VOLUME LXXX NUMBERS 152 AND 153

PROCEEDINGS

OF THE

Casualty Actuarial Society

ORGANIZED 1914

1993

VOLUME LXXX

Number 152—May 1993

Number 153—November 1993

Library of Congress Catalog No. HG9956.C3
ISSN 0893-2980

Printed for the Society by
Colortone Press
Landover, Maryland

FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow. The Society adopted its present name on May 14, 1921.

Actuarial science originated in England in 1792 in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians. Eventually, their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. The Faculty of Actuaries was founded in Scotland in 1856, followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. In 1949 the two American organizations were merged into the Society of Actuaries.

In the beginning of the 20th century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers' compensation, which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Society. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. The purpose of the Society is to advance the body of knowledge of actuarial science in applications other than life insurance, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. The Society's activities in support of this purpose include communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers' compensation were the most urgent, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published in the annual *Proceedings of the Casualty Actuarial Society.* The presidential addresses, also published in the *Proceedings,* have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the industry over the years.

The membership of the Society includes actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government. It also includes independent consultants. The Society has two classes of members, Fellows and Associates. Both classes are achieved by successful completion of examinations, which are held in February, May, and November in various cities of the United States, Canada, Bermuda, and selected overseas sites.

The publications of the Society and their respective prices are listed in the *Yearbook* which is published annually. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook,* at a charge of $40, and the *Syllabus of Examinations,* without charge, may be obtained upon request to the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

JANUARY 1, 1993

*EXECUTIVE COUNCIL

DAVID P. FLYNN . . . . *President*
IRENE K. BASS . . . . *President-Elect*
JOHN M. PURPLE . . . . *Vice President-Administration*
STEVEN G. LEHMANN . . . . *Vice President-Admissions*
DAVID N. HAFLING . . . . *Vice President-Continuing Education*
ALICE H. GANNON . *Vice President-Programs and Communications*
ALLAN M. KAUFMAN . . *Vice President-Research and Development*

THE BOARD OF DIRECTORS

**Officers*

DAVID P. FLYNN . . . . *President*
IRENE K. BASS . . . . *President-Elect*

† *Immediate Past President*

MICHAEL L. TOOTHMAN 1993

† *Elected Directors*

ROBERT A. ANKER 1993
LINDA L. BELL 1993
W. JAMES MACGINNITIE 1993
JAMES N. STANARD 1993
JAMES K. CHRISTIE 1994
HEIDI E. HUTTER 1994
GARY S. PATRIK 1994
SHELDON ROSENBERG 1994
ALBERT J. BEER 1995
PHILLIP N. BEN-ZVI 1995
MICHAEL J. MILLER 1995
SUSAN T. SZKODA 1995

* Term expires at the 1994 Annual Meeting. All members of the Executive Council are Officers. The Vice President-Administration also serves as Secretary and Treasurer.

† Term expires at Annual Meeting of year given.

1993 PROCEEDINGS
CONTENTS OF VOLUME LXXX

1993 PROCEEDINGS
CONTENTS OF VOLUME LXXX

1993 PROCEEDINGS
CONTENTS OF VOLUME LXXX

NOTICE

Papers submitted to the *Proceedings of the Casualty Actuarial Society* are subject to review by the members of the Committee on Review of Papers and, where appropriate, additional individuals with expertise in the relevant topics. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques, or have special educational value, and must not have been previously published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

The Society is not responsible for statements of opinion expressed in the articles, criticisms, and discussions published in these *Proceedings*.

Volume LXXX, Part 1 **No. 152**

PROCEEDINGS
May 9, 10, 11, 12, 1993

EMPIRICAL TESTING OF CLASSIFICATION RELATIVITIES

ROGER M. HAYNE

Abstract

There has been considerable discussion regarding the theoretical basis for insurance classifications and the calculation of classification rates. Rather than focusing on those issues, this paper presents some tests of the relative accuracy of competing rating methodologies. These tests are empirical in nature and involve comparing among classes the cost differences that actually have emerged with estimates of those differences using an alternative classification ratemaking methodology. In addition to tests of classification relativities, this paper also includes a test of the differences in excess loss experience among classes. These tests have been applied in practice and this paper includes examples of the corresponding calculations.

I would like to acknowledge the immense effort by the National Council on Compensation Insurance in providing classification rates for a substantial number of alternatives and states.

1. INTRODUCTION

There has been considerable discussion in actuarial and insurance press regarding risk classification and the calculation of relative rate differences among the various classes. Much of this discussion has centered on the theoretical basis for the classification structure and the method used to assign rates to the classes. Rather than adding to that discussion, this paper explores some techniques used to test empirically how well various methods have performed in identifying relative cost differences among classes. This analysis arose from testing specific alternative classification ratemaking methodologies as part of the 1991 examination of the National Council on Compensation Insurance (NCCI) ratemaking procedures undertaken by the National Association of Insurance Commissioners (NAIC).

Although our focus will be on applications to NCCI workers' compensation classification ratemaking, this methodology could be used for other lines of insurance. Since the methodology generally does not depend on the specific NCCI methodology, the reader should not need knowledge of current NCCI classification ratemaking methodology. This paper will, however, briefly review that methodology to the extent that it helps in the understanding of the approach. The goal is to test which of two specific alternative methodologies more accurately predicts the relative cost differences that emerge among classes. Thus, the focus is on how well a particular methodology predicts actual relative loss differences among classes.

2. TEST OF RELATIVE ACCURACY

The basic test of relative accuracy compares actual relative limited loss differences with those inherent in the rates calculated under two alternatives. For example, the NAIC examination used actual limited policy year 1987 losses by class to compare the relative accuracy of alternative methods used to calculate class rates for 1987. Since we are concerned with *relative* loss cost differences, we adjust the rates under the alternatives to generate total expected losses equal to the actual limited losses reported for the year.

The purpose of this paper is to discuss the tests used to measure differences in the relative accuracy of two sets of classification rates. Thus, it will not address the source of any specific alternative set of classification rates.

Definitions

For class i, let P_i denote the 1987 exposure, L_i denote the 1987 actual limited losses, and R_{ij} denote the 1987 rate using alternative methodology j. For workers' compensation, the experience rating plan affects the final rates charged for individual insureds and thus for classes. Since the adjustments are intended to reflect expected loss differences, they should be considered in comparing actual and expected losses. Thus, for class i, use EP_i to denote the 1987 earned premium, and MP_i to denote the 1987 manual premium.

Generally, the manual premium refers to the premium for a risk before adjustment for experience modifications, while earned premium reflects those modifications. Therefore, it could be argued that the earned premium more closely reflects exposure to loss than manual premium. Implicit below is the assumption that the adjustment to reflect this difference is the same for both methodologies. This is a practical consideration. The tests compare the current rate methodology with an alternative. The manual and earned premiums by class are available for the current methodology but not for the alternative. Note, however, that when the NCCI conducted these tests without this adjustment, the results were quite similar to those derived herein.

Finally, let E_{ij} denote expected losses for class i using alternative methodology j, calculated as follows:

$$E_{ij} = \frac{P_i \times R_{ij} \times \dfrac{EP_i}{MP_i}}{\left[\dfrac{\sum_k P_k \times R_{kj} \times \dfrac{EP_k}{MP_k}}{\sum_k L_k}\right]}.$$

In this case, the sums are taken over all classes. The numerator is simply the 1987 total premium expected for the class, after adjustment for the historical relation between manual and earned premium. Note that here and elsewhere in this paper the term "earned premium" refers to earned standard premium and not earned collected premium. The denominator is a constant that assures the total expected losses equal the total actual losses experienced. This adjustment was made since the focus, at this point, is in the evaluation of how well a particular alternative predicts *relative* loss differences among classes. Thus, the tests focus on the comparison between actual losses L_i and expected losses E_{ij}.

To accomplish this goal, consider the squared error SE_{ij} between the actual and expected losses for class i and alternative j. Define SE_{ij} as:

$$SE_{ij} = E_{ij}\left[\frac{L_i}{E_{ij}} - 1\right]^2 = \frac{(L_i - E_{ij})^2}{E_{ij}} .$$

The first representation here shows SE_{ij} as the square of the relative error between the actual losses L_i and the expected losses E_{ij}, weighted by the volume of expected losses. The second part simply rearranges and cancels terms. Readers may find this latter term familiar, since it is similar to terms in the chi-square statistic, which is sometimes used to test goodness of fit for probability distributions.

Test Statistics

An obvious choice of a test statistic would be the mean squared error, calculated as an average of the SE_{ij} values over all classes. Let MSE_j denote the mean squared error for alternative j. We could then test the difference $MSE_1 - MSE_2$. If the difference is positive, the second alternative could be judged to more accurately identify relative differences. On the other hand, if the difference is negative, the first alternative would be judged better.

However, in order to assess the significance of this difference, we would need to estimate its distribution. Given that the comparison will be made between two methods that will probably use the same data base to calculate relativities, we cannot assume that the mean squared errors observed for the methods are independent. Following Meyers [1], we use the Wilcoxon statistic, sometimes known as the Wilcoxon signed rank statistic, as one test of the significance of the difference between squared errors for the classes. This is a non-parametric test and does not depend on the underlying distribution of the squared errors. To this end define D_i for each class as:

$$D_i = \text{Rank}(|\,SE_{i\,1} - SE_{i\,2}\,|) \times \text{Sign}(SE_{i\,1} - SE_{i\,2}) .$$

Here "Rank" denotes the rank of the quantity in parentheses when the quantities are listed in order, smallest to largest, and "Sign" denotes the sign of the quantity in parentheses. Then, define the Wilcoxon statistic as

$$W = \sum_{i=1}^{n} D_i \quad ,$$

where n is the number of classes.

Under the hypothesis that it is equally likely that the differences $SE_{i\,1} - SE_{i\,2}$ are positive as it is that they are negative, we can calculate the distribution for W. For example: If $n = 1$, W can take on only one of the values -1 or 1 with equal probability; if $n = 2$, it can take one of four values $-3, -1, 1$, and 3, each with equal probability; and so forth. However, for large values of n, the statistic

$$V = \frac{W}{\sqrt{n(n+1)\,(2n+1)/6}}$$

has an approximate standard normal distribution. Although a more rigorous treatment is found in Hogg and Craig [2], this latter conclusion heuristically can be seen to follow from the law of large numbers. Under the hypothesis above, $E[W] = 0$. Thus,

$$\mathrm{Var}\,(W) = \sum_{i=1}^{n} \mathrm{Var}\,(D_i)$$

$$= \sum_{i=1}^{n} (-i)^2\,(\tfrac{1}{2}) + (i)^2\,(\tfrac{1}{2})$$

$$= \sum_{i=1}^{n} i^2$$

$$= \tfrac{1}{6}\, n\,(n+1)\,(2n+1).$$

Exhibit 1 shows the distribution for W with $n = 9$ as compared with the normal distribution approximation. They appear similar enough to use the normal distribution for values of n greater than 9. Note that in many applications, especially in workers' compensation classification ratemaking, there are more than 10 classes considered.

Exhibit 2 shows the calculation of this Wilcoxon statistic using actual NCCI data from a single state. Columns (1) through (4) show the 1987 payroll, earned premiums, manual premiums, and losses at first report for each class. Column (5) shows the final 1987 rates calculated using the current NCCI methodology, while Column (6) shows an alternate set of rates calculated using five years of data to calculate classification pure premiums. Columns (7) and (8) show the calculated "premiums," using both the current methodology [Column (7)] and the alternate [Column (8)] and adjusting for the ratio of earned to manual premiums. Columns (9) and (10) are the resulting expected losses, balancing to total reported losses, for the current and alternate methods, respectively. Columns (11) and (12) show the squared error statistics. Column (13) shows the differences, while Column (14) shows the resulting D_i values.

In this case, the statistic V has a value of 0.99. We can conclude at an approximate 84% confidence level that the alternate method, using five years of data for class rates, is relatively more accurate in identifying relative loss differences among classes than is the current method; i.e., squared errors tend to be less than under the current method. Conversely, under the above assumptions, there is an approximate 16% chance that random fluctuations could produce a Wilcoxon statistic of the observed magnitude or larger if there were actually no difference between the two distributions. Here we used a "one-tailed" test. A "two-tailed" test would have concluded that the distributions were different with an approximate 92% confidence but would not have indicated which tended to have smaller squared errors.

In the analysis for the project, there were a number of occasions when the mean squared error (calculated as the arithmetic average of the squared errors) for the current method was less than that for the alternative but in which the Wilcoxon statistic was significantly positive, indicating that the alternative was relatively more accurate. There were also cases of the converse. Although this may seem contradictory at first, it reflects different characteristics measured by the two statistics. Upon further review it became clear that these situations were caused by numerically large squared errors dominating the averages, whereas their influence in the Wilcoxon statistic was more limited.

Due to limitations in available data, the analysis of NCCI methodology focused on using limited 1987 losses at first report. However, data at second report became available later in the analysis. There may be a difference in development among classes and it is preferable to use even more mature data if available. An alternative would be to include expected development to adjust first or second report losses to their expected ultimate level. Because the goal of the test is a "proof of the pudding" analysis, using actual unadjusted data to the greatest extent possible is desirable. Tests with second report data and developed data generally provided results similar to those using first report data.

During the examination of the NCCI, this same statistic was calculated using second report data ($V = 0.88$), using first report data developed to ultimate ($V = 0.86$), and using second report data developed to ultimate ($V = 0.57$). Generally these alternatives produce roughly the same indications, though at different significance levels.

3. A SECOND TEST OF RELATIVE ACCURACY

The analysis also considered the underwriting statistic as described by Meyers [1]. Since this method is described in detail in that reference, what follows is only a brief summary of the approach and results with these data.

The approach begins by segmenting the data into two groups. Group 1 includes those classes with expected losses for the current method less than those for the alternative method, while Group 2 is comprised of all other classes. By construction, the ratios of actual to expected losses will be lower for the alternative method in the first group and higher in the second as compared to those ratios for the current method.

If, in both groups, one method produces ratios of actual to expected losses that are closer to 1.00, then that method could be considered to provide coverage to classes with better loss experience for lower rates and to classes with worse loss experience for higher rates than the other method. Thus, this method could potentially have a competitive advantage relative to the other. This test focuses on this difference and hence is called the "underwriting test."

The significance of the differences in underwriting ratios is tested by comparing ratios of actual to expected losses from similarly sized groups randomly selected from all groups using a "bootstrapping" technique. The bootstrapping approach is sometimes used in statistical analysis when the actual underlying distributions are either unknown or too complex to analyze directly. Table 1 summarizes the results of the underwriting test. It compares an alternative classification ratemaking methodology to the current methodology.

TABLE 1

SUMMARY OF RESULTS OF UNDERWRITING TEST

Group 1 Ratios of Actual to Expected Losses (217 classes):	
Current Method	1.07
Alternative Method	0.99
Group 2 Ratios of Actual to Expected Losses (210 classes):	
Current Method	0.94
Alternative Method	1.01

Using a bootstrap approach, we randomly choose 2,000 samples of 217 classes (without replacement) from the population of 427 classes and calculate the resulting ratios of actual to expected losses based on the current method. This sampling results in the Table 2 distribution of ratios of actual to expected losses.

As can be seen from Table 2, less than 5% of the samples result in ratios in excess of 1.063; i.e., there is less than a 5% chance that the 1.07 ratio generated by the current method results from random chance. In addition, the alternative method results in a lower ratio in Group 1, the classes where the current method has the greatest difference between actual and expected losses, and also can be profitable with lower prices in Group 2 where the current method is more profitable. Thus there is a significant chance that remaining with the current method could result in adverse selection if a competitor selects the alternative. We also see that the ratio generated by the alternative method, 0.99, is well within expected variation.

4. TESTS FOR EXCESS LOSSES

The current NCCI ratemaking methodology uses limited loss data and distributes a provision for losses in excess of the limitation among classes in each industry group. The NAIC study compared rel-

TABLE 2

DISTRIBUTION OF RATIOS OF ACTUAL TO EXPECTED LOSSES FOR 217 CLASSES

Ratio	Estimated Percentile
0.937	0.025
0.949	0.050
0.961	0.100
0.973	0.200
0.979	0.250
0.984	0.300
0.994	0.400
1.001	0.500
1.009	0.600
1.019	0.700
1.025	0.750
1.031	0.800
1.039	0.850
1.049	0.900
1.063	0.950
1.073	0.975
1.075	0.980

ative rate differences from the various methodologies with the loss cost differences in actual reported limited losses for the tests of relative accuracy. But the distribution of excess losses among classes in an industry group implicitly assumes that the losses above the limitation are not sufficiently different among classes to have this provision vary by class or that any real difference cannot be measured reliably due to random variation. Thus, the study included a separate test of the difference of excess loss experience among classes in an industry group.

The study also included a limited test of the validity of this hypothesis. It used both limited and unlimited loss data, by class, for three policy years at the same valuation date: 1987 at first report, 1986 at second report, and 1985 at third report. The goal here was to test whether the expected loss experience above the limit for an individual class differed significantly from that of the rest of the industry group.

For this analysis, let UL_{ij} and LL_{ij} denote unlimited and limited losses for class i and year j, respectively. The class excess loss factor implied by the data would then be:

$$ELF_{ij} = \frac{UL_{ij}}{LL_{ij}}.$$

On the other hand, the excess loss factor for the other classes in the industry group implied by the data, excluding class i, would be:

$$GELF_{ij} = \frac{\sum_{k \neq i} UL_{kj}}{\sum_{k \neq i} LL_{kj}}.$$

Here the summation is taken over all the other classes in the industry group containing the subject class.

We now test the significance of the difference between the two statistics ELF_{ij} and $GELF_{ij}$. At this point, we again use the Wilcoxon statistic to test the whether the difference $ELF_{ij} - GELF_{ij}$ is significantly different from zero.

Exhibit 3 compares the excess loss experience for class 8810 (Clerical Office Employees NOC) with that of the remainder of the "All Other" industry group. Table 3 summarizes the results shown in that exhibit.

TABLE 3

EXCESS LOSS FACTORS

Year	Class 8810	Other "All Other"	Difference	D-Value
1985	1.0000	1.0453	−0.0453	−3
1986	1.0115	1.0365	−0.0250	−1
1987	1.0000	1.0407	−0.0407	−2
Total				−6

In the case of $n = 3$, the Wilcoxon statistic can only take the values −6, −4, −2, 0, 2, 4, and 6. There is a .25 probability of a 0 value and .125 probability for each of the other values. Thus, there is a 12.5% probability that random chance could result in a value of 6 under the null hypothesis that positive and negative values of the difference are equally likely. We would thus reject this null hypothesis at any confidence level below 87.5%.

Another test statistic also suggests itself. Given the construction of the two statistics ELF_{ij} and $GELF_{ij}$ we would expect both of the statistics to be at least 1.0 and, theoretically, unlimited. Thus assume that $GELF_{ij} - 1$ and $ELF_{ij} - 1$ both form random samples of size 3 ($j = 1, 2, 3$) from independent lognormal distributions. There is the possibility of a particular class not experiencing any excess loss for a particular year. In such cases, set $UL_{ij} = LL_{ij} + 1$. We include only classes with losses experienced in each policy year. Under these assumptions, the natural logarithms of $GELF_{ij} - 1$ and $ELF_{ij} - 1$ are random samples from independent normal distributions. Therefore, we evaluate the significance of the difference between the two means:

$$M_i = \tfrac{1}{3} \sum_{j=1}^{3} \ln(ELF_{ij} - 1) \text{ ; and}$$

$$GM_i = \tfrac{1}{3} \sum_{j=1}^{3} \ln(GELF_{ij} - 1).$$

The assumptions regarding the lognormality of the statistics imply that $\ln(ELF_{ij} - 1)$ and $\ln(GELF_{ij} - 1)$ are independent random samples from normal distributions with possibly different variances.

Thus, referring to normal statistical theory, set

$$Z_{ij} = \ln(GELF_{ij} - 1) - \ln(ELF_{ij} - 1),$$

$$\bar{Z}_i = 1/3 \sum_{j=1}^{3} Z_{ij}, \text{ and}$$

$$S_i^2 = 1/2 \sum_{j=1}^{3} (Z_{ij} - \bar{Z}_i)^2 .$$

Under the null hypothesis that the two underlying means are equal,

$$H_0\text{: } \mathrm{E}[\ln(GELF_{ij} - 1)] = \mathrm{E}[\ln(ELF_{ij} - 1)] ,$$

the variable Z_{ij} has a normal distribution with mean 0.

In the development thus far we assumed that $ELF_{ij} - 1$ and $GELF_{ij} - 1$ are independent lognormal variables. However, the statistics Z_{ij} will still form a random sample from a normal distribution if we simply assume that the ratios

$$\frac{GELF_{ij} - 1}{ELF_{ij} - 1}$$

form a random sample from a lognormal distribution. This softens the requirement that the numerator and denominator be independent. The above derivation shows that independence and lognormality are sufficient to conclude that Z_{ij} is normal but they are not necessary. For example, if $ELF_{ij} - 1$ and $GELF_{ij} - 1$ are jointly lognormal (i.e., if $\ln(ELF_{ij} - 1)$ and $\ln(GELF_{ij} - 1)$ are jointly normal but are not independent), then Z_{ij} will still be normal.

From this point on, simply assume that Z_{ij} forms a random sample from a normal distribution. Then the following statistic has a t distribution with 2 degrees of freedom:

$$T_i = \frac{\bar{Z}_i}{S_i / \sqrt{3}} .$$

Then use standard tables to test the significance of the difference between the expected excess losses for a class and that of the remaining industry group by testing the significance of the difference between T_i and 0. Either accept the null hypothesis that the Z_{ij} are a random sample from a normal distribution with mean 0 or reject that hypothesis.

Some may argue that the Z_{ij} values have different distributions for different valuation years. This could be due to a different mix of open and closed claims at different maturities and the possibility that larger claims experience different development than smaller claims.

The primary hypothesis we wish to test is whether the excess loss experience of a class is significantly different than that of the remainder of its industry group. Under the null hypothesis, then, we would expect similar excess loss behavior for both the class and the remainder of the industry group, and thus possibly some positive correlation between the statistics $\ln(GELF_{ij} - 1)$ and $\ln(ELF_{ij} - 1)$. Thus, one could argue that the potential difference in variance of the Z_{ij} from one maturity to the next may not be as great as that in the two component statistics. If this argument is accepted, the assumption that the Z_{ij} statistics have the same variance may not be significantly violated if the null hypothesis is indeed true. Unfortunately, we do not have sufficient data to test which is the actual case.

Note that if we had the data at the same maturity for all years this criticism would not arise. However, such data were not readily available for the NCCI analysis. Also note that this example uses only the three most recent years of data. This restriction is primarily due to data availability rather than theoretical reasons. More years of data should be used in this test. In addition, if data for older years were available and if the test were confined to those older years, then the concern noted above regarding differing maturities would probably be of less significance. In any case, though there may be some concerns with the application of this statistic in this particular situation, there are many situations where it can be applied without such concerns.

Exhibit 3 gives an example of this calculation for class 8810 (Clerical Office Employees NOC). The first two columns give the "All Other" industry group loss data, both limited and unlimited. The next four columns give the loss, excess loss factor, and corresponding logarithmic transformation data for class 8810. Since the limited and unlimited losses for 1985 and 1987 are equal, the excess loss factors were calculated using

$$UL_{8810,1} = LL_{8810,1} + 1, \text{ and}$$

$$UL_{8810,3} = LL_{8810,3} + 1.$$

The last four columns provide the same information for the "All Other" industry group excluding class 8810. The resulting Z values and statistics are also shown there. In this case the value of the T_i is 2.3327. This value is significant at greater than 85.5% with a one-tailed test and greater than 92.8% with a two-tailed test. Thus, at a 90% confidence level, we would reject the hypothesis that the Z_{ij} form a random sample from a normal distribution with mean 0.

The limited analysis, based only on these three years of data, resulted in the Table 4 percentages of classes that were different from the remaining classes in the industry group, based on data from the same state used in the previous sections.

Table 4 shows that, based on the limited data analyzed, there is a sizable proportion of classes for which we reject the null hypothesis stated above. The NCCI study performed this test for a total of 10 states. Of the 30 industry group/state combinations tested, 17 had more than half of their classes significantly different from the group as a whole at the 99% confidence level.

TABLE 4

PERCENTAGE OF CLASSES WITH SIGNIFICANTLY DIFFERENT EXCESS LOSS EXPERIENCE THAN REMAINDER OF GROUP

	Confidence Level			
Industry Group	90%	95%	97.5%	99%
Manufacturing	1.6%	0.0%	0.0%	0.0%
Contracting	84.7%	79.7%	66.1%	49.2%
Other	84.9%	78.4%	65.5%	59.7%

Note that these results are based on only three years of experience. As such, it is possible that a class or classes may not have any excess loss experience or may have been "unlucky" enough to have excess losses during the experience period. This test, however, can be used to test the significance of differences with additional data.

REFERENCES

[1] Meyers, Glenn G., "Empirical Bayesian Credibility for Workers' Compensation Classification Ratemaking," *PCAS* LXXI, 1984, p. 96.

[2] Hogg, Robert V., and Craig, Allen T., *Introduction to Mathematical Statistics,* Third Edition, New York, Macmillan, 1970.

EXHIBIT 1
COMPARISON OF WILCOXON CUMULATIVE DENSITY AND NORMAL APPROXIMATION WITH $n = 9$

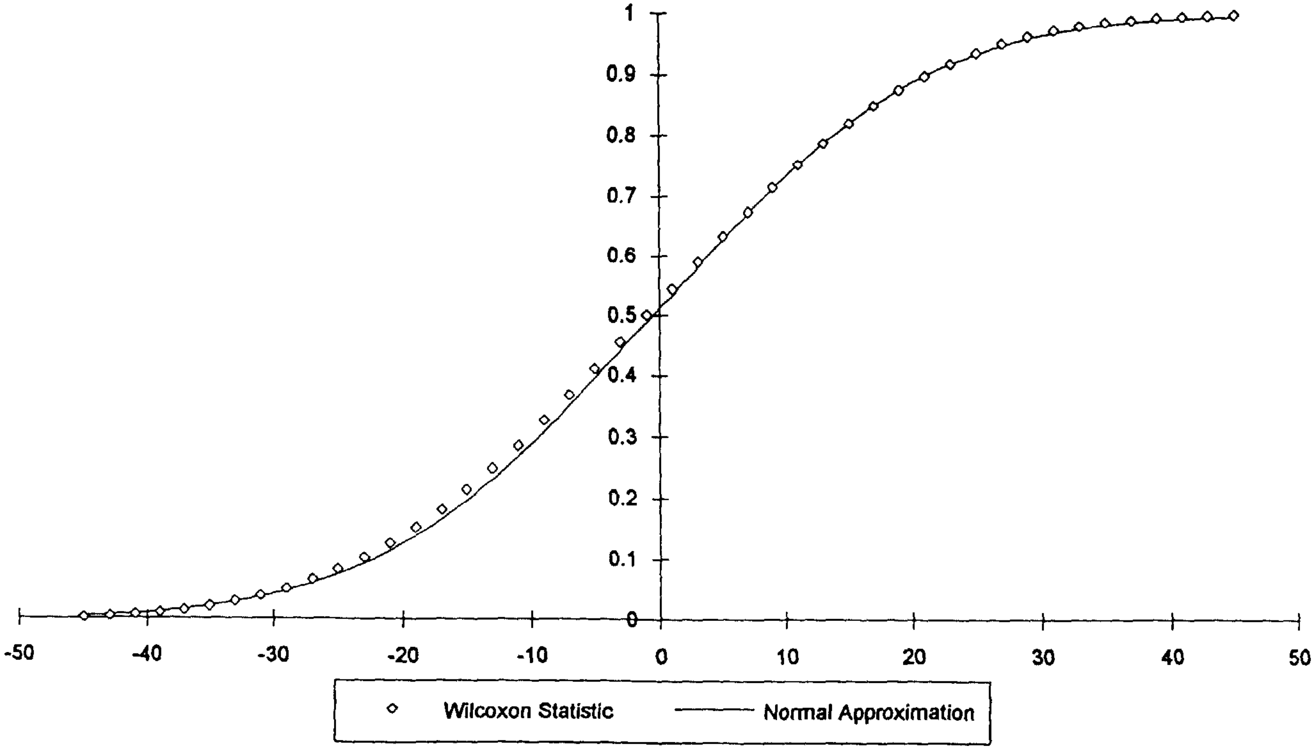


EXHIBIT 2

Part 1

CALCULATION OF WILCOXON STATISTIC

BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Class	1987 Payroll	1987 Earned Premium	1987 Manual Premium	1987 First Report Losses	Current Rate	Alternate Rate	Current Total Premiums (1)x(5)x(2)/[(3)x100]	Alternate Total Premiums (1)x(6)x(2)/[(3)x100]	Current Expected Losses	Alternate Expected Losses	Current Squared Error [(9)-(4)]^2/(9)	Alternate Squared Error [(10)-(4)]^2/(10)	Difference (11)-(12)	*D* Value
2361	1,153	10	11	0	1.23	1.23	13	13	7	7	7	7	0	1
2110	1,191	43	43	0	4.03	4.06	48	48	26	26	26	26	0	-2
4823	5,354	73	63	0	1.31	1.32	81	82	45	45	45	45	0	-3
4133	34,981	568	568	0	2.06	2.06	721	721	395	394	395	394	1	4
1438	5,039	125	125	0	2.76	2.79	139	141	76	77	76	77	-1	-5
3315	5,515	131	131	0	2.86	2.89	158	159	86	87	86	87	-1	-6
79	2,800	42	42	0	4.44	4.50	124	126	68	69	68	69	-1	-7
2790	95,223	924	924	136	1.21	1.21	1,152	1,152	631	630	389	388	1	8
4240	8,045	198	198	0	2.83	2.86	228	230	125	126	125	126	-1	-9
8203	332,198	14,607	13,762	7,999	4.61	4.62	16,255	16,290	8,907	8,914	93	94	-1	-10
9182	7,176	119	119	0	1.92	1.96	138	141	75	77	75	77	-1	-11
3385	214,627	1,559	1,604	0	0.92	0.92	1,919	1,919	1,052	1,050	1,052	1,050	1	12
4350	16,841	113	113	0	0.80	0.82	135	138	74	76	74	76	-2	-13
3118	19,519	422	422	0	2.27	2.29	443	447	243	245	243	245	-2	-14
3620	20,474,641	1,105,550	1,032,249	822,259	6.73	6.74	1,475,792	1,477,985	808,687	808,741	228	226	2	15
4923	30,835	327	327	0	1.40	1.39	432	429	237	235	237	235	2	16
4568	37,851	458	738	0	2.21	2.23	519	524	284	287	284	287	-2	-17
2600	114,484	1,660	1,660	1,040	1.89	1.91	2,164	2,187	1,186	1,197	18	20	-3	-18
4283	60,757	2,950	2,584	2,165	5.16	5.20	3,579	3,607	1,961	1,974	21	19	3	19
4431	58,109	569	569	0	1.06	1.07	616	622	338	340	338	340	-3	-20
169	3,000	222	222	0	7.28	7.46	218	224	120	122	120	122	-3	-21
4061	36,744	1,252	1,252	0	4.16	4.18	1,529	1,536	838	840	838	840	-3	-22
4053	75,843	1,183	1,183	0	2.07	2.08	1,570	1,578	860	863	860	863	-3	-23
36	76,614	3,788	3,788	1,073	6.08	6.08	4,658	4,658	2,553	2,549	858	855	3	24
4703	167,640	3,216	3,243	45	2.68	2.68	4,455	4,455	2,441	2,438	2,352	2,349	3	25
2105	19,596	498	528	0	3.36	3.40	621	628	340	344	340	344	-4	-26
2220	215,105	4,353	4,582	0	2.27	2.27	4,639	4,639	2,542	2,538	2,542	2,538	4	27
2413	43,682	1,005	1,005	0	2.99	3.01	1,306	1,315	716	719	716	719	-4	-28
7133	201,415	6,456	6,520	1,045	4.39	4.40	8,755	8,775	4,798	4,802	2,935	2,939	-4	-29
2286	123,121	2,610	2,610	521	2.51	2.52	3,090	3,103	1,693	1,698	812	816	-4	-30
4751	48,212	1,244	1,196	0	3.05	3.07	1,529	1,540	838	842	838	842	-4	-31

EXHIBIT 2
Part 2
CALCULATION OF WILCOXON STATISTIC
BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Class	1987 Payroll	1987 Earned Premium	1987 Manual Premium	1987 First Report Losses	Current Rate	Alternate Rate	Current Total Premiums (1)x(5)x (2)/[(3)x100]	Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	Current Expected Losses	Alternate Expected Losses	Current Squared Error [(9)-(4)]^2/(9)	Alternate Squared Error [(10)-(4)]^2/(10)	Difference (11)-(12)	*D* Value
3807	7,592	207	207	0	2.99	3.10	227	235	124	129	124	129	-4	-32
113	9,042	428	433	0	4.39	4.49	392	401	215	220	215	220	-5	-33
3240	129,808	6,117	5,413	0	4.57	4.57	6,704	6,704	3,673	3,668	3,673	3,668	5	34
3830	1,215,518	6,756	12,123	0	1.17	1.17	7,926	7,926	4,343	4,337	4,343	4,337	6	35
2835	396,443	10,639	11,175	6,482	3.03	3.11	11,436	11,738	6,267	6,423	7	1	7	36
2578	636,366	22,682	19,854	15,900	3.86	3.84	28,063	27,917	15,377	15,276	18	25	-8	-37
1853	1,669	70	60	289	4.63	4.66	90	91	49	50	1,162	1,154	8	38
4815	209,640	3,319	3,123	227	2.19	2.20	4,879	4,902	2,674	2,682	2,239	2,247	-8	-39
3224	61,129	1,498	1,498	0	3.23	3.26	1,974	1,993	1,082	1,090	1,082	1,090	-9	-40
2688	254,162	3,635	3,635	170	1.72	1.73	4,372	4,397	2,395	2,406	2,068	2,078	-10	-41
2388	123,790	1,750	1,750	0	1.88	1.90	2,327	2,352	1,275	1,287	1,275	1,287	-12	-42
4360	245,198	1,433	1,433	0	0.63	0.64	1,545	1,569	846	859	846	859	-12	-43
4308	125,814	1,057	1,057	0	0.83	0.85	1,044	1,069	572	585	572	585	-13	-44
2150	136,326	5,974	6,970	36	7.35	7.34	8,588	8,576	4,706	4,693	4,634	4,621	13	45
2915	461,083	13,245	12,495	2,912	3.00	3.01	14,663	14,712	8,035	8,050	3,266	3,279	-13	-46
9180	542,222	25,508	24,385	16,254	5.58	5.55	31,649	31,479	17,343	17,225	68	55	14	47
4777	76,262	2,371	2,419	0	4.12	4.16	3,080	3,110	1,688	1,702	1,688	1,702	-14	-48
2651	190,000	4,322	4,598	738	3.23	3.25	5,769	5,804	3,161	3,176	1,857	1,872	-14	-49
7230	1,198,750	29,740	35,737	18,803	3.22	3.70	32,122	36,911	17,602	20,197	82	96	-14	-50
2300	151,586	3,000	2,941	0	2.00	2.02	3,093	3,123	1,695	1,709	1,695	1,709	-15	-51
4206	168,272	3,719	3,719	0	2.62	2.64	4,409	4,442	2,416	2,431	2,416	2,431	-15	-52
4279	375,634	10,796	8,771	365	2.82	2.83	13,039	13,085	7,145	7,160	6,433	6,448	-15	-53
2065	123,755	2,998	3,069	0	2.78	2.76	3,361	3,337	1,842	1,826	1,842	1,826	16	54
4720	76,752	1,719	1,719	1,496	2.85	2.79	2,187	2,141	1,199	1,172	74	90	-16	-55
4352	561,431	5,103	4,973	1,874	0.94	0.95	5,415	5,473	2,967	2,995	403	419	-17	-56
909	410	1,403	1,377	21	43.89	44.74	1,833	1,869	1,005	1,023	963	981	-18	-57
3022	1,985,144	68,885	75,592	11,812	5.67	5.68	102,571	102,752	56,206	56,225	35,064	35,082	-19	-58
1710	48,217	2,035	2,035	156	5.93	6.01	2,859	2,898	1,567	1,586	1,270	1,289	-19	-59
2503	386,275	1,946	1,946	0	0.59	0.60	2,279	2,318	1,249	1,268	1,249	1,268	-19	-60
8606	453,192	17,340	12,145	102	3.19	3.20	20,641	20,705	11,310	11,330	11,107	11,127	-19	-61

EXHIBIT 2
Part 3
CALCULATION OF WILCOXON STATISTIC
BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

Class	(1) 1987 Payroll	(2) 1987 Earned Premium	(3) 1987 Manual Premium	(4) 1987 First Report Losses	(5) Current Rate	(6) Alternate Rate	(7) Current Total Premiums (1)x(5)x (2)/[(3)x100]	(8) Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	(9) Current Expected Losses	(10) Alternate Expected Losses	(11) Current Squared Error [(9)-(4)]^2/(9)	(12) Alternate Squared Error [(10)-(4)]^2/(10)	(13) Difference (11)-(12)	(14) *D* Value
4362	394,819	2,092	2,092	0	0.65	0.66	2,566	2,606	1,406	1,426	1,406	1,426	-20	-62
9089	349,082	5,527	5,527	2,526	1.67	1.70	5,830	5,934	3,194	3,247	140	160	-20	-63
3303	92,397	4,528	4,362	1,011	4.82	4.78	4,623	4,585	2,533	2,509	915	894	21	64
2021	118,438	940	1,516	0	1.82	1.77	1,337	1,300	732	711	732	711	21	65
1925	290,833	10,964	10,964	1,235	4.43	4.45	12,884	12,942	7,060	7,082	4,806	4,827	-21	-66
4653	144,980	3,909	6,205	0	4.22	4.27	3,854	3,900	2,112	2,134	2,112	2,134	-22	-67
3255	278,472	6,850	6,850	132	3.20	3.19	8,911	8,883	4,883	4,861	4,623	4,600	22	68
7222	82,634	7,136	7,241	0	8.08	8.04	6,580	6,547	3,606	3,583	3,606	3,583	23	69
3132	176,657	5,070	4,487	1,397	3.14	3.17	6,268	6,328	3,435	3,462	1,209	1,232	-23	-70
908	1,138	3,079	2,969	245	29.86	30.27	3,524	3,572	1,931	1,955	1,472	1,495	-23	-71
3180	95,775	2,385	2,385	5,323	2.82	2.82	2,701	2,701	1,480	1,478	9,979	10,004	-25	-72
2380	774,945	10,691	12,012	1,436	2.14	2.15	14,760	14,829	8,088	8,114	5,471	5,496	-25	-73
4717	299,280	6,045	6,045	24	2.88	2.90	8,619	8,679	4,723	4,749	4,675	4,701	-26	-74
2710	315,241	28,695	26,086	6,410	9.93	9.96	34,434	34,538	18,869	18,899	8,226	8,253	-27	-75
2702	28,835	4,072	4,072	158	17.28	17.49	4,983	5,043	2,730	2,760	2,423	2,453	-29	-76
2130	904,417	42,960	42,960	12,049	5.36	5.36	48,477	48,477	26,564	26,526	7,931	7,901	30	77
7420	134,103	6,784	15,273	0	12.04	12.15	7,172	7,237	3,930	3,960	3,930	3,960	-30	-78
1655	80,983	2,488	2,588	0	5.21	5.29	4,056	4,118	2,223	2,254	2,223	2,254	-31	-79
6206	401,927	15,885	17,283	1,583	4.38	4.37	16,180	16,143	8,866	8,834	5,983	5,951	32	80
8050	5,276,895	44,648	47,068	28,910	1.10	1.03	55,061	51,558	30,172	28,212	53	17	35	81
2587	497,138	7,175	8,750	502	2.51	2.53	10,232	10,314	5,607	5,644	4,648	4,684	-36	-82
6214	22,489	1,082	1,082	0	5.56	5.27	1,250	1,185	685	649	685	649	37	83
3881	971,699	27,531	32,252	7,715	2.39	2.41	19,824	19,990	10,863	10,938	912	950	-38	-84
170	77,880	2,890	2,890	155	3.53	3.63	2,749	2,827	1,506	1,547	1,212	1,252	-40	-85
2016	332,674	4,720	4,588	113	1.47	1.45	5,031	4,963	2,757	2,715	2,535	2,494	41	86
9019	150,254	3,989	3,989	0	3.91	3.97	5,875	5,965	3,219	3,264	3,219	3,264	-45	-87
1701	358,100	4,901	7,074	229	2.69	2.66	6,674	6,599	3,657	3,611	3,213	3,168	46	88
6836	86,533	3,864	3,864	251	4.80	4.91	4,154	4,249	2,276	2,325	1,802	1,850	-48	-89
8204	89,212	6,656	5,981	0	7.42	7.52	7,367	7,466	4,037	4,085	4,037	4,085	-49	-90
4251	2,776,423	82,735	63,516	43,776	2.92	2.93	105,603	105,964	57,867	57,983	3,431	3,481	-50	-91
3257	460,875	11,578	12,041	2,897	3.19	3.17	14,137	14,048	7,746	7,687	3,036	2,985	51	92

EXHIBIT 2
Part 4
CALCULATION OF WILCOXON STATISTIC
BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

Class	(1) 1987 Payroll	(2) 1987 Earned Premium	(3) 1987 Manual Premium	(4) 1987 First Report Losses	(5) Current Rate	(6) Alternate Rate	(7) Current Total Premiums (1)x(5)x (2)/[(3)x100]	(8) Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	(9) Current Expected Losses	(10) Alternate Expected Losses	(11) Current Squared Error [(9)-(4)]^2/(9)	(12) Alternate Squared Error [(10)-(4)]^2/(10)	(13) Difference (11)-(12)	(14) *D* Value
4809	2,371,479	28,386	44,821	5,438	1.96	1.97	29,437	29,587	16,131	16,190	7,088	7,141	-53	-93
3188	1,138,601	18,816	21,178	21,019	2.37	2.38	23,975	24,076	13,138	13,174	4,728	4,671	57	94
8832	193,998,224	405,710	403,804	191,190	0.20	0.20	389,828	389,828	213,613	213,311	2,354	2,294	60	95
34	3,071,648	130,659	118,012	35,523	4.18	4.19	142,155	142,495	77,896	77,972	23,050	23,110	-60	-96
4740	309,595	9,726	6,173	514	2.50	2.48	12,195	12,097	6,682	6,619	5,694	5,631	62	97
3307	157,576	5,417	5,610	0	3.92	3.85	5,964	5,858	3,268	3,205	3,268	3,205	63	98
913	2,607	18,895	18,784	4,356	83.66	84.30	21,939	22,107	12,022	12,097	4,888	4,953	-65	-99
4273	2,959,304	85,272	75,462	27,046	3.33	3.33	111,356	111,356	61,019	60,933	18,915	18,846	69	100
2417	271,326	7,499	7,217	2,190	3.27	3.33	9,219	9,388	5,052	5,137	1,621	1,691	-70	-101
5402	25,100	1,561	1,561	0	8.01	7.51	2,011	1,885	1,102	1,031	1,102	1,031	70	102
3126	549,664	16,105	16,105	1,544	4.30	4.33	23,636	23,800	12,952	13,023	10,048	10,118	-71	-103
1165	284,653	9,436	9,394	0	3.31	3.36	9,464	9,607	5,186	5,257	5,186	5,257	-71	-104
3827	148,160	3,022	3,022	5,341	2.57	2.59	3,808	3,837	2,087	2,100	5,076	5,003	73	105
4351	1,297,629	6,801	5,698	1,168	0.64	0.65	9,912	10,067	5,432	5,509	3,347	3,420	-74	-106
8263	33,357	2,279	2,279	266	7.74	8.18	2,582	2,729	1,415	1,493	933	1,008	-76	-107
16	135,526	5,532	5,747	717	5.28	5.40	6,888	7,045	3,774	3,855	2,477	2,554	-77	-108
5348	2,325,278	84,651	80,813	53,839	4.21	3.98	102,543	96,941	56,191	53,045	98	12	87	109
5705	218,093	10,822	10,822	9,521	8.88	8.39	19,367	18,298	10,612	10,013	112	24	88	110
8103	558,518	16,953	13,349	1,401	2.84	2.82	20,144	20,002	11,038	10,945	8,414	8,323	92	111
3082	2,818,291	66,512	83,140	44,759	3.82	3.89	86,127	87,705	47,195	47,992	126	218	-92	-112
3113	1,516,818	18,091	19,256	1,814	1.59	1.58	22,658	22,516	12,416	12,320	9,053	8,960	94	113
3111	328,428	9,987	10,022	0	3.54	3.49	11,586	11,422	6,349	6,250	6,349	6,250	99	114
2660	571,213	9,444	9,597	126	2.41	2.38	13,547	13,378	7,423	7,320	7,173	7,071	103	115
4812	1,198,578	16,546	17,755	2,037	1.45	1.47	16,196	16,419	8,875	8,985	5,268	5,372	-104	-116
4558	846,859	19,904	19,485	686	2.59	2.57	22,405	22,232	12,277	12,165	10,944	10,832	112	117
5437	5,898,232	246,117	252,385	152,644	5.43	5.42	312,320	311,745	171,141	170,584	1,999	1,887	112	118
6237	75,151	2,561	1,879	0	3.29	3.09	3,370	3,165	1,847	1,732	1,847	1,732	115	119
8010	41,352,347	471,033	431,015	228,835	1.12	1.12	506,148	506,148	277,353	276,960	8,487	8,362	125	120
7405	355,906	12,128	12,349	0	3.23	3.30	11,290	11,535	6,187	6,312	6,187	6,312	-125	-121
4307	1,927,617	28,471	28,499	21,999	1.79	1.76	34,470	33,893	18,889	18,546	512	643	-131	-122
7502	7,960,829	167,385	168,722	111,428	2.59	2.49	204,552	196,654	112,088	107,607	4	136	-132	-123

EXHIBIT 2

Part 5

CALCULATION OF WILCOXON STATISTIC

BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

+

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Class	1987 Payroll	1987 Earned Premium	1987 Manual Premium	1987 First Report Losses	Current Rate	Alternate Rate	Current Total Premiums (1)x(5)x (2)/[(3)x100]	Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	Current Expected Losses	Alternate Expected Losses	Current Squared Error [(9)-(4)]^2/(9)	Alternate Squared Error [(10)-(4)]^2/(10)	Difference (11)-(12)	*D* Value
2041	40,380	1,583	1,215	0	3.95	4.45	2,078	2,341	1,139	1,281	1,139	1,281	-142	-124
4819	1,225,557	20,224	18,262	5,861	2.06	2.04	27,959	27,687	15,321	15,150	5,841	5,696	145	125
9586	21,884,849	70,375	64,544	82,667	0.34	0.34	81,131	81,131	44,457	44,394	32,841	32,996	-155	-126
2923	294,242	5,766	5,766	701	2.58	2.69	7,591	7,915	4,160	4,331	2,876	3,043	-167	-127
5610	522,477	17,880	18,906	430	4.58	4.65	22,631	22,977	12,401	12,573	11,556	11,727	-171	-128
2841	1,197,565	42,216	37,242	63,361	3.60	3.60	48,870	48,870	26,779	26,741	49,972	50,146	-175	-129
4557	978,544	24,572	25,453	403	2.74	2.71	25,884	25,601	14,184	14,008	13,389	13,214	175	130
9505	628,476	11,736	11,226	13,671	2.23	2.26	14,652	14,849	8,029	8.125	3,965	3,785	180	131
8820	82,040,082	118,300	120,083	118,877	0.16	0.16	129,315	129,315	70,861	70,760	32,537	32,719	-183	-132
5491	212,189	6,912	6,919	62	4.11	3.95	8,712	8,373	4,774	4,582	4,651	4,458	192	133
9530	96,604	7,004	8,108	171	9.46	9.05	7,894	7,552	4,326	4,133	3,991	3,798	193	134
2759	1,219,122	56,416	53,473	49,059	5.56	5.52	71,514	70,999	39,187	38,850	2,487	2,683	-196	-135
2623	584,100	16,589	20,736	4,801	4.22	4.13	19,719	19,299	10,806	10,560	3,337	3,141	196	136
9016	2,769,858	54,152	53,152	32,730	2.40	2.34	67,782	66,088	37,143	36,163	524	326	198	137
8719	698,615	12,183	12,251	0	1.94	1.89	13,478	13,131	7,385	7,185	7,385	7,185	201	138
8350	22,154,185	468,827	279,195	473,596	2.40	2.23	526,346	489,063	288,421	267,612	295	501	-206	-139
6018	87,310	5,300	5,265	0	10.34	9.92	9,088	8,719	4,980	4,771	4,980	4,771	209	140
3383	502,808	7,294	7,294	171	1.80	1.88	9,051	9,453	4,959	5,172	4,623	4,836	-213	-141
1463	491,996	24,803	23,648	21,670	6.67	6.47	34,419	33,387	18,860	18,269	419	633	-215	-142
9186	77,117	10,149	9,908	595	17.51	17.03	13,832	13,452	7,579	7,361	6,436	6,219	217	143
1747	767,345	26,511	28,008	0	3.78	3.84	27,455	27,891	15,045	15,262	15,045	15,262	-217	-144
2836	1,074,577	21,662	23,197	22,257	2.76	2.73	27,696	27,395	15,176	14,990	3,303	3,523	-219	-145
2881	637,002	14,841	14,851	6,708	2.96	2.85	18,843	18,142	10,325	9,927	1,267	1,044	223	146
3169	293,966	10,771	9,847	2,957	3.62	3.79	11,640	12,187	6,378	6,668	1,835	2,066	-230	-147
6252	94,139	6,615	7,706	0	8.88	8.37	7,176	6,764	3,932	3,701	3,932	3,701	231	148
4484	22,896,377	605,931	541,983	404,893	2.89	2.82	737,779	721,861	405,375	394,996	1	248	-247	-149
2576	959,365	25,060	23,392	1,583	3.12	3.08	32,067	31,655	17,571	17,322	14,548	14,300	248	150
3826	5,301,706	40,299	37,311	7,975	1.20	1.21	68,715	69,288	37,654	37,914	23,393	23,641	-248	-151
8803	30,875,662	35,145	43,142	68,494	0.17	0.17	42,759	42,759	23,431	23,397	86,669	86,920	-251	-152
3145	192,955	3,170	3,512	65	1.95	2.22	3,396	3,866	1,861	2,116	1,733	1,988	-254	-153
4150	6,460,814	37,235	34,806	15,608	0.69	0.68	47,691	47,000	26,133	25,718	4,239	3,974	265	154

EXHIBIT 2
Part 6
Calculation of Wilcoxon Statistic
Based on 1987 Loss Experience From a Single State

Class	(1) 1987 Payroll	(2) 1987 Earned Premium	(3) 1987 Manual Premium	(4) 1987 First Report Losses	(5) Current Rate	(6) Alternate Rate	(7) Current Total Premiums (1)x(5)x (2)/[(3)x100]	(8) Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	(9) Current Expected Losses	(10) Alternate Expected Losses	(11) Current Squared Error [(9)-(4)]^2/(9)	(12) Alternate Squared Error [(10)-(4)]^2/(10)	(13) Difference (11)-(12)	(14) *D* Value
2570	1,181,646	67,079	69,965	29,215	10.39	10.46	117,709	118,502	64,501	64,843	19,303	19,576	-273	-155
9620	7,421,678	68,597	68,491	21,182	1.19	1.20	88,455	89,198	48,470	48,808	15,363	15,637	-274	-156
4583	15,161,662	351,932	347,167	145,342	2.47	2.48	379,633	381,170	208,027	208,573	18,889	19,169	-280	-157
8032	567,303	8,693	8,591	12,453	1.58	1.60	9,070	9,185	4,970	5,026	11,267	10,976	291	158
3220	9,133,395	130,105	168,968	105,522	3.30	3.28	232,079	230,672	127,172	126,222	3,686	3,395	291	159
4808	3,285,558	60,126	60,126	0	2.09	2.11	68,668	69,325	37,628	37,934	37,628	37,934	-306	-160
3647	23,155	1,118	1,118	0	4.69	7.14	1,086	1,653	595	905	595	905	-310	-161
1642	1,126,650	19,048	35,940	0	3.47	3.38	20,720	20,183	11,354	11,044	11,354	11,044	310	162
8235	1,008,687	30,655	32,523	1,951	3.96	4.03	37,650	38,315	20,631	20,966	16,913	17,245	-332	-163
3085	1,248,233	37,859	41,474	2,698	3.78	3.84	43,071	43,754	23,601	23,942	18,514	18,850	-336	-164
9501	1,006,554	25,151	23,363	4,744	2.54	2.48	27,523	26,873	15,082	14,705	7,086	6,747	339	165
8710	66,855	941	1,492	1,696	2.29	2.47	966	1,041	529	570	2,573	2,225	348	166
4036	1,497,638	69,455	28,006	27,327	2.74	2.72	101,768	101,025	55,765	55,280	14,503	14,135	368	167
6005	226,495	10,578	10,574	269	6.70	6.41	15,181	14,524	8,319	7,947	7,789	7,418	371	168
8745	986,891	24,719	21,978	24,934	2.66	2.62	29,525	29,081	16,179	15,913	4,738	5,114	-376	-169
2302	1,141,546	15,822	15,982	14,216	1.68	1.76	18,986	19,890	10,404	10,884	1,397	1,020	377	170
5037	196,277	43,468	41,550	7,455	30.16	29.84	61,930	61,273	33,936	33,528	20,663	20,276	388	171
4635	919,597	17,814	18,909	5,891	2.82	2.72	24,431	23,565	13,387	12,894	4,198	3,804	394	172
2731	131,281	6,146	6,084	0	4.77	5.34	6,326	7,082	3,466	3,875	3,466	3,875	-409	-173
4825	3,064,657	28,889	30,593	3,833	1.11	1.14	32,123	32,991	17,602	18,052	10,771	11,200	-429	-174
9545	168,958	9,963	14,936	342	10.24	9.55	11,541	10,763	6,324	5,889	5,658	5,225	433	175
4692	3,976,240	13,277	13,323	1,154	0.45	0.43	17,831	17,039	9,771	9,323	7,599	7,158	441	176
8058	10,445,088	143,261	145,806	124,730	1.94	1.97	199,098	202,177	109,099	110,629	2,239	1,797	442	177
5191	33,249,911	370,710	433,339	201,011	1.16	1.42	329,955	403,911	180,805	221,017	2,258	1,811	447	178
3629	1,874,898	25,585	23,546	13,155	1.64	1.55	33,411	31,577	18,308	17,279	1,450	984	466	179
7590	962,276	27,567	28,611	647	3.76	3.67	34,861	34,027	19,103	18,619	17,831	17,348	483	180
8264	797,742	67,828	71,057	24,453	13.01	13.19	99,070	100,441	54,287	54,960	16,396	16,934	-538	-181
3719	61,450	4,592	2,702	0	5.76	4.82	6,015	5,034	3,296	2,754	3,296	2,754	542	182
35	1,085,012	18,254	18,553	20,466	2.25	2.19	24,019	23,379	13,162	12,793	4,053	4,603	-549	-183
8291	4,541,480	202,143	176,716	108,643	4.76	4.82	247,279	250,396	135,501	137,015	5,324	5,875	-551	-184
917	1,270,340	65,909	66,705	2,256	6.30	6.39	79,076	80,206	43,331	43,888	38,937	39,492	-555	-185

EXHIBIT 2

Part 7

CALCULATION OF WILCOXON STATISTIC

BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

Class	(1) 1987 Payroll	(2) 1987 Earned Premium	(3) 1987 Manual Premium	(4) 1987 First Report Losses	(5) Current Rate	(6) Alternate Rate	(7) Current Total Premiums (1)x(5)x (2)/[(3)x100]	(8) Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	(9) Current Expected Losses	(10) Alternate Expected Losses	(11) Current Squared Error [(9)-(4)]^2/(9)	(12) Alternate Squared Error [(10)-(4)]^2/(10)	(13) Difference (11)-(12)	(14) *D* Value
4000	9,629,197	312,859	316,194	411,479	4.11	4.12	391,586	392,539	214,576	214,794	180,684	180,103	581	186
6217	16,050,053	843,257	895,252	533,490	6.20	6.57	937,309	993,245	513,615	543,496	769	184	585	187
8013	12,649,879	40,140	39,248	7,573	0.32	0.33	41,400	42,693	22,686	23,361	10,068	10,670	-603	-188
6504	8,696,042	207,827	201,854	145,212	2.65	2.71	237,264	242,636	130,013	132,769	1,777	1,166	611	189
8215	10,964,091	265,332	264,035	73,307	2.74	2.73	301,892	300,790	165,427	164,590	51,298	50,626	672	190
3634	612,510	7,630	7,895	1,904	1.76	1.52	10,418	8,998	5,709	4,923	2,536	1,852	684	191
4902	3,070,355	52,816	49,197	10,751	1.89	1.85	62,298	60,980	34,138	33,368	16,021	15,330	692	192
6400	768,407	38,881	43,694	41,894	7.36	7.23	50,325	49,436	27,577	27,051	7,433	8,144	-711	-193
3227	1,082,189	34,383	36,578	42,393	5.30	5.19	53,914	52,795	29,543	28,889	5,589	6,312	-723	-194
6236	98,574	13,586	15,594	151	16.59	15.07	14,248	12,942	7,807	7,082	7,508	6,783	725	195
1320	2,015,246	45,311	49,503	1,250	2.52	2.60	46,484	47,959	25,472	26,243	23,033	23,803	-770	-196
8279	2,166,922	128,669	129,495	87,923	6.19	6.36	133,277	136,937	73,031	74,931	3,036	2,253	784	197
7520	6,796,775	151,449	153,316	69,405	3.09	3.06	207,463	205,449	113,683	112,420	17,246	16,459	787	198
4361	11,259,417	111,699	113,178	47,616	1.54	1.56	171,129	173,352	93,773	94,857	22,720	23,527	-807	-199
2589	2,385,229	29,019	29,506	35,402	1.61	1.58	37,768	37,065	20,696	20,281	10,450	11,273	-823	-200
9060	14,386,175	205,827	204,590	321,595	1.79	1.79	259,070	259,070	141,962	141,761	227,301	228,134	-832	-201
7601	236,155	14,233	15,702	0	8.41	7.71	18,003	16,504	9,865	9,031	9,865	9,031	834	202
251	6,252,334	133,687	156,354	232,249	3.18	3.19	170,000	170,535	93,155	93,315	207,689	206,854	836	203
2156	198,015	7,059	6,531	26,344	4.13	4.11	8,839	8,796	4,844	4,813	95,439	96,310	-871	-204
9156	2,068,115	17,382	17,087	1,483	1.06	1.14	22,300	23,984	12,220	13,124	9,434	10,325	-891	-205
8209	1,929,783	58,194	57,480	5,749	3.74	3.66	73,070	71,507	40,040	39,128	29,368	28,475	893	206
2883	3,230,429	82,738	80,609	38,707	2.99	2.89	99,141	95,825	54,326	52,435	4,491	3,594	897	207
1322	708,802	55,006	60,167	69,668	10.56	10.68	68,429	69,207	37,497	37,869	27,602	26,701	900	208
37	11,679,317	560,062	574,036	488,489	7.03	6.98	801,069	795,371	438,960	435,221	5,589	6,520	-931	-209
9521	4,882,808	127,142	123,957	77,801	3.19	2.96	159,764	148,245	87,545	81,118	1,085	136	949	210
4665	3,797,621	196,759	211,370	178,562	6.81	6.88	240,741	243,216	131,918	133,086	16,492	15,540	953	211
2039	1,351,325	44,614	47,648	7,649	3.61	3.77	45,677	47,701	25,029	26,102	12,069	13,045	-976	-212
9033	5,324,045	102,759	105,027	104,192	2.39	2.42	124,497	126,060	68,220	68,979	18,967	17,976	991	213
4420	956,613	24,100	24,492	26,549	3.19	3.08	30,028	28,992	16,454	15,864	6,193	7,196	-1,003	-214
8500	264,119	21,155	21,762	2,615	8.39	9.16	21,541	23,518	11,804	12,869	7,153	8,170	-1,017	-215
4459	1,512,761	38,441	36,788	37,953	2.92	2.83	46,157	44,735	25,293	24,479	6,337	7,417	-1,080	-216

EXHIBIT 2
Part 8
Calculation of Wilcoxon Statistic
Based on 1987 Loss Experience From a Single State

Class	(1) 1987 Payroll	(2) 1987 Earned Premium	(3) 1987 Manual Premium	(4) 1987 First Report Losses	(5) Current Rate	(6) Alternate Rate	(7) Current Total Premiums (1)x(5)x(2)/[(3)x100]	(8) Alternate Total Premiums (1)x(6)x(2)/[(3)x100]	(9) Current Expected Losses	(10) Alternate Expected Losses	(11) Current Squared Error [(9)-(4)]^2/(9)	(12) Alternate Squared Error [(10)-(4)]^2/(10)	(13) Difference (11)-(12)	(14) *D* Value
8031	3,146,252	61,774	61,853	50,176	2.47	2.34	77,613	73,528	42,530	40,234	1,375	2,457	-1,082	-217
1803	672,457	33,608	33,582	1,214	6.84	6.55	46,032	44,080	25,224	24,120	22,854	21,753	1,101	218
2030	8,284,303	438,967	439,013	153,952	5.15	5.20	426,597	430,739	233,761	235,697	27,248	28,351	-1,103	-219
6233	830,497	56,710	65,903	43,662	9.96	9.16	71,179	65,462	39,004	35,820	556	1,717	-1,160	-220
9058	13,915,274	256,537	233,405	260,303	2.50	2.53	382,359	386,948	209,521	211,735	12,308	11,141	1,168	221
3808	280,235	7,062	7,062	23,815	2.79	2.82	7,819	7,903	4,284	4,324	89,033	87,851	1,183	222
9178	603,686	43,868	26,683	0	5.47	5.26	54,289	52,205	29,749	28,566	29,749	28,566	1,183	223
7600	17,622,321	253,473	243,212	99,703	1.82	1.84	334,258	337,931	183,162	184,913	38,029	39,266	-1,237	-224
5703	88,139	15,151	15,483	0	20.19	22.87	17,414	19,725	9,542	10,793	9,542	10,793	-1,251	-225
9220	2,827,686	96,801	94,362	17,043	4.41	4.33	127,924	125,604	70,098	68,729	40,156	38,869	1,287	226
3638	1,995,318	26,398	26,937	51,899	1.72	1.74	33,633	34,024	18,430	18,618	60,782	59,495	1,287	227
5606	40,801,432	756,184	756,486	433,919	2.11	1.76	860,567	717,819	471,563	392,784	3,005	4,308	-1,303	-228
8002	2,060,098	22,120	21,244	31,377	1.33	1.37	28,529	29,387	15,633	16,080	15,856	14,551	1,305	229
5192	5,672,275	113,950	115,227	100,120	2.43	2.49	136,309	139,674	74,693	76,429	8,656	7,344	1,312	230
9063	12,422,904	136,446	130,398	37,790	1.36	1.34	176,788	174,188	96,874	95,314	36,036	34,717	1,319	231
7855	505,800	40,002	40,353	12,488	11.69	11.13	58,614	55,806	32,118	30,537	11,998	10,668	1,330	232
7360	364,419	28,151	27,452	20,306	7.43	8.71	27,766	32,549	15,215	17,811	1,704	350	1,354	233
7422	1,754,449	64,829	70,660	2,024	3.66	3.82	58,914	61,489	32,283	33,646	28,362	29,720	-1,358	-234
9102	10,062,881	205,530	200,480	77,508	2.32	2.36	239,340	243,466	131,150	133,223	21,941	23,300	-1,360	-235
5223	699,315	30,245	29,056	2,116	5.18	4.84	37,707	35,232	20,662	19,279	16,647	15,279	1,368	236
3612	4,967,111	58,874	61,519	42,951	1.62	1.38	77,008	65,599	42,198	35,895	13	1,387	-1,373	-237
5020	1,260,575	48,497	44,756	542	3.61	3.43	49,311	46,852	27,021	25,637	25,947	24,564	1,383	238
2960	36,431	2,141	1,814	12,482	5.37	5.44	2,309	2,339	1,265	1,280	99,438	98,041	1,397	239
8116	30,099,924	689,003	689,532	797,102	3.05	3.06	917,343	920,351	502,675	503,609	172,452	171,042	1,410	240
8111	2,601,985	55,944	58,505	25,197	2.88	2.69	71,657	66,930	39,266	36,623	5,041	3,565	1,476	241
3373	2,235,703	86,162	85,085	4,696	4.31	4.44	97,579	100,522	53,470	55,005	44,490	46,014	-1,523	-242
6003	239,540	26,829	25,726	164	12.09	10.93	30,202	27,304	16,550	14,941	16,223	14,614	1,609	243
5069	131,426	25,366	27,179	39,958	22.21	22.64	27,243	27,770	14,928	15,196	41,968	40,353	1,615	244
7605	3,465,042	47,847	47,828	112,053	1.68	1.69	58,236	58,582	31,911	32,056	201,266	199,637	1,629	245
8106	2,110,785	84,989	75,338	65,447	4.30	4.03	102,391	95,962	56,107	52,509	1,555	3,188	-1,633	-246
3372	2,264,765	82,435	73,198	19,409	4.24	4.11	108,144	104,828	59,259	57,361	26,798	25,110	1,688	247

EXHIBIT 2

Part 9

CALCULATION OF WILCOXON STATISTIC

BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

Class	(1) 1987 Payroll	(2) 1987 Earned Premium	(3) 1987 Manual Premium	(4) 1987 First Report Losses	(5) Current Rate	(6) Alternate Rate	(7) Current Total Premiums (1)x(5)x (2)/[(3)x100]	(8) Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	(9) Current Expected Losses	(10) Alternate Expected Losses	(11) Current Squared Error [(9)-(4)]^2/(9)	(12) Alternate Squared Error [(10)-(4)]^2/(10)	(13) Difference (11)-(12)	(14) *D* Value
8755	3,233,485	19,120	19,810	0	0.66	0.76	20,598	23,719	11,287	12,979	11,287	12,979	-1,692	-248
4114	2,101,802	52,959	47,711	2,853	2.58	2.45	60,191	57,158	32,983	31,277	27,524	25,831	1,693	249
4130	1,859,746	53,839	51,813	4,888	3.51	3.35	67,830	64,738	37,168	35,424	28,035	26,322	1,713	250
4452	2,733,006	81,475	73,691	39,416	2.88	3.17	87,025	95,788	47,687	52,414	1,434	3,223	-1,789	-251
2586	4,264,081	53,905	44,495	18,436	1.42	1.34	73,355	69,223	40,196	37,878	11,780	9,979	1,801	252
7390	6,656,627	211,253	198,830	107,675	3.94	3.85	278,658	272,293	152,696	148,996	13,274	11,460	1,814	253
5474	12,130,377	732,297	739,183	543,925	8.04	8.82	966,197	1,059,932	529,445	579,986	396	2,242	-1,846	-254
6834	836,714	38,722	42,657	3,111	6.82	6.36	51,800	48,306	28,385	26,433	22,504	20,577	1,927	255
3114	10,041,081	142,622	132,989	102,309	1.99	1.76	214,291	189,524	117,425	103,706	1,946	19	1,927	256
7610	41,678,168	158,320	156,012	41,672	0.40	0.39	169,179	164,950	92,705	90,259	28,093	26,155	1,938	257
5	1,461,707	49,453	44,894	23,828	4.34	4.69	69,880	75,516	38,292	41,322	5,464	7,406	-1,942	-258
5480	884,982	53,794	52,722	6,412	6.56	7.00	59,235	63,208	32,459	34,587	20,902	22,952	-2,050	-259
8047	694,602	8,450	8,552	27,871	1.60	1.63	10,981	11,187	6,017	6,121	79,369	77,277	2,092	260
7421	4,778,773	125,775	126,047	46,171	3.04	2.92	144,961	139,239	79,434	76,190	13,929	11,828	2,101	261
2081	21,747,127	1,325,590	1,092,684	1,046,728	6.20	6.25	1,635,717	1,648,908	896,321	902,269	25,239	23,129	2,110	262
3040	4,789,830	189,119	189,361	103,638	4.88	4.62	233,445	221,007	127,920	120,933	4,609	2,474	2,136	263
9402	2,414,408	70,962	71,945	23,631	4.13	3.93	98,353	93,590	53,894	51,212	16,994	14,854	2,140	264
9549	1,295,403	98,550	97,470	73,362	9.25	8.40	121,152	110,020	66,388	60,202	733	2,877	-2,144	-265
8	297,516	12,943	12,130	414	3.82	5.08	12,127	16,127	6,645	8,824	5,843	8,016	-2,173	-266
9093	5,732,957	67,708	67,376	9,804	1.61	1.54	92,755	88,723	50,827	48,548	33,110	30,920	2,190	267
8392	2,176,680	51,893	40,786	63,122	2.32	2.26	64,251	62,589	35,208	34,248	22,132	24,342	-2,210	-268
6306	2,417,591	175,682	180,248	97,206	9.55	9.99	225,031	235,399	123,310	128,809	5,526	7,754	-2,227	-269
1624	2,066,236	93,668	97,380	6,523	4.56	4.82	90,629	95,796	49,662	52,419	37,472	40,185	-2,712	-270
5190	38,511,345	1,102,694	1,148,867	567,652	3.61	3.58	1,334,385	1,323,296	731,200	724,097	36,581	33,801	2,780	271
9522	3,749,914	61,511	54,163	55,812	1.55	1.65	66,009	70,268	36,171	38,450	10,665	7,840	2,825	272
7720	40,938,046	713,067	719,457	386,537	2.11	2.07	856,121	839,891	469,127	459,581	14,540	11,609	2,930	273
8381	4,916,653	95,766	96,347	44,598	2.08	2.34	101,650	114,356	55,701	62,575	2,213	5,164	-2,951	-274
7382	5,998,486	207,335	184,167	94,886	3.81	3.66	257,293	247,163	140,988	135,246	15,075	12,044	3,031	275
5102	1,841,823	68,205	67,295	58,943	4.92	4.38	91,843	81,763	50,327	44,740	1,475	4,509	-3,034	-276
4683	221,040	5,238	4,844	42,837	2.48	2.47	5,928	5,904	3,248	3,230	482,510	485,584	-3,074	-277
1164	577,974	35,146	33,795	0	5.82	6.81	34,983	40,933	19,169	22,398	19,169	22,398	-3,229	-278

EXHIBIT 2

Part 10

CALCULATION OF WILCOXON STATISTIC

BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

Class	(1) 1987 Payroll	(2) 1987 Earned Premium	(3) 1987 Manual Premium	(4) 1987 First Report Losses	(5) Current Rate	(6) Alternate Rate	(7) Current *Total* Premiums (1)x(5)x (2)/[(3)x100]	(8) Alternate *Total* Premiums (1)x(6)x (2)/[(3)x100]	(9) Current Expected Losses	(10) Alternate Expected Losses	(11) Current *Squared* Error [(9)-(4)]^2/(9)	(12) Alternate *Squared* Error [(10)-(4)]^2/(10)	(13) Difference (11)-(12)	(14) *D* Value
3179	14,974,372	209,372	214,703	219,496	1.73	1.76	252,624	257,005	138,430	140,631	47,473	44,227	3,246	279
7231	3,587,760	119,235	117,480	108,349	3.91	4.12	142,377	150,024	78,018	82,092	11,792	8,398	3,393	280
8385	3,570,890	119,238	108,730	42,226	4.04	3.83	158,206	149,982	86,692	82,069	22,807	19,343	3,464	281
3146	4,448,063	113,093	95,550	30,553	2.69	2.55	141,621	134,251	77,604	73,461	28,527	25,062	3,465	282
5160	1,698,935	50,266	51,369	13,141	4.18	3.74	69,491	62,176	38,079	34,022	16,332	12,816	3,516	283
9061	11,647,584	167,354	166,705	41,453	1.89	1.83	220,996	213,981	121,099	117,088	52,383	48,858	3,524	284
2916	255,369	7,657	7,891	73,961	3.95	3.97	9,788	9,838	5,363	5,383	877,344	873,665	3,679	285
8265	2,470,228	188,044	184,016	52,279	10.34	10.04	261,013	253,440	143,027	138,680	57,578	53,830	3,748	286
7580	3,932,413	65,261	64,360	76,162	2.16	2.06	86,129	82,142	47,196	44,947	17,777	21,678	-3,900	-287
5403	20,474,132	1,509,972	1,548,228	671,460	8.24	8.17	1,645,382	1,631,404	901,616	892,691	58,752	54,826	3,926	288
5551	7,389,684	989,646	1,011,464	728,865	19.98	19.10	1,444,611	1,380,984	791,600	755,663	4,972	950	4,021	289
8831	8,922,017	116,057	116,182	46,322	1.53	1.66	136,360	147,946	74,721	80,955	10,793	14,816	-4,023	-290
8835	6,166,499	145,983	121,203	58,087	2.70	2.56	200,536	190,137	109,887	104,042	24,418	20,298	4,120	291
3030	14,831,410	467,255	559,416	80,365	4.40	4.34	545,072	537,640	298,682	294,192	159,576	155,415	4,160	292
2111	6,661,701	205,247	157,882	106,814	3.07	2.86	265,869	247,683	145,688	135,530	10,373	6,084	4,288	293
3685	74,161,767	401,530	555,378	299,716	0.90	1.06	482,561	568,349	264,428	310,996	4,709	409	4,300	294
3648	681,110	16,034	16,687	68,086	2.72	2.75	17,801	17,998	9,754	9,848	348,820	344,396	4,424	295
5215	2,305,227	83,462	85,498	26,742	4.92	4.45	110,716	100,140	60,669	54,796	18,972	14,363	4,610	296
6216	1,162,965	73,824	83,180	17,107	7.73	6.70	79,786	69,154	43,720	37,841	16,200	11,360	4,839	297
4581	1,333,272	8,999	11,999	54,475	0.97	0.98	9,699	9,799	5,315	5,362	454,706	449,840	4,866	298
9552	2,077,533	94,954	100,436	16,970	6.35	5.86	124,723	115,098	68,344	62,981	38,618	33,613	5,004	299
7403	4,002,777	54,311	69,286	4,738	2.01	2.32	63,067	72,793	34,559	39,832	25,732	30,920	-5,187	-300
3822	279,817	10,628	10,017	54,799	4.82	4.76	14,310	14,132	7,841	7,733	281,205	286,474	-5,269	-301
4263	284,655	12,997	10,878	54,320	4.02	3.97	13,672	13,502	7,492	7,388	292,697	298,119	-5,422	-302
8232	27,719,260	990,937	994,154	371,992	3.42	3.50	944,931	967,035	517,792	529,153	41,054	46,678	-5,623	-303
5445	14,778,714	539,230	527,738	446,391	5.15	4.78	777,678	721,806	426,142	394,966	962	6,696	-5,733	-304
8006	18,410,437	292,417	296,682	169,763	2.11	1.90	382,876	344,770	209,804	188,655	7,642	1,892	5,750	305
5951	10,807,540	113,631	114,560	22,912	1.27	1.16	136,143	124,351	74,602	68,044	35,815	29,935	5,880	306
3632	29,623,254	764,463	708,643	533,501	3.41	3.17	1,089,723	1,013,027	597,133	554,320	6,781	782	5,999	307
50	8,449,341	281,065	279,868	157,116	4.52	4.19	383,544	355,542	210,170	194,549	13,392	7,203	6,190	308
5645	22,285,573	1,234,934	1,176,975	1,059,867	8.00	7.62	1,870,640	1,781,785	1,025,051	974,978	1,183	7,391	-6,209	-309

EXHIBIT 2

Part 11

CALCULATION OF WILCOXON STATISTIC

BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Class	1987 Payroll	1987 Earned Premium	1987 Manual Premium	1987 First Report Losses	Current Rate	Alternate Rate	Current Total Premiums (1)x(5)x (2)/[(3)x100]	Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	Current Expected Losses	Alternate Expected Losses	Current Squared Error [(9)-(4)]^2/(9)	Alternate Squared Error [(10)-(4)]^2/(10)	Difference (11)-(12)	*D* Value
7380	70,936,717	2,500,575	2,315,926	1,750,184	3.72	3.79	2,849,241	2,902,856	1,561,293	1,588,419	22,853	16,474	6,379	310
3635	2,923,290	87,645	71,621	27,956	2.38	2.85	85,140	101,954	46,654	55,788	7,494	13,885	-6,391	-311
8105	1,134,518	82,435	68,367	24,942	6.29	7.42	86,045	101,503	47,150	55,542	10,460	16,858	-6,398	-312
5651	1,642,158	95,003	92,858	3,064	7.24	6.55	121,639	110,046	66,654	60,216	60,667	54,244	6,423	313
4686	4,265,992	87,346	83,187	11,793	2.04	2.32	91,377	103,919	50,072	56,864	29,263	35,723	-6,460	-314
2070	16,600,823	449,562	447,170	317,559	2.96	3.24	494,013	540,744	270,703	295,891	8,110	1,587	6,523	315
6325	3,355,747	136,288	127,808	40,387	4.35	3.89	155,660	139,200	85,297	76,169	23,646	16,809	6,836	316
8001	8,462,865	86,746	83,936	107,370	1.26	1.20	110,202	104,954	60,387	57,430	36,554	43,427	-6,873	-317
6229	2,846,595	121,320	123,924	40,759	6.15	5.58	171,387	155,502	93,915	85,090	30,086	23,096	6,990	318
8748	48,854,652	267,450	268,310	154,643	0.72	0.79	350,626	384,715	192,132	210,513	7,315	14,828	-7,513	-319
5183	37,844,396	1,164,224	1,214,928	834,831	4.23	3.82	1,534,009	1,385,323	840,588	758,037	39	7,780	-7,740	-320
4304	12,948,990	249,235	267,909	88,506	2.51	2.34	302,365	281,886	165,686	154,246	35,952	28,018	7,934	321
8800	21,721,903	276,140	252,675	448,412	1.62	1.64	384,574	389,322	210,734	213,034	268,066	260,067	8,000	322
3004	20,047,116	193,414	288,678	198,090	1.80	1.72	241,768	231,023	132,481	126,414	32,492	40,640	-8,149	-323
5146	1,074,575	85,073	79,876	734	7.41	8.73	84,807	99,914	46,471	54,672	45,015	53,214	-8,199	-324
3064	257,472	9,308	9,217	37,962	4.46	4.64	11,597	12,065	6,355	6,602	157,214	148,973	8,241	325
9410	11,341,517	192,183	196,507	81,008	2.44	2.24	270,644	248,460	148,304	135,955	30,537	22,207	8,330	326
5057	407,253	77,419	91,504	8,238	32.34	27.79	111,432	95,755	61,061	52,396	45,697	37,215	8,481	327
9403	8,822,888	464,815	461,154	401,519	6.47	6.22	575,373	553,140	315,286	302,674	23,586	32,280	-8,695	-328
3365	3,066,446	194,670	200,187	81,109	8.55	7.69	254,956	229,311	139,707	125,477	24,578	15,688	8,890	329
2802	10,931,350	462,734	398,826	228,913	3.79	4.20	480,685	532,686	263,400	291,481	4,515	13,431	-8,915	-330
5213	12,589,193	1,074,887	1,096,043	749,402	9.80	10.49	1,209,927	1,295,116	663,001	708,677	11,260	2,340	8,919	331
8829	114,083,937	3,247,756	2,650,455	2,170,347	3.01	3.10	4,207,789	4,333,603	2,305,733	2,371,312	7,950	17,031	-9,082	-332
9040	36,371,375	802,337	712,533	419,576	2.42	2.52	991,122	1,032,077	543,103	564,744	28,096	37,316	-9,220	-333
8293	5,484,377	425,317	383,446	169,028	8.04	8.51	489,093	517,685	268,008	283,273	36,555	46,075	-9,521	-334
8046	15,863,946	174,136	180,816	273,671	1.53	1.50	233,751	229,168	128,088	125,399	165,466	175,317	-9,851	-335
2585	7,364,427	171,714	179,459	169,873	3.27	3.01	230,424	212,103	126,265	116,061	15,061	24,950	-9,889	-336
8292	9,731,885	313,771	294,174	269,286	3.74	3.50	388,219	363,307	212,732	198,798	15,035	24,993	-9,958	-337
3076	27,655,344	764,032	654,945	242,999	2.75	2.83	887,194	913,003	486,154	499,588	121,616	131,784	-10,168	-338
9052	28,240,721	763,424	698,457	458,060	2.83	3.17	873,551	978,501	478,678	535,428	888	11,179	-10,291	-339
5222	4,289,066	407,160	389,101	165,117	11.37	10.72	510,300	481,128	279,628	263,269	46,894	36,593	10,301	340
8901	21,192,744	77,969	70,339	114,643	0.38	0.40	89,268	93,966	48,916	51,418	88,315	77,744	10,571	341

EXHIBIT 2

Part 12

CALCULATION OF WILCOXON STATISTIC

BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Class	1987 Payroll	1987 Earned Premium	1987 Manual Premium	1987 First Report Losses	Current Rate	Alternate Rate	Current Total Premiums (1)x(5)x (2)/[(3)x100]	Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	Current Expected Losses	Alternate Expected Losses	Current Squared Error [(9)-(4)]^2/(9)	Alternate Squared Error [(10)-(4)]^2/(10)	Difference (11)-(12)	*D* Value
8833	151,142,877	1,061,273	1,054,781	654,042	0.89	0.94	1,353,451	1,429,487	741,648	782,204	10,348	20,999	-10,651	-342
7423	12,269,708	162,201	190,142	129,036	1.68	2.14	175,841	223,987	96,355	122,564	11,085	342	10,743	343
3724	16,395,020	926,733	939,196	545,405	7.43	6.98	1,201,985	1,129,187	658,649	617,882	19,471	8,501	10,969	344
9154	4,595,131	50,798	55,583	52,515	1.30	1.64	54,594	68,873	29,916	37,687	17,072	5,835	11,237	345
6319	1,898,652	163,667	172,775	50,415	9.77	11.29	175,720	203,058	96,289	111,112	21,855	33,156	-11,301	-346
8393	12,610,541	282,756	284,165	148,105	2.40	2.90	301,152	363,892	165,022	199,119	1,734	13,070	-11,335	-347
7515	3,255,925	42,087	39,458	77,074	1.36	1.29	47,231	44,800	25,881	24,514	101,260	112,692	-11,431	-348
4693	6,668,810	43,724	50,712	54,892	0.90	0.79	51,749	45,424	28,357	24,856	24,831	36,297	-11,466	-349
4034	10,669,419	419,067	430,703	169,459	5.30	4.99	550,202	518,020	301,493	283,457	57,822	45,846	11,976	350
8008	57,110,661	335,902	315,953	109,058	0.62	0.67	376,443	406,801	206,279	222,598	45,821	57,913	-12,092	-351
6204	2,081,100	146,012	157,483	57,331	10.37	8.62	200,091	166,324	109,643	91,011	24,959	12,464	12,495	352
8044	22,781,209	365,061	366,118	272,583	1.70	1.92	386,162	436,136	211,605	238,650	17,572	4,825	12,747	353
8601	30,824,624	203,941	216,892	26,483	0.67	0.76	194,193	220,279	106,412	120,535	60,036	73,387	-13,351	-354
7538	1,220,533	121,542	115,061	9,603	11.54	9.59	148,783	123,642	81,528	67,656	63,453	49,813	13,640	355
4410	74,819,108	1,024,376	793,424	713,188	1.55	1.32	1,497,264	1,275,089	820,452	697,718	14,024	343	13,681	356
5059	1,008,336	173,442	153,767	8,310	19.53	17.27	222,126	196,421	121,718	107,480	105,665	91,503	14,162	357
9519	12,159,029	269,688	277,766	193,277	2.19	2.57	258,539	303,399	141,671	166,018	18,798	4,476	14,323	358
5462	2,273,739	158,508	168,302	124,941	7.60	10.69	162,748	228,918	89,181	125,262	14,339	1	14,339	359
912	343	3,498	3,453	46,108	123.50	125.72	4,291	4,368	2,351	2,390	814,229	799,562	14,666	360
3726	876,467	74,256	60,419	107,671	7.18	7.70	77,342	82,944	42,381	45,386	100,581	85,475	15,106	361
7370	8,355,779	246,850	252,449	53,654	4.11	3.73	335,806	304,758	184,011	166,761	92,347	76,716	15,631	362
3507	34,280,108	821,087	725,738	827,496	2.88	2.98	1,116,976	1,155,760	612,067	632,422	75,824	60,171	15,653	363
4243	7,345,527	299,109	261,245	85,569	4.86	4.45	408,734	374,252	223,973	204,788	85,527	69,404	16,123	364
9101	74,785,177	1,263,538	1,253,300	718,172	2.02	2.16	1,523,001	1,628,555	834,556	891,132	16,230	33,570	-17,340	-365
3821	5,279,222	196,148	182,394	35,692	6.30	5.72	357,671	324,743	195,992	177,696	131,108	113,482	17,627	366
8017	120,202,387	1,408,479	1,298,791	598,895	1.40	1.44	1,824,955	1,877,097	1,000,017	1,027,132	160,896	178,543	-17,647	-367
3400	25,284,319	618,201	694,069	648,142	3.10	3.03	698,136	682,372	382,556	373,388	184,380	202,175	-17,794	-368
8227	11,244,683	229,559	218,265	29,884	2.18	1.89	257,818	223,521	141,276	122,309	87,830	69,843	17,987	369
9179	25,200	2,150	2,150	41,241	9.54	9.69	2,404	2,442	1,317	1,336	1,209,919	1,191,755	18,164	370
8304	33,480,668	1,541,663	1,549,973	1,316,875	6.27	6.01	2,087,983	2,001,400	1,144,148	1,095,150	26,076	44,891	-18,815	-371
5040	1,419,960	149,829	150,564	118,750	12.69	9.90	179,313	139,890	98,258	76,547	4,274	23,269	-18,995	-372

EXHIBIT 2

Part 13

CALCULATION OF WILCOXON STATISTIC

BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

Class	(1) 1987 Payroll	(2) 1987 Earned Premium	(3) 1987 Manual Premium	(4) 1987 First Report Losses	(5) Current Rate	(6) Alternate Rate	(7) Current Total Premiums (1)x(5)x (2)/[(3)x100]	(8) Alternate Total Premiums (1)x(6)x (2)/[(3)x100]	(9) Current Expected Losses	(10) Alternate Expected Losses	(11) Current Squared Error [(9)-(4)]^2/(9)	(12) Alternate Squared Error [(10)-(4)]^2/(10)	(13) Difference (11)-(12)	(14) *D* Value
6235	2,714,329	231,262	258,037	212,213	9.34	8.68	227,212	211,157	124,505	115,543	61,786	80,879	-19,093	-373
4299	61,877,038	1,051,997	1,081,123	853,852	2.07	2.20	1,246,348	1,324,621	682,959	724,822	42,762	22,970	19,792	374
8720	5,331,620	51,176	49,311	116,842	1.13	1.07	62,526	59,206	34,262	32,397	199,036	220,111	-21,075	-375
2812	19,741,799	632,119	595,328	618,932	3.57	3.44	748,337	721,087	410,065	394,573	106,387	127,573	-21,187	-376
2157	11,857,205	533,978	513,251	169,270	5.82	5.43	717,958	669,847	393,418	366,535	127,707	106,166	21,541	377
4511	7,458,308	104,054	112,477	14,790	2.08	2.68	143,515	184,914	78,642	101,184	51,843	73,765	-21,922	-378
3081	3,895,336	216,471	166,768	331,025	6.17	5.90	311,973	298,321	170,951	163,239	149,888	172,460	-22,572	-379
5022	14,476,549	992,540	1,035,977	505,008	8.64	7.76	1,198,331	1,076,279	656,647	588,931	35,018	11,959	23,059	380
3681	5,209,109	44,609	37,908	77,146	1.02	0.88	62,525	53,943	34,262	29,517	53,676	76,853	-23,176	-381
8810	2,198,799,159	3,987,787	4,004,000	2,759,148	0.23	0.21	5,036,760	4,598,781	2,759,983	2,516,415	0	23,414	-23,414	-382
3574	24,992,917	467,358	408,842	74,659	2.34	2.50	668,539	714,251	366,338	390,833	232,235	255,776	-23,541	-383
8288	23,667,045	1,180,870	1,125,199	543,608	4.72	5.22	1,172,354	1,296,544	642,412	709,458	15,196	38,771	-23,575	-384
2095	26,471,742	959,542	772,960	892,468	3.85	4.11	1,265,174	1,350,614	693,275	739,045	57,233	31,850	25,383	385
3851	21,870,744	575,678	422,954	199,827	2.20	2.43	654,897	723,363	358,862	395,818	70,479	97,046	-26,567	-386
3028	11,999,897	675,901	692,641	65,577	6.22	6.67	728,354	781,049	399,115	427,384	278,736	306,292	-27,556	-387
5507	4,170,173	254,681	238,940	37,886	10.10	8.87	448,935	394,262	246,002	215,737	176,064	146,618	29,446	388
7539	71,736,766	688,515	1,338,025	777,860	1.84	1.80	679,217	664,452	372,189	363,583	442,164	472,041	-29,877	-389
8107	9,706,527	243,543	241,581	425,485	3.02	2.93	295,518	286,711	161,934	156,886	428,933	459,860	-30,926	-390
8039	49,879,481	580,746	549,856	198,561	1.64	1.50	863,979	790,224	473,433	432,404	159,588	126,462	33,127	391
7540	15,499,925	465,161	492,514	205,370	3.17	4.03	464,059	589,956	254,290	322,819	9,411	42,731	-33,320	-392
2014	27,908,321	971,287	974,378	735,224	4.13	3.71	1,148,957	1,032,114	629,592	564,764	17,723	51,449	-33,726	-393
1430	4,883,897	307,686	307,686	77,837	7.01	8.51	342,361	415,620	187,603	227,424	64,224	98,390	-34,166	-394
5188	2,282,927	77,020	78,760	141,647	3.64	3.36	81,263	75,012	44,529	41,046	211,812	246,569	-34,757	-395
3824	3,582,250	236,743	252,612	231,706	7.53	9.11	252,798	305,842	138,525	167,354	62,679	24,745	37,934	396
2501	21,700,953	218,282	197,851	262,158	1.24	1.09	296,879	260,967	162,680	142,799	60,830	99,767	-38,937	-397
8742	533,702,988	2,669,888	2,773,835	2,412,120	0.64	0.66	3,287,699	3,390,439	1,801,553	1,855,221	206,928	167,170	39,758	398
8102	9,760,884	252,267	302,481	29,538	3.29	4.22	267,823	343,529	146,758	187,976	93,627	133,542	-39,915	-399
106	786,502	52,593	52,320	258,101	7.88	8.06	62,300	63,723	34,138	34,869	1,469,299	1,429,156	40,142	400
4244	5,367,677	145,918	168,586	188,893	3.13	3.56	145,418	165,396	79,684	90,503	149,672	106,964	42,708	401
2002	4,567,395	134,253	144,330	231,838	3.71	3.44	157,619	146,149	86,370	79,971	245,001	288,397	-43,397	-402

EXHIBIT 2

Part 14

CALCULATION OF WILCOXON STATISTIC

BASED ON 1987 LOSS EXPERIENCE FROM A SINGLE STATE

Class	(1) 1987 Payroll	(2) 1987 Earned Premium	(3) 1987 Manual Premium	(4) 1987 First Report Losses	(5) Current Rate	(6) Alternate Rate	(7) Current Total Premiums (1)x(5)x(2)/[(3)x100]	(8) Alternate Total Premiums (1)x(6)x(2)/[(3)x100]	(9) Current Expected Losses	(10) Alternate Expected Losses	(11) Current Squared Error [(9)-(4)]^2/(9)	(12) Alternate Squared Error [(10)-(4)]^2/(10)	(13) Difference (11)-(12)	(14) D Value
3110	3,207,475	124,720	122,822	193,884	6.18	5.26	201,285	171,320	110,298	93,745	63,344	106,969	-43,625	-403
4611	13,529,498	167,782	154,904	256,871	1.57	1.42	230,072	208,091	126,072	113,866	135,703	179,603	-43,900	-404
9015	34,424,103	987,609	952,461	299,418	3.38	3.07	1,206,472	1,095,819	661,108	599,623	197,879	150,300	47,580	405
5221	17,090,299	865,655	851,460	509,214	7.64	6.38	1,327,467	1,108,539	727,409	606,583	65,450	15,630	49,820	406
8033	114,278,391	1,573,161	1,561,419	1,144,095	1.47	1.82	1,692,525	2,095,507	927,450	1,146,644	50,607	6	50,601	407
4021	4,352,460	301,046	300,048	23,004	6.61	8.88	288,655	387,784	158,173	212,192	115,511	168,678	-53,167	-408
9079	221,652,376	3,452,230	3,299,226	2,373,484	1.53	1.64	3,548,554	3,803,679	1,944,494	2,081,342	94,643	41,006	53,637	409
83	16,602,987	1,201,052	1,227,372	436,074	7.29	8.32	1,184,403	1,351,746	649,015	739,664	69,865	124,607	-54,741	-410
5506	53,826,037	2,137,466	2,178,983	1,124,480	5.79	5.42	3,057,147	2,861,785	1,675,219	1,565,945	181,059	124,456	56,602	411
5538	24,980,224	1,258,449	1,330,899	1,245,459	7.38	6.86	1,743,184	1,620,358	955,209	886,647	88,196	145,206	-57,010	-412
9014	24,107,903	1,429,909	1,374,524	1,031,564	5.51	6.16	1,381,870	1,544,885	757,220	845,348	99,396	41,020	58,376	413
3643	32,109,554	348,057	324,844	579,200	1.38	1.30	474,776	447,253	260,162	244,733	391,238	457,103	-65,865	-414
7219	232,034,434	14,488,896	14,405,233	8,790,011	7.71	7.42	17,993,756	17,316,948	9,860,001	9,475,691	116,113	49,617	66,496	415
5479	3,228,231	183,246	179,091	480,088	8.31	7.92	274,490	261,608	150,412	143,150	722,594	793,069	-70,475	-416
7409	534,873	50,687	51,988	231,773	11.28	10.79	58,824	56,269	32,234	30,790	1,235,234	1,311,943	-76,709	-417
42	5,190,961	284,643	281,989	382,858	6.44	7.45	337,444	390,366	184,909	213,605	211,910	134,109	77,800	418
3066	16,725,594	511,972	512,324	545,781	2.88	3.28	481,366	548,223	263,773	299,983	301,503	201,401	100,102	419
8018	49,827,999	1,785,041	1,592,061	1,581,311	4.62	3.93	2,581,095	2,195,606	1,414,357	1,201,418	19,708	120,124	-100,416	-420
2001	249,523	8,821	6,669	167,051	3.72	3.63	12,278	11,980	6,728	6,556	3,820,565	3,929,264	-108,699	-421
7704	9,011,417	360,915	367,354	507,643	4.97	5.93	440,017	525,010	241,115	287,281	294,618	169,030	125,588	422
8868	771,043,031	1,446,502	1,424,362	1,413,984	0.21	0.24	1,644,359	1,879,267	901,056	1,028,319	291,986	144,641	147,345	423
2089	66,807,604	1,954,756	1,835,535	2,114,472	3.16	3.38	2,248,241	2,404,764	1,231,964	1,315,867	632,178	484,676	147,502	424
2003	19,155,127	666,412	564,291	577,268	2.10	2.51	475,055	567,804	260,315	310,698	385,914	228,710	157,204	425
8021	44,796,783	2,597,631	2,700,620	948,809	7.23	8.31	3,115,294	3,580,650	1,707,081	1,959,302	336,819	521,153	-184,334	-426
7431	478,944	8,574	9,035	289,670	1.52	1.55	6,908	7,045	3,786	3,855	21,589,473	21,191,372	398,101	427
Total	7,826,102,306	116,482,865	114,671,811	78,098,909			142,524,603	142,726,762	78,098,909	78,098,909				5,068
										Arithmetic Average =	114,891	113,592		
													n =	427
													V =	0.99
													Approximate Confidence =	0.84

EXHIBIT 3

TEST OF EXCESS LOSS DIFFERENCES

	"All Other" Industry Group		Class 8810—Clerical Office Employees NOC				"All Other" Excluding Class 8810			
Year	Limited Losses	Unlimited Losses	Limited Losses	Unlimited Losses	Excess Loss Factor	ln(*ELF*−1)	Limited Losses	Unlimited Losses	Excess Loss Factor	ln(*GELF* − 1)
1985	40,279,153	41,998,052	2,327,467	2,327,467	1.0000	−14.6595	37,951,686	39,670,585	1.0453	−3.0944
1986	41,989,480	43,468,233	2,180,452	2,205,452	1.0115	−4.4684	39,809,028	41,262,781	1.0365	−3.3104
1987	48,545,569	50,409,813	2,759,148	2,759,148	1.0000	−14.8372	45,786,421	47,650,665	1.0407	−3.2015
Average						−11.3217				−3.2021
									Z_1	11.5650
									Z_2	1.1580
									Z_3	11.6356
									$\bar{Z}$	8.1195
									S^2	36.3485
									T	2.3327

INJURED WORKER MORTALITY

WILLIAM R. GILLAM

I'm the one that's got to die when it's time for me to die, so let me live my life the way I want to.

—Jimi Hendrix
If Six Was Nine

Abstract

This paper discusses the NCCI Special Call for Injured Worker Mortality Data and the ensuing analysis of that data. The design of the call and companies' abilities to supply elements of the call are discussed.

The goal was to see if the mortality of pensioned workers differs significantly from that of the general population. It does appear that, at least for ages below 60, the reported injured worker mortality rate is higher than reported on standard U.S. life mortality tables. Between ages 60 and 74, the injured worker mortality rate does not differ appreciably from standard mortality.

The differences in mortality, while significant, do not imply significant redundancy or inadequacy of tabular reserves.

ACKNOWLEDGEMENTS

Alan Reynard, FSA, Travelers Insurance Company, gave advice of significant value. Hsiu-Mei Chang, Leigh Halliwell, and Jose Couret, all at NCCI, helped in the statistical analysis.

