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PROCEEDINGS

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RETROSPECTIVE RATING: EXCESS LOSS FACTORS

WILLIAM R. GILLAM

Abstract

*This paper explains how Excess Loss Factors (*ELF*s) are computed. It is organized so the essential elements of the computation are described first, then the detailed origins of these elements are added.*

The detail may be found in the many spreadsheets used in the production of ELF*s. The writer has attempted to show how each fits into the structure and leads to the final values. The calculations are quite technical; to understand the whole, it may be necessary to trace the numbers through the spreadsheets using the text as a guide.*

The procedure for computing ELF*s has changed since it was last documented, the most significant revision occurring in 1986. In describing the parts that have changed, the author has supplied justifications, or at least rationales, for the particular changes.*

1. BACKGROUND

In Workers' Compensation, the premium paid by an employer for a one-year policy is a function of the exposure—the audited payroll during the year of coverage. This, of course, is only known some time after the policy is complete, at which time the final premium is calculated.

If, at the onset of the policy, the carrier and employer agree, the final premium can be a function not only of the payroll but also of the actual losses during the coverage period. An arrangement of this sort is formalized in the Retrospective Rating Plan as approved in most states. Ultimate premium is based on actual losses, expenses, and a net insurance charge to compensate for the application of maximum and minimum aggregate amounts. A detailed description of this plan may be found elsewhere. Exhibit 1 shows the basic symbolism.

For most insureds, the maximum premium can be a burdensome amount, but an amount they are reasonably confident they won't have to pay. There remains a fear that a single disastrous accident may cause enough loss by itself to result in the maximum. So the prudent insured may wish to select a "loss limit" or cap on individual losses that enter the retrospective premium formula. This can be done for a fee.

Charges for such excess coverage can be calculated using Excess Loss Factors (*ELF*s), listed for a variety of loss limits in the *Retrospective Rating Manual*. These vary by State and Hazard Group of the insured, as well as by loss limit. Hazard Group assignments are based on the classification of the insured with the most payroll (except certain administrative classifications). The grouping of classes by Hazard Group is done on the basis of relative expected severities.

This excess coverage attaches on a per-occurrence basis. All the loss excess of the loss limit due to an occurrence (possibly multiple-claim) is excluded from the calculation of the retrospective premium.

The *ELF* for a given loss limit can be applied to Standard Premium to generate the pure loss charge for the coverage. Several adjustments must be made for use in retrospective rating. Before multiplying the *ELF* by Standard Premium, a tabular factor called the Excess Loss Adjustment

Amount ($ELAA$) is subtracted. The Loss Conversion Factor is applied to provide expenses nominally varying with loss. The Tax Multiplier applied to the final retrospective premium compensates for taxes, loss assessments, and other miscellaneous items.

Consideration of how this fixed charge may overlap with the insurance charge in the Basic Premium will not be made in this paper, but can be found in Glenn Meyers [1]. Note that, at the time he wrote this paper, the charge for excess coverage was the Excess Loss Premium Factor ($ELPF$). Partly as a result of that paper, $ELPF$s were redefined to account for overlap, using tabular factors called Excess Loss Adjustment Amounts ($ELAA$s). Then

$$ELPF = ELF - ELAA. \tag{1}$$

The next section begins the dissection of the ELF computation for hypothetical State M.

2. LOADINGS IN *ELF*s

To present the procedure for calculation of ELFs, it will be easiest to start from the end and work backwards. This is because the manipulations necessary to put the data in the correct form are quite complex and, as such, could obscure what is a fairly simple computation.

Exhibit 2 is the final calculation of the ELFs in State M. This section covers the adjustment made to pure excess loss ratios for use in the Retrospective Rating Plan.

There is a page for each Hazard Group, but only Hazard Group II is shown. Incorporation of the variation by Hazard Group is the subject of Sections 6 and 7.

Average Excess Ratio [*Column* (14) = (5) + (9) + (13)]

The average excess ratio is the sum of partial excess ratios by claim type and is the portion of the total losses expected to exceed the retention in column (1) on a per-occurrence basis. The three claim types are: Fatal, Permanent Total (PT) or Major Permanent Partial (Major), and Minor Permanent Partial (Minor) or Temporary Total (TT). These are groupings of regular statistical plan injury types.

Permissible Loss Ratio [*Column* (15)]

The Permissible Loss Ratio (*PLR*), as appears here, is the factor applicable to Standard Premium to back into expected loss. In states where pure premium rates are produced, such as State M, this factor would be closer to unity, the complement of whatever loadings there may be in published loss costs. These are typically for loss adjustment expense and loss assessments.

The *PLR* is calculated by dividing the Target Cost Ratio (*TCR*), shown at the bottom of Exhibit 2, by the sum of the Loss Adjustment Expense Factor and the (Loss) Assessment Rate. The *TCR* is less than unity in states where rates are produced. The *PLR* will be an integral part of the rate filing to which the new *ELF*s are attached.

Indicated Excess Loss Factors [*Column* (16) = (14) × (15)]

When multiplied by Standard Premium, the indicated *ELF*s produce expected loss over the selected limit. Indicated Excess Loss Pure Premium Factors (*ELPPF*s) apply to pure premium rates. In State M, the National Council on Compensation Insurance (NCCI) disseminates pure premium rates, even though the standardized computer form shows "*ELF*" at the column heading.

Flat Loading [*Column* (17)]

The flat loading is 0.005, subject to a maximum of one-half of the indicated *ELF* in Column (16). The amount was established before the changes to the procedure made in 1986. It is based on judgment and is designed to compensate the insurer for parameter risk and antiselection.

Final Excess Loss Factors [*Column* (18) = (16) + (17)]

The Final *ELF*s are updated in the *Retrospective Rating Manual* at the time of an approved rate filing.

The following section explains how the partial excess ratios by claim type in Columns (5), (9), and (13) are computed.

3. CALCULATION OF PARTIAL EXCESS RATIOS

Columns (1) through (13) of Exhibit 2 provide the elements for column (14). They are grouped as follows:

Injury Type	Columns
Fatal	(2) thru (5)
PT/Major	(6) thru (9)
Minor/TT	(10) thru (13)

Loss Limit [*Column* (1)]

Any loss limit is possible, but limits from $25,000 to $1 million are the most common. These are shown in the *Retrospective Rating Manual*. Large carriers, excess insurers, and reinsurers frequently ask for information about higher retentions, and the NCCI has obliged by providing *ELF*s for retentions up to $10 million on request. These are output from the standard procedure.

Average Cost Per Case By Injury Type (Bottom of Exhibit 2)

The derivation of these values by injury type and Hazard Group may be found in Section 7.

Ratio to Average/1.1 [*Columns* (2), (6), (10)]

Central to the procedure is the translation of each dollar retention into an entry ratio calculated by dividing the retention by the average cost per claim by type. The claim size distributions underlying the excess ratios in columns (4), (8), and (12) are normalized so their means are unity, which facilitates the application. Using entry ratios automatically indexes the final *ELF* not only for the effect of inflation, but for the differences by State and Hazard Group. This technique was first documented by Frank Harwayne [2].

