities and pure premiums:

| | Accident Year | | | Percentage Change | |
|---|---|---|---|---|---|
| | 1974 | 1975 | 1976 | AY 1975 / AY 1974 | AY 1976 / AY 1975 |
| Loss Ratio | 78.0% | 74.9% | 76.3% | −4.0% | +1.9% |
| Frequency | .0197 | .0195 | .0201 | −1.0 | +3.1 |
| Severity | $2,214 | $2,545 | $2,805 | +15.0 | +10.2 |
| Pure Premium | $43.65 | $49.57 | $56.39 | +13.6 | +13.8 |

Since the degree of variability possible in the above statistics for accident year 1974 is much smaller than for 1975 or 1976, it was chosen as the basis for comparison of the loss statistics of 1975 and 1976. The reasonableness of the percentage changes in the above statistics between accident year 1974 and 1975, and 1975 and 1976, serve to verify the accuracy of the selected loss reserve estimates for accident years 1975 and 1976. If, on the other hand, the selected estimates result in apparently unreasonable loss statistics, this should not lead to the modification of the selections unless further investigation provides sufficient justification.

## *Concluding Remarks*

We have gone to great lengths to explain *one* actuarial approach to the estimation of ultimate loss costs and the outstanding reserve associated with those costs. Although it has already been stated, we probably have not emphasized enough, that many times we do not have the luxury of obtaining *all* the data described herein. We have found, however, that by building a system which is designed to utilize such detail, we seem to obtain considerably more input data than one might expect.

Throughout this paper, we have attempted to emphasize that the successful reserving system must merge a great deal of basic information derived from "field oriented" executives *with* sophisticated actuarial methods.

Finally, it should be emphasized that however much the process of testing reserve adequacy may be refined and improved, reserve estimates will always be subject to a considerable degree of variability. The forecasting of future events is inherent in the act of estimating loss reserves. No matter how closely past events may be examined and analyzed, precise predictions of future events will never be obtainable. While this fact should serve to prevent us from becoming overconfident in our estimates, it should not, however, dissuade our profession from seeking to develop the best possible projections based on as much data and information as may be obtained at a reasonable cost. The importance of reserve analysis in safeguarding solvency and assisting the ratemaking actuary in the task of projecting ultimate losses for recent experience, should thoroughly convince us of the need for continuing advancements in this branch of actuarial science.

*EXHIBIT A*

# MEDICAL MALPRACTICE

## INCURRED LOSSES

(000's omitted)

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | Projected Ultimate |
|---|---|---|---|---|---|---|---|---|---|
| 1969 | $ 2,897 | $ 5,160 | $10,714 | $15,228 | $16,661 | $20,899 | $22,892 | $23,506 | $ 23,506 |
| 1970 | 4,828 | 10,707 | 16,907 | 22,840 | 26,211 | 31,970 | 32,216 | | 33,086 |
| 1971 | 5,455 | 11,941 | 20,733 | 30,928 | 42,395 | 48,377 | | | 52,247 |
| 1972 | 8,732 | 18,633 | 32,143 | 57,196 | 61,163 | | | | 79,634 |
| 1973 | 11,228 | 19,967 | 50,143 | 73,733 | | | | | 112,443 |
| 1974 | 8,706 | 33,459 | 63,477 | | | | | | 145,426 |
| 1975 | 12,928 | 48,904 | | | | | | | 215,275 |
| 1976 | 15,791 | | | | | | | | 175,991 |

AVERAGE INCURRED LOSS DEVELOPMENT FACTORS

| 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-96 |
|---|---|---|---|---|---|---|
| 2.532 | 1.921 | 1.503 | 1.171 | 1.205 | 1.052 | 1.027 |

*EXHIBIT B*

CASE RESERVE PER OPEN CLAIM

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 |
|---|---|---|---|---|---|---|---|---|
| 1969 | $ 3,817 | $ 5,660 | $ 9,262 | $10,151 | $11,793 | $16,627 | $19,238 | $21,423 |
| 1970 | 7,250 | 10,635 | 12,960 | 14,221 | 17,067 | 23,411 | 24,551 | |
| 1971 | 5,877 | 8,122 | 10,613 | 14,373 | 21,706 | 29,044 | | |
| 1972 | 8,324 | 11,433 | 15,499 | 25,040 | 28,019 | | | |
| 1973 | 10,124 | 13,785 | 30,223 | 33,266 | | | | |
| 1974 | 8,261 | 22,477 | 34,402 | | | | | |
| 1975 | 11,176 | 32,160 | | | | | | |
| 1976 | 13,028 | | | | | | | |
| Severity Trend | +15.3% | +29.5% | +31.1% | +34.2% | +32.8% | +32.2% | +27.6% | |

*EXHIBIT C*

PAID LOSSES PER CLOSED CLAIM

| Accident Year | MONTHS OF DEVELOPMENT | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0-12 | 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-96 |
| 1969 | $402 | $ 539 | $2,971 | $ 8,620 | $ 9,199 | $12,669 | $17,084 | $16,634 |
| 1970 | 110 | 919 | 5,487 | 9,129 | 12,403 | 18,452 | 19,533 | |
| 1971 | 706 | 1,115 | 5,644 | 4,928 | 12,994 | 14,948 | | |
| 1972 | 161 | 862 | 5,782 | 9,477 | 14,085 | | | |
| 1973 | 724 | 541 | 4,003 | 11,709 | | | | |
| 1974 | 518 | 1,394 | 7,635 | | | | | |
| 1975 | 517 | 1,494 | | | | | | |
| 1976 | 525 | | | | | | | |
| Severity Trend | +12.9% | +12.0% | +11.5% | +6.7% | +14.2% | +8.6% | +14.3% | |