Despite the existence of much supposition on the topic, the mortality of injured workers relative to the standard United States Life (USL) Tables has not been well analyzed. Interest in the subject waxes in times of deteriorating results, but then wanes as results improve. As if we needed more proof that the 1980s represented a prolonged period of less-than-satisfactory workers' compensation results, here is one more indication: a study of injured worker mortality has been completed.

1. THE CALL

In 1985, the Actuarial Committee at the National Council on Compensation Insurance (NCCI) resolved to begin such a study with a special call for data. The specifications for the call and committee sanction for its release were completed in 1986. Data elements, as described below, included several parameters of the claims, to be evaluated at two or more sequential year-end dates.

In 1987, the call was submitted to a small group of carriers who agreed to (and did) provide data. In 1988, the call was repeated, but to a larger group of carriers. Submissions were received from nine carriers in all, most in the second year only.

Exhibit 1 shows the record layout of the call. Report ID, carrier code, claim number, and state are used for identification. Injury date and age at injury are essential for the study. Pension date and sex are desirable, but fortunately not essential, as several carriers do not retain this information in the data files used to answer the call. Type of benefit code is a simplification of standard NCCI statistical plan coding. Paid and incurred amounts of indemnity and medical also are not essential, but are desirable for corollary studies and are usually easy to capture on company data files. The reason for closing field requires a choice of only three codes, which may be too simplified: permanent total (PT) claims closed for reasons other than fatality have to be handled carefully.

It probably would be useful to distinguish occupational disease from trauma cases, as allowed in the last entry, but this information is difficult for most companies to provide. In any case, the vast majority of claims reported are traumatic.

The difficulty in identifying certain claim characteristics is not critical, because the study attempts to determine which mortality table should be applied to the reserves for PT cases. If we use the experience of a random cross section of PT cases, we measure the mortality rates of exactly the group we want, whatever the profile of that group happens to be.

Workers who qualify for a life pension constitute a very select cohort. The potential for permanent injury is not usually recognized at the time of a serious accident. Certainly, no pension is established if the worker dies or, better, recovers within a short time. Even if the adjuster were able to recognize such a condition at an early stage, it usually is years before a prudent company will set up a lifetime pension and classify a claim as PT for the purpose of data reporting.

The draft specifications of the Special Call required that the earliest report be at least five years subsequent to the accident date. This requirement was dropped by the time the call was made, so that any claim recognized as PT could be submitted. Even with no maturity requirement, most of the claims submitted are at least four years old; that is, the actual accident occurred more than four years before the evaluation dates in the call. Of course, many claims are much more mature than that. We believe we have an unbiased sample of claims set up for lifetime reserves.

In summary, the call data do not allow the study of mortality rates for all seriously injured workers. Specifically, we are not able to measure the (presumably high) mortality rate of workers who have just been injured. What we can measure is the mortality rate of workers who lived long enough after their serious accidents to enter the elite group of lifetime pensioners.

2. THE DATA

We received information on nearly 13,000 injured workers from nine carriers, covering three calendar periods beginning 12/31/83 and ending 12/31/86. We believe that the data submitted represent an honest attempt to provide an unbiased sample. Minor inconsistencies in coding necessitated the following assumptions.

1) *Wrong Benefit Type*
Benefit types 0, 5, 6, 7, 8, or 9 appear on more than 3,000 claims. We assume these are regular statistical plan codes for non-serious losses and do not include them in the study. (Interestingly, inclusion of these claims in the study would increase the sample mortality rate slightly.)

2) *Reason for Closing Omitted*
There are 1,151 reports with the reason for closing field left blank. We assume they are open claims.

3) *Multiple Deaths and Life After Death*
A few claims that were closed due to death reappear, usually closed, but occasionally open. We exclude such subsequent reports.

4) *Reopened Claims*
Of the PT claims closed for reasons other than death (code 3), there are 222 that sometimes appear later as open. These claims are taken to be open the whole time.

5) *Disappearing Claims*
There are 801 claims appearing as open in one report that fail to appear in any subsequent report. These are treated as closed for reasons other than death (code 3) in the first subsequent report.

6) *Holes*
286 claims reported as open in one evaluation disappear the next, but reappear later. These claims are assumed to be open for the missing evaluation. (One claim skipped over two evaluations, and this gap was filled.)

7) *Contradictory Age Reports*
For example, a claimant may be reported at 12/31/84 to be 52 and to be 54 at 12/31/85. We use the lower of the two ages. There are 956 such reports.

Because of these inconsistencies and the resulting assumptions, we do not have strong confidence in the actual mortality rates by age in the study. Nevertheless, the patterns that emerge are believable, and the derived table is certainly better than one based on anecdotal information. The assumptions, either individually or in total, do not have much impact on the statistics derived from the sample.

3. MORTALITY RATES

The data are used to produce empirical mortality rates by age as follows:

1) As of the beginning of each year (previous year-end), there is some number of open PT cases for each age of claimant. Date of injury and age of claimant at injury are used to determine the age of a pensioner as of the evaluation date. We assume that the last birthday was six months before the accident. For each age, then, there is a sample of claimants who are followed through the calendar year to the next evaluation.

2) Claims missing or listed as closed for reasons other than fatality at the next year-end evaluation do not represent full lives. Since the exact date of closure is not coded in the call (and apparently difficult to obtain on company files), it is reasonable to assume an average mid-year closing. Using this logic, every claim closed for reasons other than fatality is counted as one-half of a life in the denominator of the mortality rate sample and zero fatalities in the numerator. This is a standard life actuarial technique.

3) The total of claims open for a year or closed due to death, plus half of the claims closed for other reasons, is denoted l_x, the lives at age x.

4) For age group x, we denote the number of deaths as d_x. For a given calendar year, the sample mortality rate, q_x, is the

number of deaths in that group during the year divided by the number of lives in the same group, so $q_x = d_x / l_x$.

5) The call spans more than a single calendar year; respondents to the call report claims evaluated at 12/31/83, 12/31/84, 12/31/85, and 12/31/86 (or some subset of those years, depending on available company data). As such, several calendar years' data can be compiled to evaluate empirical mortality rates. It should be apparent that a single claimant reported as living through several year-end evaluations would be part of the exposure for age x in the first evaluation, $x+1$ in the second, and so on. The first evaluation of a claim does not have to be 12/83, but can be 12/84 or 12/85.

Exhibit 2 shows the data and mortality rates based on this procedure. In the fitting described below, we use only the ages with more than 30 lives, which are ages 23 to 87.

4. THE FORCE OF MORTALITY

A smoothing procedure facilitates the comparison of the sample mortality rates by age to standard. Life actuaries have found that a Makeham curve of the form $M_x = A + BC^x$, where M_x is the force of mortality at age x, provides a good fit to empirical fatality statistics. We fit a Makeham curve to the injured worker mortality data, using maximum likelihood.

1) The Makeham force of mortality must first be restated as a mortality rate by age. This is done as follows:

$$Q_x = 1 - e^{-\int_x^{x+1} M_t \, dt}$$

$$= 1 - e^{-\int_x^{x+1} A + BC^t \, dt}$$

$$= 1 - e^{-[A + \frac{B(C-1) \cdot C^x}{\ln C}]} .$$

2) We use ages 23 to 87, which each have at least 30 lives. For the whole sample, the likelihood is

$$\Lambda = \prod_{x=23}^{87} \binom{l_x}{d_x} Q_x^{d_x} (1 - Q_x)^{(l_x - d_x)},$$

which is a function of independent variables A, B, and C.

It is usually easier to work with a sum, rather than a product, so we take the natural logarithm of the likelihood. The SAS function PROC NLIN is used to minimize the negative log likelihood of the sample in terms of A, B, and C.

5. THE FIT

The fit results in $A = 5.691 \times 10^{-3}$, $B = 1.156 \times 10^{-5}$, and $C = 1.115$, with a log likelihood of -136.84. Figure 1 compares the empirical and fitted injured worker mortality rates to USL rates. Figure 1 compares the graph of the mortality rates implied by the fitted curve with the data points.

The standard USL table based on 1979-1981 census data yields an excellent fit to a Makeham curve with parameters $A = 7.447 \times 10^{-4}$, $B = 5.728 \times 10^{-5}$, and $C = 1.093$. For this fit, we minimize an unweighted sum of squared differences. Figure 2 compares the empirical USL data with its fitted curve.

FIGURE 1
MAKEHAM FIT OF INJURED WORKER MORTALITY

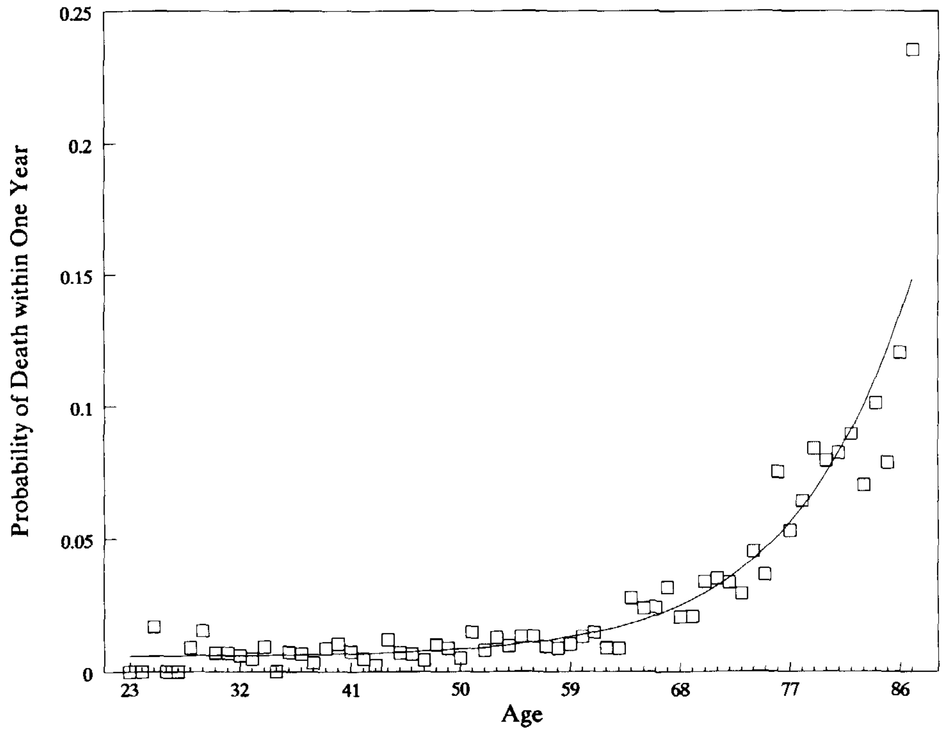
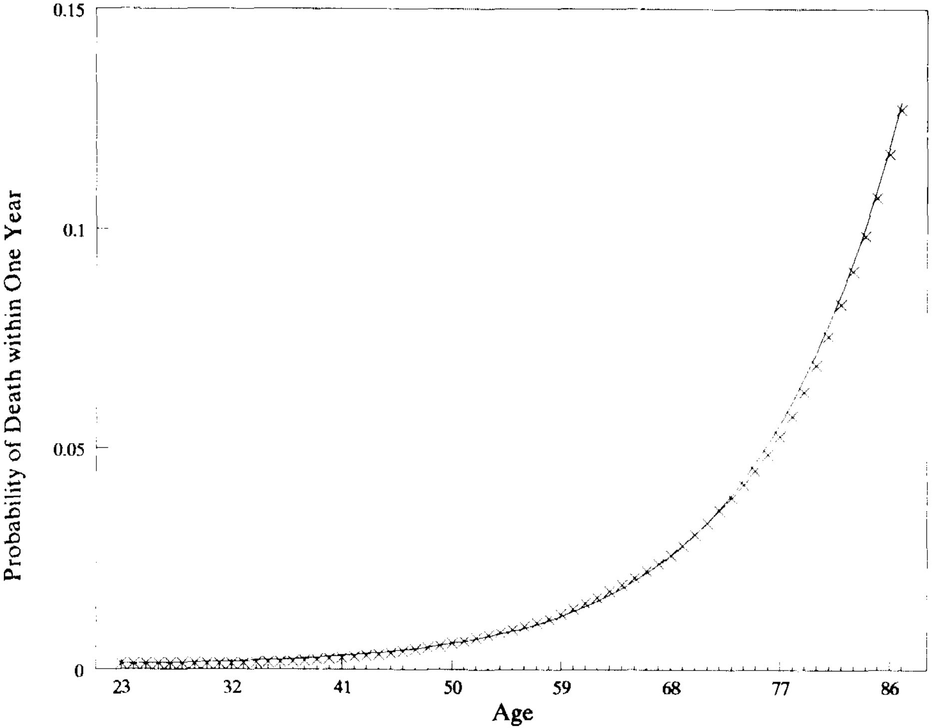


FIGURE 2
MAKEHAM FIT OF U.S. LIFE MORTALITY



6. A HYPOTHESIS TEST

To see if the sample data exhibits a mortality rate that differs from the USL rate, we use a simple likelihood ratio test. It is known that the following expression, Ω, is asymptotic chi-square, with degrees of freedom equal to the number of independent parameters, in this case three.

$$\Omega = -2 \ln\left[\frac{\Lambda_{USL}}{\Lambda_{MLE}}\right].$$

The likelihood Λ_{MLE} is that using the parameters A, B, and C estimated by maximum likelihood. The sample also has a likelihood under the USL parameters, Λ_{USL}, which is of course lower than that under the fit. In this case, we calculate $\ln \Lambda_{USL} = -152.57$.

So

$$\begin{aligned}\Omega &= -2(-152.57 + 136.84)\\ &= 31.46\,.\end{aligned}$$

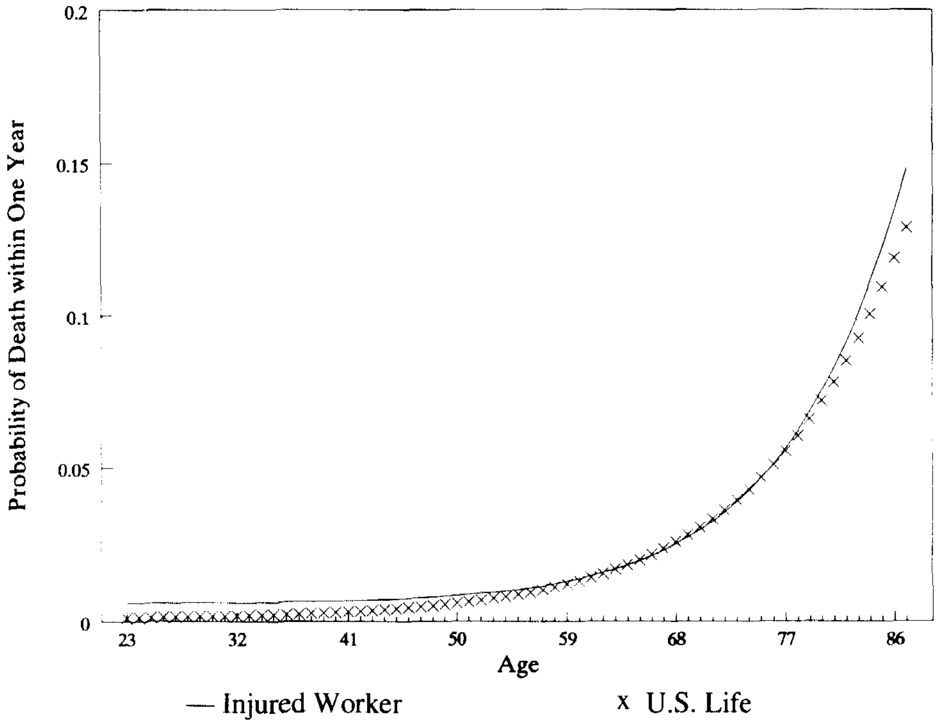
This value says that we can reject the hypothesis that the mortality rate of the sample population is USL, with a degree of confidence so large it is generally not on the table.

7. SOME CONCLUSIONS

The comparison of the injured worker mortality curve with the USL mortality value in Figure 3 is much more illuminating than a comparison of the sample data points with standard tables. The graph shows a mortality rate for injured workers that is slightly higher than standard at ages less than 60, but very slightly lower for ages 61 to 72.

Is it possible that injured worker mortality is so near standard? We think that it is possible, but it is important to remember the characteristics of the cohort in the study. An injured worker has been healthy enough to have worked in the first place. Such a person has demonstrated an ability to survive an accident long enough to be put on a

FIGURE 3
COMPARISON OF MORTALITIES



pension, which, as mentioned above, takes several years. By definition, the pensioner enjoys an annuity sufficient for lifetime support. The unfortunate worker whose workplace injury results in an immediate death, or one soon enough to preclude the need for a life pension, should not and does not enter the study.

A member of this sample population would presumably be resigned to his/her status and under relatively low stress, with the trauma of the original injury well behind him or her. It is also quite probable that older workers may qualify for permanent disability with an injury less severe than that necessary to disable a younger worker. This may account, in part, for the relatively favorable mortality of injured workers around the age of retirement.

8. THE ISSUE OF RESERVING

One of the motivations for this study was to test the propriety of using standard USL tables to reserve PT cases. We observed—and rationalized—slight differences in mortality rates by age between injured workers and the general population. The mortality found in the study implies that the average life pension on injured workers should be 1.6% lower than on standard. This finding is nominally supported by a weighted average of life pensions using sample distributions of permanently injured workers by age and wage level. The analysis is based on data from the call for detailed claim information, and may be seen in Exhibit 4.

Should action be taken on the possible 1.6% overstatement of reserves for injured workers? Perhaps, but the issue is more complicated than a simple argument about mortality rates. Pensions for permanently injured workers are subject to multiple decrements. Besides fatality, there may be other reasons for change in claim status. Such claims often change to permanent partial if the worker can resume employment in some other capacity. In fact, the worker may recover completely and be taken off the pension rolls. In some states, benefits terminate after a specified period or maximum amount. In most cases, pensions will terminate, or at least be reduced, when the claimant is eligible for Social Security. All of these things may reduce the need for a full lifetime reserve.

It should be noted that the death of the injured worker may result in a change of claim status to a benefit for the surviving spouse. This is a significant upward force on the required reserve for the permanently injured worker.

Weighing these considerations to decide whether to reduce reserves is unnecessary. The loss development analysis done in regular ratemaking almost always indicates upward reserve development. It would not be appropriate to lower reserves.

The existence of multiple decrements may indicate a need for further study of the denouement of PT claims. Certainly, the process is far more complicated than that contemplated by simple mortality tables. This study is complete, however, in that the mortality rate of pensioned workers has been reasonably determined.

The contention that the mortality rate of injured workers is higher than standard is often used in rate hearings as an argument against the need for rate increases: don't redundant reserves on pensions of short-lived injured workers overstate losses and hence the need for rate relief? Actuaries know that any systematic aggregate reserve redundancy or deficiency will result in measurable patterns of loss development, which, in turn, will be compensated for in standard methods used to project future ultimate loss levels. In that sense, then, the argument is already fallacious. Now there is direct evidence that the higher mortality in these cases does not make current reserves significantly redundant.

EXHIBIT 1

Injured Worker Mortality Study
Record Layout

Field Name	Width	Column(s)	Description
Report ID	2	1–2	Calendar year of report
Carrier Code	5	3–7	Five-digit insurer code number
Claim #	18	8–25	Alphanumeric code uniquely defining a claim
State	2	26–27	State of jurisdiction determining benefits
Injury Date	6	28–33	Date injury occurred (in MMDDYY format)
Pension Date	6	34–39	Date identified as a pension case (in MMDDYY format)
Age at Injury	2	40–41	Age on date of injury
Sex	1	42	M = male F = female U = unknown
Type of Benefit	1	43	1 = Death 2 = Permanent total 3 = Permanent partial 4 = Temporary total
Medical Paid	7	44–50	Medical benefits paid (whole dollars) as of report date
Medical Incurred	7	51–57	Medical benefits incurred (whole dollars) as of report date
Indemnity Paid	7	58–64	Indemnity benefits paid (whole dollars) as of report date
Indemnity Incurred	7	65–71	Indemnity benefits incurred (whole dollars) as of report date
Reason for Closing	1	72	1 = Open claim 2 = Death of claimant 3 = Other
OD/Trauma Code	1	73	1 = Occupational disease 2 = Traumatic

EXHIBIT 2
Part 1

INJURED WORKER MORTALITY TABLE

Age (x)	Lives (l_x)	Deaths (d_x)	Mortality (q_x)
All	29,586.5	575	0.01943
23	36.5	0	0.00000
24	45.5	0	0.00000
25	59.0	1	0.01695
26	71.0	0	0.00000
27	81.5	0	0.00000
28	112.5	1	0.00889
29	131.0	2	0.01527
30	143.5	1	0.00697
31	143.0	1	0.00699
32	167.5	1	0.00597
33	205.0	1	0.00488
34	214.0	2	0.00935
35	257.0	0	0.00000
36	282.5	2	0.00708
37	303.5	2	0.00659
38	310.5	1	0.00322
39	347.0	3	0.00865
40	387.5	4	0.01032
41	403.0	3	0.00744
42	422.5	2	0.00473
43	421.0	1	0.00238
44	415.5	5	0.01203
45	431.5	3	0.00695
46	464.5	3	0.00646
47	480.5	2	0.00416
48	510.0	5	0.00980
49	582.5	5	0.00858
50	598.0	3	0.00502
51	604.5	9	0.01489
52	631.0	5	0.00792
53	710.0	9	0.01268
54	735.0	7	0.00952

EXHIBIT 2
Part 2

Injured Worker Mortality Table

Age (x)	Lives (l_x)	Deaths (d_x)	Mortality (q_x)
55	764.5	10	0.01308
56	828.0	11	0.01329
57	848.5	8	0.00943
58	923.0	8	0.00867
59	982.0	10	0.01018
60	1,001.5	13	0.01298
61	1,017.5	15	0.01474
62	1,025.5	9	0.00878
63	1,036.0	9	0.00869
64	1,006.5	28	0.02782
65	961.5	23	0.02392
66	902.0	22	0.02439
67	849.5	27	0.03178
68	820.0	17	0.02073
69	766.0	16	0.02089
70	708.5	24	0.03387
71	624.0	22	0.03526
72	564.5	19	0.03366
73	511.5	15	0.02933
74	442.0	20	0.04525
75	383.5	14	0.03651
76	305.0	23	0.07541
77	263.5	14	0.05313
78	248.5	16	0.06439
79	202.5	17	0.08395
80	201.0	16	0.07960
81	170.0	14	0.08235
82	156.5	14	0.08946
83	128.0	9	0.07031
84	99.0	10	0.10101
85	63.5	5	0.07874
86	41.5	5	0.12048
87	34.0	8	0.23529

EXHIBIT 3

Part 1

COMPARISON OF INJURED WORKER AND U.S. LIFE MORTALITIES

Age (x)	Injured Worker Mortality Actual (q_x)	Injured Worker Mortality Fitted (Q_x)	U.S. Life Mortality
all	0.01943	0.01944	0.01787
23	0.00000	0.00548	0.00134
24	0.00000	0.00550	0.00133
25	0.01695	0.00552	0.00132
26	0.00000	0.00554	0.00131
27	0.00000	0.00557	0.00130
28	0.00889	0.00560	0.00130
29	0.01527	0.00563	0.00131
30	0.00697	0.00567	0.00133
31	0.00699	0.00571	0.00134
32	0.00597	0.00575	0.00137
33	0.00488	0.00580	0.00142
34	0.00935	0.00586	0.00150
35	0.00000	0.00592	0.00159
36	0.00708	0.00599	0.00170
37	0.00659	0.00607	0.00183
38	0.00322	0.00615	0.00197
39	0.00865	0.00625	0.00213
40	0.01032	0.00636	0.00232
41	0.00744	0.00647	0.00254
42	0.00473	0.00660	0.00279
43	0.00238	0.00675	0.00306
44	0.01203	0.00691	0.00335
45	0.00695	0.00709	0.00366
46	0.00646	0.00729	0.00401
47	0.00416	0.00751	0.00442
48	0.00980	0.00775	0.00488
49	0.00858	0.00802	0.00538
50	0.00502	0.00833	0.00589
51	0.01489	0.00866	0.00642
52	0.00792	0.00904	0.00699
53	0.01268	0.00945	0.00761
54	0.00952	0.00991	0.00830

EXHIBIT 3
Part 2
COMPARISON OF INJURED WORKER AND U.S. LIFE MORTALITIES

Age (x)	Injured Worker Mortality: Actual (q_x)	Injured Worker Mortality: Fitted (Q_x)	U.S. Life Mortality
55	0.01308	0.01042	0.00902
56	0.01329	0.01099	0.00978
57	0.00943	0.01162	0.01059
58	0.00867	0.01232	0.01151
59	0.01018	0.01310	0.01254
60	0.01298	0.01396	0.01368
61	0.01474	0.01492	0.01493
62	0.00878	0.01599	0.01628
63	0.00869	0.01717	0.01767
64	0.02782	0.01848	0.01911
65	0.02392	0.01993	0.02059
66	0.02439	0.02155	0.02216
67	0.03178	0.02334	0.02389
68	0.02073	0.02532	0.02585
69	0.02089	0.02752	0.02806
70	0.03387	0.02996	0.03052
71	0.03526	0.03267	0.03315
72	0.03366	0.03567	0.03593
73	0.02933	0.03898	0.03882
74	0.04525	0.04266	0.04184
75	0.03651	0.04673	0.04507
76	0.07541	0.05122	0.04867
77	0.05313	0.05620	0.05274
78	0.06439	0.06170	0.05742
79	0.08395	0.06777	0.06277
80	0.07960	0.07447	0.06882
81	0.08235	0.08185	0.07552
82	0.08946	0.09000	0.08278
83	0.07031	0.09896	0.09041
84	0.10101	0.10881	0.09842
85	0.07874	0.11964	0.10725
86	0.12048	0.13151	0.11712
87	0.23529	0.14452	0.12717

EXHIBIT 4
Part 1
RESERVES REQUIRED BY U.S. LIFE AND INJURED WORKER MORTALITIES FOR A SAMPLE OF PENSIONED INJURED WORKERS
(INTEREST RATE = 6.0%)

Age	Injured Workers	Average Annual Benefit	U.S. Life Annuity	Injured Worker Annuity
21	2	9,641	15.9250	15.0456
22	1	9,360	15.8719	15.0110
23	9	9,363	15.8161	14.9743
24	14	9,516	15.7575	14.9354
25	24	9,219	15.6961	14.8942
26	34	9,147	15.6316	14.8506
27	35	9,792	15.5640	14.8044
28	64	10,117	15.4932	14.7555
29	65	10,561	15.4190	14.7038
30	64	10,327	15.3413	14.6490
31	77	10,365	15.2600	14.5912
32	89	10,648	15.1749	14.5301
33	116	11,098	15.0859	14.4655
34	106	11,635	14.9929	14.3972
35	136	11,503	14.8957	14.3253
36	156	11,649	14.7943	14.2493
37	152	11,767	14.6885	14.1692
38	148	11,932	14.5781	14.0848
39	171	12,156	14.4631	13.9959
40	189	12,862	14.3434	13.9023
41	197	12,611	14.2187	13.8038
42	199	12,582	14.0890	13.7002
43	189	13,045	13.9543	13.5914
44	194	13,306	13.8143	13.4772
45	216	13,139	13.6690	13.3573
46	229	13,571	13.5184	13.2316
47	222	13,467	13.3623	13.1000
48	268	13,366	13.2007	12.9622
49	290	13,785	13.0336	12.8180
50	258	13,496	12.8609	12.6674
51	286	13,367	12.6825	12.5103
52	296	13,419	12.4986	12.3463

EXHIBIT 4
Part 2
RESERVES REQUIRED BY U.S. LIFE AND INJURED WORKER MORTALITIES FOR A SAMPLE OF PENSIONED INJURED WORKERS (INTEREST RATE = 6.0%)

Age	Injured Workers	Average Annual Benefit	U.S. Life Annuity	Injured Worker Annuity
53	336	13,607	12.3091	12.1756
54	337	13,694	12.1139	11.9979
55	356	13,631	11.9133	11.8132
56	387	13,669	11.7072	11.6214
57	369	13,439	11.4958	11.4226
58	449	13,426	11.2792	11.2168
59	449	13,459	11.0574	11.0039
60	432	13,546	10.8307	10.7841
61	444	13,433	10.5992	10.5574
62	464	13,465	10.3633	10.3241
63	449	13,127	10.1230	10.0842
64	429	13,078	9.8787	9.8381
65	384	12,930	9.6307	9.5860
66	358	12,597	9.3792	9.3282
67	342	12,347	9.1247	9.0652
68	351	12,319	8.8675	8.7972
69	288	11,778	8.6079	8.5247
70	261	11,768	8.3464	8.2484
71	233	11,406	8.0835	7.9686
72	201	11,178	7.8195	7.6860
73	188	10,738	7.5549	7.4013
74	155	10,464	7.2903	7.1151
75	126	10,141	7.0260	6.8280
76	104	10,063	6.7626	6.5408
77	100	9,678	6.5006	6.2543
78	95	9,351	6.2405	5.9692
79	70	9,400	5.9827	5.6862
80	78	8,634	5.7278	5.4062
81	59	8,256	5.4762	5.1298
82	58	8,465	5.2285	4.8578
83	40	7,869	4.9849	4.5909
84	21	7,691	4.7461	4.3298

EXHIBIT 4
Part 3

RESERVES REQUIRED BY U.S. LIFE AND INJURED WORKER MORTALITIES FOR A SAMPLE OF PENSIONED INJURED WORKERS
(INTEREST RATE = 6.0%)

Age	Injured Workers	Average Annual Benefit	U.S. Life Annuity	Injured Worker Annuity
85	16	7,275	4.5123	4.0752
86	11	6,804	4.2840	3.8276
87	14	7,481	4.0615	3.5875
88	9	6,333	3.8451	3.3555
89	3	7,041	3.6351	3.1320
90	3	6,881	3.4317	2.9173
91	4	7,043	3.2352	2.7117
92	4	6,555	3.0457	2.5155
93	1	6,803	2.8633	2.3287
95	2	5,914	2.5204	1.9839
96	2	4,994	2.3600	1.8257
97	1	5,481	2.2068	1.6770
99	1	5,406	1.9223	1.4070
100	1	5,323	1.7907	1.2853
	12,981		11.3258	11.1417

Relative Difference = (Average Injured Worker/Average US Life)−1 = −1.6%

SURPLUS—CONCEPTS, MEASURES OF RETURN, AND DETERMINATION

RUSSELL E. BINGHAM

Abstract

This paper discusses the role of surplus in an insurance company and alternative measurements of rate of return on surplus. The multi-year dimension of surplus and its linkage to liabilities over time is explained, and the concept of a calendar period balance sheet as the sum of underlying accident period balance sheets is introduced. Measures of rates of return on surplus inherent in internal rate of return and net present value discounted cash flow models are explained, and the conditions under which the returns are equivalent are demonstrated.

This paper also presents a methodology for determining a benchmark amount of surplus needed to support writings in a line of business in order to control the probability of insolvency. The methodology is based on a consideration of both the magnitude and the variability in underwriting, underwriting cash flows, and interest rates.

1. INTRODUCTION

This paper discusses several conceptual and financial aspects pertaining to surplus. It is intended to provide both a fundamental understanding of the role of surplus in an insurance company and measurements of rate of return on surplus (Section 3), as well as provide a methodology for the establishment of the proper amount of surplus (Section 4). A summary of key observations and findings is provided in Section 2 to assist the reader in assimilating the material in the paper.

Section 3 of the paper discusses the purpose of surplus, followed by the introduction of the concept of a calendar period balance sheet viewed as the sum of underlying accident period balance sheets. This discussion demonstrates the multi-year dimension of surplus and its linkage to liabilities (primarily loss reserves) over time and exposes the meaninglessness of premium to surplus relationships.

Section 3 also discusses measures of rates of return on surplus inherent in internal rate of return (IRR) and net present value (NPV) discounted cash flow models and demonstrates the conditions under which the returns are equivalent by utilizing the liability-to-surplus relationship. Section 3 also introduces the concepts of steady state and present-valued income statements, cash flow statements, and balance sheets. The effects of business growth and the commitment of surplus based on premium are demonstrated.

Finally, in Section 4, the annualized present-valued balance sheet is used as a basis for the volatility-adjusted funding approach to determine benchmark surplus requirements. Section 4 presents a methodology which determines the benchmark surplus requirement needed to control the probability of insolvency that can result from underwriting and investment volatility. This methodology is primarily based on a consideration of both the magnitude and variability in underwriting, underwriting cash flows, and interest rates. Leverage ratios are shown over an assumed range of these values.

Several pages of numerical exhibits are presented in the appendices for the reader interested in working through examples in detail. These are not required reading for this article as key figures are repeated in the text when necessary.

2. SUMMARY

The following are key observations and findings which are presented and discussed in this paper:

1) Calendar period accounting does not provide sufficient information to measure the true profitability of a given underwriting period.

2) An accident year development of income statements, cash flow statements, and balance sheets, much like a traditional loss triangle, is required to truly measure profitability.
3) Surplus is committed to support the writings of a given accident year and must run-off over a period of future years as policyholder liabilities run-off.
4) The premium-to-surplus ratio is a convenient but mostly irrelevant measure of leverage. The ratio of policyholder liabilities to surplus (or more simply, reserves to surplus) is the appropriate measure of leverage.
5) Internal rate of return and net present value cash flow models produce identical measurements of return on surplus as long as the same rules are followed for the initial contribution and subsequent withdrawal of surplus.
6) Single period financial statements (income, cash flow, balance sheet) can be created that are representative of the multi-year flows of an accident year and provide a transition to a simplified measurement of return. These are equivalent to financial statements that would exist under steady state business conditions.
7) Increasing rates of business growth will cause calendar returns on surplus to be increasingly lower than the true accident year rates of return when business is written at an underwriting loss.
8) Use of premium (via premium-to-surplus ratios) as a basis for controlling the flow of surplus for an accident year will, by itself, cause calendar rates of return to differ from the true rate of return.
9) It is possible to determine the benchmark surplus, necessary to provide a financial buffer for a line of business, that satisfies a specified probability level of insolvency.
10) The benchmark surplus needed for a line of business must recognize both the *amount* of financial exposure, which re-

sults from all cash flows, and the *volatility* expected in this financial exposure.

11) Benchmark surplus is neither SAP nor GAAP equity.

3. FUNDAMENTALS OF SURPLUS, CASH FLOW, AND RATE OF RETURN

Purpose of Surplus

Surplus exists in insurance for the same purpose as in other businesses: it serves as a financial buffer to guard against adverse business conditions during which operating losses occur. Surplus provides a cushion, at least temporarily, to cover losses and to permit business to continue to operate normally.

Insurance, however, is unique in that the major portion of its business costs (i.e., claim payments) are not known at the time the product is priced and sold. In fact, these costs may not be known for several years. Complicating the uncertainty, many factors, such as social inflation and changing tort law, limit the ability to forecast these costs with a high degree of certainty. As a result, it is difficult to determine the proper level of surplus that is required to support insurance writings.

Benchmark Surplus

Benchmark surplus is that level of surplus that will provide the proper financial buffer for a line of business or business segment. The magnitude of the benchmark surplus for a line of business must be based on a consideration of the factors unique to that line which introduce uncertainty (or volatility) in expected future results. It should also reflect the probable likelihood of the occurrence of those adverse conditions which would cause a drain on surplus.

The greater the amount of surplus, the less likely that the occurrence of adverse conditions will deplete the entire amount of a company's surplus. The concept of probability of occurrence of adverse conditions is integral to the establishment of a benchmark surplus. An amount of benchmark surplus is viewed hand-in-hand with a specified probability of insolvency.

Benchmark surplus is neither statutory surplus nor GAAP equity. Rather, it is simply the amount of assets which should be available to financially support the operations of a line of business in order to control solvency and risk. Benchmark surplus is but a measure of a necessary financial cushion, and it may or may not match a particular company's reported surplus. It does, however, reflect the realities that should be considered by a company in its operating practices.

Calendar Year Reported Surplus as the Sum of Accident Period Surplus

Policyholder Surplus, as reported on insurance company balance sheets, is often misunderstood and misused. This misuse results from a lack of understanding as to the composition of this calendar period item, which is determined by underlying current and previous accident year development activity. To understand this problem, which is somewhat unique to insurance, it helps to draw a parallel with manufacturing.

In manufacturing, a product or project is often evaluated as a unique entity with the product's revenues and expenses monitored throughout its life cycle. Management can thus make a final determination of the likely profit associated with this product. In this evaluation, capital investment in plant and equipment is linked to the product, and management can easily estimate a return on this investment.

The insurance equivalent to a product is an exposure year (or accident year) book of business. An insurance company prices policies based on an estimate of all costs, both present and future, which relate to the period for which the policy applies. Unfortunately, companies generally monitor only the cost of claim payments (i.e., losses) by accident year (and occasionally policy year).

It is important to recognize that the usual calendar period accounting does not maintain adequate detail to properly value accident year profitability. Revenues subsequent to the accident year, primarily investment income, and subsequent costs other than claims are not monitored for each originating accident period.

An ideal scenario would involve the complete segmentation of accounting records for each accident year: That is to say, income, cash flow, and balance sheet statements for each year. Under this segmentation of the accounting structure, surplus would be maintained for each accident year and it would run off along with liabilities for that year. Under this structure, the calculation of each accident year's return on investment would be relatively simple.

Since most companies do not maintain this level of detail, we can only view a combined calendar balance sheet and recognize that it represents the sum of contributions from all current and previous accident years. Thus, when one looks at a company's surplus, one must realize that it is in fact a composite of surplus amounts which are "dedicated" to these same current and previous accident years. Since surplus in most lines of business is multi-year dimensioned, to view it as a single number associated with a calendar year is incorrect. The familiar premium-to-surplus ratio has no basis in theory, although it has come to provide a convenient reference point. Certainly, surplus is not established from calendar premium-to-surplus relationships.

Cash Flow Models

In order to understand the time dimension of surplus, it is helpful to review the so-called discounted cash flow models. As discussed later, it is possible to develop a present-value based balance sheet which provides a transition from the cash flows of multiple accident years to a calendar steady-state balance sheet. First, however, a very brief review of discounted cash flow models is in order.

Cummins [1] provides a good overview of the discounted cash flow models used in insurance ratemaking. Of importance to the discussion here, he contrasts the IRR model, as used by the National

Council on Compensation Insurance (NCCI), with the Myers-Cohn NPV model used in Massachusetts.

While there are differences in the two approaches as applied, both involve recognition of insurance cash flows and surplus over time. *One of the most significant attributes of both models is that surplus is a function of policyholder funds, with its release governed by reductions in policyholder liabilities over time.* (Policyholder funds represent the net liabilities of the company which have not been settled at any point in time. These are predominantly loss reserves. Some cash flow models form a linkage between loss reserves and surplus as a simplifying assumption.)

Cummins notes a difference between the models: the NCCI's IRR model assumes that surplus additions are required to cover an initial underwriting loss, whereas the NPV model does not require this. This difference, however, has to do only with the beginning surplus requirement, and not its subsequent release. These constraints governing the initial surplus in the models are unique to these two applications. Generally, they are not part of IRR and NPV models. In fact, either model could operate under the opposite constraint. Given consistent determination of the initial surplus, measured rates of return become equivalent, as discussed later.

Some proponents of IRR are not averse to defining arbitrary surplus withdrawal schedules whose sole apparent purpose is to maximize (or minimize) the IRR. This arbitrary withdrawal is improper. By ignoring the linkage of surplus release to policyholder funds, it thereby ignores the fundamental purpose of policyholder surplus: To act as a financial buffer against the adverse development of liabilities.

As described by this author in [2], the Hartford uses a NPV approach structured to provide a calculation of total return. As part of this approach, "annualized" balance sheets are developed on both nominal and discounted bases, which include surplus. It is the development of the balance sheet from cash flows that provides the means for measuring returns. This aspect is too often overlooked in cash flow models. This will be explained in the next subsection followed by a demonstration of the equivalency of IRR and NPV measurements of return.

Controlling the Flow of Surplus

It is useful to begin by introducing an example which will demonstrate the concepts to be discussed. The appendices present an example involving a single accident year (which can be viewed as a single policy written on the first of the year) with a premium of $10,000, expense of $3,000, and ultimate loss of $8,000. The premium is received and the expenses are paid without delay; claims are paid in 25% installments at the end of the current and three following years.

The example assumes the yield rate on investments to be 8% before-tax and the tax rate on underwriting and investment income to be 34%. For simplicity, the rate used for loss discounting under the 1986 Tax Reform Act is also 8%. The example assumes one-half of premium to be unearned at the end of the first year for purposes of the premium offset provision of the tax law. In this example, all cash flows are discounted to the beginning of each respective year. Traditional accounting rules are followed to construct income statements and balance sheets. The schedule of appendices relating to this example is as follows:

Appendix A—Basic assumptions and calculations of reserves and payments

Appendix B—Nominal and discounted income statements and balance sheets for the single accident year over its four years of activity

Appendix C—Appendix B accumulated across successive accident years, reaching steady state after four years

Appendix D—Relationship of policyholder and shareholder funds

Appendix E—Shareholder flows, nominal and discounted steady state income, IRR and NPV and rates of return

Appendix F—Accident year contribution to calendar year income and return on surplus (ROS)

Appendix G—Accident year contribution to calendar year shareholder flows and IRR

Appendix H—Annualized nominal and discounted balance sheet and income statement summary

Underwriting and investment are assumed to remain constant over time. With no growth in the level of business, it takes four years to reach a steady state condition, after which all items remain the same, as shown in Appendix C.

In the example, writing the policy requires an initial capital contribution by the shareholder. Subsequently, the shareholder receives payments (i.e., return of capital) consisting of three components: 1) The return of invested capital; 2) the investment income on the invested capital while held by the company; and 3) the insurance operating earnings, which are the sum of the underwriting income and the investment income on policyholder funds.

The release of funds to the shareholder is governed by maintaining a constant 4:1 ratio of policyholder funds to shareholder funds over time. For simplification in this example, policyholder funds are assumed to consist of loss reserves only and do not include either the tax law timing items or retained earnings. (Retained earnings are, in effect, undistributed operating earnings which must be included in shareholder flows at some point, and are considered separate from surplus.)

The release of funds to the shareholder is thus a payout policy of: 1) Withdrawing investment income on capital as it is earned (i.e., annually) and 2) withdrawing the initial capital contribution and operating income as a function of loss payout. This is demonstrated in Appendix D for both the single accident year and steady state.

Under this return of capital rule, the initial surplus for the accident year is $2,000 based on the 4:1 reserve-to-surplus ratio, followed by declines to $1,500, $1,000, and $500 in years two through four, since the loss reserve is $8,000, $6,000, $4,000, and $2,000, respectively, for years one through four. At steady state, the reserve is $20,000 and the surplus $5,000. The calendar year premium-to-surplus ratio at steady state is 2:1.

The itemized shareholder flows are shown in Appendix E, page 1. Capital is withdrawn at the rate of 25% ($500) per year, matching the loss payout pattern. The shareholder receives the investment income on the contributed capital and the operating earnings in a manner that maintains the relationship to reserves.

This pattern of surplus flow results in various equivalent measurements of rates of return on surplus, the subject of the next subsection.

Rates of Return on Surplus

In Appendix E, page 1, an IRR calculation is shown for operating earnings, contributed capital, and net shareholder flows. This is repeated in Table 1.

The IRR for operating earnings and contributed capital are both 5.3%, since these flows earn 8% before-tax, or 5.3% after-tax. The shareholder receives a net IRR of 10.4%, based on the initial capital contribution of $2,000 followed by withdrawals of $708, $656, $604, and $552 in years one through four. The IRR measures the return to the shareholder from both operating earnings and investment income on surplus. It should be noted that the annual return on invested capital is also 10.4% in every year.

TABLE 1

SINGLE ACCIDENT YEAR SHAREHOLDER FLOWS

	Begin	Year 1	Year 2	Year 3	Year 4	IRR
Operating Earnings	-231	102	77	51	26	5.3%
Contributed Surplus:						
Investment Income		106	79	53	26	
Capital Withdrawal	-2,000	500	500	500	500	
Contributed Capital	-2,000	606	579	553	526	5.3%
Net Shareholder Flows	-2,000	708	656	604	552	10.4%
Annual Return		10.4%	10.4%	10.4%	10.4%	

A parallel IRR workup at steady state is shown in Appendix E, page 2. Appendix E, page 3, displays nominal and discounted calculations of return on surplus derived from the steady state balance sheet and income statements. This is summarized in Table 2.

Note that the total net income of $520 is 10.4% of the $5,000 beginning surplus. The calculation of discounted return is shown to the right and reflects the steady state figures on a basis discounted to either the beginning or the end of the initial accident year. When valued at the end of the accident year, the total return of $494 is 10.4% of the $4,755 beginning surplus.

TABLE 2

STEADY STATE SHAREHOLDER RETURN

	Nominal Basis	Discounted to Beginning of Accident Year	Discounted to End of Accident Year
Beginning Surplus	5,000	4,517	4,755
Underwriting Income	-660	-660	-695
Investment Income (or Credit)	916	891	938
Investment Income on Surplus	264	238	251
Total Net Income	520	469	494
Return on Beginning Surplus	10.4%	10.4%	10.4%

This demonstrates that all three measures of return—the IRR, the steady state nominal calendar period, and the discounted return—are equivalent. This equivalence holds under the assumption that underwriting and investment are fixed, there is no growth in business level, and policyholder and shareholder flows are linked over time.

Appendix F shows calendar and accident period net income, beginning contributed surplus, and ROS over an accumulation of eight successive accident years, including subsequent run-off after the last year, in a format similar to a loss development triangle. The ROS section on page 3 of the Appendix shows the relationship between calendar and accident period returns over the period. Initially, calendar returns are lower due to the underwriting losses from the up-front payout of expenses. At steady state, both calendar and accident returns are equal. During run-off, the presence of investment income without underwriting losses causes the calendar year returns to exceed the accident year returns. Note, however, that the overall cumulative calendar period return is 10.4%, matching the accident period return.

Appendix G demonstrates this same equivalence from the shareholder perspective by using the same calendar and accident period format to set forth shareholder flows and returns.

Transition From Multi-Year To Single Period—Steady State and Present Value Implied Balance Sheets and Income Statements

The NPV measurement of return ratios the present value of all income streams—both underwriting and investment—to the present value of surplus committed. In effect, the process creates a balance sheet which represents the annualized present value sum of individual future calendar period balance sheets. The balance sheets for future years are discounted to the present and summed. This annualized equivalent balance sheet provides the vehicle through which a rate of return can be calculated.

Returning to the example in the appendices, Appendix H demonstrates the components of both an ongoing, steady state nominal balance sheet and a discounted income and balance sheet. The exhibit displays discounted values at both the beginning and the end of the accident year. This is summarized in Table 3. For example, the ongoing steady state loss reserves are $20,000 on a nominal basis and $19,022 discounted (valued at the end of the accident year). The nominal total balance sheet consists of net liabilities of $18,707 and surplus of $5,000. The surplus commitment of $2,000, $1,500, $1,000, and $500 for years one through four, respectively, equates to an ongoing commitment at steady state of $5,000.

TABLE 3

Annualized Nominal and Discounted Balance Sheet and Income Statement Summary: Funding of Liabilities Through Committed Assets and Surplus

	Balance Sheet			Investment Income		
		Discounted			Discounted	
Committed Assets = Liabilities	Nominal	Begin Year	End Year	Nominal	Begin Year	End Year
Net Policyholder Funds	20,000	18,060	19,022	1,056	954	1,004
Net PH Liabilities (Including Tax Timing Items)	18,707	16,874	17,765	988	891	938
Net PH Liabilities (Including Retained Earnings)	17,342	15,627	16,452	916	825	869
Contributed Surplus	5,000	4,517	4,755	264	238	251
Calculation of Return:				Income		
Underwriting Income				-660	-660	-695
Operating Income				256	231	243
Total Net Income				520	469	494
Return on Surplus				10.4%	10.4%	10.4%

The corresponding discounted values are net liabilities of $17,765 and surplus of $4,755. This means that we need to set aside the equivalent of this amount today to fund future liabilities and provide the desired surplus support throughout the four year period.

The NPV investment income credit is $938 on the $17,765 policyholder related assets and $251 on the $4,755 in surplus assets. This means that the net funding requirement (i.e., assets committed) once this business is written is $17,765.

The surplus commitment is $4,755 in present value terms. This can be thought of as the one year annualized asset commitment that equates to the actual commitment of assets over the four year period.

The level of this asset commitment is a function of both the magnitude of the cash flow balances and the amount of time over which these cash flows and balances exist.

In short, *the funding commitment is the present-valued balance sheet asset commitment dictated by cash flows.* This asset commitment also represents the asset earnings base upon which the credit for future investment income is based. The annualized investment income figure is the same as the present value of the investment income stream derived from the investment of assets over the period of years, each discounted to the accident period.

The steady state present-valued balance sheet viewpoint provides a means by which transactions over several years can be translated to a single calendar period measurement. In particular, the surplus commitment over multiple calendar years sums to a single period value against which returns are calculated.

The ability to employ a single period basis is a key to simplifying discounted cash flow models and providing a single return on surplus measurement. While this measurement will equal the IRR under certain conditions, this NPV cash flow approach provides added flexibility not inherent in the IRR. For example, the approach supports the determination of the traditional operating return on premium (ROP) preferred by many in ratemaking. Appendix H shows the calculation. The ROP turns out to be 2.3% in this example.

In addition, the approach has the virtue and flexibility of separately dealing with individual cash flows, as opposed to only net shareholder flows as with the IRR. Risks associated with the component cash flows, for example, can be reflected by adjusting their respective discount rates (even though the example has used a single rate for convenience). This contrasts with the single fixed rate assumed in the typical IRR calculation.

When surplus relates to policyholder funds as in the example, it automatically responds to both the magnitude of the flows and the time frame over which flows occur. Equally important, however, is that the annualized present-valued balance sheet provides a framework for incorporating assumptions on volatility. Benchmark surplus should not only reflect the magnitude of insurance liabilities, as measured by committed assets, but also the variability that can result from the deviations in underwriting and investment results from their expected values. Section 4 discusses this in more detail.

Two particular effects on measured rates of return hold special interest: business growth, and an alternative capital withdrawal policy which does *not* maintain the relationship between policyholder and shareholder funds.

The Effect of Business Growth on Rate of Return

Appendix F, pages 4 through 6, demonstrates the effect of a 10% annual accident/exposure year rate of growth in business. In this modification of the example, each successive accident year premium grows by 10%, while the underwriting and investment assumptions remain unchanged. The example maintains surplus at the same policyholder to shareholder (reserves to surplus) ratio of 4:1.

As in the earlier version of the example, each individual accident year has the identical 10.4% return on surplus. The calendar returns are lower than before, however. On an ongoing basis, calendar returns lag behind the accident returns since the newest accident year's higher initial underwriting loss has a larger impact on the calendar returns than before. This loss offsets more heavily the previous accident year's positive investment income contributions. The calendar return now reaches 9.1% in years four through eight, rather than the previous 10.4% realized without growth.

Since this example eventually allows the business to run off the books, the *total* return does reach 10.4% after all flows are completed. But if accident year business continued at the 10% growth rate, the calendar returns would show a permanent shortfall of 1.3%. This gap becomes greater with higher rates of growth, longer loss payouts, or higher interest rates.

Table 4 demonstrates the calendar return shortfall under alternative business growth scenarios (0%, 10%, 25%, and 40%), average loss payouts ranging from one to four years, and interest rates of 8% and 10% before tax. The calendar returns which result under some of these scenarios fall significantly below the underlying 15% accident year ultimate return.

All cases in the table assume that the accident period ultimate return on surplus is 15%, the expense ratio is 30.0%, and the ratio of policyholder to shareholder funds is 4:1.

TABLE 4

CALENDAR ROS AND BUSINESS GROWTH

				Calendar ROS			
				Rate of Business Growth			
Interest Rate on Investment Before-Tax	Avg. Loss Payout (Years)	Loss Ratio	Combined Ratio	0%	10%	25%	40%
8%	1	72.6%	102.6%	15.0%	15.0%	15.0%	15.0%
8	2	75.4	105.4	15.0	14.5	13.8	13.3
8	3	78.1	108.1	15.0	14.0	12.7	11.5
8	4	80.6	110.6	15.0	13.6	11.6	9.8
10	1	74.2	104.2	15.0	15.0	15.0	15.0
10	2	78.8	108.8	15.0	14.2	13.2	12.3
10	3	83.4	113.4	15.0	13.5	11.4	9.6
10	4	88.0	118.0	15.0	12.8	9.7	6.9

The Effect of Independent Surplus Withdrawal

In order for the IRR, nominal steady state, and discounted return measures to be equal, it is necessary to maintain the linkage of shareholder and policyholder funds. To demonstrate what happens when the linkage is not maintained, Appendix E, pages 4-6, and Appendix F, pages 7-9, provide an example under which the entire surplus is withdrawn at the end of the accident year. That is, the full $5,000 is provided at the beginning of each accident year and returned to the shareholder at the end of the year. This is equivalent to setting surplus as a function of premium using a premium to surplus ratio of 2:1.

Operating earnings are distributed to the shareholder in the amount of calendar net income.

The calculated IRR is 9.5%, the nominal steady state return 11.1%, and the discounted return 10.1%. The degree to which the three return measures will differ is affected by many factors, including leverage, loss payout, and interest rates.

In the insurance industry, actual withdrawal of capital is often a function of income, or it may be designed to maintain a stable calendar year dividend payout. Certainly, historical withdrawals seldom have reflected any linkage to accident year policyholder funds and the run-off of surplus in parallel with these liabilities.

The examples in the appendices are intended to show the conditions under which the IRR, calendar period, and discounted accident period returns are equal and when they differ. If growth occurs, underwriting and investment conditions change, or capital is withdrawn without regard to a linkage with liabilities, then these measurements of return will differ.

It should be clear that rate of return measurements which are based on published calendar financial statements may not properly reflect current (i.e., accident year) profitability. Such calendar measures will likely be very poor proxies in lines of business which take many years to settle. The reported income statement, cash flow statement, and balance sheet are composites of current and prior accident years. While such calendar measures are unavoidable, the true performance picture can only be ascertained through a return measure which recognizes policyholder and shareholder flows for a given accident year over all subsequent periods during which cash flows occur.

4. DETERMINING BENCHMARK SURPLUS: THE VOLATILITY-ADJUSTED FUNDING APPROACH

Overview

Determining the "proper" surplus required to support an insurance line of business is a difficult task. Traditionally, premium/surplus leverage has been viewed from a judgmental perspective as to what constitutes a safe operating level for the financial protection of policyholders. The following discussion sets forth an analytical framework and method for determining a benchmark surplus. The method provides a structure within which judgment and knowledge are used to provide assumptions on the magnitude and volatility of underwriting and investment cash flows. The method then develops the appropriate benchmark surplus and translates this into policyholder funds/surplus and premium/surplus leverage statistics.

The following subsection discusses the purpose of surplus and presents the concepts of funding and volatility along with a methodology which utilizes funding and volatility as the foundation to determine surplus needs. The determination of the amount of assets required to fund the liabilities of a line of business and the volatility in this measure jointly produce the required level of surplus.

Table 5 presents suggested benchmark leverage ratios, for both policyholder funds-to-surplus and premium-to-surplus. Average loss payment lag and amount of loss, both their value and variability, are the key parameters in constructing this table. Variability in factors other than loss payment lag and amount also need to be evaluated but are not presented here for the sake of simplicity, since their effect is generally much less than the loss-related parameters.

The method can be utilized to determine benchmark leverage standards by line of business which reflect that line's particular characteristics. These standards and an operating return figure can produce a return on surplus for measuring an insurance company's profitability by line of business and across lines of business.

TABLE 5

BENCHMARK LEVERAGE RATIOS
(BASED ON 1% PROBABILITY OF INSOLVENCY)

Average Loss Payment Years	Loss Ratio	Variability of 5% & 10% in Avg Loss Payment Date	Variability of 5% & 10% in Loss Ratio	Suggested Leverage Ratios to Surplus: Policyholder Funds	Suggested Leverage Ratios to Surplus: Premium
1	75.0	0.05	3.75	5.8	8.2
			7.50	3.8	5.3
		0.10	3.75	3.6	5.2
			7.50	3.0	4.3
2	75.0	0.10	3.75	6.5	4.6
			7.50	3.9	2.8
		0.20	3.75	4.0	2.8
			7.50	3.2	2.3
3	80.0	0.15	4.00	6.3	2.8
			8.00	3.9	1.7
		0.30	4.00	4.1	1.8
			8.00	3.2	1.4
4	80.0	0.20	4.00	6.6	2.2
			8.00	3.7	1.3
		0.40	4.00	4.0	1.3
			8.00	3.2	1.1

The Hartford has integrated this approach into its total return methodology. This methodology also uses the concept of discounted operating return, the principles of asset/liability matching, and the assumption of "risk free" Treasury investment policies to further manage solvency risk and protect policyholder funds. An earlier paper [2] presented this methodology.

Risk and the Need for Surplus

Insolvency is the ultimate business risk. In an insurance company, the sources of this risk are the insurance operations and investment activities. *Insurance risk* has two dimensions, since it arises from both the activities of underwriting and the investing of underwriting cash flows. However, insurance risk is principally a function of underwriting, provided underwriting cash flows are invested at a "risk free" rate and the maturities of the investments match the duration of the liabilities. This restriction essentially isolates total operating income from the effects of investment policy and market volatility.

Investment risk, on the other hand, is a function of company investment policy concerning types of investments and maturities, which gives rise to yield and default risks and related volatility.

Solvency risk is the exposure of surplus to both insurance (underwriting) and investment risk. The magnitude and volatility of underwriting losses along with fluctuating investment results with their associated probabilities are key determinants of this risk.

An important aspect of the management of solvency risk lies in determining the proper minimum level of surplus. Surplus should be a function of two factors:

1) *The degree and magnitude of financial exposure.* This essentially is the amount and length of time over which funds are committed to pay the liabilities of a respective line of business. It is the funding requirement.
2) *The volatility in the funding requirement.* The variability in underwriting and investment create the risk that increased surplus may be required to maintain a low probability of insolvency in the face of increased volatility.

In summary, *the surplus associated with a line of business is a buffer whose minimum size is determined by both the magnitude and volatility of financial exposure inherent in the line in order to insure an acceptably low probability of ruin.*

Determining Benchmark Surplus

The method developed begins with a determination of funding requirements by line of business. Funding is the amount of assets that are needed to pay the liabilities at a particular level of business volume. Specifically, it is the present value equivalent in assets required to meet the liabilities inherent in all expected future cash flows. It is based on the magnitude of the cash flows and the length of time that it takes to settle them, summed across all flows after discounting to present value.

The five basic insurance cash flow components considered are: Premium receipts, loss and expense payments, and prepayment of Federal taxes due to both loss discounting and the 20% unearned premium offset. These latter two components are creations of the 1986 Tax Reform Act.

Summing the required funding across all lines of business results in the total invested assets that must be committed by a company to support all writings.

This funding provides a beginning point to establish leverage, as it provides a measure of the liability-based asset commitment when writing a line of business. The exhibits provide formulae for approximating this funding level. Exact determination of funding requires the development of multi-period balance sheets for the full period during which cash flows occur.

The next step is to set surplus initially for each line of business in direct proportion to the line's funding requirements (i.e., money at risk). If the timing and magnitude of future operating flows were known with a high degree of certainty, a line would require only a small amount of surplus. However, most insurance flows are in the future and are uncertain as to timing and magnitude, and financial volatility can be expected. (In this regard, insurance differs substantially from banking and other financial services.) This means that a line will require a larger buffer to make provision for adverse future operating flows as uncertainty increases. The degree of this cushion clearly differs among lines of business.

Further adjustment, then, is necessary to recognize the financial volatility that exists in each line of business. Characteristics such as catastrophes which introduce much of this volatility, must be reflected in the methodology employed to determine a final benchmark leverage.

As can be seen, the formula for funding involves several parameters which are subject to variability. It is the handling of the variability associated with these parameters which is the key to determination of benchmark surplus. The parameters upon which funding is based are:

- premium amount and timing of collection;
- expense amount and timing of payment;
- loss amount and timing of payment;
- tax law loss discount factor and timing;
- proportion of premium unearned at year end;
- market interest rate; and
- tax rate.

Model Simulation

The dominant factors in terms of variability typically are the magnitude of loss amounts and the timing of loss payments. The variability in all other factors, for most lines of business, has a relatively minor effect by comparison. Paid loss retrospectively rated business is a notable exception, where the longer time period over which premium flows occur becomes a consideration. A simulation model was developed to measure the volatility in total funding in the absence of an analytical algorithm which could directly quantify it.

Table 5 presents a range of suggested benchmark leverage ratios (both policyholder funds and premium in ratio to surplus) as a function of loss payment date and amount of loss, taking into account both their value and variability, corresponding to a 1% probability of insolvency. This table was developed by the simulation model utilizing the funding formula with iterative options on loss payout (1, 2, 3, and 4 years), loss ratio (75% and 80%), variability of payout (5% and

10% of payment date), and variability of loss ratio (5% and 10% of loss ratio).

The figures in the table assume an expense ratio of 30%, interest rate before tax of 8%, tax law discount rate of 8%, and no delay in premium collection or expense payment.

The variability measures for the loss payment date and amount of loss are the respective standard deviations in those parameters. Since we are dealing with book of business averages, the normal distribution was assumed for simplicity of simulation. The total variability in funding was calculated from the simulated results. A Z value of 2.33 from the normal probability curve was used to determine the amount of surplus required to cover this probability-based maximum funding requirement. In other words, required surplus is calculated as Z times the standard deviation of funding, derived through simulation.

Table 5 as presented only demonstrates approximate possible leverage ratios. To more accurately determine the required benchmark, the simulation should be performed with all parameters specified more precisely: The expense ratio, interest rate, and timing of premium and expense flows for the line of business in question. In addition, the variability (i.e., standard deviation) of a line of business's average payout and loss ratio must be provided based on historical experience and judgment as to business expectations.

Policyholder funds in ratio to surplus is the more meaningful leverage statistic, although the premium-to-surplus ratio is the traditional leverage statistic. As the figures in this table demonstrate, the premium-to-surplus ratio covers a more extreme range, because surplus itself does not directly relate to premium. Premium, for example, does not capture the dynamics of a long tail line of business and its generally greater need for surplus.

The policyholder funds-to-surplus ratio provides a more meaningful measure of leverage, since surplus does relate to policyholder liabilities. The variability in this statistic in the table is a function of the variability levels simulated. If the variability were the same in all cases, the policyholder funds to surplus leverage statistic would remain constant, regardless of the magnitude of loss or the length of its payout.

Surplus Run-off

Expressing required surplus in relation to premium via a premium-to-surplus ratio is a convenience. Use of this ratio must not hide the fact that, while the premium flows generally span a single year, the requirements for surplus exist throughout the entire run-off period for the policy cash flows, however long that may be. In other words, the need for benchmark surplus remains beyond the year that the business is written.

It is suggested that surplus committed to support business be allowed to run off in proportion to the reduction in funding over time. In much the same way that funding is the present-valued assets corresponding to future cash flows, which declines over time, required surplus should be viewed as the related present-valued assets which run off in a parallel fashion. Since loss reserves are typically the primary component of this liability funding requirement, in simpler terms this says that surplus should run off as loss reserves decline to zero.

The convenience and simplicity of the premium-to-surplus ratio encourages its widespread use. Unfortunately, it also leads to its misuse as a means of surplus allocation. A reserve-to-surplus ratio would be a far more meaningful leverage statistic than premium-to-surplus, and it would provide a more intuitive means to allocate surplus.

The method demonstrated here using average payment dates is intended to provide an estimate of normal initial surplus requirements. Insurance programs having an atypical cash flow pattern may require a more detailed cash flow model to estimate the surplus requirements over time.

In addition, the independent determination of required surplus for each of a multi-line insurer's lines of business will produce a total across all lines greater than necessary, since any line may draw on the surplus of other lines in an emergency. A multi-line insurer could, in effect, write at a higher overall leverage. The degree of truth in this depends on several factors, including the correlation in exposure to loss among lines being written.

Conclusion

This paper has discussed the role of surplus in an insurance company, measures of rate of return, and considerations which are important in the determination of a benchmark surplus requirement for a line of business.

Of particular importance is the multi-year dimension to surplus through its linkage to liabilities. Balance sheet development triangles were introduced to reinforce this concept, to demonstrate the conditions for equivalency of NPV and IRR measures of return, and to show the effects of growth and independent surplus withdrawal on calendar versus accident period rates of return.

REFERENCES

[1] Cummins, J. David, "Multi-Period Discounted Cash Flow Models in Property-Liability Insurance," *Journal of Risk and Insurance,* March, 1990.

[2] Bingham, Russell E., "Discounted Return—Measuring Profitability and Setting Targets," *PCAS* LXXVII, 1990, p. 124.

[3] Bunner, Bruce and Wasserman, David, "The Dynamics of Risk and Return Under California's Proposition 103," *Underwriter's Report,* June 15, 1989.

EXHIBIT 1

ANNUALIZED NOMINAL (FUTURE VALUE) AND DISCOUNTED (PRESENT VALUE) BALANCE SHEET AND INVESTMENT INCOME FORMULAE

APPROXIMATION FORMULAE

			Balance Sheet		Investment Income	
Committed Assets	Initial Reported Amount	Years Pay Lag	NOMINAL	DISCOUNTED Beginning of Period	NOMINAL	DISCOUNTED Beginning of Period
Premium	P	N_p	$-N_pP$	$-PD\,[N_p]/R$	$-RN_pP$	$-PD\,[N_p]$
Expense	E	N_e	N_eE	$ED\,[N_e]/R$	RN_eE	$ED\,[N_e]$
Loss	L	N_l	N_lL	$LD\,[N_l]/R$	RN_lL	$LD\,[N_l]$
Net Policyholder Funds			Sum 1	Sum 2		
Tax Law Timing Items:						
Loss Discounting			ZL/R	KL/R	ZL	KL
UPR Offset			$-0.2TPU$	$-0.2TPUD\,[1]/R$	$-0.2RTPU$	$-0.2TPUD\,[1]$
Net Timing Items			Sum 3	Sum 4		
Net Funding (including taxes)			Sum 5 = (Sum 1 + Sum 3)	Sum 6 = (Sum 2 + Sum 4)		
Contributed Surplus			(Sum 1)/M	(Sum 2)/M		

Where:

$D\,[N] = 1 - 1/(1+R)^N$
= discount factor

R = interest rate, applicable to cash flows, after tax

T = corporate tax rate, presently 34%

$Z = -RT\,[(N_l+1)/2]\,[1 - 1/(1+(R_t)^{N_l}]$
= approximate loss discount nominal investment income factor

R_t = tax law discount rate

K = loss discount investment credit factor from Exhibit 3

M = policyholder liability/shareholder surplus leverage multiple

EXHIBIT 2
General Definitions and Formulae

Underwriting Income $= (P - E - L)(1 - T)$, where

$P =$ Premium

$E =$ Expense

$L =$ Loss

$T =$ Tax Rate.

Nominal Basis

Operating Return = Underwriting Income + Investment Income on Insurance Liabilities.

Total Return = Operating Return + Investment Income on Surplus.

Discounted Basis

Operating Return = Underwriting Income + Investment Income Credit on Insurance Float.

Investment Income Credit (IIC) = Present value of investment income on all cash flows related to the accident period.

Premium IIC $= -(1 - D_p)\,P$

Expense IIC $= (1 - D_e)\,E$

Loss IIC $= (1 - D_l)\,L$

UPR Tax IIC $= -(1 - D_u)\,(0.2T)\,PU$

Disc Tax IIC: See Exhibit 3 for formula

where:

$D = 1/(1+R)^N$, i.e. discount factor

$R =$ rate for calculating discount after tax

$R_b =$ tax law discount rate before tax

$N =$ average payment date for premium, expense, or loss; for D_u, $N = 1$, UPR tax recovery payment date

$U =$ Annual premium year–end unearned factor (i.e., unearned premium/premium)

All dollar figures and discount factors are after tax except discount factor for loss discounting using R_b, the tax law discount rate.

EXHIBIT 3

Loss Discounting Investment Income Credit Factor (Factor times Loss for Dollar Impact)

Approximation Formula

1) Actual and Law Rates and Payouts Same

$$-\{(D_b - D_a) + T(1 - D_b)\}, \text{ where}$$

$D = 1/(1 + R)^N$, i.e., discount factor
R = rate for calculating discount
N = payment date
b = before tax
a = after tax
T = tax rate
$D_a = 1/(1 + R_a)^N$
$R_a = (1 - T)R_b$

2) Actual and Law Rates Different, Payouts Same

$$-\{(D_{r'b} - D_a) + T(1 - D_{r'b})\} + (D_{r'b} - D_a)(R_a - R'_a)/(R_a - R'_b)$$

(Rate Adjustment)

where ′ signifies using law rate.

3) Actual and Law Rates and Payouts Different

$$-\{(D_{n'r'b} - D_{n'a}) + T(1 - D_{n'r'b})\} + (D_{n'r'b} - D_{n'a})(R_a - R'_a)/(R_a - R'_b)$$

(Rate Adjustment)

$$+TD_a[(1 - D_{n''r'b}) - (D_{n''r'b} - D_{n''a})R'_b/(R_a - R'_b)]$$

(Date Adjustment)

where ′ signifies using law rate or payment date and
$n'' = n' - n$, i.e., difference in payment date

The effect of different rates is greater than that of payout differences, and Formula 2 is sufficiently accurate for most applications.

An approximate formula for the above is

$$-T\{(1 - D_{mra}) \times (1 - D_{n'r'b})\}, \text{ where } m = (n + 1)/2$$

$$= -T\{(1 - 1/(1 + R_a)^m) \times (1 - 1/(1 + R'_b)^{n'})\}$$

APPENDIX A

BASIC ASSUMPTIONS AND CALCULATIONS
BASELINE — FOUR YEAR PAYOUT (25% PER YEAR)
AT 4:1 RESERVE/SURPLUS RATIO

Earned Premium	10,000.00
Expense Ratio	0.30
Loss Ratio	0.80
Underwriting Tax Rate	34.00%
Investment Yield Before Tax (BT)	8.00%
Investment Yield After Tax (AT)	5.28%
Tax Law Discount Rate	8.00%

		Year				
	Total	1	2	3	4	5
Loss Payment Sched Actual	100%	25%	25%	25%	25%	0%
Loss Payment Sched Law	100%	25%	25%	25%	25%	0%
Loss Payout by Law	8,000	2,000	2000	2,000	2,000	0
Discounted		1,852	1,715	1,588	1,470	0
Beginning Reserve Before Discount		8,000	6,000	4,000	2,000	0
Tax Law Timing Items BT						
Beginning Loss Discount	1,375	1,375				
Scheduled Recovery	-1,375	-530	-412	-285	-148	0
Begin UPR Subject to Tax	1,000	1,000				
Scheduled Recovery	-1,000	-1,000				
Reserves And Payments						
Beginning Nominal Loss Reserve		8,000	6,000	4,000	2,000	0
Loss Payments		2,000	2,000	2,000	2,000	0
Begin Loss Discount Tax Reserve		-468	-288	-147	-50	0
Loss Discount Tax Recovery		180	140	97	50	0
Begin UPR Tax Reserve		-340				
UPR Tax Recovery		340				
Shareholder Cap. Flows	Begin					
From Operating Earnings[1]		102	77	51	26	0
From Investment Income on Contributed Capital		106	79	53	26	0
Capital Withdrawal	-2,000	500	500	500	500	0
Contributed Capital[2]	-2,000	606	579	553	526	0
Net Capital Flows	-2,000	708	656	604	552	0

[1] Operating earnings withdrawal: Constant calendar ROS (AT)

[2] Contributed surplus withdrawal: Proportional to reserves plus investment income

APPENDIX B
Part 1

BALANCE SHEETS AND INCOME STATEMENTS
SINGLE ACCIDENT YEAR

BASELINE—FOUR YEAR PAYOUT (25% PER YEAR)
AT 4:1 RESERVE/SURPLUS RATIO

		Year				
Income Statement	Total	1	2	3	4	5
NOMINAL						
Income BT						
Underwriting Income	-1,000	-1,000	0	0	0	0
Investment Income						
Loss Reserve	1,600	640	480	320	160	0
Loss Disc Tax Reserve	-76	-37	-23	-12	-4	0
UPR Tax Reserve	-27	-27	0	0	0	0
Retained Earnings	-109	-53	-33	-17	-6	0
Surplus	400	160	120	80	40	0
Total Income BT	787	-317	544	371	190	0
NOMINAL						
Income AT						
Underwriting Income	-660	-660	0	0	0	0
Investment Income						
Loss Reserve	1,056	422	317	211	106	0
Loss Disc Tax Reserve	-50	-25	-15	-8	-3	0
UPR Tax Reserve	-18	-18	0	0	0	0
Retained Earnings	-72	-35	-22	-11	-4	0
Surplus	264	106	79	53	26	0
Total Income AT	520	-209	359	245	125	0
DISCOUNTED						
Income AT						
Underwriting Income	-660	-660	0	0	0	0
Investment Income						
Loss Reserve	954	401	286	181	86	0
Loss Disc Tax Reserve	-46	-23	-14	-7	-2	0
UPR Tax Reserve	-17	-17	0	0	0	0
Retained Earnings	-66	-33	-20	-10	-3	0
Surplus	238	100	71	45	21	0
Total Income AT	404	-232	324	210	102	0
Total Income (Excluding Retained Earnings)	469	-199	344	220	105	0

APPENDIX B
Part 2

BALANCE SHEETS AND INCOME STATEMENTS
SINGLE ACCIDENT YEAR

BASELINE—FOUR YEAR PAYOUT (25% PER YEAR)
AT 4:1 RESERVE/SURPLUS RATIO

	Year				
Balance Sheet	1	2	3	4	5
NOMINAL					
Beginning Assets	8,532	6,795	4,638	2,376	0
Liabilities					
Loss Reserve	8,000	6,000	4,000	2,000	0
Disc Tax Reserve	-468	-288	-147	-50	0
UPR Tax Reserve	-340	0	0	0	0
Surplus					
Retained Earnings	-660	-417	-214	-73	0
Contributed	2000	1,500	1,000	500	0
Liabilities + Surplus	8,532	6,795	4,638	2,376	0
DISCOUNTED					
Beginning Assets	8,104	6,131	3,975	1,934	0
Liabilities					
Loss Reserve	7,599	5,413	3,428	1,628	0
Disc Tax Reserve	-444	-259	-126	-41	0
UPR Tax Reserve	-323	0	0	0	0
Surplus					
Retained Earnings	-627	-377	-184	-60	0
Contributed	1,900	1,353	857	407	0
Liabilities + Surplus	8,104	6,131	3,975	1,934	0

APPENDIX C

Part 1

BALANCE SHEETS AND INCOME STATEMENTS STEADY STATE BASIS, FOUR YEARS

BASELINE—FOUR YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

	Year				
Income Statement	1	2	3	4	5
NOMINAL					
Income AT					
Underwriting	-660	-660	-660	-660	-660
Investment Income					
Reserves	422	739	950	1,056	1,056
Loss Disc Tax Reserve	-25	-40	-48	-50	-50
UPR Tax Reserve	-18	-18	-18	-18	-18
Retained Earnings	-35	-57	-68	-72	-72
Surplus	106	185	238	264	264
Total Income AT	-209	149	394	520	520
DISCOUNTED					
Income AT					
Nominal Underwriting	-660	-660	-660	-660	-660
Investment Income					
Loss Reserve	401	687	868	954	954
Loss Disc Tax Reserve	-23	-37	-44	-46	-46
UPR Tax Reserve	-17	-17	-17	-17	-17
Retained Earnings	-33	-53	-63	-66	-66
Surplus	100	172	217	238	238
Total Income AT	-232	92	301	404	404
Total Income (Excluding Retained Earnings)	-199	145	364	469	469

APPENDIX C

Part 2

BALANCE SHEETS AND INCOME STATEMENTS STEADY STATE BASIS, FOUR YEARS

BASELINE—FOUR YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

	Year				
Balance Sheet	1	2	3	4	5
NOMINAL					
Beginning Assets	8,532	15,327	19,965	22,342	22,342
Liabilities					
Loss Reserve	8,000	14,000	18,000	20,000	20,000
Disc Tax Reserve	-468	-755	-903	-953	-953
UPR Tax Reserve	-340	-340	-340	-340	-340
Surplus					
Retained Earnings	-660	-1,077	-1,292	-1,365	-1,365
Contributed	2,000	3,500	4,500	5,000	5,000
Liabilities + Surplus	8,532	15,327	19,965	22,342	22,342
DISCOUNTED					
Beginning Assets	8,104	14,235	18,210	20144	20,144
Liabilities					
Loss Reserve	7,599	13,012	16,440	18,068	18,068
Disc Tax Reserve	-444	-704	-830	-871	-871
UPR Tax Reserve	-323	-323	-323	-323	-323
Surplus					
Retained Earnings	-627	-1,003	-1,187	-1,247	-1,247
Contributed	1,900	3,253	4,110	4,517	4,517
Liabilities + Surplus	8,104	14,235	18,210	20,144	20,144
DISCOUNTED END OF YEAR VALUATION					
Beginning Assets—	8,532	14,987	19,171	21,207	21,207
Liabilities					
Loss Reserve	8,000	13,699	17,308	19,022	19,022
Disc Tax Reserve	-468	-741	-874	-917	-917
UPR Tax Reserve	-340	-340	-340	-340	-340
Surplus					
Retained Earnings	-660	-1,056	-1,250	-1,313	-1,313
Contributed	2,000	3,425	4,327	4,755	4,755
Liabilities + Surplus	8,532	14,987	19,171	21,207	21,207

APPENDIX D

POLICYHOLDER/SHAREHOLDER FUNDS

BASELINE—FOUR YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

	Beginning of Year				
	1	2	3	4	5
Single Accident Year					
NOMINAL					
Policyholder Funds	8,000	6,000	4,000	2,000	0
Shareholder Funds	2,000	1,500	1,000	500	0
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	
DISCOUNTED					
Policyholder Funds	7,599	5,413	3,428	1,628	0
Shareholder Funds	1,900	1,353	857	407	0
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	
DISCOUNTED END OF YEAR VALUATION					
Policyholder Funds	8,000	5,699	3,609	1,714	0
Shareholder Funds	2,000	1,425	902	428	0
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	
Steady State Basis, Four Years					
NOMINAL					
Policyholder Funds	8,000	14,000	18,000	20,000	20,000
Shareholder Funds	2,000	3,500	4,500	5000	5,000
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	4.00
DISCOUNTED					
Policyholder Funds	7,599	13,012	16,440	18,068	18,068
Shareholder Funds	1,900	3,253	4,110	4,517	4,517
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	4.00
DISCOUNTED END OF YEAR VALUATION					
Policyholder Funds	8,000	13,699	17,308	19,022	19,022
Shareholder Funds	2,000	3,425	4,327	4,755	4,755
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	4.00

APPENDIX E

Part 1

RATE OF RETURN TO SHAREHOLDER (INCOME DISTRIBUTED/BEGINNING SURPLUS)

SINGLE ACCIDENT YEAR

BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

		Year				
Shareholder Flows	Begin	1	2	3	4	IRR
Operating Earnings [1]	-231	102	77	51	26	5.3%
Contributed Surplus Account						
Investment Income		106	79	53	26	
Capital Withdrawal	-2,000	500	500	500	500	
Contributed Capital [2]	-2,000	606	579	553	526	5.3%
Net Shareholder Flows	-2,000	708	656	604	552	10.4%
Return						
(Operating and Investment Income)		10.4%	10.4%	10.4%	10.4%	

[1] Operating earnings withdrawal: constant calendar ROS (AT)

[2] Contributed surplus withdrawal: proportional to reserves plus investment income

APPENDIX E
Part 2

RATE OF RETURN TO SHAREHOLDER (INCOME DISTRIBUTED/BEGINNING SURPLUS)
STEADY STATE BASIS
BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

		Year											
Shareholder Flows	Begin	1	2	3	4	5	6	7	8	9	10	11	IRR
Operating Earnings[1]		102	179	230	256	256	256	256	256	153	77	26	
Contributed Surplus Account													
Investment Income		106	185	238	264	264	264	264	264	158	79	26	
Capital Withdrawal	-2,000	-1,500	-1,000	-500	0	0	0	0	2,000	1,500	1,000	500	
Contributed Capital[2]	-2,000	-1,394	-815	-262	264	264	264	264	2,264	1,658	1,079	526	5.3%
Net Shareholder Flows	-2,000	-1,292	-636	-32	520	520	520	520	2,520	1,812	1,156	552	10.4%
Return (Operating and Investment Income)		10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	

[1] Operating earnings withdrawal: constant calendar ROS (AT)

[2] Contributed surplus withdrawal: proportional to reserves plus investment income

APPENDIX E

Part 3

RATE OF RETURN TO SHAREHOLDER (INCOME DISTRIBUTED/BEGINNING SURPLUS)

STEADY STATE BASIS

BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

			DISCOUNTED			
			Beginning of Year		End of Year	
	NOMINAL	% of Surplus	Valuation	% of Surplus	Valuation	% of Surplus
Beginning Surplus	$5,000		$4,517		$4,755	
Underwriting Income	-660		-660		-695	
Investment Income	916		891		938	
Oper Inc Incl Ret Earns	256	5.3%	231	5.1%	243	5.1%
Investment Income on Surplus	264	5.3%	238	5.3%	251	5.3%
Total Net Income	520	10.4%	469	10.4%	494	10.4%

APPENDIX E

Part 4

RATE OF RETURN TO SHAREHOLDER (INCOME DISTRIBUTED/BEGINNING SURPLUS)

SINGLE ACCIDENT YEAR

FOUR-YEAR PAYOUT, WITHDRAW CAPITAL AFTER ONE YEAR PLUS CALENDAR INVESTMENT INCOME

		Year				
Shareholder Flows	Begin	1	2	3	4	IRR
Operating Earnings [1]	-231	-315	302	203	103	5.3%
Contributed Surplus Account						
Investment Income		264	0	0	0	
Capital Withdrawal	-5,000	5,000	0	0	0	
Contributed Capital [2]	-5,000	5,264	0	0	0	5.3%
Net Shareholder Flows	-5,000	4,949	302	203	103	9.5%
Return						
(Operating and Investment Income)		-1.0%	0.0%	0.0%	0.0%	

[1] Operating earnings withdrawal: calendar income (U/W + investment income)

[2] Contributed surplus withdrawal: after one year

APPENDIX E

Part 5

RATE OF RETURN TO SHAREHOLDER (INCOME DISTRIBUTED/BEGINNING SURPLUS)

STEADY STATE BASIS

FOUR-YEAR PAYOUT, WITHDRAW CAPITAL AFTER ONE YEAR PLUS CALENDAR INVESTMENT INCOME

		Year											
Shareholder Flows	Begin	1	2	3	4	5	6	7	8	9	10	11	IRR
Operating Earnings [1]		-315	-13	190	293	293	293	293	293	608	306	103	
Contributed Surplus Account													
Investment Income		264	264	264	264	264	264	264	264	0	0	0	
Capital Withdrawal	-5,000	0	0	0	0	0	0	0	5,000	0	0	0	
Contributed Capital [2]	-5,000	264	264	264	264	264	264	264	5,264	0	0	0	5.3%
Net Shareholder Flows	-5,000	-51	251	454	557	557	557	557	5,557	608	306	103	9.5%
Return (Operating and Investment Income)		1.0%	5.0%	9.1%	11.1%	11.1%	11.1%	11.1%	11.1%	0.0%	0.0%	0.0%	

[1] Operating earnings withdrawal: calendar income (U/W + investment income)

[2] Contributed surplus withdrawal: after one year

APPENDIX E
Part 6

RATE OF RETURN TO SHAREHOLDER (INCOME DISTRIBUTED/BEGINNING SURPLUS)
STEADY STATE BASIS
FOUR-YEAR PAYOUT, WITHDRAW CAPITAL AFTER ONE YEAR PLUS CALENDAR INVESTMENT INCOME

			DISCOUNTED			
			Beginning of Year		End of Year	
	NOMINAL	% of Surplus	Valuation	% of Surplus	Valuation	% of Surplus
Beginning Surplus	$ 5,000		$4,749		$5,000	
Underwriting Income	-660		-660		-695	
Investment Income	953		891		938	
Oper Inc Incl Ret Earns	293	5.9%	231	4.9%	243	4.9%
Investment Income on Surplus	264	5.3%	251	5.3%	264	5.3%
Total Net Income	557	11.1%	482	10.1%	507	10.1%

APPENDIX F

Part 1

ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR

NET INCOME

BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

Accident Year	Pres Value @Year End	Net Income in Year 1	2	3	4	5	6	7	8	9	10	11	Total	Acc Year Compound Growth
1	494	-209	359	245	125	0	0	0	0	0	0	0	520	0.0%
2	494		-209	359	245	125	0	0	0	0	0	0	520	0.0%
3	494			-209	359	245	125	0	0	0	0	0	520	0.0%
4	494				-209	359	245	125	0	0	0	0	520	0.0%
5	494					-209	359	245	125	0	0	0	520	0.0%
6	494						-209	359	245	125	0	0	520	0.0%
7	494							-209	359	245	125	0	520	0.0%
8	494								-209	359	245	125	520	0.0%
9	0									0	0	0	0	0.0%
10	0										0	0	0	0.0%
11	0											0	0	0.0%
Calendar Year		-209	150	395	520	520	520	520	520	729	370	125	4,160	

APPENDIX F

Part 2

ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR CONTRIBUTED SURPLUS

BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

Accident	Pres Value	Beginning Contributed Surplus in Year												Acc Year Compound
Year	@Year End	1	2	3	4	5	6	7	8	9	10	11	Total	Growth
1	4,755	2,000	1,500	1,000	500	0	0	0	0	0	0	0	5,000	0.0%
2	4,755		2,000	1,500	1,000	500	0	0	0	0	0	0	5,000	0.0%
3	4,755			2,000	1,500	1,000	500	0	0	0	0	0	5,000	0.0%
4	4,755				2,000	1,500	1,000	500	0	0	0	0	5,000	0.0%
5	4,755					2,000	1,500	1,000	500	0	0	0	5,000	0.0%
6	4,755						2,000	1,500	1,000	500	0	0	5,000	0.0%
7	4,755							2,000	1,500	1,000	500	0	5,000	0.0%
8	4,755								2,000	1,500	1,000	500	5,000	0.0%
9	0									0	0	0	0	0.0%
10	0										0	0	0	0.0%
11	0											0	0	0.0%
Calendar Year		2,000	3,500	4,500	5,000	5,000	5,000	5,000	5,000	3,000	1,500	500	40,000	

APPENDIX F
Part 3
ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR RETURN ON SURPLUS
BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

Accident	Pres Value	ROS (Net Income/Beginning Period Contributed Surplus) in Year											
Year	@Year End	1	2	3	4	5	6	7	8	9	10	11	Total
1	10.4%	-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%
2	10.4%		-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%
3	10.4%			-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%
4	10.4%				-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	0.0%	10.4%
5	10.4%					-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	10.4%
6	10.4%						-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	10.4%
7	10.4%							-10.5%	23.9%	24.5%	25.1%	0.0%	10.4%
8	10.4%								-10.5%	23.9%	24.5%	25.1%	10.4%
9	0.0%									0.0%	0.0%	0.0%	0.0%
10	0.0%										0.0%	0.0%	0.0%
11	0.0%											0.0%	0.0%
Calendar Year		-10.5%	4.3%	8.8%	10.4%	10.4%	10.4%	10.4%	10.4%	24.3%	24.7%	25.1%	10.4%

APPENDIX F
Part 4

ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR NET INCOME

BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO, 10% ANNUAL GROWTH

Accident Year	Pres Value @Year End	Net Income in Year 1	2	3	4	5	6	7	8	9	10	11	Total	Acc Year Compound Growth
1	494	-209	359	245	125	0	0	0	0	0	0	0	520	0.0%
2	544		-230	395	269	138	0	0	0	0	0	0	572	10.0%
3	598			-253	434	296	152	0	0	0	0	0	629	21.0%
4	658				-279	478	326	167	0	0	0	0	692	33.1%
5	724					-307	525	359	184	0	0	0	761	46.3%
6	796						-337	578	394	202	0	0	837	61.0%
7	876							-371	636	434	222	0	921	77.1%
8	963								-408	699	477	244	1,012	94.6%
9	0									0	0	0	0	0.0%
10	0										0	0	0	0.0%
11	0											0	0	0.0%
Calendar Year		-209	129	387	549	605	666	733	806	1,335	699	244	5,944	

APPENDIX F

Part 5

ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR CONTRIBUTED SURPLUS

BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO, 10% ANNUAL GROWTH

Accident Year	Pres Value @Year End	Beginning Contributed Surplus in Year 1	2	3	4	5	6	7	8	9	10	11	Total	Acc Year Compound Growth
1	4,755	2,000	1,500	1,000	500	-0	0	0	0	0	0	0	5,000	0.0%
2	5,231		2,200	1,650	1,100	550	-0	0	0	0	0	0	5,500	10.0%
3	5,754			2,420	1,815	1,210	605	-0	0	0	0	0	6,050	21.0%
4	6,330				2,662	1,997	1,331	666	-0	0	0	0	6,656	33.1%
5	6,962					2,928	2,196	1,464	732	-0	0	0	7,320	46.4%
6	7,659						3,221	2,416	1,611	805	-0	0	8,053	61.1%
7	8,425							3,543	2,657	1,772	886	-0	8,858	77.2%
8	9,267								3,897	2,923	1,949	974	9,743	94.9%
9	0									0	0	0	0	0.0%
10	0										0	0	0	0.0%
11	0											0	0	0.0%
Calendar Year		2,000	3,700	5,070	6,077	6,685	7,353	8,089	8,897	5,500	2,835	974	57,180	

APPENDIX F

Part 6

ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR RETURN ON SURPLUS

BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO, 10% ANNUAL GROWTH

Accident	Pres Value	ROS (Net Income/Beginning Period Contributed Surplus) in Year											
Year	@Year End	1	2	3	4	5	6	7	8	9	10	11	Total
1	10.4%	-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%
2	10.4%		-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%
3	10.4%			-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%
4	10.4%				-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	0.0%	10.4%
5	10.4%					-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	0.0%	10.4%
6	10.4%						-10.5%	23.9%	24.5%	25.1%	0.0%	0.0%	10.4%
7	10.4%							-10.5%	23.9%	24.5%	25.1%	0.0%	10.4%
8	10.4%								-10.5%	23.9%	24.5%	25.1%	10.4%
9	0.0%									0.0%	0.0%	0.0%	0.0%
10	0.0%										0.0%	0.0%	0.0%
11	0.0%											0.0%	0.0%
Calendar Year		-10.5%	3.5%	7.6%	9.1%	9.1%	9.1%	9.1%	9.1%	24.3%	24.7%	25.1%	10.4%

APPENDIX F

Part 7

ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR NET INCOME

FOUR-YEAR PAYOUT, WITHDRAW CAPITAL AFTER ONE YEAR PLUS CALENDAR INVESTMENT INCOME

Accident Year	Pres Value @Year End	Net Income in Year 1	2	3	4	5	6	7	8	9	10	11	Total	Acc Year Compound Growth
1	507	-51	302	203	103	0	0	0	0	0	0	0	557	0.0%
2	507		-51	302	203	103	0	0	0	0	0	0	557	0.0%
3	507			-51	302	203	103	0	0	0	0	0	557	0.0%
4	507				-51	302	203	103	0	0	0	0	557	0.0%
5	507					-51	302	203	103	0	0	0	557	0.0%
6	507						-51	302	203	103	0	0	557	0.0%
7	507							-51	302	203	103	0	557	0.0%
8	507								-51	302	203	103	557	0.0%
9	0									0	0	0	0	0.0%
10	0										0	0	0	0.0%
11	0											0	0	0.0%
Calendar Year		-51	251	454	557	557	557	557	557	608	306	103	4,456	

APPENDIX F
Part 8

ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR CONTRIBUTED SURPLUS

FOUR-YEAR PAYOUT, WITHDRAW CAPITAL AFTER ONE YEAR PLUS CALENDAR INVESTMENT INCOME

Accident Year	Pres Value @Year End	Beginning Contributed Surplus in Year 1	2	3	4	5	6	7	8	9	10	11	Total	Acc Year Compound Growth
1	5,000	5,000	0	0	0	0	0	0	0	0	0	0	5,000	0.0%
2	5,000		5,000	0	0	0	0	0	0	0	0	0	5,000	0.0%
3	5,000			5,000	0	0	0	0	0	0	0	0	5,000	0.0%
4	5,000				5,000	0	0	0	0	0	0	0	5,000	0.0%
5	5,000					5,000	0	0	0	0	0	0	5,000	0.0%
6	5,000						5,000	0	0	0	0	0	5,000	0.0%
7	5,000							5,000	0	0	0	0	5,000	0.0%
8	5,000								5,000	0	0	0	5,000	0.0%
9	0									0	0	0	0	0.0%
10	0										0	0	0	0.0%
11	0											0	0	0.0%
Calendar Year		5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	0	0	0	40,000	

APPENDIX F
Part 9
ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR
RETURN ON SURPLUS
FOUR-YEAR PAYOUT, WITHDRAW CAPITAL AFTER ONE YEAR PLUS CALENDAR INVESTMENT INCOME

Accident Year	Pres Value @Year End	ROS (Net Income/Beginning Period Contributed Surplus) in Year 1	2	3	4	5	6	7	8	9	10	11	Total
1	10.1%	-1.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.1%
2	10.1%		-1.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.1%
3	10.1%			-1.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.1%
4	10.1%				-1.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.1%
5	10.1%					-1.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.1%
6	10.1%						-1.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.1%
7	10.1%							-1.0%	0.0%	0.0%	0.0%	0.0%	11.1%
8	10.1%								-1.0%	0.0%	0.0%	0.0%	11.1%
9	0.0%									0.0%	0.0%	0.0%	0.0%
10	0.0%										0.0%	0.0%	0.0%
11	0.0%											0.0%	0.0%
Calendar Year		-1.0%	5.0%	9.1%	11.1%	11.1%	11.1%	11.1%	11.1%	0.0%	0.0%	0.0%	11.1%

APPENDIX G
Part 1

ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR SHAREHOLDER FLOWS FROM CAPITAL
BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

Accident Year	Begin	Shareholder Flows From Capital (Contribution) Or Withdrawal in Year 1	2	3	4	5	6	7	8	9	10	11	Total	Acc Year Compound Growth
1	-2,000	500	500	500	500	0	0	0	0	0	0	0	0	0.0%
2		-2,000	500	500	500	500	0	0	0	0	0	0	0	0.0%
3			-2,000	500	500	500	500	0	0	0	0	0	0	0.0%
4				-2,000	500	500	500	500	0	0	0	0	0	0.0%
5					-2,000	500	500	500	500	0	0	0	0	0.0%
6						-2,000	500	500	500	500	0	0	0	0.0%
7							-2,000	500	500	500	500	0	0	0.0%
8								-2,000	500	500	500	500	0	0.0%
9									0	0	0	0	0	0.0%
10										0	0	0	0	0.0%
11											0	0	0	0.0%
Calendar Year	-2,000	-1,500	-1,000	-500	0	0	0	0	2,000	1,500	1,000	500	0	

APPENDIX G

Part 2

ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR NET SHAREHOLDER FLOWS

BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

Accident Year	Begin	Net Shareholder Flows in Year 1	2	3	4	5	6	7	8	9	10	11	Total	Acc Year Compound Growth
1	-2,000	708	656	604	552	0	0	0	0	0	0	0	520	0.0%
2		-2,000	708	656	604	552	0	0	0	0	0	0	520	0.0%
3			-2,000	708	656	604	552	0	0	0	0	0	520	0.0%
4				-2,000	708	656	604	552	0	0	0	0	520	0.0%
5					-2,000	708	656	604	552	0	0	0	520	0.0%
6						-2,000	708	656	604	552	0	0	520	0.0%
7							-2,000	708	656	604	552	0	520	0.0%
8								-2,000	708	656	604	552	520	0.0%
9									0	0	0	0	0	0.0%
10										0	0	0	0	0.0%
11											0	0	0	0.0%
Calendar Year	-2,000	-1,292	-636	-32	520	520	520	520	2,520	1,812	1,156	552	4,160	

APPENDIX G
Part 3
ACCIDENT YEAR DEVELOPMENT AND CONTRIBUTION TO CALENDAR YEAR SHAREHOLDER RETURN
BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

Accident	Shareholder Return (Operating & Investment Income/Beginning Period Capital Contribution) in Year											
Year	1	2	3	4	5	6	7	8	9	10	11	IRR
1	10.4%	10.4%	10.4%	10.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%
2		10.4%	10.4%	10.4%	10.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%
3			10.4%	10.4%	10.4%	10.4%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%
4				10.4%	10.4%	10.4%	10.4%	0.0%	0.0%	0.0%	0.0%	10.4%
5					10.4%	10.4%	10.4%	10.4%	0.0%	0.0%	0.0%	10.4%
6						10.4%	10.4%	10.4%	10.4%	0.0%	0.0%	10.4%
7							10.4%	10.4%	10.4%	10.4%	0.0%	10.4%
8								10.4%	10.4%	10.4%	10.4%	10.4%
9									0.0%	0.0%	0.0%	0.0%
10										0.0%	0.0%	0.0%
11											0.0%	0.0%
Calendar Year	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%

APPENDIX H

Part 1

ANNUALIZED NOMINAL AND DISCOUNTED BALANCE SHEET AND INVESTMENT INCOME
5.3% DISCOUNT RATE, 5.3% EARNINGS RATE, 8.0% TAX LAW DISCOUNT
BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

				Balance Sheet			Investment Income			Duration	
					DISCOUNTED			DISCOUNTED			
Committed Assets = Liabilities	Initial Reported Amount	DISCOUNTED Beginning of Period	Average Timing of Cash Flow	NOMINAL	Begin Period	End Period	NOMINAL	Begin Period	End Period	Begin Period	End Period
Premium	$10,000	$10,000	0.00	0	0	0	0	0	0	0.00	0.00
Loss & Loss Expense	8,000	7,046	2.50	20,000	18,068	19,022	1,056	954	1,004	1.97	0.97
Underwriting Expense	3,000	3,000	0.00	0	0	0	0	0	0	0.00	0.00
Net Policyholder Funds				20,000	18,068	19,022	1,056	954	1,004		
Tax Timing Items											
Tax Loss Discounting	-468	-422	2.04	-953	-871	-917	-50	-46	-48	1.75	0.75
Tax Unearned Premium	-340	-323	1.00	-340	-323	-340	-18	-17	-18	1.00	0.00
Net Liabilities (Including Timing Items)				18,707	16,874	17,765	988	891	938	2.00	1.00
Retained Earnings				-1,365	-1,247	-1,313	-72	-66	-69		
Net Liabilities (Including Retained Earnings)				17,342	15,627	16,452	916	825	869		
Contributed Surplus	2,000	1,762	2.50	5,000	4,517	4,755	264	238	251	1.97	0.97

APPENDIX H
Part 2

ANNUALIZED NOMINAL AND DISCOUNTED RETURN
5.3% DISCOUNT RATE, 5.3% EARNINGS RATE, 8.0% TAX LAW DISCOUNT
BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

	Balance Sheet			Investment Income			Duration	
		DISCOUNTED			DISCOUNTED			
	NOMINAL	Begin Period	End Period	NOMINAL	Begin Period	End Period	Begin Period	End Period
Premium [1]	$11,069	$10,000	$10,528				1.97	0.97
Underwriting Income				-660	-660	-695	0.00	-1.00
Operating Income				256	231	243	1.97	0.97
Operating Return on Premium (ROP)				2.3%	2.3%	2.3%		
Operating Return on Net Liabilities (ROL)				1.5%	1.5%	1.5%		
Total Net Income				520	469	494	1.97	0.97
Total Return on Surplus (ROS)				10.4%	10.4%	10.4%		

[1] Nominal valued at date of average total timing

APPENDIX H

Part 3

ANNUALIZED NOMINAL AND DISCOUNTED LEVERAGE RATIOS
5.3% DISCOUNT RATE, 5.3% EARNINGS RATE, 8.0% TAX LAW DISCOUNT
BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

	Balance Sheet		
		DISCOUNTED	
	NOMINAL	Begin Period	End Period
Net Policyholder Funds/Surplus	4.00	4.00	4.00
Net Liabilities (Incl Timing Items)/Surplus	3.74	3.74	3.74
Net Liabilities (Incl Retained Earnings)/Surplus	3.47	3.46	3.46
Premium/Surplus	2.21	2.21	2.21
Conventional Nominal Reported Premium/Surplus	2.00	N/A	N/A

RATE OF RETURN—POLICYHOLDER, COMPANY, AND SHAREHOLDER PERSPECTIVES

RUSSELL E. BINGHAM

Abstract

This paper discusses rate of return from the policyholder, company, and shareholder perspectives. Both net present value (NPV) and internal rate of return (IRR) variations of discounted cash flow models are used to demonstrate two important considerations: The relationship between policyholder liabilities and surplus, and the release of income and return of surplus to the shareholder. A method for determining the "insurance risk charge" implicit in insurance transactions is also presented.