The distributions underlying columns (4), (8), and (12) are of individual claims by size. The factor of 1.1, applicable to the average claim cost by type, from the bottom of the exhibit, is used to adjust the Excess Ratios from a per-claim to a per-occurrence basis. This procedure, based on judgment, is thought to be an improvement over the former procedure. In the old procedure, a flat 1.1 factor was applied to the excess ratio for every retention. The new procedure results in a loading that varies so it is about 2% or 3% at the low retentions but increases to a level of more than 10% for retentions over $1 million.

Injury Weights [*Columns* (3), (7), (11)]

The final weights vary by Hazard Group. They result from a procedure described in Section 6 that adjusts countrywide relativities using state data. Each factor is a ratio:

$$\frac{\text{Expected Loss By Injury Type}}{\text{Expected Total Loss}}.$$

There is an implicit factor for Medical Only losses, but because these losses have a negligible excess ratio for the retentions normally used in retrospective rating, it is not applied. The final weights come from Exhibit 14.

Excess Ratios [*Columns* (4), (8), (12)]

The excess ratios in columns (4), (8), and (12) are based on size of claim distributions. Exhibit 3 (Parts 3, 4, and 5) shows the excess ratios applicable in State M. Section 4 describes the development of these tables.

Partial Excess Ratios [*Columns* (5), (9), (13)]

These are respective products:

$$\begin{aligned}(5) &= (3) \times (4);\\ (9) &= (7) \times (8);\\ (13) &= (11) \times (12).\end{aligned}$$

4. EXCESS RATIOS BY CLAIM TYPE

The Excess Ratios are based on parametrized loss distributions. In keeping with the "last is first" format of this paper, Exhibit 3 shows excerpts from tables of excess ratios used. Because these values are based on probability distributions, inconsistencies that can arise in the adjustment of empirical tables for trend and development are not present. Trend and development were each considered in the selected distributions, as explained below.

These distributions are based on distributions fitted to claims from the particular injury group in each of several states. No attempt was made to combine states. To select the final curves, the loss volume in each state, type of benefits (i.e., escalating or nonescalating), goodness of fit, and degrees of freedom were each considered. How these considerations were actually applied is outlined below.

For Fatal and PT/Major separately, consideration was given to whether the state had escalating, nonescalating, or limited benefits. (In states with limited benefits, escalation or nonescalation did not seem to be relevant.) Escalation can apply to Fatal (survivor) or PT (life pension) benefits, or both types of benefits, depending on state laws. Fits to data of several sample states showed states with escalating benefits had more skewed distributions than those with nonescalating benefits.

Somewhat surprisingly, states with aggregate limits on PT benefits gave rise to fitted distributions on PT/Major with higher skewness than states with nonescalating but unlimited benefits. The average size of the claim is surely smaller than it would be with no limit on benefits, but the skewness is still high. We believe this phenomenon is due to the combined effect of unlimited medical, which can be high on PT cases, and the accumulations of claims whose indemnity is capped by the limit value. In the final selections, two distributions were chosen for PT/Major claims: one for states with nonescalating but unlimited benefits, and one for states with either escalating or limited benefits.

In a similar way, two Fatal distributions were selected, but in this case limited benefit states were paired with nonescalating benefit states. Since fatal claims generally do not have a large medical component, this pairing need not be the same as for PT/Major. A single distribution for Minor/TT sufficed, making five in all.

To estimate the impact of loss development on size of claim distributions, the curves were fit to key states' data at successive maturities. Judgment was used to estimate the impact on the shape parameter, which usually progressed in such a way as to increase variance at more mature evaluations. Since most retro plans are closed out by the fifth year, and statistical plan data is not collected beyond that maturity, the selected parameters may not reflect ultimate development. If the *ELF*s are used for pricing excess of loss coverage, some consideration of development fifth to ultimate should be made.

Numerous loss distributions were each fitted to empirical data from the 1982 policy year. These distributions included:

1) Gamma
2) Transformed Gamma
3) Inverse Gamma
4) Inverse Transformed Gamma
5) Beta
6) Transformed Beta
7) Burr
8) Weibull
9) Pareto
10) Lognormal.

The forms of these distributions may be found in Exhibit 4. More detailed information about the distributions may be found in Robert Hogg and Stuart Klugman [3] and in Gary Venter [4].

Curves were fit using the method of Maximum Likelihood. Statistics for goodness of fit, including the negative log likelihood itself, were compared. The chi-square statistic was thought to be especially good for this application, as it measures relative rather than actual squared error. For the tail of the distribution, where probabilities are small, the difference between test data and the distribution is critical if we are to measure excess ratios accurately. How well the curve fits the data around the mean and median, where probabilities are large, is of less importance than the fit in the tail. An unweighted sum of squared residuals statistic would give most weight to the many claims near the middle range of sizes, which does not seem desirable. The chi-square statistic gives a more appropriate weighting.

Maximum Likelihood may be the best way to parametrize a curve, but not necessarily the best way to choose between alternative distributions, since the (log) likelihood statistic pertains in part to the characteristics of the curve being fit, not just the fit itself. Selection of the curve, then, was based primarily on the results of the chi-square test. Frequently, both statistics were best for the same curve, facilitating the choice. (For both statistics, the best is the lowest.)

Another criterion for selection was the number of parameters in the fitted distribution. A Transformed Gamma has three, while the Gamma has only two. If the latter fits nearly as well as the former, it is preferable to use the simpler one, as the additional degree of freedom provides little more information and a greater chance for spurious results.

This principle was applied in the selection of a Fatal curve, where the simple Gamma with two parameters fit nearly as well as the Transformed Gamma, which has three. Holding the first parameters of the Transformed Gamma to unity results in a Gamma.

Two sets of statistics for fits to Fatal claims can be seen in Exhibit 5. For this and the following two exhibits, the sample states A, B, C, etc., were arranged so that A, C, and G were judged to be bellwether examples of the jurisdiction type.

For PT/Major in nonescalating benefit states, the fit of the three-parameter Inverse Transformed Gamma was nearly as good as that of the four-parameter Transformed Beta, and sometimes better. This can be seen in Exhibit 6. Also in that exhibit, it may be observed that the chi-square statistic can blow up for distributions with too low a skewness to accommodate existing large claims.

Examples of the impact of loss development for PT and Major in escalating and limited benefit states may be found in Exhibit 7. This exhibit is one of many similar exhibits produced in the study. The choices of $\alpha = 3.20$ and $\rho = 0.64$ for the Inverse Transformed Gamma were made primarily by consideration of patterns in States A and G.

Of course, the value of β (the scale parameter) in the final curve for each claim type would be adjusted so that a mean of unity would result.

The final parameters for the five curves are shown in Exhibit 8. The next section shows the derivation of the state injury weights and average cost per claim by type. These are needed to produce the figures in columns (3), (7), and (11) and the entries at the bottom of Exhibit 2.

5. STATE INJURY WEIGHTS AND AVERAGE COST PER CASE BY CLAIM TYPE

There are very few serious claims by state, and especially few Fatal or Permanent Total. Although it is possible to separate them by Hazard Group, in most states the data is so thin that usual loss development techniques do not work well and actual average values are statistically unreliable. Hence, a single set of average values and claim type weights is estimated for the state, then spread to Hazard Group using countrywide relativities. Care must be taken in this spreading to see that recombinations of the Hazard Group numbers using weights taken from state data results in the known totals. This is described in the last two sections.