*EXHIBIT D*

## MEDICAL MALPRACTICE

### OPEN CLAIMS

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 |
|---|---|---|---|---|---|---|---|---|
| 1969 | 749 | 840 | 1,001 | 1,206 | 1,034 | 765 | 533 | 359 |
| 1970 | 660 | 957 | 1,149 | 1,350 | 1,095 | 755 | 539 | |
| 1971 | 878 | 1,329 | 1,720 | 1,799 | 1,428 | 1,056 | | |
| 1972 | 1,043 | 1,561 | 1,828 | 1,894 | 1,522 | | | |
| 1973 | 1,088 | 1,388 | 1,540 | 1,877 | | | | |
| 1974 | 1,033 | 1,418 | 1,663 | | | | | |
| 1975 | 1,138 | 1,472 | | | | | | |
| 1976 | 1,196 | | | | | | | |

*EXHIBIT E*

## MEDICAL MALPRACTICE

### CUMULATIVE PAID LOSSES

(000's omitted)

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | Projected Ultimate |
|---|---|---|---|---|---|---|---|---|---|
| 1969 | $ 125 | $ 406 | $ 1,443 | $ 2,986 | $ 4,467 | $ 8,179 | $12,638 | $15,815 | $ 23,506 |
| 1970 | 43 | 529 | 2,016 | 3,641 | 7,523 | 14,295 | 18,983 | | 35,289 |
| 1971 | 295 | 1,147 | 2,479 | 5,071 | 11,399 | 17,707 | | | 46,322 |
| 1972 | 50 | 786 | 3,810 | 9,771 | 18,518 | | | | 83,220 |
| 1973 | 213 | 833 | 3,599 | 11,292 | | | | | 99,042 |
| 1974 | 172 | 1,587 | 6,267 | | | | | | 134,954 |
| 1975 | 210 | 1,565 | | | | | | | 124,997 |
| 1976 | 209 | | | | | | | | 112,042 |

*EXHIBIT F*

**ADJUSTED INCURRED LOSSES**

**(000's omitted)**

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | Projected Ultimate |
|---|---|---|---|---|---|---|---|---|---|
| 1969 | $ 3,707 | $12,085 | $18,564 | $25,924 | $23,516 | $24,979 | $24,017 | $23,506 | $ 23,506 |
| 1970 | 3,760 | 15,830 | 24,616 | 33,170 | 30,722 | 33,363 | 32,216 | | 31,539 |
| 1971 | 5,982 | 25,585 | 41,385 | 50,323 | 46,191 | 48,377 | | | 45,668 |
| 1972 | 7,819 | 33,795 | 51,362 | 64,559 | 61,163 | | | | 61,346 |
| 1973 | 9,533 | 34,586 | 49,668 | 73,733 | | | | | 68,719 |
| 1974 | 10,348 | 41,241 | 63,477 | | | | | | 78,965 |
| 1975 | 13,102 | 48,904 | | | | | | | 92,918 |
| 1976 | 15,791 | | | | | | | | 117,248 |

*EXHIBIT G*

## COMPARISON OF RESERVE ESTIMATES

(000's omitted)

| Accident Year | Before Adjustment: Incurred Projection | Before Adjustment: Paid Projection | Before Adjustment: Difference | After Adjustment: Incurred Projection | After Adjustment: Paid Projection | After Adjustment: Difference |
|---|---|---|---|---|---|---|
| 1969 | $ 7,691 | $ 7,691 | $ 0 | $ 7,691 | $ 7,691 | $ 0 |
| 1970 | 14,103 | 16,306 | −2,203 | 12,556 | 14,967 | −2,411 |
| 1971 | 34,540 | 28,615 | +5,925 | 27,961 | 25,607 | +2,354 |
| 1972 | 61,116 | 64,702 | −3,586 | 42,828 | 49,072 | −6,244 |
| 1973 | 101,151 | 87,750 | +13,401 | 57,427 | 79,665 | −22,238 |
| 1974 | 139,159 | 128,687 | +10,472 | 72,698 | 77,943 | −5,245 |
| 1975 | 213,710 | 123,432 | +90,278 | 91,353 | 88,931 | +2,422 |
| 1976 | 175,782 | 111,833 | +63,949 | 117,039 | 122,094 | −5,055 |
| | $747,252 | $569,016 | $+178,236 | $429,553 | $465,970 | $−36,417 |

*EXHIBIT H*

# AUTOMOBILE BODILY INJURY LIABILITY

## CUMULATIVE PAID LOSSES

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 |
|---|---|---|---|---|---|---|---|---|
| 1969 | $1,904 | $5,398 | $ 7,496 | $ 8,882 | $ 9,712 | $10,071 | $10,199 | $10,256 |
| 1970 | 2,235 | 6,261 | 8,691 | 10,443 | 11,346 | 11,754 | 12,031 | |
| 1971 | 2,441 | 7,348 | 10,662 | 12,655 | 13,748 | 14,235 | | |
| 1972 | 2,503 | 8,173 | 11,810 | 14,176 | 15,383 | | | |
| 1973 | 2,838 | 8,712 | 12,728 | 15,278 | | | | |
| 1974 | 2,405 | 7,858 | 11,771 | | | | | |
| 1975 | 2,759 | 9,182 | | | | | | |
| 1976 | 2,801 | | | | | | | |

*EXHIBIT I*

CUMULATIVE CLOSED CLAIMS

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 |
|---|---|---|---|---|---|---|---|---|
| 1969 | 4,079 | 6,616 | 7,192 | 7,494 | 7,670 | 7,749 | 7,792 | 7,806 |
| 1970 | 4,429 | 7,230 | 7,899 | 8,291 | 8,494 | 8,606 | 8,647 | |
| 1971 | 4,914 | 8,174 | 9,068 | 9,518 | 9,761 | 9,855 | | |
| 1972 | 4,497 | 7,842 | 8,747 | 9,254 | 9,469 | | | |
| 1973 | 4,419 | 7,665 | 8,659 | 9,093 | | | | |
| 1974 | 3,486 | 6,214 | 6,916 | | | | | |
| 1975 | 3,516 | 6,226 | | | | | | |
| 1976 | 3,230 | | | | | | | |