To render the concepts concrete and to provide a simple yet accurate method of calculating total return in most instances, the paper presents a spreadsheet model embodying all of the principles discussed. This simplified model provides an adequate basis for measuring total return for both profitability tracking and ratemaking purposes.

1. OVERVIEW

The insurance business can be viewed as a partnership among three parties—an insurance company, its policyholders, and its shareholders. In its management practices, an insurer can address the interests of its policyholders and shareholders by separately acknowledging the risks and returns that are appropriate to each of the parties within a total return framework. Total return means that income from all sources is included—from underwriting and the investment income on policyholder supplied funds and shareholder surplus.

An insurer can use a total return approach both retrospectively, to measure and analyze actual financial performance, and prospectively, in ratemaking and planning. A rate of return framework provides a consistent and most easily understood basis for reconciling these three viewpoints.

This paper discusses rate of return from the policyholder, company, and shareholder perspectives. In addition, it will present a framework utilizing discounted cash flow, both the net present value (NPV) and internal rate of return (IRR) variations, to demonstrate two important considerations: The relationship between liabilities and surplus, and the release of income and return of surplus to the shareholder.

The paper discusses rates of return applicable to the policyholder, company, and shareholder from both a balance sheet and a cash flow perspective and presents a means of determining the "insurance risk charge" implicit in insurance transactions. This charge will be explained, and the paper will demonstrate the charge's applicability to both short and long tail lines of business.

To render the concepts concrete and to provide a simple yet accurate method of calculating total return in most instances, the paper presents a spreadsheet model embodying all of the principles discussed. This simplified model provides an adequate basis for measuring total return for both profitability tracking and ratemaking purposes.

An earlier paper [1] by this author discussed several conceptual and financial aspects pertaining to surplus, including: The role of surplus in an insurance company, rates of return on surplus, and a volatility-adjusted funding approach to determining surplus requirements. Since the numerical example employed in that paper will be used here, the Appendix repeats the example in detail.

2. THE BALANCE SHEET PERSPECTIVE

The concept of "rate of return" defines the relationship between the income from an investment and the magnitude or value of the investment. An investment is a balance sheet concept, and this important fact will be explored in the following section by considering the balance sheet associated with insurance transactions.

Discounted cash flow (DCF) models often do not explicitly consider the balance sheet implied by a particular income and cash flow stream, yet this balance sheet perspective is important in order to understand the rate of return inherent in such flows. In addition, DCF models applied to insurance (other than a few IRR applications) often limit their scope of concern to insurance operations, that is to say, involving only transactions between the company and its policyholders. Total return implies by its name that the cash flows (and balance sheet implications) relating to surplus also must be included. This balance sheet perspective, inclusive of surplus, is provided in the example in the Appendix. The example follows regular accounting rules to create a nominally valued (i.e., not discounted) balance sheet along with income and cash flow statements.

Net present value (NPV) models restate the nominal income and cash flow values to present value by the process of discounting. A present-valued balance sheet is determined in the same manner—by discounting each future year's nominal balance sheet and summing to the present.

The NPV measurement of return presented here ratios the present value of all income streams—both underwriting and investment—to the present value of surplus committed. In effect, the process constructs a balance sheet that represents the annualized present value sum of individual future calendar period balance sheets for the accident period being evaluated. The process then discounts to the present the balance sheets for future years and sums them. This "annualized equivalent balance sheet" provides the vehicle through which a rate of return can be calculated.

An alternate way of presenting this concept is to consider the nominal (i.e., undiscounted) income and cash flow statements and balance sheet that would exist if business were written at a steady state (i.e., no growth). A financial steady state (i.e., static balance sheet, income, and cash flow statements) occurs after some amount of time when identical business is written in successive years without growth. This amount of time is determined by how long it takes for the last financial activity of an accident year to occur. In the case shown in the Appendix, for example, it takes four years to reach steady state, since the last activity (the final loss payment) occurs in year four. The annualized equivalent balance sheet is identical to the nominal steady state balance sheet produced in this manner, when discounted to the starting point in time. This steady state view is not an assumption of the total return approach, but is presented simply as an alternate means of explanation.

The annualized equivalent balance sheet provides a measure of the net annualized liability commitment that is made when business is written. A simple example may help explain this concept. If business were to be written at the beginning of a year with an expected loss payable in one year of $20,000, and all other cash flows were settled at inception of the policy, the balance sheet would reflect a loss reserve liability of $20,000 for the full year. On the other hand, if we expected a loss payable in two and one-half (2.5) years of $8,000, the balance sheet would reflect a loss reserve liability of $8,000 for 2.5 years. This is the annual financial equivalent of $20,000. Alternately, if this business were written for several years, the balance sheet would show a loss reserve liability of $20,000. In either case, the company is committed to a one year financial liability equivalent (not present valued) of $20,000.

Returning to the example, Table 1 demonstrates the annual components of both an ongoing, steady state nominal income and balance sheet and a discounted income and balance sheet. It displays income on an after-tax basis and all values discounted are valued at the beginning of the accident year. For example, the ongoing "Balance Sheet Liability" reflects steady state loss reserves of $20,000 on a nominal

TABLE 1

NOMINAL AND DISCOUNTED INCOME, BALANCE SHEET, AND RATE OF RETURN

	Income After Tax		Balance Sheet Liability		
Total Income Components	Nominal Future Value	Discounted Present Value	Nominal Future Value	Discounted Present Value	Rate of Return (After Tax)
INSURANCE OPERATIONS:					
Underwriting	-$660	-$660	$17,342	$17,342	-3.8% (Cost of PH Funds)
Policyholder Liabilities					
Loss & Loss Expense	1,056	954	20,000	18,068	5.3%
Tax Loss Discounting	-50	-46	-953	-871	5.3%
Tax Unearned Premium	-18	-17	-340	-323	5.3%
Net PH Liabilities	988	891	18,707	16,874	5.3%
Retained Earnings	-72	N/A	-1,365	-1,247	5.3%
TOTAL OPERATING INCOME	256	231	17,342	15,627	1.5% (Risk Charge)
SHAREHOLDER SURPLUS:					
Contributed Surplus	264	238	5,000	4,517	5.3%
TOTAL INCOME	$520	$469			10.4% (ROS)

basis and $18,068 on a discounted basis. The nominal total balance sheet liability consists of net liabilities of $18,707 and surplus of $5,000. The surplus commitment of $2,000, $1,500, $1,000, and $500 for years one through four, respectively, equates to an ongoing commitment at steady state of $5,000. The Appendix describes this example in more detail.

The corresponding discounted balance sheet values are liabilities of $16,874 and surplus of $4,517. This means that the annual equivalent of this amount must be set aside today to fund future liabilities and provide the desired surplus support throughout the four year period.

The NPV investment income credit is $891 on the $16,874 in invested assets corresponding to policyholder liabilities and $238 on the $4,517 in surplus assets.

The surplus commitment is $4,517 in present value terms. This can be thought of as the one-year, annualized asset commitment that equates to the actual commitment of assets over the four year period. The level of this asset commitment is a function of both the magnitude of the cash flow balances and the amount of time over which these cash flows and balances exist.

In short, *the funding commitment is the present valued balance sheet asset commitment dictated by cash flows.* This asset commitment also represents the asset earnings base upon which the credit for future investment income is based. The annualized investment income figure is the same as the present value of the investment income stream derived from the investment of assets over the period of years, each discounted to the accident period.

Table 1 shows three important rates of return from the company's perspective:

1) the underwriting rate of return on the assets corresponding to the liabilities assumed by the company when writing this

business (i.e., the cost to the company of policyholder supplied funds),

2) the operating return to the company on the assets corresponding to the same policyholder liabilities assumed (i.e., the insurance risk charge to the policyholder for the transfer of insurance risk to the company), and

3) the rate of return to the shareholder.

Each of these three rates of return is calculated by dividing a particular income item by its respective balance sheet liability (i.e., asset commitment). In the example, the company's underwriting return is -3.8% (-\$660 divided by \$17,342 in nominal dollars) and this is the cost of policyholder funds supplied to the company. The company's operating return is 1.5% (\$256 divided by \$17,342) and this is the insurance charge to the policyholder for the transfer of risk. The rate of return to the shareholder is 10.4%, the total net income of \$520 divided by shareholder contributed surplus of \$5,000. This is also the discounted net income of \$469 divided by the discounted surplus of \$4,517.

The steady state present-valued balance sheet viewpoint provides a mechanism to transform transactions over several years into a single annual period measurement. In particular, the surplus commitment over multiple calendar years sums to a single period value against which one can calculate shareholder returns.

The ability to employ a single period basis is a key to simplifying discounted cash flow models and providing a single return on surplus measurement. While this NPV measurement will equal the IRR under certain conditions, the NPV cash flow approach provides added flexibility not inherent in the IRR. For example, the approach supports the determination of the traditional Operating Return on Premium (ROP), a form of return preferred by many in ratemaking.

These results will be viewed from a cash flow perspective in a following section. First, the control of surplus will be discussed in order to establish the conditions under which NPV and IRR rates of return are equivalent.

3. CONTROLLING THE FLOW OF SURPLUS

Surplus exists as a financial buffer in support of business writings. The amount of the initial surplus contribution and the timing of its subsequent withdrawal is an important component of total return. For any segment of business, the accident year total return is its income as a percentage of the surplus committed, wherein both income and surplus are sums across the many years of financial activity as the liabilities run off. (It is common to view the development of loss reserves in the form of a loss triangle, and surplus is viewed analagously here).

Selecting a financial leverage factor (i.e., the ratio of liabilities to surplus) is a critical starting point, since this factor determines the initial surplus contribution and the amounts of surplus subsequently released over time as liabilities are settled. The following principles guide the flow of surplus (i.e., both initial shareholder surplus contribution and subsequent withdrawal):

1. The benchmark surplus level should be controlled over time by a direct linkage of that level to the level of net policyholder liabilities.
2. Insurance operating earnings (underwriting and investment income on policyholder supplied funds) of each accident year should be released to the shareholder (e.g., as dividends) as liabilities are settled.

It should be noted that Principle 1 implies use of "net policyholder liabilities to surplus" as the real underlying leverage ratio. The traditional "premium to surplus" leverage definition is far less meaningful.

If surplus flows are controlled following these principles, all three of the following will be identical:

a) the net present value, discounted accident year return on surplus (ROS);

b) the internal rate of return (IRR) measured for the accident year shareholder flows; and,

c) the annual increments of accident year shareholder distribution, as a rate of each year's beginning surplus.

Attribute C means that the shareholder will receive a constant annual rate of return (equal to the ROS and IRR) on each year's beginning investment for the initial and each subsequent calendar period of development of the accident year, until all accident year flows are settled.

In the example in the Appendix, the net present value return, IRR, and annual rate of income returned to the shareholder are each 10.4%. The cash flow perspective and the resultant IRR will be explained in the next section.

Note that the calendar shareholder distribution is not equivalent to the calendar ROE based net income: the latter reflects the effect of contributions from several prior accident years. During a calendar period, a shareholder is actually receiving a return of income and previously contributed surplus relating to the settlement of the current *and* previous accident year liabilities.

4. THE CASH FLOW PERSPECTIVE

Cash flow transactions occur between the policyholder and company, and between the company and shareholder. Table 2 provides a cash flow perspective demonstrating all flows involved in the insurance transaction using the same example from the Appendix. Positive cash flows are *to* the company, negative flows are *from* the company. To make the ideas more meaningful, it is best to consider the "policyholder" as actually a group of policyholders, whereby the losses that occur represent an average for the group.

The first section of Table 2 summarizes the transactions between policyholder and company and shows the "Total Underwriting

TABLE 2

CASH FLOW ANALYSIS AND IRR COMPONENTS FROM COMPANY PERSPECTIVE

	Begin Year 1	End Year 1	End Year 2	End Year 3	End Year 4	Total	IRR
POLICYHOLDER UNDERWRITING FLOWS TO COMPANY							
Premium	$10,000	$0	$0	$0	$0	$10,000	
Expense Paid	-3,000	0	0	0	0	-3,000	
Loss Paid	0	-2,000	-2,000	-2,000	-2,000	-8,000	
Tax Paid (UW, Timing)	-468	520	140	97	50	340	
Total Underwriting	6,532	-1,480	-1,860	-1,903	-1,950	-660	3.8% (To PH)
POLICYHOLDER INVESTMENT INCOME FLOWS TO COMPANY AFTER TAX							
Loss Reserves		422	317	211	106	1,056	
Tax Timing Items		-43	-15	-8	-3	-68	
Retained Earnings		-35	-22	-11	-4	-72	
Total Investment Income		345	280	192	99	916	
TOTAL POLICYHOLDER OPERATING FLOWS TO COMPANY							
UW & Investment Income	6,532	-1,135	-1,580	-1,711	-1,851	256	-1.5% (To PH)
INVESTMENT INCOME FROM CONTRIBUTED SURPLUS TO COMPANY AFTER TAX		106	79	53	26	264	
SHAREHOLDER FLOWS TO COMPANY							
Operating Earning Withdrawal		-102	-77	-51	-26	-256	
Surplus Contribution	2,000	-500	-500	-500	-500	0	
Inv Income on Surplus		-106	-79	-53	-26	-264	5.3%
Net Contributed Surplus	2,000	-708	-656	-604	-552	-520	10.4% (To SH)
NET CASH FLOW TO COMPANY FROM ALL SOURCES	$8,532	-$1,737	-$2,157	-$2,262	-$2,376	0	

Flows" net of these transactions, including taxes payable on underwriting income. In the example, the company receives a net initial cash flow of $6,532, followed by payments of $1,480, $1,860, $1,903, and $1,950 at the end of years 1, 2, 3, and 4, respectively. The total of these flows is a net payment of $660, which is the after tax underwriting loss. The IRR *to the policyholder* for this stream of cash flows is 3.8%, or -3.8% *to the company*. This is the "cost of policyholder funds" supplied to the company.

The company invests the policyholder supplied funds prior to payment of losses, and the resultant cash flows are shown separately under "Policyholder Investment Income Flows to Company—Total Investment Income." These are $345, $280, $192, and $99 for years 1, 2, 3, and 4, respectively, and total $916.

The "Total Policyholder Operating Flows to Company" is the sum of "Underwriting" and "Investment Income" flows and is $6,532 at policy inception, and -$1,135, -$1,580, -$1,711, and -$1,851, at the end of years 1, 2, 3, and 4, respectively. The total of $256 is the operating income. The IRR is -1.5% *to the policyholder*, or +1.5% *to the company*. This is the "insurance risk charge," the rate of return implicit in the transfer of underwriting risk from the policyholder to the company. In essence, the company keeps the investment income in excess of that needed to cover underwriting costs in exchange for the transfer of risk. Viewed mathematically, the market rate of return on investments of 5.3% less the 3.8% cost of policyholder funds equals the 1.5% risk charge.

Switching to the transactions between the company and the shareholder, the level of surplus is controlled so that the ratio of liabilities to surplus is 4:1 and further so that the return to the shareholder will be 10.4% in every year. The "Shareholder Flow" consists of three components: The initial contribution of surplus and its subsequent withdrawal, investment income on this surplus, and operating earnings. In this example, the company received a shareholder contribution of $2,000 initially, followed by payments to the shareholder of $708, $656, $604, and $552, in years 1, 2, 3, and 4, respectively. This totals a net payment of $520 to the shareholder, which is the total net income. The IRR *to the shareholder* is 10.4% and this is the shareholder total return in this example. The rate of return to the shareholder is also 10.4% of each year's beginning surplus.

5. NPV AND IRR RATES OF RETURN

The balance sheet and cash flow perspectives have been used to develop the NPV and IRR rates of return, respectively. In addition, rates of return have been determined at the policyholder, company, and shareholder levels. Table 3 provides a summary of the results and demonstrates the equivalency in returns.

TABLE 3

POLICYHOLDER, COMPANY, AND SHAREHOLDER RATES OF RETURN—NPV AND IRR

	Net Present Value (NPV)					Internal Rate of Return (IRR)	
	Balance Sheet Liability		Income After Tax				
	Nominal Future Value	Disc Present Value	Nominal Future Value	Disc Present Value	Rate of Return	Income After Tax Nominal Value	Rate of Return
Policyholder	$17,342	$17,342	-$660	-$660	-3.8%	-$660	-3.8%
Insurance Operations	17,342	15,627	256	231	1.5	256	1.5
Shareholder	5,000	4,517	520	469	10.4	520	10.4

As shown in Table 3, the policyholder, company, and shareholder rates of return produced by the NPV and IRR approaches are identical. This important result confirms their equivalency and demonstrates that, when surplus is controlled in the same manner, the results produced by the two approaches will be equal.

6. REFORMULATION OF THE TOTAL RETURN MODEL

The "cost of policyholder supplied funds" and "insurance risk charge" as rates of return are directly linked with the total return to the shareholder. The traditional model that describes total return is:

Total Return = Operating Return on Premium
× Premium to Surplus Ratio
+ Investment Yield on Surplus,

or notationally:

$$ROS = (ROP) \times (P/S) + R_s \, .$$

This model should be restated as follows:

Total Return = Operating Return on Liabilities
(i.e., Insurance Risk Charge)
× Liability to Surplus Ratio (i.e., Leverage)
+ Investment Yield on Surplus,

or notationally:

$$ROS = (ROL) \times (L/S) + R_s \, .$$

This new formulation reflects the direct relationship of income to the respective liabilities, but requires abandoning the old traditions of premium to surplus and return on premium. One difficulty with return on premium is that it does not provide a measure that is comparable among lines of business having different cash flow characteristics, such as long tail versus short tail lines of business. In addition and as a consequence, it is difficult to determine just what a "fair" return on premium (and profit margin) ought to be in a line of business. This difference is automatically reflected in the formulation using liabilities shown above.

As will be shown in the next section, a return on policyholder liabilities, or risk charge, can be determined that solves the short tail versus long tail problem. And perhaps more importantly, a return on liabilities provides a measurement of an insurance rate of return that can be a compromise for those who object to any "allocation" of surplus. One need simply to assume that the ratio of liabilities to surplus is the same for each line of business. This will also be discussed in the next section.

For each line of business, the linkage of surplus to liabilities controls the initial shareholder surplus contribution and its subsequent release. If there were no differences in volatility among lines of business, then the liability to surplus ratio would be constant. However, this is not the case. Therefore, the unique characteristics of each line must be reflected in the magnitude of this ratio.

While this paper and the examples used have focused on the clearly dominant loss reserves payable as the most significant liability (and thus the main factor affecting the level of surplus), *net* policyholder liabilities also include expenses payable, premiums receivable, the two tax law timing items (tax loss discount and UPR offset), and retained earnings. Some lines have unique and specialized cash flows that must be considered since this may have a significant effect on net liabilities, beyond that due to losses.

7. THE INSURANCE RISK CHARGE

The insurance charge calculation provides a means of determining what an insurance company retrospectively has charged for assuming insurance risk or prospectively should charge for assuming the risk. Furthermore, as shown in Table 4, when the principles regarding the control of surplus and the release of operating earnings discussed previously are followed, the insurance risk charge measurement "normalizes" away the differences in typical return measurements, such as ROP, that are caused by different cash flows among lines of business (i.e., the length of the tail). *Specifically, the insurance risk charge is fixed, at a given ROE level, regardless of line of business, if the liability to surplus leverage ratio does not vary.*

This insurance risk charge provides a much better way to measure operating rates of return than does the return on premium, and it is an acceptable measure to those who do not wish to allocate surplus to a line of business. Yet this is, in effect, a part of a specific total return approach that assumes a constant liability to surplus leverage ratio in all lines of business. Certainly this is more reasonable than a return on premium coupled with a constant premium to surplus ratio in all lines, a common practice by some in the industry.

As shown in the examples in Table 4, the risk charge corresponding to a 15% total rate of return is 2.43% regardless of the length of the loss payout (2.43% times 4.0 leverage plus 5.28% investment yield on surplus equals 15%). The policyholder is being paid 2.85% for the use of his or her funds. In the breakeven case, the risk charge is 0%, meaning the company is earning no return from its insurance operations.

TABLE 4

COMBINED RATIO, COST OF FUNDS, AND RISK CHARGE WITH VARYING PAYOUTS AND TARGET RETURNS

Loss Payout (Years)		Shareholder Total Return 25.0%	15.0%	Breakeven 5.28%
1	Combined Ratio (%)	100.3	102.8	105.3
	Cost of Funds (%)	0.35	2.85	5.28
	Risk Charge (%)	4.93	2.43	0.00
3	Combined Ratio (%)	101.0	109.1	117.8
	Cost of Funds (%)	0.35	2.85	5.28
	Risk Charge (%)	4.93	2.43	0.00
5	Combined Ratio (%)	101.7	115.7	132.5
	Cost of Funds (%)	0.35	2.85	5.28
	Risk Charge (%)	4.93	2.43	0.00

Each case in Table 4 assumes a liability to surplus leverage ratio of 4:1, that premium and expense are paid without delay, and that 100% of the loss is paid on the single date shown (i.e., after 1, 3, and 5 years, respectively). The expense ratio is 30%, the investment yield and tax discount rates are 8.0% before tax, and the tax rate is 34% in each case.

The supporting detail for the three year payout and 15% total return case is shown in Tables 5 and 6, which parallel Tables 1 and 2.

TABLE 5

NOMINAL AND DISCOUNTED INCOME, BALANCE SHEET, AND RATE OF RETURN
(THREE YEAR LOSS PAYOUT, 15% TARGET TOTAL RETURN)

	Income After Tax		Balance Sheet Liability		
Total Income Components	Nominal Future Value	Discounted Present Value	Nominal Future Value	Discounted Present Value	Rate of Return (After Tax)
INSURANCE OPERATIONS:					
Underwriting	-$599	-$599	$21,017	$21,017	-2.9% (Cost of PH Funds)
Policyholder Liabilities					
Loss & Loss Expense	1,253	1,131	23,723	21,422	5.3%
Tax Loss Discounting	-60	-55	-1,137	-1,043	5.3%
Tax Unearned Premium	-18	-17	-340	-323	5.3%
Net PH Liabilities	1,175	1,059	22,246	20,056	5.3%
Retained Earnings	-65	N/A	-1,229	-1,127	5.3%
TOTAL OPERATING INCOME	511	460	21,017	18,928	2.4% (Risk Charge)
SHAREHOLDER SURPLUS					
Contributed Surplus	277	250	5254	4732	5.3%
TOTAL INCOME	$788	$710			15.0% (ROS)

TABLE 6

CASH FLOW ANALYSIS AND IRR COMPONENTS FROM COMPANY PERSPECTIVE (THREE YEAR LOSS PAYOUT, 15% TARGET TOTAL RETURN)

	Begin Year 1	End Year 1	End Year 2	End Year 3	End Year 4	Total	IRR
POLICYHOLDER UNDERWRITING FLOWS TO COMPANY							
Premium	$10,000	$0	$0	$0	$0	$10,000	
Expense Paid	-3,000	0	0	0	0	-3,000	
Loss Paid	0	0	0	-7,908	0	-7,908	
Tax Paid (UW, Timing)	-585	511	184	199	0	309	
Total Underwriting	6,414	511	184	-7,708	0	-599	2.9% (To PH)
POLICYHOLDER INVESTMENT INCOME FLOWS TO COMPANY AFTER TAX							
Loss Reserves		418	418	418	0	1,253	
Tax Timing Items		-47	-20	-11	0	-78	
Retained Earnings		-32	-22	-11	0	-65	
Total Investment Income		339	375	396	0	1,110	
TOTAL POLICYHOLDER OPERATING FLOWS TO COMPANY							
UW & Investment Income	6,414	849	560	-7,313	0	511	-2.4% (To PH)
INVESTMENT INCOME FROM CONTRIBUTED SURPLUS TO COMPANY AFTER TAX		85	94	99	0	277	
SHAREHOLDER FLOWS TO COMPANY							
Operating Earning Withdrawal		-156	-173	-182	0	-511	
Surplus Contribution	1,604	173	97	-1,874	0	0	
Inv Income on Surplus		-85	-94	-99	0	-277	5.3%
Net Contributed Surplus	1,604	-67	-170	-2,155	0	-788	15.0% (To SH)
NET CASH FLOW TO COMPANY FROM ALL SOURCES	$8,018	$867	$484	-$9,368	$0	$0	

The insurance risk charge, or operating rate of return on policyholder liabilities, presented here, offers a definition of a rate of return that can be used in the establishment of a "fair" insurance return consistent (since it is mathematically part of total return) with total return as commonly accepted in the financial community.

8. THE SINGLE PAGE RATEMAKING SPREADSHEET

Cash flow models in insurance are made cumbersome due to the many periods of cash flows that must be considered for a given accident year. Fortunately, in many instances it is possible to model this process more simply through use of average settlement dates of cash flows rather than the many individual values that occur at different points in time. In the example in the Appendix, for example, this means using a single loss payment at the average payment date of 2.5 years to approximate the effects of actual flows of 25% equally at the end of years 1-4. A simplified model has been created to provide a much simplified calculation and presentation vehicle based on average settlement dates.

In Exhibit 1, "Calculation of Total Return" incorporates all of the principles discussed into a simple, concise calculation of a total rate of return. The upper portion presents the assumptions on underwriting, cash flow, investment, taxes, and leverage necessary as inputs to the determination of a total rate of return. The lower portion presents the results (at present value) for income from underwriting and investment and the rates of return that result. This exhibit is an extremely simplified, yet adequate, presentation of the rate of return that can be estimated prospectively as part of the ratemaking process. The formulas for this exhibit are presented in Exhibits 3 and 4.

It should be noted that the 10.7% total rate of return shown at the bottom of Exhibit 1 differs from the more precise 10.4% shown previously since average cash settlement dates have been used. This loss of precision is the trade-off for the gain in simplification in calculation and presentation achieved by this simplified method. The primary difference is in the loss discount provision of the tax law which does not lend itself well to the use of an average date.

A second spreadsheet page, Exhibit 2, "Nominal and Discounted Income, Balance Sheet, and Rate of Return" presented previously in format as Table 1, provides further technical backup and documents the components of the rate of return used in this simplified model in more detail. For example, the balance sheet that underlies the rate of return calculation is presented, as is the insurance risk charge. Formulas for this exhibit are presented in Exhibit 5.

9. CONCLUSION

This paper has presented a total return model of insurance that provides complementary rate of return measures that are applicable to the policyholder, company, and shareholder alike. The company operating rate of return is a direct measure of the charge for the transfer of insurance risk from the policyholder to the company. The paper offers this as a more meaningful and supportable measure than the more traditional return on premium, and it suggests a reformulation of the total return model in this regard.

Through utilization of both the balance sheet and cash flow perspectives, the paper has demonstrated equivalency of NPV and IRR rates of return *if* a company follows a set of suggested operating guidelines regarding the control of surplus: It must link surplus control to insurance liabilities and the release of operating earnings.

Finally, the paper has provided a simple rate of return calculation and presentation model as a means to approximate the more complex multi-period cash flows that exist for a given accident year. For those interested, Exhibits 3-5 provide the detailed formulas driving the model.

REFERENCES

[1] Bingham, Russell E., "Surplus—Concepts, Measures of Return, and Its Determination," *Insurer Financial Solvency,* Casualty Actuarial Society Discussion Paper Program, May 1992, p. 179.

[2] Bingham, Russell E., "Discounted Return—Measuring Profitability and Setting Targets," *PCAS* LXXVII, 1990, p. 124.

[3] Bunner, Bruce, and Wasserman, David, "The Dynamics of Risk and Return Under California's Proposition 103," *Underwriter's Report,* June 15, 1989.

[4] Cummins, J. David, "Multi-Period Discounted Cash Flow Ratemaking Models in Property-Liability Insurance," *Journal of Risk and Insurance,* March, 1990, p. 79.

[5] Feldblum, Sholom, "Pricing Insurance Policies: The Internal Rate of Return Model," Casualty Actuarial Society Exam Part 10 Study Kit, 1992.

[6] Myers, Stewart C., and Cohn, Richard A., "A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulation," *Fair Rate of Return in Property-Liability Insurance,* Kluwer-Nijhoff, 1987, p. 55.

EXHIBIT 1
Calculation of Total Return

INPUT ASSUMPTIONS	BEFORE TAX	PORTION OF PREMIUM
UNDERWRITING FINANCIALS		
Earned Premium	$10,000	
Loss & Loss Expense	$8,000	80.0%
Expense	$3,000	30.0%
Combined Loss and Expense	$11,000	110.0%
AVERAGE COLLECTION PAYMENT DELAYS (in years)		
Earned Premium	0.00	
Loss & Loss Expense	2.50	
Expense	0.00	
TAX AND INVESTMENT		
Underwriting Tax Rate	34%	
Operating Cash Flow Investment Rate—BT	8%	
Investment Income Tax Rate	34%	
Operating Cash Flow Investment Rate —AT	5.28%	
Tax Loss Discount—Average Date	2.50	
Tax Loss Discount—Discount Rate	8%	
Year End Portion Premium Unearned	50%	
SURPLUS		
Leverage Premium/Surplus	2.19	
Leverage Liability/Surplus	4.00	
Surplus Investment Rate—BT	8%	
Investment Income Tax Rate	34%	
Surplus Investment Rate —AT	5.28%	
GAAP Conversion Factor	1.00	

RESULTS AT PRESENT VALUE	BEFORE TAX	AFTER TAX
Underwriting Income	-$1,000	-$660
Investment Income Credit		
Premium	$0	$0
Loss	$1,463	$966
Expense	$0	$0
Tax Law Timing Items:		
Loss Discounting	-$63	-$42
UPR Offset	-$26	-$17
Net Investment Income Credit	$1,374	$907
Operating Income	$374	$247
Operating Return on Premium (ROP)	3.7%	2.5%
Surplus	$4,572	
Investment Income Credit on Surplus	$366	$241
Total Net Income	$740	$488
Total Return on Surplus	16.2%	10.7%
Total Return on Equity	16.2%	10.7%

EXHIBIT 2

NOMINAL AND DISCOUNTED INCOME, BALANCE SHEET, AND RATE OF RETURN

Total Income Components	Income After Tax: Nominal Future Value	Income After Tax: Discounted Present Value	Balance Sheet Liability: Nominal Future Value	Balance Sheet Liability: Discounted Present Value	Rate of Return (After Tax)
INSURANCE OPERATIONS:					
Underwriting	-$660	-$660	$17,618	$17,618	-3.7% (Cost of PH Funds)
Policyholder Liabilities					
Loss & Loss Expense	1,056	966	20,000	18,288	5.3%
Tax Loss Discounting	-45	-42	-849	-786	5.3%
Tax Unearned Premium	-18	-17	-340	-323	5.3%
Net PH Liabilities	993	907	18,881	17,180	5.3%
Retained Earnings	-63	N/A	-1,193	-1,090	5.3%
TOTAL OPERATING INCOME	270	247	17,618	16,090	1.5% (Risk Charge)
SHAREHOLDER SURPLUS					
Contributed Surplus	264	241	5,000	4,572	5.3%
TOTAL INCOME	$534	$488			10.7% (ROS)

EXHIBIT 3

FORMULAS FOR CALCULATION OF TOTAL RETURN EXHIBIT

INPUT ASSUMPTIONS	BEFORE TAX	PORTION OF PREMIUM
UNDERWRITING FINANCIALS		
Earned Premium	P	
Loss & Loss Expense	L	$100 \times L/P$
Expense	E	$100 \times E/P$
Combined Loss and Expense	$L + E$	$100 \times (L + E)/P$
AVERAGE COLLECTION PAYMENT DELAYS (in years)		
Earned Premium	N_p	
Loss & Loss Expense	N_l	
Expense	N_e	
TAX AND INVESTMENT		
Underwriting Tax Rate	T	
Operating Cash Flow Investment Rate—BT	R_b	
Investment Income Tax Rate	T_i	
Operating Cash Flow Investment Rate —AT	$R = R_b \times (1 - T_i)$	
Tax Loss Discount—Average Date	N_t	
Tax Loss Discount—Discount Rate	R_t	
Year End Portion Premium Unearned	U	
SURPLUS		
Premium/Surplus Leverage Factor	F	
Surplus Investment Rate—BT	R_{sb}	
Investment Income Tax Rate	T_i	
Surplus Investment Rate —AT	$R_s = R_{sb} \times (1 - T_i)$	
GAAP Conversion Factor	G	

RESULTS AT PRESENT VALUE	BEFORE TAX	AFTER TAX
Underwriting Income	$P - L - E$	$W = (P - L - E) \times (1 - T)$
Investment Income Credit		
Premium		$-P \times (1 - 1/(1 + R)^{N_p})$
Loss		$L \times (1 - 1/(1 + R)^{N_l})$
Expense		$E \times (1 - 1/(1 + R)^{N_e})$
Tax Law Timing Items:		
Loss Discounting		KL
UPR Offset		$-0.2TPU \times (1 - 1/(1 + R))$
Net Investment Income Credit	$C/(1 - T)$	C = sum of 5 items above
Operating Income	$(W + C)/(1 - T)$	$W + C$
Operating Return on Premium (ROP)		$(W + C)/P$
Surplus	$S = P/F$	
Investment Income Credit on Surplus	$R_{sb} \times S$	$C_s = R_s \times S$
Total Net Income	$(W + C + C_s)/(1 - T)$	$W + C + C_s$
Total Return on Surplus	$ROS/(1 - T)$	$ROS = 100 \times (W + C + C_s)/S$
Total Return on Equity	$ROE/(1 - T)$	$ROE = ROS/G$

K = Loss discount investment credit factor from Exhibit 4.

EXHIBIT 4

Loss Discounting Investment Income Credit Factor (Factor times Loss for Dollar Impact)

Approximation Formula

1) Actual and Law Rates and Payouts Same

$$-\{(D_b - D_a) + T(1 - D_b)\}, \text{ where}$$

$D = 1/(1 + R)^N$, i.e., discount factor
R = rate for calculating discount
N = payment date
b = before tax
a = after tax
T = tax rate
$D_a = 1/(1 + R_a)^N$
$R_a = (1 - T)R_b$

2) Actual and Law Rates Different, Payouts Same

$$-\{(D_r'{}_b - D_a) + T(1 - D_r'{}_b)\} + (D_r'{}_b - D_a)(R_a - R'_a)/(R_a - R'_b)$$

(Rate Adjustment)

where $'$ signifies using law rate.

3) Actual and Law Rates and Payouts Different

$$-\{(D_n'{}_r'{}_b - D_n'{}_a) + T(1 - D_n'{}_r'{}_b)\} + (D_n'{}_r'{}_b - D_n'{}_a)(R_a - R'_a)/(R_a - R'_b)$$

(Rate Adjustment)

$$+TD_a[(1 - D_n''{}_r'{}_b) - (D_n''{}_r'{}_b - D_n''{}_a)R'_b/(R_a - R'_b)]$$

(Date Adjustment)

where $'$ signifies using law rate or payment date and
$n'' = n' - n$, i.e., difference in payment date

The effect of different rates is greater than that of payout differences, and Formula 2 is sufficiently accurate for most applications.

An approximate formula for the above is

$$-T\{(1 - D_{m\,r\,a}) \times (1 - D_n'{}_r'{}_b)\}, \text{ where } m = (n + 1)/2$$

$$= -T\{(1 - 1/(1 + R_a)^m) \times (1 - 1/(1 + R'_b)^{n'})\}.$$

EXHIBIT 5
Part 1

NOMINAL AND DISCOUNTED INCOME, BALANCE SHEET, AND RATE OF RETURN MATHEMATICAL FORMULAS

Components of Total Income		Income After Tax: Nominal Future Value	Income After Tax: Discounted Present Value	Balance Sheet Liability: Nominal Future Value	Balance Sheet Liability: Discounted Present Value	Rate of Return (After Tax)	
INSURANCE OPERATIONS:		(3)	(4)	(5)	(6)		
Underwriting		$W' = W$	$W = (P - L - E) \times (1 - T)$	Nominal Sum 3	Nominal Sum 3	Column (3)/(5) or (4)/(6)	Cost of PH Funds
Policyholder Liabilities							
Premium	(a)	$-R \times Np \times P$	$-P \times (1 - 1/(1+R)^{Np})$	$-Np \times P$	$-P \times (1 - 1/(1+R)^{Np})/R$	"	
Loss & Loss Expense	(b)	$R \times Nl \times L$	$L \times (1 - 1/(1+R)^{Nl})$	$Nl \times L$	$L \times (1 - 1/(1+R)^{Nl})/R$	"	
Underwriting Expense	(c)	$R \times Ne \times E$	$E \times (1 - 1/(1+R)^{Ne})$	$Ne \times E$	$E \times (1 - 1/(1+R)^{Ne})/R$	"	
Net Policyholder Liabilities (excluding Tax Timing Items)	Sum 1	<———— Sum of (a) through (c) ————>				"	
Tax Timing Items							
Tax Loss Discounting	(d)	ZL (Note 4)	KL (Note 4)	ZL/R	KL/R	"	
Tax Unearned Premium	(e)	$-R \times 0.2TPU$	$-0.2TPU \times (1 - 1/(1+R))$	$-0.2TPU$	$-0.2TPU \times (1 - 1/(1+R))/R$	"	
Net Liabilities	Sum 2	<———— Sum of (a) through (e) ————>				"	

EXHIBIT 5
Part 2

NOMINAL AND DISCOUNTED INCOME, BALANCE SHEET, AND RATE OF RETURN MATHEMATICAL FORMULAS

Components of Total Income		Income After Tax: Nominal Future Value	Income After Tax: Discounted Present Value	Balance Sheet Liability: Nominal Future Value	Balance Sheet Liability: Discounted Present Value	Rate of Return (After Tax)	
Retained Earnings	(f)	$R \times E'$ (Note 2)	N/A	E'	E (Note 3)	Column (3)/(5) or (4)/(6)	
Net Liabilities (including Retained Earnings)	c: Sum 3	<———— Sum of (a) through (f)			————>	"	Market Inv Rate on PH Funds
TOTAL OPERATING INCOME		$W' + C'$	$W + C$	Nominal Sum 3	Discounted Sum 3	"	Insurance Charge Risk
SHAREHOLDER SURPLUS:							
Contributed Surplus		$C_S' = R_S \times S'$	$C_S = R_S \times S$	S' (Note 1)	S	"	Market Inv Rate on SH Funds
TOTAL INCOME		$W' + C' + C_S' + R \times E'$	$W + C + C_S$			Column (3)/(5) Column(4)/(6)	ROS' ROS

" ' " Denotes nominal

Note 1: Nominal Surplus $S' = S \times$ (Nominal Sum 2/Discounted Sum 2)

Note 2: $R \times E' = \text{ROS} \times S' - (W' + C' + C_S')$

Note 3 : $E = E' \times$ (Discounted Sum 2/Nominal Sum 2)

Note 4: $Z = -R \times T \times (N_l - (1 - 1/(1 + R_l)^{N_l})/R_l$

Note 5: K = Loss discount investment income credit factor determined by detail formula that reflects differing discount rates and/or payment dates of the tax law from actual

Before Tax amounts are determined by dividing After Tax amounts by $(1 - T)$.

APPENDIX

Numerical Example

This appendix presents an example involving a single accident year (which can be viewed as a single policy written on the first of the year) with a premium of $10,000, expense of $3,000, and ultimate loss of $8,000. The premium is received and the expenses are paid without delay, whereas claims are paid in 25% increments at the end of each of the current and three following years.

In addition, the example assumes the yield rate on investments to be 8%, before tax, and the tax rate on underwriting and investment income to be 34%. For simplicity, the rate used for loss discounting under the 1986 Tax Reform Act is also 8%. The example assumes one-half of premium to be unearned at the end of the first year for purposes of the premium offset provision of the tax law. In this example all cash flows are discounted to the beginning of each respective year. Traditional accounting rules are followed to construct income statements and balance sheets.

The exhibits in this appendix for this example are as follows:

Exhibit A.1—Basic assumptions and calculations of reserves and payments.

Exhibit A.2—Nominal and discounted income statements and balance sheets for the single accident year over its four years of activity.

Exhibit A.3—Exhibit A.2 accumulated across successive accident years, reaching steady state after four years.

Exhibit A.4—Relationship of policyholder and shareholder funds.

Exhibit A.5—Shareholder flows, nominal and discounted steady state income with IRR and NPV, and respective rates of return.

Underwriting and investment are assumed to remain constant over time. With no growth in the level of business, it takes four years to reach a steady state condition, after which all items remain the same as shown on Exhibit A.3.

In the example, the writing of the policy required an initial capital contribution by the shareholder. Subsequently the shareholder receives payments (i.e., return of capital) consisting of three components: 1) The return of invested capital; 2) the investment income on the invested capital while held by the company; and 3) the insurance operating earnings, which is the sum of the underwriting income and the investment income on the policyholder funds.

The release of funds to the shareholder is governed by maintaining a constant 4:1 ratio of policyholder funds to shareholder funds over time. For simplification in this example, policyholder funds are assumed to consist of loss reserves only, and do not include either the tax law timing items or retained earnings. (Retained earnings are, in effect, undistributed operating earnings that must be included in shareholder flows at some point, and are considered separate from contributed surplus).

The release of funds to the shareholder is thus a payout policy of 1) withdrawing investment income on capital as it is earned (i.e., annually) and 2) withdrawing the initial capital contribution and operating income as a function of loss payout. This is demonstrated on Exhibit A.4 for both the single accident year and steady state.

Under this return of capital rule, the initial surplus contributed for the accident year is $2,000, based on the 4:1 reserve-to-surplus ratio, followed by declines to $1,500, $1,000, and $500 in years two through four since the loss reserve is $8,000, $6,000, $4,000, and $2,000, respectively, for these years. At steady state, the reserve is $20,000 and the surplus $5,000. The calendar premium-to-surplus ratio at steady state is 2:1.

The itemized shareholder flows are shown on the upper section of Exhibit A.5. Capital is withdrawn at the rate of 25%, or $500, per year matching the loss payout pattern. The shareholder receives the investment income on the contributed capital and the operating earnings in a manner that maintains the relationship to reserves.

Rates of Return on Surplus

On Exhibit A.5, an internal rate of return (IRR) calculation is shown for "Operating Earnings", "Contributed Capital", and "Net Shareholder Return". The IRR for operating earnings and contributed capital are both 5.3%, since these flows earn 8% before tax, or 5.3%, after tax. The shareholder receives a net IRR of 10.4%, based on the initial capital contribution of $2,000 followed by withdrawals of $708, $656, $604, and $552 in years one through four. The IRR measures the return to the shareholder from both operating earnings and investment income on surplus. It should be noted that the annual return on invested capital is also 10.4% in every year.

Part 2 of Exhibit A.5 displays a nominal steady state calculation of return on surplus derived from the steady state balance sheet and income statements.

Note that the "Total Net Income" of $520 is 10.4% of the $5,000 "Beginning Surplus". The calculation of discounted return is shown to the right and reflects the steady state figures on a basis discounted to both the beginning and the end of the initial accident year. When valued at the beginning of the accident year, the "Total Return" of $469 is 10.4% of the $4,517 "Beginning Surplus."

What this demonstrates is that all three measures of return—the IRR, the steady state nominal calendar period, and the discounted return—are equivalent. This equivalence holds under the assumption that underwriting and investment are fixed, there is no growth in business level, *and* policyholder and shareholder flows are linked over time.

EXHIBIT A.1

Basic Assumptions and Calculations
Baseline — Four Year Payout (25% Per Year) at 4:1 Reserve/Surplus Ratio

Earned Premium	10,000.00
Expense Ratio	0.30
Loss Ratio	0.80
Underwriting Tax Rate	34.00%
Investment Yield BT	8.00%
Investment Yield AT	5.28%
Tax Law Discount Rate	8.00%

		Year				
	Total	1	2	3	4	5
Loss Payment Sched Actual	100%	25%	25%	25%	25%	0%
Loss Payment Sched Law	100%	25%	25%	25%	25%	0%
Loss Payout by Law	8,000	2,000	2000	2,000	2,000	0
Discounted		1,852	1,715	1,588	1,470	0
Beginning Reserve Before Discount		8,000	6,000	4,000	2,000	0
Tax Law Timing Items BT						
Beginning Loss Discount	1,375	1,375				
Scheduled Recovery	-1,375	-530	-412	-285	-148	0
Begin UPR Subject to Tax	1,000	1,000				
Scheduled Recovery	-1,000	-1,000				
Reserves And Payments						
Beginning Nominal Loss Reserve		8,000	6,000	4,000	2,000	0
Loss Payments		2,000	2,000	2,000	2,000	0
Begin Loss Discount Tax Reserve		-468	-288	-147	-50	0
Loss Discount Tax Recovery		180	140	97	50	0
Begin UPR Tax Reserve		-340				
UPR Tax Recovery		340				
Shareholder Cap. Flows	Begin					
From Operating Earnings[1]		102	77	51	26	0
From Investment Income on Contributed Capital		106	79	53	26	0
Capital Withdrawal	-2,000	500	500	500	500	0
Contributed Capital[2]	-2,000	606	579	553	526	0
Net Capital Flows	-2,000	708	656	604	552	0

[1] Operating earnings withdrawal: Constant calendar ROS (AT)

[2] Contributed surplus withdrawal: Proportional to reserves plus investment income

EXHIBIT A.2
Part 1

BALANCE SHEETS AND INCOME STATEMENTS
SINGLE ACCIDENT YEAR

BASELINE—FOUR YEAR PAYOUT (25% PER YEAR)
AT 4:1 RESERVE/SURPLUS RATIO

		Year				
Income Statement	Total	1	2	3	4	5
NOMINAL						
Income BT						
Underwriting Income	-1,000	-1,000	0	0	0	0
Investment Income						
Loss Reserve	1,600	640	480	320	160	0
Loss Disc Tax Reserve	-76	-37	-23	-12	-4	0
UPR Tax Reserve	-27	-27	0	0	0	0
Retained Earnings	-109	-53	-33	-17	-6	0
Surplus	400	160	120	80	40	0
Total Income BT	787	-317	544	371	190	0
NOMINAL						
Income AT						
Underwriting Income	-660	-660	0	0	0	0
Investment Income						
Loss Reserve	1,056	422	317	211	106	0
Loss Disc Tax Reserve	-50	-25	-15	-8	-3	0
UPR Tax Reserve	-18	-18	0	0	0	0
Retained Earnings	-72	-35	-22	-11	-4	0
Surplus	264	106	79	53	26	0
Total Income AT	520	-209	359	245	125	0
DISCOUNTED						
Income AT						
Underwriting Income	-660	-660	0	0	0	0
Investment Income						
Loss Reserve	954	401	286	181	86	0
Loss Disc Tax Reserve	-46	-23	-14	-7	-2	0
UPR Tax Reserve	-17	-17	0	0	0	0
Retained Earnings	-66	-33	-20	-10	-3	0
Surplus	238	100	71	45	21	0
Total Income AT	404	-232	324	210	102	0
Total Income (Excluding Retained Earnings)	469	-199	344	220	105	0

EXHIBIT A.2
Part 2

BALANCE SHEETS AND INCOME STATEMENTS
SINGLE ACCIDENT YEAR

BASELINE—FOUR YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

	Year				
Balance Sheet	1	2	3	4	5
NOMINAL					
Beginning Assets	8,532	6,795	4,638	2,376	0
Liabilities					
Loss Reserve	8,000	6,000	4,000	2,000	0
Disc Tax Reserve	-468	-288	-147	-50	0
UPR Tax Reserve	-340	0	0	0	0
Surplus					
Retained Earnings	-660	-417	-214	-73	0
Contributed	2000	1,500	1,000	500	0
Liabilities + Surplus	8,532	6,795	4,638	2,376	0
DISCOUNTED					
Beginning Assets	8,104	6,131	3,975	1,934	0
Liabilities					
Loss Reserve	7,599	5,413	3,428	1,628	0
Disc Tax Reserve	-444	-259	-126	-41	0
UPR Tax Reserve	-323	0	0	0	0
Surplus					
Retained Earnings	-627	-377	-184	-60	0
Contributed	1,900	1,353	857	407	0
Liabilities + Surplus	8,104	6,131	3,975	1,934	0

EXHIBIT A.3
Part 1

Balance Sheets and Income Statements
Steady State Basis, Four Years

Baseline—Four Year Payout (25% Per Year)
At 4:1 Reserve/Surplus Ratio

	Year				
Income Statement	1	2	3	4	5
Nominal					
Income AT					
Underwriting	-660	-660	-660	-660	-660
Investment Income					
Reserves	422	739	950	1,056	1,056
Loss Disc Tax Reserve	-25	-40	-48	-50	-50
UPR Tax Reserve	-18	-18	-18	-18	-18
Retained Earnings	-35	-57	-68	-72	-72
Surplus	106	185	238	264	264
Total Income AT	-209	149	394	520	520
Discounted					
Income AT					
Nominal Underwriting	-660	-660	-660	-660	-660
Investment Income					
Loss Reserve	401	687	868	954	954
Loss Disc Tax Reserve	-23	-37	-44	-46	-46
UPR Tax Reserve	-17	-17	-17	-17	-17
Retained Earnings	-33	-53	-63	-66	-66
Surplus	100	172	217	238	238
Total Income AT	-232	92	301	404	404
Total Income (Excluding Retained Earnings)	-199	145	364	469	469

EXHIBIT A.3
Part 2

BALANCE SHEETS AND INCOME STATEMENTS
STEADY STATE BASIS, FOUR YEARS

BASELINE—FOUR YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

	Year				
Balance Sheet	1	2	3	4	5
NOMINAL					
Beginning Assets	8,532	15,327	19,965	22,342	22,342
Liabilities					
Loss Reserve	8,000	14,000	18,000	20,000	20,000
Disc Tax Reserve	-468	-755	-903	-953	-953
UPR Tax Reserve	-340	-340	-340	-340	-340
Surplus					
Retained Earnings	-660	-1,077	-1,292	-1,365	-1,365
Contributed	2,000	3,500	4,500	5,000	5,000
Liabilities + Surplus	8,532	15,327	19,965	22,342	22,342
DISCOUNTED					
Beginning Assets	8,104	14,235	18,210	20144	20,144
Liabilities					
Loss Reserve	7,599	13,012	16,440	18,068	18,068
Disc Tax Reserve	-444	-704	-830	-871	-871
UPR Tax Reserve	-323	-323	-323	-323	-323
Surplus					
Retained Earnings	-627	-1,003	-1,187	-1,247	-1,247
Contributed	1,900	3,253	4,110	4,517	4,517
Liabilities + Surplus	8,104	14,235	18,210	20,144	20,144
DISCOUNTED END OF YEAR VALUATION					
Beginning Assets—	8,532	14,987	19,171	21,207	21,207
Liabilities					
Loss Reserve	8,000	13,699	17,308	19,022	19,022
Disc Tax Reserve	-468	-741	-874	-917	-917
UPR Tax Reserve	-340	-340	-340	-340	-340
Surplus					
Retained Earnings	-660	-1,056	-1,250	-1,313	-1,313
Contributed	2,000	3,425	4,327	4,755	4,755
Liabilities + Surplus	8,532	14,987	19,171	21,207	21,207

EXHIBIT A.4

Policyholder/Shareholder Funds

Baseline—Four Year Payout (25% Per Year) at 4:1 Reserve/Surplus Ratio

	Beginning of Year				
	1	2	3	4	5
Single Accident Year					
Nominal					
Policyholder Funds	8,000	6,000	4,000	2,000	0
Shareholder Funds	2,000	1,500	1,000	500	0
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	
Discounted					
Policyholder Funds	7,599	5,413	3,428	1,628	0
Shareholder Funds	1,900	1,353	857	407	0
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	
Discounted End of Year Valuation					
Policyholder Funds	8,000	5,699	3,609	1,714	0
Shareholder Funds	2,000	1,425	902	428	0
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	
Steady State Basis, Four Years					
Nominal					
Policyholder Funds	8,000	14,000	18,000	20,000	20,000
Shareholder Funds	2,000	3,500	4,500	5000	5,000
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	4.00
Discounted					
Policyholder Funds	7,599	13,012	16,440	18,068	18,068
Shareholder Funds	1,900	3,253	4,110	4,517	4,517
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	4.00
Discounted End of Year Valuation					
Policyholder Funds	8,000	13,699	17,308	19,022	19,022
Shareholder Funds	2,000	3,425	4,327	4,755	4,755
Ratio PH/SH Funds	4.00	4.00	4.00	4.00	4.00

EXHIBIT A.5
Part 1

RATE OF RETURN TO SHAREHOLDER (INCOME DISTRIBUTED/BEGINNING SURPLUS)
SINGLE ACCIDENT YEAR

BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

		Year				
Shareholder Flows	Begin	1	2	3	4	IRR
Operating Earnings [1]	-231	102	77	51	26	5.3%
Contributed Surplus Account						
Investment Income		106	79	53	26	
Capital Withdrawal	-2,000	500	500	500	500	
Contributed Capital [2]	-2,000	606	579	553	526	5.3%
Net Shareholder Flows	-2,000	708	656	604	552	10.4%
Return (Operating and Investment Income)		10.4%	10.4%	10.4%	10.4%	

[1]Operating earnings withdrawal: constant calendar ROS (AT)

[2] Contributed surplus withdrawal: proportional to reserves plus investment income

EXHIBIT A.5
Part 2

RATE OF RETURN TO SHAREHOLDER (INCOME DISTRIBUTED/BEGINNING SURPLUS)
STEADY STATE BASIS
BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

		Year											
Shareholder Flows	Begin	1	2	3	4	5	6	7	8	9	10	11	IRR
Operating Earnings[1]		102	179	230	256	256	256	256	256	153	77	26	
Contributed Surplus Account													
Investment Income		106	185	238	264	264	264	264	264	158	79	26	
Capital Withdrawal	-2,000	-1,500	-1,000	-500	0	0	0	0	2,000	1,500	1,000	500	
Contributed Capital[2]	-2,000	-1,394	-815	-262	264	264	264	264	2,264	1,658	1,079	526	5.3%
Net Shareholder Flows	-2,000	-1,292	-636	-32	520	520	520	520	2,520	1,812	1,156	552	10.4%
Return (Operating and Investment Income)		10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	10.4%	

[1]Operating earnings withdrawal: constant calendar ROS (AT)

[2]Contributed surplus withdrawal: proportional to reserves plus investment income

EXHIBIT A.5
Part 3

RATE OF RETURN TO SHAREHOLDER (INCOME DISTRIBUTED/BEGINNING SURPLUS)
STEADY STATE BASIS
BASELINE—FOUR-YEAR PAYOUT (25% PER YEAR) AT 4:1 RESERVE/SURPLUS RATIO

			DISCOUNTED			
			Beginning of Year		End of Year	
	NOMINAL	% of Surplus	Valuation	% of Surplus	Valuation	% of Surplus
Beginning Surplus	$5,000		$4,517		$4,755	
Underwriting Income	-660		-660		-695	
Investment Income	916		891		938	
Oper Inc Incl Ret Earns	256	5.3%	231	5.1%	243	5.1%
Investment Income on Surplus	264	5.3%	238	5.3%	251	5.3%
Total Net Income	520	10.4%	469	10.4%	494	10.4%

DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXIX

PARAMETRIZING THE WORKERS' COMPENSATION EXPERIENCE RATING PLAN

WILLIAM R. GILLAM

DISCUSSION BY HOWARD C. MAHLER

The author would like to thank William R. Gillam and Robert A. Bear for providing helpful comments on an earlier version of this discussion.

1. INTRODUCTION

Mr. Gillam's paper provides an excellent explanation of the detailed actuarial study that led the National Council on Compensation Insurance (NCCI) to revise the Experience Rating Plan for Workers' Compensation. This actuarial study is an example of a practical application of credibility theory to the situation where parameter uncertainty and risk heterogeneity are important.

As shown in Exhibit 1, the revised plan shares many of the features of the prior plan. Administratively, the plans are the same. There have been important actuarial changes. As Mr. Gillam states, the revised plan is a single split plan rather than a multi-split plan, and the credibilities that are determined by the parameters of the two plans are very different.

2. ACTUARIAL FORMULAS UNDERLYING EXPERIENCE RATING

Mr. Gillam's Formula 1.5 is used in both the prior plan and the revised plan in order to calculate the experience modification:

$$M = 1 + Z_p\,(P_t - \mathrm{E}\,[P_t]) + Z_x\,(X_t - \mathrm{E}\,[X_t])\ ,$$

[1] The NCCI study was also explained in Venter [1].

where:

M = experience modification;

P = primary loss divided by expected losses = A_p/E ;

X = excess loss divided by expected losses = A_x/E ;

t = (past) time period;

Z_p = primary credibility;

Z_x = excess credibility; and

E [Y] = expected value of Y.

This formula for the experience modification can also be written following Snader [2] as:

$$M = 1 + Z_p\left(\frac{A_p}{E} - \frac{E_p}{E}\right) + Z_x\left(\frac{A_x}{E} - \frac{E_x}{E}\right)$$

$$= \frac{(1 - Z_p)\,E_p + Z_pA_p + (1 - Z_x)\,E_x + Z_xA_x}{E} \; .$$

The credibilities are given by:

$$Z_p = \frac{E}{E + K_p} \text{ , and}$$

$$Z_x = \frac{E}{E + K_x} \text{ ,}$$

where E is the expected losses, and K_p and K_x are the credibility parameters to be determined.

Under the revised plan, the credibility parameters have the form $E\,\frac{\text{Linear}}{\text{Linear}}$ or in Mr. Gillam's notation $K = E\left(\frac{CE + D}{E + F}\right)$. The NCCI determined the particular coefficients used in the revised plan by the empirical testing described by Mr. Gillam.

It follows from the formulas for the credibility parameters that under the revised plan the credibilities as a function of the size of risk are of the form $\frac{\text{Linear}}{\text{Linear}}$. This can be written as:

$$Z = \frac{E + I}{JE + I + K}, \text{ where } 0 \leq I,\ 1 \leq J, \text{ and } 0 \leq K,$$

with one formula for primary credibility and one formula for excess credibility, each with different constants I, J, and K. As explained by Mr. Gillam, this is the form of credibility one expects if both parameter uncertainty and risk heterogeneity are important.[2] The more familiar formula for credibility is a special case of this formula with $I = 0$ and $J = 1$.

In the more familiar formula $Z = E/(E + K)$ the parameter K is a "scale parameter." Changing K changes the overall scale of the credibility curve without changing its shape. As will be discussed below, K, and thus the scale of the curve, depends on a state-specific inflation-sensitive parameter.

In the formula used in the revised plan, there are two additional parameters I and J which are "shape parameters." Changing I and/or J changes the shape of the credibility curve. The size of the parameter I relative to the parameter K adjusts the shape of the credibility curve for small risks. The minimum credibility is given by $I/(I + K)$, which is determined by the ratio of I to K. The parameter J adjusts the shape of the credibility curve for large risks. The maximum credibility is given by $1/J$.

---

[2] See Equation 1.6 in Mahler [3]. What was denoted as K there is denoted as $I + K$ here. This is a matter of notation rather than substance. The notation used here allows K to have the same underlying source in both the credibility formula in the revised experience rating plan and the more familiar formula for credibility.

Thus the revised plan uses a more general formula for credibility, which is better able to approximate those credibilities that would have performed well in the past and thus are expected to work well in the future. As shown in Mahler [3], one could derive an even more general formula than that used in the revised Experience Rating Plan. As a function of the size of risk, the credibilities given by formulas in Mahler [3] are of the form $\frac{\text{Quadratic}}{\text{Quadratic}}$.

As discussed in Appendix B, if one assumes the covariance of excess and primary losses is not extremely important, these formulas for the credibilities reduce to the form $\frac{\text{Linear}}{\text{Linear}}$ used in the revised Experience Rating Plan.[3]

This more general formula for credibility is somewhat better able to approximate those credibilities that would have performed well in the past. The two additional parameters can be selected so as to adjust the shape of the credibility curve for medium-size risks. In any given application, one has to decide whether the extra generality introduced by these additional parameters is worth the extra complications also introduced.

The specific formulas for Z_p and Z_x used in the revised plan are:

$$Z_p = \frac{E + 0.0028S}{1.1E + 0.01308S}, \text{ and}$$

$$Z_x = \frac{E + 0.0204S}{1.75E + 0.8357S},$$

where S is the State Reference Point.[4]

These formulas can also be stated in terms of the parameter g:[5]

[3] These covariances are discussed in more detail in a later section.

[4] The State Reference Point is calculated as 250 times the average cost per case in the particular state.

[5] The parameter g is calculated as the average cost per case in the particular state divided by 1,000; g is rounded to the nearest 0.05.

$$Z_p = \frac{E + 700g}{1.1E + 3{,}270g}, \text{ and}$$

$$Z_x = \frac{E + 5{,}100g}{1.75E + 208{,}925g}.$$

Thus, under the revised plan, the primary and excess credibilities are each given by the formula $Z = (E + I)/(JE + I + K)$, with the following parameters:

	Primary	Excess
I	$0.0028S = 700g$	$0.0204S = 5{,}100g$
J	1.1	1.75
K	$0.01028S = 2{,}570g$	$0.8153S = 203{,}825g$

If $S = \$500{,}000$ and $g = 2$, for example,[6] then the parameters would be:

	Primary	Excess
I	\$1,400	\$10,200
J	1.1	1.75
K	\$5,140	\$407,650

Note that the curves for primary and excess credibilities under the revised plan have a significantly different scale from each other due to their vastly different values of the parameter K. As is shown in Exhibit 2, the two curves also have significantly different shapes due to their different values of the parameter J and different ratios of I to K.

3. IMPLEMENTING THE ACTUARIAL FORMULAS

The values for the credibilities underlying actual experience ratings may differ slightly from those calculated using the formulas given above, due to the rounding process involved in establishing a

---

[6] These correspond to an average claim of \$2,000.

table of W and B values. Also, they will differ for small risks (those with expected losses below about \$20,000) because of the minimums imposed on the parameters W, K_p and K_x.[7]

As stated by Gillam, for the smaller risks, there are maximum values imposed on the experience rating modification under the revised plan.

Expected Losses	Maximum Modification
0 to \$5,000	1.6
\$5,000 to \$10,000	1.8
\$10,000 to \$15,000	2.0

The maximum debit and credit for small risks are compared in Exhibit 6.

The NCCI's reduction in the maximum swing for smaller risks not only makes practical sense, but also is sound from a theoretical standpoint. The inclusion of the parameter I in the credibility formula produces the larger than desired credibilities for smaller risks. However, this was based on a consideration of risk heterogeneity. Such considerations become inapplicable as risks become too small to have separate and distinct subunits.[8] Thus a credibility formula parametrized based on all sizes of risks may not fit well for the very smallest risks.

Under both plans, the W and B values vary with the expected losses and are displayed in a table. However, the formulas used to determine W and B are significantly different under the two plans. An example of W and B values for both plans is shown in Exhibit 5.

---

[7] The imposition of minimums on K_p and K_x reduces the credibility assigned to very small risks (those with expected losses below about \$6,000). The imposition of a minimum on W increases the credibility assigned to the excess losses of small risks.

[8] This is explained in Mahler [3].

The W and B values determine the credibility parameters and credibilities under both experience rating plans following the development in Snader [2].

Let $Z_p = \dfrac{E}{E+B}$, and

$$Z_x = \frac{E}{E + \dfrac{B + (1-W)\,E}{W}} = \frac{WE}{E+B} = WZ_p \,.$$

These equations can be compared to the equations given by Gillam using the credibility parameters:

$$Z_p = \frac{E}{E + K_p}, \text{ and}$$

$$Z_x = \frac{E}{E + K_x} \,.$$

The credibility parameters K_p and K_x can be calculated from the expected losses E, W, and B:

$$K_p = B \text{ , and}$$

$$K_x = \frac{B + (1-W)\,E}{W} \,.$$

As stated by Gillam, under the prior plan:

$$B = (1-W)\ 20{,}000, \text{ and}$$

$$W = \begin{cases} 0 & E \le 25{,}000 \\ \dfrac{E - 25{,}000}{S - 25{,}000} & S \ge E \ge 25{,}000 \\ 1 & E \ge S \end{cases} \,,$$

where S is the self-rating point.

Under the revised plan, the values of the credibility parameters K_p and K_x are given via formula, and then B and W follow from them. The formulas in terms of the State Reference Point S are:

$$K_p = E\left[\frac{0.1E + 0.01028S}{E + 0.0028S}\right],$$

where K_p is subject to a minimum of 7,500 (K_p subject to this minimum is labeled B by the NCCI) and

$$K_x = E\left[\frac{0.75E + 0.8153S}{E + 0.0204S}\right],$$

where K_x is subject to a minimum of 150,000 (K_x subject to this minimum is labeled C by the NCCI).

These equations can also be stated in terms of g.[9] These equations are the ones used by the NCCI:[10]

$$K_p = E\left[\frac{0.1E + 2{,}570g}{E + 700g}\right], \text{ and}$$

$$K_x = E\left[\frac{0.75E + 203{,}825g}{E + 5{,}100g}\right].$$

By solving the set of equations, one can express W and B in terms of K_p and K_x. These equations are used to determine W and B from K_p and K_x:[11]

$$B = K_p\text{, and}$$

$$W = \frac{E + K_p}{E + K_x},$$

[9] The state specific parameter g is defined by the NCCI as the average claim cost in the state divided by 1,000; g is rounded to the nearest 0.05.

[10] The NCCI has written these formulas in a slightly different form. For example, $K_p = E\,[0.1 + (2{,}500g/(E + 700g))]$.

[11] The NCCI actually defines B as K_p subject to the minimum. The NCCI defines C as K_x subject to the minimum. Then $W = (E + B)/(E + C)$.

where W is subject to a minimum of 0.07.

4. CREDIBILITIES: PRIOR PLAN VS. REVISED PLAN

The credibilities under the revised plan differ significantly from the prior plan. Therefore, the switch in experience rating plans has led to very significant impacts on individual insureds.[12] The credibilities assigned to the primary[13] and excess losses are each significantly different, as can be seen in Exhibits 3, 4, and 5:

1. For small risks, primary credibilities are larger.
2. For large risks, primary credibilities are smaller. The maximum primary credibility is 91%, rather than 100% as under the prior plan.
3. For small risks, excess credibilities are a little larger. Even very small risks have a small non-zero excess credibility, as opposed to zero under the prior plan.
4. For large risks, excess credibilities are much smaller. The maximum excess credibility is 57%, rather than 100% under the prior plan.

Thus one important change is that under the revised plan there are no longer self-rated risks. Since the primary losses are assigned a maximum credibility of 91%, while the excess losses are assigned a maximum credibility of 57%, the maximum credibility assigned to any risk is approximately 70%.[14]

---

[12] As shown in Exhibit 3, a risk's credibility can change by up to 40%. For example, if a risk with a 0.6 mod had its credibility decline by 40%, it would now get a 0.76 mod, all other things being equal. Its standard premium would then increase by 27% ($1.27 = 0.76/0.6$).

[13] Under the revised plan the definition of primary losses is changed. Thus the D-ratios, which measure the expected portion of the losses that will be primary, have to be recalculated with the adoption of the revised plan. In one state (Massachusetts) the average D-ratio decreased from about 0.35 to about 0.30. The results will vary by state, depending on the size of loss distribution, which depends heavily on the particular state workers' compensation law.

[14] Assuming a D-ratio of D, the maximum credibility is $(D \times 91\%) + ((1 - D) \times 57\%)$. For $D = 0.50$ the maximum credibility is 74%. For $D = 0.35$ the maximum credibility is 69%. For $D = 0.20$ the maximum credibility is 64%.

5. COVARIANCE OF EXCESS AND PRIMARY LOSSES

As discussed previously, the equations for credibility by size of risk underlying the revised experience rating plan can be derived from theoretical considerations, provided one assumes that the covariance of excess and primary losses is not extremely important. If this covariance is important, i.e., if excess and primary losses are highly correlated, then one expects a more complex relationship of credibilities with size of risk. (See Appendix B for the derivation of equations.)

Recall that under revised experience rating, the first \$5,000 of a loss is considered primary and the remainder of the loss that enters into the experience rating calculation is excess.[15] A simple special case will illustrate why one would expect the excess and primary losses to be significantly correlated. Assume half the losses were of size \$30,000 (with primary portion \$5,000 and excess portion \$25,000), while the other half were of size \$3,000 (with primary portion \$3,000 and excess portion of zero). Then the excess and primary losses are perfectly correlated.

While, in actuality, there are claims of all sizes, the large losses will all have \$5,000 in primary losses, while the smallest losses will all have no excess and less than \$5,000 of primary losses. Thus some positive correlation should exist. This should carry over to an examination of all the losses for an insured. For a constricted example in Mahler [3, p. 141], the primary and excess losses were highly correlated. The actual size of these correlations for actual insureds can be examined empirically. These covariances can be estimated from the data used for experience rating.

As an illustrative example, the covariances were estimated using three years of data from one state. The estimation process is described in detail in Appendix A. While there was insufficient data to arrive at a definitive conclusion, the results are interesting and should point the

[15] Recall that for very large claims, the maximum amount that enters into the calculation of the experience modification is 10% of the State Reference Point.

way for further research. As expected, the primary and excess losses were found to be significantly correlated. The "between" and "within" correlations were each greater than 50%.

6. CREDIBILITIES TAKING INTO ACCOUNT COVARIANCES

The credibilities are determined in Appendix A by using the estimated variances and covariances in the theoretical formula for the split experience rating plan. The resulting credibilities differ significantly from those under revised experience rating. As shown in Exhibit 7, the calculated primary credibilities are all 100% while the excess credibilities range from about 10% to about 45%. Both the primary and excess credibilities are significantly larger than those indicated by revised experience rating.

The data was too limited to draw any detailed information about the behavior of credibilities with size of risk, beyond the expectation that the excess credibilities increase with size of risk.[16] There are a number of reasons why the credibilities calculated here may differ from those for revised experience rating.

First, the calculation here explicitly considered the covariances between primary and excess losses.

Second, the calculation here relied upon a limited number of intrastate-rated risks from just one state from just one point in time. The credibilities are a relative measure of the informational value of the expected losses and actual losses. The informational value of the expected losses calculated from the expected loss rates depends in turn on the precision of the classification relativities. This precision will vary by state and over time depending on many factors. In addition, while most of the parameters are scaled to the average claim cost by state, the split between primary and excess losses is a fixed $5,000. Thus, the proportions of claim dollars that are primary and excess vary among states based on their differing average claim costs.

[16]Not only do the calculated excess credibilities exhibit fluctuation error, but also there is no useful information on the very largest risks.

It is likely that the variance/covariance structure also varies among these states.

Third, only three years of data were analyzed. Experience rating involves predicting a future year of experience using data generally from two, three, and four years distant. As discussed in Mahler [4] and Mahler [5], as this distance in time gets greater, the phenomenon of shifting risk parameters becomes more important. This phenomenon would act to lower the credibilities from those calculated here.

Fourth, the revised Experience Rating Plan was parametrized via an examination of which credibilities would have performed well in the past.[17] Also, the criterion used to decide which credibilities performed better differs from the least squares criterion. The "Quintiles Test" used by the NCCI and described by Gillam is a refinement of the Dorweiler criterion.[18]

For all of the above reasons, one should not draw any definitive conclusions from the work done here.

7. POSSIBLE FURTHER RESEARCH

It would be interesting to compare the more general credibility formula $\frac{\text{Quadratic}}{\text{Quadratic}}$ versus the $\frac{\text{Linear}}{\text{Linear}}$ formula using the same types of tests as performed by the NCCI.

Another area for possible research is the number of years of data used in the experience period. Currently, three years are given equal weight.[19] One could test whether some other combination of number of years and weights could produce a more accurate result.[20] Appen-

[17] This was not possible to do here due to the limited data available.

[18] These criteria are contrasted in Mahler [4].

[19] Actually since more recent years have more payroll on average, due to inflation, the most recent year on average has somewhat more weight.

[20] As pointed out in Mahler [3], the optimal set of years and weights will depend on to what extent the risk parameters of an insured are shifting over time. This subject was explored in Mahler [4] and Mahler [5].

dix D displays an example of the type of analysis needed. This preliminary analysis indicates that further investigation would be worthwhile.

8. SUMMARY

The revised Experience Rating Plan is based on significantly different credibility formulas than the prior plan. The change results in a significantly more responsive plan for small risks and a significantly less responsive plan for large risks.

While the revised Experience Rating Plan, as explained by Mr. Gillam, has a firmer theoretical and empirical basis than the prior plan, there remain areas for further actuarial research.[21]

[21]The examination of the NCCI [8] performed by the actuarial consulting firm of Milliman & Robertson, Inc. for the NAIC contains a very interesting section on further areas of research on experience rating.

REFERENCES

[1] Venter, Gary G., "Experience Rating—Equity and Predictive Accuracy," *NCCI Digest,* Vol. II, Issue I, April 1987, p. 27.

[2] Snader, Richard H., "Fundamentals of Individual Risk Rating and Related Topics," Part I of Study Note, Casualty Actuarial Society, 1980.

[3] Mahler, Howard C., Discussion of Meyers: "An Analysis of Experience Rating," *PCAS* LXXIV, 1987, p. 119.

[4] Mahler, Howard C., "An Example of Credibility and Shifting Risk Parameters," *PCAS* LXXVII, 1990, p. 225.

[5] Mahler, Howard C.,"The Credibility of a Single Private Passenger Driver," *PCAS* LXXVIII, 1991, p. 146.

[6] Meyers, Glenn G., "An Analysis of Experience Rating," *PCAS* LXXII, 1985, p. 278.

[7] Venter, Gary G., "Credibility," *Foundations of Casualty Actuarial Science,* Casualty Actuarial Society, New York, 1990, Chapter 7, pp. 432-433.

[8] Biondi, Richard S., Grannan, Patrick J., Mulvaney, Mark W., and Pestcoe, Marvin, *NAIC Examination of NCCI,* Volume IX-Section IIB -Part 7, December 5, 1991.

EXHIBIT 1

COMPARISON OF WORKERS' COMPENSATION EXPERIENCE RATING PLANS

Prior	Revised
Primary and Excess Losses	Primary and Excess Losses
Multi-Split Plan: Primary portion of a loss is determined via formula[1] or from a table.	Single Split Plan: Primary portion of a loss is the first $5,000.
Experience Modification depends on a comparison of actual losses to expected losses, taking into account credibilities.	Experience Modification depends on a comparison of actual losses to expected losses, taking into account credibilities.
W and *B* values are shown in a table, and depend on the expected losses for the risk.	*W* and *B* values are shown in a table, and depend on the expected losses for the risk.
The table of *W* and *B* values depends on a state-specific value, the *Self-Rating* Point (SRP).	The table of *W* and *B* values depends on a state-specific value, the *State Reference* Point (SRP).[2]
The per claim accident limitation is 10% of the state's *Self-Rating* Point.	The per claim accident limitation is 10% of the *State Reference* Point.
The State Multiple Claim Accident Limitation is twice the State Per Claim Accident Limitation.	The State Multiple Claim Accident Limitation is twice the State Per Claim Accident Limitation.

[1] $A_p = 10{,}000\,A/(A + 8{,}000)$. For losses less than 2,000, the whole loss is considered primary.

[2] The State Reference Point is equal to 250 times the average claim cost in the particular state. The NCCI uses the state-specific parameter g, which is defined as the average claim cost in the state divided by 1,000; g is rounded to the nearest 0.05; $g = \text{SRP}/250{,}000$.

EXHIBIT 2

NCCI REVISED EXPERIENCE RATING
PRIMARY AND EXCESS CREDIBILITIES

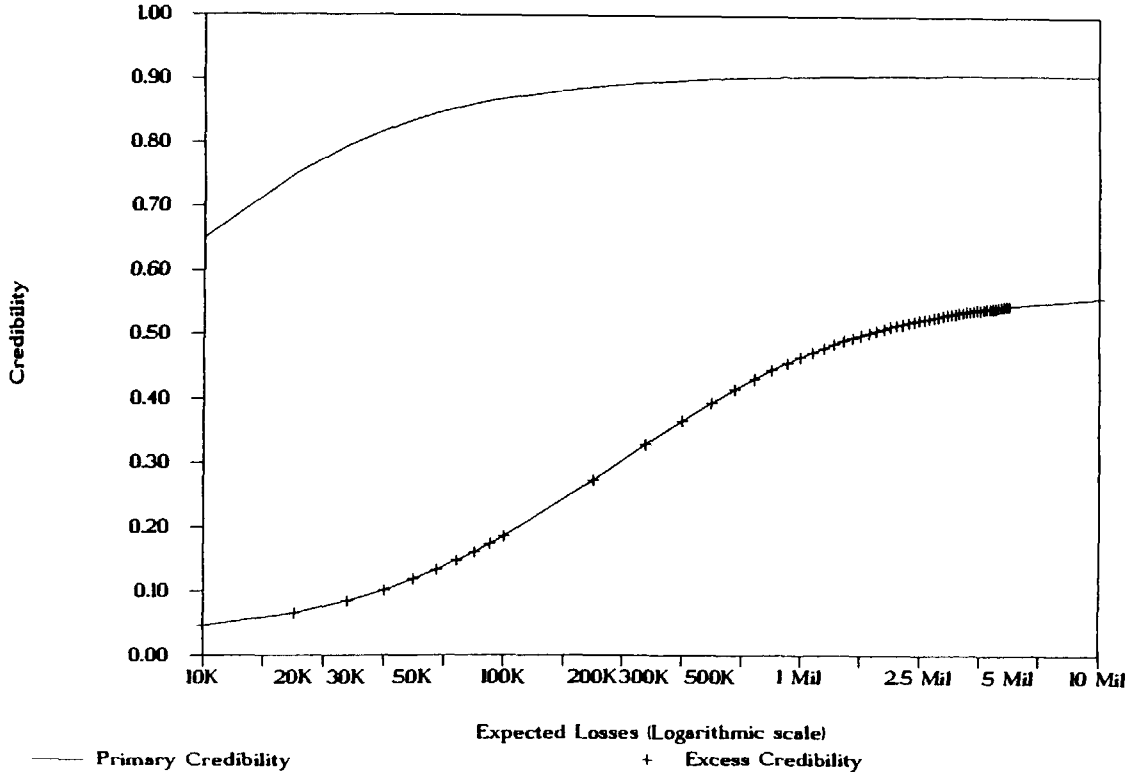


EXHIBIT 3

WORKERS' COMPENSATION EXPERIENCE RATING

Expected Losses ($000)	Credibilities (Weighted Average of Primary and Excess Credibilities)		
	Prior*	Revised**	Revised Minus Prior***
3****	5%	10%	5%
5	7	14	7
7.5	10	18	8
10	12	20	9
15	15	24	9
20	18	26	9
25	19	28	9
50	27	33	7
75	31	37	6
100	34	39	5
125	36	41	5
150	39	43	4
200	43	46	3
300	51	50	-1
400	58	53	-5
500	66	55	-11
750	83	58	-24
1,000	100	59	-41
2,000	100	63	-37
3,000	100	65	-35
4,000	100	65	-35
5,000	100	65	-35
7,500	100	66	-34
10,000	100	66	-34
∞	100	67	-33

* NCCI Experience Rating Plan prior to revision, assuming a Self-Rating Point of $1,000,000 and a *D*-ratio of 0.35.

** Revised Experience Rating Plan, assuming a State Reference Point of $500,000 and a *D*-ratio of 0.30.

*** Result may differ slightly due to intermediate rounding.

**** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

EXHIBIT 4

WORKERS' COMPENSATION EXPERIENCE RATING CREDIBILITIES

Expected	Primary		Excess	
Losses ($000)	Prior*	Revised**	Prior*	Revised**
3***	13%	29%	0%	2%
5	20	40	0	3
7.5	27	50	0	4
10	33	57	0	5
15	43	67	0	6
20	50	73	0	7
25	56	77	0	8
50	72	83	2	12
75	80	86	4	15
100	84	87	7	18
125	87	88	9	21
150	90	88	12	24
200	92	89	17	28
300	95	90	27	33
400	97	90	37	37
500	98	90	48	40
750	99	90	73	44
1,000	100	90	100	46
2,000	100	91	100	52
3,000	100	91	100	54
4,000	100	91	100	54
5,000	100	91	100	54
7,500	100	91	100	55
10,000	100	91	100	55
∞	100	91	100	57

* NCCI Experience Rating Plan prior to revision, using Self-Rating Point of $1,000,000 (assumes average serious case of $40,000).

** Revised Experience Rating Plan, using State Reference Point of $500,000 (assumes average case of $2,000).

*** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

EXHIBIT 5

WORKERS' COMPENSATION EXPERIENCE RATING
W AND *B* VALUES

Expected	*B* ($00)		*W*	
Losses ($000)	Prior*	Revised**	Prior*	Revised**
3***	200	75	0	0.07
5	200	75	0	0.08
7.5	200	75	0	0.08
10	200	75	0	0.08
15	200	75	0	0.09
20	200	75	0	0.09
25	200	75	0	0.10
50	194	99	0.03	0.14
75	190	124	0.05	0.18
100	184	149	0.08	0.21
125	180	174	0.10	0.24
150	174	200	0.13	0.27
200	164	250	0.18	0.31
300	144	350	0.28	0.37
400	124	450	0.38	0.41
500	102	550	0.49	0.44
750	52	800	0.74	0.49
1,000	0	1,050	1.00	0.51
2,000	0	2,050	1.00	0.57
3,000	0	3,050	1.00	0.59
4,000	0	4,050	1.00	0.60
5,000	0	5,050	1.00	0.60
7,500	0	7,550	1.00	0.61
10,000	0	10,050	1.00	0.61

* NCCI Experience Rating Plan prior to revision using a Self-Rating Point of $1,000,000 (assumes average serious case of $40,000).

** Revised Experience Rating Plan, using State Reference Point of $500,000 (assumes average case of $2,000).

*** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

EXHIBIT 6

Workers' Compensation Experience Rating
Revised Experience Rating Plan*

Expected	Maximum Credit**			Maximum
Losses ($000)	D-ratio = 0.40	D-ratio = 0.30	D-ratio = 0.20	Debit
3***	13%	10%	7%	60%
4	15	12	9	60
5	18	14	11	60
6	20	16	12	80
7	22	17	13	80
8	23	18	14	80
9	24	19	14	80
10	26	20	15	80
11	27	21	16	100
12	28	22	16	100
13	28	23	17	100
14	29	23	17	100
15	30	24	18	100
16	31	25	19	No Limit

* Revised Experience Rating Plan, using State Reference Point of $500,000 (assumes average case of $2,000).

** The maximum credit depends on the particular D-ratio. The maximum credit is the credibility which is equal to:

$$[D \times \text{primary credibility} + (1 - D) \times \text{excess credibility}].$$

*** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

EXHIBIT 7

CREDIBILITIES ESTIMATED TAKING INTO ACCOUNT THE COVARIANCES OF PRIMARY AND EXCESS LOSSES COMPARED TO THOSE FROM REVISED EXPERIENCE RATING

Expected	Credibilities from Appendix A[1]		Credibilities from Revised Experience Rating[2]	
Losses ($000)	Z_p	Z_x	Z_p	Z_x
7.5	100%	10%	40%	3%
12.5	100	21	48	3
17.5	100	13	53	3
22.5	100	16	58	4
37.5	100	12	66	5
62.5	100	31	74	6
87.5	100	16	78	8
125	100	35	81	10
200	100	40	84	13
375	100	27	87	19
750	100	46	89	28

[1] Based on variances and covariances estimated from experience rating data in Massachusetts. Primary credibility limited to no more than 100%.

[2] For a State Reference Point of $1,750,000 ($g = 7$), which is the current value in Massachusetts.

APPENDIX A

Massachusetts Experience Rating Data

The Data

The data examined consisted of a subset of the risks experience rated in Massachusetts during policy year 1991. Only intrastate-rated[1] risks with three years of data were examined. In addition, only risks whose expected primary losses in each of the three years were at least equal to 20% of the three year total were examined.[2] The resulting data set consisted of three years of information on each of about 16,000 risks. The information by year was expected and actual losses, each split between primary and excess losses.[3]

Estimation of Variances and Covariances

As per Gillam, we computed:

P = primary loss divided by expected loss $= A_p/E$;

X = excess loss divided by expected loss $= A_x/E$.

The covariances and variances were estimated as suggested in Venter [7]:

Let $X_{it} = X$ for risk i in year t ,

$P_{it} = P$ for risk i in year t,

N = number of risks,

n = number of years,

[1] Complete data on interstate-rated risks was not available from this source.

[2] This limitation was imposed in order to obtain a more reliable estimate of the variation from year to year in the observed results for a given risk; i.e., to make the estimates of the within variances more reliable.

[3] The split between primary and excess losses used the definition of revised experience rating, which was in effect in Massachusetts for policy year 1991. (The expected losses were computed based on the Expected Loss Rates and D-ratios in effect in Massachusetts for policy year 1991.)

$$X_{i.} = \frac{1}{n} \sum_{t} X_{it}$$

$$X_{..} = \frac{1}{nN} \sum_{i,t} X_{it} \ .$$

Then one estimates the excess variances as follows (with the analogous equations for primary variances):

$$\text{estimated within variance of excess losses} = \frac{\sum_{i} \sum_{t} (X_{it} - X_{i.})^2}{(n-1)N} \ ;$$

estimated between variance of excess losses

$$= \frac{\sum_{i} (X_{i.} - X_{..})^2}{N-1} - \frac{\begin{pmatrix}\text{estimated within variance} \\ \text{of excess losses}\end{pmatrix}}{n} \ .$$

One estimates the covariances as follows:

estimated within covariance of excess and primary losses

$$= \frac{\sum_{i} \sum_{t} (X_{it} - X_{i.})(P_{it} - P_{i.})}{(n-1)N} \ ;$$

estimated between covariance of excess and primary losses

$$= \frac{\sum_{i} (X_{i.} - X_{..})(P_{i.} - P_{..})}{N-1} - \frac{\begin{pmatrix}\text{estimated within covariance} \\ \text{of excess and primary losses}\end{pmatrix}}{n} \ .$$

The estimated variances and covariances are shown in Exhibit A.1. In each case, risks between a certain minimum and maximum size (based on expected losses) were examined separately. While there is an overall pattern observed as the risk size varies, it is clear that the limited number of risks and years of data have produced significant fluctuation errors in the individual estimates.

Estimated Credibilities

Using the estimated variances and covariances, and the equations in Appendix B, one obtains the estimated credibilities shown in Exhibit A.2. It should be noted that the estimated credibilities are for the use of three years of data, as is currently used in experience rating. The within variances and covariances estimated from the data are for a single year of data; for use in estimating credibilities, these quantities have been divided by three.[4]

Exhibit A.2 displays three different sets of credibilities. The first is the Bühlmann credibility; i.e., the least squares credibility estimated separately for primary and excess losses, ignoring their correlation. The second set of credibilities is calculated via Equations B.2 and represents the least squares credibility taking into account the correlation of primary and excess losses. The third and final set of credibilities is similar to the second set; but it has had the primary credibility set equal to unity, as discussed in Appendix C.

[4] When using many years of data or when parameters shift significantly over time, a different adjustment than performed here would be appropriate.

EXHIBIT A.1

Expected Losses ($000)*		Number	Estimated Variances				Estimated Covariances		Estimated Correlations	
			Between		Within**					
Min	Max	of Risks	Primary	Excess	Primary	Excess	Between	Within**	Between	Within
5	10	2,731	0.053	1.218	0.106	6.395	0.179	0.465	0.702	0.565
10	15	3,536	0.040	1.341	0.068	3.810	0.209	0.262	0.898	0.514
15	20	2,080	0.051	0.636	0.048	2.776	0.128	0.208	0.713	0.567
20	25	1,396	0.036	0.664	0.040	2.398	0.127	0.173	0.816	0.561
25	50	3,154	0.024	0.401	0.028	1.855	0.074	0.134	0.753	0.590
50	75	1,246	0.034	0.549	0.016	0.977	0.122	0.070	0.888	0.566
75	100	568	0.035	0.220	0.012	0.782	0.070	0.059	0.793	0.609
100	150	590	0.018	0.397	0.010	0.599	0.061	0.046	0.712	0.601
150	250	470	0.022	0.339	0.006	0.428	0.064	0.030	0.752	0.574
250	500	265	0.008	0.110	0.004	0.234	0.022	0.017	0.710	0.566
500	1,000	79	0.017	0.201	0.002	0.202	0.043	0.015	0.748	0.655

Note: Based on intrastate-rated risks in Massachusetts, as explained in the text.

* The sum of expected losses for three years of data used for experience rating.

**While all within variances and covariances were estimated using individual years of data, the values listed here have been divided by three to adjust them for the use of three years of data for experience rating.

EXHIBIT A.2

Expected Losses ($000)		Between Variance Divided by Between Variance Plus Within Variance[1]		Least Squares Credibility[2]		Alternate Credibility[3]	
Min	Max	Primary	Excess	Primary	Excess	Primary	Excess
5	10	33%	16%	109%	9%	100%	10%
10	15	37	26	164	15	100	21
15	20	51	19	157	7	100	13
20	25	48	22	184	8	100	16
25	50	47	18	166	6	100	12
50	75	69	36	280	9	100	31
75	100	75	22	222	1	100	16
100	150	65	40	181	27	100	35
150	250	77	44	221	25	100	40
250	500	69	32	192	17	100	27
500	1,000	87	50	230	28	100	46

Note: Credibilities computed based on the variances and covariances in Exhibit A.1.

[1] Bühlmann credibility, estimated least squares credibility ignoring any correlation between primary and excess losses.

[2] Estimated least squares credibility taking into account the correlation between primary and excess losses.

[3] Primary credibility limited to 100%.

APPENDIX B

DEPENDENCE OF CREDIBILITY ON SIZE OF RISK

In this appendix, the variation of credibility with size of risk will be discussed. Equations B.10 are those used in the revised experience rating plan. The theoretical underpinnings of these formulas, as well as the more general Equations B.11, are discussed.

Following the development in Mahler [3], let

a = total variance of the primary losses,

b = total variance of the excess losses,

c = variance of the hypothetical means of the primary losses ,
= "between" variance of primary losses,

d = variance of the hypothetical means of the excess losses,
= "between" variance of excess losses,

r = total covariance of hypothetical means of the primary and excess losses, and

s = covariance of hypothetical means of the primary and excess losses
= "between" covariance of primary and excess losses.

Then the optimum least squares credibilities Z_p and Z_x are derived in Appendix F of Mahler [3] and given in Equations 5.3 and 5.4 of that paper as:

$$Z_p = \frac{(c+s)b - (d+s)r}{ab - r^2}\text{, and} \qquad \text{(B.1.a)}$$

$$Z_x = \frac{(d+s)a - (c+s)r}{ab - r^2}\,. \qquad \text{(B.1.b)}$$

Thus, both the primary and excess credibilities can be written in terms of variances and covariances.

Therefore, the dependence of the credibilities on the size of the risk can be derived from the dependence of the various variances and covariances on the size of the risk.

Again following Mahler [3], let

$t = a - c =$ process variance of the primary losses

$=$ "within" variance of primary losses,

$u = b - d =$ process variance of the excess losses

$=$ "within" variance of excess losses, and

$v = r - s =$ process covariance of the primary and excess losses

$=$ "within" covariance of primary and excess losses.

Then substituting into Equations B.1, one gets:

$$Z_p = \frac{(c+s)(u+d)-(d+s)r}{(t+c)(u+d)-(v+s)^2}, \text{ and} \tag{B.2.a}$$

$$Z_x = \frac{(d+s)(t+c)-(c+s)r}{(t+c)(u+d)-(v+s)^2} . \tag{B.2.b}$$

The NCCI credibility parameters K_p and K_x are defined so that:

$$Z = \frac{E}{E+K}$$

and, therefore,

$$K = E\left(\frac{1}{Z} - 1\right). \tag{B.3}$$

Substituting into Equation B.3 the expressions for Z_p and Z_x given in Equations B.2, one obtains:

$$K_p = E\,\frac{tu+td+vd-su-sv-v^2}{cu+su+cd-s^2-sv-dv}, \tag{B.4.a}$$

$$K_x = E\frac{tu + uc + vc - st - sv - v^2}{dt + st + cd - s^2 - sv - cv}. \tag{B.4.b}$$

If the covariances between the primary and excess losses are zero, $v = s = 0$,[1] i.e., if there is no useful information about the primary losses contained in the excess losses and vice versa, then these equations are greatly simplified:

$$K_p = E\frac{t}{c}, \tag{B.5.a}$$

$$K_x = E\frac{u}{d}. \tag{B.5.b}$$

Each of the two separate pieces, which are assumed to be uncorrelated with each other, has credibility parameter given by the familiar Bühlmann result.

It is Equations B.5 that form the theoretical bases of the credibilities used by the NCCI in the revised experience rating plan, rather than the more complicated but more general Equations B.4.

It is generally assumed that process variances and covariances (so-called "within" variances and covariances) such as t, u, and v, increase proportionally with E, the size of risk:

$$t \sim E, \tag{B.6.a}$$

$$u \sim E, \tag{B.6.b}$$

$$v \sim E. \tag{B.6.c}$$

However, as shown in Meyers [6], when the phenomenon of parameter uncertainty is important, Equations B.6 do not hold. Instead, t, u, and v increase partially proportionally with E and partially pro-

[1] In fact, the covariances are observed to be significantly different from zero. The total covariance of primary and excess losses, $r = s + v$, is generally positive in actual applications.

portionally with E squared.[2] When parameter uncertainty is important:

$$t \sim E \text{ Linear } [E] \, , \tag{B.7.a}$$

$$u \sim E \text{ Linear } [E] \, , \tag{B.7.b}$$

$$v \sim E \text{ Linear } [E] \, . \tag{B.7.c}$$

It is generally assumed that variances and covariances of the hypothetical means (so-called "between" variances and covariances) such as c, d, and s increase proportionally with the square of E, the size of risk:

$$c \sim E^2, \tag{B.8.a}$$

$$d \sim E^2, \tag{B.8.b}$$

$$s \sim E^2. \tag{B.8.c}$$

However, as shown in Mahler [3], in the presence of risk heterogeneity, Equations B.8 do not hold. Instead, c, d, and s increase partially proportionally with E and partially proportionally with E squared.[3] When risk heterogeneity is important:

$$c \sim E \text{ Linear } [E], \tag{B.9.a}$$

$$d \sim E \text{ Linear } [E], \tag{B.9.b}$$

$$s \sim E \text{ Linear } [E]. \tag{B.9.c}$$

---

[2] As discussed in Mahler [3], the portion of the process variance or covariance that is proportional to the square of E represents the variation of the parameters due to the different states of the universe.

[3] As discussed in Mahler [3], the portion of the variance or covariance of the hypothetical means that is proportional to E represents the variation caused by grouping subunits together to form a single risk. For example, several separate factories might belong to a single insured.

One can substitute the behavior of the variances and covariances with size of risk into the equations for the credibility parameters K. The revised experience rating plan is based on Equations B.5, with parameter uncertainty (Equations B.7) and risk heterogeneity (Equations B.9). Substituting Equations B.7 and B.9 into Equations B.5 gives:

$$K_p \sim E \frac{\text{Linear }[E]}{\text{Linear }[E]}, \tag{B.10.a}$$

$$K_x \sim E \frac{\text{Linear }[E]}{\text{Linear }[E]}. \tag{B.10.b}$$

This is the form of the credibility parameters used in the revised Experience Rating Plan shown in the main text.[4] This form of the credibility parameters leads directly to the form of the credibilities in the main text.