Exhibit 9 shows the calculations as applied in State M. The latest three available policy years are used. They are put on the latest law level, trended, and developed to ultimate separately, then combined for the average used in the *ELF* calculation.

Indemnity and medical losses are separately trended and put on current law level in columns (1) through (8). The losses are then combined and divided by the claim count to produce an "as of" severity in column (11). PT and Major are combined at this point. Factors for severity development to ultimate are applied to produce an estimate of ultimate severity by claim type for each policy year in column (13).

Columns (14) and (15) show aggregate loss development factors to be applied to the respective indemnity losses in column (4) and medical losses in column (8). These produce one-year total developed losses by type in column (16).

The final statewide numbers are a weighted three-year average set of severities by claim type, and three-year total injury weights by type found on Part 4 of Exhibit 9.

The loss severity development factors in column (12) are calculated on Fatal, Minor, and TT separately; but for PT and Major combined. The applicable age-to-age factors (*ATAF*) are an unweighted average of three *ATAF*s, calculated from four evaluations of statistical plan data. The average of three *ATAF*s provides some year-to-year stability in the calculation, since two of three factors in the average overlap from one year to the next.

Since the most mature evaluation of statistical plan data is the fifth report, development factors from fifth to ultimate are taken from financial data. For this application, it is assumed that all loss development beyond a fifth report is severity development on serious claim types.

Until recently, aggregate losses and claim counts were separately developed. The use of severity development reduces the problems of separate loss and claim count development associated with the (possibly frequent) reassignment of claims by type between reporting dates. Such shifting of categorizations would perhaps cancel out on average if the number of claims were large, but we found excessive year-to-year *ELF* volatility in the usual case of a small number of serious claims.

Trend is applied separately to indemnity and medical, as seen in columns (3) and (7) of Exhibit 9. Exhibit 10 shows the derivation of the trend factors.

6. DISTRIBUTION OF STATE INJURY WEIGHTS TO HAZARD GROUPS

Injury weights by type start with values derived in Exhibit 9, columns (14), (15), and (16). Losses by type are put on current law level, trended, and developed, for indemnity and medical separately, then combined in the last step. Losses from the three policy periods are added to provide three-year totals by type, all Hazard Groups combined, in Part 4 of Exhibit 9.

Losses are spread to the Hazard Groups using countrywide data. This data is in the form of partial loss ratios by injury type for each Hazard Group. These loss ratios, based on countrywide statistical plan data, may be seen in Exhibit 11. In this exhibit, the partial loss ratios of each injury type are rescaled so that they sum to 1.0 across the Hazard Groups using the following formula:

$$CLR_{I,H} = \frac{CL_{I,H}/CP_H}{\sum_{\text{Hazard Groups } H} CL_{I,H}/CP_H},$$

where CP_H is the countrywide premium for Hazard Group H from the experience period used and $CL_{I,H}$ is the countrywide losses for injury type I and Hazard Group H from the same time period. (The rescaling is gratuitous as it is repeated in the next step.)

Using a state distribution of premium by Hazard Group from the latest second report, found in Exhibit 12, a state distribution of losses by Hazard Group for each injury type is found:

$$L_{I,H} = \frac{CLR_{I,H} \cdot P_H}{\Sigma_H \, CLR_{I,H} \cdot P_H},$$

where P_H is now the state premium for Hazard Group H, and $CLR_{I,H}$ is the (relative) partial loss ratio from Exhibit 11. The resulting distributions are shown in Exhibit 13.

These $L_{I,H}$, or proportions of loss dollars by Hazard Group (within injury type), are applied to actual three-year state total losses by injury type from Exhibit 9 to produce the loss dollars by type of injury and Hazard Group in column(s) 2 of Exhibit 14. After each type of loss is distributed *across* the Hazard Groups, the *downward* distribution of losses by claim type is then calculated within each Hazard Group. Subtotals give the proportion of loss in the combined groups PT/Major and Minor/TT. With the associated Fatal weights, these become the injury weights in columns (3), (7), and (11) of Exhibit 2.

7. AVERAGE COST PER CASE BY CLAIM TYPE AND HAZARD GROUP

The state input data comes from Exhibit 9, which gives the statewide three-year average claim cost by injury type. The state premium distribution by Hazard Group comes from Exhibit 12. Exhibit 15 shows countrywide severity relativities for the serious claim types by Hazard Group, which are also needed.

The distribution of claims by Hazard Group differs by state. Hence it will not be correct to apply the relativities from Exhibit 15 to the average claim costs from Exhibit 9. An adjustment must be calculated for each claim type, so that the severity relativities will produce average severities by Hazard Group that are consistent with the overall state severities by type.

The correct weights are claim counts. However, because of the small sample size; i.e., a single claim type in one Hazard Group in one state, claim counts are too volatile. The weights used are the state premiums by Hazard Group from Exhibit 12. For the PT adjustment, we calculate:

$$0.977 = (0.017)(0.813) + \cdots + (0.032)(1.245) \tag{2}$$

in Section A of Exhibit 16. The relativities from Exhibit 15 are divided by these factors, respective of injury type, and the Hazard Group differentials become those in Section B of Exhibit 16. The differentials for the combined PT/Major group are then obtained by weighting the PT and Major differentials with injury weights, respective of Hazard Group, from Exhibit 14:

$$0.943 = [(0.057)(0.976) + (0.575)(0.940)]/[(0.057) + (0.575)]. \tag{3}$$

Using this factor from Exhibit 16 and the respective state severity from Exhibit 9, Part 4, the severity for PT/Major in Hazard Group II can be found at the bottom of Exhibit 2:

$$(0.943)(108{,}997) = 102{,}784. \tag{4}$$

For Minor/TT, no differentials are calculated, and the state average cost per case, as computed in Section 5, is used for all Hazard Groups.

This is the end of the technical presentation. A few comments about the final *ELF*s are in order:

1) For higher limits, the risk component (flat loading) becomes a significant portion of the charge. State M has higher excess ratios than many other states, so this becomes evident only above the $1,000,000 loss limit.
2) For all but the lowest limits, the excess ratio for PT/Major largely determines the final *ELF*.

REFERENCES

[1] Glenn G. Meyers, "An Analysis of Retrospective Rating," *PCAS* LXVII, 1980, p. 110.

[2] Frank Harwayne, "Accident Limitations for Retrospective Rating," *PCAS* LXIII, 1976, p. 1.

[3] Robert Hogg and Stuart Klugman, *Loss Distributions*, Wiley & Sons, 1984.

[4] Gary G. Venter, "Transformed Beta and Gamma Distributions and Aggregate Losses," *PCAS* LXX, 1983, p. 156.

EXHIBIT 1

BASIC RETROSPECTIVE RATING SYMBOLISM

The Retrospective Premium R is calculated after the end of the policy period by formula:

$$H \leq R = T(B + cL) \leq G,$$

where: H is the minimum premium;
T is the Tax Multiplier;
B is the Basic Premium;
c is the Loss Conversion Factor;
L is the actual losses during the period;
G is the maximum premium.

If a loss limit is selected:

$$H \leq R = T(\hat{B} + cELPF + c\hat{L}) \leq G,$$

where: $\hat{B}$ is the Basic Premium, respective of the selected loss limitation (in the current plan, $\hat{B}$ is not affected by the choice of loss limitations so $\hat{B} = B$);

$\hat{L}$ is the actual losses during the experience period, subject to a per occurrence limit;

$ELPF$ is the (net) charge for such loss limitations after correction for overlap with the insurance charge.