*EXHIBIT J*

CUMULATIVE REPORTED CLAIMS

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | Projected Ultimate |
|---|---|---|---|---|---|---|---|---|---|
| 1969 | 6,553 | 7,696 | 7,770 | 7,799 | 7,814 | 7,819 | 7,820 | 7,821 | 7,822 |
| 1970 | 7,277 | 8,537 | 8,615 | 8,661 | 8,675 | 8,679 | 8,682 | | 8,674 |
| 1971 | 8,259 | 9,765 | 9,884 | 9,926 | 9,940 | 9,945 | | | 9,950 |
| 1972 | 7,858 | 9,474 | 9,615 | 9,664 | 9,680 | | | | 9,690 |
| 1973 | 7,808 | 9,376 | 9,513 | 9,562 | | | | | 9,590 |
| 1974 | 6,278 | 7,614 | 7,741 | | | | | | 7,810 |
| 1975 | 6,446 | 7,884 | | | | | | | 8,092 |
| 1976 | 6,115 | | | | | | | | 7,594 |

*EXHIBIT K*

# AUTOMOBILE BODILY INJURY LIABILITY

## ULTIMATE CLAIMS DISPOSED RATIOS

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 |
|---|---|---|---|---|---|---|---|---|
| 1969 | .52148 | .84582 | .91946 | .95807 | .98057 | .99067 | .99616 | .99795 |
| 1970 | .51002 | .83257 | .90960 | .95474 | .97812 | .99102 | .99574 | |
| 1971 | .49387 | .82151 | .91136 | .95658 | .98101 | .99045 | | |
| 1972 | .46409 | .80929 | .90268 | .95501 | .97719 | | | |
| 1973 | .46079 | .79927 | .90292 | .94818 | | | | |
| 1974 | .44635 | .79565 | .88553 | | | | | |
| 1975 | .43450 | .76940 | | | | | | |
| 1976 | .42534 | | | | | | | |

*EXHIBIT L*

UTILIZATION OF AN EXPONENTIAL CURVE TO ESTIMATE

CUMULATIVE PAID LOSSES

Accident Year 1969

| Months of Development | X Cumulative Closed Claims | Y Cumulative Paid Losses | Predicted Y Value ($Y = ae^{bX}$) (8 Points) | Predicted Y Value ($Y = ae^{bX}$) (7 Points) |
|---|---|---|---|---|
| 12 | 4,079 | $ 1,904 | $ 1,850 | |
| 24 | 6,616 | 5,398 | 5,885 | $ 5,443 |
| 36 | 7,192 | 7,496 | 7,653 | 7,439 |
| 48 | 7,494 | 8,882 | 8,783 | 8,762 |
| 60 | 7,670 | 9,712 | 9,518 | 9,639 |
| 72 | 7,749 | 10,071 | 9,867 | 10,061 |
| 84 | 7,792 | 10,199 | 10,062 | 10,299 |
| 96 | 7,806 | 10,256 | 10,127 | 10,377 |
| | Coefficient of Determination | | .99573 | .99821 |
| | | a | $287.741 | $150.625 |
| | | b | .000456 | .000542 |

*EXHIBIT M*

# AUTOMOBILE BODILY INJURY LIABILITY

## CUMULATIVE CLOSED CLAIMS

At Equal Percentiles of Ultimate Claims Closed

| Accident | PERCENTAGE OF ULTIMATE CLAIMS CLOSED | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Year | 42.53% | 76.94% | 88.55% | 94.82% | 97.72% | 99.05% | 99.57% | 99.80% |
| 1969 | 3,327 | 6,018 | 6,926 | 7,417 | 7,644 | 7,748 | 7,788 | 7,806 |
| 1970 | 3,693 | 6,681 | 7,690 | 8,234 | 8,486 | 8,602 | 8,647 | |
| 1971 | 4,232 | 7,656 | 8,811 | 9,435 | 9,723 | 9,855 | | |
| 1972 | 4,121 | 7,455 | 8,580 | 9,188 | 9,469 | | | |
| 1973 | 4,079 | 7,379 | 8,492 | 9,093 | | | | |
| 1974 | 3,322 | 6,009 | 6,916 | | | | | |
| 1975 | 3,442 | 6,226 | | | | | | |
| 1976 | 3,230 | | | | | | | |

*EXHIBIT N*

CUMULATIVE PAID LOSSES

At Equal Percentiles of Ultimate Claims Closed

| Accident | PERCENTAGE OF ULTIMATE CLAIMS CLOSED | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Year | 42.53% | 76.94% | 88.55% | 94.82% | 97.72% | 99.05% | 99.57% | 99.80% |
| 1969 | 1,398 | 4,222 | 6,441 | 8,506 | 9,585 | 10,066 | 10,187 | 10,256 |
| 1970 | 1,705 | 5,116 | 7,845 | 10,160 | 11,309 | 11,739 | 12,031 | |
| 1971 | 1,938 | 6,168 | 9,580 | 12,261 | 13,571 | 14,235 | | |
| 1972 | 2,191 | 7,127 | 11,034 | 13,843 | 15,383 | | | |
| 1973 | 2,523 | 7,892 | 11,943 | 15,278 | | | | |
| 1974 | 2,240 | 7,189 | 11,771 | | | | | |
| 1975 | 2,670 | 9,182 | | | | | | |
| 1976 | 2,801 | | | | | | | |