If, instead of the special case Equation B.5, one starts with the more general Equations B.4, one gets a different form for the credibility parameters. Substituting Equations B.6 and B.8 into Equations B.5 gives the following general form of the credibility parameters with parameter uncertainty and risk homogeneity:[5]

$$K_p \sim E \frac{\text{Quadratic }[E]}{\text{Quadratic }[E]}, \tag{B.11.a}$$

$$K_x \sim E \frac{\text{Quadratic }[E]}{\text{Quadratic }[E]}. \tag{B.11.b}$$

Equations B.10 are a special case of Equations B.11. Therefore, Equations B.11 will always perform at least as well as and usually perform better than Equations B.10 in any empirical tests, including

[4] This is the form for the no-split plan with parameter uncertainty and risk heterogeneity given at Mahler [3, p. 178].

5 This is the form for the split plan with parameter uncertainty and risk heterogeneity given in Mahler [3, p. 178].

the type of studies conducted by the NCCI in its development of the revised Experience Rating Plan. Practical considerations will determine whether in a particular application the extra generality represented by Equations B.11 is worth the extra complication introduced by the additional parameters contained in Equations B.11.

APPENDIX C

PRIMARY CREDIBILITY EQUAL TO UNITY

As pointed out in Mahler [3], the use of the credibilities given by Equations B.1 in Appendix B can lead to calculated credibilities greater than unity. This is the case for the Massachusetts data discussed in Appendix A.

For that data, the primary losses have a significantly smaller variance than the excess losses. In addition, the primary and excess losses are significantly positively correlated. Therefore, the observed primary losses are of value not only to predict future primary losses but also to predict future excess losses. Thus, in some sense, a portion of the credibility applied to the observed primary losses is predicting the future excess losses. Since the expected excess losses are usually greater than the expected primary losses,[1] this addition to the credibility applied to the primary losses due to taking into account the correlation with the excess losses can be very significant. It can easily result in primary credibilities greater than unity.[2]

In circumstances where the calculated primary credibility is greater than unity, one could reasonably set the primary credibility equal to unity. One can then solve for the optimal (least squares) value for the excess credibility.

Following Mahler [3], we have the following value for the efficiency:[3]

$$\text{Efficiency} = \frac{2\,Z_p(c+s) + 2Z_x(d+s) - Z_p^2 a - Z_x^2 b - 2Z_p Z_x r}{c+d+2s},$$

[1] For example, a D-ratio of 0.33 is equivalent to expected excess losses being twice expected primary losses.

[2] This same phenomenon was noted in the example discussed in Sections 6 to 9 of Mahler [3].

[3] The efficiency of an experience rating plan is defined in Meyers [6] as the reduction in the expected squared error. The higher the efficiency, the more accurate the Experience Rating Plan.

where a, b, c, d, r, and s are the variances and covariance defined in Appendix B.

If $Z_p \equiv 1$, then

$$\text{Efficiency} = \frac{2(c+s) + 2Z_x(d+s) - a - Z_x^2 b - 2Z_x r}{c+d+2s}.$$

The least squares credibility is obtained by maximizing the efficiency. Taking the derivative of the efficiency with respect to Z_x and setting it equal to zero gives:[4]

$$\frac{2(d+s) - 2Z_x b - 2r}{c+d+2s} = 0.$$

Solving for the excess credibility gives:

$$Z_x = \frac{d+s-r}{b}.$$

This equation is used in Appendix A in order to calculate the alternate credibilities with $Z_p = 1$. It may be of interest to use the fact that the total covariance equals the sum of the between and within covariances to rewrite this equation as:

$$\text{(Alternate) Excess Credibility} = \frac{\begin{matrix}\text{Between} \\ \text{Variance of} \\ \text{Excess Losses}\end{matrix} - \begin{matrix}\text{Within Covariance} \\ \text{of Primary and} \\ \text{Excess Losses}\end{matrix}}{\text{Total Variance of Excess Losses}}.$$

Except for the inclusion of the term involving the within covariance, this equation is the usual Bühlmann formula for credibility ignoring the correlation between primary and excess losses. Since that covariance is generally large and positive, this equation will produce lower excess credibilities than the usual Bühlmann formula, to go along with the higher primary credibility of 100%.

---

[4] If Z_p were not constrained to be unity, one would set the partial derivatives of the efficiency with respect to Z_p and Z_x equal to zero, and solving, obtain Equations B.1.

APPENDIX D

Shifting Risk Parameters Over Time

The phenomenon of shifting risk parameters over time can significantly alter the credibility assigned to data.[1] As shown in Mahler [4], the first step in determining the importance of this phenomenon is to examine the correlations between different years of data. If the correlations decline significantly as the separation increases, then the phenomenon of shifting risk parameters is significant.

Correlations were estimated for the three years of Massachusetts experience rating data described in Appendix A. For primary losses[2], the computed correlations were 0.22 between adjacent years and 0.16 between years with a year between. For excess losses[3], the computed correlations were 0.09 between adjacent years and 0.06 between years with a year between. Since the correlations are somewhat lower for years further apart, the phenomenon of shifting risk parameters has some significance.

Exhibit D.1 displays a more detailed breakdown by year and size category. While the overall pattern is maintained, there is significant fluctuation, particularly for excess losses.

In order to draw any conclusions, one should study more risks over a longer time span.

---

[1] See Mahler [3], Mahler [4], and Mahler [5].

[2] Actually $P = A_p/E$ as defined in Appendix A

[3] Actually $X = A_x/E$ as defined in Appendix A.

EXHIBIT D.1
ESTIMATED CORRELATIONS

Expected Losses ($000)* Min	Max	Number of Risks	Primary Year 1 and Year 2	Primary Year 2 and Year 3	Primary Year 1 and Year 3	Excess Year 1 and Year 2	Excess Year 2 and Year 3	Excess Year 1 and Year 3
5	10	2,731	0.16	0.16	0.12	0.06	0.11	0.02
10	15	3,536	0.19	0.18	0.14	0.14	0.11	0.07
15	20	2,080	0.23	0.32	0.23	0.04	0.08	0.09
20	25	1,396	0.30	0.20	0.20	0.09	0.06	0.10
25	50	3,154	0.27	0.24	0.16	0.06	0.08	0.06
50	75	1,246	0.48	0.45	0.35	0.23	0.13	0.12
75	100	568	0.45	0.55	0.49	0.11	0.05	0.11
100	150	590	0.36	0.48	0.33	0.16	0.23	0.16
150	250	470	0.55	0.55	0.49	0.17	0.24	0.22
250	500	265	0.37	0.49	0.42	0.17	0.15	0.08
500	1,000	79	0.70	0.67	0.72	0.33	0.26	0.30

* The sum of expected losses for three years of data used for experience rating.

ADDRESS TO NEW MEMBERS—MAY 10, 1993

PHILLIP N. BEN-ZVI

When Dave Flynn called me to ask me to speak to the new Fellows I had three very quick reactions. First, I realized that this meant that I could no longer characterize my gray hair as premature. Second, I realized that it has now been about a decade since I was responsible for the actuarial exams and it must now be physically safe for me to talk to new Fellows, since it's unlikely that any of you had to sit for exams for which I was responsible. Third and most importantly, however, I was honored that Dave asked me to speak because it gives me an opportunity to share with you some of the things that I hope I have learned over my 25 years as a Fellow and, hopefully, some of these thoughts may be of some assistance to you as well.

By becoming a Fellow today you have demonstrated a number of very important qualities. First, it is clear that you are all a bunch of masochists. There is certainly no other explanation for why you would go into a profession that requires such a long period of training and so many difficult exams to pass. However, you've also demonstrated how bright you are, and how hard working you are to be able to master the material and pass the exams. You've demonstrated what a good memory you must have because all too many of the exams require you to regurgitate a lot of material. You've shown that you can manage your time extremely well, both in preparing for the exams, which is quite a challenge, and in taking them. You've clearly demonstrated that you have excellent mathematical and analytical abilities, and that you have amassed a great deal of knowledge about all aspects of the insurance business. The good news is that your hard work has been recognized by your profession today and that you are now a fully credentialed member of the actuarial profession. The bad news is that the hard part is still ahead of you, that is: To become a good and successful practicing actuary.

What I'd like to do in the next few minutes is to share some thoughts with you and hopefully help you a bit in your progress to become a more successful actuary. I want to share a few dos and don'ts and offer some thoughts on each of them. Let me begin first with some of the don'ts.

We've all used the expression that a little knowledge is a dangerous thing. Well, you have amassed a great deal of knowledge about the insurance business in preparing for the exams, and that, in fact, can be an even more dangerous thing. Don't ever believe that you know everything about the business. No matter what your type of employment, you will be interacting with many other insurance professionals from other disciplines. Each of them has developed an area of expertise and each has a perspective to offer you which may be extremely valuable. Speak to them, respect their knowledge, and make yourself a better actuary as a result.

Second, in dealing with these people don't assume that others understand actuarial jargon. The thing that turns off non-actuaries fastest is when someone begins to speak to them in technical terms that they don't understand. Take the time to communicate in language that is understandable and is commonly used by those people in their disciplines. It is time well worth taking and will greatly improve your working relationship with these people.

Finally, we are in a business that largely involves predicting the future. A consultant once used an expression that I really like and which I have repeated many times. He said, "This is the only business that I know where even *actual* is an *estimate.*" A great deal of our professional work involves trying to predict the future, and, as we know, there is no precise way to do that. The thing to be learned is to try not to be overly precise. You are fooling yourself and you are potentially fooling the people with whom you deal. The best you're ever going to be able to do is to come up with a reasonable range for the quantity for which you're searching. In fact, in some cases, you may only be able to come up with an order of magnitude for that item. That's perfectly fine; that's the nature of the business in which we work. It's better to be forthright with the people with whom you interact, and to help them understand that that's the nature of the insurance business. Trying to be overly precise is a waste of your time and is terribly misleading to the people for whom you're doing the work.

Now let's turn to the positive side and talk about some of the things that you *should* do. The first is to recognize that, as a professional, what you do is to provide a service. The challenge is to identify who your client is. It may be someone in your own organization or it might be a third party. In either event, your task is to understand what that client really wants and to make sure you provide outstanding service and high quality professional advice to that client. This leads to the next "do."

Do make sure that you have effective two-way communications with your client. The first half is to find out what the client wants and the second half is to articulate clearly and carefully to that client what your analysis has indicated. If you don't do both halves, it's unlikely that you will have really satisfied that client's needs or that you will have done a truly professional job.

Next, *do* make sure that you provide vision and sound judgment in your work. Think outside the box. Don't limit yourself to standard approaches or pat answers. In fact, it may be that what you really need to do is answer a question that your client hasn't thought to ask. Get input from everyone who has something to offer in the process. Carefully consider all those inputs. Carefully consider all facets of the problem, and then apply your analytical and mathematical skills to the problem, thinking like a business person not a technician. If you do all that, you'll do a much better job for your client.

Fourth, make sure that you are consistent in everything you do—consistent in assumptions, and even in the framework for your analysis. It makes no sense for you to have an assumption of one number for pricing purposes and a totally different number for loss reserve or profitability analyses, unless there is a very good reason why different numbers would be consistent with each other. For example, you might be using limited losses in pricing and unlimited losses in reserving and, clearly, it would be consistent to have different trend assumptions in those two situations. Even if you are only involved in one aspect of actuarial work, or have responsibility for one area such as pricing, make sure that the results of your analyses are consistent with analyses that others have done and that your assumptions are consistent with the assumptions that they have used. If not, it's likely that one of the analyses is illogical and that you will have problems later on. Talk to the other people who are doing related things; don't operate in an isolated environment. Now, at the same time, consistency is not synonymous with uniformity. As I said before, it is perfectly fine to use different trend assumptions as long as the differences are appropriate for the circumstances for which they are being used.

Fifth, one of the most important things that you bring as a professional to everything you do is your objectivity. It's crucial that you guard against being either an optimist or a pessimist in your work product. Don't be influenced by what you think your customer would like to hear. Tell him what you believe to be the right answer, not what he might like it to be. At the same time, don't try to out-guess your customer and shade your answers to get that customer to do what you really believe is correct. For example, you might know that a particular person tends to regard you as too conservative and, therefore, takes actions which are lower than your recommendations. Don't raise your answers intentionally, expecting him to discount them and do what you believe is the right thing. You will do a far better job, and be a much better actuary, if you develop a reputation of always telling it like it really is, giving your best advice whether the news is good or bad. Obviously, communicating good or bad news is a very important and special skill. However, don't communicate your information in such an obfuscated way that your client can't really figure out what you truly believe. In the long run, clients value actuaries who are fair, unbiased, and can always be counted on to provide the best advice possible.

Having assured that you are consistent and objective, the next important thing for you to do always is to learn from your mistakes. None of us is perfect, although some of you are closer to perfection than others. Always be willing to admit your mistakes to yourself so that you don't repeat them. A good way to deal with this issue is to keep track of your estimates, and look back with the passage of time to see whether or not actual events are unfolding as you expected them. Keep a record of how you've changed your estimate for a particular year or particular line of business, and see whether there has been an unintentional bias, either optimism or pessimism, creeping into your analyses. By doing that you can work harder at maintaining your objectivity and eliminating as many biases as possible. This kind of approach works in almost any facet of actuarial practice, whether you're looking at pricing, at reserves, at profitability, or any of the other things that we are called upon to do. You should look at the record of your estimates and the data which has emerged in the ensuing years and you will learn an awful lot about yourself in that process.

Sixth, as much as you've learned in studying for the examinations and as much as you learn in your day-to-day activities, the world is constantly changing around us. Our business is constantly changing, and new methodologies are being developed by our peers. It's crucial that you maintain your knowledge and get involved in continuing education of all sorts. Stay on the leading edge of knowledge in your specialty, because, in the final analysis, what you are offering is expert advice and you need to make sure that you are truly the expert in the areas that you practice.

Finally, let me give you an advertisement for the CAS. *Do* give something back to the actuarial profession. As busy as you are, you owe it to the profession to take time out and share your efforts with your fellow actuaries. As you go through your work you will be developing new methodologies, learning new things, conducting research, doing all sorts of things that would be invaluable for your peers to learn about. Similarly, if you want to improve yourself, you would like your peers to be sharing their developments with you. Obviously, all of this is subject to legal constraints and the confidentiality of proprietary information.

In addition, the CAS is largely run by our volunteers. Take time out of your busy schedule to give some of your time back to the CAS.

Serve on committees, become active in the regional affiliates and special interest sections, as well as in the CAS itself, and help the CAS and the actuarial profession grow by sharing your time and knowledge with your peers.

All of this advice is intended to apply starting on Thursday when you return to your office. In the meantime, bask in the glory of what you have achieved, enjoy the next couple of days at this meeting and, above all, congratulations on achieving a very important milestone in your life and in your career. Good luck to all of the new Fellows and the new Associates as well. My suggestions apply to you, also. I look forward to working with all of you in the coming years.

LUNCHEON ADDRESS—MAY 10, 1993

CAN WE BECOME A GLOBAL PROFESSION?

JOHN H. HARDING

I am honored and delighted to have this opportunity to speak to you today. Over the past decade, I have had the opportunity, in various capacities with the American Academy of Actuaries, to work with your leadership. I have always been impressed by their professionalism and dedication to the Casualty Actuarial Society.

When I became a Fellow of the Society of Actuaries in 1965, the SOA resembled the CAS of today, in many respects. It was of similar size and, because of that, we had the advantage of knowing most of our peers and being able to share our successes and frustrations with them. We also were an organization that was tightly stretched in terms of demands on its resources. Today, the SOA has size and resources, but it also has diversity and impersonality.

Although we never leave our actuarial background behind, I traded my actuarial expertise for management responsibility over a decade ago. The actuaries in my company would claim that I am as equally qualified to practice in the casualty field as I am in the life field. I do not take that as a compliment regarding my breadth of knowledge! But I serve the AAA today because of the tremendous benefits the profession has given me and because some of the non-actuarial skills that I have acquired can benefit the profession. My focus has been, and will remain, on our profession, rather than on my affiliation to any one organization.

I am pleased and delighted with the subject matter of this meeting. While it is essential that we understand the basic skills needed to do our job and that we continually enhance those skills, true effectiveness comes when we understand the businesses we support, our obligations to those businesses, and the methods our profession can use to provide services in the most effective ways possible.

The theme of my talk today is a question: Can we become a global profession? This question is being addressed by the McCrossan Group, which meets here in Dallas this week. I would like to tell you about this group, identify our agenda, and discuss some key issues in it. I would like to finish by giving you my impression of the implications of that agenda for the future of the CAS.

The McCrossan Group, as we currently call it, was the idea of Paul McCrossan, past president of the Canadian Institute of Actuaries. He invited the leaders of the English-speaking actuarial organizations to meet just prior to the International Actuarial Association (IAA) meeting in Montreal last year. The purpose was to find the issues we may have in common and how we might address them. The result of that meeting formed the basis of the agenda for the meeting of the McCrossan Group here in Dallas this Thursday. We will focus on the needs of a global actuarial profession.

What were the forces that started to build a consensus within that group that we needed to be concerned with a global actuarial profession and how it might develop? For the Europeans, it was the evolution of the European Community and the protocols which mandated cross-border practice. For those of us in the U.S. and Canada, the implications of cross-border practice under the Free Trade Agreement were being considered and addressed. In fact, because of the cross-border implications of our existing organizations, we had developed a model that could readily support such practice.

The extension, however, to the NAFTA treaty, which may potentially add Mexico to the mix, was a far different matter. We knew very little of the Mexican actuarial organizations or of their level of professional development. As a result, the comfortable arrangements made under the Free Trade Agreement became potentially much less comfortable under NAFTA.

In the U.S.-Canadian context, we understood the answers to the questions of who is an actuary, in what fields could that actuary generally practice, and what would be the qualifications for signing statutory statements of opinion. But we knew nothing of the answers to these questions in the Mexican context.

One thing became very clear, however, under both the Free Trade Agreement and NAFTA: only one organization in each country could be identified as the actuarial organization to represent all actuaries. In the U.S. context, the decision was reached, after much discussion and thinking by each of the organizations, that the Academy is the only organization that could represent all U.S. actuaries.

But there are many other forces beyond free trade treaties that require examination of the possibility of a global profession. Within the insurance industry itself, we have a possible evolution toward international insurance businesses or, at least, joint ventures. Outside of our industry, we have many international companies that have a very difficult time in finding a coherent structure of insurance protection where it is needed. And, perhaps because I've been intimately involved with it for almost two years, I need to mention the issue of solvency. We use remarkably different methods throughout the world to address the general question of solvency management. In the first McCrossan Group discussions, one of our emerging fears was that the actuaries in one country would declare a given solution impossible or impractical, while, concurrently, the same solution would be applied in another country. We remain ignorant of those differences at our peril.

If these are the opportunities and problems of cross-border practice, what are the core questions that we must answer?

The first is, in my opinion, the essential one: who is an actuary? What are the basic credentials that can be accepted across borders? Addressing this question exposes a very difficult problem. Most of the English-speaking actuarial organizations are examination-based. We generally have rejected the notion that an alternate track toward actuarial recognition would be acceptable. Yet, most of the rest of the world either has a university-based system, a system which will accept either track, or, in some cases, no system at all.

The second question is: when is that individual we have identified as an actuary qualified to do general work within a specific practice area? Let's take a random example: Property and casualty. Only in

North America do we find a separate organization that focuses on the casualty business separate from life, pension, and health. In both the exam-based and university-based systems elsewhere, general insurance and the other specialties are considered part of the same basic education.

The next questions focus on statutory practice: what are the qualification standards necessary to sign required statements of opinion? What are the education and experience requirements of such practice, and how can they be made available to actuaries qualified in a different country?

The final core questions deal with codes of conduct. What organization has the responsibility to determine standards of practice within a given jurisdiction, and who is responsible for discipline when standards are breached? What impact would such disciplinary procedures have in the actuary's home country?

So far, the answers to most of these questions, as discussed by the members of the McCrossan Group—directly and in their home countries—tend to favor the current U.S.-Canadian model. But these are just the core questions, and on the McCrossan Group agenda are some broader questions that have yet to be thoroughly discussed.

Can there be global actuarial education? Let me suggest a hypothetical answer, based upon the 80-20 rule. However, I would suggest that this rule might be a 60-20-20 rule. There could be a generic level of actuarial education that might adequately cover 60 percent of what might be considered basic combined actuarial knowledge. However, there is probably an overlay of another 20 percent that is practice- or specialty-related. And finally, another 20 percent that is nation- or culture-specific. And, if that is the case, does it make sense for us to try to develop more uniformity in that 60 percent and make it a global core?

If we can do this with regard to education, can we not also approach it for research?

Next, we have a global supply-and-demand question. This supply and demand can be nation- and practice-specific. In Mexico, for example, demand has been broadened significantly through the recognition of the value of actuarial techniques through broader segments of financial institutions, including banks. Mexican actuaries have achieved solid progress in other areas not yet addressed in this country. The cross-border implications tend to deal with supply problems, so we may want to focus on learning from other countries where the influences of the actuary can be extended. Can we provide education and research to support that extension internationally?

And what do we do about countries where there is no formal profession but where such is needed? Do we provide guidance and support or should we let nature take its course? The answer cannot be "none of the above."

Finally, I would be remiss if I did not acknowledge that there is one global actuarial organization, the IAA. It is an organization which the North American actuaries have not heavily supported. But the pursuit of the global actuarial profession implies the need for some form of sponsoring organization. Can this pursuit be done within the boundaries of the IAA or must we go outside? The IAA, for example, does not understand the need for separate organizational structures that exist in the United States and Canada. For that reason, our organizations are significantly under-represented, particularly in view of the number of our members. We are generally looked upon as a source of funds but not as a source of leadership.

Now, let me speculate on the implications for the CAS that arise from the McCrossan Group's agenda.

Since the CAS is the only organization in the world specifically oriented to the property and casualty business, you will have many crossroads decisions. You have limited resources that you apply very effectively to research, basic education, and continuing education. However, there is a question as to how far those resources can be stretched.

Do you continue to focus your efforts on North America only and let the rest of the world take a different, more combined approach? Do you focus on providing exam support for emerging actuarial organizations in the rest of the world? How do you continue to define and retain exclusivity in the property and casualty world if the rest of the world is non-exclusive?

As you look toward the extension of the profession into other related fields, do you play a role? Where these options for non-traditional practice appear, do you support those options or do you concede those to others? Should you consider joint ventures that focus on the 60 percent basic actuarial core and be pre-eminent in the specifics of the casualty business?

None of these are easy questions, nor do they lend themselves to facile answers. But in my opinion, they are questions that can be answered as part of a careful and well-defined strategy in which your resources are deliberately and judiciously focused.

That strategy can succeed, I believe, if the benefits of your specialization and your intense knowledge of the casualty field can provide a superior product in one form or another to the rest of the world. I believe that's what you can and do provide—that you have great strengths as an organization and a great deal to offer. But I must close today with one cautionary note. Unless you will be content to have it done for you, you will need to answer these questions and define your strategy within the next few years.

KEYNOTE ADDRESS—MAY 10, 1993

GOVERNOR RICHARD LAMM

I would like to begin with a parable about a friend of mine who is a foreign service officer in Lima, Peru. As you know, those are hard living, hard drinking jobs. One night, after a particularly long evening, he was at an embassy when some beautiful music started up. Across the room he saw a lovely figure in a red velvet gown, so he went and asked her to dance. The answer was, "No, for three reasons: Number one, you are too drunk. Number two, this is the Peruvian National Anthem. And number three, I am the Archbishop of Lima."

I call this a parable because I ask you not to confuse me with someone who wouldn't like to do everything for everybody that medical science has invented. But let me suggest to you that my generation of public policymakers has incredibly compromised the life of our children.

I am increasingly convinced that ultimately it isn't companies that compete, but societies. And those societies that best educate their children, motivate their workers, settle disputes between citizens with a minimum of litigiousness, and deliver health care efficiently; those are going to be the societies that win.

In this question of health care, we have a whole new world. It is simply unsustainable. No trees grow to the sky. No element of our budget can grow at two-and-one-half times the rate of inflation. The average citizen in your state pays three times more for health care than he pays in total state taxes which fund the prison systems, the universities, and everything else. So this is simply an unsustainable curve.

When I entered politics in 1965, we spent the same amount of money on health care as we did on education. Today we spend the same amount on health care as we do on education, defense, prisons, farm subsidies, food stamps, etc. Health care is becoming the Pac

Man of all our public and private budgets, eating up the ability to do anything else in our society.

For most of our 200 years in America, medicine was practiced according to Voltaire, who said that the role of the doctor is to amuse the patient until nature affects the cure. But today we live in the day of the bionic man and the bionic woman. It is absolutely incredible what we can do to the human body. We are told by the health futurists that the average young person in this room will, sometime in the next century, meet a human being that has 50 percent of his body weight in bionic parts. There are now 24 bionic parts that we put into the human body, all the way from artificial hearts and artificial hips to contact lenses and pacemakers. There are 24 different procedures, and we are on the threshold of literally hundreds more. We are mapping genes, we're doing the autoimmune system; we have all of this medical technology. I was in Japan looking at their health care system, and I saw a wrist watch which, if I were wearing it when I had a heart attack, would automatically dial 911 and tell the ambulance where to find me.

The miracles of medicine have simply outpaced our ability to pay. The creative genius of American health care has simply run faster than our wealth and ability to keep pace. When I got out of law school we spent six percent of our Gross National Product on education, six percent on defense, and six percent on health care. Today we spend six percent on education, 4.8 percent on defense, and 14 percent on health care, and it is growing at two-and-a-half times the rate of inflation.

Look at the situation with Medicare. We are simply one recession away from insolvency in the Medicare trust fund. Nobody is projecting it is going to last that far into the future. The average person retiring in 1990 on Medicare would have contributed less than $3,000 to the fund, and his actuarial benefits would be somewhere in the range of $28,000. You can't deny that this is not a sustainable system.

At the same time, the average American is making less money in 1993 than he made back in 1973. When I entered the Colorado legislature in 1967, we were doubling our per capita wealth every 30 years. It is wonderful to be in politics when you are doubling your per capita wealth. We come from a tradition of plenty; a tradition that opposes any kind of limits. Now, however, the only way the average family can make any more money is to have more members of that family working. The average hourly wage has not increased at all.

Essentially everything that my generation of public policymakers has tried to control health care costs has failed. At the same time, my generation was inventing wonderful things in the scientific area. You can now have your gallbladder out in the morning and be at your aerobics class by evening.

We started with regulation. Typical politicians, start with regulation. To our absolute horror, we found that health care costs rose faster under regulation than they did before we started regulation.

So we moved to competition. We said we would let Adam Smith's invisible hand control health care costs. To our absolute horror, we found that towns with two hospitals have higher hospital costs than towns with one hospital. The more doctors you add to society, the higher the cost of physician services. Generally, the more specialists you have in an area, the higher the cost of that specialty. There is no evidence that competition, so far, has been able to control health care costs. One of the major reasons why competition doesn't seem to work is that most of us don't pay directly for health care. It is an economic maxim that people don't value what they don't pay for. Nine-tenths of all hospital bills and three-quarters of all physician fees are paid for by third party payers. That does not give us an incentive to shop.

So we went to DRGs: Diagnostic related groups. Never have health care costs risen faster than since we did DRGs.

Then we tried HMOs. If we all lived in the Dallas area and were members of an HMO, we would probably save money individually. But, the ability of health care providers to shift costs is so awesome, that Dallas would not save any money. This has been very well documented. An HMO will save subscribers money, but the money that is saved will be transferred to other parts of the system through cost shifting. The net result is that society doesn't save any money.

Let's look at defense spending. I am a Democrat; the Democrats are going to solve all of the world's problems by cutting defense spending. Well, I am for cutting defense spending, but even if you cut it in half, the rise of health care costs will eat up that peace dividend in three to four years. Health care is a $900 billion enterprise and we spent about $290 billion on defense. Cutting defense spending is not an answer.

What about technology assessment? I spent some time at the University of California Medical School trying to figure out what technologies we have discarded. The dilemma of medical technology is that most of it is additive. It does not substitute for something else, but rather it adds on to the array of things we can do. The X-ray machine is here, and we bring in the CAT scanner, then the MRI machine, and now we've got the PET scanner on its way. There are some substitutions; for example, a lithotripper will take the place of 60,000 surgical interventions. But the dilemma of medical technology is that it does not save us money. Welcome to this new, very upsetting world of health care costs.

When I was in Europe, I came across a study showing that the lifetime health care costs of smokers were substantially less than the lifetime health care costs of non-smokers. I asked, "How could that be? I have 53,000 employees, and every year smokers cost us more." The answer was, "Yes, every year smokers will cost you more. But *lifetime* health care costs of smokers are substantially less than those of non-smokers. Smokers generally die after the first or second disease, while the rest go on and have five or six diseases."

There is a curse that goes with our medical technology: it does not save us money. My wife had breast cancer. I am immensely grateful to our health care system and its technology and to the hospitals and the doctors. But, and I don't mean to be callous, in an actuarial sense, cured (which is the finest word I've ever heard) means "alive to die later of something else." So we've gotten ourselves into a dilemma; we have reduced mortality, but we've increased morbidity. We have had an incredible victory over acute diseases only to throw ourselves into the arms of chronic diseases.

There is a wonderful study published by the Population Reference Bureau that asked whether it would be cost effective to cure cancer, heart disease, or automobile accidents. Their findings, and I'm sure they can be argued with, were that it would not be cost effective to cure either cancer or heart disease. Why? It is because they occur, generally, when we are at or past retirement, whereas automobile accidents kill so many young males.

Does that mean I want to stop fighting cigarettes? Hell no, I don't want to stop fighting cigarettes. And it doesn't mean that I don't want to find a cure for cancer. But, we have to do this with our eyes open, recognizing that our medical miracles are too often fiscal failures. We do these wonderful things which end up prolonging life, and that is going to require us to make some tough decisions somewhere along the line. I have an 85-year-old father and he is in wonderful health. I am not arguing against this. We have added 28 years to human life expectancy, and that's wonderful. My point is not to argue against it, but to warn of its consequences. Nothing my generation has ever done will diminish in any way this volcanic rise of health care costs.

I am convinced that the United States is an immense Gulliver. We are held down by the very institutions we have set up. I look around and I ask, "What is working like it used to? Our education system doesn't educate, our health care system doesn't give us as good health care like Europe or Great Britain or Canada or Japan, and our legal system—nobody needs as many lawyers to resolve disputes among their citizens; nobody has more elected politicians in office."

So I am spending the rest of my life saying, "How do we take some of those chains off?" I think that the next new deal is to increase the efficiency and effectiveness of all of our institutions, starting with health care. This is what Bill Clinton said as governor back in 1985. He said more U.S. children die at birth, more of them die in their first year, more of them are born with low birth weight, more of them have avoidable mental and physical problems, more of them drop out of school, more of them have drug and alcohol problems, more of them wind up in prison, and more of them go into the adult workplace unable to read and write than in any major country. That's what we as a society are leaving our children.

Now where are we going? In the current system, I go to a doctor, the doctor gives me very good care, and he is paid through your organizations—the 1,500 health insurers that feed the system. One of the dilemmas is that nobody can get any leverage on the excesses of the providers. This is what was attempted by utilization review. This is where we are headed.

This is managed competition. It doesn't mean that we are not going to come out of this process with a Canadian system, but I think most of us know that there is probably going to be some version of managed competition. I continue to go to providers of care, and they continue to give me very good care. But there is now a health insurance purchasing cooperative (HIPC). HIPCs will aggregate enough lives and get enough leverage on the system to ask embarrassing questions like, "Why the hell do we have 50 percent of the hospital beds in Colorado empty? Why do we need 124 cardiologists in Denver? Why are you doing some of the things that you are doing?"

Note that the insurance function in this plan is moved to a higher level. The HIPC is an independent purchaser. Its role will be to evaluate various provider plans, and the provider plans will include the insurance function. Providers under managed competition will have to share the risk. This is key to avoiding cost overruns. An analogy here would be Kaiser Permanente where you pay a certain amount every month whether you have nothing done to you in a year or you have a heart transplant; you still have the one payment.

There are six cities that have large model HMOs that cover a substantial number of residents. I have looked at these cities and at Europe for some evidence of the ghost of health care future. What is the ghost of health care future; where are we going; and how is it going to affect your professional lives?

I would suggest to you that when you look at what Europe does and at what happens in those six cities, the first thing you notice is that there is a tremendous shrinkage of suppliers. Other societies control health care costs by limiting supply. One of the things we can do is limit supply—of doctors, specialists, medical technology, hospitals, hospital beds, and ICUs. Let's take them in order.

Right now, there are at least 50,000 too many doctors in the United States. And we are continuing, through 126 medical schools, to turn out 16,000 additional doctors a year. Eli Ginsberg, one of the grand thinkers in the health care area, said if we had increased physicians from 140 per 100,000 people in 1962 to 190 per 100,000 in 1990, instead of what we actually did which was to increase them to 250 per 100,000, we would have had a potential savings of $173 billion out of the total of $660 billion. Once a hospital is built, once a doctor is trained, once infrastructure is in place, it simply has to be paid for.

This is what other societies do. They try to limit the number of doctors they train and to use those doctors much more efficiently. That also happens in the U.S. in those six cities. HMOs operate with about 1.2 physicians per 1,000 people compared to four to five in fee-for-service medicine. Now, if you are a cardiologist practicing in America, this number is going to scare the hell out of you because you see that Kaiser and Group Health and these other places use physicians much more efficiently.

As to the question of specialists, most countries train no more than 30 percent of their doctors as specialists; we train 70 to 80 percent of our doctors as specialists. There is going to be a new world of health care after we wake up to the fact that it is crazy to train 70 to 80 percent of our physicians as specialists.

Bill Kishick at the University of Pennsylvania asks, "What if we could serve all of America like Kaiser serves America?" I understand we can't serve Texas like we can serve San Francisco or Los Angeles, but let's play around with these figures. Kaiser serves 6.8 million subscribers with 76,000 physicians at a cost of $9.8 billion. If that could be done nationwide, 38 Kaisers would provide all the primary and secondary care to 250 million Americans with 290,000 physicians, which is less than half of the physicians we have right now, at a cost of $372 billion, or five percent of the GNP. I don't mean to say that this shows that we can provide health care for five percent of GNP, but it does show that how you organize your health care system and your physicians has a lot to do with how many you need and hence the total cost of the system.

The same thing applies to hospital beds. Nationwide we have 3.8 hospital beds per 1,000 people. But how we utilize hospitals is changing due to outpatient surgery, low impact surgery, and drug therapy. We probably only need 1.8 hospital beds per 1,000 people; HMOs use 1.5. There are about 5,500 hospitals in the U.S., and I suspect that about 1,000 of them will close over the next four or five years, and then another 1,000 will close over the four or five years after that. Twenty years ago, St. Paul, Minnesota had 17 hospitals. Today it has three hospitals in six different locations. Eleven facilities just weren't needed. I think that this will happen all over America. I've got 53 percent of the hospital beds in Colorado empty. I have schools that are overcrowded, I have inadequate spending on infrastructure and highways, and yet I've got 53 percent of the hospital beds empty. That's like running an airline with 50 percent of the seats empty. It is a dreadful waste of capital and cost to run a system that way.

On any given night we have 925,000 community hospital beds in America and 310,000 of them are empty. But, as you know, there are a lot of people that are in hospital beds that don't really need to be there. There are a number of people who are there because their insurance company will pay only if they are in a bed or because it is for their doctor's convenience.

A hospital should be an institution of last resort. You should not put somebody in a hospital because that is the only way that his insurance company will pay.

This question of hospital beds and how many we need and what it means for the future is really incredible. An indemnity policy uses

519 hospital-bed-days per 1,000 members, staff model HMOs use only 340, the most efficient prepaid plan uses 180 to 200, and there is no difference in outcome. This is from the *New England Journal of Medicine.* The people who use the most efficient plan are every bit as healthy as the people with indemnity policies. In fact, because of problems in hospitals, you can argue that the people with indemnity policies might be less healthy because they are spending more time exposed to the deleterious atmosphere of the hospital.

You have the same thing in centers of excellence. Nationwide, we have 850 hospitals that do open heart surgery. Less than half of those, only 375, do what the federal government says is the minimum number to maintain proficiency. 100 do less than one a week. If you are going to get your heart operated on, you do not want to go to a hospital that does less than one a week. It is insane to have that much excess capacity in a system.

Everything we know about health care is that high volume means high quality and cheaper cost. In Denver, if everyone had their open heart surgery at our most efficient hospital, the one that does the most, we would save $12 million, and we would save a number of lives. So this 7-Eleven theory of hospitals, where everybody wants a hospital on every corner with every marvelous machine and able to do everything, is a luxury we can no longer afford and shouldn't try to.

Let's talk about medical technology. We have six percent of the world's population, with half of the CAT scanners and two-thirds of the MRI machines. I mentioned that my wife had breast cancer. As soon as she was well, she spent a lot of time promoting mammography. Recent studies show that the U.S. has 10,000 mammography machines. How many do we really need at current levels of utilization? Two thousand. If every woman would get a mammogram every time the American Cancer Society says she should, we would need 5,000. So right now we have five times as many machines as we could fully utilize. What happens when you have a machine that you are not fully utilizing? You have to raise the per capita price of its use and that, in turn, drives American women away from getting mammograms. That's public policy malpractice to have five times as many machines as you need, with the net result that you are driving American women away from getting mammograms.

Everybody wants every machine at every hospital. We have kids without Head Start programs, we have all these other needs, and instead of meeting them we are indiscriminately funding this medical arms race that is not doing any of us any good in its excesses.

We know that doctors who own X-ray or ultrasound machines have four to four-and-a-half times as many referrals as doctors who refer to independent providers. So, if a doctor has an interest in a machine, he or she may overutilize it. The cost containment people in Florida found that 75 percent of the imaging centers in Florida had doctors as full or part owners. When you go to a doctor, you should go to that doctor because you want an independent evaluation. You should not have to worry that he is running some business on the side in which he has an economic interest.

Germany does 0.7 open heart surgeries per million people. This rate is very much different than the U.S. rate (3.3) or Canada's rate (1.2). The U.S. is just way ahead and there are studies on a lot of operations that all show this pattern. Hysterectomies, prostate operations, cesarean-sections, tonsillectomies, every place we look there is an incredible variation from place to place with no difference in outcome.

I am intrigued with the question of ICU use. ICU units have about ten percent of the hospital beds in America. When I was in Europe, I found that about three percent of European hospital beds are ICU units. In the U.S., ICU beds are very expensive. Forty percent of our nurses are in the intensive care or critical care area. They make up over 20 percent of the hospitals' buildings.

Other societies ask themselves what will produce the most healthy babies. In the U.S. we spend about $158,000 per neonate. Why are we twenty-second in infant mortality, why are we eighteenth in childhood mortality, why are we eleventh in maternal mortality? I would suggest to you that it is because we spend our money here in the ICU and don't give women proper prenatal care. We fly neonates in helicopters to million-dollar neonatal centers where we put them on these expensive machines, after they are born to women who we didn't bother to give prenatal care to. That's a terrible health loss.

Every woman in Great Britain gets all the prenatal care she needs, and afterward she gets three visits by a community nurse to make sure she is recovering and that the baby is thriving. So they have asked themselves a very difficult but important question—"How do you spend your money to buy the most health for your society?" It is not on helicopters and neonatal intensive care units, but rather on ensuring that every woman gets prenatal care.

In Sweden, they give every woman whatever she needs to have a happy pregnancy—an apartment, warm clothes—whatever. But they don't try to save any baby under 600 grams. They spend their money on prenatal care rather than postnatal salvage; they are first in infant mortality and we are twenty-second. In Canada, they have a lot fewer high technology machines, but no woman goes without prenatal care.

This intensive care question is interesting. In Canada, about eight percent of all inpatient costs are for ICU units, or critical care units; in the U.S. it's 20 percent. What do we get for this? An intensive care bed costs 3.8 times a regular bed and we've got three times as many as any European society. You soon find that we are putting people in intensive care beds who are either not sick enough or are too sick, like 90-year-olds with congestive heart failure. No other society would do that, and at the same time not vaccinate its children or not give its women prenatal care.

I was at the bedside of someone in San Francisco not so long ago, a 92-year-old man who had been brought in with a very serious stroke and also had metastatic cancer of the prostate and was on kidney dialysis. Hovering over him were two specialists probably earning over $400,000 a year. Then we went down to visit the people who serve the doctors, who give them their meals and take in their laundry. We asked them if they had basic health care or health insurance. Less than half of them did. This juxtaposition between what is going on at the margin way up here at the top of the peak and way down there is children without vaccinations and women without prenatal care.

Now some doctors find that offensive. Victor Hukes says that the desire of the engineer to build the best bridge or the physician to practice in the best hospital is understandable. But, a monotechnic person fails to recognize that claims of competing units are the divergence of his or her priorities with those of other people; his advice is likely a poor guide to social policy. In other words, of course a doctor wants to practice in a wonderfully updated hospital, but the more we know about rural health care or small hospitals, we know that it is much better to have an emergency response system getting them into a big hospital where the job can be done, done right, and with less risk to the people. So that gives rise to the question about too many intensive care units, too many doctors, too many specialists, and too many hospital beds.

What are other things that I see coming? Limits on malpractice and lawyers, and gatekeepers. Many societies use gatekeepers like Kaiser Permanente. If parents take a child to Kaiser Permanente, most likely they will be seen by a pediatric nurse or a child health associate of some sort. Eighty percent of what can be done by a pediatrician can be done with equal skill by a child health care associate. So all of the thinking on health care is what is the appropriate level of delivery; you want to make sure you don't overtrain doctors when there are so many things that other societies do with a health team.

We know that every football helmet sold has a 100 percent lawyer tax or litigiousness tax. In fact, it's probably more than this right now. Every stepladder costs at least 30 percent more than you would pay for it in Canada or any other developed nation because of the risk of litigation. Every vaccine is about twice as expensive, and many of them are far more expensive, because of the risk of litigation.

Now, as you know, many doctors think that the only thing wrong with the health care system is the lawyers, and I think it's a terrible thing wrong with it. I think that it is nation-threatening to take 40,000 of our best and brightest young men and women and make them wealth-dividing lawyers. I am actually embarrassed that I am a member of a profession that has such excesses to it. Harvard Medical School and Harvard Law School did a joint study of the New York health care system, and they found that there were 16 times more people injured by negligence in the health care system than collected one dime. There are the million-dollar babies that you hear about and read about in the paper, but when you really see that most people who are injured by negligence in the health care system collect nothing, then you also see that there's a great inequity. I want to correct this system because it doesn't serve people very well. If you only have one out of 16 people who are injured by negligence collecting anything from the system, you have a bad system. But changing this system won't save us that much money.

As to the question of administration, this is where we start to get sensitive to you. We're adding three-and-a-half administrators to the system for every doctor, three-and-a-half white collars for every white coat. NBC had a special on health care which showed a 300-bed hospital in Bellingham, Washington, that had 42 billing clerks sorting out which of the 1500 insurance companies should be billed. Then they went a few hundred miles away to Canada to a 300-bed hospital that had *one* billing clerk. If I'm correct and the future belongs to the efficient, then this is one of the areas that's also going to be looked at, and this of course is where we get into your profession.

When the people that are asking hard questions are saying, "Justify what we're doing now. Why do we need 42 billing clerks? How do we reduce the billing process. We've got electronic transfers...there are a thousand more efficient ways to bill. So why do we pay for selling, advertising, underwriting and billing of health insurance?"

As to actuarial expenses, people are saying, "Look, if we're going to take care of everybody in our society anyway, why do we have to hire an actuary to say that this side of the room is going to be this much greater a risk than that side?" Don't get mad at me; you have to stand in line. The important thing here is not to be accusatory, but simply to say that this is a question that's coming. Under community rating, you're no longer going to compete on the basis of what person can do the best actuarial study. The system is going to be the insurer, remember, and they are asking, "Why should we pay for some of the things that we're now paying for? We want to be able to purchase health care on the basis of how this system provides quality health care for the best price, and we don't want to get into actuarial studies." Other expenses that will be questioned are legal fees relating to health insurance and advertising hospitals. I've never seen a society advertise a hospital anywhere in the world except in the U.S. Advertising a hospital in any other society would be like advertising the fire department.

So I would suggest to you that everybody is at risk with this system. And that's OK. As to the doctors, I think that the specialists are going to be the most at risk. There is going to be a lot of dislocation among hospitals. There is going to be a lot of dislocation among doctors. I think we're going to see the rise of nurses as primary health care providers, as they can do more and more things. But I think that your industry is going to come under some hard questioning. Jeff Goldsmith, who is a health futurist, said, "The Genome Project will destroy the population-based actuarial framework on which our health care and life insurance industry rests." I don't know that I believe this, by the way. I think certainly in life insurance and property/casualty insurance that you will continue to do the good work that you're doing right now. But I do think that when the Genome Project is going to be able to tell us so much about what we're going to get sick of when, it will open up a lot of questions about health insurance.

The most expensive piece of medical technology is a doctor's pen; 70 percent of health care costs comes out of this pen. And we know that the pen writes too often. Nine hundred thousand cesarean-sections are performed in the U.S., and it's considered that about half of these are unnecessary. About 24 percent of births are cesarean-sections now. At Kaiser Permanente it's about seven to eight percent.

This is one of the effects of defensive medicine. When you start to look at why it is that we're doing so many more cesarean-sections here than in other countries and in these HMOs, it raises some tough questions.

When you start asking tough questions about the system, you not only take on the lawyers and the insurance companies, but also the doctors. You ask yourself, "Why do we pay that much money?" A doctor should earn more money; I don't dispute that. I also don't think this is one of the biggest problems in health care costs, but you do notice the doctor's salaries come under much more pressure when you get a system that asks embarrassing questions.

In the U.S. a great deal of health care delivery is still in solo practices, whereas in most other countries and in these large group health plans, there is a team approach. Other countries found it much more efficient to put together a whole group of ancillary physician extenders.

Also in the U.S., we seem to do simultaneous diagnosis. You put somebody in the hospital and you do everything possible, whereas other societies do these things sequentially until they find out what the problem is. Most countries say, "We're happy with the most likely diagnosis," whereas in the U.S. we keep looking and trying to exclude any possibility.

There are 90 million people in the U.S. that visit their doctors with headaches. If we gave every one of them an MRI or a CAT scan, that would probably add $2 billion to $3 billion on top of what we already spend. Would it be worth it? Well, we would pick up an occasional brain tumor, but the kind of question that we're going to have to ask ourselves is, "Look, in a world of limited resources, does it make sense to give everybody with a headache a CAT scan?" Because when you look at American health care, this figure hits you in the face—we spend 70 percent of our health care costs on about 10 percent of the population. No other society would spend this much. We spend 30 percent of our health care costs on the sickest one percent of the population. Everybody spends more on sicker people, obviously, but nobody spends as disproportionately on the sick as the U.S.. When you look at American health care, you find that we've been doing more and more to fewer and fewer people at higher and higher costs for less and less benefit.

The fastest growing use of kidney dialysis is on people over 85. The second fastest growing use of kidney dialysis is on people between 75 and 84. Now, I have children and most of you have children, and we have to ask ourselves what we want to do. As we age, our bodies are fiscal black holes into which we can pour all of our children's future. We can leave them without anything. It does not make sense to me that the fastest growing use of kidney dialysis is on people over 85. I would suggest to you that in everything that isn't age-specific (a sterilization would be age-specific, for instance, or birth control), almost all of our medical technology works its way up the age ladder, and it has its biggest use at the end of life. And I question that.

There is a woman named Rita Green who was a nurse and fell into an irreversible coma the year that I was a sophomore in high school. Rita Green is still being kept alive in Washington, D.C., a town that has an infant mortality rate greater than Guatemala. Rita Green fell into an irreversible coma on October 25, 1952. For over 40 years, she's been kept comatose, vegetative, with no hint of recognition or anything else. This is what Victor Hukes calls the flat of the curve of medicine. He says when you start spending money in health care, you buy a lot of health for your dollars when you give basic health care. But he says that the dilemma of American health care is that you soon get up here on what he calls the flat of the curve of medicine; this is where you're doing fetal monitoring, or chemotherapy to somebody who is 85 years old with metastatic cancer, something no other society would do. There are many things that we're doing at the margin that other societies don't do. When you get a health care system that starts to ask questions, it will first ask, "What are we doing in the health care system that is harmful?" And there are some things we're doing that are downright harmful. Then they're going to ask, "What are we doing that is therapeutically useless or futile?" And a search for futile medicine is already on; there's a lot that we're going to do there.

The real hard ethical dilemma that we're going to face in our future is that there are some things that are clearly beneficial but we just can't afford to do them. In France they did a study that asked, "What would it cost to give all the health care that is beneficial—what they call the American yardstick?" The answer was five-and-one-half times the French Gross National Product. We've got a yardstick that's going to bankrupt us.

So welcome to the brave new world where, with limited resources, the explicit decision to pay for one procedure for one individual is an implicit decision *not* to pay for another procedure for another individual. To govern is to choose. And in a world of limited resources, we're going to have to start making some terribly difficult decisions. We have to recognize that we're living in a world of trade-offs, which is very upsetting.

A very good health ethicist says that this is the same as making the doctor a double agent or removing the Hippocratic oath from his waiting room and replacing it with a sign that reads, "Warning all ye that enter here. I generally work for your rights and welfare, but if the benefits to you are marginal and the costs are great, I will abandon you in order to protect society."

I tell you this against my own self-interest, but let me try because I feel that it's going to have to be dealt with. We do not want to make a doctor a double agent, but doctors do wear more than one hat. There's no reason that I can't ask doctors to help me sort out what we should be paying for and what we should not be paying for. There is no reason that a doctor can't be a patient advocate at the bedside and at the same time say, "Look, this is futile."

I was just at the bedside of a person in California, a drug dealer and a drug user. They would fix him up, then he'd go out on the street and use dirty needles again. He was HIV-positive. He'd soon get a terrible kidney infection or a liver disease, so he'd go back to the hospital, where they'd fix him up again and put him back on the streets. At some point we're going to have to ask, "How much can my bad habits tap your pocket?" When you get emphysema, the first thing to do is to stop smoking. If you don't stop smoking, what is our duty to you? We had a woman at Denver General not too long ago. We had given her three new valves in her heart. She was a drug user, and she had drugs blow out these valves. She showed up the fourth time, and they refused to treat her. At some point we're going to have to start making some of these decisions, where doctors are going to have to help us decide what is futile and not worth paying for.

It will be a brave new world of health care. We have to balance preventive medicine with curative medicine; improving quality of life with extending life. Young versus old, high-cost procedures for a few versus low-cost procedures for many, high-technology medicine versus basic health care, and health care versus those other things we have to do to leave our children a decent life.

A great example of this—an actuarial delight—is in Oregon. They said, "We're not going to pay for transplants until we give all women prenatal care." A seven-year-old boy named Hobie Howard came along, didn't get a transplant and tragically died. He died on the front page of all of our newspapers with stories about this terrible state, Oregon, and what they were doing in terms of prioritizing medicine. Around the same time, California voted to pay for transplants. Their politicians weren't going to make any hard choices. One week later, they knocked 270,000 low-income women off of Medicaid.

There are three studies in the *New England Journal of Medicine* showing what happened to these 270,000 women. Which state killed the most people? Which state caused the most mortality and morbidity? It wasn't Oregon. Just with the hypertension cases alone, California killed nine people, but these people didn't die on the front page of our newspapers. I normally don't quote Stalin, but he once said, "One man's death, that's a tragedy. A million men's deaths, that's a statistic." And you know, in a terrible way, he was right. Those statistics are no less; they were live American women who had dreadful things happen to them because the political system couldn't afford to make the decision.

I would suggest that this is where we're headed. An economically sound health care system must first give all Americans access to some base level of health care. But to do that we have to have a means of limiting those procedures that are ineffective or are marginally effective, and those that are effective but just too expensive. For example, I would suggest that if you're older than 85 we should not consider you for a heart transplant. We have a bigger duty to a 10-year-old in a transplant situation than we do to a 90-year-old. We have to have some consensus.

I don't expect you to take my word; we have to have a dialogue about this. There has to be some consensus about what those priorities are. Then we need limitations on the lawyers. We need some control of bureaucracy, and then some control on the supply side of health care, either competition or regulation. We have to make something work to get rid of that excess capacity. We also have to take on the issue of the elderly versus the young. We've got Leona Helmsley on Medicare and we've got children without vaccinations. Five hundred thousand millionaires get a Social Security check every month, and yet when you look at the younger people in this room, what they're going to get back from Social Security is just terrible.

These are sacred cows, but there's another issue that the elderly are 12 percent of America and they get 61 percent of our federal social spending. We give our money to who lobbies the hardest, not to who needs it the most.

We are a great pioneering society, and that's wonderful, but we also love technology. I love technology, but I think that we've got too much duplicate technology. We are going to have to ask ourselves some tough questions about what we can do and what we can't do. Here is one of the biggest things we haven't talked about. The American public seems to feel that they want all the health care somebody else's money will pay for. They want to have it all. We are a society that wants health care without taxes, and we want government without taxes and education without work. I think we have a real problem just in terms of attitude. But, we know that health care costs can't go on growing at two-and-one-half times the rate of inflation.

I end with this. Howard Nemerov was America's poet laureate. He died last year. He said, "We praise without end the go-ahead zeal of whomever it was that invented the wheel, but never a word for the poor soul's sake who thought ahead and invented the brake." If we're not going to bankrupt our children, we simply have to find some brakes.

MINUTES OF THE 1993 SPRING MEETING

May 9-12, 1993

THE LOEWS ANATOLE HOTEL, DALLAS, TEXAS

Sunday, May 9, 1993

The Board of Directors held their regular quarterly meeting from noon to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., there was a reception for new Associates and their guests. A short presentation about the CAS Committee structure was given.

A reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

A dinner for the Board of Directors and the members of the Executive Council was held from 7:45 p.m. to 10:00 p.m.

Monday, May 10, 1993

Registration continued from 7:00 a.m. to 8:00 a.m.

President David P. Flynn opened the meeting at 8:00 a.m. The first order of business was the admission of members. Mr. Flynn recognized the 101 new Associates and presented diplomas to the eleven new Fellows. The names of those individuals follow.

FELLOWS

Bruno P. Bauer	Francois Dumas	James W. Haidu
Martin L. Couture	Bradley C. Eastwood	Joanne I. Jaeger
Kevin G. Dickson	James E. Fletcher	Gordon L. Scott
Michel Dionne	Louis Gariepy	

ASSOCIATES

Rhonda K. Aikens	William P. Ayres	John A. Beckman
Craig A. Allen	Timothy J. Banick	Douglas S. Benedict
Scott C. Anderson	Philip A. Baum	Richard F. Burt, Jr.

John F. Butcher, II
Michael E. Carpenter
Benoit Carrier
Michael T. Curtis
David J. Darby
Karen L. Davies
Marie-Julie Demers
Shawn F. Doherty
Ronald R. Earls
Matthew G. Fay
John R. Ferrara
George Fescos
Kai Y. Fung
James E. Gant
Mary K. Gise
Donna L. Glenn
Marc C. Grandisson
Bradley A. Granger
Paul James Hancock
Timothy J. Hansen
Matthew T. Hayden
Lisa A. Hays
Barton W. Hedges
Noel M. Hehr
Mary B. Hemerick
Suzanne E. Henderson
Thomas H. Highet
Bernard R. Horovitz
Vincent H. Jackson
Patrick C. Jensen
Kurt J. Johnson
Mark R. Johnson
Steven A. Kelner
Joseph P. Kilroy
Craig W. Kliethermes
Terry A. Knull
Elizabeth Kolber
Howard A. Kunst
David L. Larson
Michel Laurin
Thomas L. Lee
Scott J. Lefkowitz
Elizabeth A. Lemaster
Deanne C. Lenhardt
Richard S. Light
Daniel J. Mainka
Stephen N. Maratea
Kelly J. Mathson
Robert D. McCarthy
Richard T. McDonald
Conrad O. Membrino
Paul A. Mestelle
Michelle M. Morrow
Timothy O. Muzzey
David Y. Na
Mark Naigles
Peter M. Nonken
Melinda H. Oosten
Nathalie Ouellet
Charles C. Pearl, Jr.
Edward F. Peck
Karen L. Pehrson
Daniel C. Pickens
Cathy A. Puleo
Eduard J. Pulkstenis
Mark S. Quigley
Frank J. Rau, Jr.
Thomas O. Rau
Andrew T. Rippert
James Joseph Romanowski
James B. Rowland
Kenneth W. Rupert, Jr.
James V. Russell
Stephen Paul Sauthoff
Letitia M. Saylor
Michael B. Schenk
Gordon L. Scott
Jeffrey S. Sirkin
Michael J. Steward, II
Brian M. Stoll
Katie Suljak
Todd D. Tabor
Christopher Tait
Yuan-Yuan Tang
Patrick N. Tures
Charles E. Van Kampen
Marcia C. Williams
William M. Wilt
John S. Wright
Gerald T. Yeung
Claude D. Yoder
Barry C. Zurbuchen

Mr. Flynn introduced Phillip Ben-Zvi, a past President of the Society, who addressed the new members.

Mr. Flynn introduced several guests: Morris W. Chambers from the Canadian Institute of Actuaries; John H. Harding, President of the

American Academy of Actuaries; David G. Hartman, President-Elect of the American Academy of Actuaries; James R. Kehoe from the Society of Actuaries in Ireland; Peter Milburn-Pyle and John E. Rich from the Actuarial Society of South Africa; and Terry G. Clark and Nigel R. Gillott from the Institute of Actuaries (United Kingdom).

Alice H. Gannon, Vice President-Programs and Communications, presented the highlights of the program.

Ralph Blanchard, member of the Committee on Continuing Education, presented a summary of the Discussion Paper Program.

David N. Hafling, Vice President-Continuing Education, summarized the new *Proceedings* papers.

Mr. Flynn concluded the business session by calling for any reviews of the *Proceedings* papers. Since there were none, the session was concluded at 9:00 a.m.

After a short break, Mr. Flynn introduced former Governor of Colorado, Richard Lamm. Mr. Lamm delivered an address on the issues facing us in health care reform.

Two panel presentations followed. One was "24 Hour Coverage", moderated by Michael A. McMurray, Consulting Actuary, Milliman & Robertson, Inc. The panel members were: Keith Bateman, Director of Policy Research, Alliance of American Insurers; Barry I. Llewellyn, Vice President and Actuary, National Council on Compensation Insurance; and Gary Weeks, Insurance Commissioner, State of Oregon. The other panel, presented simultaneously, was "Insurance Fraud—Remedies". It was moderated by Daniel J. Johnston, President of Automobile Insurers Bureau of Massachusetts and Executive Director, Insurance Fraud Bureau of Massachusetts. The panel members were: John B. Conners, Executive Vice President and Manager, Personal Market Department, Liberty Mutual Insurance Company and Thomas Harrington, Supervisory Special Agent, Economic Crimes Unit, Federal Bureau of Investigation

A luncheon followed from 12:15 p.m. to 1:30 p.m. John Harding, President of the American Academy of Actuaries, addressed the group with thoughts on the Global Actuarial Profession and what it means to the CAS and the Academy.

Following lunch, the remainder of the afternoon was devoted to presentation of the discussion papers, the *Proceedings* papers, and ten panel presentations.

The new *Proceedings* papers were:

1. "Empirical Testing of Classification Relativities"

   Author: Roger M. Hayne, Consulting Actuary
   Milliman & Robertson, Inc.

2. A Discussion of "Parametrizing the Workers' Compensation Experience Rating Plan"

   Author: Howard C. Mahler, Vice President and Actuary
   Workers' Compensation Rating and Inspection
   Bureau of Massachusetts

3. "Injured Worker Mortality"

   Author: William R. Gillam,
   Assistant Vice President and Actuary
   National Council on Compensation Insurance

4. "Surplus: Concepts, Measures of Return, and its Determination"

   Author: Russell E. Bingham, Director of Corporate Research
   ITT/Hartford Insurance Group

5. "Rate of Return: Policyholder, Company and Shareholder Perspectives"

   Author: Russell E. Bingham, Director of Corporate Research
   ITT/Hartford Insurance Group

The Discussion Papers presented were:

1. "Professional Ethics and the Actuary"

   Author: Sholom Feldblum,
   Liberty Mutual Insurance Company

2. "The Ethical Development of Actuaries"

   Authors: Charles S. White, MSI Insurance
   Richard V. Atkinson, MSI Insurance

3. "The Actuary as Strategist"

   Author: Sholom Feldblum,
   Liberty Mutual Insurance Company

4. "Strategic Issues: Moving Beyond the Point Estimate"

   Authors: Aaron Halpert, KPMG Peat Marwick
   Simon J. Noonan, KPMG Peat Marwick

5. "Directing Actuaries in a Localized Environment"

   Author: Roy G. Shrum, Hanover Insurance Companies

6. "Building, Structuring, and Managing an Actuarial Staff"

   Author: Gregory N. Alff, Willis Corroon

7. "How to Successfully Manage the Pricing Decision Process"

   Author: Michael J. Miller, Tillinghast/Towers Perrin

8. "The Role of the Actuary in an Insurance Brokerage Firm"

   Author: Edgar W. Davenport, Willis Corroon

9. "Turning a Bureau into a Business"

   Authors: Daniel A. Crifo, Insurance Services Office, Inc.
   Michael Fusco, Insurance Services Office, Inc.

10. "Financial Case Study of a Consulting Actuarial Firm"

    Author: James A. Kenney, Coates Kenney, Inc.

The panel presentations covered the following topics:

1. "Crossroads of Reinsurance"

   Moderator: Susan L. Cross, Consulting Actuary
   Tillinghast/Towers Perrin

   Panelists: Christopher Garand, Vice President
   General Reinsurance Corporation

   John Murad, Vice President & Chief Actuarial Officer
   Nac Re Corporation

David Spiegler, Vice President
American Re-Insurance Company

2. "Risk-Based Capital Follow-Up"

Moderator: Stephen P. Lowe, Vice President
Tillinghast/Towers Perrin

Panelists: J. David Cummins, Executive Director
S.S. Huebner Foundation, The Wharton School
University of Pennsylvania

Robert W. Klein, Ph.D., Director of Research
National Association of Insurance Commissioners

Jon W. Michelson, Consulting Actuary
Tillinghast/Towers Perrin

3. "CAS Actuarial Research Corner"

Moderator: Robert S. Miccolis,
Senior Vice President and Actuary
Reliance Reinsurance Corporation

4. "The Appointed Actuary"

Moderator: Patrick J. Grannan, Consulting Actuary
Milliman & Robertson, Inc.

Panelists: R. Michael Lamb, Casualty Actuary
Oregon Department of Insurance and Finance

Robert A. Miller III, Consulting Actuary
Milliman & Robertson, Inc.

5. "Texas Current Events"

Moderator: Steven F. Goldberg, Senior Vice President
United Services Automobile Assoc.

Panelists: George R. Busche, Manager and Assistant Actuary
CNA Insurance Companies

Mark Crawshaw, Consulting Actuary
Wakely & Associates, Inc.

Rick Gentry, Regional Vice President
Insurance Information Institute

6. "Standard of Practice—Profit Provisions"

Participants: Steven G. Lehmann, Actuary
State Farm Mutual Automobile Insurance Company

Michael J. Miller, Consulting Actuary
Tillinghast/Towers Perrin

Mark Whitman, Assistant Vice President & Actuary
Insurance Services Office, Inc.

7. "Standard of Practice—Risk Margins in Loss Reserves"

Moderator: Spencer M. Gluck, Consulting Actuary
Milliman & Robertson, Inc.

Panelists: Ralph S. Blanchard III, Associate Actuary
Aetna Life & Casualty

Robert P. Butsic, Assistant Vice President
Fireman's Fund Insurance Companies

8. "Questions and Answers with the CAS Board of Directors"

Moderator: Irene K. Bass (CAS President-Elect)
Managing Director
William M. Mercer, Inc.

Panelists: James K. Christie, President
IAO Actuarial Consulting Services

Susan T. Szkoda, Second Vice President and Actuary
The Travelers Insurance Company

W. James MacGinnitie, Consulting Actuary
Tillinghast/Towers Perrin

9. Theory of Risk Papers

Moderator: Philip E. Heckman, Senior Consulting Actuary
Ernst & Young

"Measuring the Variability of Chain Ladder Reserve Estimates"

Author: Dr. Thomas Mack,
Munich Re

"Unbiased Loss Development Factors"

Author: Daniel M. Murphy,
Argonaut Insurance Company

10. Theory of Risk Papers

Moderator: John G. Aquino, Senior Manager
KPMG Peat Marwick

"Statistical Methods for the Chain Ladder Technique"

Author: Dr. Richard J. Verrall,
Department of Actuarial Science & Statistics,
The City University

"Probabilistic Development Factor Models with Applications to Loss Reserve Variability, Prediction Intervals, and Risk-Based Capital"

Author: Ben Zehnwirth,
Insureware Pty. Ltd.

The officers held a reception for the new Fellows and their guests from 5:30 p.m. to 6:30 p.m.

A general reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 11, 1993

A panel presentation, "Risk Based Capital" started the morning at 8:30 a.m. The panel was moderated by Stephen P. Lowe, Vice President, Tillinghast/Towers Perrin. Members of the panel were: Chris D. Daykin, Government Actuary (United Kingdom), Government Actuary's Department; William McCartney, Insurance Commissioner, State of Nebraska; and Charles F. Titterton, Director, Insurance Rating Services, Standard & Poor's Corporation.

Concurrent sessions were held from 10:30 a.m. to 12:00 p.m.

Tuesday afternoon was reserved for concurrent sessions from 2:00

p.m. to 3:30 p.m. and for the various CAS committees to convene from 1:00 p.m. to 5:00 p.m.

Dinner and entertainment was a CAS barbecue and barn dance at the Austin Ranch from 6:30 p.m. to 10:00 p.m.

Wednesday, May 12, 1993

Concurrent sessions were held from 8:00 a.m. to 9:30 a.m.

A panel presentation, "Catastrophes" followed the sessions. The panel was moderated by Franklin Montross IV, Senior Vice President, General Reinsurance Corporation. Panelists were: David Hays, Actuary, State Farm Fire and Casualty Company; Ajit Jain, President of Reinsurance Division, Berkshire Hathaway Group; and Eugene L. LeComte, President and CEO, National Committee on Property Insurance.

The business session resumed at 11:30 a.m. with the presentation of the Michelbacher Award to Sholom Feldblum.

Julie Ekdom was announced as the winner of the Harold Schloss Award.

Allan Kaufman presented the prizes for the Risk Theory Prize Papers. Dr. Richard Verrall received first prize and Dr. Thomas Mack and Ben Zehnwirth tied for the second and third prizes.

Dave Flynn announced that the Board of Directors has established an annual prize to be awarded to the ASTIN paper that is judged to be of the greatest applied value to CAS members. The prize will be $500 cash, plus expenses for the author to present the paper in a workshop format at a CAS meeting. The award will be administered by the International Relations Committee. The prize will be named in honor of Charles A. Hachemeister who was a major supporter of the ASTIN committee, a frequent contributor to ASTIN meetings, and very active for many years in working to establish and encourage communication and cooperation between ASTIN and the CAS. The Charles A. Hachemeister Prize will first be awarded in the Spring of 1994.

The meeting was adjourned at 11:45 a.m. after closing remarks.

May 1993 Attendees

In attendance, as indicated by the registration records, were 170 Fellows and 130 Associates. The list of their names follows:

FELLOWS

Gregory N. Alff
Terry J. Alfuth
Charles M. Angell
John G. Aquino
Nolan E. Asch
Richard V. Atkinson
Irene K. Bass
Bruno P. Bauer
Albert J. Beer
Linda L. Bell
Abbe S. Bensimon
Phillip N. Ben-Zvi
Ralph S. Blanchard, III
LeRoy A. Boison, Jr.
Ronald L. Bornhuetter
Paul Braithwaite
George R. Busche
Jeanne H. Camp
John D. Carponter
Edward J. Carter
James K. Christie
Eugene C. Connell
John B. Conners
Mark Crawshaw
Susan L. Cross
Alan C. Curry
Daniel J. Czabaj
Robert A. Daino
Curtis Gary Dean
Jerome A. Degerness
Joseph J. Demelio
Kevin G. Dickson
Mark DiGaetano
Michel Dionne
Scott H. Dodge
Michael C. Dolan
James L. Dornfeld
Thomas J. Duffy
Francoise Dumas
Bradley C. Eastwood
Grover M. Edie
Douglas D. Eland
John S. Ewert
James A. Faber
Janet L. Fagan
Sholom Feldblum
Beth E. Fitzgerald
Nancy Gail Flannery
James E. Fletcher
David P. Flynn
Bruce F. Friedberg
Michael Fusco
Alice H. Gannon
Christopher P. Garand
Robert W. Gardner
Louis Gariepy
James J. Gebhard
Judy A. Gillam
William R. Gillam
Spenser M. Gluck
Daniel C. Goddard
Steven F. Goldberg
Patrick J. Grannan
Gary Grant
Larry A. Haefner
David N. Hafling
Malcolm R. Handte
David G. Hartman
Roger M. Hayne
David H. Hays
LeRoy E. Heer
Agnes H. Heersink
Anthony D. Hill
Carlton W. Honebein
Heidi E. Hutter
Peter H. James
Russell T. John
Kenneth R. Kasner
Allan M. Kaufman
Anne E. Kelly
Frederick W. Kilbourne
Frederick O. Kist
Douglas F. Kline
Mikhael I. Koski
Rodney E. Kreps
Jeffrey L. Kucera
Marthe A. Lacroix

Michael R. Lamb
Michael R. Larsen
Merlin R. Lehman
Steven G. Lehmann
Stuart N. Lerwick
Joseph W. Levin
Dennis J. Loper
Stephen P. Lowe
James W. MacGinnitie
Howard C. Mahler
Isaac Mashitz
Steve E. Math
Charles W. McConnell, II
Sean P. McDermott
Gary P. McDonald
Michael F. McManus
Michael A. McMurray
Robert S. Miccolis
Jon W. Michelson
Michael J. Miller
Robert A. Miller, III
Neil B. Miner
Charles B. Mitzel
Frederic James Mohl
Richard B. Moncher
Phillip S. Moore
John H. Muetterties
Todd B. Munson
Donna S. Munt
John A. Murad
Daniel M. Murphy
Thomas E. Murrin
James J. Muza
Kenneth J. Nemlick
Charles L. Niles, Jr.
Glen C. Nyce
Paul G. O'Connell
Robert G. Palm
Sylvie L. Paquette
Jacqueline E. Pasley
Bruce Paterson
Gary S. Patrik
Susan J. Patschak
Kai-Jaung Pei
Herbert J. Phillips
Joseph J. Pratt
Virginia R. Prevosto
John M. Purple
Albert J. Quirin
Steve C. Rominske
Sheldon Rosenberg
Mark A. Ruegg
Timothy L. Schilling
Jeffrey W. Schmidt
Roger A. Schultz
Debbie Schwab
Brian E. Scott
Gordon L. Scott
Roy G. Shrum
Christy L. Simon
Lisa A. Slotznick
David Spiegler
Sanford R. Squires
James N. Stanard
Lee R. Steeneck
Grant D. Steer
Chris M. Suchar
Stuart B. Suchoff
Andrea M. Sweeny
Susan T. Szkoda
Catherine H. Taylor
Karen F. Terry
Patricia A. Teufel
Kevin B. Thompson
Michael L. Toothman
Mary L. Turner
Glenn M. Walker
Patrick L. Whatley
Charles S. White
Mark Whitman
Gregory S. Wilson
James C. Wilson
Arlene F. Woodruff

ASSOCIATES

Rhonda K. Aikens
Craig A. Allen
Scott C. Anderson
James A. Andler
Karen L. Ayres
Timothy J. Banick
Philip A. Baum
John A. Beckman
Richard F. Burt, Jr.
John F. Butcher II
Arthur R. Cadorine
Michael A. Coca
Daniel A. Crifo
Guy R. Danielson
David J. Darby
Todd H. Dashoff
Edgar W. Davenport
Karen L. Davies
Marie-Julie Demers
Shawn F. Doherty
Ronald R. Earls
David L. Esposito
Matthew G. Fay
John R. Ferrara
George Fescos
David N. Fields
Terrence A. Flanagan
Howard H. Friedman
Kai Y. Fung
James E. Gant
Andrea Gardner
Mary K. Gise
Marc C. Grandisson
Bradley A. Granger
Ewa Gutman
Holmes M. Gwynn
Aaron Halpert
Paul J. Hancock
Timothy J. Hansen
Matthew T. Hayden
Thomas L. Hayes
Lisa A. Harp
Thomas F. Head
Philip E. Heckman
Noel M. Hehr
Mary B. Hemerick
Thomas H. Highet
Bernard R. Horovitz
Jeffrey R. Ill
David H. Isaac
Vincent H. Jackson
Patrick C. Jensen
Kurt J. Johnson
Mark R. Johnson
Daniel J. Johnston
Steven A. Kelner
James A. Kenney
Kevin A. Kesby
Joseph P. Kilroy
Craig W. Kliethermes
George C. Klingman
Terry A. Knull
Elizabeth Kolber
Howard A. Kunst
David W. Lacefield
Michel Laurin
Thomas L. Lee
Scott J. Lefkowitz
Elizabeth Ann Lemaster
Deanne C. Lenhardt
Barry I. Llewellyn
Daniel J. Mainka
Leslie R. Marlo
Kelly J. Mathson
Robert D. McCarthy
Donald R. McKay
Van A. McNeal
Paul A. Mestelle
Neil L. Millman
Michelle M. Morrow
Timothy O. Muzzey
David Y. Na
Mark Naigles
William A. Niemczyk
Peter M. Nonken
Dale F. Ogden
Melinda H. Oosten
Nathalie Ouellet
Ann E. Overturf
Donald D. Palmer
Jennifer J. Palo
Charles C. Pearl, Jr.
Edward F. Peck
Karen L. Pehrson
Daniel C. Pickens
Eduard J. Pulkstenis
Mark S. Quigley
James J. Romanowski
Scott J. Roth
George A. Rudduck
Kenneth W. Rupert, Jr.
James V. Russell
Stephen P. Sauthoff
Letitia M. Saylor

Michael B. Schenk
Lisa P. Schmidt
Michael L. Scruggs
Robert D. Share
Jeffrey S. Sirkin
Richard S. Skolnik
David C. Snow
John A. Stenmark
Michael J. Steward, II
Brian M. Stoll
Katie Suljak
Todd D. Tabor
Christopher Tait
Eugene G. Thompson
Patrick N. Tures
James F. Tygh
Karen P. Valenti
Charles E. Van Kamper
Jennifer A. Violette
David G. Walker
Russell B. Wenitsky
Marcia C. Williams
William M. Wilt
John S. Wright
Gerald T. Yeung
Claude D. Yoder

Volume LXXX, Part 2 **No. 153**

PROCEEDINGS
November 14, 15, 16, 17, 1993

ASSET/LIABILITY MATCHING (FIVE MOMENTS)

ROBERT K. BENDER

Abstract

It is well known that re-investment risk can be greatly reduced if the assets which are assigned to support liabilities are "matched." In particular, matching two properties of the asset and liability cash flows, the dollar duration (DD1) *and dollar convexity* (DD2), *can provide a significant reduction in re-investment risk. This paper provides a rigorous mathematical treatment of the asset/liability matching problem.*

This paper initially shows that DD1 *and* DD2 *are the first two moments of a set of cash flows* (DDn). *By means of a Taylor expansion of the present value of a set of cash flows, the paper then shows why matching individual moments of an asset flow with the corresponding moments associated with a liability flow can reduce re-investment risk.*

Finally, for every cash flow and pair of interest rates, there exists a characteristic time T. *Even if the flow is originally priced to yield the first interest rate, and it is the* *second interest rate that prevails, the initial yield rate can be achieved by selling the flow at time* T. *The paper shows how this relates to asset/liability matching, and how* T *can be expressed in terms of the generalized moments,* DDn.

The author wishes to thank William A. Bailey for his helpful comments and suggestions after reading an earlier version of this paper.

1. INTRODUCTION

Whenever a liability takes the form of future cash outflows and assets earn interest, it is reasonable to discount the liability for interest before deciding whether or not assets are sufficient to "cover" the liability. In the discounting process, several assumptions are made. One assumption is that the size and timing of the cash outflows are known. A second assumption is that the interest rate used in the discounting process can be realized in asset yield. Both of these assumptions introduce an element of risk into the matching process. The latter risk has two distinct elements: Credit risk due to possible defaults as to principal and interest, and re-investment risk due to interest rate changes during the life of the asset.

The sources of re-investment risk and ways to reduce that risk have been the subject of several recent papers and articles (see [1]-[6]). It has been demonstrated that re-investment risk can be greatly reduced if two moments of the asset and liability cash flows are matched; namely dollar duration ($DD1$), and dollar convexity ($DD2$). Another simpler moment, weighted term duration (WTD), is mentioned, but usually not considered further.

Two moments in time are also discussed when considering the reduction of re-investment risk: the initial time (implicit in the re-investment rate) and one implied by Ferguson's Table C in which a characteristic time equal to 4.13 years is shown to have special significance for a five-year, 9% par bond [1]. With the exception of Appendix B in Ferguson's paper, relationships between the five moments listed above are usually demonstrated by means of examples, rather than by a more rigorous mathematical exposition. Following the spirit of Ferguson's appendix, this paper recasts the discussion into a mathematically rigorous format, and, in Appendix B, applies the results to reflect higher order terms in Ferguson's bond example. In the process we gain some insight into the nature of the relationships and see that all of them are approximations. It is not the objective to produce a better method for reducing re-investment risk but, rather, to place the current work into a unified theoretical framework. Credit risk is beyond the scope of this paper.

2. DEFINITIONS

Assume a set of discrete cash flows $\{CF_t\}$, where CF_t is the flow at time, t. These flows may represent either an income producing asset (in which case the CF represents inflows) or a liability (in which case the CF represents outflows).

The nominal value of the flow is given by the sum of the flows over time, as follows:

$$Nom = \sum_{t=0}^{\omega} CF_t \,, \tag{2.1}$$

where ω is the largest value of t for which CF_t is non-zero. The t need not be an integer, and some CF_t with $t < \omega$ can be zero.

The present value of the flow, under an assumed interest rate, i, is

$$PV = \sum_{t=0}^{\omega} v(i)^t \, CF_t \,, \tag{2.2}$$

where

$$v(i) = 1/(1+i) \,. \tag{2.3}$$

The weighted term duration is defined by

$$WTD = \sum_{t=0}^{\omega} t\, CF_t \,/ \sum_{t=0}^{\omega} CF_t \,. \tag{2.4}$$

The dollar duration is given by

$$DD1(i) = \sum_{t=0}^{\omega} t\, v(i)^t\, CF_t \,/ \sum_{t=0}^{\omega} v(i)^t\, CF_t \,. \tag{2.5}$$

The usual notation does not explicity draw attention to the fact that $DD1$ depends upon the assumed interest rate. For much of what follows, this dependence will be significant. Dollar convexity is defined as the second moment (in time) of the cash flow, as follows:

$$DD2(i) = \sum_{t=0}^{\omega} t^2 v(i)^t\, CF_t \,/ \sum_{t=0}^{\omega} v(i)^t\, CF_t \,. \tag{2.6}$$

Again, this notation explicitly displays the dependence of the dollar convexity upon the assumed interest rate. Continuing on, higher moments of the cash flow distribution are defined by:

$$DDn(i) \equiv \sum_{t=0}^{\omega} t^n\, v(i)^t\, CF_t \,/ \sum_{t=0}^{\omega} v(i)^t\, CF_t \,. \tag{2.7}$$

As was previously mentioned, the time scale can be drawn as finely as the cash flow pattern dictates. For some flows, the payment pattern will be nearly continuous. For those flows, approximate the set of discrete flows, $\{CF_t\}$, with a flow rate $\sigma(t)$ such that $\sigma(t)dt$ represents the cash flow from time t to $t + dt$ (an infinitesimal time later). Further, define a normalized discounted flow density $\rho(i, t)$ as follows:

$$\rho(i, t) = v(i)^t\, \sigma(t) / \int_0^{\omega} v(i)^t\, \sigma(t)\, dt \,. \tag{2.8}$$

Using the definition of $\rho(i, t)$, Equations 2.1, 2.2, and 2.4-2.7 can be recast into continuous form:

$$Nom = \int_0^\omega \sigma(t)dt\,, \tag{2.9}$$

$$PV = \int_0^\omega v(i)^t \, \sigma(t)dt\,, \tag{2.10}$$

$$WTD = \int_0^\omega t\,\rho(0, t)\,dt = DD1\,(0) \tag{2.11}$$

$$DD1(i) = \int_0^\omega t\,\rho(i, t)\,dt\,, \tag{2.12}$$

$$DD2(i) = \int_0^\omega t^2\,\rho(i, t)\,dt\,, \text{ and} \tag{2.13}$$

$$DDn(i) = \int_0^\omega t^n\,\rho(i, t)\,dt. \tag{2.14}$$

In this form, the integrals for DDn ($n = 1, 2, 3....$) are clearly moments of the distribution (of cash flows) given by $\rho(i, t)$.

While the weekly payments of workers' compensation lifetime disability benefits may be reasonably approximated by a continuous cash flow, very few assets yield a nearly continuous cash flow.

A final definition allows the rigorous dealing with any discrete cash flow as if it were continuous—allowing us to work in the continuous case whenever the mathematical manipulations are easier. The device is called a Dirac delta, $\delta(x - x_0)$. Standing alone, the Dirac delta is undefined; but its action within an integral is well defined. Consider a function $f(x)$, then

$$\int_a^b f(x)\delta(x - x_0)dx = \begin{cases} f(x_0) \text{ if } a \le x_0 \le b \\ 0 \text{ if not} \end{cases} \tag{2.15}$$

If one writes, for the discrete set $\{CF_{t_0}, CF_{t_1}, CF_{t_2}, \dots CF_{t_\omega}\}$,

$$\sigma(t) = \sum_{m=0}^{\omega} CF_{t_m} \delta(t - t_m) , \tag{2.16}$$

then, for example,

$$\int_0^\omega \sigma(t)dt = \sum_{m=0}^{\omega} CF_{t_m} \text{ and} \tag{2.17}$$

$$\int_0^\omega t^n \rho(i, t)dt = \sum_{m=0}^{\omega} t^n v(i)^{t_m} CF_{t_m} / \sum_{m=0}^{\omega} v(i)^{t_m} CF_{t_m} . \tag{2.18}$$

3. ASSET/LIABILITY MATCHING: CASE 1

The usual case considered is when a discounted liability cash flow,

$$PV_L(i) = \int_0^{\omega_L} v(i)^t \sigma_L(t)\, dt , \tag{3.1}$$

is matched with (set equal to) an asset with an identical present value (but not, necessarily, identical cash flows),

$$PV_A(i) = \int_0^{\omega_A} v(i)^t \sigma_A(t)\, dt \tag{3.2}$$

at time equals zero, the interest rate changes to j. The asset and liability continue to be matched if

$$PV_L(j) = PV_A(j)\,. \tag{3.3}$$

The trivial (in a mathematical sense) solution to Equation 3.3 involves selecting an asset for which:

$$\sigma_A(t) = \sigma_L(t)\,. \tag{3.4}$$

In this case, while both $PV_L(i)$ and $PV_A(i)$ are functions of the interest rate, their difference,

$$PV_L(i) - PV_A(i) = \int_0^{\max(\omega_L,\,\omega_A)} v(i)^t \cdot 0 \cdot dt = 0 \tag{3.5}$$

is independent of i.

One could always transfer the liability to a third party in exchange for a single payment equal to the selling price of the asset (remember, we are not considering timing risk or default risks, so the price should equal the present value). The purchase of zero coupon bonds, which mature as the liabilities become due, produces just such a solution to the re-investment risk problem.

When the two $\sigma(t)$ are not identical, approximate solutions to Equation 3.3 may be found via a Taylor expansion of the present value as a function of the interest rate, i. In particular, for $j = i + \Delta i$,

$$PV_L(j) = \sum_{n=0}^{\infty} (i/n!)\,[d^n PV_L\;(k)/dk^n]\,|_{\,k=i}\,(\Delta i)^n, \tag{3.6}$$

$$PV_A(j) = \sum_{n=0}^{\infty} (i/n!)\,[d^n PV_A\;(k)/dk^n]\,|_{\,k=i}\,(\Delta i)^n, \tag{3.7}$$

or

$$PV_L(j) - PV_A(j) =$$

$$\sum_{n=0}^{\infty} (i/n!)\,[d^nPV_L\;(k)/dk^n - d^nPV_A\;(k)/dk^n]\,|_{k=i}\,(\Delta i)^n. \qquad (3.8)$$

The set $\{(\Delta i)^n\}$ for Δi not equal to zero and for $n = 0, 1, 2, 3....$ forms an independent basis for a vector space. As such, a null vector, implying $PV_L(j) = PV_A(j)$, can only be obtained if each component,

$$a_n = 1/n!\;[d^nPV_L(k)/dk^n - d^nPV_A(K)/dk^n]\,|_{k=i} \qquad (3.9)$$

is zero. We therefore conclude that the solution for Equation 3.3 obtained by setting $\sigma_L(t)$ equal to $\sigma_A(t)$ is not only the trivial solution, but it is the only exact solution (since satisfying Equation 3.9 to all orders would cause the two functions to be identical). For small i, the higher order terms in the Taylor series can be expected to decrease rapidly, allowing for an acceptable degree of error to remain if only one or two terms are matched (i.e., Equation 3.9 is satisfied).

The zero order terms are initially equal if the asset and liability have equal present values before the (time zero) interest rate change. The first order term requires a matching of (from Equation 3.9 with $n = 1$),

$$dPV_L(k)/dk\,|_{k=i} = dPV_A(k)/dk\,|_{k=i}\,. \qquad (3.10)$$

From Equation 3.1,

$$dPV_L(k)/dk\,|_{k=i} = \int_0^{\omega} \sigma_L(t)\,dv(k)^t/dk\,|_{k=i}\,dt \qquad (3.11)$$

$$= -v(i)\int_0^{\omega} tv(i)^t\,\sigma_L(t)\,dt$$

$$= -v(i)\, DD1_L(i)\, PV_L(i)\,.$$

Likewise, for the asset,

$$dPV_A(k)/dk\,|_{k=i} = -v(i)\, DD1_A(i)\, PV_A(i)\,. \tag{3.12}$$

Equation 3.10 will be satisfied, in view of Equations 3.11 and 3.12, if

$$DD1_L(i) = DD1_A(i)\,, \tag{3.13}$$

which is the usual condition that dollar durations be matched. (Note that $PV_L = PV_A$ when the asset was originally selected.)

The next term introduces convexity. Setting

$$d^2PV_L(k)/dk^2\,|_{k=i} = d^2PV_A(k)/dk^2\,|_{k=i}$$

produces matching to second order in Δi,

$$d^2PV(k)/dk^2\,|_{k=i} = \int_0^{\omega} \sigma(t)\, d^2v(k)/dk^2\,|_{k=i_{dt}} \tag{3.14}$$

$$= v(i)^2 \int_0^{\omega} (t^2 + t)v(i)^t \sigma(t)dt$$

$$= v(i)^2[DD2(i) + DD1(i)]\, PV(i)\,.$$

As long as PV and $DD1$ have been matched, Equation 3.14 adds the convexity matching requirement, or

$$DD2_L(i) = DD2_A(i) \tag{3.15}$$

for second order agreement.

While higher order terms can be matched, a small Δi raised to a large power makes the terms less significant. Nonetheless, we observe that each additional order introduces an additional moment along with the previously matched moments. Again, if all of the moments are equal, the two distributions must be equal. Practically speaking, it may be extremely difficult to match $DD2$, let alone to find assets for which higher orders of DDn are matched.

It is interesting to note that expressions for the change in price frequently omit terms and factors from the Taylor series. (Ferguson draws attention to the missing factor of $v(i)$ in the first order term.) In particular, both Babbel and Stricker [5], and Diembiec, et al [2] omit the $DD1$ contribution to the second order term, and the $v(i)$ factor at all orders. The correct expression is

$$\Delta\text{Price}/(\text{Original Price}) = [PV(j) - PV(i)]/PV(i) \qquad (3.16)$$

$$= -v(i)\, DD1(i)\Delta i$$

$$+ \tfrac{1}{2}\, v(i)^2[DD2(i) + DD1(i)](\Delta i)^2$$

$$+ R(\Delta i^3)$$

where R is a residual term of order Δi^3 and higher. The previously published residual term contains contributions of the same order as those that are explicity displayed. The expressions also appear to confuse price with Δprice/original price. Of course, the missing terms and factors are common to both the asset and the liability, so their absence in the price expansion does not introduce any errors into the matching process, or the conclusion that convexity matching is a significant improvement over dollar duration matching.

4. ASSET/LIABILITY MATCHING: CASE 2

Ferguson alludes to a second method of re-investment risk management. Given an initially matched asset and liability and an initial change of interest rate, there is some time, T, (not equal to zero) at which the asset and liability could be exchanged (assuming no intervening interest rate changes). He implies that T is equal to the duration (which is true only to first order in Δi).

Before demonstrating the degree of approximation in this assertion, it will be shown that this second time of price equality can be determined exactly in closed form. As in the previous case, assume that the asset and liability are price matched under the initial interest rate assumption,

$$PV_L(i) = PV_A(i), \tag{4.1}$$

and that at time $t = 0$, interest rates abruptly change from i to j. We have already seen that, if the change is small and $DD1(i)$ and $DD2(i)$ are equal, then $PV_L(j)$ will be approximately equal to $PV_A(j)$.

After some time has elapsed, there is a time, T, at which the asset can be sold such that the accumulated value of prior payments at the new rate, j, plus the sale price (determined at the new rate, j, for the remaining flows) yields the original rate, i. If the corresponding liability has the same characteristic T, an exchange could be made at time T without suffering the consequences of re-investment risk.

At the original yield rate and price, $PV_A(i)$ would have accumulated to $PV_A(i) \cdot (1+i)^T$ by time T. Instead, the prior payments will have accumulated to

$$\sum_{t=0}^{T} (1+j)^{T-t}\, CF_t = \sum_{t=0}^{T} v(j)^{t-T}\, CF_t, \tag{4.2}$$

using the discrete notation for simplicity. The present value of the future payments at time T are given by

$$\text{selling price} = \sum_{t=T+1}^{\omega} v(j)^{t-T}\, CF_t. \tag{4.3}$$

Combining Equations 4.2 and 4.3 to obtain the total wealth after selling the asset at time T and comparing it to the original asset price,

$$PV_A(i) * (1+i)^T = \sum_{t=0}^{T} v(j)^{t-T}\, CF_t + \sum_{t=T+1}^{\omega} v(j)^{t-T}\, CF_t \qquad (4.4)$$

$$= (1+j)^T \sum_{t=0}^{\omega} v(j)^t\, CF_t$$

$$= (1+j)^T\, PV_A(j)\,,$$

where $PV_A(j)$ is the *original* price of the asset under an assumed interest rate, j. Solving for T gives the exact solution,

$$T(i,j) = \ln[PV_A(j)/PV_A(i)]/\ln[(1+i)/(1+j)]\,. \qquad (4.5)$$

While any logarithm base could be used, we have selected the natural base. T depends upon both interest rates, so it is not a function of the original bond price alone (as one might believe after reading Ferguson's example).

To see how T is related to $DD1$ and $DD2$, expand T in a Taylor series to first order in (Δi). Here, however, the derivatives are not quite as simple as they were for the PV expansion. The Taylor series in powers of $\Delta i = j - i$ is given by

$$T(i,j) = T(i,k)\,|_{k=i} + dT(i,k)/dk\,|_{k=i}\,\Delta i + R(\Delta i^2)\,. \qquad (4.6)$$

Due to the presence of $\ln[PV(k)/PV(i)]$ in the numerator of $T(i, k)$ and $\ln[(1+i)/(1+k)]$ in the denominator, each of these terms involves the indeterminate form $0/0$ when k is set equal to i. One or more applications of l'Hopital's rule (see Appendix A) allows us to evaluate each term giving

$$T(i,j) = DD1(i) - \tfrac{1}{2}\, v(i)[DD2(i) - DD1(i)^2]\Delta i + R(\Delta i^2)\,. \qquad (4.7)$$

REFERENCES

[1] Ferguson, R. E., "Duration," *PCAS* LXX, 1983, p. 265.

[2] Dembiec, L. A., Pogorzelski, J. D., and Rowland, V. T., Jr., "The Measurement and Management of Interest Rate Risk," *Valuation Issues,* 1989 Casualty Actuarial Society Discussion Paper Program, p. 71.

[3] Feldblum, S., "Asset Liability Matching for Property/Casualty Insurers," *Valuation Issues,* 1989 Casualty Actuarial Discussion Paper Program, p. 117.

[4] Yawitz, Jess B., *Convexity: An Introduction,* Goldman Sachs, New York, 1986.

[5] Babbel, David F., and Stricker, Robert, *Asset/Liability Management for Insurance,* Goldman Sachs, New York, 1987.

[6] Jacob, David P., Lord, Graham, and Tilley, James A., *Price, Duration, and Convexity of a Stream of Interest-Sensitive Cash Flows,* Goldman Sachs, New York, 1986.

APPENDIX A

EVALUATION OF THE INDETERMINATE FORMS, $T(i, k)\mid_{k=i}$

Zero Order Term, $T(i, k)\mid_{k=i}$

Using l'Hopital's rule for the form 0/0, we replace

$$\lim_{k \to i} T(i, k) = \lim_{k \to i} \ln[PV(k)/PV(i)]/\ln[(1+i)/(1+k)] \tag{A.1}$$

with the equivalent

$$\lim_{k \to i} T(i, k) = \lim_{k \to i} d/dk \ln[PV(k)/PV(i)]/\lim_{k \to i} d/dk \ln[(1+i)/(1+k)]\,, \tag{A.2}$$

and evaluate the derivatives,

$$\lim_{k \to i} T(i, k) = \lim_{k \to i} [PV(k)^{-1}\, dPV(k)/dk]/\lim_{k \to i} [(1+k)\, d(1+k)^{-1}/dk]\,. \tag{A.3}$$

This expression can be evaluated further if the discrete form expression for $PV(k)$ is substituted, as follows:

$$\lim_{k \to i} T(i, k) = \lim_{k \to i} [(d/dk \sum_{t=0}^{\omega} v(k)^t\, CF_t)/\sum_{t=0}^{\omega} v(k)^t\, CF_t]/\lim_{k \to i} (-v(k))$$

$$= \sum_{t=0}^{\omega} tv(i)\, CF_t/\sum_{t=0}^{\omega} v(i)^t\, CF_t\,, \tag{A.4}$$

which is quickly identified as the discrete form of $DD1(i)$. Therefore,

$$T(i, k)\mid_{k=i} = DD1(i)\,. \tag{A.5}$$

First Order Term $dT(i, k)/dk\,|_{k=i}$

The first order term involves taking the first derivative of $T(i, k)$ with respect to k, or more specifically,

$$dT(i, k)/dk = \{\ln[(1 + i)/(1 + k)] \cdot d/dk \ln[PV(k)/PV(i)] - \ln[PV(k)/PV(i)] \cdot d/dk \ln[(1 + i)/(1 + k)]\} \div \{\ln[(1 + i)/(1 + k)]\}^2, \tag{A.6}$$

an expression which is rich in indeterminate forms when $k = i$.

The derivative in the first term is identical to the numerator in Equation A.2, $-v(k)DD1(k)$, and the derivative in the second term is identical to the one taken in Equation A.2, or $-v(k)$. Making these substitutions into A.6 gives

$$dT(i, k)/dk = \{-\ln[(1 + i)/(1 + k)] \cdot v(k) \cdot DD1(k) + v(k) \cdot \ln[PV(k)/PV(i)]\} \div \{\ln[(1 + i)/(1 + k)]\}^2, \tag{A.7}$$

which is clearly of the form 0/0 when $k = i$ because $v(k)$ and $DD1(k)$ are finite positive numbers for all non-negative interest rates.

L'Hopital's rule, therefore, can be applied to the right side of Equation A.7 in order to determine $dT(i, k)/dk$ as k approaches i. The application of l'Hopital's rule to Equation A.7 involves the algebraic manipulation of some rather lengthy expressions. To simplify the process we define A, B, and C as follows:

$$A(k) = v(k) \ln[PV(k)/PV(i)], \tag{A.8a}$$

$$B(k) = \ln[(1 + i)/(1 + k)]\, v(k)\, DD1(k), \tag{A.8b}$$

$$C(k) = \{\ln[(1 + i)/(1 + k)]\}^2. \tag{A.8c}$$

In terms of A, B, and C, Equation A.7 becomes

$$dT(i, k)/dk = [A(k) - B(k)]/C(k) \,, \tag{A.9}$$

and l'Hopital's rule leads to

$$dT(i, k)/dk \mid_{k=i}$$

$$= [\lim_{k \to i} dA(k)/dk - \lim_{k \to i} dB(k)/dk]/\lim_{k \to i} dC(k)/dk \,, \tag{A.10}$$

from which each term may be evaluated separately.

$$\begin{aligned} dA(k)/dk &= \ln[PV(k)/PV(i)]\, dv(k)/dk \\ &\quad + v(k)\, d/dk \ln[PV(k)/PV(i)] \\ &= -v(k)^2 \ln[PV(k)/PV(i)] - v(k)^2\, DD1(k). \end{aligned} \tag{A.11}$$

$$\begin{aligned} dB(k)/dk &= v(k)\, DD1(k)\, d/dk \ln[(1+i)/(1+k)] \\ &\quad + \ln[(1+i)/(1+k)]\, v(k)\, d \\ &\quad \div dk \left\{ \sum_{t=0}^{\omega} tv(k)^t CF_t / \sum_{t=0}^{\omega} v(k)^t CF_t \right\} \\ &= -v(k)^2\, DD1(k) - v(k)^2\, DD1(k) \ln[(1+i)/(1+k)] \\ &\quad + \ln[(1+i)/(1+k)]\, v(k)^2\, DD1(k)^2 \\ &\quad - v(k)^2\, DD2(k) \ln[(1+i)/(1+k)] \,. \end{aligned} \tag{A.12}$$

$$\begin{aligned} dC(k)/dk &= d/dk \left\{ \ln[(1+i)/(1+k)] \right\}^2 \\ &= 2 \ln[(1+i)/(1+k)]\, v(k) \,. \end{aligned} \tag{A.13}$$

The full expression becomes

$$dT(i, k)/dk \mid_{k=i} \tag{A.14}$$

$$= -\tfrac{1}{2} \left\{ \operatorname*{Limit}_{k \to i} v(k) \ln[PV(k)/PV(i)/\operatorname*{Limit}_{k \to i} \ln[(1+i)/(1+k)] \right\}$$

$$-\tfrac{1}{2}\, v(i)\, DD1(i) - \tfrac{1}{2}\, v(i)\, [DD2(i) - DD1(i)^2]\,,$$

where the first term is *still* indeterminate!