EXHIBIT 2

National Council on Compensation Insurance
State M Effective: 01/01/89
Limited Fatal Benefits—Nonescalating PT/Major Benefits
Excess Loss Factors Calculation
Hazard Group II

	Fatal				PT/Major				Minor/TT								
(1) Loss Limit	(2) Ratio To Ave. / 1.1	(3) Inj. Wgt.	(4) Excess Ratio	(5) Excess Ratio × Inj. Wt.	(6) Ratio To Ave. / 1.1	(7) Inj. Wgt.	(8) Excess Ratio	(9) Excess Ratio × Inj. Wt.	(10) Ratio To Ave. / 1.1	(11) Inj. Wgt.	(12) Excess Ratio	(13) Excess Ratio × Inj. Wt.	(14) Ave. XS Ratio	(15) PLR Excl. Asses.	(16) Ind. ELF (14) × (15)	(17) Flat Loading	(18) Final ELF (16) + (17)
$ 10,000	0.10	0.011	0.908	0.010	0.09	0.632	0.910	0.575	1.79	0.288	0.361	0.104	0.689	0.868	0.598	0.005	0.603
15,000	0,14		0.874	0.010	0.13		0.870	0.550	2.68		0.223	0.064	0.624		0.542	0.005	0.547
20.000	0.19		0.834	0.009	0.18		0.820	0.518	3.58		0.138	0.040	0.567		0.492	0.005	0.497
25,000	0.24		0.796	0.009	0.22		0.780	0.493	4.47		0.085	0.024	0.526		0.457	0.005	0.462
30,000	0.29		0.760	0.008	0.27		0.730	0.461	5.36		0.053	0.015	0.484		0.420	0.005	0.425
35,000	0.33		0.733	0.008	0.31		0.690	0.436	6.26		0.034	0.010	0.454		0.394	0.005	0.399
40,000	0.38		0.700	0.008	0.35		0.650	0.411	7.15		0.022	0.006	0.425		0.369	0.005	0.374
50,000	0.48		0.640	0.007	0.44		0.562	0.355	8.94		0.010	0.003	0.365		0.317	0.005	0.322
75,000	0.71		0.521	0.006	0.66		0.387	0.245	13.41		0.002	0.001	0.252		0.219	0.005	0.224
100,000	0.95		0.422	0.005	0.88		0.284	0.179	17.88		0.000	0.000	0.184		0.160	0.005	0.165
125,000	1.19		0.342	0.004	1.11		0.220	0.139	22.35		0.000	0.000	0.143		0.124	0.005	0.129
150,000	1.43		0.278	0.003	1.33		0.181	0.114	26.82		0.000	0.000	0.117		0.102	0.005	0.107
175,000	1.67		0.226	0.002	1.55		0.153	0.097	31.29		0.000	0.000	0.099		0.086	0.005	0.091
200,000	1.91		0.184	0.002	1.77		0.132	0.083	35.76		0.000	0.000	0.085		0.074	0.005	0.079
225,000	2.14		0.151	0.002	1.99		0.116	0.073	40.23		0.000	0.000	0.075		0.065	0.005	0.070
250,000	2.38		0.123	0.001	2.21		0.103	0.065	44.70		0.000	0.000	0.066		0.057	0.005	0.062
275,000	2.62		0.101	0.001	2.43		0.093	0.059	49.17		0.000	0.000	0.060		0.052	0.005	0.057
300,000	2.86		0.082	0.001	2.65		0.085	0.054	53.64		0.000	0.000	0.055		0.048	0.005	0.053
325,000	3.10		0.067	0.001	2.87		0.077	0.049	58.11		0.000	0.000	0.050		0.043	0.005	0.048
350,000	3.34		0.055	0.001	3.10		0.071	0.045	62.58		0.000	0.000	0.046		0.040	0.005	0.045

EXHIBIT 2

(CONTINUED)

	Fatal				PT/Major				Minor/TT								
(1) Loss Limit	(2) Ratio To Ave. / 1.1	(3) Inj. Wgt.	(4) Excess Ratio	(5) Excess Ratio × Inj. Wt.	(6) Ratio To Ave. / 1.1	(7) Inj. Wgt.	(8) Excess Ratio	(9) Excess Ratio × Inj. Wt.	(10) Ratio To Ave. / 1.1	(11) Inj. Wgt.	(12) Excess Ratio	(13) Excess Ratio × Inj. Wt.	(14) Ave. XS Ratio	(15) PLR Excl. Asses.	(16) Ind. ELF (14) × (15)	(17) Flat Loading	(18) Final ELF (16) + (17)
$ 375,000	3.57		0.045	0.000	3.32		0.066	0.042	67.06		0.000	0.000	0.042		0.036	0.005	0.041
400,000	3.81		0.037	0.000	3.54		0.062	0.039	71.53		0.000	0.000	0.039		0.034	0.005	0.039
425,000	4.05		0.031	0.000	3.76		0.058	0.037	76.00		0.000	0.000	0.037		0.032	0.005	0.037
450,000	4.29		0.025	0.000	3.98		0.054	0.034	80.47		0.000	0.000	0.034		0.030	0.005	0.035
475,000	4.53		0.021	0.000	4.20		0.051	0.032	84.94		0.000	0.000	0.032		0.028	0.005	0.033
500,000	4.77		0.017	0.000	4.42		0.048	0.030	89.41		0.000	0.000	0.030		0.026	0.005	0.031
600,000	5.72		0.008	0.000	5.31		0.039	0.025	107.29		0.000	0.000	0.025		0.022	0.005	0.027
700,000	6.67		0.004	0.000	6.19		0.033	0.021	125.17		0.000	0.000	0.021		0.018	0.005	0.023
800,000	7.63		0.002	0.000	7.08		0.029	0.018	143.05		0.000	0.000	0.018		0.016	0.005	0.021
900,000	8.58		0.001	0.000	7.96		0.025	0.016	160.93		0.000	0.000	0.016		0.014	0.005	0.019
1,000,000	9.53		0.000	0.000	8.84		0.023	0.015	178.81		0.000	0.000	0.015		0.013	0.005	0.018
2,000,000	19.06		0.000	0.000	17.69		0.011	0.007	357.63		0.000	0.000	0.007		0.006	0.003	0.009
3,000,000	28.60		0.000	0.000	26.53		0.007	0.004	536.44		0.000	0.000	0.004		0.003	0.002	0.005
4,000,000	38.13		0.000	0.000	35.38		0.005	0.003	715.26		0.000	0.000	0.003		0.003	0.002	0.005
5,000,000	47.66		0.000	0.000	44.22		0.004	0.003	894.07		0.000	0.000	0.003		0.003	0.002	0.005
6,000,000	57.19		0.000	0.000	53.07		0.003	0.002	1072.88		0.000	0.000	0.002		0.002	0.001	0.003
7,000,000	66.72		0.000	0.000	61.91		0.003	0.002	1251.70		0.000	0.000	0.002		0.002	0.001	0.003
8,000,000	76.26		0.000	0.000	70.76		0.002	0.001	1430.51		0.000	0.000	0.001		0.001	0.001	0.002
9,000,000	85.79		0.000	0.000	79.60		0.002	0.001	1609.33		0.000	0.000	0.001		0.001	0.001	0.002
10,000,000	95.32		0.000	0.000	88.45		0.002	0.001	1788.14		0.000	0.000	0.001		0.001	0.001	0.002