*EXHIBIT O*

# AUTOMOBILE BODILY INJURY LIABILITY

## A COMPARISON OF THE ESTIMATES OF METHODS I-VI PAID LOSSES PER ULTIMATE CLAIM

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 |
|---|---|---|---|---|---|---|---|---|
| 1969 | $244 | $ 447 | $ 268 | $ 177 | $ 106 | $ 46 | $ 16 | $ 7 |
| | 244 | 690 | 958 | 1,136 | 1,242 | 1,287 | 1,304 | 1,311 |
| 1970 | 257 | 464 | 280 | 202 | 104 | 47 | 32 | 8 |
| | 257 | 721 | 1,001 | 1,203 | 1,307 | 1,353 | 1,385 | 8 |
| | | | | | | | | 8 |
| | | | | | | | | 8 |
| | | | | | | | | 8 |
| | | | | | | | | 8 |
| 1971 | 245 | 493 | 333 | 200 | 110 | 49 | 26 | 8 |
| | 245 | 738 | 1,072 | 1,272 | 1,382 | 1,431 | 27 | 8 |
| | | | | | | | 27 | 9 |
| | | | | | | | 28 | 9 |
| | | | | | | | 31 | 9 |
| | | | | | | | 31 | 9 |

*EXHIBIT O (Cont'd.)*

## AUTOMOBILE BODILY INJURY LIABILITY

A COMPARISON OF THE ESTIMATES OF METHODS I-VI PAID LOSSES PER ULTIMATE CLAIM

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 |
|---|---|---|---|---|---|---|---|---|
| 1972 | 258 | 585 | 375 | 244 | 124 | 55 | 30 | 10 |
| | 258 | 843 | 1,219 | 1,463 | 1,587 | 57 | 31 | 10 |
| | | | | | | 52 | 29 | 9 |
| | | | | | | 54 | 30 | 9 |
| | | | | | | 61 | 40 | 11 |
| | | | | | | 56 | 36 | 10 |
| 1973 | 296 | 612 | 419 | 266 | 130 | 58 | 32 | 10 |
| | 296 | 908 | 1,327 | 1,593 | 138 | 62 | 34 | 10 |
| | | | | | 134 | 55 | 31 | 10 |
| | | | | | 133 | 58 | 32 | 10 |
| | | | | | 146 | 69 | 49 | 13 |
| | | | | | 137 | 61 | 41 | 11 |
| 1974 | 308 | 698 | 501 | 307 | 143 | 65 | 37 | 12 |
| | 308 | 1,006 | 1,507 | 297 | 157 | 70 | 39 | 12 |
| | | | | 283 | 145 | 58 | 33 | 11 |
| | | | | 279 | 144 | 63 | 35 | 11 |
| | | | | 320 | 172 | 83 | 64 | 16 |
| | | | | 292 | 150 | 67 | 48 | 12 |

*EXHIBIT O (Cont'd.)*

## AUTOMOBILE BODILY INJURY LIABILITY

### A COMPARISON OF THE ESTIMATES OF METHODS I-VI PAID LOSSES PER ULTIMATE CLAIM

| Accident Year | MONTHS OF DEVELOPMENT 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 |
|---|---|---|---|---|---|---|---|---|
| 1975 | 341 | 794 | 584 | 355 | 159 | 72 | 42 | 13 |
| | 341 | 1,135 | 526 | 327 | 173 | 78 | 42 | 13 |
| | | | 511 | 309 | 157 | 61 | 35 | 12 |
| | | | 503 | 303 | 156 | 68 | 38 | 12 |
| | | | 590 | 378 | 203 | 99 | 82 | 19 |
| | | | 539 | 323 | 164 | 74 | 55 | 14 |
| 1976 | 369 | 898 | 679 | 408 | 174 | 80 | 47 | 15 |
| | 369 | 819 | 550 | 342 | 181 | 81 | 44 | 14 |
| | | 826 | 561 | 337 | 170 | 64 | 37 | 12 |
| | | 822 | 548 | 328 | 168 | 73 | 41 | 13 |
| | | 907 | 697 | 447 | 239 | 119 | 106 | 23 |
| | | 855 | 606 | 357 | 179 | 81 | 64 | 15 |

*EXHIBIT P*

## AUTOMOBILE BODILY INJURY LIABILITY

### PAID LOSS DEVELOPMENT FACTORS

| Accident Year | MONTHS OF DEVELOPMENT 24 | 36 | 48 | 60 | 72 | 84 | 96 |
|---|---|---|---|---|---|---|---|
| 1969 | 2.8341 | 1.3886 | 1.1849 | 1.0934 | 1.0369 | 1.0127 | 1.0057 |
| 1970 | 2.8005 | 1.3881 | 1.2016 | 1.0865 | 1.0359 | 1.0236 | |
| 1971 | 3.0090 | 1.4511 | 1.1868 | 1.0865 | 1.0354 | | |
| 1972 | 3.2644 | 1.4450 | 1.2003 | 1.0851 | | | |
| 1973 | 3.0687 | 1.4611 | 1.2003 | | | | |
| 1974 | 3.2664 | 1.4978 | | | | | |
| 1975 | 3.3272 | | | | | | |
| Average | 3.0815 | 1.4386 | 1.1948 | 1.0879 | 1.0361 | 1.0181 | 1.0057 |
| Average Latest 4 | 3.2317 | 1.4637 | 1.1973 | 1.0879 | 1.0361 | 1.0181 | 1.0057 |
| Weighted | 3.2192 | 1.4632 | 1.1969 | 1.0868 | 1.0359 | 1.0190 | 1.0057 |
| Weighted Latest 4 | 3.2724 | 1.4782 | 1.1986 | 1.0860 | 1.0358 | 1.0197 | 1.0057 |

*EXHIBIT Q*

# AUTOMOBILE BODILY INJURY LIABILITY

## THE PROJECTED DEVELOPMENT FACTORS UTILIZED IN DETERMINING THE ESTIMATES OF METHODS I, II AND V

| Accident Year | MONTHS OF DEVELOPMENT 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-Ult. |
|---|---|---|---|---|---|---|---|
| | | | Method I | | | | |
| 1970 | | | | | | | 1.0142 |
| 1971 | | | | | | 1.0181 | 1.0142 |
| 1972 | | | | | 1.0346 | 1.0181 | 1.0142 |
| 1973 | | | | 1.0816 | 1.0339 | 1.0181 | 1.0142 |
| 1974 | | | 1.2037 | 1.0791 | 1.0332 | 1.0181 | 1.0142 |
| 1975 | | 1.5145 | 1.2066 | 1.0765 | 1.0325 | 1.0181 | 1.0142 |
| 1976 | 3.4344 | 1.5362 | 1.2096 | 1.0740 | 1.0317 | 1.0181 | 1.0142 |
| Goodness of Fit | .7969 | .8883 | .3271 | .7364 | .9548 | — | — |