A reapplication of l'Hopital's rule to the first term quickly discloses (in view of the evaluation of the zero order term) that

$$dT(i, k)/dk \mid_{k=i} = -\tfrac{1}{2}\, v(i)\, [DD2(i) - DD1(i)^2]\,. \tag{A.15}$$

APPENDIX B

A NUMERICAL EXAMPLE

Consider a five-year, par \$1,000 bond with 9% semi-annual coupons, redeemed at par. Table B.1 displays the moments necessary to price the bond to yield 9% and to determine $DD1(0.09)$ and $DD2(0.09)$. Column 2 displays the set of cash flows, with CF_5 consisting of both the final coupon and the redemption of the bond. Columns 4-6 are the components of the zero, first, and second moments of the discounted cash flow in time.

An example of the type of re-investment risk to be managed would be an abrupt change in yield rates from the 9% assumed when the bond was priced to 6.5%. Assume that the change in yield takes place at time equals zero.

Table B.2 repeats the first four columns of Table B.1, but under a 6.5% yield assumption. Had the actual re-investment rate been known when Bond 1 was priced, it would have cost \$1,109.87 rather than the \$1,007.70 purchase price.

Using the two prices and yield rates together with the exact Equation 4.5 for $T(i, j)$, we find that Bond 2 can be sold to yield the original 9% rate at $T(0.09, 0.065) = 4.1621$ years (approximately two months into the fifth year).

Solving for $T(i, j)$ to four decimal places, by means of the Taylor expansion, gives $T(i, j) =$

4.1383 years, using zero order term $DD1(i)$ (-0.57% error at zero order in $j - i$)

\+ 0.0238 years, (first order correction term)

= 4.1621 years, to first order in $j - i$ (0.00% error at first order in $j - i$)

Given the rather straightforward nature of the exact solution, there would be little reason to use the Taylor series in lieu of Equation 4.5. Assuming that $T(i, j) = DD1(i)$ would introduce an unnecessary error into the calculation. An advantage of using Equation 4.5 over the approximate $DD1(i)$ is that the sensitivity to the magnitude of change from i to j can be tested, because Equation 4.5 explicitly contains the new interest rate, j.

If Equation 3.16 is solved for the new price, one obtains

$$PV(j) = \tag{B.1}$$

$$PV(i) - v(i) \cdot DD1(i) \cdot PV(i) \cdot (j - i) + \tfrac{1}{2} \cdot v(i)^2$$

$$\cdot [DD2(i) + DD1(i)] \cdot PV(i) \cdot (j - 1)^2 + R(\Delta i^3) .$$

From Table B.2, $PV(0.065)$ should be \$1,109.87. The Taylor series produces the following approximations.

$PV(j) =$

\$1,007.70 to zero order in $(j - i)$ (-9.21% error at zero order in $(j - i)$)

+ \$95.65 (first order correction)

= \$1,103.35 to first order in $(j - i)$ (-0.59% error at first order in $(j - i)$)

+ 6.19 (second order correction)

=\$1,109.54 to second order in $(j - i)$ (–0.03% error at second order in $(j - i)$)

which verifies that, at least for this example, matching dollar convexity significantly improved the matching process.

TABLE B.1

BOND 1

Years to maturity: five years
Coupon rate: 9.00% paid semi-annually
Par value: $1,000
Redemption value: $1,000
Priced to yield i: 9.00% annually

(1) t (in years)	(2) CF_t	(3) $v(i)^t$	(4) $t^0 * v(i)^t * CF_t$	(5) $t^1 * v(i)^t * CF_t$	(6) $t^2 * v(i)^t * CF_t$
0.0	0.00	1.0000000	0.00	0.00	0.00
0.5	45.00	0.9578263	43.10	21.55	10.78
1.0	45.00	0.9174312	41.28	41.28	41.28
1.5	45.00	0.8787397	39.54	59.31	88.97
2.0	45.00	0.8416800	37.88	75.75	151.50
2.5	45.00	0.8061832	36.28	90.70	226.74
3.0	45.00	0.7721835	34.75	104.24	312.73
3.5	45.00	0.7396176	33.28	116.49	407.71
4.0	45.00	0.7084252	31.88	127.52	510.07
4.5	45.00	0.6785483	30.53	137.41	618.33
5.0	1,045.00	0.6499314	679.18	3,395.89	16,979.46
Total			1,007.70	4,170.14	19,347.57

$PV(i)$ = $1,007.70 = total (4)
$DD1(i)$ = 4.1383 = total (5) / total (4)
$DD2(i)$ = 19.1997 = total (6) / total (4)

TABLE B.2
Bond 2

Years to maturity: five years

Coupon rate: 9.00% paid semi-annually

Par value: $1,000

Redemption value: $1,000

Priced to yield j: 6.50% annually

(1) t (in years)	(2) CF_t	(3) $v(i)^t$	(4) $t^0 * v(i)^t * CF_t$
0.0	0.00	1.0000000	0.00
0.5	45.00	0.9690032	43.61
1.0	45.00	0.9389671	42.25
1.5	45.00	0.9098621	40.94
2.0	45.00	0.8816593	39.67
2.5	45.00	0.8543306	38.44
3.0	45.00	0.8278491	37.25
3.5	45.00	0.8021884	36.10
4.0	45.00	0.7773231	34.98
4.5	45.00	0.7532285	33.90
5.0	1,045.00	0.7298808	762.73
Total			1,109.87

$PV(j) =$ \$1,109.87 $=$ total (4)

MINIMUM DISTANCE ESTIMATION OF LOSS DISTRIBUTIONS

STUART A. KLUGMAN AND A. RAHULJI PARSA

Abstract

Loss distributions have a number of uses in the pricing and reserving of casualty insurance. Many authors have recommended maximum likelihood for the estimation of the parameters. It has the advantages of asymptotic optimality (in the sense of mean square error) and applicability (the likelihood function can always be written). Also, it is possible to estimate the variance of the estimate, a useful tool in assessing the accuracy of any results. The only disadvantage of maximum likelihood is that the objective function does not relate to the actuarial problem being investigated. Minimum distance estimates can be tailored to reflect the goals of the analysis and, as such, should give more appropriate answers. The purpose of this paper is to demonstrate that these estimates share the second and third desirable qualities with maximum likelihood.

1. DEFINITIONS, NOTATION, AND AGENDA

We start with a definition of a minimum distance estimate. Let $G(c;\boldsymbol{\theta})$ be any function of c that is uniquely related to $f(c;\boldsymbol{\theta})$, the probability density function (pdf) of the population. By uniquely related we mean that if you know f, you can obtain G and vice versa. Call G the model functional. Let $f_n(c)$ be the empirical density. It assigns probability $1/n$ to each of the n observations in the sample. Let $G_n(c)$ be found from f_n in the same way that G is from f. Call G_n the empirical functional. The objective function is

$$Q(\boldsymbol{\theta}) = \sum_{i=1}^{k} w_i \left[G(c_i; \boldsymbol{\theta}) - G_n(c_i) \right]^2, \tag{1.1}$$

where $c_1 < c_2 < ... < c_k$ are arbitrarily selected values and $w_1, w_2, ..., w_k > 0$ are arbitrarily selected weights. The weights can be selected either to minimize the variance of the estimate or to place emphasis on those values where a close fit is desired. The c_i will almost certainly be the class boundaries for whatever grouping was used in the initial presentation of the data. The minimum distance estimate is the value of $\boldsymbol{\theta}$ that minimizes $Q(\boldsymbol{\theta})$.

There are two functionals that appear to be appropriate for casualty work. The first is the limited expected value (LEV) which is useful in ratemaking. It is the expected loss when losses are capped at a specified value. This quantity is fundamental for calculating deductibles, limits, layers, increased limits, or the effects of inflation. This quantity is also useful for reserving if information about the distribution of outstanding claims is desired. Many practitioners make it a point to verify that the model LEVs (after estimating the parameters by maximum likelihood) and the empirical LEV match. Using the LEV as a distance measure gives this the best chance of happening.

The specific relationships are (when dealing with the LEV we will use L in place of G):

$$L(c; \boldsymbol{\theta}) = \int_0^c x f(x; \boldsymbol{\theta}) dx + c \int_c^\infty f(x; \boldsymbol{\theta}) dx \tag{1.2}$$

and

$$L_n(c) = \tfrac{1}{n} \sum_{i=1}^{n} \min(x_i, c). \tag{1.3}$$

It should be noted that to compute $L_n(c_i)$ all that is needed is the number of observations, n_i, that are between c_{i-1} and c_i (where $c_0 = 0$) and the average, a_i, of these observations. Then

$$L_n(c_i) = \left[\sum_{j=1}^{i} n_j a_j + c_i(n - \sum_{j=1}^{i} n_j) \right] / n = c_i + \sum_{j=1}^{i} n_j(a_j - c_i)/n \,. \qquad (1.4)$$

A second functional, one that makes sense for loss reserving, is the distribution function. As will be seen in the second example, loss distributions can be used to estimate the number of incurred but not reported (IBNR) claims. The key to the calculation is that the distribution function is evaluated at the highest lag for which losses have been reported. Using F for G we have

$$F(c; \boldsymbol{\theta}) = \int_0^c f(x; \boldsymbol{\theta})dx \qquad (1.5)$$

and

$$F_n(c) = \tfrac{1}{n} \,(\text{number of } x_i \leq c) \,. \qquad (1.6)$$

There are a number of steps that need to be taken to make this method practical.

1. Techniques for minimizing Q.
2. Verification that the solution possesses desirable statistical properties. This would include being unbiased, consistent, and, if not minimum variance, at least providing for calculation of the variance.
3. A demonstration that estimators obtained from this method are not unlike those obtained by maximum likelihood, at least when the data actually come from the distribution family being fitted.
4. Construction of a hypothesis test based on Q. This would allow for verification that the model selected is reasonable as well as for comparison with competing models.

This paper addresses Issues 1 and 2 in full and makes a proposal relative to Issue 4. The third issue requires a fairly substantial simulation, something we have elected not to do at this time. This paper includes two examples and a small simulation to illustrate the feasibility of the method.

2. MINIMIZATION OF Q

There are three reasonable approaches to finding the minimum. The first is the simplex method. It has been discussed in several other places; the original idea is by Nelder and Mead [4], and a comprehensive treatment can be found in the book by Walters, et al. [7]. The only input required is the function to be minimized and a starting value. It proceeds cautiously and slowly, but is almost always successful in finding the minimum. The second approach is to use a packaged minimization routine. Such routines sometimes require that partial derivatives of the function be available. The third approach is to obtain a set of equations by equating the partial derivatives to zero. The multi-variate version of the Newton-Raphson method could then be used to find the solution. When derivatives are needed they can be obtained by differentiating either Equation 1.2 or 1.5. The examples in this paper were done using the simplex method.

For the second and third approaches it is easy to write the partial derivative of Q.

$$\partial Q/\partial\theta_j = 2\sum_{i=1}^{k} w_i\Big[G(c_i;\boldsymbol{\theta}) - G_n(c_i)\Big]\,G^{(j)}(c_i;\boldsymbol{\theta}) \tag{2.1}$$

where the final factor ($G^{(j)}(c_i;\boldsymbol{\theta})$) is the partial derivative of the model functional with respect to θ_j. To simplify the notation, the model functional evaluated at c_i will be written G_i, the reference to $\boldsymbol{\theta}$ being implicit and the dependence on c_i being reflected by the subscript. Similarly, the empirical functional will be written $G_{n,i}$. Equations 1.1 and 2.1 become

$$Q = \sum_{i=1}^{k} w_i\,(G_i - G_{n,i})^2$$

and

$$\partial Q/\partial\theta_j = 2\sum_{i=1}^{k} w_i\,(G_i - G_{n,i})\,G_i^{(j)}. \tag{2.2}$$

3. STATISTICAL PROPERTIES OF MINIMUM DISTANCE ESTIMATES

The minimum distance estimate is an implicit function (as given in Equation 2.1) of $\boldsymbol{G}_n$, the vector of empirical functionals. The properties of such an estimator can be obtained by using Theorem 2 and Corollary 1 from Benichou and Gail [2]. The theorem requires that the estimator be an implicit function of random variables to which the Central Limit Theorem can be applied. This is true for both situations. The LEV is a sample average of independent observations and the empirical distribution function is a binomial proportion. We have

$$n^{1/2}(\boldsymbol{G}_n - \boldsymbol{\mu}) \to N(\boldsymbol{0}, \boldsymbol{\Sigma}) . \tag{3.1}$$

The i^{th} element of $\boldsymbol{\mu}$ is $\mu_i = \mathrm{E}(G_{n,i}) = G_i$ (at least for the two functionals used in this paper). Let the ij^{th} element of $\boldsymbol{\Sigma}$ be σ_{ij}.

The next item to be satisfied is that the k functions in Equation 2.1 have continuous first partial derivatives with respect to the elements of $\boldsymbol{\theta}$. These form a $p \times p$ matrix $\boldsymbol{A}$. The jl^{th} element is

$$a_{jl} = \partial^2 Q / \partial\theta_j \theta_l = 2 \sum_{i=1}^{k} w_i\, G_i^{(j)} G_i^{(l)} + 2 \sum_{i=1}^{k} w_i\, (G_i - G_{n,i})\, G_i^{(j,l)} . \tag{3.2}$$

So, to satisfy the conditions of the theorem, the model functional must have continuous second partial derivatives with respect to the parameters. This is true for most distributions in common use for insurance losses. It is also necessary that $\boldsymbol{A}$ have a non-zero determinant when evaluated at the true parameter value. All that is necessary to complete this analysis is that it be non-zero at the estimated value of $\boldsymbol{\theta}$.

The next matrix, $\boldsymbol{B}$ ($p \times k$), has jl^{th} element

$$b_{jl} = \partial^2 Q / \partial\theta_j\, G_{n,l} = -2 w_l\, G_l^{(j)} . \tag{3.3}$$

It is necessary that $\boldsymbol{A}^{-1}\boldsymbol{B}$ have at least one non-zero element.

The theorem then states that, as the sample size goes to infinity, there will be a unique solution, $\tilde{\boldsymbol{\theta}}$, to the equations and

$$\sqrt{n}(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}) \rightarrow N(\mathbf{0}, \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{\Sigma}\boldsymbol{B}'\boldsymbol{A}^{-1}). \tag{3.4}$$

This verifies that the minimum distance estimator is consistent and asymptotically unbiased and, even though it is not likely to have minimum variance, at least we will be able to estimate the variance.

4. EXAMPLES

Example One

The first example consists of losses from the Insurance Services Office (ISO) increased limits project for general liability (Table 2) coverage. The accident year is 1986 and the losses are those reported at Lag 1. Actual losses are given in Table 1. This example uses fewer size-of-loss intervals. For simplification, the average loss in each interval was taken as the midpoint. One problem is the existence of multiple policy limits in the ISO data set. These are difficult to deal with as it is unlikely that actual losses can be determined for those cases that exceed the upper limit. There are two such cases in this data set. One loss is known to exceed \$25,000; the other exceeds \$500,000. The easiest reasonable way to adjust for this problem may be to replace these values with the conditional (on being above the upper limit) median (as the mean may not exist) from a rough estimate of the final model. For this illustration the values \$38,865 and \$769,061 were used. They were incorporated in the calculation of the empirical LEVs in Table 1.

For this illustration, the only distribution being considered is the Pareto distribution. ISO rejected it as a useful model (opting for a mixture of two Pareto distributions), but it will serve as a good example mostly because all the required derivatives are easy to compute. About the only other distributions that have this property are the lognormal and inverse Gaussian. Should analytical derivatives not be available, approximate differentiation must be employed. This example also proves to be somewhat simple, as there is no deductible involved. The relevant quantities for the Pareto distribution where $\boldsymbol{\theta} = (\alpha, \lambda)'$ are:

$$F(x;\boldsymbol{\theta}) = 1 - \left(\frac{\lambda}{\lambda + x}\right)^{\alpha}, x, \alpha, \lambda > 0 ,$$

$$L(c;\boldsymbol{\theta}) = \frac{\lambda}{\alpha - 1}\left[1 - \left(\frac{\lambda}{\lambda + c}\right)^{\alpha - 1}\right],$$

$$L^{(1)}(c;\boldsymbol{\theta}) = -\frac{L(c;\boldsymbol{\theta})}{\alpha - 1} - \frac{\lambda}{\alpha - 1}\left(\frac{\lambda}{\lambda + c}\right)^{\alpha - 1}\ln\left(\frac{\lambda}{\lambda + c}\right),$$

$$L^{(2)}(c;\boldsymbol{\theta}) = \frac{L(c;\boldsymbol{\theta})}{\lambda} - \frac{c\lambda^{\alpha - 1}}{(\lambda + c)^{\alpha}} . \tag{4.1}$$

Maximum likelihood estimation produced the estimates $\hat{\alpha} = 1.482595$ and $\hat{\lambda} = 705.785$. The estimated covariance matrix of these estimators is

$$\begin{bmatrix} 0.0020473 & 1.3680 \\ 1.3680 & 1{,}090.5 \end{bmatrix}.$$

Minimization of Q using weights of 1 at all endpoints (the value 10,000,000 was arbitrarily selected to replace ∞) produced the minimum LEV estimates of $\tilde{\alpha} = 1.3388257$ and $\tilde{\lambda} = 590.32670$. The value of Q at the minimum is 8,619 compared to a value of 196,244 using the maximum likelihood estimates (which were used as a starting point for the simplex method). Table 2 shows the LEVs for both maximum likelihood and minimum LEV estimation. The wide discrepancy between these two estimators may well indicate that the Pareto model is not suitable for these data.

TABLE 1

ISO LOSS DATA

Lower Limit	Upper Limit	Number of Losses	LEV (at upper limit)
$ 0	$ 50	482	$ 48.19
50	100	574	92.41
100	150	478	132.68
150	200	431	169.54
200	250	343	203.49
250	300	337	234.89
300	400	616	290.52
400	500	518	337.64
500	600	311	378.53
600	700	263	415.10
700	800	256	447.78
800	900	170	477.26
900	1,000	212	503.86
1,000	1,500	501	610.12
1,500	2,000	297	686.41
2,000	2,500	181	744.74
2,500	3,000	116	791.91
3,000	3,500	93	831.24
3,500	4,000	72	864.37
4,000	4,500	40	893.29
4,500	4,999	32	919.45
4,999	5,000	18	919.50
5,000	6,000	59	962.39
6,000	7,500	53	1,014.12
7,500	9,999	60	1,079.07
9,999	10,000	6	1,079.09
10,000	12,000	21	1,117.10
12,000	15,000	27	1,163.30
15,000	20,000	22	1,221.89
20,000	25,000	23	1,263.58
25,000	35,000	15	1,318.42
35,000	50,000	15	1,366.87
50,000	75,000	6	1,408.19
75,000	100,000	3	1,432.60
100,000	250,000	3	1,511.48
250,000	500,000	0	1,586.60
500,000	1,000,000	2	1,661.72
1,000,000	∞	0	1,661.72
Total		6,656	

TABLE 2

LEVs

Limit	Empirical LEV	MLE LEV	MinLEV
$ 50	$ 48.19	$ 47.58	$ 47.34
100	92.41	90.59	89.97
150	132.68	129.88	128.66
200	169.54	165.90	164.00
250	203.49	199.09	196.47
300	234.89	229.80	226.44
400	290.52	284.92	280.14
500	337.64	333.10	327.03
600	378.53	375.70	368.49
700	415.10	413.72	405.53
800	447.78	447.93	438.91
900	477.26	478.93	469.23
1,000	503.86	507.19	496.93
1,500	610.12	618.64	607.11
2,000	686.41	697.87	686.67
2,500	744.74	757.95	747.94
3,000	791.91	805.55	797.21
3,500	831.24	844.47	838.05
4,000	864.37	877.08	872.70
4,500	893.29	904.92	902.63
4,999	919.45	929.02	928.82
5,000	919.50	929.06	928.87
6,000	962.39	969.06	972.98
7,500	1,014.12	1,014.86	1,024.62
9,999	1,079.07	1,068.76	1,087.18
10,000	1,079.09	1,058.77	1,087.20
12,000	1,117.10	1,100.01	1,124.49
15,000	1,163.30	1,135.25	1,167.65
20,000	1,221.89	1,176.11	1,219.33
25,000	1,263.58	1,201.50	1,256.47
35,000	1,318.42	1,242.33	1,307.84
50,000	1,366.87	1,276.61	1,356.64
75,000	1,408.19	1,309.30	1,405.70
100,000	1,432.60	1,329.01	1,436.75
250,000	1,511.48	1,376.53	1,518.03
500,000	1,586.60	1,400.93	1,564.89
1,000,000	1,661.72	1,418.41	1,601.99
10,000,000	1,661.72	1,457.70	1,712.80

To estimate the asymptotic variance we need the variance of the empirical LEVs which are computed using:

$$E[\min(X, c_i)^2] - \{E[\min(X, c_i)]\}^2.$$

They are:

$$\sigma_{ii} = \text{Var}(\min(X, c_i))$$

$$= \int_0^{c_i} x^2 f(x; \boldsymbol{\theta})dx + c_i^2\left[1 - F(c_i; \boldsymbol{\theta})\right] - L_i^2 = {}_2L_{ii} - L_i^2,$$

$$\sigma_{ij} = \text{Cov}(\min(X, c_i), \min(X, c_j))$$

$$= \int_0^{c_i} x^2 f(x; \boldsymbol{\theta})dx + \int_{c_i}^{c_j} c_i\, x f(x; \boldsymbol{\theta})dx + c_i c_j\left[1 - F(c_j; \boldsymbol{\theta})\right] - L_i L_j$$

$$= {}_2L_{ij} - L_i L_j, \text{ for } i < j. \tag{4.2}$$

Note that if there is a deductible, d, the integral must start at d and the pdf and cumulative density function (cdf) must be modified to reflect the truncation.

For the Pareto distribution, with $i \leq j$,

$${}_2L_{ij} = \frac{2\lambda^2}{(\alpha-2)(\alpha-1)} - \frac{\lambda^\alpha(\lambda + c_i)^{-\alpha+1}(\alpha c_i + 2\lambda)}{(\alpha-2)(\alpha-1)} - \frac{\lambda^\alpha(\lambda + c_j)^{-\alpha+1} c_i}{(\alpha-1)}. \tag{4.3}$$

Using the 38 intervals and the estimated parameters produces a 38×38 matrix, which will not be presented here. The square root of the diagonal elements measures the standard deviation of the empirical LEVs based on a single observation. The standard deviation of the actual empirical LEVs can be estimated by dividing these values by the square root of the sample size (81.58). These standard deviations are presented for selected values in Table 3.

Calculation of the matrix $\boldsymbol{B}$ is relatively simple as Equation 3.3 requires only the first partial derivatives of the model LEVs. These were given in Equation 4.1. This matrix is not presented here.

Calculation of $\boldsymbol{A}$ requires the second partial derivatives of the model LEV. They are

$$L_i^{(1,1)} = -2\frac{L_i^{(1)}}{(\alpha-1)} - \frac{\lambda}{(\alpha-1)}\left(\frac{\lambda}{\lambda+c_i}\right)^{\alpha-1}\left[\ln\left(\frac{\lambda}{\lambda+c_i}\right)\right]^2,$$

$$L_i^{(1,2)} = L_i^{(2,1)} = \frac{L_i^{(1)}}{\lambda} - \frac{c_i}{\lambda+c_i}\left(\frac{\lambda}{\lambda+c_i}\right)^{\alpha-1}\ln\left(\frac{\lambda}{\lambda+c_i}\right),$$

$$L_i^{(2,2)} = \frac{L_i^{(2)}}{\lambda} - \frac{L_i}{\lambda^2} + \frac{c_i\lambda^{\alpha-2}(\lambda+c_i-\alpha c_i)}{(\lambda+c_i)^{\alpha+1}}. \tag{4.4}$$

For the data of the example, the matrix is

$$\boldsymbol{A} = \begin{bmatrix} 204{,}021{,}910 & -169{,}261.81 \\ -169{,}261.81 & 148.34278 \end{bmatrix}.$$

The estimated covariance matrix, $\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{\Sigma}\boldsymbol{B}'\boldsymbol{A}^{-1}/6{,}656$ (the denominator is the sample size for this problem), is

$$\begin{bmatrix} 0.034751 & 33.571 \\ 33.571 & 32{,}765 \end{bmatrix}.$$

As expected, the minimum LEV estimator is inferior to maximum likelihood.

TABLE 3

STANDARD DEVIATIONS OF EMPIRICAL LEVS

Limit	LEV	Std. Dev.
$ 100	$ 92	0.3
250	203	1.0
500	338	2.3
1,000	504	4.6
2,500	745	9.9
5,000	920	15.7
10,000	1,079	23.2
25,000	1,264	36.0
50,000	1,367	48.3
100,000	1,433	63.3
500,000	1,587	113.4
1,000,000	1,662	144.2

Example Two

The second example concerns medical malpractice claim count development. The data are from Accomando and Weissner [1]. Cumulative numbers of claims were recorded at intervals of six months through 168 months. The data are presented in Table 4.

Maximum likelihood estimation revealed that the Burr distribution provides a good fit. The distribution function is

$$F(x) = \frac{1 - \left(\dfrac{\lambda^\tau}{\lambda^\tau + x^\tau}\right)^\alpha}{1 - \left(\dfrac{\lambda^\tau}{\lambda^\tau + 168^\tau}\right)^\alpha} . \tag{4.5}$$

TABLE 4

MEDICAL MALPRACTICE CLAIM COUNT DEVELOPMENT

Lag	Claims	Fn	F – MLE	F – Min$F$
6	4	0.0086	0.0020	0.0026
12	10	0.0216	0.0173	0.0194
18	18	0.0389	0.0574	0.0604
24	56	0.1210	0.1257	0.1276
30	101	0.2181	0.2142	0.2139
36	137	0.2959	0.3101	0.3079
42	199	0.4298	0.4025	0.3998
48	232	0.5011	0.4860	0.4838
54	261	0.5637	0.5585	0.5576
60	285	0.6156	0.6207	0.6212
66	307	0.6631	0.6736	0.6754
72	331	0.7149	0.7188	0.7216
78	352	0.7603	0.7574	0.7611
84	369	0.7970	0.7907	0.7949
90	380	0.8207	0.8195	0.8241
96	389	0.8402	0.8447	0.8493
102	396	0.8553	0.8668	0.8714
108	409	0.8834	0.8863	0.8907
114	414	0.8942	0.9036	0.9077
120	416	0.8985	0.9190	0.9229
126	423	0.9136	0.9329	0.9363
132	440	0.9503	0.9454	0.9484
138	445	0.9611	0.9567	0.9592
144	453	0.9784	0.9669	0.9690
150	455	0.9827	0.9763	0.9778
156	461	0.9957	0.9849	0.9859
162	463	1.0000	0.9927	0.9933
168	463	1.0000	1.0000	1.0000

The denominator is required to reflect the truncation of the data at 168 months. The maximum likelihood estimates of the parameters are $\hat{\alpha} = 0.40274$, $\hat{\lambda} = 34.224$, and $\hat{\tau} = 3.1181$. The values of $F(x)$ for this model are presented in Table 4.

The asymptotic covariance matrix of the maximum likelihood estimates is

$$\begin{bmatrix} 0.017336 & 0.57018 & -0.035566 \\ 0.57018 & 20.656 & -1.2135 \\ -0.035566 & -1.2135 & 0.10703 \end{bmatrix}.$$

For minimum distance estimation, the weights were selected as follows: if $F_{n,i} < 0.5$ the weight is 4, while if $F_{n,i} \geq 0.5$ the weight is $1/[F_{n,i}(1 - F_{n,i})]$. This places the smallest emphasis on the early durations and makes the weights proportional to the reciprocal of the variance at later durations (due to the omission of the sample size). Because the value of F_n at the last duration (162) is 1, the weight here is set equal to the one at duration 156. An alternative is to use the model distribution for the weights, changing them at each iteration as the parameters change. The minimum distance estimates are $\hat{\alpha} = 0.48798$, $\hat{\lambda} = 36.989$, and $\hat{\tau} = 2.9496$. These turn out to be very similar to the maximum likelihood estimates. A look at the distribution function in Table 4 verifies that this model does a better job of matching the distribution function, especially after the 95$^{\text{th}}$ percentile.

Estimation of the variance is messier than for the previous example due to the additional parameter and the complexity added by the denominator in Equation 4.5. For this illustration, the elements of $\boldsymbol{A}$ and $\boldsymbol{B}$ were obtained by numerical differentiation. When this approximation was applied to the previous example, the answers matched to two significant digits. The elements of $\boldsymbol{\Sigma}$ are much easier to obtain. The ij^{th} element is

$$\sigma_{ij} = F_i(1 - F_j), \; i \leq j \,. \tag{4.6}$$

The estimated covariance matrix is

$$\begin{bmatrix} 0.081077 & 2.6655 & -0.16625 \\ 2.6655 & 89.507 & -5.5313 \\ -0.16625 & -5.5313 & 0.33525 \end{bmatrix}.$$

This is about four to five times greater than the variances for the maximum likelihood estimate.

The goal of this application is to forecast the number of claims that will be reported after Lag 168. Using the Burr distribution it can be estimated as

$$\hat{\rho} = 463[1/F(168;\hat{\boldsymbol{\theta}}) - 1] = \frac{463}{\left[1 + \left(\frac{168}{\lambda}\right)^{\tau}\right]^{\alpha} - 1}, \tag{4.7}$$

where F is the untruncated Burr distribution. Inserting the maximum likelihood estimates yields $\hat{\rho} = 72.3998$, while doing the same for the minimum distance estimates yields $\hat{\rho} = 58.7556$. An estimate of the variance of these estimators can be obtained by finding the vector of partial derivatives (with respect to the parameters) of $\hat{\rho}$, $\boldsymbol{\delta}$, and then computing $\boldsymbol{\delta}\boldsymbol{\Sigma}\boldsymbol{\delta}$ where $\boldsymbol{\Sigma}$ is the covariance matrix of the parameter estimates. For the maximum likelihood estimate, the variance is 60.703 while for the minimum distance estimate it is 103.09. In the latter case, we can be about 95% confident that there are between 39 and 79 unreported claims.

5. A GOODNESS-OF-FIT TEST

If the model selected is correct, the empirical $G_{n,i}$ will have an approximate multivariate normal distribution with a mean equal to the model G and a covariance matrix given by $\boldsymbol{\Sigma}/n$. If the true parameters were known,

$$n(\boldsymbol{G}_n - \boldsymbol{G})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{G}_n - \boldsymbol{G}), \tag{5.1}$$

where $\boldsymbol{G}$ is the vector of functionals at the true parameter value, would have a chi-square distribution with k degrees of freedom. With the parameters being estimated, it is not so clear what to do. The remainder of this section addresses that problem. The approach used here is similar to the one used to derive the distribution of the chi-square goodness-of-fit test statistic. An excellent exposition can be found in Moore [3]. It is based on the work of Rao [5].

Let $\boldsymbol{V}_n(\boldsymbol{\theta})$ be a $k \times 1$ vector with i^{th} element $w_i^{1/2}[G_n(c_i) - G(c_i \mid \boldsymbol{\theta})]$ so $Q = \boldsymbol{V}_n'(\boldsymbol{\theta})\boldsymbol{V}_n(\boldsymbol{\theta})$. Let $\boldsymbol{\theta}_0$ be the true parameter value and $\boldsymbol{R}$ be a $k \times p$ matrix with ij^{th} element

$$r_{ij} = w_i^{1/2} \frac{\partial G(c_i \mid \boldsymbol{\theta})}{\partial \theta_j} \Big|_{\theta_j = \theta_{0j}} . \tag{5.2}$$

From Equation 2.2, we have

$$\sum_{i=1}^{k} w_i^{1/2} \Big[G_n(c_i) - G(c_i \mid \tilde{\boldsymbol{\theta}}) \Big] \tilde{r}_{ij} = 0, \quad j = 1, \ldots, p \tag{5.3}$$

where $\tilde{r}_{ij}$ is r_{ij} except with the derivative evaluated at $\tilde{\theta}_j$. Next write

$$G_n(c_i) - G(c_i \mid \tilde{\boldsymbol{\theta}}) = G_n(c_i) - G(c_i \mid \boldsymbol{\theta}_0) + G(c_i \mid \boldsymbol{\theta}_0) - G(c_i \mid \tilde{\boldsymbol{\theta}}) \tag{5.4}$$

$$= G_n(c_i) - G(c_i \mid \boldsymbol{\theta}_0) - \sum_{j=1}^{p} \Big[w_i^{-1/2} r_{ij} + o_p(1) \Big] (\tilde{\theta}_j - \theta_{0j})$$

using a Taylor series approximation. Multiplying both sides by $w_i^{1/2}$ and arranging the elements in a $k \times 1$ vector produces

$$\boldsymbol{V}_n(\tilde{\boldsymbol{\theta}}) = \boldsymbol{V}_n(\boldsymbol{\theta}_0) - \boldsymbol{R}(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) - o_p(1)\,(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) . \tag{5.5}$$

Assuming continuity of the elements of $\boldsymbol{R}$ as a function of $\boldsymbol{\theta}$, $\tilde{r}_{ij} = r_{ij} + o_p(1)$. Substituting this and Equation 5.5 into Equation 5.3 gives

$$\boldsymbol{0} = (\boldsymbol{R}' + o_p(1))\boldsymbol{V}_n(\tilde{\boldsymbol{\theta}})$$

$$= (\boldsymbol{R}' + o_p(1))\,[\boldsymbol{V}_n(\boldsymbol{\theta}_0) - \boldsymbol{R}(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) - o_p(1)\,(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}_0)]$$

$$= \boldsymbol{R}'V_n(\boldsymbol{\theta}_0) - \boldsymbol{R}'\boldsymbol{R}(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) + o_p(1)\,. \tag{5.6}$$

Rearranging gives

$$\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}_0 = (\boldsymbol{R}'\boldsymbol{R})^{-1}\boldsymbol{R}'V_n(\boldsymbol{\theta}_0) + o_p(1)\,. \tag{5.7}$$

Substituting Equation 5.7 into Equation 5.5 yields

$$V_n(\tilde{\boldsymbol{\theta}}) = V_n(\boldsymbol{\theta}_0) - \boldsymbol{R}(\boldsymbol{R}'\boldsymbol{R})^{-1}\boldsymbol{R}'V_n(\boldsymbol{\theta}_0) + o_p(1)$$

$$= [\boldsymbol{I} - \boldsymbol{R}(\boldsymbol{R}'\boldsymbol{R})^{-1}\boldsymbol{R}']V_n(\boldsymbol{\theta}_0) + o_p(1)$$

$$= \boldsymbol{C}V_n(\boldsymbol{\theta}_0) + o_p(1)\,. \tag{5.8}$$

Note that $\boldsymbol{C}$ is idempotent and assume that it is of rank $k - p$, as will most certainly be the case. Next observe that

$$V_n(\boldsymbol{\theta}_0) \sim N(\boldsymbol{0}, n^{-1}\boldsymbol{W}^{1/2}\boldsymbol{\Sigma}\boldsymbol{W}^{1/2})\,, \tag{5.9}$$

where $\boldsymbol{W}^{1/2}$ is diagonal with i^{th} diagonal element $w_i^{1/2}$ and $\boldsymbol{\Sigma}$ is as in Equation 3.1. Therefore

$$V_n(\tilde{\boldsymbol{\theta}}) \sim N(\boldsymbol{0}, \boldsymbol{S} = n^{-1}\boldsymbol{C}\boldsymbol{W}^{1/2}\boldsymbol{\Sigma}\boldsymbol{W}^{1/2}\boldsymbol{C})\,. \tag{5.10}$$

In general, if $\boldsymbol{X} \sim N(\boldsymbol{0}, \boldsymbol{S})$ then $\boldsymbol{X}'\boldsymbol{S}^{-}\boldsymbol{X} \sim \chi^2(m)$ where m is the rank of $\boldsymbol{S}$ and $\boldsymbol{S}^{-}$ is a generalized inverse (Moore [3], Theorem 2). One definition (among many that are equivalent) of a generalized inverse is that $x = \boldsymbol{S}^{-}y$ solves $y = \boldsymbol{S}x$ provided y is in the column space of $\boldsymbol{S}$. That is, if there is a solution to the equation, then $\boldsymbol{S}^{-}$ will provide it. A discussion of generalized inverses can be found in Searle [6]. At first it appears that this test is arbitrary, because the generalized inverse is not unique. But, for $\boldsymbol{X}$ in the column space of $\boldsymbol{S}$, $\boldsymbol{X}'\boldsymbol{S}^{-}\boldsymbol{X}$ will take on the same value, regardless of the form of the generalized inverse selected. For the normal distribution, the probability that this will happen is 1. Because the normal distribution in Equation 5.10 is approximate, it is possible that in practice, the value will depend slightly on the form of the generalized inverse selected. The test statistic is then

$$V_n(\hat{\theta})'S^- V_n(\hat{\theta}) = (G_n - G)'W^{1/2}S^- W^{1/2}(G_n - G), \tag{5.11}$$

which is very similar to Equation 5.1.

For the second example, the value using Equation 5.1 is 70.53; it is 70.115 using Equation 5.11 with the Moore-Penrose generalized inverse. Both values clearly exceed the 5% critical value for 24 degrees of freedom (36.42).

This test indicates that a better choice of weights would have been appropriate. One such choice, from pure statistical (as opposed to actuarial) considerations, would be the reciprocals of the diagonal elements of $\boldsymbol{\Sigma}$. Aside from being an advance attempt to pass the hypothesis test, it makes sense in that the expected value of each term of $\boldsymbol{Q}$ is $1/n$. Thus, each term is making an approximately equal contribution to the criterion. For the Pareto example, a look at Table 3 shows that the weights would be decreasing with c_i. Again, this makes statistical sense, as for low limits virtually any reasonable model will produce an LEV that is just a little bit below c_i, and the empirical LEV will also be in that range. At the larger limits, there is likely to be much more sampling error and, therefore, wider variations should be tolerated. However, for actuarial purposes, one might come to the opposite conclusion. Once put to use, the model will be evaluated only at the larger limits, and so it is there where deviations from the sample should be small.

A more direct form of hypothesis test would be one based on $\boldsymbol{Q}$. This would be similar to the Cramer-von Mises test for comparing a model cdf to the empirical cdf. It has the advantage of being independent of the weights in the sense that the parameter estimate is, by definition, the one that minimizes the test statistic. However, this involves extra work as the distribution of $\boldsymbol{Q}$ under the null hypothesis is not so easy to obtain and depends heavily on the unknown $\boldsymbol{\theta}$.

6. SIMULATION

The theorem and hypothesis test are both asymptotic results. Also, both employ the replacement of the true parameter value by the estimate to complete the calculations. In this section, a simulation study is conducted to provide some feel for the accuracy of the method.

The true model selected for the study is Pareto with $\alpha = 3$ and $\lambda = 500$. The empirical LEV is obtained at 12 points: 20, 40, 65, 90, 130, 180, 250, 350, 575, 850, 1,300, and 2,000. At each simulation, 500 observations were generated. The parameters are then estimated by the minimum LEV method using weights of 1, 1, 1, 1, 1, 1, 1, 2, 4, 8, 16, and 16. The covariance matrix was also estimated, using Equation 3.4. Finally, the chi-square goodness-of-fit test statistic was computed using both Equations 5.1 and 5.11. The latter was done with two different algorithms for the generalized inverse, the Moore-Penrose and a sweep method. If the results in Sections 3 and 5 hold, the following should be observed:

1. The sample mean of the parameter estimates should be close to the true value. This will indicate that the estimator is unbiased.
2. The sample covariance matrix of the parameter estimates should be close to the matrix given by Equation 3.4 using the true parameter values. This will indicate that the theorem gives reasonable results for samples of size 500.
3. The estimated covariance matrices should have an average that is close to the matrix given by Equation 3.4 using the true parameter values. This will indicate that the replacement of the true values by the estimates does not distort the covariance estimation (on average).
4. The goodness-of-fit test statistics should have a sample mean of 10 and a sample variance of 20. This will indicate that the chi-square distribution with 10 degrees is reasonable. Also, 95% of the time the test statistic should be less than 18.307, and 99% of the time it should be less than 23.209. This will confirm that the significance level is as advertised.

A run of 1,000 simulations was conducted. The asymptotic covariance matrix for maximum likelihood estimation is

$$\begin{bmatrix} 0.3217 & 66.89 \\ 66.89 & 14{,}750 \end{bmatrix}.$$

The asymptotic covariance matrix for minimum LEV estimation is

$$\begin{bmatrix} 0.6640 & 120.3 \\ 120.3 & 21{,}830 \end{bmatrix}.$$

The sample means of the minimum LEV estimates were 3.161 for α and 535.1 for λ. The standard errors for α and λ are 0.023 and 4.9, respectively, indicating that, for a sample size of 500, there is bias in these estimates. The sample variances were 0.5133 for α and 24,150 for λ, and the sample covariance was 108.8. These are close to those given by the asymptotic approximation, indicating that Point 2 holds for this problem. With both estimates having a positive bias, there is some cancellation of error. For example, the true mean is $500/2 = 250$ while the mean of the Pareto distribution using the sample means is $535.1/2.161 = 247.62$. Using the approximation for the covariance matrix yielded average variances of 1.178 and 54,196. These considerably overstate the true values, and so Point 3 does not hold. Finally, the basic chi-square test (Equation 5.1) accepted the model 94.8% of the time when a 5% significance level was used and 99.3% of the time when a 1% level was used. Using Equation 5.11 with the Moore-Penrose inverse yielded acceptance rates of 95.5% and 99.4%, while the sweep inverse accepted the model 95.4% and 99.4% of the time. Another indication of accuracy is the mean and variance of the chi-square statistics. They were 10.002 and 19.542 for Equation 5.1, 9.843 and 18.849 for the Moore-Penrose inverse, and 9.846 and 18.847 for the sweep inverse. Finally, the absolute differences in the chi-square statistics were averaged for each of the three possible comparisons. For Equation 5.1 versus Moore-Penrose, the average absolute difference was 0.158; and versus the sweep inverse, it was 0.162. The two versions of Equation 5.11 had an average absolute difference of 0.016. It appears that any of the three tests are likely to be valid.

REFERENCES

[1] Accomando, F., and Weissner, E., "Report Lag Distributions: Estimation and Application to IBNR Counts," *Transcripts of the 1988 Casualty Loss Reserve Seminar,* 1988, p. 1038.

[2] Benichou, J., and Gail, M., "A Delta Method for Implicitly Defined Random Variables," *The American Statistician,* Vol. 43, 1989, p. 41.

[3] Moore, D., "Chi-Square Tests," in *Studies in Statistics,* R.V. Hogg, ed., Mathematical Association of America, 1978, p. 66.

[4] Nelder, J., and Mead, R., "A Simplex Method for Function Minimization," *The Computer Journal,* Vol. 7, 1965, p. 308.

[5] Rao, C.R., "Theory of the Method of Estimation by Minimum Chi-Square," *Bulletin of the International Statistics Institute,* Vol. 35, 1955, p. 25.

[6] Searle, S., *Linear Models,* New York, Wiley, 1971.

[7] Walters, F., Parker, L., Morgan, S., and Deming, S., *Sequential Simplex Optimization,* Boca Raton, Florida, CRC Press, 1991.

ESTIMATING SALVAGE AND SUBROGATION RESERVES—ADAPTING THE BORNHUETTER-FERGUSON APPROACH

GREGORY S. GRACE

Abstract

With the recent Internal Revenue Service and NAIC interest in salvage and subrogation reserves, insurance companies must develop methods of estimating anticipated recoveries. This paper examines two traditional methods and proposes an adapted Bornhuetter-Ferguson method for the projection of salvage and subrogation recoveries.

1. INTRODUCTION

Salvage and subrogation reserves recently have become a more prominent issue than ever before. The Internal Revenue Service's new rules requiring an explicit adjustment to the losses for these reserves is at the heart of this piqued interest. Previously the tax calculation was based on loss reserves as reported in the statutory statement adjusted for discounting. According to the statutory rules, salvage and subrogation recoveries were not to be anticipated in these reserves. Beginning with the 1990 tax year, insurance companies have been required to specifically reflect salvage and subrogation recoverable on unpaid losses. In addition, beginning with the 1992 Annual Statement, reserves may be shown net of anticipated recoveries in the statutory statement.

Many companies which previously had never addressed the issue of estimating salvage and subrogation reserves are now faced with the task of determining this amount.

It is difficult to ascertain the impact of salvage and subrogation on an industry-wide basis at this time since there is no source that shows total recoveries including all anticipated recoveries. Also, there are not sufficient data available to independently determine this amount.

2. THE TRADITIONAL METHODS

Ideally, a company would have a sufficient volume of data regarding recoveries so the recoveries could be estimated independently using a paid or incurred loss projection method (Method 1). Salvage would be projected independently of subrogation since there is no expectation that the two types of recoveries develop similarly. In practice, this is seldom the usual case, especially for smaller companies. For some lines, particularly liability lines, there may be few recoveries received until an accident year has reached two or three years of development. In this instance, there is no base of recovery data from which to project.

Exhibit 2 shows a reserve estimate using the paid projection method on the hypothetical data in Exhibit 1. In the example, the 12-to-24 month factor is indeterminable since, historically, there have been no recoveries in the first 12 months. Therefore it is impossible to project a reserve for the current accident year ($ay - 0$) utilizing a strict adherence to this method.

The next most logical approach (Method 2) would be to perform two separate loss projections; one excluding, or gross of, the recoveries and one including, or net of, the recoveries. The difference between the ultimates resulting from these two projections would be the projected recoveries. This is similar to establishing ceded IBNR as the difference between the separate projections of direct IBNR and net IBNR (assuming there are no assumed losses). But, as in the prior case, there can be problems using this method without adjustments.

Exhibit 3 illustrates the major problem which can occur when using this method. In this example, the projected ultimate for $ay - 2$ including recoveries is \$92 less than the projected ultimate excluding recoveries, yielding anticipated ultimate recoveries of \$92. However, we already have received \$100 in recoveries so our reserve indication is \$92 – \$100 = (\$8). Thus our projected ultimate salvage and subrogation recovery is negative. Obviously, this normally would not be acceptable.

3. THE ALTERNATIVE

To avoid these problems, an adaptation of the Bornhuetter-Ferguson method can be used. Briefly, in its original form this method utilizes three input factors to determine an IBNR reserve. These three factors are earned premium, anticipated loss ratio, and a loss reporting pattern. IBNR is then calculated as the product of the premium, loss ratio, and expected percent of ultimate losses which are unreported as of the current valuation.

Adaptation of this method to establish salvage and subrogation reserves (Method 3) involves a substitution of variables. Projected ultimate incurred losses are used in place of earned premium, the anticipated ultimate recovery ratio (ultimate salvage and subrogation divided by ultimate losses) is substituted for the anticipated loss ratio, and the salvage and subrogation reporting pattern is substituted for the loss reporting pattern.

It is assumed that the ultimate losses have been estimated elsewhere. Note that this method is not dependent on whether the losses are gross or net of recoveries, as long as the anticipated recovery ratio utilizes losses on the same basis in the denominator. These two projections will not necessarily yield the same reserves, but, in most cases, the projections should be reasonably close.

Exhibit 4 illustrates this proposed method using the same hypothetical data as above. One immediate benefit of this method can be seen in Column 2. The fact that we cannot calculate an age-to-age factor for the 12-24 month period does not cause a problem for us. We know that no recoveries are anticipated to have been made as of 12 months, so the percent reported is 0%. The percentages in Column 2 are subtracted from 100% to yield the expected portion of recoveries which have not yet been reported, Column 3. It is this column that will be used in the calculation.

By incorporating the projected ultimate losses excluding recoveries from Exhibit 3, we have two of the three necessary factors. Only one factor needs to be determined, the anticipated recovery ratio. This can be estimated in a variety of ways, from historical information to pure judgment. This ratio may vary from year to year. In the example we have no reason to expect that this ratio should not be relatively static. It is therefore assumed that our a priori estimate is the same for each accident year, as shown in Column 6. The derivation of this factor will be explained later. The product of these three factors is the estimated salvage and subrogation reserve, shown in Column 7. Columns 10 through 14 repeat the procedure using losses including recoveries. In this example, the results are equivalent.

It is possible to stop here, but there are a few additional steps necessary to explain the choice of 0.111 (and 0.125) as the ratio of recoveries to losses. Column 4 displays recoveries received to date. Adding these actual recoveries to the reserves gives us the estimated ultimate recoveries, shown in Column 8. Column 9 is the ratio of ultimate recoveries to ultimate losses. It is the expected value of this ratio that we needed as the third input factor. Thus, it is the average of this column, or 0.111 in the example, which was selected as the input factor. The twist is that Column 9 depends on the value placed in Column 6 which, in turn, depends on Column 9. Obviously, this is not a straightforward computation.

If we assume that the anticipated recovery ratio should be constant, this factor can be mathematically determined as follows:

Since

$$R/L = \Sigma\, [UL \times PRU \times R/L + SS]/\Sigma\, UL,$$

where:

R/L = expected ratio of recoveries to losses,

UL = ultimate losses,

PRU = percent of recoveries unreported, and

SS = salvage and subrogation recoveries to date.

Then

$$R/L \times \Sigma\, UL = R/L \times \Sigma\, [UL \times PRU] + \Sigma\, SS$$
$$R/L \times [\, \Sigma\, UL - \Sigma\, [UL \times PRU]] = \Sigma\, SS$$
$$R/L = \Sigma\, SS/[\Sigma\, UL - \Sigma\, [UL \times PRU]]\,.$$

Thus, in the example,

$$\begin{aligned} R/L &= 600/[7{,}500 - (1{,}500 \times 0\% + 1{,}500 \times 0\% + 1{,}500 \times 0\% \\ &\quad + 1{,}500 \times 40\% + 1{,}500 \times 100\%)] \\ &= 600/[7{,}500 - 2{,}100] \\ &= 0.111. \end{aligned}$$

The 0.125 is similarly determined.

This variation of the Bornhuetter-Ferguson method is similar to the Stanard-Bühlmann method of loss development as described in Chapter 6 of the *Foundations of Casualty Actuarial Science* text.

4. APPLICATION TO REAL DATA

Exhibits 5 through 7 take real data extracted from *Best's Aggregates and Averages* Consolidated Industry Schedule P for Other Liability and project the indicated salvage and subrogation reserve using Methods 1 through 3, respectively. All data are from the 1990 Annual Statement reproduction except for salvage and subrogation received as of 12/31/89. These data are from Part 1H, Column 9 of the 1989 reproduction. Although it is technically incorrect to match this data with the corresponding data from the 1990 statement due to the change in the mix of companies included in the consolidated statements, I have done so for demonstration purposes. The data shown for paid loss and ALAE are from Part 3H, Columns 10 and 11. The projected ultimate loss and ALAE shown in Column 6 of Exhibit 7 is from Part 2H, Column 11.

Columns 5 and 10 of Exhibit 6 highlight another potential problem with Method 2. While these two columns of projections are consistent with one another and make sense within the context of estimating the salvage and subrogation reserves, there is a large disparity if they are compared to the actual estimated ultimate losses, shown in Column 6 of Exhibit 7. It would be difficult to explain why the estimated ultimates used for establishing salvage and subrogation reserves were so different from the Statement ultimates. The only alternative would be to adjust the projections so that they reconcile, which would require modifying both sets of factors, which, in turn, may distort the salvage and subrogation projection process.

Exhibit 8 compares the projections from each of the methods. While we cannot know at this time which method is closest to being correct, it appears that the proposed method (Method 3) yields a result which is at least as reasonable as the others without any of the potential drawbacks.

5. CONCLUSION

Salvage and subrogation reserves can be computed simply even when the available data are limited. All that is required are three factors. One of these, ultimate incurred losses, should already be available. Another, the expected ratio of ultimate recoveries to ultimate losses, can be determined within the process. The only other requirement, a salvage and subrogation reporting pattern, can be computed by using a loss triangle approach. If a triangle is unavailable, it is possible that data on recoveries from Schedule P, Part 1 of two consecutive statutory blanks could be used to derive the needed development factors.

REFERENCES

[1] Bornhuetter, Ronald L., and Ferguson, Ronald E., "The Actuary and IBNR," *PCAS* LIX, 1972, p. 181.

[2] Patrik, Gary S., "Reinsurance," *Foundations of Casualty Actuarial Science,* Casualty Actuarial Society (First Edition), 1990, Chapter 6.

[3] A. M. Best Company, *Best's Aggregates & Averages Property-Casualty,* 1991 Edition.

[4] A. M. Best Company, *Best's Aggregates & Averages Property-Casualty,* 1990 Edition.

EXHIBIT 1

Loss Triangles

Actual Salvage and Subrogation Recoveries

Accident Year	Months of Development 12	24	36	48	60
$ay-4$	0	100	200	200	200
$ay-3$	0	100	200	200	
$ay-2$	0	100	100		
$ay-1$	0	100			
$ay-0$	0				
Age-to-Age Factor		xx	1.667	1.000	1.000
Factor to Ultimate		xx	1.667	1.000	1.000

Paid Losses Excluding Recoveries

Accident Year	Months of Development 12	24	36	48	60
$ay-4$	1,000	1,200	1,400	1,500	1,500
$ay-3$	1,000	1,200	1,400	1,500	
$ay-2$	1,000	1,200	1,400		
$ay-1$	1,000	1,200			
$ay-0$	1,000				
Age-to-Age Factor		1.200	1.167	1.071	1.000
Factor to Ultimate		1.500	1.250	1.071	1.000

Paid Losses Including Recoveries

Accident Year	Months of Development 12	24	36	48	60
$ay-4$	1,000	1,100	1,200	1,300	1,300
$ay-3$	1,000	1,100	1,200	1,300	
$ay-2$	1,000	1,100	1,300		
$ay-1$	1,000	1,100			
$ay-0$	1,000				
Age-to-Age Factor		1.100	1.121	1.083	1.000
Factor to Ultimate		1.336	1.215	1.083	1.000

EXHIBIT 2

Salvage and Subrogation Reserve Projection

Method 1—Paid Recoveries Development Method

Accident Year	(1) Salvage & Subrogation Received to Date	(2) Factor to Ultimate	(3) Projected Ultimate Recoveries [(1) x (2)]	(4) Salvage & Subrogation Reserve [(3) - (1)]
$ay-4$	200	1.000	200	0
$ay-3$	200	1.000	200	0
$ay-2$	100	1.000	100	0
$ay-1$	100	1.667	167	67
$ay-0$	0	xx	xx	xx
Total	600		xx	xx

EXHIBIT 3

SALVAGE AND SUBROGATION RESERVE PROJECTION

METHOD 2—DIFFERENCE BETWEEN LOSSES PROJECTED WITH AND WITHOUT RECOVERIES

Accident Year	(1) Losses Paid to Date Excluding Recoveries	(2) Factor to Ultimate	(3) Projected Ultimate Losses Excl. Recoveries [(1) x (2)]
$ay - 4$	1,500	1.000	1,500
$ay - 3$	1,500	1.000	1,500
$ay - 2$	1,400	1.071	1,500
$ay - 1$	1,200	1.250	1,500
$ay - 0$	1,000	1.500	1,500
Total	6,600		7,500

Accident Year	(4) Losses Paid to Date Including Recoveries	(5) Factor to Ultimate	(6) Projected Ultimate Losses Incl. Recoveries [(4) x (5)]
$ay - 4$	1,300	1.000	1,300
$ay - 3$	1,300	1.000	1,300
$ay - 2$	1,300	1.083	1,408
$ay - 1$	1,100	1.215	1,336
$ay - 0$	1,000	1.336	1,336
Total	6,000		6,680

Accident Year	(7) Projected Ultimate Recoveries [(3) - (6)]	(8) Salvage & Subrogation Received to Date	(9) Salvage & Subrogation Reserve [(7) - (8)]
$ay - 4$	200	200	0
$ay - 3$	200	200	0
$ay - 2$	92	100	(8)
$ay - 1$	164	100	64
$ay - 0$	164	0	164
Total	820	600	220

EXHIBIT 4

SALVAGE AND SUBROGATION RESERVE PROJECTION

METHOD 3—ADAPTED BORNHUETTER-FERGUSON

	(1)	(2)	(3)	(4)
Accident Year	Salvage and Subrogation Factor to Ultimate	Percent of Recoveries Reported [1/(1)]	Percent of Recoveries Unreported [100% − (2)]	Salvage and Subrogation Rec'd to Date
ay − 4	1.000	100%	0%	200
ay − 3	1.000	100	0	200
ay − 2	1.000	100	0	100
ay − 1	1.667	60	40	100
ay − 0	xx	0	100	0

PROJECTION USING LOSSES EXCLUDING RECOVERIES AS A BASE

	(5)	(6)	(7)	(8)	(9)
Accident Year	Projected Ultimate Losses Excluding Recoveries [Exh. 3, Col (3)]	Expected Recovery Ratio	Projected Salvage and Subrogation Reserve [(3) × (5) × (6)]	Projected Ultimate Recoveries [(4) + (7)]	Indicated Recovery Ratio [(8) / (5)]
ay − 4	1,500	0.111	0	200	0.133
ay − 3	1,500	0.111	0	200	0.133
ay − 2	1,500	0.111	0	100	0.067
ay − 1	1,500	0.111	67	167	0.111
ay − 0	1,500	0.111	167	167	0.111
Total	7,500		234	834	0.111

PROJECTION USING LOSSES INCLUDING RECOVERIES AS A BASE

	(10)	(11)	(12)	(13)	(14)
Accident Year	Projected Ultimate Losses Including Recoveries [Exh. 3,Col (6)]	Expected Recovery Ratio	Projected Salvage & Subrogation Reserve [(3)×(10)×(11)]	Projected Ultimate Recoveries [(4)+(12)]	Indicated Recovery Ratio [(13)/(10)]
ay − 4	1,300	0.125	0	200	0.154
ay − 3	1,300	0.125	0	200	0.154
ay − 2	1,408	0.125	0	100	0.071
ay − 1	1,336	0.125	67	167	0.125
ay − 0	1,336	0.125	167	167	0.125
Total	6,680		234	834	0.125

EXHIBIT 5

SALVAGE AND SUBROGATION RESERVE PROJECTION
METHOD 1—PAID RECOVERIES DEVELOPMENT METHOD

	(1)	(2)	(3)	(4)	(5)	(6)
Accident Year	Salvage and Subrogation Received @ 12/31/89	Salvage and Subrogation Received @ 12/31/90	Age-to-Age Factor [(2) / (1)]	12/31/90 Factor to Ultimate	Projected Ultimate Salvage and Subrogation [(2) × (4)]	Estimated Salvage and Subrogation Reserve [(5) − (2)]
Prior	33,261(a)	22,146(b)	1.6658(c)			
1981	44,596	49,701	1.1145	1.6658	82,793	33,092
1982	51,216	58,268	1.1377	1.8565	108,175	49,907
1983	45,169	52,804	1.1690	2.1121	111,530	58,726
1984	54,051	61,176	1.1318	2.4692	151,053	89,877
1985	42,666	62,954	1.4755	2.7946	175,934	112,980
1986	29,999	38,052	1.2684	4.1235	156,908	118,856
1987	20,291	28,045	1.3821	5.2304	146,688	118,643
1988	13,697	20,971	1.5311	7.2292	151,604	130,633
1989	9,801	17,834	1.8196	11.0684	197,394	179,560
1990		6,452		20.1404	129,944	123,492
Total						1,015,766

(a) Accident year 1980
(b) 1990 recoveries for all years prior to 1981
(c) 1 + (2)/(1)

EXHIBIT 6
Part 1
SALVAGE AND SUBROGATION RESERVE PROJECTION
METHOD 2—DIFFERENCE BETWEEN LOSSES PROJECTED WITH & WITHOUT RECOVERIES

	(1)	(2)	(3)	(4)	(5)
Accident Year	Paid Loss & ALAE @ 12/31/89 Including Recoveries	Paid Loss & ALAE @ 12/31/90 Including Recoveries	Age-to-Age Factor [(2)/(1)]	12/31/90 Factor to Ultimate	Projected Ultimate Loss & ALAE Incl. Recoveries [(2) ×(4)]
Prior	4,210,629(a)	1,285,126(b)	1.3052(c)	1.0000	1,285,126
1981	4,538,025	4,708,084	1.0375	1.3052	6,145,038
1982	5,034,356	5,260,011	1.0448	1.3541	7,122,695
1983	5,374,654	5,809,967	1.0810	1.4148	8,220,043
1984	5,886,801	6,538,530	1.1107	1.5294	10,000,085
1985	5,837,150	6,915,234	1.1847	1.6987	11,747,115
1986	4,741,568	6,086,522	1.2837	2.0125	12,248,970
1987	3,413,265	4,975,690	1.4578	2.5833	12,853,781
1988	2,377,818	4,116,062	1.7310	3.7658	15,500,397
1989	1,129,567	2,688,211	2.3799	6.5187	17,523,770
1990		1,118,052		15.5137	17,345,130

EXHIBIT 6
Part 2
SALVAGE AND SUBROGATION RESERVE PROJECTION
METHOD 2—DIFFERENCE BETWEEN LOSSES PROJECTED WITH & WITHOUT RECOVERIES

	(6)	(7)	(8)	(9)	(10)
Accident Year	Paid Loss & ALAE @ 12/31/89 Excluding Recoveries	Paid Loss & ALAE @ 12/31/90 Excluding Recoveries	Age-to-Age Factor [(7)/(6)]	12/31/90 Factor to Ultimate	Projected Ultimate Loss & ALAE Excl. Recoveries [(7) × (9)]
Prior	4,243,890(a)	1,307,272(b)	1.3080(d)	1.0000	1,307,272
1981	4,582,621	4,757,785	1.0382	1.3080	6,223,355
1982	5,085,572	5,318,279	1.0458	1.3580	7,222,404
1983	5,419,823	5,862,771	1.0817	1.4202	8,326,163
1984	5,940,852	6,599,706	1.1109	1.5362	10,138,749
1985	5,879,816	6,978,188	1.1868	1.7066	11,909,083
1986	4,771,567	6,124,574	1.2836	2.0254	12,404,820
1987	3,433,556	5,003,735	1.4573	2.5997	13,008,396
1988	2,391,515	4,137,033	1.7299	3.7886	15,673,594
1989	1,139,368	2,706,045	2.3750	6.5538	17,734,971
1990		1,124,504		15.5656	17,503,604

EXHIBIT 6
Part 3
SALVAGE AND SUBROGATION RESERVE PROJECTION
METHOD 2—DIFFERENCE BETWEEN LOSSES PROJECTED WITH & WITHOUT RECOVERIES

Accident Year	(11) Projected Ultimate Recoveries [(10) – (5)]	(12) Salvage and Subrogation Received @ 12/31/90	(13) Salvage and Subrogation Reserve [(11) – (12)]
1981	78,317	49,701	28,616
1982	99,709	58,268	41,441
1983	106,120	52,804	53,316
1984	138,664	61,176	77,488
1985	161,967	62,954	99,013
1986	155,850	38,052	117,798
1987	154,614	28,045	126,569
1988	173,197	20,971	152,226
1989	211,201	17,834	193,367
1990	158,474	6,452	152,022
Total	1,438,113	396,257	1,041,856

(a) Accident year 1980
(b) 1990 payments for all years prior to 1981
(c) 1 + (2)/(1)
(d) 1 + (7)/(6)

EXHIBIT 7
Part 1
SALVAGE AND SUBROGATION RESERVE PROJECTION
METHOD 3—ADAPTED BORNHUETTER-FERGUSON

Accident Year	(1) Salvage and Subrogation Received @ 12/31/89	(2) Salvage and Subrogation Received @ 12/31/90	(3) Age-to-Age Factor [(2)/(1)]	(4) 12/31/90 Factor to Ultimate	(5) Percent of Recoveries Unreported [1 − 1/(4)]
Prior	33,261(a)	22,146(b)	1.6658(c)	1.0000	0%
1981	44,596	49,701	1.1145	1.6658	40
1982	51,216	58,268	1.1377	1.8565	46
1983	45,169	52,804	1.1690	2.1121	53
1984	54,051	61,176	1.1318	2.4692	60
1985	42,666	62,954	1.4755	2.7946	64
1986	29,999	38,052	1.2684	4.1235	76
1987	20,291	28,045	1.3821	5.2304	81
1988	13,697	20,971	1.5311	7.2292	86
1989	9,801	17,834	1.8196	11.0684	91
1990		6,452		20.1402	95

EXHIBIT 7
Part 2
SALVAGE AND SUBROGATION RESERVE PROJECTION
METHOD 3—ADAPTED BORNHUETTER-FERGUSON

	(6)	(7)	(8)	(9)	(10)
Accident Year	Projected Ultimate Loss & ALAE	Expected Recovery Ratio	Estimated Salvage and Subrogation Reserve [(5) × (6) × (7)]	Estimated Ultimate Salvage and Subrogation [(2) + (8)]	Indicated Recovery Ratio [(9)/(6)]
1981	5,405,329	0.016	33,514	83,215	0.015
1982	6,197,270	0.016	44,188	102,456	0.017
1983	7,101,341	0.016	58,339	111,143	0.016
1984	8,432,516	0.016	78,424	139,600	0.017
1985	9,734,415	0.016	96,568	159,522	0.016
1986	11,018,491	0.016	129,801	167,853	0.015
1987	12,441,103	0.016	156,201	184,246	0.015
1988	13,278,817	0.016	177,011	197,982	0.015
1989	13,416,986	0.016	189,251	207,085	0.015
1990	13,696,887	0.016	201,691	208,143	0.015
Total	100,723,155		1,164,986	1,561,243	0.016

(a) Accident year 1980
(b) 1990 recoveries for all years prior to 1981
(c) 1 + (2)/(1)

EXHIBIT 8
Part 1
SALVAGE AND SUBROGATION RESERVE PROJECTION
COMPARISON OF ESTIMATES

Accident Year	Ultimate Recoveries Method 1	Ultimate Recoveries Method 2	Ultimate Recoveries Method 3
1981	82,793	78,317	83,215
1982	108,175	99,709	102,456
1983	111,530	106,120	111,143
1984	151,053	138,664	139,600
1985	175,934	161,967	159,522
1986	156,908	155,850	167,853
1987	146,688	154,614	184,246
1988	151,604	173,197	197,982
1989	197,394	211,201	207,085
1990	129,944	158,474	208,143
Total	1,412,023	1,438,113	1,561,243

EXHIBIT 8
Part 2
SALVAGE AND SUBROGATION RESERVE PROJECTION
COMPARISON OF ESTIMATES

Accident Year	Salvage and Subrogation Reserve Method 1	Salvage and Subrogation Reserve Method 2	Salvage and Subrogation Reserve Method 3
1981	33,092	28,616	33,514
1982	49,907	41,441	44,188
1983	58,726	53,316	58,339
1984	89,877	77,488	78,424
1985	112,980	99,013	96,568
1986	118,856	117,798	129,801
1987	118,643	126,569	156,201
1988	130,633	152,226	177,011
1989	179,560	193,367	189,251
1990	123,492	152,022	201,691
Total	1,015,766	1,041,856	1,164,986

EXHIBIT 8
Part 3
SALVAGE AND SUBROGATION RESERVE PROJECTION
COMPARISON OF ESTIMATES

Accident Year	Ultimate Recovery Ratio Method 1	Ultimate Recovery Ratio Method 2	Ultimate Recovery Ratio Method 3
1981	0.015	0.014	0.015
1982	0.017	0.016	0.017
1983	0.016	0.015	0.016
1984	0.018	0.016	0.017
1985	0.018	0.017	0.016
1986	0.014	0.014	0.015
1987	0.012	0.012	0.015
1988	0.011	0.013	0.015
1989	0.015	0.016	0.015
1990	0.009	0.012	0.015

MERIT RATING FOR DOCTOR PROFESSIONAL LIABILITY INSURANCE

ROBERT J. FINGER

Abstract

Merit rating is the use of the insured's actual claim experience to predict future claim experience. This paper discusses merit rating for professional liability insurance for both individual doctors and group practices. The paper presents several different theoretical formulations for merit rating. Credibilities are stated in terms of the parameters of the risk process. The paper discusses several methods of estimating the key parameters, along with sample data. Finally, the paper discusses several practical considerations in the design of a merit rating formula.

1. INTRODUCTION

The use of an insured's past claim experience for prospective premium determination can variously be called experience rating or *merit rating*. Merit rating is common for workers' compensation and commercial liability coverages. Merit rating for individual insureds is less common, although "claim-free discounts" or accident surcharges for personal automobile insurance are widely used. Several insurers now use merit rating for doctor professional liability insurance.

Section 2 of this paper provides a general statement of the merit rating problem. Section 3 presents the mathematical formulation of the risk process. It also discusses alternative merit rating formulations in terms of the parameters of the risk process. Section 4 provides several methods for estimating the required parameters. It applies these methods to actual data. Finally, Section 5 discusses various practical problems in implementing a merit rating program. The paper deals with two related situations: (1) Claim-free discounts and surcharges for individual doctors, and (2) merit rating for group practices.

2. GENERAL STATEMENT OF THE PROBLEM

We assume that there is some classification plan that will determine a premium for a given doctor (or group of doctors). The classification variables may include medical specialty, types of procedures, geography, and teaching or part-time status. For groups, there may also be schedule rating credits.

Why do we also want to use merit rating? Generally speaking, we want to use the insured's actual claim experience if (and only if) it is an efficient predictor of future claim costs. The insured's own claim experience may provide additional information that is not included in the other rating variables. Below, we give some reasons why the class rating variables may not have captured all of the relevant information. Using additional information may produce more accurate rates.

In a competitive environment, more accurate rates will generate greater profitability for the insurer. From the insured's point of view, more accurate rates are also fairer. Better doctors (in the sense of being less claims prone) will pay less and poorer doctors will pay more. From society's point of view, merit rating (and more accurate rating, generally) will provide an incentive for loss prevention.

Merit rating should be considered to be a complement to the classification plan (i.e., other rating variables). The more accurate the class plan, the less meaningful individual claim experience will be, and vice versa. Assume, for example, that the presence of a particular factor makes an insured 10% more expensive. If that variable is used in the classification plan, every insured with that factor will pay 10% more. If that variable is omitted, insureds with that factor who are merit rated will pay somewhat more than those without the factor, but most likely they will not pay 10% more. This follows from the concept that most insureds will receive less than 100% credibility.

Why do Individual Costs Differ?

Why would we expect doctors to have different loss costs? It is well recognized that different specialties have widely differing costs. This probably results from a variety of reasons. Certain specialties, such as surgeons, perform a higher percentage of procedures that can have devastating results, if done improperly. For certain specialties, such as psychiatrists, it may be very difficult to prove the causal connection between negligent practice and adverse results for the patient. For certain specialties, such as general practitioners with no surgery, the average patient is much healthier and any negligence is less likely to do damage. Thus, most insurers classify doctors by specialty. For physicians, most insurers also classify by the type or amount of surgery performed.

This classification plan does not cover all possible variations in costs among doctors in the same specialty. Costs may also vary for three general reasons: (1) Limitations in the class plan, (2) exposure, and (3) competence. Each will be discussed below.

Limitations in the Class Plan

Most class plans group specialties into about 10 different rate groups. In addition to specialty, the grouping may depend upon whether a doctor performs various procedures. The reason for this grouping is a lack of credibility for many specialties and procedures. That is, the number of insured doctors and the number of claims for many specialties and procedures are low. The volatility of claim experience for these low-volume categories makes it difficult to determine their cost. It is also difficult to determine how many of a certain type of procedure were performed during a given year. Doctors are usually classified by whether or not they perform a procedure, not on the number of procedures.

This classification scheme can result in significant cost variation within a given rate group. For example, Group 0 may have a rate relativity of 70%; Group 1, 100%; and Group 2, 150%. Within Group 0, there may be specialties that have relativities of 50%, 60%, 70%, and 80%. Within Group 1, there may be specialties with relativities of 90%, 100%, 110%, and 125%. In addition, the exposure to certain procedures may vary significantly. For example, the performance of procedure A may shift a doctor's classification from Group 1 to Group 2. Some doctors may perform 10 A's a year and some may perform 50 A's a year. A more exact classification plan might base the premium on the number of A procedures during the year.

The classification plan also may not consider other cost variations. Costs vary significantly from state to state. Some of this is due to differences in statutory or case law. Some of the difference may also be due to differences in the liberality of juries, the quality of the plaintiff's bar, and the claims consciousness of patients. These latter differences may exist within a state. In particular, there may be differences between urban, suburban, and rural areas.

Exposure

There may also be cost differences among doctors related to differences in exposure. For example, some doctors may treat more patients or may engage in more high-risk procedures. In addition, the type of patient may be different. Some doctors may have richer or poorer clients, who may have higher or lower damages, should negligence occur. Some doctors may also accept higher-risk patients, which could affect both the frequency and severity of loss costs.

Competence

Finally, doctors undoubtedly differ in competence, which has many causes. Training and experience differ. Doctors vary in their adherence to continuing education and changing practice standards. Doctors vary in their dexterity, judgment, attention to detail, bedside manner, and supervisory skills. The style of practice (e.g., number of patients, number of prescribed tests) may vary. Some doctors may have alcohol, drug, or other psychological problems.

Generalized Mathematical Structure

Now that we recognize that costs can vary significantly within the classification plan, how do we structure the merit rating plan? Virtually all merit rating plans use an adjustment to the class rate. In many lines, this is called a "modification factor." The adjustment could also be a credit or surcharge, which is expressed as a percentage of the class rate.

Modification Factor Formula

Virtually all merit rating plans calculate the modification factor according to the following generalized formula:

$$M = Z\frac{A}{E} + 1 - Z,$$

where

$M =$ the modification factor, which is multiplied against the class
rate;

$Z =$ the credibility factor;

$A =$ the insured's actual claim experience; and

$E =$ the average claim experience for the class.

In practice, virtually always the credibility is limited to values between and including 0 and 1. Thus M is a weighted average of the insured's relative experience (to the class average) and the class rate. (We could have written the right-hand term as $(1 - Z) \times 1$.)

We can express the same concept in terms of a discount or surcharge, as a percentage of the class rate. The adjustment to the class rate, as a factor of the class rate, can be calculated by subtracting 1 from M:

$$\text{Adjustment} = M - 1 = \frac{A - E}{E} Z.$$

When $M < 1$, the adjustment will be negative, or a discount from the class rate. When $A = 0$, the insured has no claims. The "claim-free" discount is thus Z, the credibility.

Indeed, this may often be the easiest way to measure credibility. If we have claim data for two experience periods, with a substantial number of claim-free insureds in the first period, the cost of these insureds in the second period relative to the average cost for all insureds in the second period is the empirical claim-free discount and the empirical credibility.

The formula for M is a linear function of the insured's actual claim experience. It would theoretically be possible for M to be some other type of function. However, other functions do not seem to have been used in actual practice. Perhaps the linear function is the most intuitively reasonable function. In addition, where a linear function might not be useful, the definition of A is modified. For example, it seems unreasonable in some cases to charge the entire amount of a large claim; very often, the maximum chargeable claim size is limited in some manner. An advantage of the linear formulation comes in the estimation and interpretation of Z.

Merit rating plans differ in defining A, in calculating E, and in determining Z. The usual process is to first define A, or what data are to be used for the insured's claim experience. Once this is done, E usually can be handled in a straightforward manner; it represents the class average claim experience for the given definition of A. The specification of Z can be done in at least three ways: (1) Ad hoc, (2) risk theory, and (3) direct estimation.

Ad Hoc Credibility

First, credibility can be established on an ad hoc basis. For example, we could decide that 100 expected claims represented "full" or 100% credibility, and partial credibility was the square root of the ratio of expected claims to 100. We might inject some actuarial or statistical theory into the selection of the full credibility standard. (See, e.g., Longley-Cook [5] or Venter [14].)

Risk Theory Credibility

Second, Z can be developed from risk theory. We can use the famous credibility formula:

$$Z = \frac{P}{P+K}, \tag{2.1}$$

where P is a measure of exposure and K can be determined from the following equation:

$$K = \frac{\sigma^2}{\tau^2}, \tag{2.2}$$

where σ^2 is defined as the "process variance" and τ^2 is defined as the "variance of the hypothetical means." The process variance is the variance we would expect for the class average insured's experience, given P units of exposure. The variance of the hypothetical means is the inherent variability of mean claim costs for the insureds within the given class, adjusted for P units of exposure. Depending on our definition for A, it may be possible to determine numerical equivalents for the process variance and the variance of the hypothetical means.

Direct Estimation of Credibility

Third, we can estimate Z statistically from actual data. Potentially, we could use any statistical estimation procedure. It happens, however, that the use of linear regression results in the same credibility formula and parameter explanation as the risk theory approach.

Although the risk theory and regression approaches are very similar, it should be realized that actual results may differ. The real world may differ from our theory or our theory may only approximate the real world. The theoretical approach allows us to apply knowledge from one context to another context. For example, measurement of the variance of the hypothetical means for one company, state, or line of business may be a useful input to another company, state, or line of business. The theoretical approach also allows us to generalize actual findings. For example, we may extrapolate three-year data to a four-year experience period. We should remember, however, that the real test of merit rating is how accurately it prices insureds in practice.

Alternative Forms for Modification Factor

There are several general considerations in the design of a merit rating plan. (See, e.g., Tiller [11].) First, it should be readily understood by insureds, agents, and company personnel. Second, it should be reasonably simple to administer. Third, it should not allow for manipulation by insureds. Finally, it should strike a balance between stability and responsiveness. On the last point, any formula can be adjusted to give greater or lesser weight (i.e., credibility) to the insured's own experience. If too much weight is given, rates may fluctuate too much from year to year. If too little weight is given, the pricing system may not be as accurate as possible and loss prevention incentives are reduced.

Definition of Actual Experience

The first decision in formulating a merit rating formula is the definition of A, the insured's actual claim experience. Choices involve the length of the experience period and whether to use counts or amounts. The length may be thought of as the number of years of experience, but could also include exposure from multiple locations or states. If the actual claim count is used, it could be defined as the reported count, the closed-paid count, or some definition of a non-nuisance claim. For example, a non-nuisance claim could be a settlement for more than \$5,000 (CP5). If amounts are used, there may be some limitation on the maximum chargeable claim; there is also an option of including or excluding allocated loss adjustment expense, loss development, and incurred but not reported (IBNR) claims.

In the National Council on Compensation Insurance (NCCI) revised Experience Rating Plan, A is defined in terms of loss amounts, usually for three policy years. A is divided into primary and excess losses, with the first \$5,000 of each loss being primary, and the remainder excess. There is also a per claim limit of 2.5 times the average cost per serious claim, a per occurrence limit of twice the per claim limit, and a limit on the total cost of diseases. Experience generally is pooled for all NCCI states and all entities with at least 50% common ownership. E, the expected loss, is divided into primary and excess portions. E must also be adjusted for loss development and the loss limitations.

The Insurance Services Office (ISO) has similar experience rating plans for general and automobile liability. A is limited to basic limits loss amounts. There is an additional limitation on the maximum claim size, based on premium size. A provision for IBNR, based on exposure, is added to A. E is adjusted for the loss limits and loss development.

Existing Plans for Doctors

Several insurers use merit rating for doctors. The typical plan offers an individual doctor a certain percentage discount for each claim-free year. Chargeable claims usually are limited to non-nuisance settlements (e.g., claim closed for more than $5,000). There is usually a maximum discount, which applies after five or six claim-free years. One insurer offers lower discounts for physicians than surgeons. A doctor loses the entire discount when a claim is charged; the discounts accumulate thereafter for each new claim-free year.

Rules may differ according to the insurer of the claim. For example, some insurers give credit for claim-free experience with other insurers. The experience period may be actual policy experience or it may be any settlements during a given period, regardless of the occurrence or reporting date.

Several insurers offer merit rating discounts to groups of doctors, based on the following generalized formula:

$$\text{Adjustment} = M - 1 = \frac{A - E}{JE + K},$$

where

$E =$ the expected claim count,

$A =$ the actual claim count,

$J =$ a constant (e.g., 2), and

$K =$ a constant (e.g., 1).

E is calculated from the number of insureds by rating class for the group; there is a separate claim frequency factor for each rating class.

Some Truisms

Finally, we consider some implications of merit rating. In workers' compensation there is the concept of the "off-balance" in the merit rating plan. That is, the average modification factor is not necessarily 1.0. The average collectible rate for a class will not necessarily be the same as the class manual rate. Thus the manual rate must be adjusted for off-balance. This concept is important for doctor professional liability insurance, particularly if we adopt a claim-free discount-only approach. With only discounts and no surcharges, the average collectible rate will be less than the manual rate.

Taking another perspective, it is necessary for those who do not receive the discounts to pay for the discounts. If some insureds pay less than the average cost, some must pay more. Even if we do not call it a surcharge, the difference between the claim-free discount and the manual rate is the cost of not qualifying for the claim-free discount. For example, the claim-free discount might be 25%. A doctor who loses the discount will pay an additional 33%. Whether we call this a surcharge or the manual rate, the cost of a claim is still 33%.

Although we will estimate credibilities in a later section of the paper, it is worthwhile to consider the tradeoffs between discounts of various sizes. Exhibit 1 shows the required manual rate increase, given discounts of various sizes (10%, 20%, 30%, 40%, and 50%). The manual rate increase is dependent upon the percentage of insureds receiving the discounts. For example, if 90% of insureds receive a discount of 10%, the manual rate must be increased 9.9%. In other words, 10% of insureds pay 109.9% of the average and 90%

pay 98.9% of the average. We give a discount of 1.1% to the 90% that are claim-free and require the other 10% to pay an additional 9.9%.

3. ACTUARIAL THEORY

As we have seen, the first step in formulating a merit rating plan is to define A, the insured's actual claim experience. Once that is done, usually it is straightforward to determine E, the average claim experience for the insured's class. The most complicated and difficult part is to determine Z, the credibility to attach to the insured's experience.

This section discusses various risk theory formulations for credibility. Although these formulations may not replicate the real world, they are useful in several ways. First, they provide a conceptual basis for understanding the statistical validity (i.e., credibility) of claim experience. Second, they provide a means to formulate credibilities when directly relevant claim experience is not available. Finally, they provide insight into the process of estimating credibilities.

In developing the following formulas, we will want to consider both claim counts and claim amounts. We also will want formulas for a single exposure period as well as multiple periods. There is no limit to the number and sophistication of formulas that can be developed; even so, we probably have included formulas that may be too difficult to test in practice.

The Basic Risk Process

We begin with a simple risk process and add various layers of complexity. We will develop formulas for variances. With few exceptions, the means are obvious and therefore omitted.

Assume that we have one doctor insured for one exposure unit (of time). We define N as a random variable for the number of claims for the period. We assume that N has a mean of λ. We assume that each claim has a claim size distribution S, with mean μ and coefficient of variation squared α. We also define T as the sum of individual claim amounts, or the total losses for that doctor for that exposure unit. If we assume that N and S are independent, we can calculate the variance of T from the moments of N and S.

$$\mathrm{Var}(T) = \mathrm{E}\,[N]\,\mathrm{Var}(S) + \mathrm{Var}(N)\,\mathrm{E}^2\,[S]\,.$$

We use the notation E[x] as the expected value of x. We previously defined α as Var $(S)/\mathrm{E}^2\,[S]$. If we make the additional assumption that N is Poisson distributed, then Var $(N) = \mathrm{E}\,[N] = \lambda$. Thus we have a fundamental risk theory formula:

$$\mathrm{Var}(T) = \lambda\mu^2\,(1+\alpha)\,. \tag{3.1}$$

We can extend this formula to P exposure units. We assume that the same parameters apply to each exposure unit. Generally speaking, we can replace λ by $P\lambda$, if we assume that N is Poisson. Thus for P exposures, we have:

$$\mathrm{Var}(T) = P\lambda\mu^2\,(1+\alpha)\,.$$

There are two important assumptions in this formulation: That the count and amount distributions are independent, and that the count distribution is Poisson. To the extent these are not true in practice, our use and interpretation of these formulas may be faulty. If we do not assume independence, we can still calculate the variances using covariance terms. This will be complicated, particularly when we make the formulas more complex. It seems reasonable in practice to assume independence, as long as we remove nuisance or closed-without-payment claims.

The Poisson assumption is very significant, particularly for the property that its mean equals its variance. The Poisson distribution arises from a process that satisfies three conditions: (1) events in two different time intervals are independent, (2) the number of events in an interval is dependent only on the length of the interval, and (3) the probability of more than one event occurring at the same time is zero. (See Beard [1], Chapter 2.) In practice, these conditions might be violated if there were some catastrophe (or contagion) or if an individual's claim frequency depended on its past history. As an example of the first case, we might have suits for breast implants or for the transmission of AIDS (acquired immune deficiency syndrome). As an example of the second case, we might have a plaintiff's attorney developing a series of suits against a practitioner, related to multiple incidents of unnecessary surgery or sexual misconduct with patients. For the most part, the Poisson assumption seems reasonable in practice, but we must be aware when it does not apply.

It would be possible to assume that N followed some other distribution with two parameters. The practical consequence of this, however, would be to add one more parameter that we would need to estimate. The interpretation of this parameter likely would overlap with the interpretation of other parameters, to be explained below. In addition, the estimation of this parameter might require data from an additional time period, which might be difficult to obtain.

Heterogeneity in the Insured Population

The above formulations assume that we know the parameters for the given doctor. We have calculated the "process variance." By the nature of merit rating, we assume that doctors will vary in their inherent claim costs. Thus we need to expand the formulation to add this heterogeneity. Conceivably, any of the above parameters could vary among the doctor population. We will assume that only the mean claim frequency varies among doctors; this should add sufficient complexity for practical purposes. We define a new random variable, χ, to have a mean of 1 and a variance of β. We will refer to β as the "structure variance." It is the (weighted average) variance of the insured population means (relative to the overall population mean). β probably varies from insurer to insurer. β also may change over time. For use in merit rating, β must be defined for the given insurer for the given experience period.

For any given doctor, the mean claim frequency is assumed to be $\lambda\chi$. We can incorporate these assumptions into our formulation by using a fundamental property of conditional probabilities:

$$\mathrm{Var}(N) = \mathrm{E}_{\chi}\,[\mathrm{Var}(N \mid \chi)] + \mathrm{Var}_{\chi}\,(\mathrm{E}\,[N \mid \chi])\,.$$

If we assume a Poisson process, we have $\text{Var}(N \mid \chi) = \lambda\chi$. We can rewrite the last equation as:

$$\text{Var}(N) = E_\chi[\lambda\chi] + \text{Var}_\chi(\lambda\chi).$$

With the expectations taken over the variable χ, λ is a constant and can be taken outside of the operator. The variance of a scalar times a random variable is the scalar squared times the variance of the random variable. We previously defined $E[\chi] = 1$ and $\text{Var}(\chi) = \beta$. Thus we can rewrite the previous equation as:

$$\text{Var}(N) = \lambda + \beta\lambda^2.$$

For P exposure units with the same parameters, we have:

$$\text{Var}(N) = P\lambda + \beta(P\lambda)^2.$$

For the total amount, T, for a single exposure unit, we have:

$$\text{Var}(T) = E_\chi[\text{Var}(T \mid \chi)] + \text{Var}_\chi(E[T \mid \chi]).$$

This can be written as:

$$\text{Var}(T) = E_\chi[\lambda\chi\mu^2(1+\alpha)] + \text{Var}_\chi(\lambda\chi\mu);$$

$$\text{Var}(T) = \lambda\mu^2(1+\alpha) + \beta(\lambda\mu)^2. \quad (3.2)$$

For P exposure units with the same parameters, we have:

$$\text{Var}(T) = P\lambda\mu^2(1+\alpha) + \beta(P\lambda\mu)^2.$$

Although we used the same notation, β, for the population heterogeneity for both counts and amounts, in reality there may be a different value in the two different contexts. For example, there may be differences in the inherent claim size distribution among insureds, as well as in claim frequency. Indeed, there may be a different numerical value for β, depending upon what claim data is used, such as reported count, CP5 count, or indemnity amounts limited to \$100,000. We will refer to β_C as the structure function, when the claim data is claim counts. We will refer to β_A as the structure variance, when the claim data is amounts.

For Equation 3.2, we note that the first quantity is the "process variance," or the variance given one exposure unit and known parameters, from Equation 3.1. The second quantity is the product of β, the variability in the insured population (given a mean of 1), and the square of $\lambda\mu$, which is the mean. This second quantity is the "variance of the hypothetical means." The $\lambda\mu$ term is a scalar that results from the variance calculation. Indeed, we can rewrite the first term, dividing by the square of the scalar, as:

$$\frac{(1+\alpha)}{\lambda} .$$

This quantity represents the process variance relative to the mean, just as β is the structure variance relative to the mean. We will use the term "relative variance" to be the ratio of a variance to the square of the mean. It is the coefficient of variation squared.

The Basic Credibility Formula

Using the fundamental formula for conditional probabilities, we can write Var (T) as:

$$\mathrm{Var}(T) = \mathrm{E}_\chi\,[\mathrm{Var}(T \mid \chi)] + \mathrm{Var}_\chi\,(\mathrm{E}\,[T \mid \chi]) .$$

This is the same form as:

$$\mathrm{Var}(T) = \sigma^2 + \tau^2 .$$

Here σ^2 is the average process variance and τ^2 is the variance of the means of the insured population. If we define τ^2 and σ^2 in terms of one exposure unit, our credibility Formula 2.1 becomes:

$$Z = \frac{\tau^2}{\sigma^2 + \tau^2} . \qquad (3.3)$$

It is important to note that the denominator of the credibility formula is the total variance for the insured experience. Thus we have a general formula for credibility that conforms to our risk theory model of the claim process. For claim counts, we have $\sigma^2 = \lambda$ and $\tau^2 = \beta_C \lambda^2$. Dividing through by λ we have:

$$Z = \frac{\beta_C \lambda}{1 + \beta_C \lambda} . \tag{3.4}$$

If we divide through by $\beta_C \lambda$, we get the generalized formula, $1/(1 + K)$, with:

$$K = \frac{1}{\beta_C \lambda} .$$

For P exposure units, we substitute $P\lambda$ for λ above. This gives us an extra P in the τ^2 terms. By the same operations, we arrive at the generalized formula for $Z = P/(P + K)$, with the same K as above.

It will be useful to write the credibility in terms of the expected claim count, $E = P\lambda$. Thus we have:

$$Z = \frac{E}{E + K'} , \tag{3.5}$$

where $K' = 1/\beta_C$.

If A is defined in terms of amounts, then $\sigma^2 = \lambda\mu^2 (1 + \alpha)$ and $\tau^2 = \beta_A (\lambda\mu)^2$. Dividing through the general formula for Z by $\lambda\mu^2$ yields:

$$Z = \frac{\beta_A \lambda}{(1 + \alpha) + \beta_A \lambda} .$$

Dividing this through by $\beta_A \lambda$ leads to the formula for K:

$$K = \frac{1 + \alpha}{\beta_A \lambda} .$$

We can also see that the scalar term for the mean will appear, squared, in both the σ^2 and τ^2 terms. These items will cancel in the credibility formula. We will be left with a formula for K that is the following ratio:

$$K = \frac{\text{(Relative) Process Variance}}{\text{(Relative) Structure Variance}}.$$

For counts, the numerator is $1/\lambda$ and the denominator is β_C. For amounts, the numerator is $(1 + \alpha)/\lambda$ and the denominator is β_A.

It also will be useful to analyze the total relative variance. We remember that the total variance is $\sigma^2 + \tau^2$ and the relative variance is calculated by dividing the variance by the square of the mean. For the above credibility formulation, for counts, we have the following formula:

$$\text{Total Relative Variance} = \frac{1}{\lambda} + \beta_C .$$

We know that the Poisson relative variance is $1/\lambda$. Thus the excess relative variance, for this formulation, is β_C.

Risk-Shifting

One of the limitations mentioned in connection with the Poisson assumption was the changing of an individual's mean costs over time. This can be handled formally by an adjustment to the credibility formula. This phenomenon has been called by various names, such as "parameter uncertainty" (see Meyers [10]) or "risk-shifting" (see Mahler [6], [7] and Venezian [13]). An interesting application is presented by Meyers [10] concerning the merit rating of Canadian automobile insurance.

In effect, the basic risk theory formulation breaks down when exposure is added for a given insured. Instead of credibility increasing approximately in proportion to P, in the general credibility formula the increase is significantly less. There is an intuitive explanation. Since the insured's mean costs may change over time, there is uncertainty that its historical mean may be the same as its future mean.

This phenomenon can be modeled in the same manner that we modeled heterogeneity among different insureds. The heterogeneity parameter, of course, should be different. Instead of reflecting the differences among the insured population, it reflects the differences for a given individual over time.

We define δ as the variance of the individual insured's mean costs over time. We should note that it may be difficult to differentiate between β and δ. Both parameters reflect the differences in individual insured experience: β reflects those differences between individuals in the same period, and δ reflects differences between the same individuals in different periods. Since we do not have the opportunity to observe different experience for the same individual in the same period, there may be some ambiguity in the measurement process. We should also note that δ may have different numerical values, depending on the definition of the claim experience.

The main difference in the mathematics from the previous formulation is that the process variance is different. Instead of being λ for counts, it now becomes:

$$\sigma^2 = \lambda + \delta\lambda^2 .$$

For amounts, the process variance is:

$$\sigma^2 = \lambda\mu^2 (1 + \alpha) + \delta (\lambda\mu)^2 .$$

The formula for credibility, $\tau^2/(\sigma^2 + \tau^2)$, for counts, becomes:

$$Z = \frac{\beta_C \lambda}{1 + \delta\lambda + \beta_C \lambda} .$$

The total relative variance is $1/\lambda + \delta + \beta_C$. The excess relative variance is $\delta + \beta_C$. Dividing through by $\beta_C \lambda$, we can rewrite the last equation as:

$$Z = \frac{1}{1 + \frac{\delta}{\beta_C} + \frac{1}{\beta_C \lambda}}. \tag{3.6}$$

If we let $K = 1/\beta_C \lambda$ and we define $J = 1 + \delta/\beta_C$, then we have a general credibility formula, $Z = 1/(1 * J + K)$. For P exposure units, we can derive the equation:

$$Z = \frac{P}{PJ + K}.$$

We can also state the credibility in terms of E, the expected claim count:

$$Z = \frac{E}{EJ + K'}, \tag{3.7}$$

where J has the same definition as above and $K' = 1/\beta_C$, as before in the basic credibility formulation, Equation 3.5.

For amounts, we derive the credibility formula:

$$Z = \frac{1}{1 + \frac{\delta}{\beta_A} + \frac{1 + \alpha}{\beta_A \lambda}}.$$

This has the same form for J as for counts, $(1 + \delta/\beta_A)$, and the same K as for amounts in the basic credibility formulation.

We have the following changes from the basic formulation. The process variance is now larger, since there will be more variability in the individual insured's experience. The excess relative variance is the sum of δ and β. When we estimate β, we will have a smaller structure variance. Thus σ^2 is now larger and τ^2 is now smaller. The credibility will be reduced.

We should note that the maximum credibility is $1/J$. In effect, we are saying that, since the individual's mean cost may be different in the future than it was in the past, we may not be insuring the same risk and, hence, we will always give some credibility to the class average.

Heterogeneity within the Insured

The rationale for the next generalization in the credibility formula does not apply to individual doctor experience. It may be useful, however, in developing formulas for group experience. This generalization has been used by NCCI. As with risk-shifting, we have a situation where adding exposures does not yield as much credibility as if all exposures had the same underlying risk parameters.

In the first credibility formulation, we developed a parameter, β, which described the variance in the insured population. We now want to develop credibility for groups. If all of the doctors in the group were equally good or equally bad, we could apply the first credibility formulation, using P to represent the exposure for the number of doctors in the group. In all likelihood, however, the group will have some better doctors and some poorer doctors. Some of the underlying risk factors, such as geography, might apply to the entire group; other risk factors, such as training and experience, would be different for different members. If the composition of the group were entirely random with respect to the insured population, we could rate each doctor individually; there would be no additional statistical validity to the group experience, apart from the individual doctor experience.

We define γ as the variance of mean costs (adjusted by class) within a given group or insured. We expect that $0 < \gamma < \beta$. In other words, the variability within the group is not as large as the insured population, but it is not zero. As with β and δ, γ may have different numerical values for different formulations of claim experience, such as counts or amounts.

The variance of the insured population means is different than before. Here the "insured population" is groups with a degree of heterogeneity. Some part of the variance will be proportional to the number of exposures (i.e., each exposure has the same parameters, for which the variances are additive) and some part will be proportional to the square of the number of exposures. We can write this as:

$$\tau^2 = \lambda\gamma + (\beta - \gamma)\,\lambda^2 .$$

We know from the previous development that, for counts:

$$\sigma^2 = \lambda + \varepsilon\,\lambda^2 .$$

We also know that the total variance, ignoring the possibility of $\delta > 0$, is $\lambda + \beta_C\,\lambda^2$. From this we can solve for $\varepsilon = \gamma\,(\lambda - 1)/\lambda$. Thus we have:

$$\sigma^2 = \lambda + \gamma\,(\lambda - 1)\,\lambda .$$

Using the general formula for credibility and dividing by $\beta_C\,\lambda^2$, we have:

$$Z = \frac{\left(1 - \dfrac{\gamma}{\beta_C}\right) + \dfrac{\gamma}{\beta_C\,\lambda}}{1 + \dfrac{1}{\beta_C\,\lambda}} .$$

For P exposure units, we have:

$$Z = \frac{\left(1 - \dfrac{\gamma}{\beta_C}\right)P + \dfrac{\gamma}{\beta_C\,\lambda}}{P + \dfrac{1}{\beta_C\,\lambda}} .$$

In terms of the expected count, E, we have:

$$Z = \frac{\left(1 - \dfrac{\gamma}{\beta_C}\right)E + \dfrac{\gamma}{\beta_C\,\lambda}}{E + \dfrac{1}{\beta_C\,\lambda}} . \tag{3.8}$$

We can write this in a more general form:

$$Z = \frac{(1 - I)\,E + I}{E + K'}, \tag{3.9}$$

where $I = \gamma/\beta_C$ and K' has the same form as the previous formulations for E.