Fatal Average Cost Per Case:	$95,372	Target Cost Ratio:	1.0000
PT/Major Average Cost Per Case:	$102,784	Loss Adjustment Expense:	1.120
Minor/TT Average Cost Per Case:	$5,084	Assessment Factor:	0.032

EXHIBIT 3, PART 1

State Type: Escalating Benefits
Injury: Fatal
Distribution: Gamma (1.667, 0.6)

Mean = 1, Var. = 1.667, Coef. of Var. = 1.291,
Skewness = 2.582

Entry Ratio	Excess Ratio
0.25	.804
0.50	.659
0.75	.544
1.00	.452
1.25	.377
1.50	.315
1.75	.264
2.00	.222
2.25	.187
2.50	.157
2.75	.133
3.00	.112
3.25	.095
3.50	.080
3.75	.068
4.00	.058
4.25	.049
4.50	.041
4.75	.035
5.00	.030
5.25	.025
5.50	.022
5.75	.018
6.00	.016
6.25	.013
6.50	.011
6.75	.010
7.00	.008
7.25	.007
7.50	.006
7.75	.005
8.00	.004
9.00	.002
10.00	.001

EXHIBIT 3, PART 2

State Type: Escalating and Limited Benefits
Injury: Permanent Total and Major Permanent Partial
Distribution: Inverse Transformed Gamma
(3.2, 0.515, 0.64)

Mean = 1, Var. = 11.465, Coef. of Var. = 3.386,
Skewness: Undefined

Entry Ratio	Excess Ratio
1	.269
2	.132
3	.086
4	.064
5	.050
6	.042
7	.035
8	.031
9	.027
10	.024
11	.022
12	.020
13	.019
14	.017
15	.016
16	.015
17	.014
18	.013
19	.012
20	.012
25	.009
30	.008
35	.007
40	.006

EXHIBIT 3, PART 3

State Type: Nonescalating and Limited Benefits
Injury: Fatal
Distribution: Gamma (1.25, 0.8)

Mean = 1, Var. = 1.250, Coef. of Var. = 1.118,
Skewness = 2.236

Entry Ratio	Excess Ratio
0.25	.789
0.50	.628
0.75	.513
1.00	.404
1.25	.325
1.50	.262
1.75	.211
2.00	.170
2.25	.138
2.50	.112
2.75	.090
3.00	.073
3.25	.059
3.50	.048
3.75	.039
4.00	.032
4.25	.026
4.50	.021
4.75	.017
5.00	.014
5.25	.011
5.50	.009
5.75	.007
6.00	.006
6.25	.005
6.50	.004
6.75	.003
7.00	.003
7.50	.002
8.00	.001
9.00	.001
10.00	.000

EXHIBIT 3, PART 4

State Type: Nonescalating
Injury: Permanent Total and Major Permanent Partial
Distribution: Transformed Beta (7.0, 0.513, 1.28, 0.30)

Mean = 1, Var. = 5.045, Coef. of Var. = 2.246,
Skewness: Undefined

Entry Ratio	Excess Ratio
1	.247
2	.115
3	.074
4	.054
5	.042
6	.034
7	.029
8	.025
9	.022
10	.020
11	.018
12	.016
13	.015
14	.014
15	.013
16	.012
17	.011
18	.010
19	.010
20	.009
25	.007
30	.006
35	.005
40	.004

EXHIBIT 3, PART 5

State Type: All
Injury: Minor Permanent Partial and Temporary Total
Distribution: Transformed Beta (2.2, 7.24, 0.12, 2.9)

Mean = 1, Var. = 2.574, Coef. of Var. = 1.604,
Skewness = 2.914

Entry Ratio	Excess Ratio
1	.554
2	.322
3	.188
4	.110
5	.065
6	.039
7	.023
8	.015
9	.009
10	.006
11	.004
12	.003
13	.002
14	.001
15	.001
20	.000

EXHIBIT 4

Loss Distributions

For the following definitions, all of α, β, ρ, θ, and X are greater than zero.

1. Transformed Gamma

$$F(X;\alpha,\beta,\rho) = \int_0^{(X/\beta)^\alpha} \frac{\mu^{\rho-1}e^{-u}}{\Gamma(\rho)}\, du,\ X > 0$$

$$\mathrm{E}[F(X)] = \frac{\beta \cdot \Gamma(\rho + 1/\alpha)}{\Gamma(\rho)}$$

β is a scale parameter
If $\alpha = 1$, this is the Gamma Distribution, $\Gamma(X;\beta,\rho)$
If $\rho = 1$, this is the Weibull Distribution

2. Inverse Transformed Gamma

$$G(X;\alpha,\beta,\rho) = 1 - \int_0^{(\beta/X)^\alpha} \frac{\mu^{\rho-1}e^{-u}}{\Gamma(\rho)}\, du,\ X > 0$$

$$\mathrm{E}[G(X)] = \frac{\beta \cdot \Gamma(\rho - 1/\alpha)}{\Gamma(\rho)}$$

β is a scale parameter
If $\alpha = 1$, this is an Inverse Gamma Distribution
If $\rho = 1$, this is an Inverse Weibull Distribution

3. Transformed Beta

$$\hat{\beta}(X;\alpha,\beta,\rho,\theta) = \frac{\Gamma(\theta + \rho)}{\Gamma(\theta)\Gamma(\rho)} \int_0^{(X/\beta)^\alpha} t^{\rho-1}(1 + t)^{-(\rho+\theta)}\, dt,\ X > 0$$

$$\mathrm{E}[\hat{\beta}(X)] = \frac{\beta\Gamma(\rho + 1/\alpha)\Gamma(\theta - 1/\alpha)}{\Gamma(\rho)\Gamma(\theta)}$$

β is a scale parameter
If $\alpha = 1$, this is the Beta Distribution
If $\rho = 1$, this is the Burr Distribution
If $\rho = 1$, $\alpha = 1$, this is the Shifted Pareto

4. Lognormal

$$L(X;\alpha,\beta) = \frac{1}{\sqrt{2\pi}} \int_0^{(\ln X-\alpha)/\beta} e^{u^2/2}\, du,\ X > 0$$

$$\mathrm{E}[L(X)] = e^{[\alpha+\beta^2/2]}$$

EXHIBIT 5, PART 1

Fatal Loss Distribution
Curve Fit by Maximum Likelihood
Negative Log Likelihood

	DISTRIBUTION					
State	Gamma	T. Gamma	T. Beta	Pareto	Lognormal	Weibull
ESCALATING BENEFITS						
A	137	137	136	153	146	138
B	13	13	13	15	13	13
NONESCALATING BENEFITS						
C	144	142	143	154	156	145
D	111	111	120	131	124	120
E	88	85	86	91	97	87
LIMITED BENEFITS						
F	418	418	421	439	439	421
G	205	197	197	207	221	201
H	115	114	113	117	122	115