*EXHIBIT Q (Cont'd.)*

# AUTOMOBILE BODILY INJURY LIABILITY

## THE PROJECTED DEVELOPMENT FACTORS UTILIZED IN DETERMINING THE ESTIMATES OF METHODS I, II AND V

| Accident Year | MONTHS OF DEVELOPMENT 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-Ult. |
|---|---|---|---|---|---|---|---|
| | | | Method II | | | | |
| 1970 | | | | | | | 1.0142 |
| 1971 | | | | | | 1.0190 | 1.0142 |
| 1972 | | | | | 1.0359 | 1.0190 | 1.0142 |
| 1973 | | | | 1.0868 | 1.0359 | 1.0190 | 1.0142 |
| 1974 | | | 1.1969 | 1.0868 | 1.0359 | 1.0190 | 1.0142 |
| 1975 | | 1.4632 | 1.1969 | 1.0868 | 1.0359 | 1.0190 | 1.0142 |
| 1976 | 3.2192 | 1.4632 | 1.1969 | 1.0868 | 1.0359 | 1.0190 | 1.0142 |
| | | | Method V | | | | |
| 1970 | | | | | | | 1.0145 |
| 1971 | | | | | | 1.0219 | 1.0148 |
| 1972 | | | | | 1.0384 | 1.0245 | 1.0152 |
| 1973 | | | | 1.0916 | 1.0399 | 1.0273 | 1.0155 |
| 1974 | | | 1.2120 | 1.0939 | 1.0414 | 1.0305 | 1.0159 |
| 1975 | | 1.5201 | 1.2191 | 1.0963 | 1.0430 | 1.0341 | 1.0163 |
| 1976 | 3.4574 | 1.5467 | 1.2264 | 1.0988 | 1.0447 | 1.0381 | 1.0167 |
| Goodness of Fit | .7966 | .8831 | .3304 | .7417 | .9567 | 1.0000 | — |

*EXHIBIT R*

## AUTOMOBILE BODILY INJURY LIABILITY

### DETERMINATION OF THE ESTIMATES OF PAID LOSSES PER ULTIMATE CLAIM BY METHODS III, IV AND VI FOR ACCIDENT YEAR 1974 DURING CALENDAR YEAR 1977

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| Accident Year | Average Paid Losses Calendar Year AY+3 | Factor to Adjust Claim Costs to Calendar Year 1977 Level | Estimated Average Paid Losses at Calendar Year 1976 Cost Level | Weights | Weighted Average of Column (3) |
| | | | **Method III** | | |
| 1969 | $177.17 | $1.0912^5$ | $274.15 | 9/135 | $ 18.28 |
| 1970 | 201.73 | $1.0912^4$ | 286.06 | 16/135 | 33.90 |
| 1971 | 200.18 | $1.0912^3$ | 260.13 | 25/135 | 48.17 |
| 1972 | 244.14 | $1.0912^2$ | 290.72 | 36/135 | 77.53 |
| 1973 | 265.87 | $1.0912^1$ | 290.13 | 49/135 | 105.31 |
| | | | Method III Estimate = | | $283.19 |

*EXHIBIT R (Cont'd.)*

## AUTOMOBILE BODILY INJURY LIABILITY

### DETERMINATION OF THE ESTIMATES OF PAID LOSSES PER ULTIMATE CLAIM BY METHODS III, IV AND VI FOR ACCIDENT YEAR 1974 DURING CALENDAR YEAR 1977

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| Accident Year | Average Paid Losses Calendar Year AY+3 | Factor to Adjust Claim Costs to Calendar Year 1977 Level | Estimated Average Paid Losses at Calendar Year 1976 Cost Level | Weights | Weighted Average of Column (3) |
| | | Method IV | | | |
| 1969 | 177.17 | $1.0842^5$ | 265.44 | 9/135 | 17.70 |
| 1970 | 201.73 | $1.0842^4$ | 278.76 | 16/135 | 33.04 |
| 1971 | 200.18 | $1.0842^3$ | 255.13 | 25/135 | 47.25 |
| 1972 | 244.14 | $1.0842^2$ | 286.99 | 36/135 | 76.53 |
| 1973 | 265.87 | $1.0842^1$ | 288.26 | 49/135 | 104.63 |
| | | Method IV Estimate = | | | $279.15 |

*EXHIBIT R (Cont'd.)*

## AUTOMOBILE BODILY INJURY LIABILITY

### DETERMINATION OF THE ESTIMATES OF PAID LOSSES PER ULTIMATE CLAIM BY METHODS III, IV AND VI FOR ACCIDENT YEAR 1974 DURING CALENDAR YEAR 1977

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| Accident Year | Average Paid Losses Calendar Year AY+3 | Factor to Adjust Claim Costs to Calendar Year 1977 Level | Estimated Average Paid Losses at Calendar Year 1976 Cost Level | Weights | Weighted Average of Column (3) |
| | | | Method VI | | |
| 1969 | 177.17 | $1.1061^{5}$ | 293.30 | 9/135 | 19.55 |
| 1970 | 201.73 | $1.1061^{4}$ | 301.93 | 16/135 | 35.78 |
| 1971 | 200.18 | $1.1061^{3}$ | 270.88 | 25/135 | 50.16 |
| 1972 | 244.14 | $1.1061^{2}$ | 298.68 | 36/135 | 79.65 |
| 1973 | 265.87 | $1.1061^{1}$ | 294.07 | 49/135 | 106.74 |
| | | Method VI Estimate = | | | $291.88 |

NOTE:
1. Factors shown in Column (2) are rounded. The actual factors used in the calculations are 1.091241 (Method III), 1.084217 (Method IV) and 1.106077 (Method VI).