The interpretation of this formula depends on the specific values for the given parameters. As we will see below, this formula may produce higher credibilities than the previous two formulations, when the expected claim count is low. Excepting this situation, however, we can relate this formula to the previous formulations. We see that the $(1 - I)$ term reduces the effectiveness of additional exposures. Since the exposures within a group are heterogeneous, we would not expect to generate as much credibility per additional exposure, compared to the situation where all exposures had the same parameters. We can also see that τ^2 is generally lower than it is in the other formulations, because we have incorporated some of the population heterogeneity into the process variance for the insured.

The NCCI credibility formulation includes both risk-shifting and insured heterogeneity. The credibility may be developed from the formulations for σ^2 and τ^2. As a practical matter, the sample data we used for this paper is not sufficient to separately estimate all of the required parameters.

4. PARAMETER ESTIMATION

There are several different approaches that we can take to estimate the appropriate credibility. We can estimate the credibility directly or we can estimate the credibility parameters. We can estimate credibility directly by using claim-free discount data or a regression method.

Direct estimation basically requires that we have data for the same insureds during at least two different experience periods. This is probably the best approach to estimating credibility, because our theoretical models may not always apply to the real world. We may also estimate credibility by estimating the parameters in the formulas that we developed above. This may be our only alternative if we do not have sufficient data. Even if we estimate credibilities directly, we may want to estimate the theoretical parameters, in order to gain more insight into the process.

Direct Estimation of Credibilities

We will define some generalized notation to simplify the estimation equations. Assume that we can measure the experience of Q insureds over two different experience periods. For each insured, i, we define x_i , the *relative* cost ratio for the first period. For example, if we have 10 claims for 100 insureds, the average claim frequency is 0.1. For an insured with one claim, $x_i = 10$. For an insured with no claims, $x_i = 0$. We define y_i as the *relative* cost ratio for the second period. We also define w_i as the weight that we will apply during the estimation process. We can think of w_i as being the relative exposure of that insured to the total group of insureds. Some of the following equations will have a special meaning where the sum of the w_i is 1.0.

We want the x_i to be defined in the same manner as A, the actual claim experience that we are using in the modification factor formula. We want to test the predictability of the actual experience. It is possible that different definitions of x_i will give similar values for certain parameters, such as β. For example, rating based on reported counts might produce the same value for β_C as rating based on CP5 counts. We would expect the level of credibility to be different, however, since the reported count frequency will be much higher than the CP5 frequency.

We can use *any* y_i data to test the validity of the modification factor. Since, ideally, we want to test the actual cost of insured experience, our preference is to use insured amounts for y_i. As we saw above, however, the variability in results likely will be much higher using amounts than counts. Using amounts may give too much weight to outliers and render the estimation process ineffective. The y_i using counts, however, may not be directly related to insurance costs. For example, the inherent claim size distribution might differ among insureds.

Thus, we want the x_i to reflect the definition of A and the y_i to reflect the actual costs of insurance. We can make substitutions, if we understand the limitations that this might produce.

The simplest estimate for Z is the claim-free discount. Our notation can be made simpler by grouping all insureds by their claim experience in the first period. x_0 would be the relative cost in the first period for insureds with no claims. x_1 would be the relative cost for insureds with one claim, etc. y_0 would be the second period relative cost for insureds with no claims in the first period. Similar definitions would follow for y_1, etc. The weights would represent the percentage of insureds with no claims, etc. in the first period.

The empirical claim-free discount is $1 - y_0$. This is the credibility that applies to this group of insureds. We have assumed that the credibility is the same for all insureds in the group. If this is not the case, the estimated credibility will be an "average" credibility for the individual in the group.

The stability of our estimate will depend upon how many insureds were claim-free in the first period, as well as how volatile the claim experience is in the second period. Note that there is no particular requirement for measuring y_i in the same manner as x_i. We could try several measures of y_i, such as pure premium and different count definitions. (The y_i may not be directly related to the cost of insurance, however.)

This claim-free discount formulation is somewhat limiting, however, in that we do not use the experience of non-claim-free insureds. We could expect to get a better estimate by using more information.

Least Squares Regression Formulation

A more generalized formulation uses the modification factor, M_i, to estimate the second period experience:

$$\hat{y}_i = Zx_i + (1 - Z) .$$

In effect, we want the most appropriate credibility, Z, to convert the insured's first period experience into a prospective rate for the second period. We can derive a mathematically appropriate Z by selecting some criteria to minimize the differences between the predicted experience (M_i) and the actual experience (y_i). Although it is not the only possible criterion, least squares minimization is commonly used to determine Z. Thus we have the following formulation:

$$\min_Z C = \sum_i w_i (\hat{y}_i - y_i)^2 ;$$

$$C = \sum_i w_i (Zx_i + 1 - Z - y_i)^2 .$$

We can solve for Z by taking the partial derivative of C with respect to Z and setting the result equal to 0.

$$\frac{\partial C}{\partial Z} = \sum_i 2w_i (Z (x_i - 1) + 1 - y_i)(x_i - 1) .$$

We can separate out the terms that have Z and those that do not.

$$\frac{\partial C}{\partial Z} = 2 \sum_i w_i Z (x_i - 1)^2 + w_i (1 - y_i)(x_i - 1) .$$

When we set this equal to zero, the 2 drops out. We can put all the Z terms on one side of the equation and the non-Z terms on the other side. Since Z is a constant, we wind up with a ratio for Z:

$$Z = \frac{\sum_i w_i (x_i - 1)(y_i - 1)}{\sum_i w_i (x_i - 1)^2} .$$

If the sum of the w_i is 1.0, the denominator is the total relative variance and the numerator is the relative variance of the means of the insured population, the structure variance. If the w_i are the exposures for both x_i and y_i, and the sum of the w_i is 1.0, then the formula simplifies to:

$$Z = \frac{\left(\sum_i w_i x_i y_i\right) - 1}{\left(\sum_i w_i x_i^2\right) - 1} .$$

We can also use this formula for grouped rather than individual insured data, but we must define the groups by the first period experience. For example, we might divide the data into 10 groups, the first having the lowest loss ratios in the first period, etc. This approach can remove the undue impact of outliers. Strictly speaking, Z will be optimal for the selected group means, not for every insured.

Figure 1 graphically depicts the regression process. It shows the prior relative frequencies (x_i), the subsequent relative frequencies (y_i), and the modification factors (M_i), which are the fit of the regression line between the prior and subsequent experiences. The estimate based on the claim-free discount is almost the same as the regression estimate; it can be different, in some cases, because the regression considers the experience of all of the insureds.

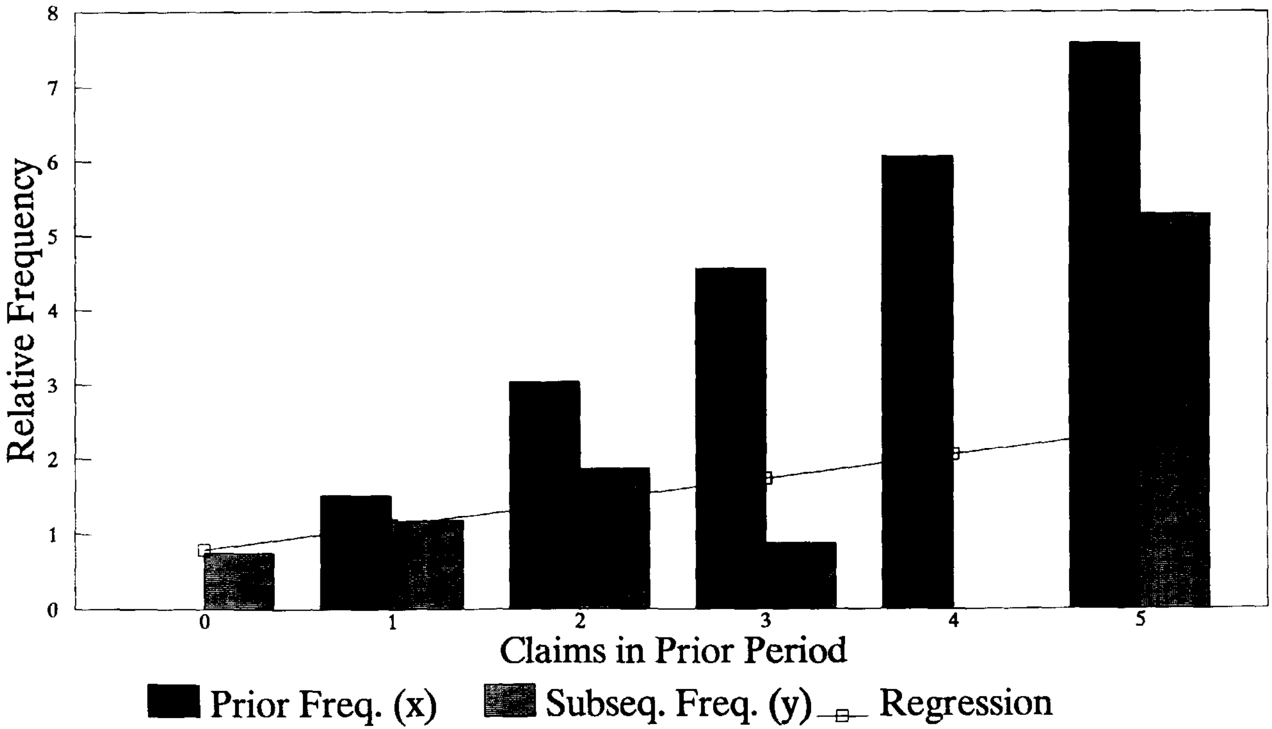
In certain cases, we may wish to pool data for which we know that the credibility is different for different insureds. This formulation would be:

$$\hat{y} = Z_i x_i + 1 - Z_i .$$

Since the Z_i vary for each insured, we cannot solve for a single value of Z. If we can formulate a reasonable function for Z_i, however, we can use the least squares approach to solve for the parameters of our Z_i function. Reasonable candidates for the credibility function can be developed from risk theory, as we showed in an earlier section. Given two periods of data, we would be limited to estimating one

FIGURE 1

RELATIVE CLAIM FREQUENCY



parameter. For example, we may assume that the appropriate credibility function is:

$$Z_i = \frac{\beta_C \lambda_i}{1 + \beta_C \lambda_i} . \tag{4.1}$$

where λ_i is the expected (mean) frequency for class i. We may use the regression approach to solve for β_C. In effect, we are determining the optimal β_C , if credibility does indeed follow the postulated function. If the selected function is not appropriate, we may not get a reasonable estimate for β_C. If the credibility function is complicated, we may not be able to calculate the optimal parameter from a simple equation. We might have to resort to numerical methods.

Estimation of Credibility Parameters

The parameters λ, α, and μ can be estimated from single-period experience. In fact, we do not even need individual insured experience to estimate them. (We do need individual claim experience to estimate α, but λ and μ may be readily available from aggregate data or other projections.) If we can somehow obtain estimates for β, δ, or γ and we also have confidence in the correct form for the credibility function, we do not need to obtain two periods of individual risk data to test the credibilities.

Estimates for the Structure Variance, β

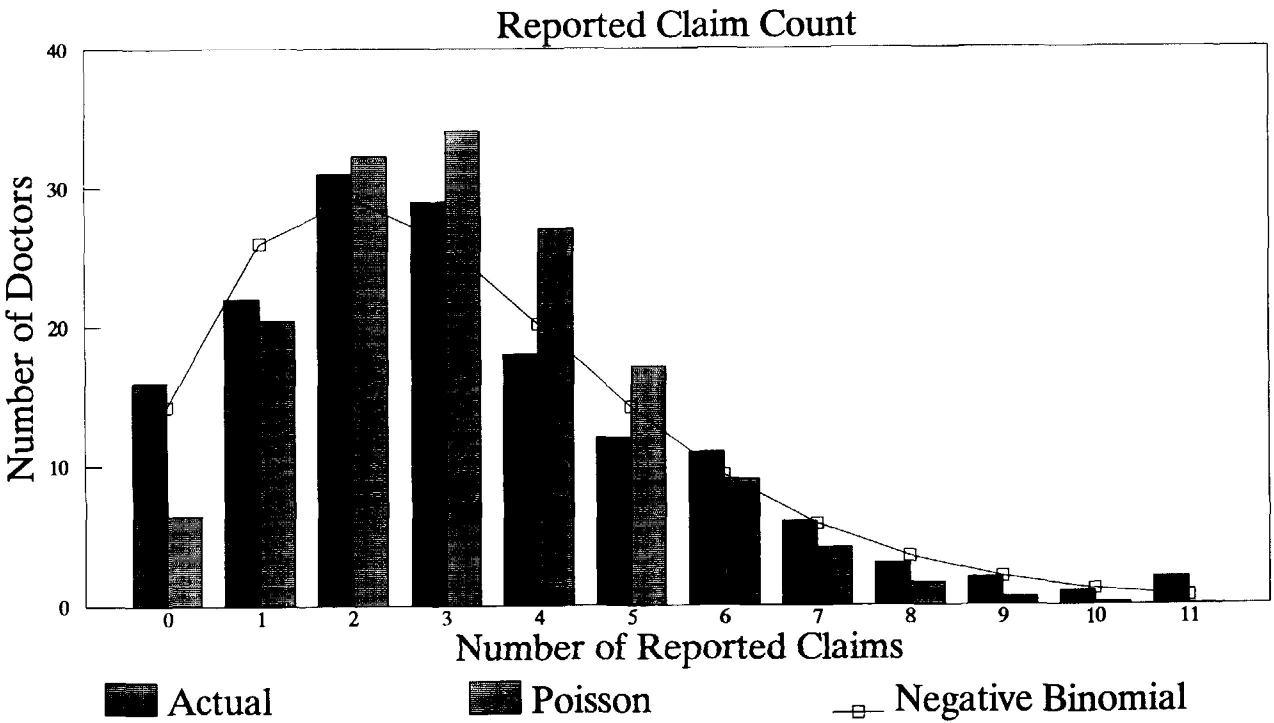
The simplest estimate for the structure variance comes from the basic properties of the Poisson distribution. Since we know that the mean and variance of the Poisson are the same, any "excess" variance in the data can be thought of as being the structure variance.

$$\beta_C = \frac{\mathrm{Var}(N) - \lambda}{\lambda^2} . \tag{4.2}$$

Figure 2 displays an example. It shows the actual number of doctors with a given number of claims. It also shows the theoretical number of doctors who would have had that many claims, had the

FIGURE 2

FREQUENCY DISTRIBUTIONS



distribution been Poisson. Under some generalized assumptions, incorporating the excess variance yields a negative binomial distribution, which is also shown. We see that the actual distribution is more dispersed than the Poisson assumption. There are far more doctors with no claims, and more doctors with only one claim, than the Poisson assumption would indicate. Of course, to balance out, there are also more doctors with large numbers of claims than the Poisson assumption would indicate.

The negative binomial provides a reasonably good fit to the data. It should be noted, however, that the excess variance method is greatly affected by the small number of insureds that will have very unusual experience. If we have a relatively limited sample, we would expect the excess variance estimates to be volatile.

Unfortunately, the structure variance may not be the only component of the excess variance. Other credibility formulations, such as risk-shifting and within-insured heterogeneity, also affect the excess variance. We can think of the excess variance as being a combination of all of these effects. Given a reliable estimate, the excess variance is probably an upper bound on the structure variance.

We obtained another estimate for the structure variance from the numerator in the regression approach, where the sum of the w_i is 1.0:

$$\hat{\beta} = \sum_i w_i (x_i - 1)(y_i - 1) .$$

If the w_i are the exposures, the formula simplifies to:

$$\hat{\beta} = \left(\sum_i w_i x_i y_i \right) - 1 .$$

This regression formulation probably is more reliable than the excess variance approach, because it is based on the predictability of actual data. This formula can be found in Woll [15] and can apply to any claim data (i.e., counts or amounts).

We can also apply this formula to grouped data, although we must group by the loss experience in the first period. We also would expect the grouping process to bias the estimate on the low side, since we are taking differences of group means. We could correct for this bias by multiplying by the ratio of the total relative variance for the individual insureds to the total relative variance of the groups.

Another estimator for the structure variance can be developed from the following relationship:

$$\beta = Z \frac{\mathrm{Var}(T)}{\mathrm{E}^2[T]} .$$

This can be used with a variety of inputs. The estimate for Z can come from claim-free discount data. The ratio on the right is the total relative variance. This can be calculated from one-period data. We can adjust the claim experience for all insureds by the mean experience and then calculate the variance over all insureds. This estimator is based on the general credibility formula, $Z = \tau^2/(\sigma^2 + \tau^2)$. It can be used for either count or amount data.

Another estimator is taken from Woll [15]. This was developed for count data where the structure function, χ, has a gamma distribution.

$$\hat{\beta} = \frac{y_1 - y_0}{y_0} . \tag{4.3}$$

Numerical Examples

We will present various numerical calculations, based on actual data. The data was developed from the experience of one insurer in one state, for insureds that were continuously insured for seven years on an occurrence form. The "prior" period consisted of the first five years and the "subsequent" period consisted of the last two years. The evaluation date was about four years after the inception of the last policy year.

For this insurer, most claims have been reported for the subsequent period, but many of these remain open. The large majority of claims from the "prior" period are closed. Data was available for the reported count, the closed-paid count, the CP5 count, and the basic limits amount, for both periods. Data was segregated for nine different class groups, based on the current classification plan by specialty. There are some rating variables that are not reflected in the class groupings.

Exhibit 2 shows numerical calculations for a number of the methods described above. This data includes the experience of 153 doctors in a particular rating group. For this exhibit, we have defined A, the actual claim experience, to be the number of CP5 claims in the five-year experience period. Ninety-one of the doctors (59.5%) had no CP5 claims in the first period. These doctors had 13 CP5 claims in the second period, for a frequency of 14.3%. The entire class had 29 claims in the second period, for a class frequency of 19.0%. The relative frequency for the claim-free doctors is 75.4%. Thus the claim-free discount, based on CP5 count, is 24.6%. (A claim-free discount can also be calculated for the other data items, such as reported count and pure premium.)

The CP5 frequency for the group is 0.660 and the CP5 variance is 0.969. The variance for a Poisson process also would be 0.660; thus the excess variance is 0.309. All of these numbers reflect the frequency of the actual data. For analysis purposes, it is easier to work with the "relative" variances, which are the actual variances divided by the square of the frequency. The total relative variance is 2.225. The Poisson relative variance is 1.515 (the reciprocal of the frequency). Thus, the excess relative variance is 0.710 ($= 2.225 - 1.515$). We could also calculate the excess relative variance as the actual excess variance (0.309) divided by the frequency squared ($0.660 * 0.660$).

If we use the basic credibility formulation, β_C can be estimated from the excess relative variance, by Equation 4.2, as 0.710. This would imply a credibility of 0.319, from the formula $Z = \beta_C \lambda / (1 + \beta_C \lambda)$. If we use the risk-shifting credibility formulation, the excess relative variance is the sum of β_C and δ_C. Thus, if we believe there is some risk-shifting, the excess variance method will overstate the estimate for β.

The regression method produces a credibility of 0.208. This estimate can be interpreted as the ratio of an estimate of β_C and the total relative variance, which is 2.225, as above. Based on the regression method, the estimate of β_C is thus 0.463 (= 0.208 * 2.225). This might indicate that either: (1) δ_C is 0.247 (= 0.710 − 0.463), or (2) the data is relatively unstable. Normally, we would think that the regression approach, which is based on two-period data, would produce better estimates for β and the credibility.

The claim-free discount data indicates a credibility of 0.246. This may imply a β_C of 0.548 (= 2.225 * 0.246). We can also derive another estimate of β_C from the relative costs of claim-free and one-claim insureds in the second period, from Equation 4.3. This estimate is 0.556, as shown. As can be seen, the results for this class are relatively similar among the different methods above.

We also used first period reported count experience. We would expect the numerical amount of the credibilities to be different (because the frequency was different). The β_C estimates could be similar or different, depending upon whether the use of reported counts has the same predictability as the use of CP5 counts. (For example, does the fact of a CP5 claim imply a higher prospective cost than the fact of a reported claim?) For this data set, the β_C estimates were quite similar for both reported counts and CP5 counts.

We also used claim-free discount data based on reported counts and pure premiums. As we might expect from risk theory concepts, the pure premium data was more volatile.

In merit rating, we want to vary premiums based on differences in prospective costs. Ideally, we would measure the cost differences in terms of pure premiums. Due to the volatility of claim size data, however, estimates based on pure premiums will be much more volatile than estimates based on claim counts. It may be more efficient to estimate credibilities or parameters, such as β and δ, from claim count data. We can either use these parameter estimates directly, by assuming that there is no inherent variation in claim sizes among insureds within the given class, or we can use adjusted values.

We can think about the optimal estimation procedure by considering the regression approach. There, the x_i are best defined by the claim experience used for merit rating. For example, we may use CP5 counts. The y_i are best defined by actual insurance costs. Our structure function estimate for this situation could be given the following notation: β_{CA}, where the first subscript defines the prior period data and the second subscript defines the subsequent period data.

For some of the classes, the number of insureds was small or the actual claim experience was erratic. This raised dual questions: (1) how do we determine β for the smaller classes, and (2) does β vary by class?

Exhibit 3 shows the calculation of the excess relative variance by class for reported counts. Assuming the basic credibility formulation, the excess relative variance is an estimate of β. Several classes have β_C of about 0.6 or 0.7 and several are in the 0.2 to 0.35 range. This might indicate that the β_C vary by class. Class 6, however, has the lowest excess relative variance of 0.215 for reported counts. We saw in Exhibit 2 that its β_C for the CP5 count was about 0.5. Thus the variations by class may be due to random fluctuations in the data.

We can also estimate β by the other methods. Exhibit 4 estimates β using the claim-free discount method. For two classes, the subsequent claim experience for claim-free insureds was actually worse than the average. This would imply a negative value for β. We also note from Exhibit 4 that the claim-free discount based on CP5 counts is significantly different from the claim-free discount based on pure premiums, for several of the classes. Part of this probably is explained by the greater volatility of pure premium data. We also obtained varying β estimates by class from the regression approach.

In reviewing the individual calculations, it appears that much of the volatility is caused by the relatively low number of insureds and claims. We should also note that variance methods give exceptional weight to outliers. There may be a difference in β from class to class, but it does not appear to be statistically significant.

We also pooled the data, for all classes, for the regression and claim-free discount methods. We assumed that the credibility function was the same as Equation 4.1, with λ_i being the expected claim frequency for the class. For the claim-free data, for insureds grouped by CP5 in the first period, the estimate of β_{CC} was 0.54, based on CP5 counts in the second period, and β_{CA} was 0.59, based on pure premiums in the second period. For insureds grouped by reported count in the first period, β_{CC} was 0.54, based on CP5 counts in the second period, and β_{CA} was 0.36, based on pure premiums in the second period.

For the regression approach, for insureds grouped by CP5 in the first period, β_{CC} was 0.51, based on CP5 count in the second period. When insureds were grouped by the reported count in the first period, β_{CC} was 0.50, based on the reported count in the second period.

Estimates for δ and γ

We have mentioned that all three parameters, β, δ, and γ, arise in a similar manner, to explain additional variance beyond a Poisson process. The basic formulation for δ is a shifting of relative claim costs for the individual insured over time. With more years of data, it might be possible to estimate this parameter. The basic formulation for γ is heterogeneity among different doctors within the same insured group. We could estimate this parameter if we had credible data for at least several different size groups and if we assumed that the same heterogeneity applied to all size groups. In fact, the NCCI has used a similar approach to calibrate all of its credibility parameters. It divided risks into various size groups; it estimated optimal credibilities for the different groups; and it fitted these optimal credibilities to a credibility function.

We can use the above numerical example to see whether δ might be significant. If the risk-shifting formulation is correct, the total variance will include a provision for β and δ, as well as the usual

Poisson variance. The excess variance estimate should be the sum of β and δ. The numerator of the regression credibility estimate, however, should include only β. Thus we can compare the two estimates to see if the excess variance estimate is significantly larger. Exhibit 5 shows this comparison for the classes for which the individual estimates were satisfactory. In some cases the excess variance estimate is higher and in some cases it is lower! It does not appear that the excess variance estimate is consistently higher. In practical terms, this might imply that an individual doctor's inherent (relative) risk does not change appreciably over time.

Other Published Data

Two published papers, Ellis [2] and Venezian [12], give some estimates of credibility parameters. The Ellis data included the number of closed-paid claims against doctors in various specialties, for four years, 1980 through 1983, in New York State. It is not clear what the authors used for exposure, but it would appear to be licensed doctors. The authors published theoretical prospective mean frequencies for doctors, in a given specialty, that had various numbers of closed-paid claims within a five year experience period. Comparing the prospective frequencies, for doctors with no claims and all doctors, yields the five-year claim-free discount, or credibility, for the five-year experience.

Except for some minor differences, probably caused by slightly different methods of estimation, we can generate the same credibilities using the procedures outlined above. The Ellis method is equivalent to a credibility formula of $\beta\lambda/(1+\beta\lambda)$, where β is the excess relative variance and λ is the five-year mean frequency. We have estimated the excess relative variance from the claim count distribution given in the paper. The results are shown in Exhibit 6.

For most of the specialties, the excess relative variances are much higher than those estimated from the data set used in this paper. There are several reasons for this. First, it is not clear what exposure was used. If it was licensed doctors, which includes retired, part-time, and government-employed doctors, a substantial number of the doctors would have virtually no claim exposure; we would expect the excess variance to be higher than that for full-time doctors in private practice.

Second, the exposure does not differentiate among other class variables. An insurer's premiums could vary significantly within a given specialty, due to class relativities, geographical relativities, and other rating variables. It is interesting to note that the specialties that are more likely to be grouped into one insurance class, such as anesthesiology, general surgery, neurosurgery, obstetrics, and urology, have much lower excess variances.

Third, New York State could have more geographical variation in costs than the state our data was taken from. Fourth, some doctors are not insured voluntarily. These doctors may have an extreme number of claims, which would produce a much higher excess variance than an insured population. In any case, we might use this data as an upper bound on β.

The Venezian data was taken from the Pennsylvania Medical Professional Liability Catastrophe Loss Fund, which covers both excess losses (attachment points have varied over time) and late reported claims (over four years). Although this data came from insured doctors, the exposures were estimated by the authors. The excess relative variance was estimated from the data in the paper and is shown by specialty in Exhibit 6. With one exception, the excess variances are smaller than in Ellis. Most of the above comments apply to these estimates, as well.

5. PRACTICAL CONSIDERATIONS

This section will consider several practical considerations in the design of a merit rating plan. These include:

- Is it better to use counts or amounts?
- Is it better to use the reported count or the CP5 count?
- What is the best length of the experience period?

- Is the credibility different if we offer only discounts and have no surcharges?
- How do we calibrate the expected costs?
- What if we use non-optimal credibilities?
- How do we establish a formula for insured groups?

Counts or Amounts?

The NCCI and ISO use amounts, rather than counts, in their merit rating plans. The situation for doctor professional liability insurance, however, may call for a different approach. We can analyze the situation by reference to the formula for K, in the basic credibility formulation:

$$K = \frac{1+\alpha}{\beta_A \lambda} .$$

The K for counts is similar, but a 1 replaces the $(1+\alpha)$ in the numerator and β_C may be different from β_A.

For amounts, the K will be $(1+\alpha)$ times larger, if the β are the same. For one exposure unit, the credibility of claim amount experience will be only about $1/(1+\alpha)$ times as much. To the extent an individual's experience is relatively better or worse than the average, it will receive credit for only about $1/(1+\alpha)$ of that difference. The claim-free discount also will be only about $1/(1+\alpha)$ as much.

It is likely that claim severity varies among insureds within the same class. If so, and if frequency and severity are not negatively correlated, we would expect the β to be larger for amounts than for counts. Most likely, however, the β will not increase by as much as $(1+\alpha)$. If we use indemnity amounts limited to \$100,000, $(1+\alpha)$ may be about 2 for doctors. For indemnity amounts limited to \$200,000, $(1+\alpha)$ may be about 2.5. We would expect that β_A for indemnity amounts would be only marginally higher than β_C for counts. Thus using indemnity amounts rather than counts would cut the credibility and the claim-free discounts about in half.

We could also use combined indemnity and allocated loss adjustment expense, limited to various amounts. The $(1 + \alpha)$ terms would be somewhat lower when allocated expenses are included. Credibilities would be much closer to those for indemnity only amounts than those for counts.

Which Count?

There are several choices for claim counts. We could use reported claims, closed with indemnity claims, closed with either indemnity or expense claims, or possibly some non-nuisance claim definition, such as CP5. We can analyze this situation by reference to the basic credibility formula, defined in terms of the expected count, E:

$$Z = \frac{E}{E+K} ,$$

where $K = 1/\beta_C$. We note that credibilities generally will be higher for higher expected counts. We saw from the sample data above that the β's for reported counts and CP5 counts tended to be about the same. This result might not be universally applicable, but we might conclude that the β's would not increase in the same proportion. Thus reported counts would generate more credibility and higher claim-free discounts. If the β's happened to be the same, the credibility for reported count experience might be three to five times higher, depending on the claim frequency for the class and the length of the experience period.

Using reported counts, however, may cause consumer relations problems. It is common for every surgeon in the operating theater to be named in a suit, even if only one is likely to be responsible. Most claims will be closed without a payment or for a nuisance-value payment. Even if more costly doctors are sued more often (which is the logical consequence of the β's being the same), it may be difficult to charge an individual doctor more, just for being named in a suit.

On occurrence policies, in particular, charging for reported claims may also deter or delay the reporting of claims. This could have adverse consequences for both the claim settlement process and the ratemaking process.

From a pricing perspective, using reported counts probably is preferred. Practical considerations, however, may favor a CP5 program.

What Should be the Length of the Experience Period?

Both the NCCI and ISO use a three-year experience period as a standard. Claim frequency for doctors, however, is quite low, particularly when using CP5 counts. Current doctor merit rating programs typically give a certain discount for each year of claim-free experience. This is a reasonable approach, although the discount percentages should vary by specialty. Recall that the basic credibility formula is

$$Z = \frac{\beta_C P\lambda}{1 + \beta_C P\lambda}$$

for counts, for P exposure units. For each additional year of claim-free experience, the credibility will increase about $\beta_C \lambda$. Assuming $\beta_C = 0.5$ and $\lambda = 0.02$ (for one year), the claim-free discount would be about 1% per year. After 10 years, the discount would be 9.1%. For a higher-rated specialty, where $\lambda = 0.1$, the first year discount would be about 4.8%, the second year, an additional 4.3%, the third, 3.9%, the fourth, 3.7%, and the fifth, 3.3%, for a total of 20%.

The above credibility formulation assumes that the doctor's relative cost remains the same over time; i.e., there is no risk-shifting. If there is risk-shifting, and the δ parameter is relatively high compared to β, the additional discounts for additional claim-free years will decline quickly.

Discount Only Plans

Current merit rating plans for individual doctors have claim-free discounts but no surcharges. What should the credibilities be for this type of program?

We can use the same regression formulation to select an optimal credibility. Let w_0 be the percentage of doctors with no claims in the first period and w_1 be the remaining doctors. The modification factors are $1 - Z$ and 1, respectively. Using these modification factors, however, will lead to an "off-balance." That is, the collectible premium will be less than the manual premium. The amount of the off-balance will be $w_0 Z$. The manual rates will be:

$$\hat{y}_0 = \frac{1 - Z}{1 - w_0 Z} \text{ , and}$$

$$\hat{y}_1 = \frac{1}{1 - w_0 Z} \text{ .}$$

We can write the optimization function as:

$$\min_{Z} C = \sum_i w_i \, (\hat{y}_i - y_i)^2 \text{ .}$$

Taking the partial derivative with respect to Z and setting it equal to zero, we obtain the optimal $Z = (1 - y_0)/(1 - y_0 \, w_0)$. This result can also be obtained in another manner. Since $y_0 \, w_0 + y_1 \, w_1 = 1$, it follows that $y_1 = (1 - y_0 \, w_0)/(1 - w_0)$. The above formula for Z makes the prospective rates proportional to the ratio of the actual second period experience, y_0/y_1.

The given credibility is optimal for the postulated pricing policy. It would be more accurate, however, to charge a higher premium for every additional claim in the experience period. The above pricing policy produces a single rate for all insureds with one or more claims. This rate will be relatively too high for the one-claim doctors and relatively too low for the more-than-one-claim doctors.

This can be demonstrated from another perspective. When there are only discounts, and no surcharges, the loss of the claim-free discount is essentially the surcharge for one or more claims. Recalling the general modification factor formula, and assuming that the average experience period frequency for the given class is λ, the appropriate amount to surcharge for each claim is:

$$\text{Surcharge} = \frac{Z}{\lambda}.$$

Given the basic credibility formula, with $Z = \beta\lambda/(1 + \beta\lambda)$, the surcharge becomes $\beta/(1 + \beta\lambda)$. If λ is relatively small, the surcharge will be approximately equal to β.

Calibrating the Expected Costs

Once we have defined the actual claim experience, A, we determine E, the expected claim experience, as the corresponding class average experience. If E is not calibrated to the class average, most likely we will generate an off-balance. (There also may be an off-balance due to other factors.) We briefly discuss some issues with respect to reported counts and CP5 counts.

First assume that A is defined as the reported count, for claims-made coverage, and that the insurer offers a certain fixed discount for each claim-free year. If claim frequency has changed over time, the optimal discount may be different for each year of experience. We may want to select an average frequency for the maximum number of years that credits are offered. We also may want to add an adjustment for the step of the insured policy, if we use the experience on non-mature years.

We may not have class frequencies or we may want to use our rate relativities. In this case, we should remove that part of the relativity that reflects differences in severities by class. We should also reflect other rating variables in the discounts. For example, if we give teaching doctors a 25% discount, logically their claim frequency should be about 75% of the class average and their credits should be 75% of regular doctors. The same adjustment would apply for territorial rate relativities.

We also may want to apply claim-free discounts to occurrence coverage. In this case, we should adjust for the reporting pattern of claims. Assume, for example, that 10% of claims are reported in the first year, 40% in the second year, 20% in the third year, and 10% in the fourth and fifth years. Thus the cumulative percentage of claims reported would be 10%, 50%, 70%, 80%, and 90%. We also assume that the average doctor in this class has an annual occurrence claim frequency, $\lambda = 0.20$, that has remained relatively constant for the past five years. The average doctor would have a reported claim frequency of 0.18 for the fifth prior year, 0.16 for the fourth prior year, and 0.14, 0.10, and 0.02, respectively. For the five-year experience period, the expected frequency is 0.60. If $\beta = 0.5$ and we use the basic credibility formulation, $Z = 23.1\%$ for the five years of experience. If we round off and simplify, we could give a 5% discount for each claim-free year. We should note, however, that after the first year the expected claim frequency is only 0.01 and the appropriate claim-free discount is only 1%. (The appropriate discounts for each successive year of claim-free experience would be 4.7%, 5.8%, 5.9%, and 5.7%.)

If we define the actual claim experience, A, in terms of non-nuisance claims, such as CP5, there is an additional problem in trying to match claim experience to exposure. Even on claims-made forms, the average claim may take three years or so to be settled. On occurrence forms, the average claim may take six years to be settled. One solution is to define A as being any CP5 claim closed within the last five years, regardless of policy period or occurrence date. This approach would be biased in favor of newer doctors, who would not have had as much chance to have had closed claims.

Non-Optimal Credibilities

For various reasons, we may design a plan that has non-optimal credibilities. For example, we may have the same discount per year for every class, even though we know that classes with higher frequencies should receive larger discounts (if their β's are the same). We may also use a discount only program.

With non-optimal credibilities, most likely there will be an off-balance. An off-balance can also arise if the book of business changes over time. (For example, those insureds that would have received stiff surcharges may move to a residual market program or another insurer.) A negative off-balance causes the class rate to be higher than the average class cost. This may cause problems in ratemaking and in analyzing claim experience. If off-balances are different by class, the ratemaking procedure for class relativities should adjust for these off-balances. Profitability analysis should focus on collectible premiums, rather than manual premiums.

Non-optimal credibilities imply an inaccuracy in pricing. This may place the insurer at a competitive disadvantage compared to an insurer that has more accurate pricing. An example may help to clarify this point.

Assume that the optimal credibility for claim-free insureds is 10%, that the insurer gives a 25% discount and no surcharges, that claim-free insureds constitute 80% of the class, that insureds with one claim constitute the other 20% of the class, and that all insureds have the same experience period. The insurer's off-balance would be 20% (80% of insureds receive a 25% discount), implying a manual rate of 125% $(1/(1-0.2))$ of the average cost. The claim-free insureds would pay 93.75% (0.75×1.25) of the average cost and the non-claim-free insureds would pay 125%.

The most accurate cost estimate for a claim-free doctor would be 90% of the manual rate. The off-balance would be 8% (80% times 10%) and the manual rate would be 108.7% $(1/(1-0.08))$ of the average cost. The claim-free doctor would pay 97.8% of the average cost $(0.9 \times 108.7\%)$ and others would pay 108.7%. The optimal competitor could insure all the one-claim doctors at a profit, while the given insurer would be left with all of the claim-free doctors, at a loss.

As a general rule, if claim-free discounts are higher than the optimal credibility, claim-free doctors will be under-priced and the non-claim-free insureds will be over-priced. The insurer will be vulnerable to price competition for the non-claim-free doctors. Another way of looking at this is as follows. When a doctor has a claim, he or she loses his or her claim-free discount and his or her premium increases. The additional premium is more than the insurer needs to profitably insure that doctor.

Group Formulations

Finally, we consider merit rating formulas for groups of doctors. To a large extent, the practical problems discussed above will also apply to groups. Given that the claim frequency may be much larger for groups, we may prefer a plan that looks more like the NCCI or ISO plans. We discuss the components of the merit rating formula, A, E, and Z, in turn.

The choices for the actual claim experience, A, include all of the possible choices for individual doctors plus several more. Since groups are likely to have several experience period claims, the claim-free discount approach may not be practical. Most likely we will use a fixed experience period of three, five, or more years. The credibility we can assign to the group's experience will increase for each additional year of experience. The amount of the increase will depend upon several factors, such as: Whether there is risk-shifting among individual insureds over time, whether the composition of the group changes over time, and the extent to which there is heterogeneity within the group.

If we use claim counts for A, we may want to define them in terms of occurrences. That is, more than one member of a group may be sued for a given incident; the statistical validity of this multiple-claim single incident is probably not much different than that for a single-claim single incident.

We may want to consider using loss amounts. The reduction in credibility that we saw above, for the variability in the claim size distribution, should be more than offset by the increased number of doctors within the average group. If we use loss amounts, we might want to consider a limit on the amount of a chargeable claim, as is done in the ISO plans. The limit could be determined so that the increase in the modification factor for a maximum claim might be a given percentage (e.g., 25%). Logically, this would reduce the credibility that could be given for the group's experience, since α would be lower for lower claim limits. An adjustment also would need to be made to the expected losses, E. Both of these adjustments could be determined from claim size distribution data.

The calibration of E depends upon the definition of A. If we use reported counts for occurrence policies for a five-year experience period, for example, we would need to adjust for the reporting pattern. The expected frequency might be calculated as the annual occurrence frequency times the number of years in the experience period times an adjustment for the reporting pattern (e.g., 60% in the above example). If A is defined in terms of loss amounts, we need to consider loss development and IBNR.

The determination of Z is more difficult, unless we have two-period claim experience for large numbers of groups of varying sizes. There are several approaches that can be taken. First, we could use the same K that we used for individual doctors. Most likely, this is not appropriate because all of the doctors within the group will not have the same relative cost. This approach would overstate credibilities, because the heterogeneity among groups is less than the heterogeneity among individuals. (Mathematically, the τ^2 for groups is lower than the τ^2 for individuals.)

Second, we could use the basic credibility formulation (e.g., Equation 3.4) and estimate the β from group experience. Since the groups (j) for which we have data most likely will have different claim frequencies (λ_j), we must use a generalized formula for Z, such as $Z_j = (\beta\lambda_j)/(1 + \beta\lambda_j)$. This approach has a few problems. If there is risk-shifting among individuals or a change in the group's composition over time, the appropriate credibility formula would have an additional term in the denominator, e.g., $\delta\lambda_j$. Thus our estimate for

β may not be entirely accurate. In addition, to the extent there is risk-shifting, the credibilities for very large groups should be less than those given by the basic credibility formulation. If we do not insure very many large groups and if there is reasonable homogeneity among the group, this approach may be a reasonable approximation to optimality.

Third, we could build in risk-shifting and insured heterogeneity. In order to measure the appropriate parameters, however, we would need additional data. This could be additional years of data for the same groups or a segmentation of group data by size. If we do not have the necessary data, we may make some educated guesses about the values of δ and γ.

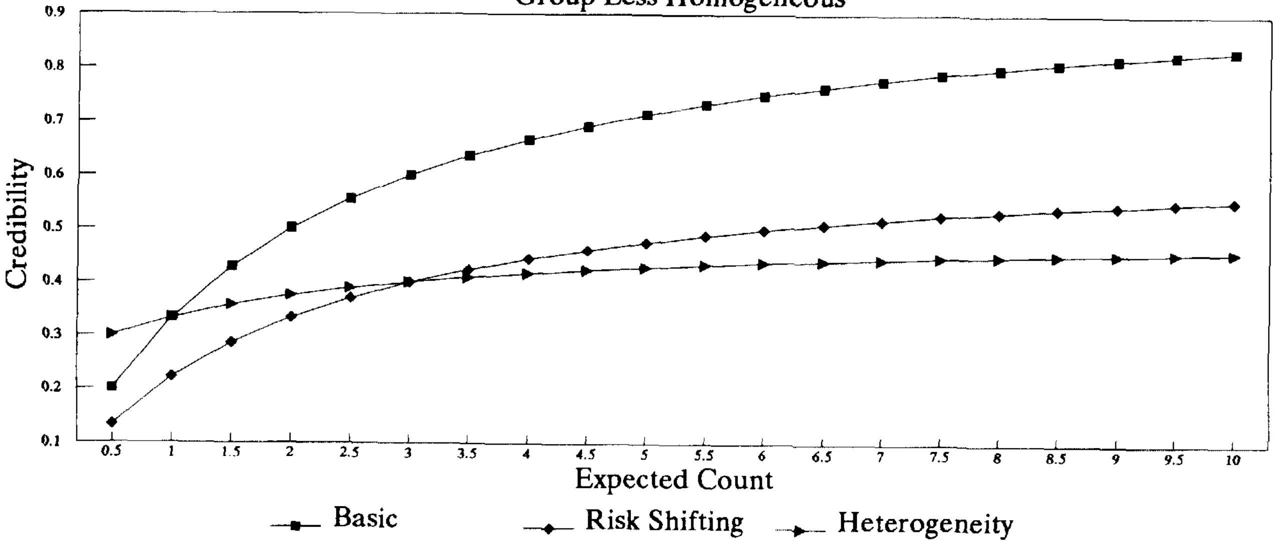
We can compare the results we get with the three different credibility formulations, Formulas 3.5, 3.7, and 3.9. We assumed that the excess variance was 0.5. For the first and third formulations, $\beta = 0.5$. For the second formulation, $\beta + \delta = 0.5$. We think there is a conceptual similarity between the δ parameter in the risk-shifting formulation and the γ parameter in the insured heterogeneity formulation. We think of risk-shifting as how different subsequent years of exposure are to each other. We think of insured heterogeneity as how different sub-exposures within the same experience are to each other.

We have prepared two graphs, Figures 3 and 4. Figure 3 shows the case where $\delta = 0.1$, which is relatively small compared to β. This would occur for groups that are relatively homogeneous. Figure 4 shows the case where $\delta = 0.167$, where the group is less homogeneous. We see that the credibility is always lower for the risk-shifting formulation. For less homogeneous groups, the credibility will be lower. We also see that the risk heterogeneity formulation generally produces lower, though similar, credibility to the risk-shifting formulation. For very low expected counts, the risk heterogeneity formulation may produce higher credibility than the simple formulation. Exhibit 7 gives the numerical credibilities for these two cases.

FIGURE 3

GROUP CREDIBILITIES FOR VARIOUS FORMULATIONS

Group Less Homogeneous

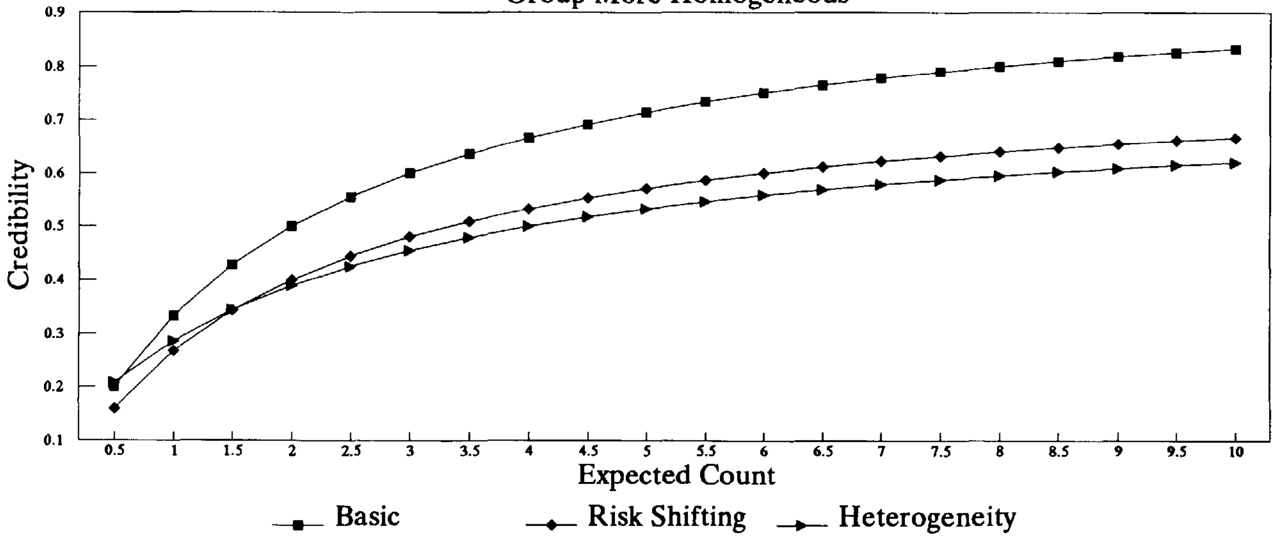


Beta = .4, Delta = .1, Gamma = .125.

FIGURE 4

GROUP CREDIBILITIES FOR VARIOUS FORMULATIONS

Group More Homogeneous



Beta = .333, Delta = .167, Gamma = .25.

6. SUMMARY

Merit rating is the use of the insured's actual claim experience to predict future losses. Merit rating modifies the otherwise applicable class rate. The modification depends on two factors: (1) how much better or worse the insured's experience is relative to the class average, and (2) how credible (i.e., statistically significant) the insured's experience is.

Merit rating formulas can differ in what claim experience is used. Variations include counts or amounts and different lengths of insured experience. There are several generic theoretical formulations for credibility that have been used in insurance pricing. Given sufficient actual data, the appropriate credibility can be estimated.

Merit rating is a complement to the rating plan. It will pick up statistically valid information that is not already reflected in other rating variables. The remainder of the rating structure must be considered in calibrating and applying the merit rating plan.

If the merit rating system creates a collectible premium "off-balance," class rates must be adjusted. If merit rating produces non-optimal discounts or surcharges, there will be inaccurate pricing. If claim-free discounts are too high, for example, those receiving the discounts will be relatively under-priced and those not receiving the discounts will be relatively over-priced.

The statistical validity of an insured's claim experience can be quantified by "credibility" and used in a merit rating formula. Many formulations for credibility are available. Under virtually all formulations, credibility will increase with: (1) The increasing expected claim frequency of the insured's actual experience (λ_i), and (2) the heterogeneity of the insured population, or structure variance, β, remaining after the application of all of the other rating variables. Credibility will decrease with: (1) Increasing variability in the claim size distribution, α; (2) changes in the insured's mean costs over time, or risk-shifting, δ; and (3) heterogeneity within the insured (e.g., with group practices), γ.

In practice, it is relatively easy to determine the expected claim frequency and the variability in the claim size distribution. The structure variance can be determined from single-period data (i.e., from the excess variance), but this requires the assumption that risk-shifting and within-insured heterogeneity are not significant. It is better to estimate the structure variance from two-period data. That is, we must know the relative costs of insureds, within the same rating class, in two different time periods. We would expect the structure variance to be different for different insurers (because they have different underwriting standards), for different states, and for different classes.

Risk-shifting and within-insured heterogeneity are important with respect to the merit rating of group practices. Since all doctors within the group will not be equally good or equally bad, credibility may not increase with additional exposure as it would for an individual doctor. For example, the credibility for one doctor's five-year experience is probably higher than the credibility of five different doctors' combined one-year experience. To measure these factors we need two-period or multi-period data for insured groups of several different sizes.

There are several practical conclusions that can be based on the general theoretical developments and the actual data presented above. Using claim count data will generate more credibility and, hence, larger discounts or surcharges than claim amounts. Using reported count data will generate more credibility than closed-paid count data, but this may cause consumer relations and other problems. Claim-free discounts seem to be a reasonable merit rating plan for individual doctors, subject to two limitations. The amount of the discount should vary with the class expected claim frequency and, generally, the amount should decline for each successive claim-free year.

REFERENCES

[1] Beard, R. E., Pentikänen, T., and Pesonen, E., *Risk Theory* (Third Edition), Chapman & Hall, New York, 1984.

[2] Ellis, R. P., Gallup, C. L., and McGuire, T. G., "Should Medical Professional Liability Insurance Be Experience Rated?" *Journal of Risk and Insurance,* Vol. 57, 1990, p. 66.

[3] Finger, Robert J., "Risk Classification," *Foundations of Casualty Actuarial Science* (First Edition), Casualty Actuarial Society, New York, 1990, Chapter 5, p. 231.

[4] Gillam, William R., "Parametrizing the Workers' Compensation Experience Rating Plan," *PCAS* LXXIX, 1992, p. 21.

[5] Longley-Cook, L. H., *An Introduction to Credibility Theory*, Casualty Actuarial Society, New York, 1962.

[6] Mahler, Howard C., Discussion of Meyers: "An Analysis of Experience Rating," *PCAS* LXXIV, 1987, p. 119.

[7] Mahler, Howard C., "An Example of Credibility and Shifting Risk Parameters," *PCAS* LXXVII, 1990, p. 225.

[8] Mahler, Howard C., "An Actuarial Analysis of the NCCI Revised Experience Rating Plan," *Casualty Actuarial Society Forum,* Winter 1991, p. 37.

[9] Meyers, Glenn G., and Schenker, Nathaniel, "Parameter Uncertainty in the Collective Risk Model," *PCAS* LXX, 1983, p. 111.

[10]Meyers, Glenn G., "An Analysis of Experience Rating," *PCAS* LXXII, 1985, p. 278.

[11]Tiller, Margaret W., "Individual Risk Rating," *Foundations of Casualty Actuarial Science* (First Edition), Casualty Actuarial Society, New York, 1990, Chapter 3, p. 91.

[12]Venezian, Emilio C., Nye, B. F., and Hofflander, A. E., "The Distribution of Claims for Professional Malpractice: Some Statistical and Public Policy Aspects," *Journal of Risk and Insurance*, Vol. 56, 1989, p. 686.

[13]Venezian, Emilio C., "The Distribution of Automobile Accidents—Are Relativities Stable Over Time?" *PCAS* LXXVII, 1990, p. 309.

[14]Venter, Gary G., "Credibility," *Foundations of Casualty Actuarial Science* (First Edition), Casualty Actuarial Society, New York, 1990, Chapter 7, p. 375.

[15]Woll, Richard G., "A Study of Risk Assessment," *PCAS* LXVI, 1979, p. 84.

EXHIBIT 1

REQUIRED MANUAL RATE INCREASES FOR GIVEN CLAIM-FREE DISCOUNTS

Percentage Claim-free	Discount (Z)				
(P_o)	10%	20%	30%	40%	50%
10%	1.0%	2.0%	3.1%	4.2%	5.3%
20	2.0	4.2	6.4	8.7	11.1
30	3.1	6.4	9.9	13.6	17.6
40	4.2	8.7	13.6	19.0	25.0
50	5.3	11.1	17.6	25.0	33.3
60	6.4	13.6	22.0	31.6	42.9
70	7.5	16.3	26.6	38.9	53.8
80	8.7	19.0	31.6	47.1	66.7
90	9.9	22.0	37.0	56.3	81.8

EXHIBIT 2
Part 1

PARAMETER ESTIMATION EXAMPLE

I. RAW DATA AND BASIC CALCULATIONS

	Prior Period						Subsequent Period			
Count	Doctors	Percentage of Doctors	Claims	Extension	Relative Frequency	Relative Variance	Claims	Frequency	Relative Frequency	Extension
N	P	w	NP	wNN	x	wxx	q	q/P	y	wxy
0	91	59.5%	0	0.000	0.000	0.000	13	0.143	0.754	0.000
1	36	23.5	36	0.235	1.515	0.540	8	0.222	1.172	0.418
2	17	11.1	34	0.444	3.030	1.020	6	0.353	1.862	0.627
3	6	3.9	18	0.353	4.545	0.810	1	0.167	0.879	0.157
4	2	1.3	8	0.209	6.059	0.480	0	0.000	0.000	0.000
5	1	0.7	5	0.163	7.574	0.375	1	1.000	5.276	0.261
Total	153	100.0%	101	1.405		3.225	29			1.463
	(a)		(b)	(c)	$N/(1)$	(d)	(e)		$q/[P(2)]$	(f)
Frequency			0.660				0.190			
			$(1) = (b)/(a)$				$(2) = (e)/(a)$			

EXHIBIT 2
Part 2

PARAMETER ESTIMATION EXAMPLE

II. EXCESS VARIANCE METHOD

		Nominal			Relative to Mean	
		Source	Value		Source	Value
(3)	Frequency	(1)	0.660	(7)	By Definition	1.000
(4)	Total Variance	$(c) - (1)(1)$	0.969	(8)	$(d) - 1$	2.225
(5)	Poisson Variance	= (3)	0.660	(9)	1/(1)	1.515
(6)	Excess Variance	(4) − (5)	0.309	(10)	(8) − (9)	**0.710** $= \hat{\beta}_{CC}$

III. REGRESSION METHOD

Credibility, Z	$[(f) - 1]/(8)$	0.208
$\hat{\beta}_{CC}$	$(f) - 1$	**0.463**

IV. CLAIM-FREE DISCOUNT METHOD

(11) Claim-Free Discount, Z	$1 - y_0$	0.246
$\hat{\beta}_{CC}$	(11) (8)	**0.548**

V. OTHER METHOD—EQUATION (4.3)

$\hat{\beta}_{CC}$	$(y_1 - y_0)/y_0$	**0.556**

EXHIBIT 3

EXCESS VARIANCE METHOD

Class	No. of Doctors (1)	No. of Reported Claims (2)	Frequency (3) = (2)/(1)	Total Relative Variance (4)	Poisson Relative Variance (5) = 1/(3)	Excess Relative Variance (6) = (4) − (5)
0	98	64	0.653	2.206	1.531	0.675
1	725	674	0.930	1.429	1.076	0.353
2	208	187	0.899	1.837	1.112	0.725
3	297	413	1.391	1.352	0.719	0.633
4	198	236	1.192	1.161	0.839	0.322
5	170	386	2.271	0.903	0.440	0.463
6	153	485	3.170	0.530	0.315	0.215
7	41	145	3.537	0.605	0.283	0.322
8	28	85	3.036	0.670	0.329	0.341

EXHIBIT 4

CLAIM-FREE DISCOUNT METHOD

Class	No. of Doctors (1)	No. Claim-Free (2)	Class CP5 Frequency (3)	Claim-Free Discount CP5 Count (4)	Total Relative Variance (5)	$\hat{\beta}_{CC}$ (6) = (4) (5)	Claim-Free Discount Pure Premium (7)	$\hat{\beta}_{CC}$ (8) = (7) (5)
0	98	88	0.102	-11.4%	8.800	-1.003	-11.0%	-0.968
1	725	624	0.154	3.7	6.860	0.254	3.5	0.240
2	208	172	0.183	12.1	5.050	0.611	3.7	0.187
3	297	233	0.285	4.1	5.971	0.245	44.4	2.651
4	198	155	0.261	-1.4	4.004	-0.056	-2.1	-0.084
5	170	105	0.547	30.6	2.322	0.711	31.3	0.727
6	153	91	0.660	24.6	2.225	0.547	16.1	0.358
7	41	22	0.829	33.4	1.696	0.566	20.6	0.349
8	28	17	0.464	58.8	1.817	1.068	52.0	0.945
Total	1,918	1,507						

EXHIBIT 5

IS THERE RISK-SHIFTING?

Class	Excess Relative Variance	Regression Estimate for β	Difference	Percentage Difference
1	0.353	0.318	0.035	9.9%
2	0.725	0.570	0.155	21.4
3	0.633	0.868	-0.235	-37.1
4	0.322	0.371	-0.049	-15.2
5	0.463	0.370	0.093	20.1
6	0.215	0.228	-0.013	-6.0
Sum	2.711	2.725	-0.014	-0.5%

Note: Based on reported counts.

EXHIBIT 6

OTHER DOCTOR EXPERIENCE

I. ELLIS, GALLUP, AND MCGUIRE

Specialty	Five-Year Claim-Free Discount	Excess Relative Variance	Five-Year Mean Frequency
Anesthesiology	3.4%	0.20	16.3%
Dermatology	28.4	4.04	9.2
Family Practice	17.6	2.88	7.1
General Surgery	20.2	0.90	35.2
Internal Medicine	24.1	3.87	8.3
Neurosurgery	30.5	1.07	42.8
Obstetrics/Gynecology	29.4	1.08	39.9
Ophthalmology	37.0	3.46	15.2
Orthopedic Surgery	52.6	4.22	26.0
Otolaryngology	38.2	2.64	24.5
Pediatrics	23.6	4.65	7.0
Plastic Surgery	59.6	6.78	34.2
Psychiatry	24.2	22.89	1.7
Radiology	21.0	2.92	9.1
Urology	19.2	1.22	15.9
All Other	10.0	5.22	2.5

II. VENEZIAN, NYE, AND HOFFLANDER

Specialty	Mean Frequency	Excess Relative Variance
Anesthesiology	7.5%	0.46
General Surgery	14.4	1.10
Internal Medicine	3.6	0.19
Neurosurgery	50.0	0.72
Obstetrics/Gynecology	18.7	0.62
Ophthalmic Surgery	3.0	5.34
Orthopedic Surgery	25.7	1.37

EXHIBIT 7
Part 1

Comparison of Different Group Credibility Formulas
Group More Homogeneous

$\beta = 0.400$
$\delta = 0.100$
$\gamma = 0.125$

Expected Count	Basic (1)	Risk-Shifting (2)	Heterogeneity (3)
0.5	20.0%	16.0%	20.8%
1.0	33.3	26.7	28.6
1.5	42.9	34.3	34.4
2.0	50.0	40.0	38.9
2.5	55.6	44.4	42.5
3.0	60.0	48.0	45.5
3.5	63.6	50.9	47.9
4.0	66.7	53.3	50.0
4.5	69.2	55.4	51.8
5.0	71.4	57.1	53.3
5.5	73.3	58.7	54.7
6.0	75.0	60.0	55.9
6.5	76.5	61.2	56.9
7.0	77.8	62.2	57.9
7.5	78.9	63.2	58.8
8.0	80.0	64.0	59.5
8.5	81.0	64.8	60.2
9.0	81.8	65.5	60.9
9.5	82.6	66.1	61.5
10.0	83.3	66.7	62.0

Notes: (1) $Z = E/(E + 2)$. Equation (3.5).
(2) $Z = E/(1.25E + 2.5)$. Equation (3.7)
(3) $Z = (0.75E + 0.25)/(E + 2)$. Equation (3.9).

EXHIBIT 7
Part 2

COMPARISON OF DIFFERENT GROUP CREDIBILITY FORMULAS
GROUP LESS HOMOGENEOUS

$\beta = 0.333$
$\delta = 0.167$
$\gamma = 0.250$

Expected Count	Basic (1)	Risk-Shifting (2)	Heterogeneity (3)
0.5	20.0%	13.3%	30.0%
1.0	33.3	22.2	33.3
1.5	42.9	28.5	35.7
2.0	50.0	33.3	37.5
2.5	55.6	37.0	38.9
3.0	60.0	40.0	40.0
3.5	63.6	42.4	40.9
4.0	66.7	44.4	41.7
4.5	69.2	46.1	42.3
5.0	71.4	47.6	42.9
5.5	73.3	48.8	43.3
6.0	75.0	50.0	43.8
6.5	76.5	50.9	44.1
7.0	77.8	51.8	44.4
7.5	78.9	52.6	44.7
8.0	80.0	53.3	45.0
8.5	81.0	53.9	45.2
9.0	81.8	54.5	45.5
9.5	82.6	55.0	45.7
10.0	83.3	55.5	45.8

Notes: (1) $Z = E/(E + 2)$. Equation (3.5).
(2) $Z = E/(1.5E + 3)$. Equation (3.7)
(3) $Z = (0.5E + 0.5)/(E + 2)$. Equation (3.9).

DISCUSSION OF PAPER PUBLISHED IN VOLUME LXI

THE CALIFORNIA TABLE L

DAVID SKURNICK

DISCUSSION BY WILLIAM R. GILLAM

It's di Lemma, it's de limit, it's de lovely . . .

(apologies to Cole Porter)

"The California Table L" is as pertinent today as it was when it was published almost twenty years ago. It is well-constructed, rigorous and easy to follow.

The subject, Table L, provides great simplicity in calculating retrospectively rated plans with a prescribed individual accident limit. It enables a built-in correction for the overlap of the charge for the per-accident limit and the aggregate loss limit. In the days when retros were calculated by hand, the simplification was highly desirable.

THE NEED FOR GREATER FLEXIBILITY

But the plan based on Table L is not the reason for the enduring value of the paper. Since the charge for a pre-determined accident limit is built into the table, it cannot be used for alternate accident limits. The table must be updated regularly to account for changes in the incremental charge for that accident limit, as well as for changes in the aggregate loss distribution resulting from a fixed cap on accidents during a time of loss size inflation. Further, the need for calculations simple enough to do by hand has been obviated by the revolution in electronic data processing.

The need for a more flexible plan has been addressed recently in a more general manner. The Revised Retrospective Rating Plan[1] of the National Council on Compensation Insurance (NCCI) allows a degree of choice not available in the California plan. The plan is built around a Table of Insurance Charges (previously called Table M, as it will be in this review) which lists excess pure premium ratios for loss distributions when there is no per accident limit. Like Table L, Table M is indexed by entry ratio as well as size of risk. Setting $K = 0$ and removing the asterisks in Mr. Skurnick's definition of $\varphi^*(r)$ results in the Table M Charge $\varphi(r)$.[2]

Incremental charges for the accident limit (Excess Loss Factors or ELFs) are separate from Table M charges and are updated with each state rate filing. Table M is updated regularly, at least for claim size inflation, by changes in the Expected Loss Size Ranges used to select the appropriate column of the table for the specific insured. In rating a specific plan, the insurance charge is now calculated respective of any selected accident limit. Inherent in this calculation is the correction for overlap with the ELF; hence the mnemonic ICRLL for Insurance Charge Reflecting Loss Limitations.

The ICRLL equations equivalent to Skurnick's formulas (20) and (21) are easily derived if one realizes the actual losses subject to the plan are limited on a per accident basis.[3] Given the expected unlimited loss ratio, E, the expected limited loss ratio is

$$\hat{E} = E - ELF \, .$$

---

[1] Principles of this plan are described in "Overlap Revisited—the 'Insurance Charge Reflecting Loss Limitation' Procedure," by Ira Robbin, 1990 CAS Discussion Paper Program, *Pricing*, p. 809, as well as "Fundamendals of Individual Risk Rating," by William R. Gillam and Richard H. Snader, © 1992, National Council on Compensation Insurance.

[2] A nostalgic description of the construction of Table M may be found in "The 1965 Table M," by LeRoy J. Simon, *PCAS* LII, 1965, p. 1. A more generic, if less detailed, description may be found in "Fundamentals of Individual Risk Rating," op. cit.

[3] Fundamentals of Individual Risk Rating," op. cit.

Entry ratios $\hat{r}_H$ and $\hat{r}_G$ are ratios of actual limited to expected limited losses. Then

$$\hat{r}_H - \hat{r}_G = \frac{G - H}{c\hat{E}T} ,$$

and

$$\hat{\varphi}(\hat{r}_H) - \hat{\varphi}(\hat{r}_G) = \frac{P - PD - H}{c\hat{E}T} ,$$

where the reviewer has added a Tax Multiplier, T, and hats, ^ , to notation taken from the paper. NCCI uses discounted expense ratios, e, to Standard Premium, not including tax. Using $P - PD = T(e + E)$, the latter equation can be written:

$$\hat{\varphi}(\hat{r}_H) - \hat{\varphi}(\hat{r}_G) = \frac{e + E - H/T}{c\hat{E}} .$$

An absolutely correct ICRLL calculation would require multiple Limited Loss Tables M, i.e., one for each possible accident limit. Limited Loss Table M should be distinguished from Table L. The former lists excess pure premium ratios (charges) appropriate for the aggregate loss distribution of the insured risk when a per accident loss limit is elected, but includes no charge for the loss limit; the incremental charge for this limit must be included as a separate item in plans with such a limit.

The NCCI plan uses a formula shift in Table M columns to approximate a limited loss Table M. Specifically, the selection of a loss limit reduces the skewness of the claim size distribution and hence the loss ratio distribution. This can be modeled by a column of Table M for a larger size risk. The NCCI plan specifies a multiplier, K, to apply to standard expected losses to determine the Expected Loss Size Group (ELG) of the risk:

$$K = \frac{1 + (0.8)\, LER}{1 - LER}$$

$$= \frac{1 + (0.8)\, ELF/E}{1 - ELF/E} .$$

The loss elimination ratio, *LER*, for the selected accident limit is calculated by dividing the *ELF* by the expected loss ratio, *E*.

THE LEMMA

This reviewer would like to highlight a seemingly trivial portion of Skurnick's paper: Lemma 1. Skurnick uses this lemma the way one normally uses a lemma: To prove theorems. The theorems relate to the important relationships in Table L and the balance in the Retrospective Rating Plan. The longevity of the paper is due in part to the elegance of these proofs. But it is easy to overlook the power of the lemma.[4]

The lemma looks simple enough. Let A be a loss process with expectation, E[A], and L the same loss process except that aggregate loss amounts are capped at $r_2 E$ and subject to a minimum value of $r_1 E$. In a retrospective rating plan with minimum and maximum premium factors, L would be the *ratable* losses. Then:

$$E[L] = E[A] \bullet (1 - \varphi(r_2) + \psi(r_1)) ,$$

where $\varphi(r)$ is the Table M (or L) charge for entry ratio r and $\psi(r)$ is the corresponding savings.

Skurnick uses the lemma to prove the useful formula, $r = 1 + \psi(r) - \varphi(r)$, as well as derive the balance equations used in the plan. This reviewer shows how to use the lemma to evaluate retrospective rating plans that *do not necessarily balance to guaranteed cost.*

Example 1

An example of such an application follows. (This is adapted from 1989 CAS Examination 9, question 26.) The question describes an

4 Others have recognized the value of the lemma. See, for instance, "The Mathematics of Excess of Loss Coverage and Retrospective Rating—A Graphical Approach," by Yoong-Sin Lee, *PCAS* LXXV, 1988, p. 67.

unbalanced retro plan such as used in the Residual Market in some states.

The workers' compensation assigned risk pool has promulgated a retrospective rating plan for assigned risks with $50,000 to $59,999 of Standard Premium.

	Retrospective Premium	= 0.28 × Standard Premium + 1.00 × Incurred Losses
SUBJECT TO:	Minimum premium	= Standard Premium
	Maximum premium	= 1.50 times Standard Premium

Suppose that the expected standard loss ratio for these risks is 120% and the following Table M applies to this group of risks.

ENTRY RATIO	CHARGE	SAVINGS
0	1.00	0.00
0.20	0.80	0.00
0.40	0.62	0.02
0.60	0.46	0.06
0.80	0.32	0.12
1.00	0.18	0.18
1.20	0.10	0.30
1.40	0.06	0.46
1.60	0.04	0.64
1.80	0.02	0.82
2.00	0.00	1.00

a. In terms of Standard Premium, what is the expected ultimate premium for a risk in this group?

b. Assume that 28% of Standard Premium is needed for expense including loss adjustment expenses and taxes. Compute the maximum premium factor needed (instead of the 1.5 given above) so that the expected ultimate premium will be adequate for these risks.

It should be clear that the expected premium of this plan is at least the Standard Premium, which is, in turn, greater than guaranteed cost (assuming premium discounts would otherwise apply). We would say the plan does not *balance* to guaranteed cost.

Part b. of the question asks for a plan that does balance, not to guaranteed cost, but to expected losses and expenses.

The answer to Part a. may be obtained easily using the lemma. Using ratios to Standard Premium, the plan looks like the following:

$$1 \le RP = 0.28 + A \le 1.5$$

or

$$1 \le RP = 0.28 + r\,\mathrm{E}[A] \le 1.5$$

where RP is the retrospective premium, and r is the entry ratio of actual to expected actual losses.

The losses leading to the maximum premium result from entry ratio r_2.

$$0.28 + r_2\,\mathrm{E}[A] = 1.5$$

$$r_2\,\mathrm{E}[A] = 1.22$$

$$r_2 = \frac{1.22}{1.20}$$

$$r_2 \approx 1.00$$

Similarly, minimum losses are represented by r_1.

$$0.28 + r_1\,\mathrm{E}[A] = 1.00$$

$$r_1\,\mathrm{E}[A] = .72$$

$$\text{and } r_1 = 0.60$$

Now,

$$\begin{aligned} \mathrm{E[RP]} &= 0.28 + \mathrm{E[L]} \\ &= 0.28 + \mathrm{E}[A] \bullet (1 - \varphi(1.0) + \psi(0.6)) \\ &= 0.28 + 1.2\,(1 - (0.18) + (0.06)) \\ &= 1.336 \end{aligned}$$

The answer to Part a. is 133.6% of Standard Premium. The answer to Part b. requires finding a maximum premium factor leading to a maximum entry ratio where the charge is offset by the known savings for the minimum.

Example 2

The lemma can be used to answer another question of practical interest: what is the premium impact of an update to Expected Loss Size Ranges in the Retrospective Rating Plan? The Expected Loss Size Ranges are shown in a table that relates expected losses of a risk to columns of Table M. The expected losses of a risk are first adjusted by a factor based on its state and hazard group assignment, called the (state) Hazard Group Differential. Typically, an update accounts for one year's inflation in the average cost per case of workers' compensation claims. For the last several years, this has been about +10%, so the size range endpoints have increased by that amount. In order to estimate this impact, it would be extremely difficult—if not impossible—to check the results of policies actually retrospectively rated. It is particularly difficult to estimate the impact of loss development on individual insured loss ratios. Rather, it makes sense to assume Table M was adequate last year, and the proposed update is needed to keep the plan in balance.

An example will clarify the idea. Suppose an insured is rated according to 1992 size ranges. Assume the following values:

$E = 0.62$	Expected Loss Ratio (This is expected actual losses, E[A], as a ratio to adequate standard premium)
$T = 1.07$	Tax Multiplier
$e = 0.220$	Expense Ratio
$D = 0.101$	Premium Discount Factor
risk $ELG = 60$	Indicated column of Table M (Columns of Table M are indexed by

the charge at entry ratio 1.0, which in this case is 0.60).

A not atypical plan for a risk this size would be as follows:

$G = 1.20$ (Maximum Premium)

$H = 0.7$ (Minimum Premium)

$c = 1.125$ (Loss Conversion Factor)

We find a basic premium factor of $B = 0.576$, with $r_G = r_2 = 0.78$ and $r_H = r_1 = 0.11$ and an insurance charge.

$$E \bullet (\varphi(r_2) - \psi(r_1)) = (0.62)(\varphi(0.78) - \psi(0.11))$$
$$= (0.62)(0.653 - 0.031)$$
$$= 0.386$$

Expected ratable losses are given by

$$\mathrm{E[L]} = E \bullet (1 - \varphi(0.78) + \psi(0.11))$$
$$= 0.62\,(1 - 0.653 + 0.031)$$
$$= 0.234$$

It is no accident $0.386 + 0.234 = 0.62$, which is to say the plan is balanced with respect to loss. Thus,

$$\mathrm{E[RP]} = T(B + c\mathrm{E}[L])$$
$$= 1.07(0.576 + (1.125)(0.234))$$
$$= 0.898$$

if the 1992 size ranges apply. Notice that $0.898 \approx 1\text{-}0.101$, the Standard Premium minus Premium Discount Ratio.

In due course of time, the loss process is better described by the 1993 Expected Loss Size Ranges; the insured should be in *ELG* 61. We evaluate this 1992 *ELG* 60 plan according to the distributions underlying column 61 of Table M. First evaluate expected ratable losses.

$$\begin{aligned} E'[L] &= E \bullet \ (1 - \varphi'(0.78) + \psi'(0.11)) \\ &= 0.62\ (1 - 0.662 + 0.032) \\ &= 0.229 \end{aligned}$$

The primes denote expectation according to the updated loss distribution.

Now

$$\begin{aligned} E'[RP] &= T\,(B + cE'[L]) \\ &= 1.07(0.576 + (1.125)(0.229)) \\ &= 0.892 \end{aligned}$$

The change in net expected retrospective premium will be the following amount.[5]

$$\begin{aligned} \frac{E'[RP] - E[RP]}{E[RP]} &= \frac{0.892 - 0.898}{0.898} \\ &= -\,0.007 \end{aligned}$$

This is not to say the update of size ranges *reduces* expected retrospectively rated premium, but rather *failure* to make the indicated update causes an expected 0.7% *shortfall*, at least on this specific plan.

We calculate an expected aggregate impact by grouping premium according to size of insured, calculating the expected impact on typical plans within each size range, and weighting these impacts by the respective premium volume. The actual volume of premium retro-

---

5 The expected change as a percent of standard premium turns out to be nothing more than a factor times a difference in Table M values:

$$\begin{aligned} E'[RP] - E[RP] &= T(B + cE'[L]) - T(B + cE[L]) \\ &= Tc\,(E'[L] - E[L]) \\ &= Tc\,(E((1 - \varphi'(r_2) + \psi'(r_1)) - E(1 - \varphi(r_2) + \psi'(r_1))) \\ &= Tc\,E(\varphi(r_2) - \varphi'(r_2) - \psi(r_1) + \psi'(r_1)) \end{aligned}$$

= Tax Multiplier × Loss Conversion Factor × Expected Loss Ratio × (Difference in Table M charges - Difference in Table M Savings) spectively rated within each size range must be estimated, but the average impact is not very sensitive to the weights. This is because the expected impact on typical plans within each size range, as calculated above, does not differ much between sizes.

The expected impact on any individual risk depends more on the number of columns shifted in Table M. Given an inflationary update of 10%, a risk may shift 0, 1 or 2 columns in the table. Most of the larger size risks likely to be written on retro shift only one group.

Exhibit 1 shows an application of this procedure to the Size Range Update effective July 1, 1993. This is done using a computer program we call "square peg, round hole," a name which reminds us that balance is not a foregone conclusion. The second line of the exhibit corresponds to the example above. Notice most risks shift one size group, but one of the smaller ones shifts two. We believe this is a fair depiction of the actual distribution of changes. A more accurate treatment of the shifting of size groups is described in the Appendix.

EXHIBIT 1
Part 1

CALCULATION OF THE PREMIUM IMPACT OF CHANGES IN RETRO PARAMETERS
SAMPLE PLANS-STATE X
(AVERAGE SHG RELATIVITY 0.774)

Risk Distributions			Effective Parameters				Hypothetical Plan Values		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Premium Range	Risks	Avg Std Prem	*E*	*T*	*D*	*e*	*c*	*H*	*G*
25,001-50,000	58	35,874	0.620	1.070	0.094	0.227	1.125	0.80	1.20
50,001-100,000	71	72,371	0.620	1.070	0.101	0.220	1.125	0.70	1.20
100,001-250,000	89	154,037	0.620	1.070	0.111	0.210	1.125	0.65	1.10
250,001-500,000	53	360,223	0.620	1.070	0.120	0.203	1.125	0.55	1.10
over 500,000	27	1,290,138	0.620	1.070	0.135	0.188	1.125	0.45	1.10
	298	**251,187**	**0.620**	**1.070**	**0.123**	**0.199**	**1.125**	**0.54**	**1.11**

EXHIBIT 1
Part 2

CALCULATION OF THE PREMIUM IMPACT OF CHANGES IN RETRO PARAMETERS
SAMPLE PLANS-STATE X
(AVERAGE SHG RELATIVITY 0.774)

Rating According To Current Plan									Evaluation With Indicated Changes					
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
ELG	*RG-RH*	*XH-XG*	*RG*	*RH*	*XG*	*SH*	*B*	EXP(*R*)	*E′*	*ELG′*	*XG′*	*SH′*	*EXP*(*R′*)	((24)-(19))/(19)
69	0.54	0.142	0.81	0.27	0.724	0.136	0.560	0.906	0.620	70	0.733	0.140	0.903	-0.003
60	0.67	0.266	0.78	0.11	0.653	0.031	0.576	0.898	0.620	61	0.662	0.032	0.892	-0.007
50	0.60	0.320	0.70	0.10	0.595	0.014	0.538	0.889	0.620	52	0.611	0.016	0.878	-0.012
39	0.74	0.442	0.87	0.13	0.435	0.009	0.422	0.880	0.620	40	0.444	0.010	0.874	-0.007
29	0.87	0.556	1.04	0.17	0.276	0.003	0.301	0.865	0.620	30	0.286	0.003	0.858	-0.008
								0.877	**0.620**				**0.869**	**-.009**

APPENDIX[6]

For a given update, the distribution of possible column movement (within each Expected Loss Size group or *ELG*) should be determined. Then impacts for 0, 1 or 2 columns can be weighted by the appropriate probabilities to obtain the expected impact.

A good example may be found by starting with a risk in 1992 *ELG* 40. In 1993, the risk may find itself in *ELG* 41 or *ELG* 42, with probabilities determined below:

1992 Group	1993 Group			Probabilities
1992 *ELG* 40 $153,035 to 165,308	1993	*ELG* 41	$156,077 to 168,490	75%
	1993	*ELG* 42	$144,650 to 156,076	25%

The probability of a 1992 *ELG* 40 risk arriving in *ELG* 42 is calculated as a ratio of intervals. For instance:

$$\frac{\text{The segment of old } ELG \text{ 40 in new } ELG \text{ 42}}{\text{old } ELG \text{ 40}} = \frac{156{,}077 - 153{,}035}{165{,}309 - 153{,}035} = 0.25 \ .$$

6 This degree of care in the estimation was suggested by Howard Mahler.

DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXVII

RISK LOADS FOR INSURERS

SHOLOM FELDBLUM

DISCUSSION BY STEPHEN PHILBRICK

VOLUME LXXVIII

AUTHOR'S REPLY TO DISCUSSION

Abstract

Insurance risk has moved to the forefront of the actuary's concerns. Three other papers on this topic by Fellows of the Casualty Actuarial Society, all written independently, have appeared at the same time as this one: Kreps [14], Venter [24], and Meyers [18]. Insurance risk is the foundation of the NAIC risk-based capital requirements (Hartman, et al. [11]; Kaufman and Liebers [12]). It is also the subject of prize paper competitions by the CAS Loss Reserve Committee and the CAS Risk Theory Committee.

It is appropriate, therefore, that two actuaries deeply involved in the current deliberations, Glenn Meyers and Stephen Philbrick, have written a discussion of, and articles closely related to, this paper. The following remarks from their articles, along with my response to their remarks, provide the reader with a more complete perspective on the issues.

1. THE ACTUARY AND THE DJINN

Stephen Philbrick [21] takes issue with the statement that "the standard deviation of the individual's loss distribution is no guide even to the process risk faced by the insurer." Philbrick notes that when a risk is added to an insurer's book of business, the increase in either the aggregate variance or the aggregate standard deviation of the insurance portfolio is proportional to the variance of the added risk. He concludes that the variance of the individual risk is indeed a guide to the insurer's process risk.

Shortly after the CAS meeting at which this paper was presented, Philbrick wrote a marvelous column for the *Actuarial Review* [22], which should help the reader understand both his criticism and the reply here. A magical djinn offers to replace a lackluster portfolio with a larger and more profitable one, if only the actuary can answer certain questions. The djinn and the actuary agree that surplus requirements should be proportional to the aggregate standard deviation of the portfolio, and the djinn then asks:

> "You can either write an additional risk of Type A or an additional risk of Type B. Risk A has an expected loss of \$1 million, a standard deviation of \$100,000, and a variance of 1 times 10 to the 10^{th}. Risk B is identical to Risk A except that each of the individual losses is exactly twice that associated with Risk A. Consequently, Risk B has expected losses of \$2 million, a standard deviation of \$200,000, and a variance of 4 times 10 to the 10^{th}. You can also assume that both of these risks are independent of the rest of the portfolio."
>
> . . . If you decide to add Risk B to your portfolio instead of Risk A, how much additional surplus would you require to write Risk B relative to the additional surplus you would require for Risk A?"

The answer is four times as much, since the increase in the aggregate standard deviation is proportional to the variance of the marginal risk, not to the standard deviation of the marginal risk.

Philbrick is correct that if a risk is added to a portfolio, the *relative* increase in aggregate standard deviation is proportional to the *relative* variance of the added risk. But the actual increase in aggregate standard deviation, either in absolute terms or relative to the standard deviation of the existing portfolio, is *not* proportional to the variance of the added risk.

Philbrick's example, reproduced and expanded in Exhibit 1, clarifies this. In this example, Risk A has an expected loss of $1,000, a standard deviation of $9,950, and a variance of 99 million; Risk B has an expected loss of $1,000, a standard deviation of $31,607, and a variance of 999 million.

Risk B has a standard deviation about three times greater than Risk A's and a variance about 10 times greater. Whether one begins with 10,000 risks of Type A or 5,000 risks of Type A, both the marginal variance and the marginal standard deviation are about ten times greater when one adds a risk of Type B than when one adds a risk of Type A. If a standard deviation method were used for risk loads, *and we knew the appropriate risk load for adding a Type A risk*, then we could derive the corresponding risk load for adding a Type B risk.[1]

But the argument in the paper is that the loss distributions of individual risks tell us neither the appropriate risk load for the portfolio nor the additional risk load for adding another risk. The ratio of the marginal standard deviation of the portfolio to the variance of the added risk depends on the composition of the portfolio. If the begin-

[1] In general, the marginal standard deviation is approximately equal to the variance of the added risk divided by twice the standard deviation of the portfolio. Letting

Var_{bk} = the variance of the portfolio,

Var_{risk} = the variance of the added risk, and

SD_{bk} = the standard deviation of the portfolio,

we have

$$SD_{bk} = \sqrt{(Var_{bk})};$$

$$\partial\sqrt{(Var_{bk})}/\partial Var_{bk} = 1/[2\sqrt{(Var_{bk})}];$$

$$\Delta\sqrt{(Var_{bk})} \approx \Delta(Var_{bk})/[2\sqrt{(Var_{bk})}].$$

But $\Delta(Var_{bk}) = Var_{risk}$, and $SD_{bk} = \sqrt{Var_{bk}}$, so the marginal standard deviation $\approx Var_{risk}/[2\,(SD_{bk})]$. I am indebted to Dr. Eric Brosius for this formula as well as for explanations of these concepts.

EXHIBIT 1

MARGINAL STANDARD DEVIATION OF AN INSURANCE PORTFOLIO

	10,000 Type A	10,000 Type A + 1 Type A	10,000 Type A + 1 Type B
Expected Losses	10,000,000	10,001,000	10,001,000
Variance	990,000,000,000	990,099,000,000	990,999,000,000
Marginal Variance		99,000,000	999,000,000
Standard Deviation	994,987.44	995,037.19	995,489.33
Marginal Std. Dev.		49.75	501.89

	5,000 Type A	5,000 Type A + 1 Type A	5,000 Type A + 1 Type B
Expected Losses	5,000,000	5,001,000	5,001,000
Variance	495,000,000,000	495,099,000,000	495,999,000,000
Marginal Variance		99,000,000	999,000,000
Standard Deviation	703,562.36	703,632.72	704,271.96
Marginal Std. Dev.		70.36	709.60

ning portfolio consists of 10,000 Type A risks, the ratio is about 5×10^{-7} [$\approx 49.75/99{,}000{,}000 = 501.89/999{,}000{,}000$]. If the beginning portfolio consists of 5,000 Type A risks, the ratio is about 7×10^{-7} [$\approx 70.36/99{,}000{,}000 = 709.60/999{,}000{,}000$].

Philbrick is correct that if we use a standard deviation risk load method, then the relative variances of the additional risks are a guide to the relative increases in the aggregate risk load. But the variance of the additional risk does not tell us what the increase in the aggregate risk load should be.

2. UNDERWRITING RISK AND RESERVING RISK

Philbrick writes:

> "Because . . . the risk load in pricing is inextricably linked to the risk margins in reserving, this paper will also add to the literature on that important subject."

The coming implementation of risk-based capital requirements for property/casualty insurers highlights the need for careful analysis of risk, both pricing and reserving. Philbrick is correct: pricing and reserving risks are linked. A few comments may further clarify the relationship between the two.

Pricing risk is an economic risk. When the actuary prices a policy, the premium has not yet been earned nor the losses incurred. The risk load is the additional profit required to induce the insurer to underwrite the policy. The risk load is a market transaction: the insurer actually receives the risk load from the policyholder.

Reserving risk is primarily an accounting risk. When the reserve is booked, the loss has already occurred. The risk is that the insurer's reserve estimates are inaccurate. The reserve margin is the additional capital the insurer must hold to protect policyholders and to satisfy regulators that its reserves will suffice to settle the claims. The reserve margin is *not* a market transaction: no cash passes hands, and there is no profit or loss to the insurer.[2]

Yet a partial connection remains between pricing risk and reserving risk. Pricing risk reflects the uncertainty in operating ratios. Reserving risk reflects the uncertainty in reserve adequacy. Lines with highly volatile reserves have volatile operating ratios as well.

Some actuaries proceed further along this path and presume that duration of reserves is a suitable proxy for both reserving risk and pricing risk. This last statement is an oversimplification. The relationship between reserve duration, pricing risk, and reserving risk in four lines of business should clarify this.[3]

- *Property:* Large property exposures, as in earthquake insurance or commercial fire insurance, may have great pricing risks. (Note that commercial multi-peril has a high standard deviation of operating ratios and a large β.) But reserves are paid quickly, and there is generally little doubt about the insurer's liability once the accident occurs. Both reserve duration and reserving risk are low.
- *Products Liability:* Products liability includes asbestos and pollution exposures, in addition to other toxic torts and potentially harmful operations. Reserve duration is long, because liability is so uncertain. In fact, much of the litigation in environmental impairment issues has been on coverage disputes: who (if anyone) must pay the costs of clean-up? Similarly, pricing risk is great, because liability may be imposed, even retroactively, in contravention of underwriters' intent in issuing the insurance contract [10, 16]. Products liability fits the simple scheme: reserve duration is long, and both reserving risk and pricing risk are great.

---

[2] As an anonymous referee for the *Proceedings* has pointed out, there are also instances in which reserve margins may affect pricing or cash transactions, such as where "individual risks are retrospectively rated, individual risks are experience rated, or underwriting acceptability is a function of experience."

[3] Actuaries have different opinions about the relative risks by line of business. The subsequent statements in the text are one perspective; other views are also possible.

- *Automobile:* personal automobile underwriting problems often stem from regulatory or statutory enactments. California's Proposition 103 changed a competitive insurance marketplace to one characterized by prior approval of rates, severe restrictions on underwriting freedom, prohibitions on cancellation or non-renewal of insureds, mandated classification systems with no allowance for various traditional dimensions, and rollbacks of rates. New Jersey insurance regulators have depressed rate levels, flattened classification systems, imposed penalties on servicing carriers for the involuntary market, and now seek to recoup Joint Underwriting Association losses from insurance companies. Massachusetts personal automobile regulation has been so onerous and unpredictable that many carriers have paid large fees simply to leave the state.

  Regulatory problems heighten pricing risk. But reserve duration is short (less than one year for all coverages combined), and there is little reserving risk.

- *Workers' Compensation:* fixed statutory benefits and an administered pricing system left workers' compensation with little pricing risk for indemnity coverage from the mid-1970s to the mid-1980s.[4] (The advent of open competition and a multiplicity of statutory "reforms" have increased pricing risk since the late 1980s.) Disability benefits are paid only as the income loss accrues; the benefits may extend over the injured worker's lifetime in permanent total disability cases. Workers' compensation reserve duration is the longest among all Annual Statement lines, except for medical malpractice and casualty excess-of-loss reinsurance. Yet reserving risk is moderate, since the slowest paying claims are often quite certain. In sum, workers' compensation has long dura-

[4] Medical costs for catastrophic cases, however, are hard to predict and pose greater pricing risk.

tion reserves, below average pricing risk, and below average reserving risk.[5]

There are many types of risk which the actuary must consider. Philbrick, of course, is well aware of the interrelationships between these risks. Other readers should be equally careful not to confuse them, but to separately measure each one.

3. CAPM AND RISK DIVERSIFICATION

The Capital Asset Pricing Model (CAPM) posits that only systematic risk, or non-diversifiable risk, is rewarded by higher expected returns. Firm-specific risk can be eliminated by diversification, and it is not rewarded by increased returns.

Glenn Meyers [19] reproduces a derivation of the CAPM from Copeland and Weston [4], which concludes that

$$E[R_j] = R_f + \lambda \,\mathrm{Cov}\,[R_j, R_m]\,,$$

where

$\lambda = (E[R_m] - R_f)/\mathrm{Var}\,[R_m]$,

R_f is the risk-free return,

R_j is the rate of return on the j^{th} asset, and

R_m is the market rate of return.

Meyers then comments: "CAPM proponents claim that the market should not reward [diversifiable] risks. . . . The flaw in these

[5] Similarly, traditional whole life policies and fixed benefit life annuity contracts have long reserve durations, but little reserving risk. (Disintermediation risks, which are present in these contracts, are not applicable to workers compensation loss reserves.) Note also that the April 1991 NAIC risk-based capital reserving risk charges are high for other liability but are nil for workers compensation [15]. Other liability has had high and unpredictable adverse loss development in the 1980s. The implicit interest discount in the long duration workers compensation reserves outweighs the moderately adverse loss development. Subsequent developments have partially changed these relationships; see [8, 9].

statements can be addressed by the CAPM itself. Nowhere in the above development of the CAPM is one required to label a particular risk as being diversifiable or non-diversifiable."

On the contrary, the CAPM derivation indeed reflects the diversifiability of risk. A diversifiable risk refers to the portion of the return that is independent of the market return. That is, for diversifiable risk, Cov $[R_j, R_m] = 0$, so $E[R_j] = R_f$ in the equation above. Systematic risk is not independent of the market, so Cov $[R_j, R_m] \neq 0$. If Cov $[R_j, R_m] > 0$, as is normally the case, then $E[R_j] > R_f$.

Similarly, the formula provided in the text of the paper has

$$E[R_j] = R_f + \beta\,(E[R_m] - R_f), \text{ where}$$
$$\beta = \text{Cov}\,[R_j, R_m]/\text{Var}\,[R_m].$$ [6]

Again, if the risk's return is independent of the market return, then $\beta = 0$, and $E[R_j] = R_f$.

If the risk's return is positively correlated with that of the market, then $\beta > 0$ and $E[R_j] > R_f$.

4. SURPLUS ALLOCATION

Meyers notes that an application of the CAPM to insurance operations requires an allocation of surplus. He argues that this allocation is inappropriate, and he quotes Charles McClenahan's remarks at the 1990 CAS Ratemaking Seminar.[7]

---

[6] Using Meyers's notation, $\beta = \lambda\,\text{Cov}\,[R_j, R_m]\,/\,(E[R_m] - R_f)$. Both the "lambda" and the "beta" expressions may be found in the theoretical literature, although the latter is now more common.

[7] Other actuaries have expressed similar reservations. In testimony regarding California's Proposition 103, Bass [2] says: "By its fundamental nature, surplus is not allocatable, whether to line of business, to jurisdiction, or to any other segment of an insurer's operation" (page 231). After reviewing several allocation methods, Kneuer [13] concludes that not one "addresses the philosophical questions that underlie any attempt to allocate surplus" (page 224). Roth [23], in a discussion of Proposition 103, argues against surplus allocation and proposes an alternative measure of return.

The application of the CAPM to insurance operations does not require an allocation of surplus. The analysis in the text of the paper deals with operating ratios by line of business, not returns on equity. The propriety of surplus allocation has no bearing on the usefulness of the Capital Asset Pricing Model to estimate insurance risk loads.[8]

There remains much confusion on the issue of surplus allocation, so a few more comments may be worthwhile. Actuaries correctly argue that surplus should not be allocated for solvency examinations. This is the gist of McClenahan's statement that Meyers quotes: "The protection against solvency afforded by a $100 million surplus for a free-standing automobile insurance company is not comparable to the protection afforded by a multi-line insurance company with $100 million of surplus allocated to automobile insurance." McClenahan emphasizes: "The fact is that the entire surplus of an insurer stands behind each and every risk" [17, page 152].

But solvency is different from pricing. Many actuarial pricing methods relate net income to some measure of net worth, such as statutory surplus or GAAP equity (see [7, 5, 6]). As Murdza [20] notes, "allocation for ratemaking purposes only does not mean that surplus is actually allocated for solvency or other purposes." That is, the actuary uses the allocation procedure to measure profitability, not to limit the company's legal obligations. Similarly, Callaghan and Derrig [3] say:

> "A company's surplus is not in fact or in law allocated by line and state. A company's entire surplus is available to meet the losses on any line in any state. . . .
>
> "The fact that surplus is not actually allocated by line and state does not, however, mean that it need not be allocated for purposes of determining an appropriate underwriting

[8] One might argue that the determination of operating income uses the Insurance Expense Exhibit Formula for spreading investment income by line of business. But this is not an allocation of surplus. In fact, the end of the paper suggests methods of improving the analysis and notes that "...cash flow discounting should be used instead of spreading investment income to that line of business."

> profit provision for each line. As noted above . . . Massachusetts law requires the determination of rates by line. Thus it is not only appropriate but required that the ratemaker . . . consider surplus by line, just as other elements of the rate-making methodology must be considered by line.
>
> "Such consideration requires that surplus be allocated by line and state for purposes of rate-making, even though it is not allocated by line and state by law. Indeed, such allocation is unavoidable. Any profit methodology which purports to determine profit provisions by line assumes an allocation of surplus by line and state."

The issue of surplus allocation is vexing. But surplus allocation is not needed for applying the CAPM to insurance operations, so it is not germane to this discussion of risk loads.

5. CORRECTIONS

William Bailey has pointed out that the figures in Tables 4 and 5 for fire and commercial multi-peril are in error. The standard deviations in Table 4 should be 6.48 for fire and 13.49 for CMP, and the β in Table 5 should be 0.92 for fire and 2.79 for CMP. For a method of measuring the stability of the β estimates, see [1].

REFERENCES

[1] Bailey, William, "Interval Estimates for Risk Loads for Insurers," 1993.

[2] Bass, Irene, "Excerpts from Proposition 103 Testimony," *Casualty Actuarial Society Forum*, Spring 1990, pp. 223-278.

[3] Callaghan, Acheson H., Jr., and Derrig, Richard A., "Position Paper on Surplus," Hearing on October 1, 1981 Workers' Compensation Rate Filing, Exhibit 69, June 1982.

[4] Copeland, Thomas E., and Weston, J. Fred, *Financial Theory and Corporate Policy*, Addison-Wesley, 1979.

[5] Cummins, J. David, "Discounted Cash-Flow Ratemaking Models in Property-Liability Insurance," in Borba, Philip S., and Appel, David, (eds.), *Benefits, Costs, and Cycles in Workers' Compensation*, Boston: Kluwer Academic Publishers, 1990, pp. 163-182.

[6] Cummins, J. David, "Multi-Period Discounted Cash Flow Ratemaking Models in Property-Liability Insurance," *Journal of Risk and Insurance*, Volume 57, No. 1, March 1990, pp. 79-109.

[7] Feldblum, Sholom, "Pricing Insurance Policies: The Internal Rate of Return Model," Casualty Actuarial Society Part 10A Examination Study Note, May 1992.

[8] Feldblum, Sholom, "Workers' Compensation Underwriting and Reserving Risk Charges," memorandum to David G. Hartman, August 26, 1992.

[9] Feldblum, Sholom, and Brosius, Eric, "Workers' Compensation RBC Charges," memorandum to Elise C. Liebers, October 8, 1992.

[10] Hamilton, Thomas M. and Routman, Eric L., "Cleaning Up America: Superfund and Its Impact on the Insurance Industry," *CPCU Journal*, Volume 41, No. 3, September 1988, pp. 172-184.

[11] Hartman, David G., et al., "Property-Casualty Risk-Based Capital Requirement: A Conceptual Framework," 1992.

[12] Kaufman, Allan M., and Liebers, Elise C., "NAIC Risk-Based Capital Efforts in 1990-91," *Insurer Financial Solvency,* Casualty Actuarial Society 1992 Discussion Paper Program, Vol. I, pp. 123-178.