Lowest Statistics are Best

EXHIBIT 5, PART 2

Fatal Loss Distribution
Curve Fit by Maximum Likelihood
Chi-Square Statistics

State	DISTRIBUTION Gamma	T. Gamma	T. Beta	Pareto	Lognormal	Weibull
ESCALATING BENEFITS						
A	8.9	9.9	11.6	68.1	23.5	12.3
B	5.9	5.6	5.5	33.8	5.6	5.9
NONESCALATING BENEFITS						
C	19.9	19.3	21.1	83.5	40.5	23.1
D	22.7	22.6	25.2	80.2	34.0	25.1
E	28.1	25.7	27.3	47.4	53.0	31.0
LIMITED BENEFITS						
F	23.9	24.2	27.1	66.9	48.8	26.9
G	69.8	58.6	57.3	83.6	111.9	70.2
H	20.1	19.5	18.6	26.8	35.6	21.6

Lowest Statistics are Best

EXHIBIT 6, PART 1

PT & Major Loss Distribution
Curve Fit by Maximum Likelihood
Negative Log Likelihood

	DISTRIBUTION							
State	Burr	Gamma	T. Gamma	I.T. Gamma	T. Beta	Pareto	Lognormal	Weibull
ESCALATING BENEFITS								
A	4,311	4,735	4,696	4,287	4,287	5,688	4,582	4,828
B	979	1,021	980	969	969	991	977	1,030
NONESCALATING BENEFITS								
C	2,272	2,702	2,521	2,260	2,260	2,398	2,329	2,580
D	2,620	3,123	3,110	2,625	2,619	2,821	2,765	3,110
E	724	856	851	728	724	782	770	863
F	9,027	9,906	9,393	9,020	9,025	9,857	9,338	10,290
LIMITED BENEFITS								
G	1,455	1,791	1,757	1,456	1,454	1,607	1,560	1,763
H	1,737	1,832	1,806	1,743	1,736	1,751	1,744	1,847

Lowest Statistics are Best

EXHIBIT 6, PART 2

PT & Major Loss Distribution
Curve Fit by Maximum Likelihood
Chi-Square Statistics

	DISTRIBUTION							
State	Burr	Gamma	T. Gamma	I.T. Gamma	T. Beta	Pareto	Lognormal	Weibull
ESCALATING BENEFITS								
A	95	3.35×10^{11}	2.35×10^{10}	65	65	21,191	9.67×10^{6}	1.66×10^{8}
B	39	494	54	24	24	96	44	591
NONESCALATING BENEFITS								
C	32	3,121	1.57×10^{9}	13	13	5,047	9,696	2.1×10^{9}
D	41	45,108	1.43×10^{12}	904	42	458	3,470	1.47×10^{12}
E	11	975	24,846	113	12	456	429	22,328
F	89	2.07×10^{9}	16,465	96	112	6,637	4,334	1.5×10^{13}
LIMITED BENEFITS								
G	5	906	1.74×10^{10}	8	4	362	1,892	1.2×10^{8}
H	31	1,029	2.41×10^{6}	90	31	175	84	7.8×10^{8}

Lowest Statistics are Best

EXHIBIT 7

Parameter Development
Inverse Transformed Gamma
Permanent Total and Major Permanent Partial

Escalating Benefits

	State A		
Parameters	α	β	ρ
1st Report	3.4725	23,638	.6948
2nd Report	3.0598	24,323	.7062
3rd Report	3.2537	23,627	.6392

	State B		
Parameters	α	β	ρ
1st Report	.7720	194,021	4.1473
2nd Report	.5156	1,348,999	6.3716
3rd Report	.7077	195,658	3.2564

Limited Benefits

	State G		
Parameters	α	β	ρ
1st Report	3.76	16,827	.5727
2nd Report	3.99	16,571	.5526
3rd Report	3.76	16,827	.5727

	State H		
Parameters	α	β	ρ
1st Report	.18	1.37×10^{15}	84.6606
2nd Report	.19	1.37×10^{15}	102.7719
3rd Report	.18	1.37×10^{15}	84.6606

EXHIBIT 8

Loss Distribution Models
Parameters Chosen

Type of State/Injury	Distribution	α	β	ρ	θ
1. Escalating Benefit/ Fatal	Gamma		1.667	.60	
2. Escalating and Limited Benefit/Permanent Total and Major PP	I.T.G.	3.20	.515	.64	
3. Nonescalating and Limited Benefit/Fatal	Gamma		1.250	0.80	
4. Nonescalating Benefit/ Permanent Total and Major PP	T. Beta	7.00	.513	1.28	0.3
5. All/Minor PP and Temporary Total	T. Beta	2.20	7.24	0.12	2.9

EXHIBIT 9, PART 1

National Council on Compensation Insurance

State M
Effective: 1/1/89
Policy Period: 4/1/85–3/31/86
Report: First

Excess Loss Factor Calculation
Average Cost Per Case

Type of Injury	(1) Indemnity Losses	(2) Amend. Factor	(3) Trend in Ind. Cost Per Case	(4) Indem. Trend on Level (1) × (2) × (3)	(5) Medical Losses	(6) Law Level Factor	(7) Medical Trend	(8) Medical Trended on Level (5) × (6) × (7)
Fatal	$8,904,969	1.073	1.292	$12,345,101	$472,879	1.000	1.326	$627,038
PT	3,372,190	1.022	1.292	4,452,721	3,714,911	1.000	1.326	4,925,972
Major	114,956,442	1.017	1.292	151,048,626	46,784,854	1.000	1.326	62,036,716
Minor	14,573,805	1.017	1.292	19,149,455	9,794,077	1.000	1.326	12,986,946
TT	61,806,919	1.016	1.292	81,132,212	57,376,307	1.000	1.326	76,080,983
Med. Only					27,520,731	1.000	1.326	36,492,489

Type of Injury	(9) Total Losses (4) + (8)	(10) No. of Claims	(11) Average Severity (9)/(10)	(12) Severity Dev. to Ult. Rpt.	(13) Developed Severity (11) × (12)	(14) Indemnity Dev. to Ult. Rpt.	(15) Medical Dev. to Ult. Report	(16) Total Developed (4) × (14) + (8) × (15)
Fatal	$12,972,139	103	125,943	0.869	109,444	1.027	1.532	$13,639,041
PT	9,378,693	41	83,508	1.333	111,316	4.711	3.638	38,897,455
Major	213,085,342	2,623				2.060	1.904	429,278,077
Minor	32,136,401	2,628	12,228	0.761	5,174	0.908	0.959	29,842,186
TT	157,213,195	30,998	5,072	0.951		0.959	0.959	150,767,454
Med. Only	36,492,489	xx	xx	xx	xx	1.000	0.959	34,996,297

EXHIBIT 9, PART 2

National Council on Compensation Insurance

State M
Effective: 1/1/89
Policy Period: 4/1/84–3/31/85
Report: Second

Excess Loss Factor Calculation
Average Cost Per Case

Type of Injury	(1) Indemnity Losses	(2) Amend. Factor	(3) Trend in Ind. Cost Per Case	(4) Indem. Trend on Level (1) × (2) × (3)	(5) Medical Losses	(6) Law Level Factor	(7) Medical Trend	(8) Medical Trended on Level (5) × (6) × (7)
Fatal	$6,989,165	1.090	1.370	$10,436,920	$1,287,604	1.000	1.419	$1,827,110
PT	6,951,686	1.026	1.370	9,771,429	11,951,469	1.000	1.419	16,959,135
Major	182,012,327	1.021	1.370	254,593,383	59,464,731	1.000	1.419	84,380,453
Minor	17,083,444	1.021	1.370	23,895,809	10,947,760	1.000	1.419	15,534,871
TT	54,847,614	1.020	1.370	76,644,056	48,598,297	1.000	1.419	68,960,983
Med. Only					29,022,162	1.000	1.419	41,182,448