*EXHIBIT S*

# AUTOMOBILE BODILY INJURY LIABILITY

## FINAL CALCULATIONS IN THE DETERMINATION OF THE ESTIMATES OF METHODS III, IV AND VI

| Accident Year | Calendar Year | | | | | | |
|---|---|---|---|---|---|---|---|
| | AY + 1 | AY + 2 | AY + 3 | AY + 4 | AY +5 | AY + 6 | AY + 7 |
| | | | **Method III** | | | | |
| 1970 | | | | | | | 11.84 |
| 1971 | | | | | | 27.36 | 12.47 |
| 1972 | | | | | 52.21 | 29.07 | 14.26 |
| 1973 | | | | 133.87 | 55.04 | 30.89 | 15.49 |
| 1974 | | | 283.19 | 144.92 | 58.02 | 32.82 | 17.31 |
| 1975 | | 511.04 | 309.02 | 156.88 | 61.16 | 34.87 | 18.86 |
| 1976 | 825.61 | 560.59 | 337.22 | 169.82 | 64.47 | 37.06 | 20.20 |
| Growth Rate | +8.82% | +9.70% | +9.12% | +8.25% | +5.42% | +6.25% | +7.76% |
| Goodness of Fit | .9668 | .9764 | .9695 | .9337 | .9991 | 1.0000 | — |

*EXHIBIT S (Cont'd.)*

## AUTOMOBILE BODILY INJURY LIABILITY

### FINAL CALCULATIONS IN THE DETERMINATION OF THE ESTIMATES OF METHODS III, IV AND VI

| Accident Year | Calendar Year AY + 1 | AY + 2 | AY + 3 | AY + 4 | AY +5 | AY + 6 | AY + 7 |
|---|---|---|---|---|---|---|---|
| | | | | Method IV | | | |
| 1970 | | | | | | | 11.84 |
| 1971 | | | | | | 27.89 | 12.47 |
| 1972 | | | | | 54.18 | 30.09 | 14.29 |
| 1973 | | | | 133.33 | 58.31 | 32.45 | 15.53 |
| 1974 | | | 279.14 | 144.05 | 62.74 | 35.01 | 17.33 |
| 1975 | | 503.06 | 302.65 | 155.63 | 67.52 | 37.76 | 18.81 |
| 1976 | 822.09 | 548.04 | 328.14 | 168.15 | 72.66 | 40.73 | 20.07 |
| Growth Rate | +8.62% | +8.94% | +8.42% | +8.04% | +7.61% | +7.86% | +7.97% |
| Goodness of Fit | .9668 | .9764 | .9695 | .9337 | .9991 | 1.0000 | — |

*EXHIBIT S (Cont'd.)*

## AUTOMOBILE BODILY INJURY LIABILITY

### FINAL CALCULATIONS IN THE DETERMINATION OF THE ESTIMATES OF METHODS III, IV AND VI

| Accident Year | Calendar Year | | | | | | |
|---|---|---|---|---|---|---|---|
| | AY + 1 | AY + 2 | AY + 3 | AY + 4 | AY + 5 | AY + 6 | AY + 7 |
| | | | Method VI | | | | |
| 1970 | | | | | | | 11.84 |
| 1971 | | | | | | 30.62 | 12.50 |
| 1972 | | | | | 56.04 | 35.50 | 14.36 |
| 1973 | | | | 136.74 | 61.43 | 41.15 | 15.67 |
| 1974 | | | 291.89 | 149.53 | 67.35 | 47.69 | 17.64 |
| 1975 | | 539.36 | 322.85 | 163.52 | 73.83 | 55.28 | 19.58 |
| 1976 | 854.84 | 605.64 | 357.10 | 178.82 | 80.94 | 64.08 | 21.47 |
| Growth Rate | +10.43% | +12.29% | +10.61% | +9.36% | +9.63% | +15.91% | +10.65% |
| Goodness of Fit | .9734 | .9827 | .9321 | .7330 | .9723 | 1.0000 | — |

*EXHIBIT T*

## AUTOMOBILE BODILY INJURY LIABILITY

### ESTIMATION OF ADJUSTED GROWTH RATES FOR

### METHOD IV

| Months of Development | (1) Slope of Trend Line | (2) Initial Growth Rate EXP (1) | (3) Weights | (4) Overall Growth Rate | (5) Adjusted Growth Rate |
|---|---|---|---|---|---|
| 12 | .0611 | 1.0630 | 64/203 | .3351 | 1.0630 |
| 24 | .0845 | 1.0882 | 49/203 | .2627 | 1.0862 |
| 36 | .0926 | 1.0970 | 36/203 | .1946 | 1.0894 |
| 48 | .0873 | 1.0912 | 25/203 | .1344 | 1.0842 |
| 60 | .0793 | 1.0825 | 16/203 | .0853 | 1.0804 |
| 72 | .0528 | 1.0542 | 9/203 | .0467 | 1.0761 |
| 84 | .0606 | 1.0625 | 4/203 | .0209 | 1.0786 |
| | | | | 1.0797 | |