[13] Kneuer, Paul J., "Allocation of Surplus for a Multi-Line Insurer," *Financial Analysis of Insurance Companies,* Casualty Actuarial Society 1987 Discussion Paper Program, pp. 191-228.

[14] Kreps, Rodney E., "Reinsurer Risk Loads from Marginal Surplus Requirements," *PCAS LXXVII,* 1990, pp.196-203.

[15] Laurenzano, Vincent, "Draft Risk-Based Capital Model," Memorandum to members of the NAIC Property/Casualty Risk-Based Capital Working Group, April 1991.

[16] Manta, Joseph G., and Welge, Mark A., *Toxic Tort Environmental Hazards Litigation Workshop,* 1990.

[17] McClenahan, Charles L., "Risk Theory and Profit Loads—Remarks," Casualty Actuarial Society *Forum,* Spring 1990, pp. 145-162.

[18] Meyers, Glenn G., "The Competitive Market Equilibrium Risk Load Formula,"*PCAS LXXVIII,* 1991, pp. 163-200.

[19] Meyers, Glenn G., Discussion of Feldblum: "Risk Loads for Insurers," Unpublished Manuscript, 1993.

[20] Murdza, Peter J., Jr., Private Correspondence on Internal Rate of Return Pricing Analyses, February 1992.

[21] Philbrick, Stephen W., Discussion of Feldblum: "Risk Loads for Insurers," *PCAS LXXVIII,* 1991, pp. 56-63.

[22] Philbrick, Stephen W., "BRAINSTORMS—I Dream of Surplus," *Actuarial Review,* February 1991, pp. 10-11.

[23] Roth, Richard J., Jr., "Observations on the California Proposition 103 Debate Over Profitability and Surplus," Casualty Actuarial Society *Forum*, Spring 1992, pp. 1-30.

[24] Venter, Gary G., "Premium Calculation Implications of Reinsurance Without Arbitrage," ASTIN Bulletin, Vol. 21, No. 2, November 1991, pp. 223-230.

DISCUSSIONS OF PAPERS PUBLISHED IN VOLUME LXXVIII

AN EXPOSURE RATING APPROACH TO PRICING PROPERTY EXCESS-OF-LOSS REINSURANCE

STEPHEN J. LUDWIG

DISCUSSION BY SHOLOM FELDBLUM

1. INTRODUCTION

Stephen Ludwig's paper provides numerous improvements to the exposure rating procedure first introduced by Ruth Salzmann. In particular, he

- provides up-to-date size-of-loss distributions,
- considers damages besides property losses,
- considers perils in addition to fire, and
- constructs size-of-loss distributions for commercial property risks.

Exposure rating methods are particularly important for pricing property excess-of-loss reinsurance treaties. This discussion provides a brief background and then comments on three topics addressed in Ludwig's paper:

- the relative advantages of exposure rating versus other pricing techniques for reinsurance excess-of-loss treaties;
- several variables affecting exposure rating procedures that Ludwig discusses: Size of risk, peril, deductibles, jurisdictional differences, and data availability; and
- the principles of exposure rating.

The importance of exposure rating for excess-of-loss reinsurance pricing is sometimes unnoticed, since the actuarial literature on this subject is sparse. Casualty actuaries have much to gain from the thorough analysis provided by Stephen Ludwig.

2. BACKGROUND

Property/casualty losses vary in severity, and the distribution of losses by size directly influences the pricing of insurance contracts. In life insurance, a $100,000 policy costs twice as much as a $50,000 policy, since the benefit is fixed. But in property/casualty insurance, a $100,000 policy costs less than twice as much as a $50,000 policy, since most claims are less than the policy limit.

Liability insurance ratemaking assumes that the distribution of losses by size depends on factors external to the insurance transaction: Factors such as the class of business, the hazard, and the state. The policy limit in the contract may curtail the amount of reimbursement, but it should not affect the size of the loss. The distribution of losses by size is therefore determined from dollar amounts. The policy limits purchased by insureds are sometimes used by pricing actuaries to adjust the distribution for truncation of benefits. They are not usually assumed to be correlated with the size of the claim.[1]

In property insurance, the size of the claim depends on the insured value in addition to other factors such as construction, protection (both internal and external), peril, and occupancy. If a building and its contents are worth $100,000, a fire cannot cause damage of $1 million. Thus, there are two influences on property size-of-loss distributions:

- since losses vary in severity, the distribution of insured losses by layer is not uniform; and
- since damages depend on the insured values, the distribution of insured losses varies by size of risk.

[1] The prevalence of suits against "deep pockets" raises questions about this assumption: insureds with large assets are more likely to be sued for large amounts, and so they purchase high limit liability policies. Thus, the policy limit and the size of loss may indeed be correlated.

3. SALZMANN

To model the distribution of property losses by size, Ruth Salzmann [12] uses two assumptions:

1. The amount of insurance in homeowners policies is a good proxy for the "sound value," or the value of the building before the loss. She notes that

   - in the 1950s, most homeowners policies were on new buildings, for which mortgagees demanded full coverage; and

   - the replacement cost provision in the policy encouraged insureds to purchase amounts of insurance equal to at least 80% of the sound value.

2. The distribution of losses by size is directly proportional to the amount of insurance. If there is a 10% probability that a fire loss on a $50,000 building will exceed $25,000, then there is a 10% probability that a fire loss on a $100,000 building will exceed $50,000.

The first assumption seems valid, particularly for homeowners. Salzmann shows that the second assumption, although far from perfect, is reasonable, at least for fire losses on buildings (but see Hurley's review of Salzmann's paper, as well as the discussion below). She constructs loss distributions by percentage of amount insured for four classifications: frame-unprotected, frame-protected, brick-unprotected, and brick-protected. She notes that, "There may be few direct applications of the loss cost data, but such statistics could well serve as a useful yardstick in evaluating other fragmentary size of loss data" [12, page 18].

Enter the reinsurer.

As Salzmann comments, "In the reinsurance area, the potential for further exploration in rating by layer of insurance is tremendous." Reinsurers quickly began using "Salzmann Tables," or "first-loss scales," to price excess-of-loss property reinsurance treaties. Stephen

Ludwig has now provided us with a lucid description of the "exposure rating" method, along with significant improvements in the statistical tables and procedures.

4. PRICING EXCESS-OF-LOSS REINSURANCE TREATIES

Pricing excess-of-loss reinsurance is difficult, for both property and casualty coverages. American insurers that provided general liability coverage in the 1970s are facing unexpected asbestos and pollution claims, but the London reinsurers that provided excess-of-loss treaties are facing even more severe liabilities. The pricing difficulties are not just due to the low loss frequency in high layers. Equally important is the sparse information available to the reinsurer. The reinsurer may not know the mix of property business written by the primary carrier: Amounts of insurance, classes of commercial risks, types of construction, protection classes, and territories. Similarly, the London reinsurer may not be fully aware of liability standards being developed in American courts.

Reinsurers use a variety of pricing procedures: Experience rating, expected loss distributions, and exposure rating. Reinsurance experience rating, or the "burning cost" method, is called by Ludwig "the natural alternative to exposure rating." It is similar to experience rating used by primary companies. Historical losses are adjusted for trend and development and then related to an exposure base (subject premium) to provide a rate for the future treaty period. The adjustments must be made carefully, since both trend factors and development factors increase with the retention (Roberts [11]; Ferguson [1]; Pinto and Gogol [9]; and Gerathewohl, et al [2, pp. 269-278]). Three problems, however, limit the usefulness of experience rating:

1. *Credibility:* For high reinsurance layers (that is, layers above working covers), there may be little historical experience. Moreover, the observed loss frequency and severity in high layers are influenced by random loss occurrences, and they may not be good predictors of future losses. Experience rating plans used by primary carriers, such as the revised

National Council on Compensation Insurance workers' compensation experience rating plan, give little credibility to excess losses, even for large insureds (Venter [13]; King and Gillam [6]), so the manual (or class) rate is used to complement the insured's experience. But the reinsurer has no "manual rate" with which to credibility weight the historical experience, since each reinsurance treaty is different. As Ludwig notes, "Generally . . . experience rating is only useful on working layers."

2. *Information:* Since nominal loss amounts increase with inflation, a \$100,000 loss one year may be a \$150,000 loss several years later. Experience rating requires historical losses below the present retention if the trended value of these losses would exceed the retention during the future policy period (Gilmore [3]). If such data are not available to the reinsurer, and no adjustment is made, the treaty may be underpriced.
3. *Changes in Mix of Business:* Experience rating presumes that the hazards have not changed significantly between the past experience period and the future policy period. This assumption is often valid for workers' compensation, since workplace hazards in a given factory usually change slowly, or for general liability premises/operations risks, where hazards may also be stable. The assumption is poor for reinsurance treaties, since the primary carrier may have changed its underwriting philosophy or may be targeting different markets.

Another reinsurance pricing procedure uses expected loss distributions. These "curve-fitting" methods model claim frequency and claim severity to forecast future losses (Patrik and John [8], Patrik [7]). The reinsurance pricing actuary chooses a family of curves to represent the loss process and selects parameters to fit observed data. At low severities, there are enough observations to fit the curve. At high severities (the tail of the distribution), there may be few or no observations, but the fitted curve forecasts the expected loss amounts.

Two problems limit the usefulness of this technique:

1. *Subjectivity:* There are many curve families that can model the loss process (Hogg and Klugman [5]). Some actuaries use a Pareto curve to model loss severity; some prefer a lognormal; and some like a Weibull or an inverse Gaussian. The curves all seem to fit the observations well at low severities, but they provide different forecasts for the tail. Two actuaries using this technique may come up with vastly different rates for high excess layers.
2. *Complexity:* The pricing actuary must explain the derivation of the rate to the reinsurance underwriter, as well as to representatives of the primary carrier. Curve fitting methods are obscure to some actuaries and incomprehensible to many underwriters. The problem is exacerbated when different actuaries provide different rates, none of whose derivations can be understood by the layman.[2]

The third pricing procedure is exposure rating: First-loss scales for property insurance and increased limits tables for liability insurance. The method can be easily explained to non-technical underwriters and brokers. Size-of-loss distributions can be obtained from industry data or from carriers with large primary books of business, so the credibility problems are mitigated. Finally, the method uses information about the current mix of business, so changes in underwriting philosophy or marketing strategy should not distort the indicated rates.

Exposure rating, of course, is not without problems. Several issues are discussed below, and perhaps Ludwig can mention in an author's response how his company deals with each one.

---

[2] Gilmore [3, page 351] cautions, "be wary of approaches which are too 'actuarial' in nature....If...the retention level has been set high on the theory that the business is well spread and not really subject to a significant catastrophe loss, it is difficult if not impossible to defend the wisdom of the decision after a large loss occurs."

5. SIZE OF RISK

The first-loss scales, or Salzmann Tables, presume that the distribution of losses as a percentage of the amount of insurance does not vary much by size of risk. Salzmann's data actually were sparse and showed counter-intuitive reversals, so she graduated her scales.[3] Ludwig does not show the actual distributions by size of risk for homeowners, though he provides exhibits for certain classes of commercial property.

Four years ago, I examined homeowners size-of-loss distributions, using a vast book of business, for the same purpose as Ludwig's: To update the first-loss scales for reinsurance treaty exposure rating. The data were divided by: a) Size of risk; b) construction class; c) protection class; d) peril; e) state; and f) policy year. The loss distributions by percentages of insured value were sufficiently similar across risks of different size to justify the use of first-loss scales for reinsurance exposure rating.

The difficulties arise with commercial property risks. The homogeneity of homeowners risks, both in size and in hazards, makes the distributions of loss by percentage of insured value sufficiently similar across different sizes of risk to allow exposure rating. Commercial property risks, even the small "businessowners" risks, are less homogeneous.

Small risks are more likely to have losses that are a large percentage of the insured value than large risks are. Head [4] provides several loss distributions to support this, and he concludes:

---

[3] Salzmann [12, page 17] writes: "The actual data was then graduated by the method of adjusting second differences to an orderly downward progression. In addition, the brick-protected distribution was adjusted so that the increments in the upper portion of the distribution were no greater than those in the frame-protected distribution. This adjustment was made entirely on the basis of the author's judgment." Even so, reversals exist. Note particularly Exhibit A on page 20, where the $20,000 policy amount shows higher loss distribution percentages than either the $15,000 or the $25,000 policy amounts for both frame and brick construction.

> ". . . the probability of a loss of a given size is inversely related to its dollar size as well as to the fraction of full property value lost" (page 95), and
>
> ". . . small properties tend to suffer a greater proportion of total or severe losses than do large properties" (page 99).

For commercial property risks, much of the damage is to contents, not just to the building. The flammability of the goods affects the distribution of losses by percentage of insured value: the more flammable the goods, the greater the likelihood that a fire will spread. I have not examined commercial property risks, since our treaties covered only homeowners. Ludwig notes that "the relationship between size of loss and insured value is not constant for any cause of loss." This is particularly true for wind losses, which are often small, regardless of the insured value. Perhaps Mr. Ludwig can comment further on

- the effects of size of risk and flammability of contents on the distribution of losses by percentage of insured value for commercial property risks, and
- the relative usefulness for reinsurance treaty pricing of distributions by percentage of insured value versus by dollar amounts of loss for perils (such as wind) or classes of business where the relationship between size of loss and insured value is not consistent.[4]

6. DEDUCTIBLES

First-loss scales work well when the average deductible in the policies from which the scale is formed is similar to the average deductible for the book of business covered by the treaty. (The "aver-

---

[4] Gerathewohl, et al [2, pp. 296-305], in contrast, uses simulated experience in which the frequency of severe losses increases as the size of risk increases; see particularly his exhibit on page 299. Presumably, this is caused by higher average deductibles on large risks; see the following section of this discussion.

age deductible," as used here, refers to the percentage of insured value, not the dollar amount.) When the deductible level changes, exposure rating is distorted, for two reasons:

- If losses below the deductible are not reported to the insurer, the first-loss scales depend on the deductible level.
- The subject premium reported to the reinsurer, and from which the reinsurance treaty rate is derived, varies with the deductible level.

A first-loss scale uses "ground-up" losses. If the insured has a $500 deductible and incurs a $1,000 loss, the full $1,000 is used in the first-loss scale. If the same insured has a $200 loss, and so receives no indemnification from the insurer, the $200 must still be entered in the first-loss scale. But if the insured never files a claim for the $200 loss, since it is below the deductible, the first-loss scale compiled by the insurer depends on the deductible level.

Alternatively, the first-loss scale may use net losses, i.e., losses adjusted for the deductible. If so, a difference in the average deductible level as a percentage of insured value between the experience used for the first-loss scale and the book of business being reinsured impairs the accuracy of exposure rating.

The relationship of deductible to subject premium is a more severe impediment to exposure rating. The reinsurance cost for a $100,000 excess of $100,000 treaty does not depend much on the size of the deductible, as long as it is small. Whether the insured has full coverage, a $200 deductible, or a $1,000 deductible, there is little effect on the expected losses in the reinsured layer. But the subject premium varies greatly between full coverage and a $1,000 deductible. If a full coverage first-loss scale is used to exposure rate a block of business with an average $1,000 deductible, the reinsurance rate will be inadequate.

This problem is particularly severe for commercial property risks, where deductibles are large and vary widely among risks. Reinarz, et al [10, Vol. 2, p. 46], commenting on the problems of applying exposure rating to commercial property insureds, note that "this type of risk characteristically has a larger deductible, and the larger the deductible, the greater the segment of the premium charged for the catastrophic loss."

If the first-loss scale is derived from losses net of deductibles, and the deductible level has not changed between the policies used to derive the scale and the block of business for which the treaty rate is formed, the deductible problem does not arise. Unfortunately, it is not just a timing problem. The first-loss scale may be derived from the experience of one insurer and applied to the subject premium of another insurer. If the two insurers have different average deductible levels, the exposure rate may be distorted. Ludwig's paper does not explicitly address deductible problems in exposure rating. Perhaps he will comment on how he deals with this issue in pricing applications.

7. PERIL

Salzmann's 1963 paper dealt with fire losses only; Ludwig extends the analysis to other perils. Ludwig's results are consistent with my own study. Fire causes the greatest frequency of severe losses. The catastrophic perils, such as hurricanes and earthquakes, have a great effect on "per occurrence" treaties, but the average loss to the typical risk is often small.

Ludwig shows not just that windstorm losses are more concentrated at lower percentages of insured value than are fire losses. Even the distribution of losses from a severe catastrophe, such as those from the 1989 Hurricane Hugo, lies between the fire and windstorm distributions. Similarly, most earthquakes in California have caused only a small percentage of severe losses. To some extent, this reflects the time period and the jurisdiction:

- I used data from 1982 through 1987, so the 1989 earthquake was not included; and
- the California courts often endorse expansive interpretations

of policy language, allowing numerous small earthquake claims.

But the general observation remains true: earthquakes and hurricanes cause fewer total losses, as a percentage of all losses from the peril, than fires do.

8. JURISDICTION

The novice actuary might presume the following: the states with the highest primary homeowners rates should have the highest excess-of-loss reinsurance homeowners rates. In fact, the opposite is true: high primary rates are often associated with low excess-of-loss reinsurance rates.

The exposure base for the primary rate is the amount of insurance. Rate differences by territory are affected by the "claims consciousness" of the population and by the frequency of small losses, such as vandalism or small windstorm losses. In some areas, insureds file insurance claims for every loss, even when the coverage is of questionable legitimacy. In other locations, insureds file claims only when a true covered loss occurs. Similarly, small losses (theft, vandalism, malicious mischief) are common in some areas, but they are rare in other locations.[5]

The exposure base for the reinsurance rate is the subject premium. These small losses do not affect the reinsurance recoveries, but they increase the subject premium. Thus, the higher the primary rate, the lower the ratio of reinsurance recoveries to subject premium and the lower the reinsurance treaty rate.

9. INFORMATION

Both Salzmann and Ludwig note that the distribution of losses by percentage of insured value varies with construction class and protec-

[5] See Weisberg and Derrig [14] on build-up and fraud in Massachusetts automobile insurance claims.

tion class. Ludwig says that ideally the reinsurer should obtain the mix of business by construction class and protection class in the primary carrier's book to properly exposure rate the treaty.

As Ludwig comments, this information is not always available:

> ". . . reinsurers often have difficulty obtaining information regarding a ceding company's distribution of homeowners business by construction type or protection class."

Even the percentage of premium attributable to each peril is not always provided to the reinsurer. Generally, the primary carrier can provide the subject premium, the type of business (e.g., homeowners, small commercial property), and the location. Location is important because per-occurrence excess-of-loss treaties require geographic information. Primary carriers generally keep track of data by location when purchasing reinsurance.[6]

For exposure rating, construction class, protection class, and the premium attributable to each peril may be associated with location. In a certain section of one state, most homes may be frame, towns may have poor fire protection, and windstorms may be relatively frequent; in another section, most homes are masonry, municipal fire protection is good, and windstorms are rare. Different first-loss scales may be constructed for each state or section of a state. These are the first-loss scales that the reinsurer can use in actual treaty pricing.

Location is being used here as a proxy for other variables. In theory, the first-loss scales should depend on construction, protection, and peril; in practice, the only information the reinsurer may have is location. Perhaps Ludwig will comment on what information his

---

[6] See Gilmore [3, page 362, Exhibit 2B] "Homeowners Direct Written Premium by County," for an example of data by location used in reinsurance treaty negotiations.

company has when it prices a property excess-of-loss reinsurance treaty, and what types of first-loss scales would be most helpful.[7]

10. PRINCIPLES OF EXPOSURE RATING

The discussion above may be summarized in the eight principles listed below. There are exceptions to every rule, though. The reinsurance actuary may begin with these principles, but he or she must then carefully examine the proposed treaty and the book of reinsured business to adjust the rate if necessary.

1. Size-of-loss distributions for a homogeneous book of homeowners business can be modeled as a percentage of insured value.
2. The less homogeneous the book, and the wider the range of insured values, the greater will be the disparity in distributions of loss by percentage of insured value across sizes of risk. In general, smaller risks have a greater proportion of severe losses than larger risks.
3. Higher deductibles increase the percentage of net losses in higher layers. As the deductible increases, the primary

---

[7] The considerations of using the primary carrier's distribution and mix of business versus that of the industry are similar. Ideally, the reinsurer wants to know the premium attributable to each peril in the primary carrier's book of business. Ludwig recommends: "Obtain the ceding company's historical distribution of homeowners losses by cause of loss." In practice, the mix of premium for another insurer, or information for the members of a rating bureau, may be the only data available. Countrywide data for the industry's mix of business is not too helpful, since the reinsured's book may be concentrated in areas where certain perils are more common. But industry data, or data from another insurer, broken down by state and territory may be sufficient.

For deductibles, one needs data from the reinsured; another carrier's data are not appropriate. Deductible levels reflect underwriting practices, which vary widely by carrier. Average size of risk is similar: some carriers target high-priced homes, whereas others serve wider markets. The underwriting philosophy of the ceding company, its marketing strategy, and the types of risks it insures are discussed in the reinsurance treaty negotiations.

carrier's premium rate decreases and the reinsurance excess-of-loss treaty rate increases.

4. The relative rates by peril for per-risk excess-of-loss and catastrophe excess-of-loss are different. For instance, fire has a higher per-risk excess-of-loss rate than windstorm, but windstorm has the higher catastrophe excess-of-loss rate.
5. Primary rates depend greatly on claim frequency; reinsurance rates depend on claim severity. Jurisdictions with high claim frequency, and therefore high primary rates, often have low reinsurance excess-of-loss treaty rates.
6. Reinsurers rarely have all the information needed for ideal exposure rating. The reinsurance actuary must find proxies (such as location) for the attributes that influence the excess-of-loss treaty rate (such as construction class, protection class, and peril).

To these should be added two principles from Ludwig's paper:

7. The amount of insurance is not the limit for the size of the claim. To the amount of insurance for Coverage A (building) must be added the limit for contents losses, losses on other structures, and loss of use.
8. For small commercial property risks, first-loss scales vary by classification and occupancy. In general, "people-oriented" classes, such as restaurants, have a lower frequency of severe losses; properties with flammable contents have a higher frequency of severe losses.

As Ludwig's paper makes clear, exposure rating of excess-of-loss reinsurance treaties contains numerous pitfalls for the unwary actuary. Yet the advantages of exposure rating are strong: the method is sound and it can be explained to nontechnical underwriters and brokers. By considering the influences discussed above, the actuary can ensure the accuracy of the reinsurance treaty rate.

REFERENCES

[1] Ferguson, Ronald E., "Actuarial Note on Loss Rating," *PCAS* LXV, 1978, pp. 50-55; Discussion by Gary. S. Patrick, pp. 56-63.

[2] Gerathewohl, Klaus, Bauer, Wolf Otto, Glotzmann, H. Peter, Hosp, Ernst, Klein, Julius, Kluge, Harold and Schimming, Werner, *Reinsurance Principles and Practice*, translated by John Christofer La Bonté, Karlsruhe,: Verlag Versicherungswirtschaft e.V., Volume I, 1980, and Volume II, 1982.

[3] Gilmore, Richard F., "Planning and Managing a Reinsurance Program," *Reinsurance*, Robert W. Strain (ed.), New York, The College of Insurance, 1980, pp. 347-418.

[4] Head, George L., *Insurance to Value*, Homewood, Illinois, Richard D. Irwin, Inc., 1971.

[5] Hogg, R. V., and Klugman, S. A., *Loss Distributions*, Somerset, New Jersey, John Wiley and Sons, 1984.

[6] King, Robert L., and Gillam, William R., "Revisions to the Workers' Compensation Experience Rating Plan: An Industry Perspective," *NCCI Digest*, Volume 5, Issue 1, March 1990, pp. 1-8.

[7] Patrik, Gary S., "Reinsurance," *Foundations of Casualty Actuarial Science,* New York, Casualty Actuarial Society, 1990, Chapter 6, pp. 277-374.

[8] Patrik, Gary S., and John, Russell, "Pricing Excess-of-Loss Casualty Working Cover Reinsurance Treaties," *Pricing Property and Casualty Insurance Products*, Casualty Actuarial Society 1980 Discussion Paper Program, pp. 399-474; Discussion by Jerry A. Miccolis, pp. 475-484.

[9] Pinto, Emanuel, and Gogol, Daniel F., "An Analysis of Excess Loss Development," *PCAS* LXXIV, 1987, pp. 227-255; Discussion by George M. Levine, pp. 256-271.

[10] Reinarz, S. J., Schloss, J. O., Patrik, G. S., and Kensicki, P. R., *Reinsurance Practices*, 2 Volumes, Insurance Institute of America, 1990.

[11] Roberts, Lewis H., "The Impact of Inflation on Reinsurance Costs," *Reinsurance and Reinsurance Management*, Andrew J. Barile and Peter R. Barker (eds.), Oklahoma City, Oklahoma, Interstate Service Corporation, 1981, Part 8, Section 11.

[12] Salzmann, Ruth E., "Rating by Layer of Insurance," *PCAS* L, 1963, pp. 15-26; Discussions by Robert L. Hurley, pp. 27-30; and Robert Pollack, pp. 30-31.

[13] Venter, Gary, "Experience Rating—Equity and Predictive Accuracy," *NCCI Digest*, Volume 2, Issue 1, April 1987, pp. 27-35; revised version appears as a Casualty Actuarial Society Part 9 Study Note.

[14] Weisberg, Herbert I., and Derrig, Richard A., "Fraud and Automobile Insurance: A First Report on Bodily Injury Liability Claims in Massachusetts," *Journal of Insurance Regulation*, Volume 9, No. 4, June 1991, pp. 497-541.

THE COMPETITIVE MARKET EQUILIBRIUM RISK LOAD FORMULA FOR INCREASED LIMITS RATEMAKING

GLENN MEYERS

DISCUSSION BY IRA ROBBIN

VOLUME LXXIX

AUTHOR'S REPLY TO DISCUSSION

Abstract

In his discussion of the author's paper, Ira Robbin takes issue with several aspects of the proposed risk load formula. In this response, the author seeks to clarify some of these differences and expand upon the role of reinsurance in the pricing of high limit policies. In particular, he shows how the risk load formula can be used to develop an efficient reinsurance program.

1. INTRODUCTION

Ira Robbin [2] has provided a thought-provoking article on the subject of risk loads. The subject has historically been a controversial one among actuaries since it attempts to describe one of the more subjective elements of insurance pricing with a mathematical formula.

Part of the problem has been a confusion in the terminology used to describe the pricing of insurance. Terms include expected losses, various insurer expenses, investment income, risk loads, and profit loads, all of which can be overridden by marketing considerations. I believe many of the differences between Robbin and myself can be attributed to differences in terminology. But when he combines these differences in terminology with an improper interpretation of the role of an advisory organization (which he calls a rating bureau), he draws conclusions about my paper which I neither *implicitly* (his word, my italics) nor explicitly intended.

2. THE ROLE OF AN ADVISORY ORGANIZATION

It should be kept in mind that the Competitive Market Equilibrium (CME) risk load formula was developed for use in ISO advisory increased limits filings. We (ISO) do not view our role as simply to provide increased limits factors on a "take it or leave it" basis. We recognize that our increased limits factors will not be appropriate for every situation, yet the development of these factors contains information of value to all insurers. We view our job as providing information to aid the insurer in deciding what its increased limits factors (or more generally, rates) should be. To do this job effectively, we must explicitly identify the various components that make up the increased limits factors so that insurers can more easily implement whatever changes they want to make.

This becomes particularly important when reinsurance is involved.

Robbin defines the risk load so that it contains a provision for reinsurance expenses, while in my definition there is no such provision. My risk load is for "pure" risk, and the reinsurance expenses are addressed separately. Since the purpose of reinsurance is to spread risk, Robbin's definition might be considered reasonable. However, it presents problems because there are many purposes of reinsurance, and a diverse population of reinsurance buyers. For this reason we decided to presume as little as possible about the nature of an insurer's reinsurance arrangements, and to provide information that will aid the insurer to account effectively for the use of reinsurance in increased limits pricing.

Thus, in our advisory increased limits filings we explicitly assume the insurer is retaining the entire risk. If an insurer wishes to obtain excess of loss reinsurance, it can use the filed factors to obtain its price up to the amount it retains, and then add on the price of reinsurance. In addition, we provide circulars and software that may be useful in planning for excess of loss reinsurance. The software handles reinsurance expense in the manner described in Section 8 of the paper.

My definition of risk load is motivated by institutional, rather than fundamental, reasons. While I have no fundamental objection to Robbin's definition of risk load, when he combines it with his interpretation of a "rating bureau," he draws inferences with which I strongly disagree. For example, he writes that I believe "that the bureau should file ILFs under the hypothesis that layering is not allowed," or that "implicitly, Meyers has prohibited insurers from entering into transactions that his theory says are beneficial."

Instead, the theory provides a tool to aid in the development of an efficient reinsurance program, and to incorporate reinsurance into the pricing of increased limits. However, we feel the responsibility for doing this lies with the insurer, and not with an advisory organization.

3. REINSURANCE PLANNING

In spite of our differences, I would like to recommend many of the ideas in Robbin's section on "Putting Reinsurance into the Model" for serious consideration in reinsurance planning. The exercise of finding the reinsurance program that results in the most competitive rate should be a regular activity for the insurer. He offered a solution for quota share reinsurance. Here I give an example which illustrates how an insurer might proceed when both excess of loss and quota share reinsurance are available. This example will be a continuation of the example started in Section 7 of the paper. Table 1 gives the ground up increased limits factors derived in this example.

Let us assume that the reinsurer charges the risk load indicated by the CME formula and charges an additional charge, which is expressed as a percentage of the expected loss for the layer, to cover expenses.

TABLE 1

GROUND UP INCREASED LIMITS FACTORS

Policy Limit	Average Severity	Process Risk	Parameter Risk	ILF with Risk Load
$25,000	$ 8,202	$28	$253	1.000
500,000	18,484	659	575	2.324
1,000,000	20,579	1,262	641	2.650
2,000,000	22,543	2,391	703	3.022
5,000,000	24,943	5,513	779	3.682

Note that the total average severity and parameter risk will be the same for all possible primary insurer retentions. However, the total process risk and the reinsurer expense charge will depend upon the retention. Thus, the search for the best retention leads to the question: *what retention will minimize the sum of the process risk and the reinsurance charge?* In the case of a single reinsurer, trial and error will quickly provide the answer. Tables 2 and 3 provide results for our example. In this case we assume that the reinsurance charge for expenses is 10% of the expected loss for the layer.

In the following tables, the increased limits factor is given by:

$$\frac{\text{Average Severity} + \text{Process Risk} + \text{Parameter Risk} + \text{Reinsurance Charge} \quad \text{for the increased limit}}{\text{Average Severity} + \text{Process Risk} + \text{Parameter Risk} \quad \text{for the basic limit}}$$

Table 2 illustrates the kind of search that can be taken to find the most economical reinsurance program with a single reinsurer for a $5,000,000 policy limit.

TABLE 2

SEVERAL SINGLE EXCESS LAYER PROGRAMS FOR $5,000,000 POLICY LIMIT

Layers	Average Severity	Process Risk	Parameter Risk	Reinsurance Charge	Proc. Risk + Reins. Charge	ILF
0 – 1,000,000	$20,579	$1,262	$641			2.650
1,000,000 – 5,000,000	4,364	2,506	137	$436		0.877
Combined		3,768			$4,204	3.527
0 – 1,900,000	22,402	2,281	699			2.992
1,900,000 – 5,000,000	2,541	1,301	80	254		0.492
Combined		3,582			3,836	3.484
0 – 2,000,000	22,543	2,391	703			3.022
2,000,000 – 5,000,000	2,400	1,202	76	240		0.462
Combined		3,593			3,833	3.484
0 – 2,100,000	22,676	2,500	707			3.051
2,100,000 – 5,000,000	2,267	1,109	71	227		0.433
Combined		3,609			3,835	3.484
0 – 3,000,000	23,632	3,463	738			3.281
3,000,000 – 5,000,000	1,311	476	41	131		0.231
Combined		3,939			4,070	3.512

Table 3 shows the results of a similar search for a single reinsurer program with other limits.

TABLE 3

SELECTED SINGLE EXCESS LAYER PROGRAMS

Layers	Average Severity	Process Risk	Parameter Risk	Reinsurance Charge	Proc. Risk + Reins. Charge	ILF
0 > 350,000	$17,353	$469	$540			2.165
350,000 > 500,000	1,131	32	36	$113		0.155
Combined		501			$614	2.319
0 > 600,000	19,048	783	593			2.408
600,000 > 1,000,000	1,532	111	48	153		0.217
Combined		894			1,047	2.625
0 > 900,000	20,269	1,144	632			2.599
900,000 > 2,000,000	2,273	428	72	227		0.354
Combined		1,572			1,800	2.952
0 > 2,000,000	22,543	2,391	703			3.022
2,000,000 > 5,000,000	2,400	1,202	76	240		0.462
Combined		3,593			3,833	3.484

By examining Table 3 one can see that excess of loss reinsurance can be used to reduce increased limits factors. It is tempting to ask if one can further reduce increased limits factors by using more than one reinsurer. A problem is that more reinsurers mean more administrative and transaction expenses. In Tables 4 and 5, we assume that the reinsurance charge for two and three reinsurers is respectively 15% and 20% of the expected losses for each reinsurer.

Tables 4 and 5 were derived by a systematic search for the least expensive reinsurance program for two and three excess reinsurers.

Note that the increased limits factor decreased for only the top two limits. For the lower two limits, using multiple reinsurers in this example did not reduce the process risk by an amount sufficient to cover the extra expense involved. We press on and add another reinsurer for these top two limits.

TABLE 4

SELECTED DOUBLE EXCESS LAYER PROGRAMS

Layers	Average Severity	Process Risk	Parameter Risk	Reinsurance Charge	Proc. Risk + Reins. Charge	ILF
0–350,000	$17,353	$469	$540			2.165
350,000–425,000	620	9	20	$93		0.087
425,000–500,000	511	7	16	77		0.072
Combined		486		170	$655	2.324
0–500,000	18,483	659	575			2.324
500,000–700,000	1,034	39	33	155		0.149
700,000–1,000,000	1,062	60	33	159		0.155
Combined		758		314	1,072	2.628
0–800,000	19,919	1,025	621			2.542
800,000–1,300,000	1,419	129	45	213		0.213
1,300,000–2,000,000	1,205	155	38	181		0.186
Combined		1,310		394	1,704	2.941
0–1,400,000	21,549	1,722	672			2.823
1,400,000–2,900,000	1,993	519	63	299		0.339
2,900,000–5,000,000	1,401	530	44	210		0.258
Combined		2,772		509	3,281	3.419

TABLE 5

SELECTED TRIPLE EXCESS LAYER PROGRAMS

Layers	Average Severity	Process Risk	Parameter Risk	Reinsurance Charge	Proc. Risk + Reins. Charge	ILF
0 > 800,000	$19,919	$1,025	$621			2.542
800,000 > 1,100,000	938	53	30	$188		0.142
1,100,000 > 1,500,000	887	67	28	177		0.137
1,500,000 > 2,000,000	799	76	25	160		0.125
Combined		1,221		525	$1,746	2.946
0 > 1,000,000	20,579	1,262	641			2.650
1,000,000 > 2,000,000	1,963	343	62	393		0.326
2,000,000 > 3,300,000	1,339	316	42	268		0.232
3,300,000 > 5,000,000	1,062	334	33	212		0.193
Combined		2,255		873	3,128	3.401

Here we see a reduced increased limits factor for only the $5,000,000 policy limit. Table 6 summarizes the results we have obtained so far. The boldface numbers represent the lowest increased limit factor obtained for each policy limit.

TABLE 6

SUMMARY OF INCREASED LIMITS FACTORS

Policy Limit	Without Reinsurance	1 Excess Layer	2 Excess Layers	3 Excess Layers	Without Risk Load
$25,000	1.000	1.000	1.000	1.000	1.000
500,000	2.324	**2.319**	2.324	—	2.254
1,000,000	2.650	**2.625**	2.628	—	2.509
2,000,000	3.022	2.952	**2.941**	2.946	2.748
5,000,000	3.682	3.484	3.419	**3.401**	3.041

In this example we see that for the $500,000 and $1,000,000 policy limits the lowest increased limits factor comes as a result of using a single reinsurer. For the $2,000,000 and $5,000,000 policy limits, the lowest increased limits factor comes as a result of using, respectively, two and three reinsurers.

In examining various reinsurance agreements, one often finds a single excess layer shared by two or more reinsurers on a quota share basis. It is demonstrated in Appendix A that if r reinsurers share an excess layer equally on a quota share basis, the total process risk gets reduced by a factor of $1/r$, while the total parameter risk remains the same.

In a final example we examine the effect of quota share for the single excess layer. For the $2,000,000 policy limit, the retention of the primary insurer was $800,000, the reinsurance charge was 15% of the expected losses, and two reinsurers were involved. For the $5,000,000 policy limit, the retention of the primary insurer was $1,000,000, the reinsurance charge was 20% of the expected losses, and three reinsurers were involved. In this example we keep the same assumptions except that each reinsurer shares the excess loss equally on a quota share basis. The results are in Table 7.

TABLE 7

QUOTA SHARE FOR EXCESS LAYER

Layers	Average Severity	Process Risk	Parameter Risk	Reins. Charge	Proc. Risk + Reins. Charge	ILF
0 > 800,000	$19,919	$1,025	$621			2.542
800,000 > 2,000,000	2,624	175	83	$394		0.386
Combined		1,200			$1,594	2.928
0 > 1,000,000	20,579	1,262	641			2.650
1,000,000 > 5,000,000	4,364	627	137	873		0.707
Combined		1,889			2,762	3.357

Here we see that sharing the excess layer on a quota share basis produces even lower increased limits factors. The results of this example are summarized in Table 8.

TABLE 8

SUMMARY OF INCREASED LIMITS FACTORS

Policy Limit	Without Reins.	1 Excess Layer	2 Excess Layers	3 Excess Layers	Quota Share Excess	Without Risk Load
$25,000	1.000	1.000	1.000	1.000	1.000	1.000
500,000	2.324	**2.319**	2.324	—	—	2.254
1,000,000	2.650	**2.625**	2.628	—	—	2.509
2,000,000	3.022	2.952	2.941	2.946	**2.928**	2.748
5,000,000	3.682	3.484	3.419	3.401	**3.357**	3.041

These examples do not illustrate the entire story. While quota share reinsurance may exhibit superior risk load reduction, it also involves more administrative expense since all reinsurers must look at every claim. However, sound reinsurance underwriting may remove the need to examine every claim.

This certainly explains why quota share reinsurance is often used on excess layers. But at some level, the risk-sharing advantages of quota share reinsurance and the effect of reinsurance underwriting may overcome the additional administrative expense.

Another common feature of reinsurance contracts is that the primary insurer can take a pro-rata share of the excess layer. The possible reduction in the "morale hazard" may make the contract more attractive to reinsurers, but it comes at the expense of higher total risk load.

How to balance all these aspects of reinsurance contracts is not clear. What is clear is that there are many problems involved in making an advisory filing which attempts to build all this into its increased limits factors.

4. CONSISTENCY

Another "definition" problem between Robbin and myself involves the notion of consistency. Consistency means that the price of a layer of insurance of a given width does not increase as the initial attachment point increases. It is argued that since a loss covered by the insurance does not increase as the initial attachment increases, the price should not increase. Robbin believes that this definition should restrict the pricing formula to taking the difference between the ILFs of the layer limit and the initial attachment point. I believe one should use the method that is actually used in pricing the layer. Since the motivation for consistency refers to price, the definition should refer to price.

At this point I would like to confess to an error in my original paper. Robert Bear, a CAS member, recently pointed out that my proof of consistency for the CME formula (by my definition) contained an error in the part that involved parameter uncertainty for the severity distribution. Upon further investigation I discovered conditions when the CME formula can produce inconsistent layer prices. Conditions under which the CME formula will be consistent are given in Appendix B. Generally speaking, inconsistency can occur for low layers when most of the parameter uncertainty is in the severity distribution. Thus the status of consistency with respect to excess layers can be summarized as follows: (1) the expected loss is consistent; (2) process risk is consistent; (3) the part of parameter risk due to uncertainty in the claim count distribution is consistent; but (4) the part of parameter risk due to uncertainty in the severity distribution can be inconsistent. However, the consistency of the first three parts can overpower the inconsistency in the fourth part.

Table 9 gives an example of the CME formula producing inconsistent layer pricing. This example was produced by modifying the previous example by putting all the parameter uncertainty into the severity, and increasing the risk load multiplier, λ, by a factor of 100. This produces inconsistency for the parameter risk up to \$5,000 and for the total price up to \$2,000. It was necessary to increase the risk load multiplier drastically to produce the inconsistency in the total price.

TABLE 9

Layer	Average Severity	Process Risk	Parameter Risk	ILF
0 > 25,000	$8,202	$2,811	$12,012	1.000
0 > 1,000	903	17	226	0.050
1,000 > 2,000	751	15	492	0.055
2,000 > 3,000	641	12	606	0.055
3,000 > 4,000	559	11	649	0.053
4,000 > 5,000	494	10	659	0.050
5,000 > 6,000	443	9	651	0.048
6,000 > 7,000	400	8	634	0.045
7,000 > 8,000	365	7	613	0.043

This example is extreme. But occasionally it is instructive to push a theory to its extreme cases to examine its theoretical foundations. Here, we examine it from the viewpoint of utility theory.[1]

Let X_1 and X_2 be losses for layer a_1 to $a_1 + h$ and a_2 to $a_2 + h$, respectively, where $a_1 < a_2$. Let P_1 and P_2 be the premium obtained for insuring against X_1 and X_2. If $P_1 = P_2 = P$, an insurer, I, with utility function u_I, will prefer to sell a policy for X_2 since:

$$\mathrm{E}\left[u_I(P - X_1)\right] < \mathrm{E}\left[u_I(P - X_2)\right].$$

Each insurer, with its own utility function, will prefer to sell a policy for X_2. Thus you should expect P_2 to be less than P_1. Here we have a case where the CME risk load formula and utility theory disagree.

Suppose we have an insured, G, with utility function u_G. If insurance is being bought for X_2, we must have:

$$u_G(-P_2) \geq \mathrm{E}\left[u_G(-X_2)\right].$$

[1] The utility of an insurance policy depends upon many variables, such as initial wealth. Here I will not write down variables which are the same for all situations.

Now let's suppose we have inconsistency, i.e. $P_1 < P_2$. Then one of the following three cases must happen.

$$\text{Case 1: } \mathrm{E}\,[u_G(-X_2)] \geq u_G(-P_1) > u_G(-P_2)$$

$$\text{Case 2: } u_G(-P_1) > \mathrm{E}\,[u_G(-X_2)] \geq u_G(-P_2)$$

$$\text{Case 3: } u_G(-P_1) > u_G(-P_2) \geq \mathrm{E}\,[u_G(-X_2)]\,.$$

In Case 1, no insurance will be bought for either layer. In Case 2, no insurance will be bought for the second layer. In Case 3, insurance will be bought for the second layer in spite of the inconsistency. Note that the derivation of the CME risk load formula assumes that the demand for insurance is fixed; i.e., it only considers Case 3 where inconsistency can be tolerated.

5. WHERE DO WE GO FROM HERE?

The CME is derived using a variance constraint on insurance portfolios. I consider utility theory to be a better measure of risk than variance. However, risk loads derived from variance principles often provide good practical approximations to the results that are obtained using utility theory. I regard the results on inconsistency discussed above as evidence that the approximation is not perfect.

For now anyway, the inconsistency appears to be a theoretical rather than a practical problem. The lengths to which one has to go to produce inconsistent results seem far removed from real pricing decisions. Should real life cases where this becomes a problem arise in the future, I offer the following avenues of research to deal with these and other problems.

1. Replace the insurer's maximum variance constraint in the CME derivation with a minimum utility constraint.

2. Allow for flexibility in the demand for insurance. The assumption of constant demand has problems at both the very high and the very low layers.

3. The parameter uncertainty in the current CME formulation is very restricted. It allows only for uncertainty in the scale of the severity distribution. A potentially bigger problem is uncertainty in the shape of the severity distribution. Work on this needs to be done. Also, it is conceivable that incorporating other kinds of parameter uncertainty may make the consistency issue more pressing.

The problem of determining risk loads is perhaps one of the most difficult in all of actuarial science. Its solution will not come about with any single brilliant insight, but will evolve slowly after much trial and error. I would like to think that my work, as well as the work of Dr. Robbin and Mr. Bear, makes a positive contribution to this effort.

REFERENCES

[1] Meyers, Glenn, "The Competitive Market Equilibrium Formula for Increased Limits Ratemaking," *PCAS* LXXVIII, 1991

[2] Robbin, Ira, Discussion of Meyers: "The Competitive Market Equilibrium Formula for Increased Limits Ratemaking," *PCAS* LXXIX, 1992.

APPENDIX A

QUOTA SHARE REINSURANCE

The CME risk load is given by $\bar{\lambda}\,(\boldsymbol{U} + 2\boldsymbol{V}\,\bar{\boldsymbol{n}})$ with:

$$u_i = \mathrm{E}_\alpha[\mathrm{E}[Z_i^2 \mid \alpha]\,] + \mathrm{E}_\alpha[\mathrm{E}[Z_i \mid \alpha]^2\,] \bullet d$$

$$v_{ij} = (1 + c) \bullet \mathrm{E}_\alpha[\mathrm{E}[Z_i \mid \alpha] \bullet \mathrm{E}[Z_j \mid \alpha]\,] - \mathrm{E}_\alpha[\mathrm{E}[Z_i \mid \alpha]\,] \bullet \mathrm{E}_\alpha[\mathrm{E}[Z_j \mid \alpha]\,]\,.$$

If we multiply the loss in the i^{th} line of insurance by $1/r$ we get:

$$u_i' = \mathrm{E}_\alpha[\mathrm{E}[(Z_i/r\,)^2 \mid \alpha]\,] + \mathrm{E}_\alpha[\mathrm{E}[Z_i/r \mid \alpha\,]^2] \bullet d$$

$$= u_i/r^2.$$

$$v_{ij}' = (1{+}c) \bullet \mathrm{E}_\alpha\,[\mathrm{E}[Z_i/r \mid \alpha] \bullet \mathrm{E}[Z_j \mid \alpha]\,] - \mathrm{E}_\alpha[\mathrm{E}[Z_i/r \mid \alpha]\,] \bullet \mathrm{E}_\alpha[\mathrm{E}[Z_j \mid \alpha]\,]$$

$$= v_{ij}/r\,.$$

The total risk load contributed by the r reinsurers in the i^{th} line of insurance is:

$$r\bar{\lambda}(\boldsymbol{U}' + 2\boldsymbol{V}'\bar{\boldsymbol{n}})_i = r\bar{\lambda}(\boldsymbol{U}/r^2 + 2\boldsymbol{V}\,\bar{\boldsymbol{n}}/r) = \bar{\lambda}(\boldsymbol{U}/r + 2\boldsymbol{V}\,\bar{\boldsymbol{n}}).$$

Thus the total process risk is reduced by a factor of $1/r$ and the total parameter risk is unchanged.

APPENDIX B

CONSISTENCY

In this appendix, it is assumed that the reader is familiar with the results in Appendices C and E of the original paper.

Let the claim severity distribution be given by $S(z)$ and the expected claim cost for the layer from a to $a + h$ be given by $M_1(a,h)$.

Recall from Lemma E.1 of the original paper that:

$$M_1(a,h) = \int_a^{a+h} (1 - S(z)) \bullet dz .$$

Now:

$$M_1(a,h \mid \alpha) = \int_a^{a+h} \left(1 - S\left(\frac{z}{\alpha}\right)\right) \bullet dz = \alpha \bullet \int_{a/\alpha}^{(a+h)/\alpha} (1 - S(z)) \bullet dz ,$$

with the second equality being derived by substituting z for z/α.

Lemma B.1:

$\frac{\partial}{\partial\alpha} M_1(a,h \mid \alpha)$ is positive.

Using the product rule and the fundamental theorem of calculus we get:

$$\frac{\partial}{\partial\alpha}\left(\alpha \bullet \int_{a/\alpha}^{(a+h)/\alpha} (1-S(z)) \bullet dz\right)$$

$$= \left[\int_{a/\alpha}^{(a+h)/\alpha} (1-S(z)) \bullet dz - \frac{h}{\alpha}\left(1-S\left(\frac{a+h}{\alpha}\right)\right)\right] + \frac{a}{\alpha}\left(S\left(\frac{a+h}{\alpha}\right) - S\left(\frac{a}{\alpha}\right)\right). \quad \text{(B.1)}$$

The bracketed expression is positive since $S(z) < S\left(\frac{a+h}{\alpha}\right)$ for all z in the interval from a/α to $(a+h)/\alpha$. The interval is of length h/α. The remainder of the expression is also positive since S is an increasing function.

Lemma B.2:

If $zS'(z)$ is decreasing for $z > a_0$, then $\frac{\partial}{\partial\alpha}M_1(a,h \mid \alpha)$ is a decreasing function of a for $a > \alpha a_0$.

$$\frac{\partial}{\partial a}\frac{\partial}{\partial\alpha}M_1(a,h \mid \alpha) = \frac{\partial}{\partial a}\left[\int_{a/\alpha}^{(a+h)/\alpha}(1-S(z)) \bullet dz - \frac{h}{\alpha}\left(1-S\left(\frac{a+h}{\alpha}\right)\right) + \frac{a}{\alpha}\left(S\left(\frac{a+h}{\alpha}\right) - S\left(\frac{a}{\alpha}\right)\right)\right]$$

$$= \frac{1}{\alpha}\left[\frac{a+h}{\alpha}S'\left(\frac{a+h}{\alpha}\right) - \frac{a}{\alpha}S'\left(\frac{a}{\alpha}\right)\right] < 0 \text{ for } a > \alpha a_0.$$

Thus $\frac{\partial}{\partial\alpha}M_1(a,h \mid \alpha)$ is a decreasing function of a for $a > \alpha a_0$.

The condition that $zS'(z)$ be decreasing for $z > a_0$ is a common property of severity models. Consider the Pareto distribution:

$$S(z) = 1 - \left(\frac{b}{z+b}\right)^q \quad \text{and} \quad zS'(z) = \frac{zqb^q}{(z+b)^{q+1}}.$$

Now

$$\frac{d}{dz}(zS'(z)) = qb^q\left(\frac{1}{(z+b)^{q+1}} - \frac{z(q+1)}{(z+b)^{q+2}}\right)$$

is negative if and only if $z > b/q$. The reader can verify that this property holds for many other distributions such as the Weibull and the lognormal.

Theorem B.3:

If the severity distribution satisfies the property that $zS'(z)$ is decreasing for $z > a_0$, then there exists a limit D so that increased limits factors are consistent for retentions $a > D$.

It is instructive to consider the (incorrect) "proof" given in my paper. Let $a_2 > a_1$.

$$v_{1j} = (1+c) \bullet \mathrm{E}_\alpha[M_1(a_1,h \mid \alpha) \bullet \mathrm{E}[Z_j \mid \alpha]] - \mathrm{E}_\alpha[M_1(a_1,h \mid \alpha)] \bullet \mathrm{E}_\alpha[\mathrm{E}[Z_j \mid \alpha]]$$

$$> (1+c) \bullet \mathrm{E}_\alpha[M_1(a_2,h \mid \alpha) \bullet \mathrm{E}[Z_j \mid \alpha]] - \mathrm{E}_\alpha[M_1(a_2,h \mid \alpha)] \bullet \mathrm{E}_\alpha[\mathrm{E}[Z_j \mid \alpha]]$$

$$= v_{2j} .$$

It then follows that:

$$(\boldsymbol{V} \bullet \overline{\boldsymbol{n}})_1 > (\boldsymbol{V} \bullet \overline{\boldsymbol{n}})_2 .$$

Robert Bear's contribution was to point out that while

$$c\mathrm{E}_\alpha[M_1(a_1,h \mid \alpha) \bullet \mathrm{E}[Z_j \mid \alpha]]$$

is greater than

$$c\mathrm{E}_\alpha[M_1(a_2,h \mid \alpha) \bullet \mathrm{E}[Z_j \mid \alpha]] ,$$

it does not follow that

$$\mathrm{E}_\alpha[M_1(a_1,h \mid \alpha) \bullet \mathrm{E}[Z_j \mid \alpha]] - \mathrm{E}_\alpha[M_1(a_1,h \mid \alpha)] \bullet \mathrm{E}_\alpha[\mathrm{E}[Z_j \mid \alpha]]$$

is greater than (B.2)

$$\mathrm{E}_\alpha[M_1(a_2,h \mid \alpha) \bullet \mathrm{E}[Z_j \mid \alpha]] - \mathrm{E}_\alpha[M_1(a_2,h \mid \alpha)] \bullet \mathrm{E}_\alpha[\mathrm{E}[Z_j \mid \alpha]].$$

It is this last inequality, Equation B.2, that we must demonstrate in order to make the claim of consistency.

It will help if we introduce some shorthand notation. Let:

$$m_1(\alpha) = M_1(a_1, h \mid \alpha) \text{ and } \overline{m}_1 = \mathrm{E}_\alpha[m_1(\alpha)],$$

$$m_2(\alpha) = M_1(a_2, h \mid \alpha) \text{ and } \overline{m}_2 = \mathrm{E}_\alpha[m_2(\alpha)], \text{ and}$$

$$e(\alpha) = \mathrm{E}[Z_j \mid \alpha] \text{ and } \overline{e} = \mathrm{E}_\alpha[\mathrm{E}[Z_j \mid \alpha]].$$

Equation B.2 can then be written as:

$$\int_0^\infty (m_1(\alpha)-\overline{m}_1)(e(\alpha)-\overline{e})f(\alpha)d\alpha > \int_0^\infty (m_2(\alpha)-\overline{m}_2)(e(\alpha)-\overline{e})f(\alpha)d\alpha .$$

The left hand side of this expression can be evaluated using integration by parts:

$$u = m_1(\alpha)-\overline{m}_1 \qquad dv = (e(\alpha)-\overline{e})f(\alpha)d\alpha$$

$$du = m_1'(\alpha)d\alpha \qquad v = \int_0^\alpha (e(t)-\overline{e})f(t)dt$$

$$\text{LHS} = \underbrace{\lim_{r\to\infty} (m_1(r)-\overline{m}_1)\int_0^r (e(t)-\overline{e})f(t)dt}_{=0} - \lim_{r\to\infty} \int_0^r \underbrace{m_1'(\alpha)}_{>0} \underbrace{\int_0^\alpha (e(t)-\overline{e})f(t)dt}_{<0}\, d\alpha$$

$$= -\lim_{r\to\infty} \int_0^r m_1'(\alpha) \int_0^\alpha (e(t)-\overline{e})f(t)dt\, d\alpha \qquad \text{(which is positive).}$$

Similarly:

$$\text{RHS} = -\lim_{r\to\infty} \int_0^r m_2'(\alpha) \int_0^\alpha (e(t)-\overline{e})f(t)dt\, d\alpha \qquad \text{(which is positive).}$$

If we evaluate the outer integrals by a numerical integration formula (as I do in my paper):

$$\text{LHS} = \sum_{i=1}^{L} m_1'(\alpha_i) \int_0^{\alpha_i} (e(t)-\bar{e})f(t)dt \text{ and}$$

$$\text{RHS} = \sum_{i=1}^{L} m_2'(\alpha_i) \int_0^{\alpha_i} (e(t)-\bar{e})f(t)dt.$$

If we choose $D = \max\{\alpha_i\}a_0$, then by Lemma B.2, $m_1'(\alpha_i) > m_2'(\alpha_i)$ for all i. Thus LHS is greater than RHS. This proves Equation B.2 and consequently, Theorem B.3.

A close examination of the above proof reveals that if $zS'(z)$ is increasing for $z < a_0$, as it does for the Pareto distribution with $a_0 = b/q$, then it is possible to have inconsistent increased limits factors for $a_2+h < \min\{\alpha_i\}a_0$. Our example can be modified to produce inconsistent increased limits factors as follows. Change c from .02 to 0, a from .001 to .02, and $\bar{\lambda}$ from 2×10^{-7} to 2×10^{-5}. This yields $\min\{\alpha_i\}a_0 = 3{,}432$ and $\max\{\alpha_i\}a_0 = 5{,}659$. The results are in Table 9. Note that the parameter risk shifts from inconsistent to consistent in the interval from 3,432 to 5,659. The shift from inconsistent to consistent for increased limit factors occurs at a lower level because of the consistency of the average severity and the process risk.

ADDRESS TO NEW MEMBERS—NOVEMBER 15, 1993

GUARDIANS OF GREED OR ANGELS OF COOPERATION

C.K. "STAN" KHURY

Today I would like to talk to you about money. History tells us that money was invented to serve as a tool to facilitate bartering in particular, and human endeavor in general. Over the millennia this tool has grown in importance. So much so that the net object of nearly all human transactions, in one way or another, has become the accumulation of money. In fact, the process of accumulating money has been given a number of code names including benevolent sounding labels such as "securing one's future," indifferent labels such as "creating wealth," and unpretentious labels such as "greed."

A consequence of this preoccupation has been the emergence of a number of professions to serve the idea of money. Examples abound: Accountants, auditors, economists, financial planners, brokers, bankers, and (yes) actuaries. All of these professions exist primarily to make possible the idea of money on the scale of importance it has assumed in daily life.

What I would like to do right now is to put the actuary and his or her work in two contexts: The context of the original motivation for the invention of money and the context of the accumulation of money as a degeneration of that original purpose. To the extent an actuary is applying his or her skills to the process of facilitating human endeavor, then the actuary is dedicating his or her work to facilitating human cooperation. On the other hand, to the extent an actuary is applying his or her skills for the purpose of accumulating money, then the actuary, in doing his or her work, is serving at the altar of greed. In the former case, when an actuary is serving to promote cooperation, I would call the actuary an "angel of cooperation." In the latter case, I would call the actuary serving the purpose of accumulating money a "guardian of greed."

The question I would like to leave with you today is "whom will you represent as you apply your actuarial craft, an angel of cooperation or a guardian of greed?"

PRESIDENTIAL ADDRESS—NOVEMBER 15, 1993

THE CHALLENGE OF CHANGE

DAVID P. FLYNN

As I was organizing my thoughts to prepare for this occasion, it was a natural part of that process to consider the recent activities that your leadership has been involved in and to speculate on the likely effects of current economic trends on the future position of our Society.

I found myself musing about many things—musing is a safer word than "daydreaming" which, while not as safe, may be a more accurate word! It's been very clear to me for some time now that the affairs of the CAS today are much more complex and rapidly changing than at any previous time in at least the last thirty years. These changes have occurred in virtually every area of our activities and ranged from building stronger relationships with the worldwide actuarial community to strengthening the structures of professionalism within our Society.

A moment's reflection leads one to conclude that this increased level of activity is not surprising given the equally dramatic changes taking place in the larger community and particularly in those areas involving economic matters. The activity within the Casualty Actuarial Society is in fact largely reflecting the changes being experienced by our primary customers—the insurance industry.

I do not intend today in these brief moments with you to review in any detail all of the activities that our Society has been involved in during the past year. It simply would be impossible to recognize individually the effective work of our members. At different times in the past on these occasions, retiring presidents have dramatically underscored the level of participation of the members of our Society by asking members of the various committees and task forces to stand and be recognized. It's not a good sampling technique, but it effectively makes the point.

Many times during my two years of service as president and president-elect, I have heard outsiders describe the CAS as the model of an effective actuarial organization. I know that it is; it's that way only because of your voluntary efforts and our Society is indebted to you for your commitment.

I do want to assure you that your Society continues to be active and financially healthy. This morning John Purple in his report to the membership laid out some of the financial facts that can give us some comfort in the viability of our organization. I will not repeat them here except to focus on one aspect of John's report—the growth of our Society.

The recent dramatic growth of our student population led me to the theme of my report to you today—the challenge of change.

You have all heard the statistics regarding our membership levels. The number of new Associates, new Fellows, and new students compared to previous years continues to increase. I'm particularly pleased to report to you that my exposure to these new members over the past year or so has convinced me that there are truly outstanding candidates entering the CAS today and that our profession will benefit for many years to come through their contributions. I welcome them—they are truly our Society's future!

Some members of our Society have expressed concern about employment opportunities available to our members, both today and in the future. Regarding this issue, it is beneficial to look back at past membership levels and recall the times that similar concerns have been expressed in the past. I'm only surprised at how far back these alarms have been sounded!

When I first entered the CAS in 1967, total membership stood at 432 and soared to 461 three years later when I received my Fellowship! I can assure you that there was talk at that time about the need to make the exams tougher and that we were growing too fast! That was twenty-five years ago!

Recently, Walt Rugland of the Society of Actuaries shared a letter with me from an FSA who received his Fellowship fifty years ago this year. The now-retired actuary was reminiscing that when he took his first actuarial exam in 1933, the pass mark was deliberately set very high—resulting in a 12.5% pass rate—because the then-Fellows were concerned that their society was growing too rapidly!

I suspect that alarms were sounded even further back than the sixty years involved in this anecdote, but they have now been lost to history. By the way, for the new Fellows and Associates, this letter also provides some evidence to support our older members' claims that the exams were much tougher in the past!

The point is clear that change and challenge have been with us in the actuarial profession for many years. Most often, the change is beyond our control or direct influence; but we have always in the past met the challenge of change to become an even stronger and more rewarding profession. I believe that we can meet that challenge and continue to do so—but only with some effort on our part!

Let's look briefly at some of the elements of change today.

One element is the changing character of the insurance business itself. More companies are now announcing downsizing actions, or "right-sizing" in the current vernacular. The proportion of commercial business written by standard carriers has been dropping steadily for years as more and more businesses become self-insured, join risk-retention groups, or develop other innovative ways to manage risk. Some insurance companies are dramatically reducing or completely dropping their personal lines exposure. The industry has been under stress for years as higher insurance costs are not being met with higher prices or more efficient risk sharing techniques.

Many of you will recall that just a few short years ago, it was possible for most companies to make an underwriting profit. Later, most made an operating profit using relatively small amounts of capital gains rather than an underwriting profit. In recent years, capital gains taken by companies have been responsible for most, if not all, of the operating profit. The economy is telling us that the risk transfer systems that used to work very effectively are being replaced by either more efficient systems or by less efficient, subsidized systems.

This time of stress in the business community presents dangers and opportunities for the actuarial profession. The danger comes from a possible failure on our part to increase the efficiency of the actuarial processes used to serve our business customers. The opportunity comes from the fact that all of these changes in the company environment dictate that in order to compete effectively, companies must have a thorough understanding of the risks and rewards associated with all of the different risk transfer opportunities.

The margin of forgiveness that used to exist in the insurance system is no longer there. While there is no one better than an actuary to evaluate the risks and rewards of these mechanisms, we cannot be blind to the fact that we must deliver our services in an economically sensible and competitive way.

Another element of change is the developments in the North American economy as the Cold War ends and, God willing, an opportunity for a long period of peace begins. Government spending patterns are changing and these changes will have profound effects on the economy in the years ahead.

How these changes will manifest themselves is impossible to predict with any precision. But simply because of this lack of precision, our Society must become more adept at recognizing new opportunities for our members in the *overall* economy, *not just in the insurance business*, and providing our members with the resources to exploit these new opportunities.

The Casualty Actuarial Society today is strong and continues to grow but we must recognize that we are not immune to the laws of supply and demand. We must continue to enhance our abilities to meet our customer's needs if we are to continue to enjoy the favorable position that we hold today.

Our past successes in the education of casualty actuaries should not lull us into complacency. We must become more flexible in our educational efforts or in this era of change we risk educating our members to meet demands that no longer exist. Our Education Policy Committee should become one of the most important of our operating committees and our educational practices must be recertified on a frequent basis to keep pace with the changing world.

The need for a separate continuing education effort is a relatively recent one. I can readily foresee the time when the continuing education function will become the dominant educational force in our Society.

A third element of change is the changing demographics of the North American population, which is directly or indirectly the source of our earning capacity. Senior citizens are the fastest growing component of our population. What will these super-senior citizens mean for the demand of property/casualty products. The newest type of actuary—actuaries of the fifth type—are the continuing care retirement plan actuaries. These are the actuaries that meld the disciplines of all facets of the actuarial profession into their specialty. These actuaries have a very strong foothold in this growing part of our population.

Other innovations in actuarial practice are sure to arise as new medical products and treatment techniques give rise to new sources of liability. Is the concept of "enterprise liability," which was raised in the discussions of medical care in the United States by the Clinton Administration, a viable one? Enterprise liability would transfer the financial responsibility for medical care incidents from the individual medical practitioners and hospitals to the organizations, such as insurance companies, that finance the care. If this concept survives, what are its ramifications for the CAS and for our customers?

In another facet of President Clinton's proposal, how large of an intersection will exist between universal health insurance and the workers' compensation system? Will the underwriters of workers' compensation coverage be able to control the political momentum behind a universal medical coverage package and its inherent basis in community rating with the workplace safety, vocational injuries, and experience rating which are inherent to the workers' compensation system? Casualty actuaries can and should play key roles in the articulation of the issues and costs associated with these choices!

The final element of change that I want to bring to your attention is what I will call the new era of aggressive regulation. Today regulators are under intense pressure from the public and from various legislative forums to not only prevent the pain of insolvency, but to do so while keeping prices as low as possible! Even the issue of data collection by industry statistical agents is being reconsidered.

By and large our regulators are intelligent and honorable people doing a demanding job under difficult circumstances. In the years ahead, as the voices of change grow louder and more persistent, we must make sure that we are not so caught up in the service of our customers that critical facts are twisted or abandoned. That is not to say that we should not always represent our customers as effectively as possible but simply that we must always adhere to the precepts of professional integrity contained in the Code of Professional Conduct. To do otherwise will cause fatal damage to our profession.

Lest I wander too far from my theme of the challenge of change, let me emphasize that whatever shape the future takes in terms of the needs of our customers, the CAS must become increasingly flexible in the triad of our goals of education, research, and continuing education if the CAS is to continue to be successful in the future.

Let me assure you that our Long Range Planning Committee has not been asleep at the switch in anticipating changes in our environment. This committee delivers an annual report to the Board of Directors which suggests the planning priorities for our Society. This year's report identified the appointed actuary as our number one priority. The appointed actuary in this context is an actuary who would opine on the adequacy or sufficiency of the surplus of a company to meet current and perhaps even future obligations. I bring this to your attention not only because of the material implications of this concept in the regulatory and business arenas but also because it emphasizes the immediate need of our members to be exposed to the investment concepts that are embedded in the work of the appointed actuary.

As a result of this report, the Board of Directors has determined that over the next five years every member of our Society must be given the opportunity through our educational processes to become familiar with the investment concepts necessary to function as an appointed actuary. These investment concepts are important to us as casualty actuaries in order that we may better serve our current and future clients!

Keep an eye out for the educational opportunities to be made available and be sure to take advantage of them. Your first opportunity will come April 20 to 22, 1994, at the AFIR Colloquium to be held in Orlando. AFIR is the section of the International Actuarial Association which is devoted to the evaluation and management of financial risk. The CAS is cosponsoring this meeting and our Continuing Education Committee is working hard to provide many learning opportunities on casualty-related investment issues at the Colloquium.

There are four additional actions that the CAS should consider to better respond to the challenges that lie ahead of us that I want to raise for your consideration:

The first is the need to shorten the examination process without lessening its rigor. One way to do this may be by making greater use of the educational opportunities available in the colleges and universities.

The second is to enhance the flexibility of the education required for Fellowship by developing options for different career paths.

The third is to strengthen our continuing education efforts by making greater use of the research and educational capabilities available in the academic and business communities and in actuarial societies throughout the world.

The fourth is to continue working through the Academy of Actuaries to devise new ways to strengthen our relationships with the regulatory community. We have a long way to go to build lasting partnerships with this key segment of our industry.

In closing, there are many individuals that I could name in this short time to whom go my thanks for their great work in the past year. Please keep up that same level of effort in the year ahead.

I also want to express my thanks to some special people—to my family for their tolerance and understanding of my frequent absences from home, to Tim Tinsley and the CAS office staff who make this position much easier than it might otherwise be, and to the members of the Executive Council who truly bear the largest burden within the Society.

In addition to the significant honor of having served, it's been great fun to have been the President of the Casualty Actuarial Society! I sincerely thank you for the privilege.

MINUTES OF THE 1993 ANNUAL MEETING

November 14 - 17, 1993

THE POINTE HILTON RESORT ON SOUTH MOUNTAIN

PHOENIX, ARIZONA

Sunday, November 14, 1993

The Board of Directors held their regular quarterly meeting from 1:00 p.m. until 5:00 p.m.

Registration for the Annual Meeting occurred from 4:00 p.m. until 6:00 p.m.

From 5:30 p.m. until 6:30 p.m., new Associates and their guests attended a reception that featured a presentation on the CAS committee structure.

A welcome reception for all members and guests was held from 6:30 p.m. until 7:30 p.m.

Monday, November 15, 1993

Registration continued from 7:00 a.m. until 8:00 a.m.

CAS President David P. Flynn opened the meeting at 8:00 a.m. and recognized past CAS presidents in the audience, as well as special guests: Larry Baber, President-Elect, Conference of Consulting Actuaries; John O'Connor, Executive Director, Society of Actuaries; and Kurt von Schilling, President-Elect, Canadian Institute of Actuaries.

Mr. Flynn announced the results of the election of CAS officers. The members of the 1994 Executive Council will be Vice President-Administration, John M. Purple; Vice President-Programs and Communications, Alice H. Gannon; Vice President-Research and Development, Michael J. Miller. President-Elect will be Allan M. Kaufman. New Board members will be Steven F. Goldberg, Patrick J. Grannan, Anne E. Kelly, and Robert S. Miccolis.

Mr. Flynn asked the Fellows in attendance to confirm the appointment by the Board of Brian E. Scott to fill the unexpired term of Michael J. Miller on the Board of Directors, which they did.

Mr. Flynn thanked outgoing Executive Council and Board members for their service to the CAS.

Allan Kaufman, Steven Lehmann, and John Purple introduced the 60 new Associates, and Irene Bass introduced the 85 new Fellows at the meeting. The names of these individuals follow:

FELLOWS

Marc J. Adee
Kristen M. Albright
Guy A. Avagliano
Katharine Barne
Gregory S. Beaulieu
Xavier Benarosch
Lisa M. Besman
Cara M. Blank
Alicia E. Bowen
Christopher K. Bozman
Yaakov B. Brauner
Mark D. Brissman
J. Eric Brosius
Paul A. Bukowski
Mark J. Cain
David S. Cash
Chyen Chen
Peter J. Collins
David B. Cox
Manon Debigare
Germain Denoncourt
Pierre Dionne
Victor G. dos Santos
William F. Dove
Michael C. Dubin
Patrick Dussault
Charles C. Emma
Philip A. Evensen
Randall A. Farwell
Barry A. Franklin
Scott F. Galiardo
Andrea Gardner
Richard J. Gergasko
Bruce R. Gifford
Richard S. Goldfarb
Charles T. Goldie
Odile Goyer
Edward M. Grab
Carleton R. Grose
George M. Hansen
Gordon K. Hay
Kathleen A. Hinds
Beth M. Hostager
Laura A. Johnson
Brian A. Jones
Changseob J. Kim
Gerald S. Kirschner
Jerome F. Klenow
D. Scott Lamb
Alan E. Lange
Nicholas J. Lannutti
Nicholas M. Leccese, Jr.
Eric F. Lemieux
Stephanie J. Lippl
Mark J. Mahon
Heidi J. McBride
Dennis T. McNeese
John H. Mize
William A. Niemczyk
Kathleen C. Nomicos
Kathy A. Olcese
Sarah Louise Petersen
Michael Petrocik
Michael D. Poe
Stuart Powers
Jeffrey C. Raguse
Donald K. Rainey
A. Scott Romito
Allen D. Rosenbach
Jean Roy
Stuart G. Sadwin
Yves Saint-Loup
Thomas E. Schadler
David A. Smith
Linda D. Snook
David B. Sommer

Susan M. Treskolasky
Jennifer A. Violette
Bryan C. Ware
Peter A. Weisenberger
John P. Welch
Kevin L. Wick
Gnana K. Wignarajah
Teresa J. Williams
Chung-Ye Yen

ASSOCIATES

Jonathan D. Adkisson
Michael E. Angelina
Bhim D. Asdhir
Gary Blumsohn
Ann M. Bok
Maurice P. Bouffard
Tracy L. Brooks-Szegda
Peter V. Burchett
Michael W. Cash
Tania J. Cassell
Kevin James Cawley
Debra S. Charlop
Kay A. Cleary
Jo Ellen Cockley
Joyce A. Dallessio
Ronald E. Glenn
Russell H. Greig, Jr.
Richard J. Haines
Robert L. Harnatkiewicz
Christopher L. Harris
Paul D. Henning
Robert J. Hopper
Sadagopan S. Iyengar
Janet S. Katz
Michael F. Klein
Jason A. Kundrot
Robert J. Larson
David R. Lesieur
Shu C. Lin
Richard J. Marcks
Lawrence F. Marcus
Sharon L. Markowski
Dee Dee Mays
Stephen V. Merkey
Douglas H. Min
Kimberly J. Mullins
Giovanni A. Muzzarelli
Douglas J. Onnen
Paul S. Osborn
Joseph W. Pitts
Yves Provencher
Regina M. Puglisi
Steven J. Regnier
Ellen J. Respler
Al J. Rhodes
Sallie S. Robinson
David A. Rosenzweig
Kevin L. Russell
Peter R. Schwanke
Calvin C. Spence, Jr.
Victoria G. Stachowski
John P. Stefanek
Richard A. Stock
Kimberley A. Ward
Gayle L. Wiener
Calvin Wolcott
Robert F. Wolf
Cheng-Sheng P. Wu
Edward J. Yorty
Jeffery M. Zacek

Mr. Flynn introduced C.K. Khury who gave the Address to New Members.

Mr. Flynn then presented the 1993 Matthew S. Rodermund Service Award to Robert A. Bailey. As part of the presentation ceremony, Mr. Flynn read a letter from Mr. Rodermund.

Mr. Flynn requested a moment of silence to mark the passing of three members of the CAS during the past year.

John Purple read the Vice President-Administration's Report.

Alice Gannon covered the highlights of the program. Chairman of the Committee on Review of Papers, John Kollar, summarized the four new *Proceedings* papers, two discussions, and two authors' replies being presented.

The Dorweiler Prize was awarded to Michael G. Wacek.

Mr. Flynn made a call for reviews of *Proceedings* papers previously presented and received no responses.

Featured speaker, Daniel Burrus, one of the nation's leading science and technology forecasters, spoke from 10:00 a.m. until 11:00 a.m.

A general session panel on "Solvency and Regulation" took place from 11:00 a.m. until 12:30 p.m. W. James MacGinnitie, Consulting Actuary with Tillinghast/Towers Perrin, moderated the session, with panelists Dawn Bardwell, Partner, Coopers & Lybrand; Mary Moore Hamrick, Minority Counsel, House Energy and Commerce Committee; and Frank Nutter, President, Reinsurance Association of America.

Following the panel, there was a luncheon from 12:30 p.m. until 1:30 p.m., highlighted by the Presidential Address from David P. Flynn.

The afternoon's concurrent sessions ran from 1:30 p.m. until 5:00 p.m. and consisted of various panels and presentations of papers.

The panel presentations covered the following topics:

1. "Data Management and the Actuary"

Moderator: Philip D. Miller,
Senior Vice President and Actuary
Insurance Services Office, Inc.

Panelists: Gary Knoble, Assistant Vice President
ITT/Hartford Insurance Group

Marc B. Pearl, Vice President and Actuary
Continental Insurance

2. "Quasi-Regulation"

Moderator: Dale F. Ogden, Senior Manager
KPMG Peat Marwick

Panelists: Paul Malvasio,
Vice President, Chief Financial Officer
NAC Re Corporation

Alice Schroeder, Analyst
Paulsen, Dowling Securities

3. "Outlook on Health Care Reform"

Moderator: Cecily A. Gallagher, Consulting Actuary
Tillinghast/Towers Perrin

Panelists: David Appel, Director of Economic Consulting
Milliman & Robertson, Inc.

Daniel W. McAdams, Jr.
National Practice Leader, Workers' Compensation
Tillinghast/Towers Perrin

4. "Winning in the Public Eye: A Communications Primer for Actuaries"

Moderator: Erich Parker, Director of Public Relations
American Academy of Actuaries

5. "Statement of Opinion Requirements for December 1993"

Moderator: Patrick J. Grannan, Consulting Actuary
Milliman & Robertson, Inc.

Panelists: Terrence M. O'Brien, Partner
Coopers & Lybrand

Richard J. Roth, Jr., Assistant Commissioner
California Department of Insurance

6. "Actuarial Standards Board"

Moderator: LeRoy A. Boison, Jr., Vice President
Insurance Services Office, Inc.

Panelists: Spencer M. Gluck, Consulting Actuary
Milliman & Robertson, Inc.

Gary Grant, Vice President and Actuary
State Farm Fire and Casualty Company

Michael J. Miller, Consulting Actuary
Tillinghast/ Towers Perrin

7. "Benchmarking Corporate Actuarial Departments"

Moderator: Lee R. Steeneck, Vice President
General Reinsurance Corp.

Panelists: Philip N. Ben-Zvi, Executive Partner
Coopers & Lybrand

Charles A. Bryan, Partner
Ernst & Young

8. "Worldwide MIS"

Moderator: Arthur R. Cadorine,
Assistant Vice President and Associate Actuary
Insurance Services Office, Inc.

Panelists: Adrienne B. Kane, Vice President and Actuary
Chubb Group of Insurance Companies

Dominic A. Weber, Vice President and Actuary
Empire Insurance Group

9. "Risk-Based Capital"

Moderator: David G. Hartman,
Senior Vice President and Actuary
Chubb Group of Insurance Companies

Panelists: Paul Braithwaite, Senior Vice President
Zurich Reinsurance Centre, Inc.

Frederick O. Kist, Managing Partner
Coopers & Lybrand

Stephen P. Lowe, Vice President
Tillinghast/Towers Perrin

10. "Insurance Research Council—1992 Auto Liability Closed Claim Study"

Moderator: Gregory L. Hayward, Actuary
State Farm Mutual Automobile Insurance Company

Panelist: Jerry W. Rapp, Consulting Actuary
Tillinghast/Towers Perrin

Donald Segraves, Executive Director
Insurance Research Council

11. "CAS Actuarial Research Corner"

    Moderator: Robert S. Miccolis,
    Senior Vice President and Actuary
    Reliance Reinsurance Corp.

The new *Proceedings* papers presented were:

1. "Asset/Liability Matching (Five Moments)"

   Author: Robert K. Bender,
   Kemper National Insurance Group

2. Author's Reply to a Discussion of "The Competitive Market Equilibrium Risk Load Formula for Increased Limits Ratemaking"

   Author: Glenn G. Meyers,
   Insurance Services Office, Inc.

3. Author's Reply to Discussions of "Risk Loads for Insurers"

   Author: Sholom Feldblum,
   Liberty Mutual Insurance Company

4. A Discussion of "An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance"

   Author: Sholom Feldblum,
   Liberty Mutual Insurance Company

The officers held a reception for new Fellows and their guests from 5:30 p.m. until 6:30 p.m. There was a general reception for all members and guests from 6:30 p.m. until 7:30 p.m.

Tuesday, November 16, 1993

From 8:00 a.m. until 9:30 a.m., simultaneous general sessions were offered.

One general session, "Expert Witness," was led by Mavis A. Walters, Executive Vice President, Insurance Services Office, Inc. Panelists included Martin Brown, Counsel, State Farm Insurance Company; Michael A. LaMonica, Vice President and Actuary, Allstate

Insurance Company; and Martin M. Simons, Chief Casualty Actuary, South Carolina Insurance Department.

The other general session, "Capacity," was moderated by David L. Wasserman, President and Chief Executive Officer, Centre Reinsurance of New York. Panelists were Herbert E. Goodfriend, Director of Insurance Analysis, KPMG Peat Marwick, and Michael Morrissey, Chairman & CEO, Firemark Group.

From 10:00 a.m. until 11:30 a.m., several concurrent sessions were conducted. The panel presentations, in addition to repeats of some of the subjects covered on Monday, were:

1. "Basic Asset Concepts"

   Moderator: James W. Yow,
   Vice President and Corporate Actuary
   Aetna Life & Casualty

   Panelists: Sholom Feldblum,
   Assistant Vice President and Associate Actuary
   Liberty Mutual Insurance Company

   Oakley E. Van Slyke, President
   Oakley E. Van Slyke, Inc.

2. "Pareto Soup"

   Panelists: Clive L. Keatinge, Manager and Associate Actuary
   Insurance Services Office, Inc.

   Glenn G. Meyers, Assistant Vice President, Actuary
   Insurance Services Office, Inc.

3. "Questions and Answers with the CAS Board of Directors"

   Moderator: Irene K. Bass (CAS President-Elect)
   Managing Director
   William M. Mercer, Inc.

   Panelists: Phillip N. Ben-Zvi, Executive Partner
   Coopers & Lybrand

   Michael J. Miller, Consulting Actuary
   Tillinghast/Towers Perrin

   Sheldon Rosenberg, Vice President and Chief Actuary
   Continental Insurance

The *Proceedings* papers presented were:

1. "Estimating Salvage and Subrogation Reserves—Adapting the Bornheutter-Ferguson Approach"

   Author: Gregory S. Grace
   American Reliance Insurance Companies

2. A Discussion of "The California Table L"

   Author: William R. Gillam
   National Council on Compensation Insurance

The afternoon was reserved for committee meetings.

A Country & Western Round-up was held from 6:00 p.m. until 10:00 p.m.

Wednesday, November 17, 1993

From 8:00 a.m. until 9:30 a.m., concurrent sessions were held. Sessions not offered on Monday or Tuesday included the following topics:

1. "The Appointed Actuary"

   Moderator: Allan M. Kaufman, Principal
   Milliman & Robertson, Inc.

   Panelists: Sholom Feldblum,
   Assistant Vice President and Associate Actuary
   Liberty Mutual Insurance Company

   Brian E. Scott, Vice President and
   Senior Corporate Actuary, Aetna Life & Casualty

   Michael A. Walters, Consulting Actuary
   Tillinghast/Towers Perrin

2. "CAS Membership Survey"

   Panelists: Regina M. Berens, Actuarial Director
   Prudential Reinsurance Company

   Deborah M. Rosenberg
   Assistant Chief Casualty Actuary
   New York State Insurance Department

The *Proceedings* papers presented were:

1. "Minimum Distance Estimation of Loss Distributions"

   Author: Stuart A. Klugman and A. Rahulji Parsa, Drake University

2. "Merit Rating for Doctor Professional Liability"

   Author: Robert J. Finger, Milliman & Robertson, Inc.

From 10:00 a.m. until 11:30 a.m., a general session was held on "Affordability/Availability — Public Challenges to Insurance Underwriting and Pricing Practices." Charles A. Bryan, Partner, Ernst & Young, moderated the session. Panelists included Robert Pike, General Counsel, Allstate Insurance Company; Lynn Schubert, General Counsel, American Insurance Association, and Selwyn Whitehead, President, Economic Empowerment Foundation.

After the general session, Mr. Flynn introduced Mr. MacGinnitie, who commented on the AFIR meeting in April, 1994. Mr. Flynn announced the schedule of CAS sponsored or co-sponsored meetings for 1994.

With the transfer of the presidency, Irene Bass closed the meeting at 11:45 a.m.

November 1993 Attendees

In attendance, as indicated by the registration records, were 425 Fellows, and 181 Associates. The list of members' names follows.

FELLOWS

Ralph L. Abell	Clarence R. Atwood	Albert J. Beer
Barbara J. Addie	Guy A. Avagliano	Stephen A. Belden
Marc J. Adee	Robert A. Bailey	Linda L. Bell
Kristen M. Albright	D. Lee Barclay	David M. Bellusci
Mark S. Allaben	Katharine Barnes	Phillip N. Ben-Zvi
Charles M. Angell	Raymond Barrette	Xavier Benarosch
Kenneth Apfel	Irene K. Bass	Robert K. Bender
Nolan E. Asch	Gregory S. Beaulieu	Regina M. Berens
Richard V. Atkinson	Allan R. Becker	James R. Berquist

Lisa M. Besman
John R. Bevan
Richard M. Beverage
Richard A. Bill
James E. Biller
Richard S. Biondi
Gavin C. Blair
Roberto G. Blanco
William H. Bland
Cara M. Blank
Michael P. Blivess
LeRoy A. Boison, Jr.
Martin Bondy
Joseph A. Boor
Ronald L. Bornhuetter
Alicia E. Bowen
David S. Bowen
Wallis A. Boyd
Christopher K. Bozman
John G. Bradshaw, Jr.
Paul Braithwaite
James F. Brannigan
Yaakov B. Brauner
Paul J. Brehm
Mark D. Brissman
Dale L. Brooks
J. Eric Brosius
Charles A. Bryan
Gary S. Bujaucius
George Burger
Patrick J. Burns
Mark J. Cain
John E. Captain
Ruy A. Cardoso
Christopher Carlson
Jeffrey R. Carlson
Sanders B. Cathcart
Michael J. Caulfield
Lisa G. Chanzit
David R. Chernick
James K. Christie
Allan Chuck
Gregory J. Ciezadlo
Mark M. Cis
Howard L. Cohen
Jeffrey R. Cole
Peter J. Collins
Martin L. Couture
Michael D. Covney
David B. Cox
Alan M. Crowe
Patrick J. Crowe
Richard M. Cundy
Kathleen F. Curran
Diana M. Currie
Alan C. Curry
Ronald A. Dahlquist
Robert N. Darby, Jr.
Curtis G. Dean
Manon Debigare
Daniel Demers
Howard V. Dempster
Germain Denoncourt
Michel Dionne
Pierre Dionne
Michael C. Dolan
James L. Dornfeld
Victor G. dos Santos
William F. Dove
John P. Drennan
Eric T. Drummond-Hay
Michael C. Dubin
Diane Symnoski Duda
Timothy B. Duffy
Patrick Dussault
N. Paul Dyck
Myron L. Dye
Richard D. Easton
Dale R. Edlefson
Bob D. Effinger, Jr.
Darrell W. Ehlert
Nancy R. Einck
James Ely
Charles C. Emma
Jeffrey A. Englander
David Engles
Catherine E. Eska
Glenn A. Evans
Philip A. Evensen
James A. Faber
Doreen S. Faga
Richard J. Fallquist
William G. Fanning
Randall A. Farwell
Dennis D. Fasking
Sholom Feldblum
Mark E. Fiebrink
Robert J. Finger
William G. Fitzpatrick
Daniel J. Flaherty
David P. Flynn
James M. Foote
John R. Forney, Jr.
Richard L. Fox
Jacqueline B. Frank
Barry A. Franklin

Patricia A. Furst
Michael Fusco
Scott F. Galiardo
Cecily A. Gallagher
Thomas L. Gallagher
Alice H. Gannon
Andrea Gardner
Roberta J. Garland
Richard J. Gergasko
Thomas L. Ghezzi
John A. Gibson, III
John F. Gibson
Richard N. Gibson
Bruce R. Gifford
William R. Gillam
Bryan C. Gillespie
Richard S. Goldfarb
Charles T. Goldie
Odile Goyer
Susan M. Gozzo
Edward M. Grab
Gregory S. Grace
David J. Grady
Timothy L. Graham
Patrick J. Grannan
Gary Grant
Nancy A. Graves
Ronald E. Greco
Eric L. Greenhill
Cynthia M. Grim
Linda M. Groh
Carleton R. Grose
Denis G. Guenthner
Christy H. Gunn
Larry A. Haefner
David N. Hafling
Allen A. Hall
James A. Hall, III
Malcolm R. Handte
George M. Hansen
David C. Harrison
David G. Hartman
Douglas S. Haseltine
Gordon K. Hay
Gregory L. Hayward
E. LeRoy Heer
Mary R. Hennessy
Dennis R. Henry
Teresa J. Herderick
Richard J. Hertling
Todd J. Hess
David R. Heyman
Kathleen A. Hinds
Alan M. Hines
Mark J. Homan
Carlton W. Honebein
Mary T. Hosford
Beth M. Hostager
Paul E. Hough
Ruth A. Howald
George A. Hroziencik
Robert P. Irvan
David H. Isaac
Russell T. John
Larry D. Johnson
Laura A. Johnson
Thomas S. Johnston
Brian A. Jones
Jeffrey R. Jordan
Edward M. Jovinelly
Adrienne B. Kane
Allan M. Kaufman
Clive L. Keatinge
Wayne S. Keller
Anne E. Kelly
C.K. "Stan" Khury
Changseob J. Kim
Gerald S. Kirschner
Richard O. Kirste
Frederick O. Kist
Joel M. Kleinman
Jerome F. Klenow
John J. Kollar
Gary I. Koupf
Ronald T. Kozlowski
Israel Krakowski
Gustave A. Krause
Rodney E. Kreps
David J. Kretsch
Kenneth R. Krissinger
Jane Jasper Krumrie
John R. Kryczka
Andrew E. Kudera
Michael A. LaMonica
Paul E. Lacko
David A. Lalonde
D. Scott Lamb
Dean K. Lamb
John A. Lamb
Alan E. Lange
Nicholas J. Lannutti
Joseph R. Lebens
Nicholas M. Leccese, Jr.
Robert H. Lee
Steven G. Lehmann
Eric F. Lemieux
Pierre Lepage

Jean-Marc Leveille
Allen Lew
Peter M. Licht
Stephanie J. Lippl
Barry C. Lipton
Richard W. Lo
Jan A. Lommele
Edward P. Lotkowski
William D. Loucks, Jr.
Robert F. Lowe
Stephen P. Lowe
Stephen J. Ludwig
Aileen C. Lyle
W. James MacGinnitie
Mark J. Mahon
Steven D. Marks
Blaine C. Marles
Burton F. Marlowe
Robert W. Matthews
Kevin C. McAllister
Heidi J. McBride
Charles L. McClenahan
Charles W. McConnell
William G. McGovern
Dennis T. McNeese
Dennis C. Mealy
William T. Mech
Robert E. Meyer
Glenn G. Meyers
Robert S. Miccolis
David L. Miller
Mary Frances Miller
Michael J. Miller
Philip D. Miller
William J. Miller
John H. Mize
Frederic James Mohl
Richard B. Moncher
Brian C. Moore
William S. Morgan
Jay B. Morrow
Robert V. Mucci
William F. Murphy
Thomas G. Myers
James R. Neidermyer
Chris E. Nelson
Karen L. Nester
Richard T. Newell, Jr.
Richard W. Nichols
Gary V. Nickerson
William A. Niemczyk
Kathleen C. Nomicos
James W. Noyce
G. Christopher Nyce
Terrence M. O'Brien
Kathy A. Olcese
Joanne M. Ottone
Richard D. Pagnozzi
Robert G. Palm
Gary S. Patrik
Susan J. Patschak
Marc B. Pearl
Sarah L. Petersen
Michael Petrocik
George N. Phillips
Emanuel Pinto
Arthur C. Placek
Michael D. Poe
Jennifer A. Polson
Jeffrey H. Post
Stuart Powers
Philip O. Presley
Ronald D. Pridgeon
John M. Purple
Richard A. Quintano
Albert J. Quirin
Jeffrey C. Raguse
Kay K. Rahardjo
Donald K. Rainey
Andrew J. Rapoport
Jerry W. Rapp
Ralph L. Rathjen
Pamela Sealand Reale
Kurt A. Reichle
Ronald C. Retterath
Kevin B. Robbins
John P. Robertson
Richard D. Robinson
Sharon K. Robinson
Diane R. Rohn
A. Scott Romito
Allen D. Rosenbach
Deborah M. Rosenberg
Sheldon Rosenberg
Gail M. Ross
Richard J. Roth, Jr.
Jean Roy
Stuart G. Sadwin
Yves Saint-Loup
Thomas E. Schadler

Peter J. Schultheiss
Joseph R. Schumi
Brian E. Scott
Mark R. Shapland
Alan R. Sheppard
Harvey A. Sherman
Ollie L. Sherman
Richard E. Sherman
Edward C. Shoop
Jerome J. Siewert
Melvin S. Silver
Christy L. Simon
LeRoy J. Simon
David Skurnick
Christopher M. Smerald
David A. Smith
Richard A. Smith
Richard H. Snader
Linda D. Snook
David B. Sommer
Joanne S. Spalla
Bruce R. Spidell
Daniel L. Splitt
Elisabeth Stadler
Stephen D. Stayton
Lee R. Steeneck
Elton A. Stephenson
Edward C. Stone
Douglas N. Strommen
Stuart B. Suchoff
Russel L. Sutter
John A. Swift
Susan T. Szkoda
Frank C. Taylor
Michael T.S. Teng
Kathleen W. Terrill
Karen F. Terry
Margaret Wilkinson Tiller
Michael L. Toothman
Warren B. Tucker
Gail E. Tverberg
Jean Vaillancourt
William R. Van Ark
Oakley E. Van Slyke
Anne-Marie Vanier
Gary G. Venter
Ricardo Verges
Jennifer A. Violette
Gerald R. Visintine
Steven M. Visner
Joseph L. Volponi
William J. VonSeggern
Robert H. Wainscott
Thomas A. Wallace
Albert J. Walsh
Mavis A. Walters
Michael A. Walters
Patrick M. Walton
Bryan C. Ware
Thomas V. Warthen, III
David L. Wasserman
Nina H. Webb
Dominic A. Weber
Patricia J. Webster
Marjorie C. Weinstein
Peter A. Weisenberger
John P. Welch
Jonathan White
Kevin Wick
Peter W. Wildman
Teresa J. Williams
Ernest I. Wilson
James C. Wilson
Martha A. Winslow
Michael L. Wiseman
David A. Withers
Susan K. Woerner
Richard G. Woll
Patrick B. Woods
Paul E. Wulterkens
Chung-Ye Yen
Hank Youngerman
Heather E. Yow
James W. Yow
John D. Zicarelli

ASSOCIATES

Jonathan D. Adkisson
Scott C. Anderson
Michael E. Angelina
Bhim D. Asdhir
Martha E. Ashman
Joanne Balling
Ina M. Becraft
Douglas S. Benedict
Jennifer L. Biggs
Gary Blumsohn
Thomas S. Boardman
Ann M. Bok
Maurice P. Bouffard
George P. Bradley
John O. Brahmer
Tracy L. Brooks-Szegda
Richard S. Brutto
William E. Burns
Arthur R. Cadorine
Kenrick A. Campbell
Michael E. Carpenter
Michael W. Cash
Tania J. Cassell
Carol A. Cavaliere
Paul A. Chabarek
Debra S. Charlop
Gary T. Ciardiello
David G. Clark
Kay A. Cleary
Michael A. Coca
Jo Ellen Cockley
Vincent P. Connor
Daniel A. Crifo
Michael T. Curtis
Joyce A. Dallessio
Michael K. Daly
James R. Davis
Jeffrey F. Deigl
William Der
Gordon F. Diss
David A. Doe
Frank H. Douglas
Thomas R. Fauerbach
Denise A. Feder
Kendra M. Felisky-Watson
David N. Fields
Ross C. Fonticella
Howard H. Friedman
Mary B. Gaillard
Kim B. Garland
Felix R. Gerard
Scott B. Gerlach
Donna L. Glenn
Ronald E. Glenn
Terry L. Goldberg
Russell H. Greig
Ewa Gutman
Richard J. Haines
Leigh J. Halliwell
Robin A. Harbage
Jonathan M. Harbus
Robert L. Harnatkiewicz
Christopher L. Harris
Barton W. Hedges
Renee Helou
Joseph P. Henkes
Paul D. Henning
Joseph A. Herbers
David D. Hu
Jeffrey R. Hughes
Paul R. Hussian
Sadagopan S. Iyengar
John J. Javaruski
Daniel J. Johnston
James W. Jonske
Janet S. Katz
David L. Kaufman
Michael F. Klein
James J. Kleinberg
Timothy F. Koester
Adam J. Kreuser
Jason A. Kundrot
David R. Kunze
James W. Larkin
David L. Larson
Michael D. Larson
Robert J. Larson
Carl J. Leo
David R. Lesieur
Sam F. Licitra
Shu C. Lin
Joseph R. Liuzzi
Paul R. Livingstone
Barry I. Llewellyn
William G. Main
Sudershan Malik
Donald E. Manis
Richard J. Marcks
Lawrence F. Marcus

Sharon L. Markowski
Dee Dee Mays
John W. McCutcheon, Jr.
Richard T. McDonald
Stephen J. McGee
Eugene McGovern
Donald R. McKay
James P. McNichols
Stephen V. Merkey
Linda K. Miller
Douglas H. Min
Madan L. Mittal
Stanley K. Miyao
Andrew W. Moody
Stephen T. Morgan
Raymond D. Muller
Kimberly J. Mullins
Donald R. Musante
Rade T. Musulin
John K. Nelson
Henry E. Newman
Kwok C. Ng
Keith R. Nystrom
Leigh S. Oates
Dale F. Ogden
Douglas J. Onnen
Timothy A. Paddock
Teresa K. Paffenback
Willard W. Peacock
Joseph W. Pitts
Ruth Youngner Poutanen
Yves Provencher
Regina M. Puglisi
Cathy A. Puleo
R. Stephen Pulis
Eric K. Rabenold
James E. Rech
Donna J. Reed
Steven J. Regnier
Ellen J. Respler
Al J. Rhodes
Sallie S. Robinson
David A. Rosenzweig
Kevin L. Russell
Maureen S. Ruth
Robert M. Sandler
Michael Sansevero, Jr.
Melodee J. Saunders
Susan C. Schoenberger
Peter R. Schwanke
Michael L. Scruggs
Martin M. Simons
Rial R. Simons
Byron W. Smith
David C. Snow
Calvin C. Spence, Jr.
Victoria G. Stachowski
John P. Stefanek
Elissa M. Sturm
Scott J. Swanay
R. Glenn Taylor
Joseph O. Thorne
Charles F. Toney, II
Darvin A. Torgrimson
David B. Van Koevering
John J. Varca
Kimberley A. Ward
Stephen D. Warfel
James D. Watford
Robert A. Weber
Russell B. Wenitsky
Alan E. Wickman
Gayle L. Wiener
Oliver T. Wilson
William F. Wilson
Calvin Wolcott
Robert F. Wolf
Vincent F. Yezzi
Edward J. Yorty
Sheng Hau Yu
Ronald J. Zaleski
Barry C. Zurbuchen

REPORT OF THE VICE PRESIDENT-ADMINISTRATION

The purpose of this annual report is to provide the membership with a brief overview of CAS activities that have occurred since the last annual meeting.