Type of Injury	(9) Total Losses (4) + (8)	(10) No. of Claims	(11) Average Severity (9)/(10)	(12) Severity Dev. to Ult. Rpt.	(13) Developed Severity (11) × (12)	(14) Indemnity Dev. to Ult. Rpt.	(15) Medical Dev. to Ult. Report	(16) Total Developed (4) × (14) + (8) × (15)
Fatal	$12,264,030	105	116,800	0.896	104,653	0.963	1.340	$12,499,081
PT	26,730,564	61	94,254	1.190	112,162	2.651	2.917	75,373,855
Major	338,973,836	3,819				1.315	1.404	453,260,455
Minor	39,430,680	4,020	9,809	0.864	4,760	0.975	0.867	36,767,147
TT	145,605,039	33,794	4,309	1.002		0.990	0.867	135,666,788
Med. Only	41,182,448	xx	xx	xx	xx	1.000	0.867	35,705,182

EXHIBIT 9, PART 3

National Council on Compensation Insurance

State M
Effective: 1/1/89
Policy Period: 4/1/83–3/31/84
Report: Third

Excess Loss Factor Calculation
Average Cost Per Case

Type of Injury	(1) Indemnity Losses	(2) Amend. Factor	(3) Trend in Ind. Cost Per Case	(4) Indem. Trend on Level (1) × (2) × (3)	(5) Medical Losses	(6) Law Level Factor	(7) Medical Trend	(8) Medical Trended on Level (5) × (6) × (7)
Fatal	$6,257,156	1.109	1.452	$10,075,698	$624,136	1.000	1.518	$947,438
PT	6,086,216	1.027	1.452	9,075,790	2,863,209	1.000	1.518	4,346,351
Major	186,520,691	1.023	1.452	277,057,088	57,815,100	1.000	1.518	87,763,322
Minor	17,093,885	1.022	1.452	25,366,368	10,177,879	1.000	1.518	15,450,020
TT	49,286,232	1.021	1.452	73,066,445	42,863,196	1.000	1.518	65,066,332
Med. Only					29,338,432	1.000	1.518	44,535,740

<table>
<tr><th></th><th>(9) Total Losses (4) + (8)</th><th>(10) No. of Claims</th><th>(11) Average Severity (9)/(10)</th><th>(12) Severity Dev. to Ult. Rpt.</th><th>(13) Developed Severity (11) × (12)</th><th>(14) Indemnity Dev. to Ult. Rpt.</th><th>(15) Medical Dev. to Ult. Report</th><th>(16) Total Developed (4) × (14) + (8) × (15)</th></tr>
<tr><td>Fatal</td><td>$11,023,136</td><td>107</td><td>103,020</td><td>0.982</td><td>101,166</td><td>1.006</td><td>1.274</td><td>$11,343,188</td></tr>
<tr><td>PT</td><td>13,422,141</td><td>44</td><td rowspan="2">97,510</td><td rowspan="2">1.069</td><td rowspan="2">104,238</td><td>1.490</td><td>1.834</td><td>21,494,135</td></tr>
<tr><td>Major</td><td>364,820,410</td><td>3,835</td><td>1.119</td><td>1.337</td><td>427,366,443</td></tr>
<tr><td>Minor</td><td>40,816,388</td><td>3,664</td><td>11,140</td><td>0.912</td><td rowspan="2">5,364</td><td>0.994</td><td>0.989</td><td>40,494,240</td></tr>
<tr><td>TT</td><td>138,132,777</td><td>29,309</td><td>4,713</td><td>1.011</td><td>0.989</td><td>0.989</td><td>136,613,316</td></tr>
<tr><td>Med. Only</td><td>44,535,740</td><td>xx</td><td>xx</td><td>xx</td><td>xx</td><td>1.000</td><td>0.989</td><td>44,045,847</td></tr>
</table>

EXHIBIT 9, PART 4

Three-Year Statewide Totals

State M

Losses by Injury Type	
Fatal	$37,481,310
PT	135,765,445
Major	1,309,904,975
Minor	107,103,573
TT	423,047,558

Average Cost Per Case	
Fatal	$105,035
PT/Major	108,997
Minor/TT	5,084

EXHIBIT 10

National Council on Compensation Insurance

State M
Effective: 1/1/89

Limited Fatal Benefits—Nonescalating PT/Major Benefits

Calculation of *ELF* Trend

Policy Period:	4/1/85–3/31/86 First Report	4/1/84–3/31/85 Second Report	4/1/83–3/31/84 Third Report
(1) Effective Date of Filing		1/1/89	
(2a) Midpoint of Filing		1/1/90	
(2b) Midpoint of Policy Period	4/1/86	4/1/85	4/1/84
(3) Benefit Level		1/1/89	
(4a) Yrs. from (2b) to (3)	2.75	3.75	4.75
(4b) Yrs. from (3) to (2a)	1	1	1
(5) Indemnity Trend (1.060**(4a)) × (1.101**(4b))	1.292	1.370	1.452
(6) Medical Average Charge—3/31/88		321.95	
(7) Medical Average Charge—3/31/83		230.93	
(8) Change over 5 Yrs. (6)/(7)		1.394	
(9a) Indicated Change Per Year (8) ** .2		1.069	
(9b) Limit on Change Per Year		1.070	
(10) Medical Trend ((9)**(4a)) × (1.101**(4b))	1.326	1.419	1.518

EXHIBIT 11

Type of Injury Loss Distribution Table

Countrywide

$CLR_{I,H}$

Hazard Group

Injury Type	I	II	III	IV
Fatal	0.086	0.128	0.282	0.504
PT	0.158	0.208	0.278	0.355
Major	0.224	0.228	0.288	0.260
Minor	0.310	0.283	0.226	0.181
TT	0.308	0.281	0.240	0.171
Med. Only	0.331	0.297	0.201	0.171

Based on countrywide Unit Statistical Plan summaries, policy year 1981 at second and third reports.

EXHIBIT 12

Premium Distribution by Hazard Group*

State M

Hazard Group	(1) Standard Premium	(2) Total Standard Premium	(3) P_H (1) ÷ (2)
I	35,912,865	2,095,858,472	0.017
II	988,939,212		0.472
III	1,003,721,317		0.479
IV	67,285,078		0.032

* Based on Unit Statistical Data excluding stevedoring for policy periods 4/1/82–3/31/83, 4/1/83–3/31/84, 4/1/84–3/31/85 (second reports).