*EXHIBIT U*

## AUTOMOBILE BODILY INJURY LIABILITY

### COMPARISON OF LOSS RESERVE ESTIMATES

**(000's omitted)**

| | Initial Estimates | | | Adjusted Estimates | | | |
|---|---|---|---|---|---|---|---|
| | | | | Mean of Methods I-VI | | | |
| | | | | | Incurred Loss Development | | |
| Accident Year | Mean of Methods I-VI | Incurred Loss Development | Hindsight Average O/S Losses | Data Adjusted for Shifts in Settlement Rates | Projections Adjusted for Bias | Constant Adequacy Level | Selected Estimate |
| 1969 | $ 87 | $ 87 | $ 87 | $ 87 | $ 87 | $ 87 | $ 87 |
| 1970 | 173 | 192 | 192 | 187 | 174 | 192 | 192 |
| 1971 | 493 | 549 | 514 | 522 | 494 | 520 | 517 |
| 1972 | 1,092 | 1,256 | 1,231 | 1,282 | 1,147 | 1,240 | 1,233 |
| 1973 | 2,496 | 2,908 | 2,994 | 3,092 | 2,650 | 2,961 | 2,953 |
| 1974 | 4,592 | 6,161 | 5,688 | 5,950 | 4,904 | 5,714 | 5,576 |
| 1975 | 9,716 | 12,697 | 11,677 | 12,198 | 10,446 | 11,589 | 11,406 |
| 1976 | 16,653 | 19,678 | 18,704 | 19,196 | 18,044 | 18,267 | 18,618 |
| 1969-76 | $35,301 | $43,526 | $41,087 | $42,512 | $37,945 | $40,570 | $40,582 |

*EXHIBIT V*

## AUTOMOBILE BODILY INJURY LIABILITY

### A RETROSPECTIVE TEST OF METHOD II

| Accident Year | Calendar Year AY + 1 | AY + 2 | AY + 3 | AY + 4 | AY + 5 |
|---|---|---|---|---|---|
| | **Historical Averages** | | | | |
| 1971 | 493.02 | 333.13 | 200.18 | 109.94 | 48.94 |
| 1972 | 585.08 | 375.30 | 244.14 | 124.45 | |
| 1973 | 612.34 | 418.83 | 265.87 | | |
| 1974 | 698.12 | 500.83 | | | |
| 1975 | 793.75 | | | | |
| | **Estimates Derived From Prior Calendar Years** | | | | |
| 1971 | 443.52 | 286.66 | 209.55 | 113.41 | 50.12 |
| 1972 | 500.48 | 356.72 | 233.01 | 128.61 | |
| 1973 | 625.02 | 393.45 | 258.80 | | |
| 1974 | 644.43 | 447.04 | | | |
| 1975 | 737.08 | | | | |
| | **Percentage Deviations** | | | | |
| 1971 | −10.04% | −13.95% | +4.68% | +3.16% | +2.41% |
| 1972 | −14.46 | −4.95 | −4.56 | +3.34 | |
| 1973 | +2.07 | −6.06 | −2.66 | | |
| 1974 | −7.69 | −10.74 | | | |
| 1975 | −7.14 | | | | |

# APPENDIX A

## RELEVANT DATA FOR A RESERVE ANALYSIS

*I. Incurred, Paid and Outstanding Losses*

This data may be provided by accident year, report year, policy year or calendar year (in descending order of preference) by year of development for the latest five to twenty years. The number of years of experience should be great enough to assure that any further development in reported counts or incurred losses will be negligible for the oldest years. Accident quarters, report months, etc., and quarters or months of development may be used in place of years. The loss history may include or exclude allocated loss adjustment expenses or may provide a separate history of such expenses in the same detail as losses. Paid losses should either include partial payment or paid losses on closed claims and partial payments should be shown separately and in the same detail. The losses may be direct, gross or net with respect to reinsurance and gross or net of salvage and subrogation recoveries.

The loss history should be provided separately for each line of business and, if possible, for major blocks of business within each line which represent more homogeneous groupings of risks or types of claims. These may include loss experience by subline, state, underwriting or claims office, size of risk, policy limit or deductible amount. They may also include separate loss experience for personal versus commercial risks, voluntary versus assigned risks and prospectively versus retrospectively rated risks. The loss history should also provide separate detail by size of loss or layer of loss, although this information is usually available only in terms of a listing of large claims or catastrophic losses. If possible, the loss history should be provided separately by kind of claim (fast track versus regular, medical versus indemnity for workers' compensation, or by status of the law suit). Where the book of business consists of a large number of small risks and a few large risks, it may be necessary to review the loss experience for the large risks separately.

*II. Reported, Closed, Reopened and Outstanding Counts*

This data should be provided in the same detail as the history of incurred, paid and outstanding losses. Closed counts may also be broken down into claims closed with payment and without payment.

### *III. Earned or Written Premium and Earned or Written Exposures*

Earned or written premium may be provided at collected levels or at current rates. Premiums or exposures may be provided by year, quarter or month to match the detail of the loss history (but not its periods of development). Premiums or exposures should be provided separately for each line of business, for each significant subline, state, underwriting office, policy limit or deductible, and for each significant subdivision of business (e.g., personal versus commercial, voluntary versus assigned risk and prospectively versus retrospectively rated risks) even if a history of losses is not available in the same detail. Where a few large risks are underwritten, separate premium data for such risks may be useful.

### *IV. Miscellaneous Documents*

A history of reinsurance treaties, the latest NAIC examination report, and annual statements as well as quarterly and annual reports to stockholders and/or policyhoders for the most recent three or four years are examples of documents which may provide useful additional information and insight into company operations.

### *V. Industrywide Frequency and Severity Data and External Indices*

Such information is often available by line of business and may prove useful where company experience is not fully credible or marked changes in company operations or procedures have significantly altered or distorted frequency or severity trends. This type of data may be helpful even where reserves from specialty underwriting (such as substandard business) are being analyzed.

## APPENDIX B

### SAMPLE QUESTIONS FOR DEPARTMENT EXECUTIVES

*Questions for a Claims Executive:*

1. What specific objectives and guidelines does your department have in setting case reserves? Are case reserves established on the basis of what it would cost to settle the case today, or has a provision for inflation between now and the estimated time of settlement of the claim been included in the case reserve?

2. Have there been any significant changes in the guidelines for setting and reviewing case reserves during the last five years?