For purposes of this year's report, I have chosen to first cover the activities that supported the three significant continuing themes that President Flynn identified in his February 1993 *Actuarial Review* article. I'll then briefly outline other significant activities and close with a summary of the current state of the CAS.

The first of David P. Flynn's themes was to continue the efforts to build on the existing structures of professionalism. Toward this end, the formal legal review of the CAS bylaws and procedures was completed and the Board adopted the recommended Antitrust Compliance Policy. A revised working agreement among the actuarial organizations in the U.S. and Canada was reviewed and approved. Also, a revised Common Code of Professional Conduct was adopted and will become effective on January 1, 1994. This document, prepared at the direction of the Council of Presidents, will harmonize differences among the codes of the various U.S-based actuarial organizations.

Following establishment of the Actuarial Board for Counseling and Discipline (ABCD) rules of procedure in June, the CAS Discipline Committee prepared their own Rules of Procedure and Operating Guidelines which were reviewed and adopted at the November 14 Board meeting. These rules of procedure will govern the consideration of recommendations for public disciplinary action that will come from the ABCD. Of course, the CAS Board of Directors will continue to have ultimate responsibility for any disciplinary actions taken.

Focusing on prospective members, the CAS Board also adopted a policy on examination discipline. In addition, the course on professionalism, which provides training on ethics and standards, was given six times during the year at five different locations with 279 students in attendance.

The second theme revolved around preparing ourselves and our students for the future. The major initiative in this area was the Appointed Actuary Task Force, chaired by Robert A. Miller, III. The Task Force's report, which was formally presented at the May Board meeting, listed 51 recommendations for actions required to prepare CAS members to function as appointed actuaries. The report identified needs in the key areas of organization, education, research, and responsibility.

The Board, acting on the Task Force's recommendation on organization, has approved the creation of an Advisory Committee that will report to the President-Elect and oversee implementation of the Appointed Actuary concept. This committee will be composed of a small number of senior actuaries, responsible for reviewing the implementation projects, priorities, and schedules, and counseling the President-Elect regarding appropriate changes.

The most immediate needs identified by the Task Force are in the areas of research and continuing education. A master plan for goals in both of these areas is being developed and was reported on by Allan M. Kaufman at the November 14 Board meeting.

Ongoing communication to the membership and others will be critical to the success of the Appointed Actuary concept. CAS members should look forward to frequent updates on activities during the upcoming year.

Another future-related initiative that took place during the year was the Membership Survey. This survey was intended to reassess the needs and attitudes of CAS members via a detailed questionnaire patterned after the original survey first taken in 1987. Input was gathered on members' views regarding the future educational, organizational, professional and other needs of the CAS.

The Membership Survey Task Force, chaired by Regina M. Berens, presented its final report to the Board on November 14, 1993. A summary of the survey results will be distributed to the membership, and a session is being held in Phoenix to go over the results with attendees. The results and conclusions from the survey are already being considered in the development of committee goals for the upcoming year.

The third theme addressed the continued enhancement of the contributions of the CAS office toward a more efficiently operating Society. A number of strides were made in this area with the most significant being the completion of Phase I of the MIS project. Office capabilities are now significantly enhanced in the areas of meeting registration and examination support. Detailed exam histories are now available in the database in order to evaluate travel time or project exam sittings. The new meetings module was utilized for this meeting and allowed us to replace outside vendor services.

The presence of a full-time staff editor for the entire year led to further enhancements to our various publications. Printing and mailing costs have been reduced, while the overall quality and timeliness have improved.

In general, more and more of the day-to-day administrative functions formerly performed by CAS committees are now handled by the office staff. It's a trend that will continue in 1994.

There were a number of other significant activities for the CAS during 1993.

The first full audit of the CAS financial records and procedures was completed in early 1993. You'll be pleased to know that we received an unqualified opinion on our finances and that there were only a few audit recommendations, all of which have now been implemented.

One of the most significant changes adopted was to revise the role of the audit committee to support a financial review process that provides for an annual audit of the CAS books by an outside CPA firm. A new audit committee structure, which will include at least two Board members, has been established. The 1994 committee will be chaired by Sheldon Rosenberg.

The year-end financial statement, prepared in accordance with the CPA's recommendations, is attached to this report. An article on the audit and its recommendations will appear in the February 1994 issue of the *Actuarial Review*.

Continuing Education opportunities provided by the CAS this year were numerous and varied. In addition to the spring meeting in Dallas and the annual meeting in Phoenix, a number of other educational seminars were held including:

- The 1993 Seminar on Ratemaking, held in Arlington, Virginia, had 631 registrants;
- The Casualty Loss Reserve Seminar in Orlando, Florida, of which the CAS is a co-sponsor, was attended by 752;
- The special interest Seminar on Underwriting in April was attended by 48;
- The special interest Seminar on Dynamic Solvency held October 1993 in Newark, New Jersey, was attended by 158;
- The Reinsurance Seminar in June attracted 280 attendees to Boston; and
- The P&C Insurance Liability Seminar in Montreal, sponsored by the Canadian Institute of Actuaries and the CAS, was attended by 200.

There were also numerous research efforts during the year in our continuing effort to expand the existing body of knowledge for education. Research projects are in progress or being planned on risk margins, variability in loss ratios and loss reserves, and appraisal techniques. A call for papers on Environmental Liability went out in September, while a discussion draft of Risk Classification Principles was released this summer. The Management Data and Information Committee surveyed the members and is planning a follow-up seminar.

The Theory of Risk Prize Paper Program was successfully completed and it is expected that a publication including roughly 10 papers will be released in 1994.

Publications released in 1993 included the 1992 *Proceedings*, a special edition of the *Forum* for the 1993 Ratemaking Call Papers, the *1993 Discussion Paper Program* on the Actuary as Business Manager, and the summer *Forum*.

Several key activities took place with respect to admissions. A task force was created to develop a system for evaluating the success of exam partitioning. Working with the CAS office, they have created the necessary database and system to enable us to measure travel time and other related parameters. An exam waiver policy for foreign actuaries was developed and approved by the Board in November. In addition, the ongoing process of exam administration continued as over 6,000 candidates registered for our exams this year.

Efforts in the international arena also continued. Our leadership continued its active involvement in the McCrossan Group discussions. The CAS will co-sponsor the AFIR conference next April. An exam waiver policy with the Institute of Actuaries has been approved for CAS members and students.

Lastly, a brief status of our membership and financial condition: our size continued its rapid increase as we added 160 new Associates, and 96 new Fellows were named. Our membership now stands at 2,083.

Your Board of Directors met four times during 1993. New members elected to the Board for next year include Steven F. Goldberg, Patrick J. Grannan, Anne E. Kelly, and Robert S. Miccolis. The membership elected Allan M. Kaufman to the position of President-Elect, while Irene K. Bass will assume the presidency.

The Executive Council, with primary responsibility for day-to-day operations, met either by teleconference or in person at least once a month during the year. The Board of Directors elected the following Vice Presidents for the coming year:

> Vice President-Administration, John M. Purple; Vice President-Admissions, John J. Kollar; Vice President-Continuing Education, David N. Hafling; Vice President-Programs and

Communications, Alice H. Gannon; Vice President-Research and Development, Michael J. Miller.

In closing, here are some comments on our financial status. The CPA firm of Feddeman & Company examined the CAS books for fiscal year 1993 and found the accounts to be properly stated. The fiscal year ended with net income of $73,620 which compares favorably to a budgeted amount of approximately $44,000. Members' equity now stands at $1,050,650, subdivided as follows:

Michelbacher Fund	$85,336
Dorweiler Fund	6,624
CAS Trust	3,208
Scholarship Fund	7,715
Rodermund Fund	14,894
CLRS Fund	5,000
Research Fund	97,665
CAS Surplus	830,207
TOTAL MEMBERS' EQUITY	$ 1,050,650

This represents an increase in equity of $99,425 over the audited amount last year. There were several accounting changes from the fiscal year 1992 and fiscal year 1993 financial audits that resulted in a surplus increase of $243,766.

For 1993-94, the Board of Directors has approved a budget of approximately $2.2 million.

Members' dues for next year will be $240, an increase of $10, while fees for the invitational program will increase by $15 to $290. Examination fees for Parts 4 through 10 will remain the same.

Respectfully submitted,

JOHN M. PURPLE

Vice President-Administration

November 15, 1993

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/93

OPERATING RESULTS BY FUNCTION

FUNCTION	INCOME	EXPENSE	DIFFERENCE
Member Services	$ 496,983	$ 700,312	($ 203,329)
Seminars	505,357	316,754	188,603
Meetings	436,693	443,038	(6,345)
Exams	732,458	600,198	132,260
Publications	61,096	61,699	(603)
TOTAL	$ 2,232,587	$ 2,122,001	$ 110,586*

*Note: Change in CAS Surplus before interfund transfers

BALANCE SHEET

ASSETS	9/30/92	9/30/93	DIFFERENCE
Checking Account	$ 255,199	$ 165,981	($ 89,218)
T-Bills, Notes	972,768	1,084,207	111,439
Accrued Interest	12,348	16,421	4,073
CLRS Deposit	5,000	5,000	0
Prepaid Expenses	20,225	17,383	(2,842)
Account Receivable	50,000	50,000	0
Computers, Furn.	185,370	223,533	38,163
Less: Acc. Deprec.	(53,309)	(84,770)	(31,461)
TOTAL ASSETS	$ 1,447,601	$ 1,477,754	$ 30,153
LIABILITIES			
Exam Fees Deferred	$ 278,507	$ 236,765	($ 41,742)
Nov. Mtg. Deferred	84,332	61,563	(22,769)
Sem. Fees Deferred	40,434	42,100	1,666
Subscriber Programs	3,925	624	(3,301)
Accounts Payable	24,173	20,967	(3,206)
Deferred Rent	60,510	53,145	(7,365)
Accrued Pension	4,495	11,940	7,445
TOTAL LIABILITIES	$ 496,376	$ 427,104	($ 69,272)
MEMBERS' EQUITY			
Michelbacher Fund	$ 83,895	$ 85,336	$ 1,441
Dorweiler Fund	7,402	6,624	(778)
CAS Trust	3,115	3,208	93
Scholarship Fund	7,976	7,715	(261)
Rodermund Fund	15,551	14,894	(657)
CLRS Fund	5,000	5,000	0
Research Fund	58,665	97,665	39,000
CAS Surplus	769,621	830,207	60,586
TOTAL EQUITY	$ 951,225	$ 1,050,650	$ 99,425

John M. Purple, Vice President-Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

Audit Committee: William J. Rowland, Chairman; Anthony J. Grippa, Albert J. Quirin, Charles Walter Steward, Russel L. Sutter

1993 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 3B, 4A, 4B, 6, 8, 8C (Canadian), and 10 of the Casualty Actuarial Society were held on May 3, 4, 5, 6, and 7. Examinations for Parts 3B, 4A, 4B, 5A, 5B, 7, and 9 were held November 1, 3, 4, and 5.

Examinations for Parts 1, 2, 3A and 3C (SOA courses 100, 110, 120, and 135) are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. Parts 1 and 2 were given in February, May, and November of 1993, and Parts 3A and 3C were given in May and November of 1993. Candidates who were successful on these examinations were listed in joint releases of the two societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the Part 1 examination.

For the February 1993 examination, the $200 first prize was awarded to Harry T. Pearce. The $100 prize winners were James L. Auld, Keeheng Ng, Ho L. Ng, and Alexander Volokh.

For the May 1993 examination, three $200 first prizes were awarded to Chiung M. Chen, Emmanuel Montini, and Hong Sheng. The $100 prize winners were Ibrahim Abdullah, Brian C. Alvers, Mei-Yu Chen, Tsu-Yueh Shueh, and Kao-Yung Shih.

For the November 1993 examination, the $200 first prize was awarded to Michael C. Rotkowicz. The $100 prize winners were Aaron T. Bono, John W. Slipp, Walter Sun, and Ru-Fang Yeh.

The following candidates were admitted as Fellows and Associates at the May 1993 meeting as a result of their successful completion of the Society requirements in the November 1992 examinations.

FELLOWS

Bruno P. Bauer	Francois Dumas	James W. Haidu
Martin L. Couture	Bradley C. Eastwood	Joanne K. Ikeda
Kevin G. Dickson	James E. Fletcher	Gordon L. Scott
Michel Dionne	Louis Gariepy	

ASSOCIATES

Rhonda K. Aikens
Craig A. Allen
Scott C. Anderson
William P. Ayres
Timothy J. Banick
Philip A. Baum
John A. Beckman
Douglas S. Benedict
Richard F. Burt, Jr.
John F. Butcher, II
Michael E. Carpenter
Benoit Carrier
Michael T. Curtis
David J. Darby
Karen L. Davies
Marie-Julie Demers
Shawn F. Doherty
Ronald R. Earls
Matthew G. Fay
John R. Ferrara
George Fescos
Kai Y. Fung
James E. Gant
Mary K. Gise
Donna L. Glenn
Marc C. Grandisson
Bradley A. Granger
Paul J. Hancock
Timothy J. Hansen
Matthew T. Hayden
Lisa A. Hays
Barton W. Hedges
Noel M. Hehr
Mary B. Hemerick
Thomas H. Highet
Bernard R. Horovitz
Vincent H. Jackson
Patrick C. Jensen
Kurt J. Johnson
Mark R. Johnson
Steven A. Kelner
Joseph P. Kilroy
Craig W. Kliethermes
Terry A. Knull
Elizabeth Kolber
Howard A. Kunst
David L. Larson
Michel Laurin
Thomas L. Lee
Scott J. Lefkowitz
Elizabeth A. Lemaster
Deanne C. Lenhardt
Richard S. Light
Daniel J. Mainka
Stephen N. Maratea
Kelly J. Mathson
Robert D. McCarthy
Richard T. McDonald
Conrad O. Membrino
Paul A. Mestelle
Michelle M. Morrow
Timothy O. Muzzey
David Y. Na
Mark Naigles
Peter M. Nonken
Melinda H. Oosten
Nathalie Ouellet
Charles C. Pearl, Jr.
Edward F. Peck
Karen L. Pehrson
Daniel C. Pickens
Cathy A. Puleo
Eduard J. Pulkstenis
Mark S. Quigley
Frank J. Rau, Jr.
Thomas O. Rau
Andrew T. Rippert
James J. Romanowski
James B. Rowland
Kenneth W. Rupert, Jr.
James V. Russell
Stephen Paul Sauthoff
Letitia M. Saylor
Michael B. Schenk
Suzanne E. Schoo
Jeffrey S. Sirkin
Michael J. Steward, II
Brian M. Stoll
Katie Suljak
Todd D. Tabor
Christopher Tait
Yuan-Yuan Tang
Patrick N. Tures
Charles E. Van Kampen
Marcia C. Williams
William M. Wilt
John S. Wright
Gerald T. Yeung
Claude D. Yoder
Barry C. Zurbuchen

The following is a list of successful candidates in examinations held in May 1993.

Part 3B

Iliana Adamidis
Scott J. Altstadt
Michelle L. Andrew
Andrew F. Anschell
Mohammed Q. Ashab
Scott P. Augutis
Richard J. Babel
Maureen A. Barnes-Kellman
Cheryl L. Barnett
Thomas C. Bates
Stephanie N. Baum
Lisa A. Baynon
Darryl R. Benjamin
David C. Benton
Sarah J. Billings
Mario Binetti
Kevin M. Bingham
Cindy M. Bloemer
Gary Blumsohn
Daniel R. Boerboom
Thomas S. Botsko
Joel L. Braatz
Travis L. Brank
Linda M. Brockmeier
Robert Lindsay Brown
David V. Bruce
Ron Brusky
Hugh E. Burgess
Alan Burns
Brian P. Bush
Michael W. Cash
Jean-Francois Chalifoux
Nathalie Charbonneau
Jean-Francois Charest
Joyce Chen
Joyce C. Chen
Heng Seong Cho
Christopher J. Claus
Jeffrey J. Clinch
Kimberly S. Coles
Nancy J. Collings
Elizabeth J. Conley
Peter J. Cooper
Edgar Corredor
Greg M. Costelloe
Jose R. Couret
Stephen M. Couzens
Angela T. Cuonzo
Kendra S. Cupp
Mujtaba H. Datoo
Allison J. Dekker
John T. Devereux
Peter F. Drogan
Raymond S. Dugue
Sophie Dulude
Sally C. Dunlap
Ruchira Dutta
Kevin M. Dyke
Robert E. Farnam
John D. Ferraro
Benedick Fidlow
Stephen C. Fiete
Brenda L. Finlen
Chauncey E. Fleetwood
Candy Ann Flynn
Jack A. Frank
Walter H. Fransen
Julie R. Fregeau
Timothy J. Friers
James M. Gallagher
Paul Gauthier
Theresa Giunta
David Patrick Glenn
Lynn E. Golas
Elizabeth A. Grande
Daniel E. Greer
Marc S. Hall
Lynne M. Halliwell
Barbara Hallock
Marlene M. Hardison
Bernadette M. Hare
Michael S. Harrington
Shrinivas Havaldar
Sonja M. Heiberg
Peter A. Heinrichs
Ronald J. Herrig
Thomas E. Hettinger
Carrie L. Higgins
Stephen J. Higgins, Jr.

Part 3B (cont'd)

Martin M. Houlihan
David D. Hudson
Elizabeth J. Hudson
Julie A. Hungerford
Mangyu Hur
Rusty A. Husted
Christopher R. Jarvis
Philip J. Jennings
Paul J. Johnson
Derek A. Jones
Yuhanis Kamil
Panayotis N. Karambelas
Robert B. Katzman
Jennifer Kelly
Scott A. Kelly
Ruta V. Kher
Linda I. Kierenia
Debra L. Kocour
Jonelle A. Kohne
Linda Kong
Sarah Krutov
Julia M. Kuks
Jean-Sebastien Lagarde
Jin-Mei J. Lai
Timothy J. Landick
Marc LaPalme
James C. Lastinger
Normand Lavallee
Sue Jean Lee
Robin E. Lemke
Karen A. Lerner
Philip Lew
Jacqueline M. Lewis
Eric F. Liland
Christina Link
William F. Loyd
Kenneth T. Lui
William R. Maag
Alexander P. Maizys
Laura S. Marin
Archibald G. Mattis
Sarah K. McNair-Grove
Scott A. McPhee
Kathleen C. Miller
Stephen A. Moffett
Quynh-Nhu T. Morse
Gwendolyn D. Moyer
Sherry L. Mueller
Mihaela L. O'Leary
Colleen M. Ohle
Leo M. Orth, Jr.
Paul S. Osborn
David A. Ostrowski
David J. Otto
Alan M. Pakula
Rebecca W. Palmer
Jennifer L. Paris
Moshe C. Pascher
Nicholas H. Pastor
Carole K. Payne
Jeremy P. Pecora
Leslie C. Pelecovich
Miriam E. Perkins
Luba Pesis
Andrea L. Phillips
Anthony G. Phillips
Lind R. Pratt
Edward R. Press
Mark Priven
Ni Qin-Feng
Amy M. Quinn
Robert Rachlow
Daniel D. Rath
Jennifer M. Rath
James J. Rehbit
Dean R. Reigner
Ellen K. Rein
Juan V. Restrepo
Al J. Rhodes
Rebecca J. Richard
Lian Z. Rohsner
Brian P. Rucci
Julie C. Russell
John C. Ruth
Catherine L. Ryan
Michelle R. Safiran
Romel G. Salam
Daniel V. Scala
Raymond G. Scannapieco
Gregory J. Schoener
Jeffery W. Scholl
Peter R. Schwanke
William H. Scully, III
Ronald G. Sevold
Kevin H. Shang
Glenn D. Shippey
Maria Shlyankevich
Gregory M. Smith

Part 3B (cont'd)

Laura Smith
George D. Sparks
Sandra L. Spiroff
Aaron J. Srugis
Beth A. Stahelin
Nathan R. Stein
Curt A. Stewart
Marion C. Stone
Joy M. Suh
Adam M. Swartz
Josephine L. C. Tan
Michel Theberge
Patricia Therrien
Abraham Thomas
Kai L. Tse
Arthur J. Turner
Steven J. Vercellini
Lidia E. Villasenor
Janet K. Vollmert
Douglas M. Warner
Patricia A. Warrington
Keith M. Waskom
Matthew J. Wasta
Brent G. Watson
William H. Watson, III
Kelly M. Weber
Christopher B. Wei
Dean A. Westpfahl
Trevar K. Withers
Amy M. Wixon
Brandon L. Wolf
Donald S. Wroe
Wei Wu
Mindy Yu
Robin Zinger
Eric E. Zlochevsky

Part 4A

Michael D. Adams
Anthony L. Alfieri
Kristine M. Anderson
Larry D. Anderson
Robert J. Anderson
Richard J. Babel
Phillip W. Banet
Christine Landon Barker
Frank J. Barnes
Keith M. Barnes
Karen L. Barrett
Victoria A. Beltz
Bruce J. Bergeron
Michael J. Bluzer
Christina M. Bond
Hobart F. Bond, III
Caleb M. Bonds
Lloyd J. Bouchard
Pierre Boucher
Lee M. Bowron
Douglas J. Bradac
Patrice Brassard
Margaret A. Brinkmann
Jeffery M. Brobjorg
Linda M. Brockmeier
Audrey W. Broderick
Ron Brusky
Elise S. Burns
Anthony R. Bustillo
Pamela J. Cagney
James E. Calton
Ann Marie L. Cariglia
William Brent Carr
Heather L. Chalfant
Sharon L. Chapman
Joyce C. Chen
Sigen Chen
David M. Cherniawsky
Kathy A. Christensen
Lori Anne Cieri
Laura R. Claude
Frank S. Conde
Matthew D. Corwin
Sharon A. Crosson
Christopher G. Cunniff
Malcolm H. Curry
Sheri De la Boursodiere
John D. Deacon
Raymond V. DeJaco
Michael B. Delvaux
Raymond Demers
Dina M. Deschino
Giuseppe C. Di Tullio
Kelly D. Dickens
Gayle L. Dittrich

Part 4A (cont'd)

Sara P. Drexler
Jennifer S. Ebert
Annette M. Eckhardt
Donna L. Emmerling
Linda S. Eveland
Tracy M. Fleck
Joyce M. Frank
Mark A. Fretwurst
J'ne E. Furrow
Isabelle Gaumond
Michael L. George
Barry A. Gertschen
Neil P. Gibbons
James B. Gilbert
Serge Girard
Allen J. Gould
David J. Gronski
Phil D. Haddad
Steven K. Haine
Scott T. Hallworth
Kenneth J. Hammell
Brian D. Haney
Joel D. Hanson
David S. Harris
Lise A. Hasegawa
Jason B. Heissler
Daniel J. Henderson
Daniel F. Henke
Shohreh Heshmati
Jay T. Hieb
Barbara A. Higdon
David D. Hudson
Philip M. Imm
Brian L. Ingle
Henry J. Itri
Sadagopan S. Iyengar
Jean-Claude J. Jacob
James B. Kahn
Daniel R. Kamen
Robert B. Katzman
Claudine H. Kazanecki
James M. Kelly
Michael D. Kemp
Thomas P. Kenia
Young Y. Kim
Deborah M. King
James F. King
Diane L. Kinner
Wendy A. Knopf
Paul W. Kollner
Brian S. Krick
Sarah Krutov
John R. Kunstman
Steven M. Lacke
Jean-Sebastien Lagace
Ravikumar Lakshminarayan
Karen M. Lancour
Douglas W. Latimer
Manuel Alberto T. Leal
Guy Lecours
Kevin A. Lee
Neal M. Leibowitz
Maria T. Leonard
Chu-Minn Leu
Adrienne Levine
John N. Levy
Cheng-Te Liang
Janet G. Lindstrom
Michael Lipkin
Richard B. Lord
Laura J. Lothschutz
Michael B. Love
Kenneth T. Lui
Sak-Man Luk
Xinhong Luo
Susan I. Lynch
Betsy F. Maniloff
William J. Mantemach
James P. Mathews
Laura A. Maxwell
Robert B. McCleish, IV
Deborah L. McCrary
Kathleen A. McMonigle
Jeffrey S. McSweeney
William E. McWithey
James R. Merz
Richard E. Meuret
Alison M. Milford
Lando M. Milligan
Michael J. Miraglia
Mark J. Moitoso
Matthew S. Mrozek
Kimberly J. Mullins
Charles P. Neeson
Jennifer L. Nelson
Michael D. Neubauer

Part 4A (cont'd)

Tina T. Ni
Chris M. Norman
James L. Nutting
Mark A. O'Brien
Mihaela L. O'Leary
Andres F. Ochoa-Gomez
Josephine M. Oliver
Richard D. Olsen
David A. Ostrowski
Serge A. Ouellette
Michael G. Owen
Dmitry Papush
James A. Partridge
Curtis D. Pederson
Bruce G. Pendergast
John M. Pergrossi
Miriam E. Perkins
Sylvain Perrier
Mary K. Plassmeyer
Darlene Pogrebinsky
Josee Pomerleau
Aleksey Popelyukhin
Christopher J. Poteet
Michael D. Price
Warren T. Printz
Julie Privman
Lewis R. Pulliam
Patricia A. Pyle
Jacqueline M. Ramberger
Scott Reynolds
Brad M. Ritter
Charles J. Ryherd
Shama S. Sabade
Samuel E. Sackey
Romel G. Salam
Suzanne Salvatori
Robert Sanche
James C. Sandor
Steven M. Schienvar
Matt J. Schmitt
Michael C. Schmitz
Terry M. Seckel
Joyce E. Segall-Lopez
Darrel W. Senior
Anastasios Serafim
Jennifer M. Shantz
Kelli D. Shepard-El
Jeffrey P. Shirazi
Paul O. Shupe
Raleigh R. Skaggs, Jr.
Jeffery J. Smith
Lori A. Snyder
Jay M. South
Klayton N. Southwood
Christine L. Steele-Koffke
Christopher M. Steinbach
Barry P. Steinberg
Curt A. Stewart
Jayme P. Stubitz
Joy M. Suh
Jay M. Sussman
Clifford Steven Swalley
Duc M. Ta
Michael J. Tempesta
Glenda O. Tennis
Harlan H. Thacker
Paul W. Thorpe, III
Ruijue Tong
Huong Thi Tran
Laura M. Turner
Valerie J. Vandewetering
Edward H. Wagner
Benjamin A. Walden
Joseph W. Wallen
Robert J. Walling, III
Isabelle T. Wang
Kimberley A. Ward
Linda F. Ward
Keith M. Waskom
Todd A. Weber
Petra L. Wegerich
Joel D. Whitcraft
Jeffrey D. White
Matthew M. White
Trevar K. Withers
Brandon L. Wolf
Chi-Chih Woo
Virginia R. Young
Benny S. Yuen
Darci L. Zelenak
Edward J. Zonenberg
Paul W. Zotti

Part 4B

Larry D. Anderson
Mark B. Anderson
Jean-Francois Angers
Mario G. Arguello
Mohammed Q. Ashab
Karen N. Babcock
Karen L. Babitt
Amy L. Baranek
Karen L. Barrett
Christopher N. Bartholow
Mary P. Bayer
Michel Bazinet
Christopher Beke
Alvin F. Beltramo
Joseph M. Bernardi
Wayne F. Berner
Frank J. Bilotti
Mariano R. Blanco
Ming Y. Blinn
Daniel E. Block
Carol A. Blomstrom
Josee Bolduc
Donna M. Bono
Winfred N. Botchway
Lloyd J. Bouchard
Pierre Boucher
Martin Bourassa
Michael D. Brannon
Kirk P. Braunius
Margaret A. Brinkmann
Stephane Brisson
Robert F. Brown
Kirsten R. Brumley
Keith A. Bucich
Mark E. Burgess
Sandra J. Callanan
Heather L. Chalfant
Ching-Jen Chang
Rosvita L.J. Chang
Sonia Chatigny
Alice M. Cheung
Joey-Margaret Christner
John Clara
Alan R. Clark
Derek A. Clark
Sally M. Cohen
Kimberly S. Coles
Patricia M. Coless
Kevin A. Cormier
Brian C. Cornelison
Michael H. Crawford
Paul T. Cucchiara
Marc-Andre Dallaire
Amos R. Darrisaw
Steven F. Delfino
Sharon D. Devanna
Jocelyn Dion
Kenneth R. Dipierro
Louis G. Doray
Michael E. Doyle
Stephanie S. Dubose
Charles P. Dugas
Sophie Duval
Thomas J. Dwyer
Kevin M. Dyke
Wayne W. Edwards
David J. Englemayer
Thomas L. Fagan
Fang-Yuan Fan
Gregory G. Fann
Denise M. Farnan
David A. Fennell
Stephen C. Fiete
Tracy M. Fleck
David M. Flitman
Tracy L. Fogel
Sy Foguel
Sally M. Forsythe
Christian Fournier
David Fournier
Mark A. Fretwurst
Shina N. Fritz
Patricia A. Galeazza
Christian Gaouette
Brad P. Gardner
Nicolas Genois
Derek M. Gerard
Eric J. Gesick
Jie Gong
Allen J. Gould
Jennifer Graunas
Caroline Gregoire
Christopher G. Gross
Stephen J. Gruber
Neil E. Gundel
Alessandrea C. Handley
Susan J. Heinzelman
Chi Yiu Ho
Laura K. Hobart
Christopher T. Hochhausler

Part 4B (cont'd)

Brett Horoff
Hsienwu Hsu
Jen-Hsiao Huang
Zhen Huang
Hsiao-Hsia A. Hung
Chain-Yea Monica In
Naveed Irshad
Joseph M. Izzo
Suzanne M. James
Stephen L. Jauss
Brian E. Johnson
Daniel K. Johnson
Jill C. Johnson
Charles N. Kasmer
Craig B. Keizur
Scott A. Kelly
William J. Keros
Ruta V. Kher
Chiao-hwa Kiang
He-Jung Kim
John H. Kim
Ung M. Kim
James F. King
Alexander E. Kirimov
Jason T. Klawonn
John P. Kmetic
Keat-Ling Koay
Elina L. Koganski
Robert A. Kranz
Brian S. Krick
Arunod Kumar
Bobb J. Lackey
Jocelyn Laflamme
Michel Lafrance
Jean-Sebastien Lagarde
Yaohsien Lai
Michel Lalonde
Laura L. Lankin
Jean-Francois Larochelle
Sebastien Lauziere
Guy Lecours
Betty F. Lee
Rebecca A. Leeb
Robert Leikums
Christiane Lemieux
Charles R. Lenz
Chu-Minn Leu
Philip Lew
Michael Leybov
Xiaoying Liang
Han-Hsin Lin
Yuan Long Liu
Lee C. Lloyd
John C. Louko
Nora J. Lovall
Wayne L. Lowe
Yih-Jiuan B. Lu
Vincent Y. Lui
Michelle Luneau
William R. Maag
James M. MacPhee
Barbara D. Majcherek
Suzanne E. Maki
Richard J. Manship
Catherine Marcotte
Stephen P. Marsden
Julie Martineau
James P. Mathews
William J. Mazurek
Michael B. McKnight
Scott A. McNabb
Jeffrey A. Mehalic
Bruce R. Menzel
Michelle L. Merkel
Yury Mezhebovsky
Stephanie J. Michalik
Jean Michel
Camille D. Minogue
Stephen A. Moffett
Mark J. Moitoso
Matthew A. Monson
Lisa J. Moorey
Jarow G. Myers
Michael D. Neubauer
Tang-Tri Nguyen
Susan K. Nichols
Haripaul Pannu
Moshe C. Pascher
Chantal Pelletier
Stephane Pelletier
Michael W. Phillips
Jonathan M. Piper
Christopher J. Poteet
Karen L. Queen
John L. Quigley
Janelle P. Ridder
Sallie S. Robinson
William E. Rockwell
Dave H. Rodriguez

Part 4B (cont'd)

William P. Rudolph
Thomas A. Ryan
Shama S. Sabade
James C. Sandor
Mary Catherine Sandro
Nicolas A. Santa Gadea
Ronald J. Santacroce
Sheila A. Schroer
Steven G. Searle
Deven N. Shah
Scott A. Sheldon
James S. Shoenfelt
Joyce A. Simmons
Gregory T. Snider
Lori A. Snyder
Robert Sokol
Brian E. Speight
Jonathan C. Stavros
Christine L. Steele-Koffke
David T. Steen
Christopher M. Steinbach
Patrick E. Sutherland
James C. Tai
Mike K. S. Tam
Josephine L. C. Tan
Frank Tancredi
Ming Tang
Mark L. Thompson
Serena EE IK Tiong
Joseph D. Tritz
Kris D. Troyer
Bonnie J. Trueman
Chung-Sen Tsai
Lei Tsui
Matthew L. Uhoda
Charles R. Updike
Eric Vaith
Rasa T. Varanka
Marc-Andre Vinson
Amy R. Waldhauer
Robert J. Walling, III
David W. Warren
Karen E. Watson
Shu-Mei Wei
Matthew M. White
Michael J. Williams
Karin H. Wohlgemuth
Barbara A. Wolinski
Kah-Leng Wong
Kai-Yip Wong
Chi-Chih Woo
Jeffrey S. Wood
Jun Yan
Shih-Hsin Yeh
Shwu-Yuann J. YouChow
Virginia R. Young
Benny S. Yuen
Ruth Zea
Fengming Zhang
Steven B. Zielke

Part 6

Rimma Abian
Shawna S. Ackerman
Jonathan D. Adkisson
Elise M. Ahearn
K. Athula P. Alwis
Timothy P. Aman
Michael E. Angelina
Bhim D. Asdhir
William M. Atkinson
Lewis V. Augustine
Robert S. Ballmer, II
James M. Bartie
Brian P. Beckman
Steven L. Berman
Lisa A. Bjorkman
Suzanne E. Black
Ann M. Bok
Lesley R. Bosniack
Maurice P. Bouffard
Tracy L. Brooks-Szegda
Lisa J. Brubaker
Christopher G. Brunetti
Peter V. Burchett
Mark E. Burgess
Marian M. Burkart

Part 6 (cont'd)

Mark W. Callahan
Douglas A. Carlone
Martin Carrier
Tania J. Cassell
Julia C. Causbie
Maureen A. Cavanaugh
Kevin J. Cawley
Francis D. Cerasoli
Debra S. Charlop
Gary C. Cheung
Thomas J. Chisholm
Rita E. Ciccariello
Brian A. Clancy
Laura A. Claude
Kay A. Cleary
James P. Cochran
Jo Ellen Cockley
Frank S. Conde
Pamela A. Conlin
Catherine Cresswell
Joyce A. Dallessio
Francis L. Decker
Jean A. DeSantis
Dawn M. DeSousa
Kurt S. Dickmann
Behram M. Dinshaw
Mary Jane B. Donnelly
Dean P. Dorman
John P. Doucette
William A. Dowell, Jr.
Robert G. Downs
Kimberly J. Drennan
David L. Drury
Bernard Dupont
Tammy L. Dye
Jeffrey Eddinger
Martin A. Epstein
Dianne L. Estrada
James G. Evans
Joseph G. Evleth
Charles V. Faerber
Michael A. Falcone
John D. Ferraro
Daniel B. Finn
Ginda K. Fisher
Daniel J. Flick
Kirsten A. Frantom
Richard A. Fuller
Susan T. Garnier
Lynn A. Gehant
John T. Gleba
Ronald E. Glenn
John E. Green
Steven A. Green
Russell H. Greig
Charles R. Grilliot
Richard J. Haines
Joyce G. Hallaway
William D. Hansen
Robert L. Harnatkiewicz
Christopher L. Harris
Paul D. Henning
Amy J. Himmelberger
Wayne Hommes
Robert J. Hopper
Linda M. Howell
Po-Wo Hsieh
Sandra L. Hunt
Fong-Yee J. Jao
Christian Jobidon
Michael S. Johnson
Janet S. Katz
Mark J. Kaufman
Brian D. Kemp
Michael B. Kessler
Robert W. Kirklin
Michael F. Klein
Brandelyn C. Klenner
Joan M. Klucarich
Louis K. Korth
Eleni Kourou
Gary R. Kratzer
Jason A. Kundrot
Kenneth A. Kurtzman
Bertrand J. LaChance
Blair W. Laddusaw
Gregory D. Larcher
Robert J. Larson
Steven W. Larson
John P. Lebens
Lewis Y. Lee
Ramona C. Lee
Robin R. Lee
Brian P. LePage
David R. Lesieur
Paul B. LeStourgeon
Aaron S. Levine
Kenneth A. Levine
Shu C. Lin
Yuan Long Liu
Andrew M. Lloyd
Victoria S. Lusk

Part 6 (cont'd)

Barbara S. Mahoney
Robert G. Mallison, Jr.
Joseph A. Malsky
Richard J. Marcks
Lawrence F. Marcus
Sharon L. Markowski
Joseph Marracello
Robert F. Maton
Tracey L. Matthew
Deann M. Mays
Camley A. Mazloom
Michael K. McCutchan
Mark Z. McGill, III
Charles L. McGuire, III
Kelly S. McKeethan
David W. McLaughry
Robert F. Megens
Daniel J. Merk
Stephen V. Merkey
Claus S. Metzner
Stephen J. Mildenhall
Scott M. Miller
Douglas H. Min
Kenneth B. Morgan
Benoit Morissette
Robert J. Moser
Kimberly J. Mullins
Giovanni A. Muzzarelli
Catherine A. Neufeld
Denis P. Neumann
Mark A. O'Brien
James D. O'Malley
Marc F. Oberholtzer
Kathleen C. Odomirok
Denise R. Olson
Douglas J. Onnen
Jean-Francois Ouellet
Ajay Pahwa
John E. Pannell
Abha B. Patel
Wende A. Pemrick
Anne M. Petrides
Genevieve Pineau
Mark A. Piske
Gregory J. Poirier
Tracey S. Powers
Arlie J. Proctor
Yves Provencher
David S. Pugel
Regina M. Puglisi
Kiran Rasaretnam
Darin L. Rasmussen
Steven J. Regnier
Ellen J. Respler
Meredith G. Richardson
Donald A. Riggins
Brad M. Ritter
Anthony V. Rizzuto
John R. Rohe
John W. Rollins
Jay A. Rosen
David A. Rosenzweig
Christine R. Ross
Robert A. Rowe
Douglas A. Rupp
David A. Russell
Kevin L. Russell
Sean W. Russell
Linda M. Saunders
Christina L. Scannell
Marc Shamula
Nathan I. Shpritz
Gerson Smith
John B. Sopkowicz
Carl J. Sornson
Kendra D. South
Michael P. Speedling
Calvin C. Spence, Jr.
Michael J. Sperduto
Victoria G. Stachowski
Christina L. Staudhammer
John P. Stefanek
Richard A. Stock
Judy L. Stolle
Ilene G. Stone
Arumugam Suthanthiranathan
Siu Cheung S. Szeto
Francois Tardif
David M. Terne
Tony Tio
Dom M. Tobey
Glenn A. Tobleman
Robert C. Turner
Robert W. Van Epps
Jeffrey A. Van Kley
Mark D. van Zanden

Part 6 (cont'd)

Trent R. Vaughn
Martin Vezina
Robert J. Vogel
Wittie O. Wacker
Annette H. Wade
Lisa Marie Walsh
Erica L. Weida
Geoffrey T. Werner
James C. Whisenant
Steven B. White
Gayle L. Wiener
Calvin Wolcott
Robert F. Wolf
Tad E. Womack
Cheng-Sheng P. Wu
Hwamei Yen
Edward J. Yorty
Jeffery M. Zacek
George H. Zanjani
Joshua A. Zirin
Rita M. Zona

Part 8

Rhonda K. Aikens
Scott C. Anderson
Timothy J. Banick
Philip A. Baum
John A. Beckman
Douglas S. Benedict
Daniel D. Blau
Betsy L. Blue
Christopher K. Bozman
Paul A. Bukowski
Richard F. Burt, Jr.
Mark J. Cain
Daniel G. Carr
David S. Cash
Carol A. Cavaliere
Jessalyn Chang
Wei Chuang
Kasing L. Chung
Mary L. Corbett
Gregory L. Cote
Timothy J. Cremin
M. Elizabeth Cunningham
Michael T. Curtis
Jeffrey F. Deigl
Michael L. DeMattei
Stephen R. DiCenso
Jeffrey L. Dollinger
Mary Ann Duchna-Savrin
Madelyn C. Faggella
Matthew G. Fay
Denise A. Feder
Judith M. Feldmeier
Carole M. Ferrero
George Fescos
Daniel B. Finn
Brian C. Fischer
Russell Frank
Cynthia J. Friess
James E. Gant
Kim B. Garland
Michael A. Ginnelly
Bradley J. Gleason
Donna L. Glenn
Matthew E. Golec
Michele P. Gust
Jonathan M. Harbus
Matthew T. Hayden
Lisa A. Hays
Mary B. Hemerick
David L. Homer
Paul R. Hussian
Hou-wen Jeng
Patrick C. Jensen
Stephen H. Kantor
Timothy P. Kenefick
Deborah E. Kenyon
Ann L. Kiefer
Changseob J. Kim
Craig W. Kliethermes
Terry A. Knull
Timothy F. Koester
Adam J. Kreuser
Cheung S. Kwan
Michael D. Larson
Diana Lee
Richard S. Light
John J. Limpert
Stephanie J. Lippl
Paul R. Livingstone
Robert G. Lowery
Mark J. Mahon
Kelly J. Mathson

Part 8 (cont'd)

Malkie Mayer
John W. McCutcheon, Jr.
Stephen J. McGee
M. Sean McPadden
Lynne S. McWithey
Brian J. Melas
John P. Mentz
Timothy Messier
Paul A. Mestelle
Stephen J. Meyer
Linda K. Miller
Paul W. Mills
Stacy L. Mina
Russell E. Moore
Robin N. Murray
Victor A. Njakou
Kathleen C. Nomicos
Jennifer J. Palo
Charles C. Pearl, Jr.
Karen L. Pehrson
Joseph W. Pitts
Brian D. Poole
Kathy Popejoy
Stuart Powers
Mark Priven
Cathy A. Puleo
Eduard J. Pulkstenis
Mark S. Quigley
Eric K. Rabenold
Karin M. Rhoads
Elizabeth M. Riczko
James J. Romanowski
Allen D. Rosenbach
Kevin D. Rosenstein
Stuart G. Sadwin
Lisa P. Schmidt
Susan C. Schoenberger
Jeffery J. Scott
David M. Shepherd
Jeffrey S. Sirkin
David B. Sommer
Keith R. Spalding
Thomas N. Stanford
Brian M. Stoll
Elissa M. Sturm
Scott J. Swanay
Jeanne E. Swanson
Georgia A. Theocharides
Charles F. Toney, II
Patrick N. Tures
John V. Van de Water
Charles E. Van Kampen
David B. Van Koevering
Kenneth R. Van Laar, Jr.
Bryan C. Ware
John P. Welch
Kevin Wick
Gnana K. Wignarajah
Marcia C. Williams
John S. Wright
Floyd M. Yager
Claude D. Yoder
Ralph T. Zimmer

Part 8C

Xavier Benarosch
Cindy C.M. Chu
Nathalie Gamache
Mylene J. Labelle
Benoit Laganiere
Michel Laurin
Marc-Andre Lefebvre
Francois Morin
Jean Roy
Yves Saint-Loup
Michael Toledano

Part 10

Marc J. Adee
Kristen M. Albright
Danny M. Allen
Richard R. Anderson
Katharine Barnes
Todd R. Bault
Gregory S. Beaulieu
Lisa M. Besman
Jennifer L. Biggs
Cara M. Blank
Alicia E. Bowen
Yaakov B. Brauner
Mark D. Brissman
J. Eric Brosius
Robert N. Campbell
Benoit Carrier
David S. Cash
Ralph M. Cellars
Chyen Chen
Peter J. Collins
David B. Cox
Edgar W. Davenport
Manon Debigare
Germain Denoncourt
Pierre Dionne
Shawn F. Doherty
Victor G. dos Santos
William F. Dove
Yves Doyon
Michael C. Dubin
Patrick Dussault
Charles C. Emma
Paul E. Ericksen
Philip A. Evensen
Randall A. Farwell
Yves Francoeur
Barry A. Franklin
Scott F. Galiardo
Andrea Gardner
Richard J. Gergasko
Bruce R. Gifford
Richard S. Goldfarb
Charles T. Goldie
Odile Goyer
Edward M. Grab
Carleton R. Grose
George M. Hansen
Gordon K. Hay
Kathleen A. Hinds
Deborah G. Horovitz
Beth M. Hostager
Laura A. Johnson
Brian A. Jones
Allan A. Kerin
Michael G. Kerner
Gerald S. Kirschner
Jerome F. Klenow
Gilbert M. Korthals
John M. Kulik
D. Scott Lamb
Alan E. Lange
Nicholas J. Lannutti
Christopher Lattin
Paul W. Lavrey
Nicholas M. Leccese, Jr.
Eric F. Lemieux
Elise C. Liebers
John J. Limpert
Stephanie J. Lippl
Daniel J. Mainka
Donald F. Mango
Blair E. Manktelow
Katherine A. Mann
Suzanne Martin
Heidi J. McBride
James B. McCreesh
Thomas S. McIntyre
Dennis T. McNeese
Brett E. Miller
Robert L. Miller
John H. Mize
Andrew W. Moody
Robert A. Mueller
Rade T. Musulin
William A. Niemczyk
Stephen R. Noonan
William L. Oostendorp
Sarah Louise Petersen
Michael Petrocik
Daniel C. Pickens
Michael D. Poe
Jeffrey C. Raguse
Donald K. Rainey
A. Scott Romito
Jean Roy
David M. Savage
Thomas E. Schadler
Gregory R. Scruton
David A. Smith
Linda D. Snook
Paul J. Struzzieri

Part 10 (cont'd)

Susan M. Treskolasky
Dale G. Vincent, Jr.
Jennifer A. Violette
Peter A. Weisenberger
Teresa J. Williams
William M. Wilt
Beth M. Wolfe
Kathy A. Wolter
Chung-Ye Yen
Charles J. Yesker

The following candidates were admitted as Fellows and Associates at the November 1993 meeting as a result of their successful completion of the Society requirements in the May 1993 examinations.

FELLOWS

Marc J. Adee
Kristen M. Albright
Guy A. Avagliano
Katharine Barnes
Gregory S. Beaulieu
Xavier Benarosch
Lisa M. Besman
Cara M. Blank
Alicia E. Bowen
Christopher K. Bozman
Yaakov B. Brauner
Mark D. Brissman
J. Eric Brosius
Paul A. Bukowski
Mark J. Cain
David S. Cash
Chyen Chen
Peter J. Collins
David B. Cox
Manon Debigare
Germain Denoncourt
Pierre Dionne
Victor G. dos Santos
William F. Dove
Michael C. Dubin
Patrick Dussault
Charles C. Emma
Philip A. Evensen
Randall A. Farwell
Barry A. Franklin
Scott F. Galiardo
Andrea Gardner
Richard J. Gergasko
Bruce R. Gifford
Richard S. Goldfarb
Charles T. Goldie
Odile Goyer
Edward M. Grab
Carleton R. Grose
George M. Hansen
Gordon K. Hay
Kathleen A. Hinds
Beth M. Hostager
Laura A. Johnson
Brian A. Jones
Changseob J. Kim
Gerald S. Kirschner
Jerome F. Klenow
D. Scott Lamb
Alan E. Lange
Nicholas J. Lannutti
Nicholas M. Leccese, Jr.
Eric F. Lemieux
Stephanie J. Lippl
Mark J. Mahon
Heidi J. McBride
Dennis T. McNeese
John H. Mize
William A. Niemczyk
Kathleen C. Nomicos
Sarah Louise Petersen
Michael Petrocik
Michael D. Poe
Stuart Powers
Jeffrey C. Raguse
Donald K. Rainey
A. Scott Romito
Allen D. Rosenbach
Jean Roy
Stuart G. Sadwin

Yves Saint-Loup
Thomas E. Schadler
David A. Smith
Linda D. Snook
David B. Sommer
Susan M. Treskolasky
Jennifer A. Violette
Bryan C. Ware
Peter A. Weisenberger
John P. Welch
Kevin Wick
Gnana K. Wignarajah
Teresa J. Williams
Kathy A. Wolter
Chung-Ye Yen

ASSOCIATES

Jonathan D. Adkisson
Michael E. Angelina
Bhim D. Asdhir
Gary Blumsohn
Ann M. Bok
Maurice P. Bouffard
Tracy L. Brooks-Szegda
Peter V. Burchett
Michael W. Cash
Tania J. Cassell
Kevin J. Cawley
Debra S. Charlop
Kay A. Cleary
Jo Ellen Cockley
Joyce A. Dallessio
Ronald E. Glenn
Russell H. Greig
Richard J. Haines
Robert L. Harnatkiewicz
Christopher L. Harris
Paul D. Henning
Robert J. Hopper
Sadagopan S. Iyengar
Janet S. Katz
Michael F. Klein
Jason A. Kundrot
Robert J. Larson
David R. Lesieur
Shu C. Lin
Richard J. Marcks
Lawrence F. Marcus
Sharon L. Markowski
Deann M. Mays
Stephen V. Merkey
Douglas H. Min
Kimberly J. Mullins
Giovanni A. Muzzarelli
Douglas J. Onnen
Paul S. Osborn
Joseph W. Pitts
Yves Provencher
Regina M. Puglisi
Steven J. Regnier
Ellen J. Respler
Al J. Rhodes
Sallie S. Robinson
David A. Rosenzweig
Kevin L. Russell
Peter R. Schwanke
Calvin C. Spence, Jr.
Victoria G. Stachowski
John P. Stefanek
Richard A. Stock
Kimberley A. Ward
Gayle L. Wiener
Calvin Wolcott
Robert F. Wolf
Cheng-Sheng P. Wu
Edward J. Yorty
Jeffery M. Zacek

Linda D. Snook
David B. Sommer
Peter A. Weisenberger
John P. Welch
Kathy A. Wolter
Chung-Ye Yen

ASSOCIATES

Jonathan D. Adkisson
Paul D. Henning
Regina M. Puglisi

The following is the list of successful candidates in examinations held in November 1993.

Part 3B

Kristine M. Adamson
Paul D. Anderson
Wendy L. Artecona
Douglas L. Atlas
Jane L. Attenweiler
Timothy W. Atwill
Bruce D. Ballentine
Robert S. Ballmer
Dana Barre
Elizabeth F. Bassett
Julie Bennett
Wayne F. Berner
Kristen M. Bessette
Jennifer L. Blackmore
Jodi L. Bohac
Thomas L. Boyer, II
Bernardo Bracero, Jr.
Cary J. Breese
Laura G. Brill
James D. Buntine
Kevin D. Burns
Mark W. Callahan
Matthew R. Carrier
Richard M. Chiarini
Hong Choi
Michael J. Christian
Maryellen J. Coggins
Danielle G. Comtois
Margaret E. Conroy
Michelle J. Cooper
David C. Coplan
Sandra Creaney
Michael T. Cronin
Sharon A. Crosson
Andrew S. Dahl
Kathleen M. Daly
Sheri L. Daubenmier
Mari A. Davidson
Nancy K. DeGelleke
Michael B. Delvaux
Jonathon M. Deutsch
Sean R. Devlin
Michele E. Dimmick
Christopher S. Downey
Kevin F. Downs
Michael E. Doyle
David L. Driscoll
Lucy Drozd
John A. Duffy
Jane Eichmann
Melita M. Elinon
Andrew C. Erlewein
Brian A. Evans
Carolyn M. Falkenstern
Richard B. Federman
Tracy M. Fleck
Mary E. Fleischli
Sy Foguel
Christian Fournier
Douglas E. Franklin
W. Derrick Fung
Nicholas G. Garbis
Natasha C. Garten
Isabelle Gaumond
Robert W. Geist
Salvatore J. Giambrone
Sanjay Godhwani
Ann Marie Grassucci
Mari L. Gray
Christopher G. Gross
Scott T. Hallworth
Alex A. Hammett
Brian D. Haney
Gerald D. Hanlon
Gregory Hansen
Michelle L. Harnick
Judith F. Hausman
Jason B. Heissler
Daniel F. Henke
David E. Heppen
Anna M. Hnateyko
Kevin E. Hobbs
Daniel L. Hogan
Steven A. Hornacek
Sandra L. Hunt
Christopher D. Jacks
Joseph W. Janzen
Walter L. Jedziniak
Kathleen M. Johnson
Philip A. Kane, IV
Ira M. Kaplan
Chad C. Karls
Kathryn E. Keehn

Part 3B (cont'd)

William N. Kocken
Janet M. Krehel
Alexander Krutov
John R. Kunstman
Margaret J. Kuperman
Dennis H. Lawton
Brian P. LePage
Bradley R. Leblond
Guy Lecours
Lewis Y. Lee
Diane Lesage
Julia Leung
Steven E. Levitt
Herman Lim
Shiu-Shiung Lin
Richard B. Lord
Mark A. Lowis
James R. Lyter
Jeffrey Margasak
Thomas D. Martin
Michael J. Mastricolo
Alison L. Matsen
Stanislav D. Maydan
Patrice McCaulley
Timothy T. McKee
Claus S. Metzner
Alison M. Milford
Shannon A. Miller
Richard G. Millilo
Monica J. Monaghan
Melissa R. Montante
Benoit Morissette
Korri A. Morsey
Thomas M. Mount
Yinay Nadkarni
John-Giang L. Nguyen
John E. Noble
Dianna B. Norman
Martin J. O'Connell
Christopher E. Olson
Wade H. Oshiro
Jean-Francois Ouellet
Nancy L. Owen
Dmitry Papush
Genevieve Pare
Bruce G. Pendergast
Wendy W. Peng
Claude Penland, IV
Amy S. Polashuk
Mitchell S. Pollack
Josee Pomerleau
Dale S. Porfilio
Robert K. Prescott
Gariguin E. Prilepski
Arlene M. Richardson
Kathleen F. Robinson
Richard A. Rosengarten
Robert R. Ross
Chet James Rublewski
Jason L. Russ
David A. Russell
Manalur S. Sandilya
Glenn R. Scharf
Michael C. Schmitz
Lawrence M. Schober
Ronald J. Schuler
Michael R. Schummer
Timothy D. Schutz
Peter A. Scourtis
Robin M. Seifert
David J. Shaloiko
Allison M. Skolnick
Mark A. Smith
Steven A. Smith, II
Jody L. Sneed
Jennifer A. Sovell
Kristen L. Sparks
Stephen J. Streff
Andrea E. Sufke
Helaina I. Surabian
Arumugam Suthanthiranathan
Julie Ann Swisher
Varsha A. Tantri
Mark L. Thompson
Dom M. Tobey
Glenn A. Tobleman
Mollie J. Toole
David M. Towriss
Stephanie J. Traskos
Laura M. Turner
Timothy J. Ungashick
Robert W. Van Epps
Michael O. VanDusen
Jeffrey J. Voss
Karen L. Wajda
Tice R. Walker
Frances K. Wallace
Christopher J. Waller
Helen R. Wargel
Jacob Wechsler

Part 3B (cont'd)

Elizabeth A. Wentzien
Scott Werfel
Jean (Patti) P. West
Kendall P. Williams
L. Alicia Williams
Dean M. Winters
Mark L. Woods
Linda Yang
Adam M. Yasan
Christopher H. Yaure
Milton F. Yee
Chris Seung H. Yu
Kathermina Lily Yuen
William M. Yuen
Fengming Zhang
Luqun Zhang
Zhiming Zou

Part 4A

Tamela Alamo
Christopher R. Allan
Nancy S. Allen
Denise M. Ambrogio
Mario G. Arguello
Frank A. Aritz
Timothy W. Atwill
Nathan J. Babcock
Karen L. Babitt
David B. Bassi
Andrew S. Becker
David J. Belany
Jody J. Bembenek
Mario Binetti
Nicole P. Bitros
Linda J. Bjork
Ming Y. Blinn
Kristen R. Bond
Elizabeth S. Borchert
Raju Boura
Audrey L. Bridgewater
Kirsten R. Brumley
James A. Bull
Elliot R. Burn
Alan Burns
Kevin D. Burns
Kimberly Burrows
Sandra L. Cagley
Sandra J. Callanan
Donna L. Callison
Milissa D. Carter
Patrick J. Causgrove
Todd D. Cheema
Lisa C. Chen
Zhong Ling Chen
Theresa A. Christian
Stephen D. Clapp
Jason T. Clarke
Susan M. Cleaver
Carolyn J. Coe
Brian R. Coleman
Kimberly S. Coles
Peter J. Cooper
David C. Coplan
Sharon R. Corrigan
David E. Corsi
Catherine Cresswell
Stephanie Csintyan
Gregory E. Daggett
Andrew S. Dahl
Kenneth S. Dailey
Jill A. Davis
Robert E. Davis
Robin M. Davis
Dawn M. DeSousa
Anne M. DelMastro
Karen D. Derstine
Jonathon M. Deutsch
Denis Dubois
Sophie Duval
Marianne E. Dwyer
Jennifer L. Ehrenfeld
Sylvain Fauchon
Robert C. Fox
Michael A. Fradkin
James E. Frye
Hannah Gee
Steven L. Gibbs
Bernard H. Gilden
Peter S. Gordon
Judith M. Gottesman
Stephanie A. Gould
Mari L. Gray
Daniel C. Greer
Michael K. Griffin
David T. Groff
John A. Hagglund
Barry R. Haines

Part 4A (cont'd)

Lynne M. Halliwell
Alessandrea C. Handley
Bernadette M. Hare
Jason C. Head
Jodi J. Healy
Sara L. Helgeson
Rhonda R. Hellman
Sherry L. Hess
Thomas E. Hettinger
Christopher T. Hochhausler
Corine Huey
Rebecca R. Hunt
Rusty A. Husted
Tina T. Huynh
Suzanne M. James
Christopher Jamroz
John F. Janssen
Joseph W. Janzen
Philip J. Jennings
Tricia L. Johnson
William Rosco Jones
John P. Kannon
Charles N. Kasmer
Ruta V. Kher
Susan L. Klein
Brian R. Knox
Kathryn L. Kritz
Denise A. Kuhl
Renu A. Kumar
Salvatore T. LaDuca
Jocelyn Laflamme
Laura L. Lankin
Sylvie Lanoix
Patricia N. Laracuente
Jean-Francois Larochelle
Stephane Leduc
Joan K. Lee
Thomas C. Lee
James P. Leise
Bradley H. Lemons
Steven J. Lesser
Charles Letourneau
Aaron S. Levine
Michael Leybov
Xiaoying Liang
Frank K. Ling
Kim D. Litwack
Paul Liu
Rita M. MacIntyre
James M. MacPhee
Elaine J. Malupa
Richard J. Manship
Kelly E. Martin
Emma Macasieb McCaffrey
Cassandra M. McGill
Smith W. McKee
William A. Mendralla
Stephanie J. Michalik
Susan A. Minnich
Catherine E. Moody
Kimberly A. Moran
Michael W. Morro
Janice C. Moskowitz
Ethan Mowry
Kathleen V. Najim
Kari S. Nelson
Phann J. Nhem
Michael A. Nori
Brett S. Oakley
Christopher E. Olson
Lowell D. Olson
Leo M. Orth, Jr.
David J. Otto
Alan M. Pakula
Gerard J. Palisi
Moshe C. Pascher
Javanika Patel
Andrea L. Phillips
David R. Picking
Donna M. Pinetti
David J. Pochettino
Igor Pogrebinsky
Alan D. Potter
Matthew H. Price
Anthony E. Ptasznik
Rhonda A. Puda
William D. Rader, Jr.
Vinayak Ranade
Mary S. Rapp
Beth A. Rasmussen
Frank S. Rau
Timothy O. Reed
James J. Rehbit
Raymond J. Reimer
Ellen K. Rein
Peggy-Anne K. Repella
Cynthia L. Rice

Part 4A (cont'd)

David C. Riek
Brad E. Rigotty
Karen L. Rivara
Bo Rubin
Chet James Rublewski
William P. Rudolph
Jason L. Russ
Joanne E. Russell
Rachel Samoil
Natalie J. Sanders
Nicolas A. Santa Gadea
Barbara A. Satsky
Christina L. Scannell
Christine E. Schindler
Lothar Schneider
Jeffrey A. Schreiber
Dov Schwartz
Vladimir Shander
Michael Shane
Dawn M. Shannon
Scott A. Shapiro
Brad J. Sherfey
James S. Shoenfelt
Katherine R. Smith
Robert Sokol
Michele L. Spale
Glenda M. Stalkfleet
Deborah L. Stone
Marion C. Stone
William J. Stone
William M. Stringfellow
Christopher S. Strohl
Mark R. Strona
Patricia A. Sullivan
Adam M. Swartz
Todd D. Tabor
Mark L. Thompson
Jeffrey S. Trichon
Bonnie J. Trueman
Kai L. Tse
Martin Turgeon
Dennis R. Unver
Joel A. Vaag
Janet K. Vollmert
Mary E. Waak
Amy R. Waldhauer
Robert J. Wallace
Jon S. Walters
Helen R. Wargel
Karen E. Watson
Dean A. Westpfahl
Karin H. Wohlgemuth
Terry C. Wolfe
Gretchen L. Wolfer
Milton K. Wong
Jun Yan
Michael L. Yanacheak
Robert S. Yenke
Anthony C. Yoder
George H. Zanjani
Ruth Zea

Part 4B

Shawna S. Ackerman
Jeffrey R. Adcock
Joseph J. Allard
John M. Allen
Robert E. Allen
Steve B. Altemeier
Barry L. Bablin
Melissa M. Bados
Keith M. Barnes
David B. Bassi
William M. Batchelder
Anna Marie Beaton
Michael J. Bednarick
Victoria A. Beltz
Louis Bernatchez
Duane L. Bernt
Kofi Boaitey
Gregory Bomash
Hobart F. Bond, III
Raju Boura
Douglas J. Bradac
Glen R. Bratty
Audrey W. Broderick
Karen A. Brostrom
Ron Brusky
Hugh E. Burgess
Elliot R. Burn
Sandra L. Cagley
Mark W. Callahan
William Brent Carr
James H. Carson
Jin S. Chang
Yong Seok Choi

Part 4B (cont'd)

Lisa V. Clarke
Carolyn J. Coe
Geert Coene
Maryellen J. Coggins
Brian R. Coleman
Kendall Albert Collins
Steve J. Cornell
Luc Croteau
Nathan E. Cultice
Christopher G. Cunniff
Mark A. Davenport
James D. Davis
Christopher DeMeo
Raymond Demers
Karin R. Doerr
John C. Dougherty
Sara P. Drexler
Ewa Duma
Annette M. Eckhardt
Judith A. Edwards
Donald M. Elbaum
Dawn E. Elzinga
Donna L. Emmerling
Jui-Chuan Fan
Junko K. Ferguson
Heather A. Ford
Hugo Fortin
Stephane G. Fortin
Lilane L. Fox
Simon D. Frechet
Mauricio Freyre
J'ne E. Furrow
Robert J. Garbus
Bruce A. Georgenson
Siddhartha Ghosh
Nicholas P. Giuntini
Karl Goring
Jay C. Gotalaere
Daniel E. Greer
Gary J. Griesmeyer
William A. Guffey
Tuvy Guss
Kastity Ha
John A. Hagglund
Marc S. Hall
Lynne M. Halliwell
Scott T. Hallworth
Michael R. Hamilton
Mary H. Hartsoch
Lise A. Hasegawa
Jean-Francois Hebert
Rhonda R. Hellman
Daniel J. Henderson
Tina M. Henninger
Michael E. Hermary
Thomas E. Hettinger
Brook A. Hoffman
Geoffrey W. Horton
Paul N. Houston
Long Fong Hsu
David D. Hudson
Kathy K. Huong
Jean-Claude J. Jacob
Michel Jacques
Anaar Jessa
Philip A. Kane, IV
John P. Kannon
Ruby S. Kao
Anthony N. Katz
Claudine H. Kazanecki
Russell G. Kirsch
Vineet Kochhar
William N. Kocken
Eric James Kohli
Kathryn L. Kritz
W. Scott Kupchinsky
Cheung S. Kwan
Salvatore T. LaDuca
Steven M. Lacke
Jen-Rong Lai
Ravikumar Lakshminarayan
Clifford Lam
Gregory D. Larcher
Steven W. Larson
Terry Yeow Lee
Thomas C. Lee
Glen A. Leibowitz
Charles Letourneau
Jacqueline M. Lewis
Xiangdong Li
Xianglin Li
Xiaodong Lin
Michael Lipkin
Serge M. Lobanov
Cara M. Low
Jean C. Lu
Sasi D. Mahesan
Archibald G. Mattis
Susan M. McCormick
Patrick A. McGoldrick
Jeffrey M. McLeRoy
Kathleen A. McMonigle
Jeffrey S. McSweeney

Part 4B (cont'd)

Hernan L. Medina
Brian J. Melas
Daniel J. Merk
Thomas C. Messer
Christopher G. Mighty
Alison M. Milford
Sherry L. Mueller
Linda D. Nagle
Ronald C. Neath
Jennifer A. Nelson
Kari S. Nelson
Michael A. Nori
Harry P. Norman
James L. Nutting
Mark A. O'Brien
Mihaela L. O'Leary
Scott P. Odierno
Kelly K. Olmen
Kristen A. Olsson
Diane L. Pedersen
James Michael Petrone
Anthony G. Phillips
Gwen C. Polston
Donald S. Priest
Benoit Primeau
Martin Raymond
James J. Rehbit
Scott Reynolds
Brad E. Rigotty
Kecia G. Rockoff
Jean Marie Rosa
Jay A. Rosen
Christine R. Ross
Christina Ann Ryan
Cheryl Y. Sabiston
Christina L. Scannell
Nancy J. Scheiring
Margot A. Schmid
Samuel G. Schmirler
Christy B. Schreck
Jay M. Schwartz
Craig J. Scukas
Vladimir Shander
Kevin H. Shang
Scott A. Shapiro
Theodore J. Shively
Nathan I. Shpritz
Jill C. Sidney
Ronald A. Signore
Tracey A. Silagy
Michael N. Singer
M. Kate Smith
Carl J. Sornson
Benoit St. Aubin
Kenneth W. Stam
Tracey A. Stark-Baldere
Gregory J. Stevenson
Deborah L. Stone
William M. Stringfellow
Kevin D. Strous
Mark Sturm
Bayad Sulaiman
Adam M. Swartz
Glenda O. Tennis
Francois Theberge
Jo D. Thiel
Philippe Trahan
Thomas A. Trocchia
Sonia L. Trudeau
Eric Trudel
Tzong-chou Tsai
Turgay F. Turnacioglu
Laura M. Turner
Jordan N. Uditsky
Robert J. Vogel
Janet K. Vollmert
Edward H. Wagner
Michael A. Wallace
Joseph W. Wallen
Xiaochuan Wang
Keith M. Waskom
Matthew J. Wasta
Norman E. Watkins
Todd A. Weber
Mark S. Wenger
Jeffrey D. White
Chi Wai Wong
Chun S. Wong
Yoke W. Wong
Boll Wu
Lang Wu
Benjamin J. Yahr
Linda Yang
Shang You Zeng
Luqun Zhang
Zhiming Zou

Part 5A

Fred S. Allsbrook
Steven D. Armstrong
Nathan J. Babcock
Richard J. Babel
Phillip W. Banet
Karen L. Barrett
David M. Baxter
LaVerne J. Biskner, III
Carol A. Blomstrom
Douglas J. Bradac
Charles Brindamour
Margaret A. Brinkmann
Jeffrey H. Brooks
Robert F. Brown
David V. Bruce
Jacqueline M. Campbell
Francine Cardi
Douglas A. Carlone
Heather L. Chalfant
Daero Choi
Wanchin W. Chou
Christopher P. Coehlo
Sally M. Cohen
Brian C. Cornelison
Kenneth S. Dailey
Jeffrey W. Davis
Behram M. Dinshaw
Martin W. Draper
Barry P. Drobes
Denis Dubois
Louis Durocher
Wayne W. Edwards
Jeffrey S. Ellis
Vicki A. Fendley
Stephen C. Fiete
William P. Fisanick
Daniel J. Flick
Walter H. Fransen
Bethany L. Fredericks
Timothy J. Friers
Gary J. Ganci
Michael A. Garcia
Eric J. Gesick
Matthew L. Gossell
John E. Green
Steven A. Green
Christopher G. Gross
Barbara Hallock
Alessandrea C. Handley
David S. Harris
Scott J. Hartzler
Ronald J. Herrig
Jay T. Hieb
Jason N. Hoffman
Todd H. Hoivik
Dave R. Holmes
Marie-Josee Huard
Tina T. Huynh
Suzanne M. James
Walter L. Jedziniak
Donna G. Jockers
Rishi Kapur
Kimberly S. Kaune
James M. Kelly
Thomas P. Kenia
John H. Kim
Bobb J. Lackey
Jean-Sebastien Lagarde
Steven W. Larson
Khanh M. Le
Todd W. Lehmann
Neal M. Leibowitz
Philip Lew
Katherine E. Lewis
Lee C. Lloyd
Cara M. Low
Robb W. Luck
Kyra D. Lynn
Stephen P. Marsden
Meredith J. Martin
Kelly S. McKeethan
William E. McWithey
Constance M. Mika
Lisa J. Moorey
Matthew K. Moran
Sek Ngai Ngai
Darci L. Noonan
James L. Nutting
Mihaela L. O'Leary
Brett S. Oakley
Dmitry Papush
Thomas Passante
Nicholas H. Pastor
Prabha Pattabiraman
Michael A. Pauletti
Lisa M. Pawlowski
Luba Pesis
Kathy A. Poppe
Warren T. Printz
Mark Priven
Richard B. Puchalski

Part 5A (cont'd)

Patrice Raby
Brentley J. Radeloff
Alan T. Reynard
Scott Reynolds
Lian Z. Rohsner
Jean-Denis Roy
Julie C. Russell
Margaret J. Sanchez
Christine E. Schindler
Michael C. Schmitz
Terry M. Seckel
Joyce E. Segall-Lopez
Jennifer M. Shantz
Paul O. Shupe
Jill C. Sidney
Raleigh R. Skaggs, Jr.
Cindy W. Smith
Laura Smith
Michelle Smith
Lori A. Snyder
Scott G. Sobel
Jay M. South
Sandra L. Spiroff
Theodore S. Spitalnick
Curt A. Stewart
Lori E. Stoeberl
Deborah L. Stone
Stephen J. Streff
Mark R. Strona
Roman Svirsky
Rachel R. Tallarini
Michel Theberge
Laura L. Thorne
Jennifer M. Tornquist
Joseph D. Tritz
Kai L. Tse
Karen E. Watson
Jeffrey D. White
Jennifer N. Williams
Kirby W. Wisian
Trevar K. Withers
Amy M. Wixon
Brandon L. Wolf
Barbara A. Wolinski
Tad E. Womack
Kah-Leng Wong
Jeffrey F. Woodcock
Rick A. Workman
Xuening Wu
Mindy M. Yu
Robin Zinger

Part 5B

Steve B. Altemeier
John D. Arendt
Steven D. Armstrong
Nathan J. Babcock
Richard J. Babel
Karen L. Babitt
Phillip W. Banet
Karen L. Barrett
Carol A. Blomstrom
Charles Brindamour
Margaret A. Brinkmann
Robert F. Brown
Douglas A. Carlone
William Brent Carr
Heather L. Chalfant
Brian C. Cornelison
Jose R. Couret
Luc Croteau
Kendra S. Cupp
Jeffrey W. Davis
Douglas L. Dee
Steven F. Delfino
Behram M. Dinshaw
Martin W. Draper
Stefvan S. Drezek
Barry P. Drobes
Jennifer L. Ehrenfeld
Brian M. Fernandes
Daniel J. Flick
Michael A. Garcia
Hannah Gee
Eric J. Gesick
Neil P. Gibbons
James B. Gilbert
David Patrick Glenn
Allen J. Gould
John E. Green
Steven A. Green
Daniel E. Greer
Kenneth J. Hammell
Alessandrea C. Handley
Brian T. Hanrahan
Marlene M. Hardison

Part 5B (cont'd)

Michael S. Harrington
Scott J. Hartzler
Jason N. Hoffman
David D. Hudson
Henry J. Itri
Paul Ivanovskis
David R. James
Suzanne M. James
Christopher R. Jarvis
Robert B. Katzman
James M. Kelly
Thomas P. Kenia
Ruta V. Kher
John H. Kim
Ung M. Kim
Young Y. Kim
Jennifer E. Kish
Henry T. Lee
Kevin A. Lee
Todd W. Lehmann
Philip Lew
Lee C. Lloyd
Robb W. Luck
Michelle Luneau
James M. MacPhee
Leslie A. Martin
William J. Mazurek
Michael B. McKnight
Scott A. McPhee
James R. Merz
Stephanie J. Michalik
Constance M. Mika
Brenda D. Miller
Matthew S. Mrozek
Randy J. Murray
James L. Nutting
Brett S. Oakley
Thomas Passante
Nicholas H. Pastor
Abha B. Patel
Curtis D. Pederson
David M. Pfahler
Michael W. Phillips
Mary K. Plassmeyer
Gene Z. Qiann
Patrice Raby
Sandra J. Rickel
Mary B. Rios-Gandara
Jean-Denis Roy
William P. Rudolph
Raymond G. Scannapieco
Steven M. Schienvar
Kelvin B. Sederburg
Jerelyn K. Seeger
Bipin J. Shah
Jennifer M. Shantz
Raleigh R. Skaggs, Jr.
Lori A. Snyder
Scott G. Sobel
Jay M. South
Kendra D. South
Sandra L. Spiroff
Theodore S. Spitalnick
Susan D. Stieg
Lori E. Stoeberl
Mark Sturm
Joy M. Suh
Colleen M. Sullivan
Daniel G. Sutcliffe
Roman Svirsky
Jennifer M. Tornquist
Joseph D. Tritz
Kai L. Tse
Arthur J. Turner
Matthew L. Uhoda
Steven J. Vercellini
Jacqueline J. Verfurth
Keith A. Walsh
Jon S. Walters
Laura M. Williams
Tad E. Womack
Kah-Leng Wong
Simon Wong
Mindy M. Yu
Steven B. Zielke

Part 7

Mark A. Addiego
Kay L. Allen
John P. Alltop
Larry D. Anderson
Michael J. Andring
Mohammed Q. Ashab
William M. Atkinson
Lewis V. Augustine
Jack Barnett
Rose D. Barrett
James M. Bartie
Andrea C. Bautista
Martin Beaulieu
Brian P. Beckman
Richard Belleau
Cynthia A. Bentley
Eric D. Besman
Suzanne E. Black
Michael G. Blake
Barry E. Blodgett
Erik R. Bouvin
Christopher L. Bowen
Lori M. Bradley
Robert E. Brancel
Kevin J. Brazee
Christopher G. Brunetti
Russell J. Buckley
Mark E. Burgess
Robert N. Campbell
Daniel G. Carr
Martin Carrier
Richard J. Castillo
Julia C. Causbie
Maureen A. Cavanaugh
Francis D. Cerasoli
Julie S. Chadowski
Daoguang E. Chen
Peggy Cheng
Gary C. Cheung
John S. Chittenden
Kuei-Hsia R. Chu
Darrel W. Chvoy
Susan D. Ciardiello
Rita E. Ciccariello
Brian A. Clancy
Alan R. Clark
Laura R. Claude
J. Paul Cochran
Frank S. Conde
Pamela A. Conlin
Pamela A. Connors
Nancy L. Cooper
Matthew D. Corwin
William F. Costa
Kirsten J. Costello
Catherine Cresswell
Richard J. Currie
Charles A. Dal Corobbo
Smitesh Dave
Jean A. DeSantis
Francis L. Decker
Laura B. Deterding
Kurt S. Dickmann
Andrew J. Doll
John P. Doucette
Robert G. Downs
Mary Ann Duchna-Savrin
Bernard Dupont
Tammy L. Dye
David M. Elkins
Martin A. Epstein
Dianne L. Estrada
Charles V. Faerber
Michael A. Falcone
Bruce D. Fell
Karen M. Fenrich
Janine A. Finan
Stephen A. Finch
Daniel B. Finn
Brian C. Fischer
Ginda K. Fisher
Robert F. Flannery
Kirsten A. Frantom
Mark A. Fretwurst
Cynthia J. Friess
Nathalie Gamache
Christopher H. Geering
Christine A. Gennett
Margaret W. Germani
Julie T. Gilbert
Nicholas P. Giuntini
Michael D. Green
Joyce G. Hallaway
Julie K. Halper
Elizabeth E. Hansen
William D. Hansen
Steven T. Harr

Part 7 (cont'd)

Jonathan B. Hayes
Amy J. Himmelberger
Wayne Hommes
Thomas A. Huberty
Fong-Yee J. Jao
June V. Jarvis
Christian Jobidon
Mark J. Kaufman
Brian D. Kemp
Rebecca A. Kennedy
Michael B. Kessler
Joseph P. Kirley
Paul H. Klauke
Joan M. Klucarich
Steven M. Koester
Louis K. Korth
Debra K. Kratz
Gary R. Kratzer
Mary C. Kroggel
Kenneth A. Kurtzman
Edward M. Kuss
Bertrand J. LaChance
Mylene J. Labelle
Blair W. Laddusaw
Elaine Lajeunesse
William J. Lakins
Josee Lambert
Matthew G. Lange
Paul B. LeStourgeon
John P. Lebens
Claude Lefebvre
Julie Lemieux-Roy
Daniel E. Lents
Kenneth A. Levine
Andrew M. Lloyd
Ronald P. Lowe, Jr.
Robert G. Lowery
Christopher J. Luker
Barbara S. Mahoney
Gary P. Maile
Robert G. Mallison, Jr.
Gabriel O. Maravankin
Anthony G. Martella, Jr.
Peter R. Martin
Michael B. Masters
Robert F. Maton
Charles L. McGuire
David W. McLaughry
Robert F. Megens
Daniel J. Merk
Timothy Messier
Stephen J. Mildenhall
David J. Miller
Scott M. Miller
Camille D. Minogue
Mark J. Moitoso
David Molyneux
Gregory A. Moore
Kenneth B. Morgan
Robert J. Moser
Turhan E. Murguz
Prakash Narayan
Charles P. Neeson
Aaron W. Newhoff
Mark A. O'Brien
Marc F. Oberholtzer
Kathleen C. Odomirok
Denise R. Olson
Milary N. Olson
John E. Pannell
Charles Pare
Wende A. Pemrick
Robert L. Penick
Anne M. Petrides
Beverly L. Phillips
Genevieve Pineau
Mark A. Piske
Igor Pogrebinsky
Gregory J. Poirier
Tracey S. Powers
Michael D. Price
Arlie J. Proctor
Sally A. Qiann
Karen L. Queen
Kathleen M. Quinn
Peter S. Rauner
Yves Raymond
Natalie J. Rekittke
Donald A. Riggins
Brad M. Ritter
Douglas S. Rivenburgh
Douglas A. Roemelt
Paul J. Rogness
Jay A. Rosen
Christine R. Ross
Sean W. Russell
Stephen P. Russell
Linda M. K. Saunders
Risa R. Schattin
Matt J. Schmitt
Lisa M. Scorzetti
Jeffrey P. Shirazi
Kerry S. Shubat

Part 7 (cont'd)

Gerson Smith
Gina L. Smith
Klayton N. Southwood
Angela K. Sparks
Louis B. Spore
Beth A. Stahelin
Laurence H. Stauffer
Avivya S. Stohl
Judy L. Stolle
Ilene G. Stone
Collin J. Suttie
Jeanne E. Swanson
Steven J. Symon
John L. Tedeschi
Josephine T. Teruel
Daniel A. Tess
John P. Thorrick
Tony Tio
Theresa A. Traynor
Robert C. Turner
Ching-Hom R. Tzeng
Jeffrey A. Van Kley
Mark D. Van Zanden
Trent R. Vaughn
Wittie O. Wacker
Lisa Marie Walsh
Alice M. Wang
Gregory S. Wanner
Linda F. Ward
Michelle M. Wass
Geoffrey T. Werner
James C. Whisenant
David L. Whitley
Mark L. Woods
Jeanne Lee Ying
Benny S. Yuen
Doug A. Zearfoss
Guangjian Zhu
Joshua A. Zirin
Rita M. Zona

Part 9

Jonathan D. Adkisson
Rhonda K. Aikens
Richard R. Anderson
Timothy J. Banick
John A. Beckman
Wayne E. Blackburn
Annie Blais
Gary Blumsohn
Ward M. Brooks
Richard F. Burt, Jr.
Mark W. Callahan
Benoit Carrier
Tania J. Cassell
Kevin J. Cawley
Cindy C.M. Chu
Wei Chuang
Mary L. Corbett
Gregory L. Cote
Michael T. Curtis
Michael L. DeMattei
Herbert G. Desson
Stephen R. DiCenso
Shawn F. Doherty
Jeffrey L. Dollinger
Jeffrey D. Donaldson
Yves Doyon
Alan J. Erlebacher
Matthew G. Fay
Denise A. Feder
Kendra M. Felisky-Watson
George Fescos
Daniel B. Finn
Mary K. Gise
Donna L. Glenn
Ronald E. Glenn
Anne G. Greenwalt
Russell H. Greig
Steven J. Groeschen
Richard J. Haines
Ellen M. Hardy
Christopher L. Harris
Matthew T. Hayden
Lisa A. Hays
Suzanne E. Henderson
Patrick C. Jensen
Stephen H. Kantor
Timothy P. Kenefick
Allan A. Kerin
Warren A. Klawitter
Michael F. Klein
Gilbert M. Korthals
Robert J. Larson
Michel Laurin

Part 9 (cont'd)

Paul W. Lavrey
Elizabeth A. Lemaster
Giuseppe F. Lepera
David R. Lesieur
Aaron S. Levine
John J. Limpert
Paul R. Livingstone
James M. Maher
Daniel J. Mainka
Blair E. Manktelow
Lawrence F. Marcus
Dee Dee Mays
Robert D. McCarthy
Jeffrey F. McCarty
M. Sean McPadden
Paul M. Merlino
Paul A. Mestelle
Stephen J. Meyer
Robert L. Miller
Paul W. Mills
Francois L. Morissette
Michelle M. Morrow
David A. Murray
Donald R. Musante
Stephen R. Noonan
Mary Beth O'Keefe
Douglas J. Onnen
Nathalie Ouellet
Donald D. Palmer
Jennifer J. Palo
Charles C. Pearl, Jr.
Karen L. Pehrson
Daniel C. Pickens
John F. Rathgeber
Elizabeth M. Riczko
Gregory Riemer
Sallie S. Robinson
James J. Romanowski
Bradley H. Rowe
Kenneth W. Rupert, Jr.
James V. Russell
Susan C. Schoenberger
Peter R. Schwanke
Tom A. Smolen
Keith R. Spalding
Victoria G. Stachowski
Richard A. Stock
Joseph W. Tasker, III
Barbara H. Thurston
Michael J. Toth
Charles E. Van Kampen
David B. Van Koevering
Robert J. Vogel
Leigh M. Walker
Patricia K. Walker
Joseph W. Wallen
Marcia C. Williams
William M. Wilt
Beth M. Wolfe
John M. Woosley
John S. Wright
Cheng-Sheng P. Wu
Gerald T. Yeung
Jeffery M. Zacek

Fellows Admitted in May 1993

The 11 new Fellows presented at the 1993 CAS Spring Meeting in Dallas included: *(from left)* Gordon Scott, Francois Dumas, Louis Gariepy, Bruno Bauer, **CAS President David P. Flynn,** Kevin Dickson, Brad Eastwood, Joanne Ikeda, James Fletcher, and Michel Dionne. Not pictured are Martin Couture and James Haidu.

ASSOCIATES ADMITTED IN MAY 1993

First row (left to right): Michel Laurin, Karen Davies, Mary Hemerick, Deanne Lenhardt, Elizabeth Lemaster, Philip Baum, Elizabeth Kolber, **CAS President David P. Flynn,** Kelly Mathson, Mary Gise, Marie-Julie Demers, John Butcher, Rhonda Aikens, Lisa Hays, Marc Grandisson, Noel Hehr. *Second row (left to right):* Richard Burt, Bernard Horovitz, Patrick Jensen, Scott Lefkowitz, Howard Kunst, Timothy Banick, Timothy Hansen, John Ferrara, James Gant, John Beckman, Matthew Hayden, Scott Anderson, Bradley Granger, David Darby. *Third row (left to right):* Matthew Fay, Shawn Doherty, Kurt Johnson, Paul Hancock, Vincent Jackson, Joseph Kilroy, Mark Johnson, William Ayres, Thomas Highet, Craig Kliethermes, Craig Allen, George Fescos, Ronald Earls, Richard Light, Steven Kelner, Terry Knull.

ASSOCIATES ADMITTED IN MAY 1993

First row (left to right): Nathalie Ouellet, Todd Tabor, Michael Steward, Melinda Oosten, Marcia Williams, Michelle Morrow, **CAS President David P. Flynn,** Charles Pearl, Karen Pehrson, Katie Suljak, Letitia Saylor, Mark Naigles, Peter Nonken. *Second row (left to right):* Edward Peck, Charles VanKampen, Jeffrey Sirkin, Patrick Tures, James Romanowski, Paul Mestelle, Mark Quigley, Kenneth Rupert, Stephen Sauthoff, John Wright, Suzanne Schoo, Brian Stoll. *Third row (left to right):* David Na, James Russell, Eduard Pulkstenis, Michael Schenk, Daniel Pickens, Claude Yoder, Christopher Tait, William Wilt, Timothy Muzzey. New Associates not pictured: Douglas Benedict, Michael Carpenter, Benoit Carrier, Michael Curtis, Kai Fung, Donna Glenn, Barton Hedges, David Larson, Thomas Lee, Daniel Mainka, Stephen Maratea, Robert McCarthy, Richard McDonald, Conrad Membrino, Cathy Puleo, Frank Rau, Thomas Rau, Andrew Rippert, James Rowland, Yuan-Yuan Tang, Gerald Yeung, and Barry Zurbuchen.

FELLOWS ADMITTED IN NOVEMBER 1993

Bottom Row (left to right): Stuart G. Sadwin, Andrea Gardner, Lisa M. Besman, Kathy A. Olcese, David A. Smith, **CAS President David P. Flynn,** J. Eric Brosius, Dennis T. McNeese, Mark D. Brissman, Heidi J. McBride, Stephanie J. Lippl. *Second Row (left to right):* Richard S. Goldfarb, Marc J. Adee, William A. Niemczyk, Peter J. Collins, Bryan C. Ware, Philip A. Evensen, George M. Hansen, Kevin Wick, Brian A. Jones, Michael Petrocik. *Third Row (left to right):* Nicholas J. Lannutti, Randall A. Farwell, Yaakov B. Brauner, William F. Dove, Michael C. Dubin, Alicia E. Bowen, Carleton R. Grose, Alan E. Lange, D. Scott Lamb, Christopher K. Bozman. *Top Row (left to right):* Gregory S. Beaulieu, Sarah Louise Petersen, Beth M. Hostager, Laura A. Johnson, Kathleen A. Hinds, Kathleen C. Nomicos, Eric F. Lemieux, Bruce R. Gifford, Jeffrey C. Raguse, Nicholas M. Lecesse, Jr., Allen D. Rosenbach, Scott F. Galiardo.

FELLOWS ADMITTED IN NOVEMBER 1993

Bottom Row (left to right): John P. Welch, Kristen M. Albright, Gerald S. Kirschner, Victor G. dos Santos, Mark J. Mahon, **CAS President David P. Flynn,** Richard J. Gergasko, Linda D. Snook, Peter A. Weisenberger, Cara M. Blank, Edward M. Grab. *Middle Row (left to right):* Mark J. Cain, Pierre Dionne, Barry A. Franklin, David B. Cox, Katharine Barnes, Odile Goyer, David B. Sommer, Germain Denoncourt, Xavier Benarosch, Jean Roy, Yves Saint-Loup, A. Scott Romito, Jerome F. Klenow. *Top Row (left to right):* Jennifer A. Violette, Teresa J. Williams, Donald K. Rainey, Guy A. Avaglioano, Changseob J. Kim, John H. Mize, Patrick Dussault, Manon Debigare, Charles T. Goldie, Michael D. Poe, Gordon K. Hay, Paul A. Bukowski. Not Pictured: David S. Cash, Chyen Chen, Charles C. Emma, Stuart Powers, Thomas E. Schadler, Susan M. Treskolasky, Gnana K. Wignarajah, Kathy A. Wolter, and Chung-Ye Yen.

ASSOCIATES ADMITTED IN NOVEMBER 1993

Bottom Row (left to right): Michael F. Klein, David A. Rosenzweig, Robert J. Larson, Richard J. Haines, Jo Ellen Cockley, **CAS President David P. Flynn,** Sharon L. Markowski, Kay A. Cleary, Janet S. Katz, Kimberley A. Ward, Victoria G. Stachowski. *Middle Row (left to right):* Stephen V. Merkey, Michael E. Angelina, Robert L. Harnatkiewicz, Calvin C. Spence, Jr., Ellen J. Respler, Steven J. Regnier, Al J. Rhodes, John P. Stefanek, Dee Dee Mays, Ann M. Bok. *Top Row (left to right):* David R. Lesieur, Tracy L. Brooks-Szegda, Joyce A. Dallessio, Christopher L. Harris, Russell H. Greig, Douglas J. Onnen, Sallie S. Robinson, Ronald E. Glenn, Peter V. Burchett, Shu C. Lin, Gary Bluhmsohn, Calvin Wolcott.

ASSOCIATES ADMITTED IN NOVEMBER 1993

Bottom Row (left to right): Debra S. Charlop, Tania J. Cassell, Gayle L. Wiener, Sadagopan S. Iyengar, Bhim Asdhir, **CAS President David P. Flynn,** Kevin L. Russell, Kimberly J. Mullins, Peter R. Schwanke, Maurice P. Bouffard, Regina M. Puglise. *Top Row (left to right):* Jonathan D. Adkisson, Lawrence F. Marcus, Edward J. Yorty, Joseph W. Pitts, Richard J. Marcks, Paul D. Henning, Robert F. Wolf, Douglas H. Min, Jason A. Kundrot, Kevin J. Cawley. Not Pictured: Michael W. Cash, Robert J. Hopper, Giovanni A. Muzzarelli, Yves Provencher, Richard A. Stock, Cheng-Sheng P. Wu, and Jeffery M. Zacek.

OBITUARIES

William H. Bittel
Charles C. Fung
Charles A. Hachemeister
Lawrence W. Scammon

WILLIAM HAROLD BITTEL
1904—1993

W. Harold Bittel, an Associate of the Casualty Actuarial Society since 1925, died in Trenton, New Jersey on May 21, 1993. He was 88 years old.

Born in Peoria, Illinois on September 18, 1904, Mr. Bittel graduated from the University of Michigan in 1926. Shortly after graduation, he began working for the Peoria Life Insurance Company, where he spent the next eight years. During that time, Mr. Bittel attained his Fellowship status in the Society of Actuaries. Mr. Bittel then moved to New York City where he was a consulting actuary at the firm of Woodward, Ryan, Sharp & Davis. In 1943, he joined the New Jersey Insurance Department to work in insurance regulation. Mr. Bittel worked at the New Jersey Insurance Department for 29 years until he retired in 1972. At the time of his retirement, he served as Chief Actuary.

Mr. Bittel was influential in the work of the National Association of Insurance Commissioners, especially regarding annual statements and revision of terminology, and played key roles in developing insurance legislation. He represented the New Jersey Commissioner on committees that developed several major programs, including accident and health uniform provisions, benchmark accident and health loss ratios and advertising standards, group and credit insurance studies and requirements, variable annuity laws and contracts, mandatory securities valuation reserves, and insider trading regulations. He was indeed a force for the betterment of insurance regulation in New Jersey and throughout the United States.

Harold Bittel worked with various committees of the SOA and the CAS and on the Board of Directors for the American Academy of Actuaries. He also served the Academy by working with accountants on financial reporting matters.

He is survived by his son, William H. Bittel, Jr., his daughter-in-law, three grandchildren and four great-grandchildren. His wife Elizabeth predeceased him in 1979.

CHARLES C. FUNG
1949—1993

Charles Fung, an Associate of the Casualty Actuarial Society and actuary at Simcoe Erie Group in Burlington, Ontario, died on May 7, 1993 after a year-long battle with lung cancer. He was 44 years of age.

Mr. Fung was born in Hong Kong on December 15, 1949 and came to Canada in 1971 to further his studies at Simon Fraser University in British Columbia. While studying there, he met his wife, Phoebe, and they were married in 1975.

The Fungs then moved near the University of Waterloo in Ontario where Mr. Fung began working toward his Master's Degree in Statistics. In 1979, after graduating from the University of Waterloo, he began working as an actuarial student in the Actuarial Department at the Royal Insurance Company.

Mr. Fung's former co-workers at Royal remember Charles for his strong, lively, and frequently entertaining conversations, as well as his sense of humor. They describe him as "a very caring person, devoted to his family, work and church. He was also a loyal employee: dedicated, hard working and well-respected. He enjoyed his work and was very good at it. He was gifted both in the areas of finance and languages."

Mr. Fung served as a consulting actuary for Eckler Partners, Ltd., in 1986, then became an Actuary for the Simcoe Erie Group in 1987. He achieved Associateship status in the Casualty Actuarial Society in May 1992.

He is survived by his wife Phoebe, a daughter Melodie, and a son Adrian.

CHARLES A. HACHEMEISTER
1937—1993

A Fellow of the Casualty Actuarial Society since 1968, an Associate of the Society of Actuaries since 1986, and a Member of the American Academy of Actuaries since 1969, Charles A. Hachemeister died on September 9, 1993, at the age of 55.

Mr. Hachemeister was a 1959 graduate of Wagner College on Staten Island, New York. He later engaged in graduate studies at the University of Pennsylvania. In the early 1960s, Mr. Hachemeister served for two years in the United States Army.

When he attained his Associate status in the CAS in 1965, Mr. Hachemeister was employed as an Actuarial Assistant for Royal-Globe Insurance Companies in New York City. In 1966, he was promoted to Senior Actuarial Assistant for Royal-Globe. The following year, Mr. Hachemeister joined the Insurance Company of North America, in Philadelphia, as Director, Actuarial Research. In 1971 he was named Associate Actuary.

He moved to California in 1973 and began working for Allstate Insurance Company in Menlo Park, California, as an Associate Actuary. Returning to the East Coast in 1980, Mr. Hachemeister joined Prudential Reinsurance Company in Newark, New Jersey, as an Actuary. The following year he was named Vice President and Actuary for the firm. In 1986, he became President of Pruco Managers, Inc., in New York City.

In 1987, he joined F&G Re, Inc., in Morristown, New Jersey, where he held the positions of Vice President (1987-1992) and Senior Vice President (1992-1993).

Mr. Hachemeister's contributions to the Casualty Actuarial Society were substantial. He authored two reviews of papers which were published in the Society's *Proceedings*: "Loss Ratio Distributions—A Model," in 1967, and "Stochastic Theory of a Risk Business," in 1970. He was a familiar face at CAS meetings and seminars and over the years he participated in a total of nine panel discussions and led three workshops on a variety of topics.

Mr. Hachemeister also served as a member of the CAS Board of Directors from 1976-78, and again in 1988. He was extensively involved in CAS committee activities. Over the course of 23 years, Mr. Hachemeister served as a member, vice chairman, or chairman of 21 committees. Many of the committees for which he served worked to advance and promulgate standards of actuarial science. They included the Education and Examinations, Education Policy, Continuing Education, Textbook, Risk Theory, Statistical Theory, Loss Reserves, Risk Classification, and Reserves committees. In addition, Mr. Hachemeister worked to increase public awareness of the actuarial profession through his work on the Actuarial Communications and Public Relations committees.

The efforts he may best be remembered for, however, are his work with other actuarial organizations in North America and around the world, including the Society of Actuaries, the International Actuarial Association, and especially ASTIN, which he served as U.S. representative. Through the CAS International Relations committee, he was particularly involved in the hosting of the XXIst ASTIN Colloquium in New York City in 1989. As a tribute to his outstanding contributions in this area, the CAS recently established the Charles A. Hachemeister Prize for papers published in the November, 1992 or April, 1993 ASTIN *Bulletin* or presented at the 1993 ASTIN Colloquium.

Mr. Hachemeister is survived by his wife, Lana James, of South Orange, New Jersey; and three daughters from a previous marriage: Lauren Ruth Hachemeister of San Francisco, California; Adrianne Lee Hachemeister of New York City; and Meredith Jane Hachemeister of Burlington, Vermont.

LAWRENCE WHIDDEN SCAMMON
1905—1993

Lawrence W. Scammon, an Associate of the Casualty Actuarial Society since 1947, died of chronic cardiovascular disease on May 24, 1993, in St. Petersburg, Florida.

A native of Stratham, New Hampshire, he served as valedictorian at Exeter High School in New Hampshire, then graduated from Dartmouth College in 1927. After graduating from Dartmouth, Mr. Scammon worked briefly for IBM in sales, then began working for the Automobile Rating Bureau of Massachusetts in Boston in 1947. That same year, Mr. Scammon became an Associate of the Casualty Actuarial Society. In 1960, he was promoted to Manager of the Rating Bureau and, in 1967, he became a Member of the American Academy of Actuaries.

He served as Manager of the Rating Bureau for 10 years before retiring in 1970. Mr. Scammon and his wife subsequently moved to New London, New Hampshire, and spent winters in St. Petersburg. Mr. Scammon served on several alumni committees for Dartmouth and was presented an Alumni Award in 1982. He served as head agent for the Dartmouth Alumni Fund from 1977 to 1981, and treasurer for his class's 60-year reunion.

Mr. Scammon is survived by his wife, Ora, a son, two daughters, a brother, seven grandchildren, and three great-grandchildren.

INDEX TO VOLUME LXXX

INDEX—CONTINUED

Page

INDEX—CONTINUED

INDEX—CONTINUED

INDEX—CONTINUED