EXHIBIT 13

Distribution of Losses by Hazard Group for Each Injury Type

State M

$L_{I,H}$

Hazard Group

Injury Type	I	II	III	IV
Fatal	0.007	0.284	0.633	0.076
PT	0.011	0.400	0.543	0.046
Major	0.015	0.418	0.535	0.032
Minor	0.021	0.528	0.428	0.023
TT	0.020	0.514	0.445	0.021
Med. Only	0.023	0.566	0.389	0.022

These numbers are derived from the state premium distribution and countrywide loss distribution. For each Hazard Group, the following procedure is utilized to obtain the distribution of losses within each injury type:

> The percentage of countrywide losses by Hazard Group (see Exhibit 11) is multiplied by the corresponding statewide ratio of standard earned premium to total (Exhibit 12). This is then divided by the sum of these calculations for all four Hazard Groups.

EXHIBIT 14

Combined Injury Weights

State M

Hazard Group I

(1) Type of Injury	(2) Total Incurred Losses	(3) Injury Weights
Fatal	262,369	0.008
PT	1,493,420	0.043
Major	19,648,575	0.565
PT/Major	21,141,995	0.608
Minor	2,249,175	0.065
TT	8,460,951	0.243
Minor/TT	10,710,126	0.308
Med. Only	2,639,188	xx
Total	34,753,678	xx

Hazard Group II

(1) Type of Injury	(2) Total Incurred Losses	(3) Injury Weights
Fatal	10,644,692	0.011
PT	54,306,178	0.057
Major	547,540,280	0.575
PT/Major	601,846,458	0.632
Minor	56,550,687	0.059
TT	217,446,445	0.229
Minor/TT	273,997,132	0.288
Med. Only	64,946,987	xx
Total	951,435,269	xx

Hazard Group III

(1) Type of Injury	(2) Total Incurred Losses	(3) Injury Weights
Fatal	23,725,669	0.022
PT	73,720,637	0.068
Major	700,799,162	0.651
PT/Major	774,519,799	0.719
Minor	45,840,329	0.043
TT	188,256,163	0.175
Minor/TT	234,096,492	0.218
Med. Only	44,636,710	xx
Total	1,076,978,670	xx

Hazard Group IV

(1) Type of Injury	(2) Total Incurred Losses	(3) Injury Weights
Fatal	2,848,580	0.044
PT	6,245,210	0.096
Major	41,916,959	0.646
PT/Major	48,162,169	0.742
Minor	2,463,382	0.038
TT	8,883,999	0.137
Minor/TT	11,347,381	0.175
Med. Only	2,524,441	xx
Total	64,882,571	xx

For each Hazard Group, the following procedure is utilized to obtain the distribution of losses:

> The injury type total incurred losses from Exhibit 9, Part 4 are spread across the Hazard Groups using the distributions from Exhibit 13. Within each Hazard Group, percentages of loss by type of total are then calculated in column (3).

EXHIBIT 15

Severity Differential to Unweighted Average

Countrywide

	Hazard Group			
Injury Type	I	II	III	IV
Fatal	0.771	0.911	1.087	1.231
PT	0.813	0.954	0.988	1.245
Major	0.898	0.930	1.041	1.131

Based on 1981 statistical plan data, latest second and third reports.

EXHIBIT 16

Severity Differentials by Claim Type and Hazard Group

State M

(A) Adjustment Factors

Injury Type	
Fatal	1.003164
PT	0.977201
Major	0.989057

(B) Normalized Differentials

	Hazard Group			
Injury Type	I	II	III	IV
Fatal	0.769	0.908	1.084	1.227
PT	0.832	0.976	1.011	1.274
Major	0.908	0.940	1.053	1.144
PT/Major	0.903	0.943	1.048	1.161

(A) For each serious injury type, the countrywide Hazard Group unweighted average cost per case differential from Exhibit 15 is multiplied by percent of premium in the Hazard Group for that state from Exhibit 12. These products are summed to form the factors in (A).

(B) For each Hazard Group, the factors from Exhibit 15 are divided by the appropriate adjustment factor in Section A of this exhibit to produce differentials appropriate for State M.
For PT and Major injury types, combined differentials are derived by calculating weighted averages of two differentials by Hazard Group, using the factors from Exhibit 14 as weights.

EFFECTS OF VARIATIONS FROM GAMMA-POISSON ASSUMPTIONS

GARY G. VENTER

Abstract

Two types of variations from negative binomial frequency are considered: mixtures of Poisson other than Gamma, and Poisson parameters that change over time. Any severity distribution can be used instead of the Gamma as a mixing distribution, and Bayesian estimators are easy to calculate from the mixed probabilities. In the case of changing frequencies over time, the Gerber-Jones model is illustrated for calculating credibilities. The Bailey-Simon method is found to be useful for testing model assumptions.

1. INTRODUCTION

A model often used for experience rating assumes that each individual risk has its own Poisson distribution for number of claims, with a Gamma distribution across the population for the Poisson mean. This model has been known since at least 1920 (M. Greenwood and G. Yule [7]), and has been applied to insurance experience rating since at least 1929 (R. Keffer [10]). However, there is meager theoretical support for the Gamma distribution as a mixing function, and the main empirical support given in many studies is that it provides a better fit to the aggregate claim frequency distribution than that given by the assumption that all individuals have the same Poisson distribution; e.g., see Lester B. Dropkin [4], B. Nye and A. Hofflander [12], or R. Ellis, C. Gallup, and T. McGuire [5]. The Poisson assumption for each individual does have theoretical support, but not enough to be regarded as certain. For example, the Poisson parameter could vary over time in random ways, to be discussed further below.

Several alternative models, which, in many cases, fit better than the Poisson, have been presented in the literature; e.g., Gordon Willmot [19, 20], M. Ruohonen [14], W. Hürlimann [8]. Many of these are mixtures of the Poisson by other distributions, such as the inverse Gaussian, reciprocal inverse Gaussian, beta, uniform, noncentral chi-squared, and three-parameter origin shifted Gamma distributions.

The purpose of this paper is to explore the adequacy of the Poisson and Gamma assumptions, the information needed to verify them, and the experience rating consequences of using these assumptions when they do not apply. As will be seen below, there are substantial differences in the experience rating implications of models which have very similar predictions of the aggregate claim frequency distribution. Thus, a model which gives a good fit to this distribution does not necessarily give proper experience rating adjustments. In other words, a model that just fits better than the Poisson is not enough for experience rating use. More detailed records which track individuals over time are needed to determine how much credibility should be given to individual claim experience.

2. PRELIMINARY BACKGROUND

Suppose each risk has its own claim frequency distribution, constant over time, and that the mean of the individual risk annual claim frequency variances is s^2, and the variance of the risk means is t^2. Among linear estimators, the expected squared error in subsequent observations is minimized by the credibility estimator $zx + (1 - z)m$, where m is the overall mean, x is the individual risk annual frequency observed, and for n years of observations, $z = n/(n + K)$, with $K = s^2/t^2$. See, for example, A. Bailey [1], H. Bühlmann [3], W. Jewell [9]. If the restriction to linear estimators is removed, then the Bayesian predictive mean minimizes the expected squared error. Thus, when the Bayes estimator is linear in the observations, it must be the same as the credibility estimator.

This is the case with the Gamma-Poisson model. In fact, if the Gamma has parameters α and β, with mean α/β and variance α/β^2, the Bayesian predictive mean is

$$\frac{\alpha + nx}{\beta + n} = \frac{\alpha}{\beta} \cdot \frac{\beta}{\beta + n} + x \cdot \frac{n}{\beta + n},$$