3. Have there been any changes in the definitions of or rules for establishing bulk or formula reserves for reported claims in the last five years?

4. Are any special procedures or guidelines applied in the reserving of large or catastrophic claims? If so, please describe.

5. Has the size of the caseload of the average claims adjuster changed significantly in the past several years?

6. When, in the sequence of events, is a claim file established?

7. Is a claim file established for each claimant or for each accident? What procedures are followed when there are multiple claimants from the same accident? Is a claim file established for each coverage or for all coverages combined?

8. What procedures are followed in recording reopened claims? Are such claims coded to the report date of the original claim or to the date of reopening? How will the reopening of a claim affect aggregate data for paid, open or reported claims and paid, outstanding or incurred losses?

9. Have there been any noticeable shifts in the reporting or nonreporting of very small or trivial claims? In the procedures for the recording of such?

10. Has there been any shift in emphasis in settling large versus small

claims? In the relative proportion of such claims? In attitudes in adjusting such claims?

11. Have there been any changes in the guidelines on when to close a claim? For example, is a P.D. claim kept open until the associated B.I. claim is closed, or only until the P.D. portion is settled?

12. Have there been any noticeable changes in the rate of settlement of claims recently?

13. Has there been any shift from the employment of company adjusters to independent adjusters? Or vice versa? If so, how has this affected the operations of the claims department?

14. Has there been any change in the timing of the payment of allocated loss adjustment expenses? For example, are such payments made as these expenses are accrued (or incurred) or when the claim is closed?

15. Has there been any change in the definition and limit for one-shot or fast-track claims in recent years? What is that limit?

16. What safeguards against fraudulent claims are now employed? Are any special procedures followed in the event of the filing of apparently questionable or non-meritorious claims? Have these safeguards changed in recent years?

17. Have there been any shifts toward (or away from) the more vigorous defense of suits in recent years?

18. Could you provide copies of all bulletins to the field issued in the last five years in which details of the changes in claims procedures are provided?

*Questions for an Underwriting Executive:*

1. What significant changes have occurred in your company's book of business and mix of business in the past five to seven years? How are the risks insured today different from those of the past?

2. Do you underwrite any large risks which are not characteristic of your general book of business?

3. Have any significant changes occurred in your underwriting guidelines in recent years?

4. Has the proportion of business attributable to excess coverages for self-insurers changed in recent years? Can a distribution of such business be obtained by line, retention limit, class, etc.? Is a record of self-insured losses and claims available?

*Questions for a Data Processing or Accounting Executive:*

1. Has there been any change in the date on which the books are closed for the quarter? the year?
2. How are loss payments handled for claims which have already been paid, but which have not yet been processed to the point where they can be allocated to accident quarter? Are they excluded from the loss history until they are allocated to accident quarter or are they loaded into an arbitrary quarter?
3. Have new data processing systems been implemented in recent years? Have they had a significant impact on the rate of processing claims or on the length of time required from the reporting to the recording of a claim?
4. To what extent have each of the data sources supplied (see Appendix A) been crosschecked and audited for accuracy and for balancing to overall company statistics? Comment on the degree of accuracy with which each kind of statistic has been properly allocated to accident quarter, to line of business, to size of loss, etc.
5. Have there been any changes in coding procedures which would affect the data supplied?
6. Would it be possible for partial payments to exceed the case reserve on a claim? In such an event, what adjustments are made? Are case reserves taken down by the amount of partial payments?

*Questions for Actuaries Specializing in Ratemaking:*

1. Have there been any changes in company operations or procedures which have caused you to depart from standard ratemaking procedures? If so, please describe those changes and how they were treated.
2. What data which is currently used for ratemaking purposes could also be used in testing loss reserves?
3. Have you noted any significant shifts in the composition of business by type of risk or type of claim within the past several years?

4. Do you have any of the following sources of information which may be of value in reserve testing:

   a. External economic indices,

   b. Combined loss data for several companies (e.g., data obtainable from bureau rate filings),

   c. Special rating bureau studies,

   d. Changes in state laws or regulations, and

   e. Size of loss or cause of loss studies?

# APPENDIX C

## PROBLEMS AFFECTING THE UNDERLYING ASSUMPTIONS OF LOSS RESERVE METHODS

*Significant internal changes in company operations:*

A. Claims procedures (See Questions for a Claims Executive in Appendix B)

B. Data processing and recording procedures (See Questions for a Data Processing or Accounting Executive in Appendix B)

C. Loss experience by heterogeneous groups of exposures or types of claims:

1. Utilization of total limits experience when large claims comprise a significant portion of losses.

2. Inclusion of catastrophic losses in the loss experience.

3. Loss experience for the multi-peril coverages (a mixture of property and liability claims).

4. Major changes in the rate of growth of earned exposures — causing a shift in the average accident date within a given accident period and producing distortions in development factors.

5. Utilization of combined data for two or more types of risks when each type of risk comprises a significant portion of the experience.

   Examples would include combined data for:

   A. Large versus small risks.

   B. Personal versus commercial risks.

   C. Voluntary versus assigned risks.

   D. Direct business versus pooled risks.

   E. Prospectively versus retrospectively rated risks.

   F. Primary versus excess or umbrella business

   G. Different states, sublines, classes, territories, policy limits or deductibles.

D. Changes in the mix of business — utilization of combined loss data when there have been major changes in the composition of business by type of risk or type of claim.

*External changes:*

A. Legislation or court decisions significantly modifying claimant's legal rights.

B. High rates of inflation or wide fluctuations in inflation rates.

C. Changes in the social climate producing shifts in claims consciousness.

D. Impact of publicity of any kind regarding an insurer (e.g., lack of solidity or withdrawal from a given state).

E. Seasonal or cyclical fluctuations in loss experience.

F. Changes in the liberality of juries in granting awards.

G. Changes in the incidence of fraudulent claims and in the insurer's safeguards against such claims.

H. Changes in state regulations affecting company practices.